THE DYNAMICS OF PENSION REFORM

David Sundén
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David Sundén
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Franz Kafka, Der Proceß
Contents

Acknowledgements ix

Summary xiii

Chapter 1: The Swedish pension reform and projected demographics 1

Chapter 2: Pareto improving privatization of PAYG old-age social security 43

Chapter 3: Projections of the reformed Latvian public pension system 63
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Stockholm in October 2002

David Sundén
Summary
Summary

This thesis consists of three essays, which all concern the dynamics of pension reform. The first essay analyzes the dynamic properties of the recently reformed Swedish pay-as-you-go pension system. The second essay discusses the possibilities of a Pareto-improving reform of pay-as-you-go social security. The third essay evaluates the dynamic adjustability of the new Latvian pay-as-you-go pension system.

I. The Swedish pension reform and projected demographics.

This essay evaluates the financial balance and the demographic adjustability of the reformed Swedish pay-as-you-go pension system in a dynamic general equilibrium model of overlapping generations. The main findings are that the demographic adjustability of the system is poor. Furthermore, the financial balance and pension levels are, to a large degree, dependent on the pension fund and its returns. Making some alterations to the system's benefit formula may improve the adjustability of the system, as well as decreasing its pension fund dependency. It is also shown that the new public system imposes an age-dependent implicit tax on labor earnings that is falling with age. Within the pay-as-you-go system, this tax is large for younger workers for whom almost the whole contribution is regarded as a tax. By introducing a public defined contribution system, the total implicit tax may be reduced since the defined contribution system implies a negative implicit tax because savings are subsidized within the defined contribution system.

II. Pareto improving privatization of PAYG old-age social security

In the second essay a three-generation OLG model for analyzing a privatization of PAYG old-age social security is developed. Furthermore, it proposes an explicit reform for how the privatization transition may be undertaken. The set of government policy instruments is limited to debt issuing and proportional labor income taxation. The possibilities of a Pareto-improving privatization, given the proposed reform, are then analyzed. Contrary to models where a two-generation OLG framework is used, the three-generation framework creates possibilities for a Pareto improving privatization of old-age social security, since the PAYG system induces a non-optimal implicit tax over the life cycle. By shifting to an optimal tax policy cannot only the pension claims accrued under the PAYG system be financed, but the shift will also be Pareto-improving.
III. Projections of the reformed Latvian public pension system

In this essay the performance of the reformed Latvian pay-as-you-go pension system is evaluated against the background of an exceptional projected decrease in the Latvian labor force. The pension system is designed to handle the upcoming difficulties, and special attention has been given in the design to keep the expenditures low relative to the revenues, by introducing rules dampening the increase in the pension expenditures. In the light of the pessimistic projection of the Latvian demography, the newly reformed PAYG system performs remarkably well. The expenditure reducing rules introduced have significant effects on the system's financial balance. The pension reform also includes the launch of a publicly run defined contributions pension system. It is shown that the resulting implicit tax imposed by the public pension system imposes on labor earnings is negative and increasing with age. That is, savings are subsidized in the public pension system. It is also shown that private savings are fully crowded out as individuals try to offset their savings in the pension system. Since individuals are capital constrained, they will have no private assets at all. From a welfare perspective, this suggests the overall contribution rate to the public pension system to be too high.
Chapter 1: The Swedish pension reform and projected demographics.
The Swedish Pension Reform and Projected Demographics*

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October 30, 2002

Abstract

The main goal of the Swedish pension reform was to construct a public PAYG pension system, which is more or less autonomous and where no or only a minimum level of political intervention is needed to guarantee the financial stability of the system. Despite this goal, some features of its design may jeopardize the desired political autonomy. This paper evaluates the financial balance and the demographic adjustability of the new PAYG system in a dynamic general equilibrium model of overlapping generations. The main findings are that the demographic adjustability of the system is poor. Furthermore, the financial balance and pension levels are, to a large degree, dependent on the pension fund and its returns. The adjustability may be improved by making some alterations to the system’s benefit formula and also decreasing the system’s pension fund dependency. It is also shown that the new public system imposes an age-dependent implicit tax on labor earnings that is falling with age. Within the PAYG system, this tax is large for younger workers for whom almost the whole contribution is regarded as a tax. By introducing a public defined contribution system, the total implicit tax may be reduced since the defined contribution system implies a negative implicit tax because savings are subsidized within the defined contribution system.

Keywords: PAYG, pension reform, demography.

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1 Introduction

A successive increase in the old-age pension contribution rate to a level exceeding 30 percent of the total payroll in 2025 would have been necessary to secure a long-run financial balance in the old Swedish pay-as-you-go pension system, SOU (1994). This increase would, to a large extent, be driven by an overly generous defined benefit formula, especially during periods of low growth, combined with a rapidly increasing old-age dependency ratio. The age dependency ratio and the old-age dependency ratio are given in table 1\textsuperscript{1}. In the upcoming decade, the increase will be rather modest. But with the retirement of the baby boom generations born in the 1940’s, both dependency ratios demonstrate substantial increases, mainly driven by the increase in longevity among the elderly.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
Year & Dependency ratio & Old-age dependency ratio \\
\hline
1999 & 0.71 & 0.30 \\
2010 & 0.72 & 0.33 \\
2020 & 0.79 & 0.40 \\
2030 & 0.86 & 0.45 \\
2040 & 0.87 & 0.47 \\
2050 & 0.86 & 0.46 \\
\hline
\end{tabular}
\caption{Dependency ratio and old-age dependency ratio.}
\end{table}

To counteract the financial pressure on the social security system due to the aging of the population, a first legislation to reform the Swedish old-age pension system was passed in 1994. The new system consists of three parts; a pay-as-you-go pension system with individual accounts, a small mandatory defined contribution (DC) scheme, and a minimum pension guarantee to secure the old-age income of people with a low or no pension at all. The system is fully functional from the beginning of 2001. The old pension system will gradually be phased out and thus, for a long period of time, a large part of the pension

\textsuperscript{1}The dependency ratio is defined as the number of individuals aged 0 to 19 and 65 and above, to the number of individuals in the age group 20 to 64. The old-age dependency ratio is calculated as the ratio of people aged 65 and above to people aged 20 to 64.
expenditures will consist of pensions calculated according to the old and more generous rules.

The main goal of the reform was to make the public pay-as-you-go pension system adaptable to the economic and demographic development, in order to create a more or less financially stable system with no or only a minimum requirement of political intervention. The intention to create an autonomous public pension system is apparent in the choice of pension system. The pay-as-you-go system is defined benefit but is couched in the vocabulary of a defined contribution system and referred to as notional defined contributions or an NDC-system, first described by Buchanan (1968). One important aspect of the NDC system is to establish a full link between contributions and benefits. This link or the earnings-relatedness of the benefit formula has been stressed by Auerbach & Kotlikoff (1992) as key parameter governing the distortions generated by the old-age social security system. A second feature is to equate the return on contributions paid to some measure of the growth rate of the economy. This construction allows for a better adjustment of the expenditures to the development of revenues. Among the countries which have or will introduce the NDC-system, Sweden has chosen a rate of return that equals the per capita wage-sum growth rate (Prop 1997/98:151) while Italy has chosen the growth rate in GDP, (Hamann (1997)). Latvia and Poland have chosen the growth rate in the wage-sum (Fox and Palmer (1999) and Chlon et al. (1999)). A third feature of the NDC-system is that the benefit formula basically mimics a traditional annuity purchase, where longevity and the projected future rate of return are taken into consideration. With an increasing life expectancy among the elderly, this feature of the benefit formula becomes a crucial part in sustaining the financial stability of the pension system, without increasing the contribution rate.

Even though long-run financial stability was one of the main goals of the reform, the Swedish design of the pay-as-you-go includes two features that may potentially threaten the balance of the system and consequently, the desired political autonomy. First, as stated above the Swedish design includes a legis-
lateral rate of return within the pay-as-you-go system, which equals the per capita wage-sum growth. This implies that the growth of the pension payments, or the expenditures of the system is determined, to a large extent, by the per capita wage-sum growth. In contrast, the growth of contributions, or revenues, follows the change in the contribution base, i.e. the wage-sum growth. This means that changes in the size of the labor force have instantaneous effects on the revenue side of the system while expenditures, on the other hand, only respond with a lag. The labor force is projected to decrease in the long-run, thereby creating a negative financial pressure on the pay-as-you-go system. Second, the pension annuity is calculated using cross-section mortality tables. An increasing longevity among retirees implies that the expected remaining life span upon retirement is underestimated, the consequence being a higher pension annuity than would be the case if projected mortality risk where used. This choice of design introduces a deficit tendency.

The legislator recognized both problems and an automatic balancing mechanism was introduced to guarantee the stability of the system. Projections made by the National Social Insurance Board (NSIB) show that the system basically balances over time.

In this paper, I will incorporate the reformed pension system in a dynamic general equilibrium model of overlapping generations, which creates a possibility for interaction between the pension system and the rest of the economy. It also allows individuals to respond not only to the change in the old-age social security rules, but also to the strong link between contributions paid and benefits received by changing their labor supply choices. I will also consider some alternative formulations of the pay-as-you-go benefit formula to quantify the negative instability effects created by the rules described above. However, I will not consider the automatic balancing mechanism. Instead, I will quantify the magnitude of possible imbalances as if no automatic balancing took place, thus giving an estimate of the imbalances of the system before any automatic balancing is used.

The model used builds on the tradition of Auerbach & Kotlikoff (1987) and
is similar to Storesletten (2000). In my model, individuals only face a life-span risk and retire at an exogenously given age. The demographic structure is exogenously determined. The financial balance of the new Swedish NDC-pension system is evaluated, given the policy experiment of changing the tax rate on labor earnings and pension income from 1999 to a level such that the new tax rate balances the intertemporal budget of the public-sector, and secures a public-sector debt to output ratio that does not accelerate over time. While the current tax system is highly progressive, the balancing tax rate will be proportional. To evaluate the consequences of the aging problem by only evaluating its effects on the old-age income replacement would be misleading. An aging population also changes the pattern of government purchases and transfers. A shift towards a situation where a larger share of government consumption is directed to the elderly is to be expected. In Sweden, 64 percent of the total government purchases in 1991 were directed at specific age groups. Consequently, I model government consumption with both a non-age-dependent as well as an age-dependent part.

2 The model

The economy consists of overlapping generations who live for a maximum of $H + 1$ years. During the first $h^w$ years individuals are considered to be children and do not work at all and parents finance their consumption. At the age $h^w$, individuals start working in exchange for wages. They are assumed to work for the following $h^p - h^w$ years and retire at the age of $h^p$. During the final $H - h^p$ years, the individuals are retired and receive old-age social security benefits. Over the life-cycle, individuals may smooth consumption by investing in government bonds or physical capital. Production in the economy is given by a constant return to scale Cobb-Douglas production function with constant labor-augmenting technological progress. Even though Sweden is a small open country, the economy is closed. But with a similar aging pattern in Sweden as in the other developed countries, prices are assumed to develop such that
they reflect the general process in the rest of the world. The government taxes consumption, bequests, and income from capital, labor and pensions to finance debt, government consumption and transfers to households. The budgets of the public NDC and DC pension systems are separated from the government budget and from each other. The systems collects contributions, pays old-age social security benefits, services the pension fund in the NDC system and administers the assets in the DC system. For any variable \( x \), a subscript \( h \) denotes age, and an argument \( t \) in parentheses denotes calendar time.

### 2.1 Demographics

At the beginning of any time period \( t \), \( N_0(t) \) individuals are born. Individuals within a generation are identical and face an exogenously given mortality risk at the beginning of each period. The unconditional probability of surviving to period \( t \) if born in period \( t - h \) is denoted \( s_h(t) \), where \( s_0(t) = 1 \). Individuals in each generation are assumed to live for a maximum of \( H + 1 \) periods, hence \( s_{H+1}(t) = 0 \). Let \( N_h(t) \) be the number of age \( h \) individuals at time \( t \). Then the total population alive at \( t \), denoted \( N(t) \), is

\[
N(t) = \sum_{h=0}^{H} N_h(t),
\]

and the labor force in period \( t \), denoted \( N^L(t) \), is the sum of the population between age \( h^w \) and age \( h^p - 1 \),

\[
N^L(t) = \sum_{h=h^w}^{h^p-1} N_h(t). \tag{2}
\]

### 2.2 Production

Output is given by a standard CRS Cobb-Douglas production function

\[
Y(t) = (K(t-1))^{\alpha} (z(t) L(t))^{1-\alpha}, \tag{3}
\]

where \( \alpha \) is the income share of capital. The labor argument is given by

\[
L(t) = \sum_{h=h^w}^{h^p-1} \epsilon_h (1 - l_h(t)) N_h(t). \tag{4}
\]
Here, $\epsilon_h$ is an exogenous age-efficiency index and $1 - l_h$ the labor supply decision of an age $h$ individual. The exogenous and labor-augmenting technological progress is given by

$$z(t) = z(1 + \gamma)^t,$$

where $\gamma$ is the assumed constant rate of technological progress. Product and factor markets are assumed to be competitive and the rentals are then given by

$$r(t) = \alpha \left( \frac{K(t-1)}{z(t) L(t)} \right)^{\alpha - 1} - \delta,$$

$$w(t) = (1 - \alpha) z(t) \left( \frac{K(t-1)}{z(t) L(t)} \right)^{\alpha},$$

where $\delta$ is the physical depreciation rate of capital.

### 2.3 Agents

The agents derive utility from consumption and leisure in each period and the instantaneous utility function for an age $h$ individual in period $t$ is given by a CES class with a unit elasticity of substitution between consumption and leisure

$$u(c_h(t), l_h(t)) = \frac{(c_h(t))^\rho (l_h(t))^{1-\rho} \beta^{-1/\theta}}{1 - 1/\theta},$$

where $c_h(t)$ represents consumption and $l_h(t)$ the leisure choice of the individual, where $0 \leq l_h \leq 1$. Here, $\theta$ is the intertemporal elasticity of substitution. The expected lifetime utility is given by

$$U = \sum_{h=h^w}^{H} \beta^{h-h^w} u(c_h(t), l_h(t)) s_h(t),$$

where $s_h(t)$ is the unconditional probability for an age $h$ person of being alive at time $t$, and $\beta$ the subjective discount factor. Individuals have no bequest motives but they may leave accidental bequests. Negative asset holdings at any age are ruled out, so that the expected utility in $9$ is maximized subject to the borrowing constraint $a_h(t) \geq 0 \forall h, \forall t$, and the recursive budget constraint

$$c_h(t) + a_h(t)$$

$$\geq 0 \forall h, \forall t,$$
= (1 + r(t)) a_{h-1} (t - 1) + w(t) \epsilon_h (1 - l_h (t))
+ \text{transfer} (t) + \text{bequest}_h (t) + p_h (t) - \Upsilon^g_h (t) - \Upsilon^{ndc}_h (t) - \Upsilon^{dc}_h (t) .

Here, \text{transfer} (t) is the government transfer to the individual in the period and \text{bequest}_h (t) the bequest received at age \( h \) in period \( t \). The old-age pension is denoted \( p_h (t) \), where \( p_h (t) = 0 \ \forall h < h^p, \forall t \). The calculation of the pension is described in the next section. Total taxes paid by the agent are represented by \( \Upsilon^g_h (t) \)

\[
\Upsilon^g_h (t) = (\tau^e + \tau^p) \omega_h (t) \epsilon_h (1 - l_h (t)) + \tau^c c_h (t) + \tau^e p_h (t)
+ \tau^a r (t) a_{h-1} (t - 1) + \tau^b \text{bequest}_h (t).
\] (11)

The pension reform makes all pension income subject to income taxation. In the case of publicly provided pensions, the tax rate coincides with the tax rate on labor earnings, denoted \( \tau^e \). The consumption tax is represented by \( \tau^c \), the capital income tax by \( \tau^a \), and the payroll tax (here assumed to be paid by the employee) by \( \tau^p \). Bequests are taxed by \( \tau^b \). \( \Upsilon^{ndc}_h (t) \) and \( \Upsilon^{dc}_h (t) \) represent contributions to the respective part of the pension system,

\[
\Upsilon^{ndc}_h (t) = \tau^{ndc} \omega_h (t) \epsilon_h (1 - l_h (t)),
\] (12)

\[
\Upsilon^{dc}_h (t) = \tau^{dc} \omega_h (t) \epsilon_h (1 - l_h (t)),
\] (13)

where \( \tau^{ndc} \) is the NDC system contribution rate on taxable earning and \( \tau^{dc} \) the defined contribution counterpart.

Taxable labor earnings, i.e. the tax base for \( \tau^e, \tau^p, \tau^{ndc}, \) and \( \tau^{dc} \), are given by \( \omega_h (t) \epsilon_h (1 - l_h (t)) \) for an age \( h \) individual in period \( t \). Here, \( \omega_h (t) \) is the wage rate after payroll tax and contributions. In the Swedish system, in addition to the payroll tax, the employer pays half of the pension contributions while the employee pays the other half. Consequently, in order to find an expression for the wage rate after payroll tax and contributions, the wage rate, \( w(t) \), must be adjusted in the following way

\[
\omega_h (t) = \frac{w(t)}{1 + \tau^p + (\tau^{ndc} + \tau^{dc})/2}.
\] (14)
where \(1 + \tau^p + (\tau^{ndc} + \tau^{dc})/2\) reflects the contributions and payroll payment structure. Observe the distinction between labor income \(w(t) \epsilon_h (1 - l_h (t))\), which reflects labor compensation, and labor earnings, \(\omega_h (t) \epsilon_h (1 - l_h (t))\), that is labor income when payroll taxes and half of the pension contributions have been paid. This distinction is made to be able to use the labor and pension earnings tax rate \(\tau^e\) as an endogenous variable for balancing the public-sector budget over time.

In each period, total accidental bequests in the economy are divided among individuals in the age group \(h_b\). The bequests to an individual in this age group are denoted \(\text{bequest}_{h_b}(t)\) and are given by

\[
\text{bequest}_{h_b}(t) = \frac{\sum_{j=h}^{H} a_{j-1} (t-1) N_{j-1} (t-1) (1 + (1 - \tau^a) r (t)) \left(1 - \frac{s_j(t)}{s_{j-1}(t-1)}\right)}{N_{h_b} (t)}.
\]  

(15)

Here, the \(a_{j-1} (t-1) N_{j-1} (t-1)\) is the amount of assets for the age group \(j - 1\) in period \(t - 1\), multiplied with \((1 + (1 - \tau^a) r (t))\) to obtain the period \(t\) asset value after the asset income tax has been deducted. \(1 - s_j (t)/s_{j-1} (t-1)\) gives the relative share of the number of non-survivors between period \(t - 1\) and period \(t\) for age group \(j\). Summing over all relevant ages gives the total amount of assets that should be redistributed to \(N_{h_b} (t)\) individuals in age group \(h_b\).

### 2.4 Pension system and government

The pension system and the government are divided into three parts; the fully funded defined contribution system, the pay-as-you-go notional defined contributions system and the government. The public-sector is defined as the compound of the pay-as-you-go system and the government.

#### 2.4.1 The Pension System

 Suppressing the subscript for age and the time argument, the entry pension at retirement in the defined contribution system is calculated as,
\[ p^{dc} = \frac{a^{dc}}{\lambda^{dc}}, \tag{16} \]

where \(a^{dc}\) are the accumulated assets at retirement. Contributions made in the working life are added to the individual's defined contribution account. In the accumulation phase, the account assets earn the market rate of return. Asset returns are not subject to asset income taxation. Furthermore, assets of individuals who die before retirement are inherited by individuals in the same age group and are referred to as gains of inheritance. The denominator in expression (16), \(\lambda^{dc}\), is the life expectancy at retirement, adjusted for future interest rates. This factor converts the assets at retirement into a fixed pension annuity in real terms.

The NDC-assets are accumulated in the same way as the DC-assets, including the gains of inheritance, with the difference that the rate of return instead follows the per capita wage-sum growth. The pension is calculated in a similar fashion, but as a variable annuity, where \(\lambda^{ndc}\) is life expectancy at retirement adjusted for future expected growth in the per capita wage-sum. In the Swedish system, the "expected" future growth in average wages is set to 1.6 percent per annum by legislation and referred to as the norm. The NDC pension is subject to indexation, and is adjusted according to the deviation of the actual per capita wage growth from the expected growth. Accordingly, the pension annuity will be variable over the retirement period and increase in periods when the per capita wage growth exceeds the expected growth, and vice versa. \(\lambda^{ndc}\) is calculated using cross-section mortality tables, not projected mortality rates. The effect on the aggregate level is that if longevity among the retirees increases (decreases) over time, the system will have a tendency to run deficits (surpluses). The reason is that the longevity is underestimated (overestimated) using cross-section data and consequently, the pension annuity is higher (lower) than if projected mortality risks were used.

One reason for introducing the NDC-system is that it is supposed to create a strong link between contributions and pensions by mimicking parts of the
DC-system features. By allowing for a direct relation between the contribution made and the pension received, the individual can affect her pension level by her labor supply choice. Thus, contributions made to the NDC system may be considered as savings, but at a rate of return lower than the market rate of return. Consequently, only a part of the contribution is considered as savings while the rest may be considered as a tax. Call the part of the contribution not perceived as savings the *implicit tax* in the NDC-system. Observe that since the rate of return on contributions follows the growth in the per capita wage-sum, it follows that the later in the life-cycle an NDC-contribution is made, the shorter is the period in which the contribution is accrued to the relatively lower rate of return. Consequently, the lower is also the loss due to the rate of return difference and hence, the lower the implicit tax, which is thus age-dependent and falling with age. Define the implicit tax rate in the NDC-system, $\phi_{h}^{ndc}(t)$, for an individual of age $h$ in period $t$, where $20 \leq h \leq 64$, as

$$\phi_{h}^{ndc}(t) = \pi_{h}^{ndc}(t) - \pi_{h}^{ndc}(t) .$$  \hspace{1cm} (17)

Here, $\pi_{h}^{ndc}(t)$ represents the part of the contribution, at age $h$ in period $t$, considered as savings. This is calculated as the present value, at $t$, of the contribution in terms of the ensuing pension payments, as a share of the individual’s earnings in period $t$. Subtracting this savings rate from the contribution rate yields the implicit tax rate.

At a first glance the DC-system may seem neutral from a tax perspective, but the tax and the accrual construction in the defined contribution system are such that there will be an implicit tax. Three factors make the implicit tax in the DC-system deviate from zero. First, savings within the pension system are not subject to asset income taxation, which implies that savings within the DC-system are subsidized compared to ordinary savings, thus imposing a negative implicit tax rate. The effect is the same as in the NDC-system, but with the opposite sign; contributions made earlier in life are subject to the relatively better rate of return for a longer period of time than later contributions. An increasing, but negative, implicit tax over the life-cycle is then the result. Sec-
ond, to compensate for not taxing the asset return during the accrual phase, pension payments are taxed in the same way as ordinary labor earnings, which imposes a positive implicit tax rate. Third, the effective accrual rate within the DC-system is higher than the market rate of return, since survivors from one period to another inherit the assets of non-survivors in the same age group. This makes the implicit tax rate negative but increasing with age. The age dependent implicit tax in the DC-system, \( \phi_{h}^{dc}(t) \), is defined as

\[
\phi_{h}^{dc}(t) = \tau^{dc} - \pi_{h}^{dc}(t).
\]

A more detailed and mathematical representation of the Swedish system can be found in Appendix A.

A pension fund is linked to the NDC-system to accommodate yearly imbalances in the cash flow. The budget constraint of the NDC-system can be written as

\[
B(t) + \sum_{h=h^P}^{H} p_{h}^{ndc}(t) N_h(t) = (1 + r(t)) B(t - 1) + \sum_{h=h^P}^{h^P - 1} \Upsilon_{h}^{ndc}(t) N_h(t).
\]

Here, \( B(t) \) represents the end of period pension fund and \( p_{h}^{ndc}(t) \) the pension payments to age \( h \) individuals. The budget of the pay-as-you-go system is separated from the governmental budget purely for bookkeeping purposes. None of them need to balance in themselves but together they must.

### 2.4.2 Government

The government budget constraint at \( t \) is

\[
G(t) + T(t) + (1 + r(t)) D(t - 1) = D(t) + \sum_{h=h^P}^{H} \Upsilon_{h}^{g}(t) N_h(t),
\]

where \( D(t) \) is government debt at the end of period \( t \). Following Storesletten (2000) government consumption, \( G(t) \), is given by

\[
G(t) = y(t) \left[ G + \sum_{h=0}^{H} g_{h} N_h(t) \right],
\]
where \( y(t) \) is GDP per capita and \( G \) is fixed government consumption as a fraction of per capita GDP. The second part within brackets reflects age-dependent government consumption where \( g_i \) is government consumption targeted to an individual in age group \( i \) as a share of per capita GDP. The specification of government purchases in (21) implies that changes in \( G(t) \) are driven by changes in the demographic structure, as well as by output per capita growth.

Transfers at \( t \) are determined according to

\[
T(t) = tr(t) \sum_{h=h^\omega}^H N_h(t) = y(t) tr \sum_{h=h^\omega}^H N_h(t),
\]

where \( tr(t) \) is the transfer per recipient and \( tr \) the assumed constant transfer per recipient as a share of GDP per capita. Observe that no individuals below the age \( h_w \) receive any transfers. The above specification of \( T(t) \) implies that the growth in total transfers is determined by the growth in output per capita, as well as by demographic changes.

### 2.4.3 Balancing the public-sector budget over time

The public-sector is defined as the compound of the NDC-system budget and the government budget. Combining the budget constraint for the pension system in expression (19) with that for the government in expression (20), an intertemporal budget constraint for the public-sector is obtained

\[
B(t) - D(t) + \sum_{h=h^p}^H p_{h}^{ndc}(t) N_h(t) + G(t) + T(t)
\]

\[
= (1 + r(t))(B(t-1) - D(t-1)) + \sum_{h=h^\omega}^H (\tau_h^g(t) + \tau_h^{ndc}(t)) N_h(t).
\]

The financial balance of the public-sector as a whole is achieved by finding the tax on labor and pension earnings, \( \tau^* \), which intertemporally balances equation (23), so that the consolidated debt to output ratio, \( (B(t) - D(t))/Y(t) \) is non-accelerating over time. In practice, this means that the debt output ratio in the final steady state is constant over time.
3 Parameterization and calibration

3.1 Demography

The demographic projection of Statistics Sweden from the year 2000 is used. The projection ends in year 2100; up to that year the population size for ages 0-20 and the unconditional survival probabilities for ages 21-105 are directly taken from the projection. Cohort size over time is calculated by using the number of 20 year olds and the unconditional survival probabilities. Statistics Sweden assumes a positive net immigration for the whole projection period. If this is not considered the model population will be lower than the projected, a problem avoided by increasing the number of 20-year old individuals by 5500 for each cohort. This yields a good approximation to the Statistics Sweden population projection.

The cohort size is extrapolated beyond year 2100 by setting the number of newborn and the mortality risks in each period after 2100 to their respective averages during the period 2091-2100. Mortality risks and the number of newborn are thus constant after 2100 and, as a consequence, the population will converge to a stationary state over time.

Individuals are assumed to start working at the age of $h^w = 20$, and retire at the age of $h^p = 65$. Bequests are received at the age of $h_b = 55$. Individuals are all assumed to die before the age of 106, i.e. $H = 105$.

3.2 Government and the pension system

The government taxes labor, capital and pension income. The payroll tax, $\tau^p$, is set to 22.7 percent of the labor earnings$^2$. The asset income tax rate, $\tau^a$, is set to 30 percent, the consumption tax rate, $\tau^c$, to 25 percent$^3$, and the tax rate for bequests, $\tau^b$, to 20 percent.

The tax on labor and pension earnings, $\tau^e$, is endogenous in the model and used to balance the intertemporal budget for both the government and the

---

$^2$This figure excludes contributions to the pension system paid by the employer.

$^3$This figure is based on the total taxes on goods and services.
pension system.

The age specific government consumption levels, \( \{g_i\}_{i=0}^H \) are based on Ekberg and Andersson (1995) and are given in table 2.

Table 2: Government consumption per individual in the age group. Share of GDP per capita.

<table>
<thead>
<tr>
<th>Age group</th>
<th>Consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 6</td>
<td>17.6%</td>
</tr>
<tr>
<td>7 - 15</td>
<td>36.3%</td>
</tr>
<tr>
<td>16 - 19</td>
<td>27.8%</td>
</tr>
<tr>
<td>20 - 24</td>
<td>13.4%</td>
</tr>
<tr>
<td>25 - 34</td>
<td>8.2%</td>
</tr>
<tr>
<td>35 - 44</td>
<td>7.6%</td>
</tr>
<tr>
<td>45 - 54</td>
<td>7.5%</td>
</tr>
<tr>
<td>55 - 64</td>
<td>8.8%</td>
</tr>
<tr>
<td>65 - 74</td>
<td>15.8%</td>
</tr>
<tr>
<td>75 +</td>
<td>50.0%</td>
</tr>
</tbody>
</table>

64 percent of total government consumption is age dependent\(^4\). The average ratio of general government purchases to GDP during the period 1960-1996 is 27 percent. The remaining 36 percent constitute non-age dependent consumption and consequently, the non-age dependent consumption is set to 9.7 percent of GDP.

Transfers to households are set to match the 1985-1997 average of transfers per recipient as a share of GDP per capita, which is 21.4 percent. Here, housing subsidies to private enterprises for reducing the cost of living of households are included and old-age pension transfers excluded.

The contribution rate to the NDC system, \( \tau^{ndc} \), is 13.5 percent of the labor earnings and 2.1 percent for the DC system. Thus, in total, the public pension contribution rate amounts to 15.6 percent of labor earnings. Using labor income as the tax base, the total contribution rate amounts to 12 percent.

\(^4\)The age-related expenditures mainly concern social welfare, medical services, education and labor market programs.
3.3 Preferences, technology and age-efficiency

The intertemporal elasticity of substitution, $\theta$, is set to 0.5. The parameter $\rho$ is set to 0.25. The time preference parameter $\beta$ is set to 0.99. Capital is assumed to depreciate by 10 percent per year, $\delta = 0.10$. The growth rate of labor-augmenting technological progress, $\gamma$, is kept constant at a level of 2 percent per annum. This relatively high growth rate is used since the pension data provided by the NSIB are calculated using this growth rate. The results for a growth rate of 1 percent are also reported. The age-efficiency profile is based on statistics from the Household Income Survey (HINK).

With Cobb-Douglas production technology, the capital income share and the contribution rate determine the total pension contributions as a share of GDP. Total contributions to the NDC-system as a share of output can be stated as

$$\frac{\tau^{ndc}}{1 + \tau/2 + \tau^p} \frac{w(t) L(t)}{Y(t)}.$$  \hfill (24)

A Cobb-Douglas production function implies that the second term in expression (24) is equal to the labor income share, $1 - \alpha$. Accordingly, contributions to the NDC-system as a share of GDP can be expressed as

$$(1 - \alpha) \frac{\tau^{ndc}}{1 + \tau/2 + \tau^p}. \hfill (25)$$

With all parameters in the second term in expression (25) specified according to their legislated values, the extension of the public pension system in the model will be a direct consequence of the choice of income capital share, $\alpha$. With short-run pension expenditures to a large extent determined by the current demographic structure and pension liabilities, which are only affected by the reform to a small extent, the choice of $\alpha$ also affects the short-run net cash flow of the pension system. The lower the capital income share, the larger the contribution to output ratio and hence, the better the short-run positive net cash flow of the system will be, and vice versa. Here, a capital income share of $\alpha = 0.33$ is used in the main scenario but simulation results for $\alpha = 0.30$, and $\alpha = 0.36$ are also reported.
3.4 Initial conditions

The distribution of natives for 1998 is used as the initial condition for the population. The initial public net debt, $D - B$, is set to 12.3 percent of GDP\(^5\), and the initial size of the pension fund, $B$, to 34.3 percent of GDP; accordingly, the initial size of the net government debt, $D$, is 46.6 percent of GDP. Three annual transfers of assets from the pension fund to the governmental budget are scheduled and amount to 2.4 percent of GDP in 1999, 2.3 percent in 2000, and 7.5 percent in 2001. The initial distributions of pension assets, both for the NDC and the DC-system are from 1998. Since the old pension system will only gradually be phased out, all income related pension rights earned within the old system are converted into notional assets. This conversion is made by the NSIB and amounts to 84.2 percent of GDP in 1998\(^6\). The initial household asset structure is taken from the final steady state and scaled so that the capital to output ratio equals 2.0, (de Nardi (2000)). A capital to output ratio of 2.5 is used as a robustness test in two scenarios. In these scenarios, the 2.5 capital to output ratio is also assumed to hold in the steady-state by calibrating the preference parameter, $\beta$.

4 Results

This section opens with a discussion of the results from the main scenario where the benefit formula is based on per capita wage-sum indexation and cross-section mortality risks. The section then continues to discuss some alternative formulations of the benefit formula in the pay-as-you-go system. It concludes with a section on the robustness of the results.

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\(^5\)The official figure for 1998 is 15.3, but this includes assets in the defined contribution system, which amount to 3.0 percent of GDP.

\(^6\)This total amount is distributed among the relevant generations according to their relative population size in 1998 and the transition rules.
4.1 Main scenario

The results for the main scenario when using a labor-augmenting growth rate of 1 percent are summarily described at the end of this section. All results until then are for the case when the growth rate is 2 percent.

In order to balance the public intertemporal budget without an accelerating debt over time, the tax on labor and pension earnings, \( \tau^e \), must be 31.4 percent. Using labor income, instead of the labor earnings, as the tax base, labor income taxes amount to 41.5 percent, adding another 12.0 percent in pension contributions. The public debt in the final steady state is 0.2 percent of GDP, compared to the initial level of 12.3 percent, thus leaving the debt to output level basically unchanged.

The pension fund is positive in the final steady state. In present value terms, the steady state value of the pension fund, or the positive imbalance of the system, amounts to 1.9 percent of GDP in the year 2000, or 35.9 billion SEK.

4.1.1 The final steady state

The interest rate in the final steady state is 6.4 percent, which corresponds to 4.5 percent net of capital income tax. The capital to output ratio is 2.0, which is the same as the initial level.

Table 3 provides a comparison of the distribution of age-dependent government consumption in 1999 and in the steady state. For age groups above 55, government outlays have increased, for the group 75 + they are as large as 50 percent, and for younger age groups, the outlays are about the same or lower. In total, the demographic shift towards a more elderly population results in an increase in government consumption from 27.4 in 1999 to 28.9 percent of GDP in the steady state. Government transfers basically remain unchanged at a level of 16.6 percent of GDP (16.2 percent in 1999), since the total population is about the same. Total pension payments amount to 11.6 percent of GDP, where 4.6 percentage points are defined contribution. Consequently, while the
defined contribution system amounts to 13.5 percent of the total contributions, its share of total pension payments is 40.1 percent.

Table 3: Age dependent government consumption in the steady-state, and the percentage change since 1999. Share of GDP and change in percent.

<table>
<thead>
<tr>
<th>Age group</th>
<th>Steady-state</th>
<th>Change since 1999</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 6</td>
<td>1.4%</td>
<td>-6.9%</td>
</tr>
<tr>
<td>7 - 15</td>
<td>3.6%</td>
<td>-12.3%</td>
</tr>
<tr>
<td>16 - 19</td>
<td>1.3%</td>
<td>-0.3%</td>
</tr>
<tr>
<td>20 - 24</td>
<td>0.8%</td>
<td>0.7%</td>
</tr>
<tr>
<td>25 - 34</td>
<td>1.0%</td>
<td>-14.3%</td>
</tr>
<tr>
<td>35 - 44</td>
<td>0.9%</td>
<td>-9.6%</td>
</tr>
<tr>
<td>45 - 54</td>
<td>0.9%</td>
<td>-16.5%</td>
</tr>
<tr>
<td>55 - 64</td>
<td>1.0%</td>
<td>8.9%</td>
</tr>
<tr>
<td>65 - 74</td>
<td>1.7%</td>
<td>26.3%</td>
</tr>
<tr>
<td>75+</td>
<td>6.6%</td>
<td>50.1%</td>
</tr>
</tbody>
</table>

This result is based on a difference in the rate of return between the NDC-system (2 percent annually) and the DC-system (6.4 percent annually) of 4.4 percent in steady-state.

The implicit tax rates from the respective pension system and overall are reported in figure 1.
Over the life-cycle, the total implicit tax rate falls from 5.9 to 2.8 percent of labor earnings, as compared to a total contribution rate of 15.6 percent. In the NDC-system, the implicit tax falls from an initial level of 9.7 percent at the age of 20 to 3.1 percent at the age of 64, compared to a contribution rate of 13.5 percent. Consequently, for a 20 year old, almost the entire NDC-contribution is considered as tax, while for a 64 year old, the main part is considered as savings. In the DC-system, the tax is negative, i.e. savings are subsidized the whole active period. The results present an interesting feature by combining two different public pension systems. By introducing a public DC-scheme, where savings are subsidized, it is possible to reduce the the implicit tax wedge of the system as a whole.

4.1.2 Transition

Figure 2 shows some selected growth rates of the economy during the transition.

Figure 2: Growth rates.

The per capita and the wage-sum growth rate converge to the assumed tech-
nological growth rate, 2 percent, in the long-run, as the economy approaches the steady state. The labor force stabilizes as the demography approaches its stationary state where the population and the labor force are constant. On average, the per capita wage-sum growth is slightly higher than 2 percent per annum, calculated for the whole transition period, while the same figure for the wage-sum is slightly below 2 percent. The difference in growth rates also approximates the difference in how the revenues and the expenditures of the NDC-system change over time. With revenues and expenditures basically balancing at the beginning of the simulation, the difference indicates a significant long-run deficit tendency within the system. The difference between the growth rate in the per capita wage-sum and the wage-sum is explained by changes in the labor force. Here, an increasing labor force implies that the growth rate in the per capita wage-sum is lower than the growth rate in the wage-sum and vice versa. The labor force increases in some periods of the simulation but, in total, it falls by 12 percent during the whole interval, even though the total population remain fairly constant. The reason can be found in the demographic shift towards a more elderly population.

Figure 3 depicts the capital to output ratio over time.

Figure 3: Capital to output ratio.
At the beginning of the interval, the ratio is increasing, it then levels out and assumes a cyclical pattern until it reaches its steady state level slightly above 2.0. Government consumption increases from 27.4 to 28.9 percent of GDP during the transition.

Basically, this increase is the net effect of two opposing demographic changes, depicted in figure 4. First, the number of people in the age group 7-15 decreases, resulting in a drop in government consumption by almost 1 percentage unit of GDP. Second, the number of individuals in age group 75+ increases drastically due to the increased longevity among the elderly, the effect being that government consumption increases by more than 2 percentage units of GDP.

The demographic impact of the age-dependent government consumption is shown in figure 5, where the government primary savings are plotted. Without the age-dependency, government consumption would be a constant fraction of GDP over the period. Here, it increases by approximately 1.5 percentage units over the simulation period and largely explains the fall in the government’s
primary savings from a surplus of just above 3 percent of GDP to 0. With only a small increase in the transfers from 16.2 to 16.6 percent of GDP, the residual fall in the primary savings is explained by a fall in tax revenues.

Figure 5: Government primary savings. Share of GDP.

Figure 6 shows the primary savings of the NDC-pension system. The figure presents a rather dark picture where the system runs a deficit in the whole simulation period until the economy reaches the steady state. Until year 2050, the deficits must be considered as large. The deficits during this period are, to a large extent, due to the fact that the old pension system, with its more generous rules, is phased out only gradually. Although the deficits are reduced over time, the adjustment of the system to the demographic development must be said to be fairly poor. Not until the demography becomes stationary do the contributions match the expenditures of the system. When also considering the pension fund, the deficit is more than covered by the return on assets. Given the projected demographic development, the system is highly dependent on funding in order to secure the long-run balance without triggering the automatic balancing mechanism. Thus, the expenditure side and consequently, the level of
pension payments, are then, to a certain degree, directly determined by factors such as the market rate of return as well as the size of the pension fund.

Figure 6: Primary savings in the NDC - system. Share of GDP.

The public net assets are plotted in figure 7.

Figure 7: Public sector assets. Share of GDP.
During the first 20 years of the simulation period, the net assets are increasing, and after a long period of decrease, they stabilize on a level of $-0.2$ percent of GDP.

In figure 8, the primary savings of the DC-system are displayed. It takes about 60 years for the system to mature to a point where the asset size of the system amounts to 82 percent of GDP. Here, the effect of introducing the DC-system basically only results in a crowding out of private life-cycle savings and locking them into old-age savings.

Assuming an annual labor-augmenting technological growth rate of 1 percent, instead of 2 percent, does not significantly alter the results, which indicates that the system is robust to differences in the growth rate. The major differences worth noting are the higher capital to output ratio of 2.3 (as compared to 2.0). The resulting interest rate is consequently lower, 4.8 percent, and thus 3.4 percent net of taxes (as compared to 6.4 and 4.5 percent).
4.2 Wage-sum indexation and projected mortality risks

The choice of using the per capita wage-sum growth rate as the internal rate of return for the NDC-system implies that the PAYG pension expenditures approximately change according to the average wage growth. At the same time, the system revenues change according to the wage-sum growth a difference in growth paths which implies a deficit (surplus) tendency when the per capita wage growth exceeds (falls short of) wage-sum growth. Using the change in the wage-sum as the base for the internal rate of return does not imply period­wise budget balance, but imposes a better adjustment of the expenditures to the development of revenues. With decreasing mortality risks among the elderly, the use of cross-section mortality tables has the effect of life expectancy at retirement being underestimated. Consequently, the NDC pension annuity calculation will result in a higher pension than if projected mortality risks were used. With increasing longevity among retirees over time, the legislated rules for calculating the pension benefit will result in a bias towards deficits within the NDC system.

In figure 9, the primary savings of the NDC system are plotted when wage­sum indexation instead of per capita wage-sum indexation\textsuperscript{7} is used. The result when wage-sum indexation is used in combination with projected mortality tables is also included. The labor-augmenting growth rate is 2 percent for all simulations. In the first 20 years, the wage-sum indexation approach only has a limited effect on pension expenditures, since the expenditure level, to a large extent, is predetermined by pensions calculated by the old rules. After 2020, the effect of the wage-sum indexation is clearly observable; a much better adjustability of the expenditures to revenues. This is especially evident for the period after 2050, when the old pension system is completely phased out. The system even runs surpluses in certain periods.

\textsuperscript{7}The use of the wage-sum growth as an indexation method here only applies to the period simulated. Before 1999 all notional capital are indexed with per capita wage sum growth. This means that the initial size of the NDC - assets is the same irrespective of the type indexation method during the simulation period.
The effects of changing the benefit formula to instead incorporate projected mortality tables are visible from year 2015. During the period 2015 to 2040, the yearly improvement in the primary balance is approximately 0.3 percent of GDP per annum. Then the demography starts to converge to its stationary state, which implies that projected and cross-section mortality tables will coincide and hence, the gains from using projected mortality tables will diminish.

The welfare effects of changing the benefit rules are measured as the percentage change in consumption that an individual would need to be as well off as under the allocation with the alternative benefit rule. Following Jonsson and Klein (2002), this measure is denoted by $\Delta$ and is defined as

$$
\Delta = \sum_{h=1}^{H} \beta^{h-h^w} u (c^* h (t), l^* h (t)) s_h (t) = \sum_{h=1}^{H} \beta^{h-h^w} u ((1 + \Delta) c_h (t), l_h (t)) s_h (t)
$$

where $c^* h (t)$ and $l^* h (t)$ are the consumption and leisure, respectively, in the economy under the alternative benefit rules and $c_h (t)$ and $l_h (t)$ are the consumption respectively leisure under the legislated rules. In figure 10, the welfare effects of the two alternative benefit rules are reported for each generation separately.
Both alternatives result in a small but positive welfare improvement for all generations, but later generations gain relatively more since their pensions are not affected by the change in benefit rules. In the transition the wage-sum growth rate, on average, is lower than the per capita wage-sum growth rate. Thus, wage-sum indexation yields a lower rate of return and consequently, a lower pension. In the steady-state the per capita wage-sum and the wage-sum growth rate coincide, leaving later generations unaffected by the change in indexation method.

Projected mortality risks are lower than the cross-section risks, for the early generations, which implies a higher remaining life expectancy upon retirement, and consequently a lower pension. In the steady-state, the projected mortality risks coincide with the cross-section risks, thus later generations are unaffected. The reduction in pension expenditures has a positive effect on the primary savings of the NDC-system, which results in a lower public-sector debt in the steady-state and hence a lower lower labor earnings tax rate to service the debt. In the main scenario the labor earnings tax rate is 31.4 percent, with wage-
sum indexation the tax rate can be reduced to 31.2 percent, if also projected mortality risks are used it can be reduced further, to 31.1 percent. For the generations living during the transition the positive welfare effects of a lower labor earnings tax rate are large enough to outweigh the negative consequences of changing the benefit formula.

4.3 Comparing some alternative scenarios

In table 4, some results from different scenarios are compared; the name of the scenario indicates the change from the main scenario. The size of the labor-augmenting productivity growth does not seem to have any significant effect on the resulting pension fund size, here calculated as the its present value in the year 2000 as a share of GDP. This indicates a significant robustness of the NDC-system vis-à-vis the growth of the economy. The higher growth rate consistently results in a 1.1 percentage unit lower earnings tax rate, when comparing the same scenarios with different growth rates.

Comparing the scenarios shows a higher pension fund present value when using the wage-sum indexation than in the main scenario. The pension fund present value is even larger when also altering the benefit formula to use projected mortality tables. For the case of a 1 percent growth rate, the reduction in NDC expenditures is 7.4 percent of GDP (8.4% - 1.0%) in present value. The same figure for the case of a 2 percent growth rate is 6.8 percent of GDP (8.7% - 1.9%). Using a different capital income share, the short-run primary savings of the system are directly affected as described earlier, while the long-run primary savings remain unchanged. A lower share implies that the pension contribution base is broadened, since the labor income share is increased. The effect of both increasing and decreasing the capital income share is rather modest when comparing the present value of the pension fund, which indicates that the capital income share only has a limited effect on the short-run revenues of the NDC-system. In the scenarios where a capital to output ratio of 2.5 is used for size of the initial capital stock, the steady state value is forced to 2.5 by cal-
ibrating the discount factor, \( \beta \). This change results in a higher discount factor of individuals, \( \beta = 1.00 \) in the 1 percent growth rate scenario and \( \beta = 1.01 \) in the 2 percent scenario, than assumed in the other simulations.

Table 4: The public sector balancing earnings tax rate, and some selected steady state variables.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Earnings tax</th>
<th>Year 2000 present value of the steady state buffer fund. Share of GDP</th>
<th>K/Y</th>
<th>Interest rate</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1 % growth rate</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Main</td>
<td>32.5%</td>
<td>1.0%</td>
<td>2.3</td>
<td>4.8%</td>
</tr>
<tr>
<td>Wagesum indexation</td>
<td>32.3%</td>
<td>4.6%</td>
<td>2.3</td>
<td>4.7%</td>
</tr>
<tr>
<td>Wagesum indexation and projected mortality tables</td>
<td>32.2%</td>
<td>8.4%</td>
<td>2.3</td>
<td>4.7%</td>
</tr>
<tr>
<td>( \alpha = 0.3 )</td>
<td>30.2%</td>
<td>0.8%</td>
<td>2.1</td>
<td>4.6%</td>
</tr>
<tr>
<td>( \alpha = 0.36 )</td>
<td>34.9%</td>
<td>1.2%</td>
<td>2.4</td>
<td>5.0%</td>
</tr>
<tr>
<td>K/Y = 2.5</td>
<td>35.8%</td>
<td>-14.7%</td>
<td>2.5</td>
<td>3.3%</td>
</tr>
<tr>
<td><strong>2 % growth rate</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Main</td>
<td>31.4%</td>
<td>1.9%</td>
<td>2.0</td>
<td>6.4%</td>
</tr>
<tr>
<td>Wagesum indexation</td>
<td>31.2%</td>
<td>5.0%</td>
<td>2.1</td>
<td>6.4%</td>
</tr>
<tr>
<td>Wagesum indexation and projected mortality tables</td>
<td>31.1%</td>
<td>8.7%</td>
<td>2.1</td>
<td>6.4%</td>
</tr>
<tr>
<td>K/Y = 2.5</td>
<td>35.8%</td>
<td>-28.1</td>
<td>2.5</td>
<td>3.5%</td>
</tr>
</tbody>
</table>

It also implies a significantly lower market rate of return, which results in the returns on the pension fund not being able to cover the deficits in the pension system. As a consequence, the pension fund is negative in the steady-state; hence the negative present values in year 2000. Compared with the other scenarios, the differences in the present values of the pension fund when increasing the capital to output ratio are large, implying that the balance of the system is
sensitive to changes in the market rate of return, and assumptions affecting the market rate of return.

5 Concluding remarks

Using a dynamic overlapping generations model, I have found that the Swedish pay-as-you-go pension system adjusts poorly to the projected demographic changes. The result largely depends on three factors. First, the slow phase out of the old system leads to pension expenditures that, to a large extent, are determined according to the old and more generous rules for a long period of time. Second, using cross-section mortality risks in the benefit formula results in a deficit tendency, when longevity among the elderly increases. Third, the chosen indexation method, per capita wage-sum indexation, only reflects the change of revenues to the system to a limited degree. The problem with the index is that pension expenditures are poorly adjusted to demographic changes, a feature demonstrated in the presented simulations.

The reason why the system can sustain such high pension levels in the long-run is that the deficits are covered by the pension fund. Figure 10 shows the share of pension expenditures financed by the pension fund when a 2 percent growth rate is used. At most, more than 20 percent of the pension expenditures in the reformed system are financed in this way, which makes the financial balance of the system and the pension levels directly dependent on the size of the pension fund and its returns. Consequently, the PAYG-system becomes dependent on funding to a high degree. Even after the obligations of the old pension system have been phased out, the negative strain on the balance of the system is considerable. By adopting the wage-sum approach and using projected mortality tables, the deficit pressure on the pension fund may be reduced. With a system basically balancing of its own accord in the long-run, the need for a large-scale pension fund is reduced.
Another finding is that the implicit tax imposed by the pension system is highly age-dependent, where the young face a substantially higher marginal tax on labor than the old. One explicit desire when the basic fundaments of a new system were formulated was the introduction of a strong link between contributions and the pension received. In the NDC-system, the link seems to be weak for the young and a large portion of the contribution is perceived as tax. However, by adding a DC-scheme to the public pension system, the implicit tax as a whole may be significantly reduced.

The model includes a range of simplifications and shortcuts that it would be interesting to avoid in a more elaborate model. Concerning the pay-as-you-go pension system, modeling the automatic balancing mechanism would open up for the possibility of evaluating its long-run properties, as well as its possible intergenerational redistributional effects. A model with intragenerational heterogeneity would make it feasible to study the minimum pension guarantee and its effect on income distribution and on the government budget. Such a model would possibly also make the defined contribution system have significant real
effects on individual behavior. Furthermore, in the Swedish pension system, there is a maximum amount of labor income on which contributions are paid. Contributions paid on income above this amount are considered as taxes, and are instead included in the government budget. This is not included in my model with the effect of too large contributions to the pension system at the expense of government revenue. On the other hand, in the reformed system, some of the transfers to households are treated as labor income and contributions to the pension system are paid. This is not modeled either, but the effect on the pay-as-you-go system goes in the opposite direction and is roughly of the same magnitude as the taxes above the ceiling. Some other extensions would be to include age-dependent transfers, allow a flexible retirement age and include a bequest motive.
References


Appendix A: The pension system in detail.

The defined contribution system

Assets in the DC-system accumulate according to

\[ a^\text{dc}_h(t) = a^\text{dc}_{h-1}(t-1)(1+r(t)) + \tau^\text{dc}\omega_h(t) \]
\[ + \frac{a^\text{dc}_{h-1}(t-1)(1+r(t))(N_{h-1}(t-1) - N_h(t))}{N_h(t)}. \]  

(26)

The first term on the RHS reflects the interest on accumulated assets. Observe here that DC-assets are not subject to asset income taxation. The second term is the contribution during the period (where \( \omega_h(t) \) is taxable labor earnings), and the third term the gains of inheritance. Substituting the number of individuals for the survival probabilities and rearranging yield the following expression

\[ a^\text{dc}_h(t) = a^\text{dc}_{h-1}(t-1)(1+r(t)) \frac{s_{h-1}(t-1)}{s_h(t)} + \tau^\text{dc}\omega_h(t). \]  

(27)

Since individuals are identical, the gains from inheritance can be reformulated and incorporated into the interest expression, giving an additional boost to the rate of return on defined contribution assets above the market rate.

Upon retirement, the assets are converted into a fixed annuity, in real terms, according to the formula

\[ p^\text{dc} = \frac{a^\text{dc}_h(t)}{\lambda^\text{dc}(t)}. \]  

(28)

Here, \( p^\text{dc} \) is the fixed pension annuity, \( a^\text{dc}_h(t) \) the accumulated DC-assets at retirement date \( t \), and \( \lambda^\text{dc}(t) \) the annuity conversion factor at retirement date \( t \).

Basically, the above annuity formula equates the assets at retirement with the sum of discounted expected payments, i.e. a conventional annuity calculation. For an individual retiring in period \( t \) at the age \( h^p \), the assets, \( a^\text{dc}_h(t) \), must equal the discounted expected payments

\[ a^\text{dc}_h(t) = \sum_{h=h^p}^{H} p^\text{dc}_h(t+h-h^p) \prod_{i=t}^{t+h-h^p} (1 + r(i))^{-1}. \]  

(29)
Here, $\sigma_h = \frac{s_h}{s_{hp}}$ is the probability of being alive at age $h$, conditional on being alive at retirement, i.e. the probability that the payment will occur. The product on the RHS discounts the expected payment to the period of retirement. Combining expression (28) and (29), the annuity conversion factor can be stated as

$$\lambda^{dc}(t) = \sum_{h=h^p}^H \sigma_h (t + h - h^p) \prod_{i=t}^{t+h-h^p} (1 + \tau(i))^{-1}. \quad (30)$$

The notional defined contribution system

Assets in the NDC-system accumulate similarly to assets in the DC-system, and using the same approach as in the above description of the DC-system, it can be expressed as

$$a^{ndc}_h(t) = a^{ndc}_{h-1}(t-1)(1 + \mu(t)) + \tau^{ndc} \omega_h(t)$$

$$+ \frac{a^{ndc}_{h-1}(t-1)(1 + \mu(t))(N_{h-1}(t-1) - N_h(t))}{N_h(t)}$$

$$= a^{ndc}_{h-1}(t-1)(1 + \mu(t)) \frac{s_{h-1}(t-1)}{s_h(t)} + \tau^{ndc} \omega_h(t). \quad (31)$$

Instead of assets earning a market rate of return, assets in the NDC-system earn a rate of return determined by the per capita wage-sum growth $\mu(t)$, defined as

$$\mu(t) \equiv \frac{w(t)L(t)/N^L(t)}{w(t-1)L(t-1)/N^L(t-1)} - 1. \quad (32)$$

The NDC-system mimics the annuity calculation in expression (28), with the difference that instead of using the market rate of return, the per capita wage-sum is used to discount future expected payments. In the Swedish system, the expected growth of the per capita wage-sum is legislated to 1.6 percent per annum and referred to as the "norm". Using the norm as the yearly discount rate, observe that with Cobb-Douglas production technology, the per capita wage sum growth rate coincides with the growth rate of GDP per person in the labor force and that the growth rate in the wage sum coincides with the growth rate in GDP, this since $w(t)L(t)/Y(t) = 1 - \alpha$. 

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8Observe that with Cobb-Douglas production technology, the per capita wage sum growth rate coincides with the growth rate of GDP per person in the labor force and that the growth rate in the wage sum coincides with the growth rate in GDP, this since $w(t)L(t)/Y(t) = 1 - \alpha$. 

36
rate implies an annuity conversion factor for the NDC-system of the form

$$\lambda_{ndc}^{h}(t) = \sum_{h=h^p}^{H} \sigma_h (t + h - h^p) \prod_{i=t}^{t+h} (1 + \text{norm})^{-1}.$$  \hspace{1cm} (33)

During retirement, the pensions are also subject to a yearly price indexation and an additional indexation capturing the deviation of the per capita wage-sum indexation from the expected "norm". In practice, this means that the NDC-pension is variable over the retirement period. Observe that in the NDC-system, $\sigma_h(.)$ for the retiring generation is based on an average of mortality risks over the 5 years preceding the period of retirement for all ages, $h$. Life expectancy is not calculated according to the projected mortality of the retiring cohort.

Calculating the implicit tax rate

To simplify the calculations, assume the economy to be in a steady-state. Suppressing the time argument, the implicit tax in the NDC-system at age $h$, $\phi_{ndc}^{h}$, is defined as

$$\phi_{ndc}^{h} = \tau_{ndc} - \pi_{h}^{ndc}.$$  \hspace{1cm} (34)

Here, $\pi_{h}^{ndc}$ is the present value at age $h$ of that part of the pension payments that can be accredited the contribution made at age $h$ as a share of the labor earnings at that age. This part of the pension payments can be found by first separating out the corresponding part of the assets at retirement that are due to that contribution, that is

$$\tau_{ndc} \omega_h (1 + \mu)^{h^p-(h+1)} \frac{s_h}{s_{h^p}},$$  \hspace{1cm} (35)

where $\tau_{ndc} \omega_h$ is the contribution made in period $h$, $(1 + \mu)^{h^p-(h+1)} s_h/s_{h^p}$ is the effective accrual rate between age $h$ and retirement at age $h^p$, which includes the gains of inheritance, and the product gives the total accrual between the contribution and the retirement date. The part of the initial pension due to that contribution is then found by dividing expression (35) by the annuity conversion factor. The pension is then indexed during the retirement, according to the
difference between expected growth, the norm, and actual growth; thus the initial pension must be adjusted by

\[
\left( \frac{1 + \mu}{1 + \text{norm}} \right)^{i-(hP-1)},
\]

(36)
to yield the payment at age \( i \). To obtain the present value at age \( h \), the pension payment is discounted with the net after tax interest rate. This yields the following expression for the present value at age \( h \) of a contribution at age \( h \) in terms of the generated pension payment stream, net of the earnings tax, \( \tau^e \),

\[
\sum_{i=hP}^{H} (1 - \tau^e) \frac{\tau^{ndc} P_{h}^i (1 + \mu)^{hP-(h+1)} \frac{S_h}{s_{hP}}}{\lambda^{ndc}} \left( \frac{1 + \mu}{1 + \text{norm}} \right)^{i-(hP-1)} \frac{1}{(1 + (1 - \tau^e \tau) r)^{i-h}}.
\]

(37)

Dividing by the earnings at age \( h \) and rearranging give \( \pi_h^{ndc} \)

\[
\pi_h^{ndc} = \frac{\tau^{ndc} (1 - \tau^e)}{\lambda^{ndc}} \sum_{i=hP}^{H} \left( \frac{1 + \mu}{1 + (1 - \tau^e) \tau} \right)^{i-h} \frac{1}{(1 + \text{norm})^{i-hP+1}} \frac{S_h}{s_{hP}}.
\]

(38)

Thus, the implicit tax in the NDC-system is

\[
\phi_h^{ndc} = \tau^{ndc} \left( 1 - \frac{1 - \tau^e}{\lambda^{ndc}} Q_h^{ndc} \right),
\]

(39)

where

\[
Q_h^{ndc} = \sum_{i=hP}^{H} \left( \frac{1 + \mu}{1 + (1 - \tau^e) \tau} \right)^{i-h} \frac{1}{(1 + \text{norm})^{i-hP+1}} \frac{S_h}{s_{hP}}.
\]

(40)

For the DC-system, the implicit tax rate is defined as

\[
\phi_h^{dc} = \tau^{dc} - \pi_h^{dc}.
\]

(41)

The assets at retirement due to a contribution at \( h \) are

\[
\tau^{dc} \omega_h (1 + \tau)^{hP-(h+1)} \frac{S_h}{s_{hP}}.
\]

(42)

Dividing the above assets with the annuity conversion factor in the DC-system results in the part of the pension that can be attributed to the age \( h \) contribution. The pension is calculated as a fixed annuity over time, thus it is not indexed and changed over time. To obtain the present value at age \( h \) the pension payment is
discounted with the net after tax interest rate. It is then possible to calculate $\pi_h^{dc}$ as

$$\pi_h^{dc} = r^{dc} \frac{1 - \tau^e}{\lambda^{dc}} \sum_{i=h}^{H} \frac{(1 + \tau)^{i+1} - (h+1)^{i+1}}{1 + (1 - \tau^a) r^{i-h}} \frac{s_h}{sh_P}.$$  \hspace{1cm} (43)

The implicit tax at age $h$ then becomes

$$\phi_h^{dc} = r^{dc} \left( 1 - \frac{1 - \tau^e}{\lambda^{dc}} Q_h^{dc} \right),$$ \hspace{1cm} (44)

where

$$Q_h^{dc} = \sum_{i=h}^{H} \frac{(1 + \tau)^{i+1} - (h+1)^{i+1}}{1 + (1 - \tau^a) r^{i-h}} \frac{s_h}{sh_P}.$$  \hspace{1cm} (45)
Chapter 2: Pareto improving privatization of PAYG old – age social security
Pareto-improving privatization of PAYG old-age social security.*

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Abstract

This paper provides a three-generation OLG model for analyzing a privatization of PAYG old-age social security with linear benefit formulas. Furthermore, it proposes an explicit reform for how the privatization transition may be undertaken. The paper then continues to analyze the possibilities of a Pareto-improving privatization, given the proposed reform. The set of government policy instruments is limited to debt issuing and proportional labor income taxation. Contrary to models where a two-generation OLG framework is used, the three-generation framework creates possibilities for a Pareto improving privatization of old-age social security, since the PAYG system induces a non-optimal implicit tax over the life cycle. By shifting to an optimal tax policy can not only the pension claims accrued under the PAYG system be financed but the shift will also be Pareto-improving.

JEL-classification: H55, H6

Keywords: Social security reform, pay-as-you-go pension systems, overlapping generations.

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1 Introduction

The issue of pension system reform derives its importance from the distortions caused by unfunded old-age social security. Back of the envelope calculations made by Feldstein (1996) suggest that the resulting yearly deadweight loss in the US is about one percent of GDP, or nearly one fifth of the total social security payroll tax revenue due to labor market distortions only. There is no reason to believe the situation to be any better in most other developed countries. On the contrary, the problem seems more serious in Europe than in the US, since the overall tax rates are substantially higher in Europe. This is an indication of large possible efficiency gains and welfare improvements if the distortions were to be reduced or removed.

A large literature has been devoted to finding possible ways of reforming the existing unfunded social security systems and analyzing the welfare effects of these reforms. Breyer (1989) develops a model for analyzing the Pareto efficiency of PAYG systems. In his model, the economy is small and open with exogenously determined factor prices. He uses a two-generation OLG framework where individuals provide an inelastic labor supply. The pension is provided as a lump-sum transfer to the old generation and is financed by contributions from the young generation, which are collected as lump-sum taxes. His conclusion is that this type of PAYG scheme is Pareto efficient and there are no efficiency gains to be made by moving from an unfunded PAYG system to a funded pension system. Indeed, this kind of lump-sum redistribution of income is Pareto efficient and consequently, the result is intuitively understandable; a Pareto-improving transition from such an unfunded PAYG pension scheme to a funded one is impossible.

Homburg (1990) uses the same model as Breyer (1989), but allows for an elastic labor supply and finances the PAYG pensions by a proportional tax on labor income instead of lump-sum taxation. His conclusion is that this type of PAYG-scheme is not Pareto efficient, due to the labor supply distortions introduced by the proportional tax. Breyer and Straub (1993) extend the analysis
to cover the closed economy case. They show that Homburg's results are also valid when factor prices are endogenous. That the PAYG system is not Pareto efficient is due to the inefficiencies arising with the taxation of labor income to finance the pensions. Since the pension is lump-sum, there is no link between taxes paid as young and pensions received when old.

By allowing for a link between taxes and pensions, the distortion of labor supply can be reduced. Using a proportional benefit formula where the pension is dependent on earlier contributions, part of the contribution will be regarded as savings and will thus reduce the tax wedge imposed by the PAYG system. Homburg and Richter (1990) analyze this kind of PAYG system in a two-generations OLG model with elastic labor supply where the economy is assumed to be small and open. They come to the conclusion that a Pareto-improving transition to a pre-funded pension system is indeed possible. Breyer and Straub (1993) arrive at the same conclusion in a model where they assume the economy to be closed. In both cases, their findings are dependent on the use of lump-sum taxation to finance the transition.

Fenge (1995) criticizes the use of lump-sum taxation. Instead, he assumes the only possible tax instrument to be a proportional labor income tax. The model he uses is a two-generation OLG model with elastic labor supply, in a small open economy. He allows for the pension to be dependent on earlier contributions so that the effective rate of return in the PAYG system equals the wage-sum growth rate. His conclusion is that a Pareto-improving transition to a fully funded system is impossible. All the efficiency gains arising from the reform are needed to honor the pension claims of the last generation covered by the PAYG system.

All the described models above use a two-generation OLG framework for the analysis. In this paper, a three-generation OLG model will be developed to analyze the Pareto efficiency of a PAYG-system with a proportional benefit formula. This is then an extension of Fenge's model in the sense of individuals working for two periods instead of one.

The main question of the paper is whether a Pareto-improving transition
from an unfunded pension system, with a linear benefit formula, to a fully funded system is possible in a three-generation OLG framework. The pension system is explicitly modeled with a linear benefit formula, where the internal rate of return of the system is set to equal the wage-sum growth rate in the economy. The economy is small and open and factor prices are determined on the world market. Individuals have perfect foresight and capital markets are perfect. The role of the government is to manage the pension system and service public debt. The policy instruments of the government are restricted to debt issuing and an age-dependent proportional labor income tax. The reform of the old-age social security is assumed to be such that all living generations are unaffected by the reform when it is implemented. This is done by fully paying the pensions of the currently retired generation, but also by honoring the accrued pension claims of working generations by a lump-sum transfer of assets. These transfers are financed by issuing debt.

The paper is organized as follows. The next section develops the three-generation overlapping generations model. It also includes the specifics of the unfunded PAYG old-age social security system and the effect on individuals in terms of how labor supply is distorted. The third section describes the pension reform and the main results of the paper. The last section concludes.

2 The model

I consider an overlapping generations model with two working and one retired generation. The working generations supply labor on a continuous scale. The demographic development is assumed to be exogenous and independent of the economic development. The economy is assumed to be small and open, where the interest rate is determined on the world market. The government collects taxes in order to service government debt, but also manages an unfunded pay-as-you-go pension system. Both the contribution and the pension benefit are assumed to be proportional to labor earnings. Individuals have perfect foresight and capital markets are perfect.
For variables denoted by a capital letter, $X_t$ the subscript denotes time. For any other variable, $x_{i,t}$ the subscript $i = 0, 1, 2$ denotes the age of the individual i.e., young worker, old worker or retiree. Subscript $t \in I$ denotes the time period and will occasionally be suppressed for notational convenience. One period may be considered as approximately 20 years.

2.1 Demography

All individuals live for three periods, one period as a young worker one as an old worker, and one as a retiree. They face no life span risk. The growth rate in the number of young workers, $n$, is exogenous and constant over time. Normalize the number of young workers in period $t = 0$ to unity, and let $N \equiv 1 + n$. The number of young workers in any period, $t$, is then given by $N^t$, the number of old workers by $N^{t-1}$ and the number of retirees by $N^{t-2}$.

2.2 Factor prices

The wage rate, $w$, is assumed to be constant and is normalized to unity. The interest rate is determined on the world market and assumed constant over time and denoted $r$. Let $R \equiv 1 + r$. Constant factor prices greatly simplify the calculations so that the basic mechanism driving the results may easily be highlighted, since factor prices will be unaffected by the reform. The growth of the wage sum, $G_t$, is defined as $G_t \equiv W_t / W_{t-1}$, where $W_t$ denotes the wage sum in period $t$.

2.3 Agents

Individuals born in period $t$ solve the following decision problem

$$\max_{\{c_{i t+1}\}_{i=0}^{1}, \{x_{i t+1}\}_{t=0}^{1}} U \left( c_{0, t}, c_{1, t+1}, c_{2, t+1}, 1 - l_{0, t}, 1 - l_{1, t+1} \right),$$

subject to the budget constraint

$$c_{0, t} + \frac{c_{1, t+1}}{R} + \frac{c_{2, t+1}}{R^2} = l_{0, t} + \frac{l_{1, t+1}}{R} - x_{0, t} - \frac{x_{1, t+1}}{R} + \frac{p_{2, t+2}}{R^2}.$$
Here, $c$ is consumption, $l$ labor supply where the sum of labor and leisure time is normalized to one and $0 \leq l \leq 1$. Taxes and contributions to the pension system are given by $x$, and $p$ is the pension. $U(\cdot)$ is assumed to be strictly quasiconcave and twice differentiable with all first partial derivatives positive. Workers may smooth consumption over the three periods of life by investing in government bonds or physical capital. They have no bequest motive, so the only reason to save is to smooth consumption and provide income for the retirement period. Workers are assumed not to be capital constrained.

2.4 The government and the pension system

The role of the government is to collect labor income taxes and contributions to the pension system, pay pensions and service the government debt. The government intertemporal budget constraint is given by

$$D_t = RD_{t-1} + P_t - X_t + B_t,$$

(3)

where $D_t$ is the end of period government debt, $X_t$ is total government revenues consisting of taxes and pension contributions collected in the period, and $P_t$ is total pension expenditures. Net bond issuing during the period is $P_t - X_t + B_t$, where $B_t$ is bond issuing not attributable to cover net cash flow deficits, $P_t - X_t$, during the period.

The only tax instrument available for debt service is an age-dependent proportional tax on labor income, where young workers are taxed at the rate $\tau_{0,t}$, whereas old workers are taxed at the rate $\tau_{1,t}$.

Pension contributions are collected proportionally to labor earnings, where the contribution rate, $\psi$, is assumed to be constant. Taxes and contributions paid by an individual of age $i$ are then given by $x_{i,t} = (\tau_{i,t+i} + \psi)l_{i,t+i}$ for $i = 0, 1$.

The unfunded PAYG pension system has a proportional benefit formula, where the pension benefit is linear in the contributions made. The internal rate of interest of the system equals the wage-sum growth rate. Consequently, for
individuals born in period $t$, the benefit formula can be stated as

$$ p_{2, t+2} = \psi l_{0, t} G_{t+1} G_{t+2} + \psi l_{1, t+1} G_{t+2}, \quad (4) $$

where $\psi l_{0, t}$ and $\psi l_{1, t+1}$ are the individual’s contribution to the pension system as a young and an old worker respectively. Both contributions earn a rate of return equal to the wage-sum growth rate, the first contribution in two periods and the second in one period, hence $G_{t+1} G_{t+2}$ and $G_{t+2}$ are the interest factors for the respective contribution. Using the fact that $x_{i,t} = (\tau_{i,t+i} + \psi) l_{i,t+i}$ for $i = 0, 1$ and inserting the benefit formula (4) into the budget constraint of the individual (2) reduces the RHS of the individual’s budget constraint to the following

$$ (1 - \tau_{0, t} - \phi_{0, t}) l_{0, t} + \frac{(1 - \tau_{1, t+1} - \phi_{1, t+1})}{R} l_{1, t+1}, \quad (5) $$

where

$$ \phi_{0, t} = \psi \left( 1 - \frac{G_{t+1} G_{t+2}}{R^2} \right) \quad \text{and} \quad (6) $$

$$ \phi_{1, t+1} = \psi \left( 1 - \frac{G_{t+2}}{R} \right). \quad (7) $$

Here, $\phi_{0, t}$ and $\phi_{1, t+1}$ are called the \textit{implicit tax rates} for young and old workers, respectively. Observe that while the government tax and contribution \textit{collection rate} is $\tau_0 + \psi$ for young workers and $\tau_1 + \psi$ for old workers, (suppressing the time index), only part of the collection rate is perceived as a distortionary tax, namely $\tau_0 + \phi_0$ for the young and $\tau_1 + \phi_1$ for the old. Call this the \textit{effective} labor income tax rate. The difference, $\psi - \phi_0$ for the young and $\psi - \phi_1$ for the old, can be interpreted as the share of labor income constituted by forced implicit savings. Consequently, the total effective labor income tax rate the individual faces consists of a debt servicing tax and an implicit tax. The fact that the distortion decreases with age is an effect of the market rate of return being higher than the internal rate of return of the pay-as-you-go system. Contributions made early in life receive a rate of return below the market rate for a longer period than later contributions. The higher the difference in returns between the wage-sum growth rate and the interest rate, the higher the distortion difference between the two period will be.
Total pension expenditures in period $t$ are given by $P_t = p_{2,t}N^{t-2}$, and total pension contributions by $\psi W_t$, where the wage sum is given by $W_t = l_{0,t}N^t + l_{1,t}N^{t-1}$.

3 Pareto-improving reform

Assume that the economy is in a steady-state in period $s$ and that there is no outstanding government debt. With no outstanding debt, the debt servicing tax rates are zero, $\tau_{0,s} = 0$ and $\tau_{1,s} = 0$. The no debt assumption does not alter the result, but simplifies the exposition. Suppressing the time index, let $l_0^u$ and $l_1^u$ denote the labor supply choice of young and old workers respectively in the steady-state, where the superscript $u$ indicates that the labor supply is the outcome under unfunded PAYG. Focusing first on the government intertemporal budget constraint, it can easily be verified that the pension system is period-wise budget balanced. Recognizing that $G = N$ in steady-state, inserting the per capita pension given by expression (4) into the expression for total pension expenditures, and using the steady-state labor supply choices, yields

$$P_t = \psi l_0^u N^t + \psi l_1^u N^{t-1} = \psi W_t. \quad (8)$$

Consequently, total expenditures equal contributions in steady-state. Since the debt servicing taxes are zero, the total effective labor income tax rate only consists of the implicit tax rate. For young workers, the distortion amounts to

$$\phi_0 = \psi \left( 1 - \left( \frac{N}{R} \right)^2 \right), \quad (9)$$

whereas for the old workers, it is

$$\phi_1 = \psi \left( 1 - \frac{N}{R} \right). \quad (10)$$

Given that the economy is dynamically efficient, $R > N$, the implicit tax introduced by the pension system when young is higher than when old, $\phi_0 > \phi_1$.

Under these circumstances, the government proposes an old-age social security reform implying the abolishment of the unfunded PAYG pension system.
and an introduction of a laissez-faire policy where individuals must provide for their own old-age income. Currently living generations are to be unaffected by the reform which implies that only the welfare of future generations is relevant in evaluating whether the reform is welfare increasing.

Current retirees, in period $s$, are to be paid according to the existing rules, financed by the contributions collected in the period. From equation (8), we know that the pension expenditures are covered by the contributions. The working generations will be compensated for their loss of future pension payments by a lump-sum transfer of assets at the end of period $s$. In the case of the young working generation, which will also be working in period $s + 1$, an additional feature is that, as old workers, they will face a labor income tax rate of the same size as the implicit tax rate they would have faced in the no reform case, that is $\phi_1$. In order to keep the young generation fully unaffected by the reform, this additional feature must be satisfied.

For the old working generation, the lump-sum transfer equals the capital value of the pension they would receive if no reform was undertaken, $P_{s+1}/R = \psi (l_{0}^{0} N_{s+1}^{0} + l_{1}^{0} N_{s}^{0}) / R$. For the young working generation, the transfer equals the capital value of their accrued pension claims$^1$, which is $\psi l_{0}^{0} N_{s+2}^{0}/R$. The government issues debt to finance these transfers such that $B_s = \psi l_{0}^{0} N_{s+2}^{0}/R^2 + \psi (l_{0}^{0} N_{s+1}^{0} + l_{1}^{0} N_{s}^{0}) / R$. This debt is to be serviced by a proportional tax scheme on labor income that is constant over time. The tax rates are set so that the debt is non-accelerating over time. The scheme is age dependent, where young workers face the tax rate $\tau_0$ and old workers $\tau_1$. As an effect of the constant tax policy; the labor supply choices of future generations are constant over time and denoted $l_0$ and $l_1$ for the young and the old working generation, respectively. The economy will thus move from one steady-state in period $s$ to a new steady-state in period $s+3$ when the last generation, born in period $s$, which is partially covered by the abolished social security system, is dead. With this background,

$^1$The total contribution at $s$ is $\psi l_{0}^{0} N_{s}^{0}$. The pension is due for payment in period $s + 2$, so the accrued value of the contribution at $s + 2$ is $\psi l_{0}^{0} N_{s+2}^{0}$; discounting this back to date $s$ gives the capital value $\psi l_{0}^{0} N_{s+2}^{0}/R^2$.  

51
it is now possible to arrive at the following conclusion.

**Lemma 1** Given that the above described reform is undertaken, the government intertemporal budget constraint in period $s$ can be reduced to

$$\tau_0 l_0 + \tau_1 l_1 \frac{1}{R} = \phi_0 l_0^0 + \phi_1 l_1^0 \frac{1}{R}, \tag{11}$$

**Proof.** See appendix A. ■

Equation (11) is deceptively simple and almost self-evident, but it incorporates all that is required to show that a Pareto-improving transition may indeed be possible. The lemma simplifies the government budget constraint so that it can be interpreted at the individual level. It states that the capital value of lifetime taxes paid by an individual in the reformed economy must equal the capital value of implicit taxes paid by an individual in the pre-reformed economy. Except for guaranteeing that the debt is non-explosive, that is that the government budget constraint is satisfied, equation (11) also implies that all generations living in period $s$ are indifferent to the reform. Furthermore, it also follows that a transition, without hurting any generation, indeed is possible. Call the tax policy where the age-dependent debt servicing tax rates in the reformed system equals the implicit tax rates in the pre-reformed system, that is $\tau_0 = \phi_0$ and $\tau_1 = \phi_1$, the PAYG tax policy.

**Lemma 2** With the above reform proposal, it is possible to privatize old-age social security without hurting any future generation by implementing the PAYG tax policy to service government debt.

**Proof.** Assume that the above described reform is undertaken and that the government implements the PAYG tax policy to service government debt. This implies that prices are the same as if no reform had been undertaken; furthermore the optimal choices in the reformed economy are the same as in the pre-reformed one. Consequently, the welfare is the same as in the unreformed economy. It is also easily verified that the PAYG tax policy is feasible in the sense of satisfying the government budget condition (11) by substituting for the PAYG tax policy and using the fact that $l_0 = l_0^0$ and $l_1 = l_1^0$. ■
Lemma 2 not only shows that it indeed is possible to move to a fully funded scheme without hurting anyone. Since the PAYG tax policy balances the government intertemporal budget, the problem may be simplified; the question of whether a Pareto-improving privatization of old-age social security is possible is now reduced into a question of whether the PAYG tax policy is Pareto optimal for servicing government debt in the reformed economy, which is an optimal taxation problem. Unless the answer is unconditionally no, there is scope for a Pareto-improving reform where a privatization of social security combined with a change in labor income tax policy will produce a Pareto superior outcome, which leads to the following proposition.

**Proposition 3** There is scope for a Pareto-improving privatization of old-age social security with a linear benefit formula and a constant contribution fee, unless the optimal tax structure in the reformed economy coincides with the PAYG tax policy.

**Proof.** Since the PAYG tax policy belongs to the set of feasible tax policies in the reformed economy, in the sense of satisfying the government intertemporal budget condition, an optimal tax rate policy deviating from this must be such that welfare is higher than under the PAYG tax policy.

Note here that the implicit tax rates, $\phi_0$ and $\phi_1$, are given by the contribution rate, the population growth rate and the interest rate alone, while the optimal tax rate scheme will also depend on preferences. The case where the PAYG tax policy is also the optimal policy is therefore purely coincidental.

The result may be easily understood if contrasted with the two-generations OLG model. In the two generations case, the tax rate needed to service the government debt is uniquely determined by the need to balance government debt. Considerations of when to pay taxes and how these should be distributed over the life-cycle is not an issue. In the three-generations model, the debt servicing tax policy is not uniquely determined by the need for servicing the debt. Instead, the three-period model introduces an additional instrument by allowing for taxing the individual in two separate periods. How taxes should
be distributed over the life cycle is an optimal taxation problem and, as such, dependent on preferences.

In the following, I will give an example where a Pareto-improving transition is possible. This example will also serve as an illustration of the proposition. Assume the following explicit form to describe the utility given in expression (1)

\[ U = \ln c_0 (1 - l_0) + \beta \ln c_1 (1 - l_1) + \beta^2 \ln c_2. \]  \hspace{2cm} (12)

where \( \beta \) is the discount factor. Recalling that the income side of the budget constraint in (2) can be rewritten as (5) and using the fact that there is no outstanding debt that needs to be serviced, the budget constraint for an individual living under the PAYG scheme is given by

\[ c_0 + \frac{c_1}{R} + \frac{c_2}{R^2} = (1 - \phi_0) l_0 + \frac{(1 - \phi_1) l_1}{R}. \]

For an individual living in the reformed economy who will face the debt servicing tax rates, \( \tau_0 \) and \( \tau_1 \), the constraint instead becomes

\[ c_0 + \frac{c_1}{R} + \frac{c_2}{R^2} = (1 - \tau_0) l_0 + \frac{(1 - \tau_1) l_1}{R}. \]

Specify the parameter values of the model in the following way. Let the population be constant, that is \( n = 0 \), future utility is discounted by \( \beta = 0.5 \) per period, the interest rate is set so that \( R = 2 \), and the contribution rate is \( \psi = 0.3^2 \). With \( \beta R = 1 \), the euler equation is simplified so that \( c_0 = c_1 = c_2 \) and \( l_0 = l_1 \). The resulting implicit tax rates generated by the PAYG system are \( \phi_0 = 0.225 \) and \( \phi_1 = 0.150 \). Let \( V^u (\phi_0, \phi_1) \) denote the indirect utility function for the case of unfunded PAYG and \( V (\tau_0, \tau_1) \), its counterpart for the reformed system.

With the above parameters specified, the maximum utility under the unfunded PAYG regime is \( V^u (\phi_0 = 0.225, \phi_1 = 0.150) = -2.902 \). This is indicated

\footnote{One period may be approximated to 20 years. A discount factor of 0.5 per period is then equivalent to an annual discount rate of 0.965. \( R = 2 \) is approximately equivalent to an interest rate of 3.5 percent per annum. The contribution rate of 0.3 results in a replacement rate of 58.7 percent.}
in figure 1, which is a plot in tax space, by the intersection of $\phi_0 = 0.225$ and $\phi_1 = 0.150$. The thick curve represents the tax pairs $\tau_0$ and $\tau_1$ which yield the same utility for an individual in the reformed economy as for an individual living under unfunded PAYG, that is $V(\tau_0, \tau_1) = -2.902$. Observe that utility increases towards origo. The thin curve represents tax pairs $\tau_0$ and $\tau_1$ which satisfy the government budget constraint. Observe that also this curve passes through the point given by $\phi_0 = 0.225$ and $\phi_1 = 0.150$, which is in accordance with lemma 2; a reform can indeed be finance without hurting anyone.

Following Diamond and Mirrles (1971a) and (1971b), the optimal tax policy for servicing the debt in the reformed economy may easily be solved for. The optimal taxation rule is very close to a uniform rule over the life-cycle with $\tau_0 = 0.198$ and $\tau_1 = 0.198$, which is indicated in the figure. The utility under
this debt servicing regime is $V(\tau_0 = 0.198, \tau_1 = 0.198) = -2.900$, which is larger than $V^u (\phi_0 = 0.225, \phi_1 = 0.150) = -2.902$.

4 Conclusions

This paper has developed a three-generation OLG model incorporating unfunded PAYG old-age social security with a linear benefit formula. The linear benefit formula introduces an age-dependent distortion of the labor supply, where the resulting distortionary tax rate faced by individuals is falling with age. The paper has also provided an explicit reform proposal for a transition from the PAYG system to a fully funded laissez-faire system, where the government instruments are limited to debt issuing and a proportional labor income tax. It has been shown that it is always possible to privatize the unfunded PAYG scheme without hurting any generation. Furthermore, if the implicit taxes induced by the unfunded PAYG system are non-optimal from a life-cycle perspective, there is scope for a Pareto-improving privatization of the PAYG system. This result distinctly diverges from earlier results where the privatization of old-age social security is analyzed in models with two overlapping generations. Further research on this topic includes a close economy assumption with a general specification of production technology, where factor prices will also adjust to the reform.
References


A Proof of Lemma 1

For the government intertemporal budget not to explode over time, current debt in period \( s \) must equal the present value of future tax revenues. The government recursive budget constraint can thus be stated as

\[
D_s = \frac{\tau_0 l_0 N^{s+1} + \phi_1 l_1^N N^s}{R} + \sum_{t=s+2}^{\infty} \frac{\tau_0 l_0 N^t + \tau_1 l_1 N^{t-1}}{R^{t-s}}. \tag{13}
\]

Here, the left-hand side is the government debt at the end of period \( s \) and the right-hand side the present value of future tax revenues. The first ratio on the RHS is the present value of the tax revenues in period \( s + 1 \), where the tax rate old workers face coincide with the implicit tax rate of old workers in the unfunded pay-as-you-go-system \( \phi_1 \), in order to ensure that they are unaffected by the reform.

The debt in steady-state at the end of period \( s \) is given by

\[
D_s = \psi l_0^u \frac{N^2}{R} \frac{N^s}{R} + \psi l_0^u N \frac{N^s}{R} + \psi l_1^u \frac{N^s}{R}. \tag{14}
\]

Using the relationship between the implicit tax rates in steady-state and the contribution rates

\[
\phi_0 = \psi \left( 1 - \left( \frac{N}{R} \right)^2 \right), \tag{15}
\]

\[
\phi_1 = \psi \left( 1 - \frac{N}{R} \right) \tag{16}
\]

and rearranging terms, the debt can be rewritten as

\[
\left( \phi_0 l_0^u + \phi_1 l_1^u \frac{1}{N} \right) \frac{N}{R - N} N^s. \tag{17}
\]

The RHS of (13) can be restated as

\[
\left( \tau_0 l_0 N + \phi_1 l_1^u \right) \frac{N^s}{R} + \left( \tau_0 l_0 N + \tau_1 l_1 \right) \frac{N^s}{R^2} \sum_{t=s+2}^{\infty} \frac{N^{t-s-2}}{R^{t-s-2}}, \tag{18}
\]

which may be further modified into

\[
\left( \phi_1 l_1^u \frac{1}{N} \frac{R - N}{R} + \tau_0 l_0 + \tau_1 l_1 \frac{1}{R} \right) \frac{N}{R - N} N^s. \tag{19}
\]
Equating expression (17) with (19) yields,

\[ \phi_0 l_0^u + \phi_1 l_1^u \frac{1}{N} = \phi_1 l_1^u \frac{1}{N} \frac{R - N}{R} + \tau_0 l_0 + \tau_1 l_1 \frac{1}{R}, \]  

(20)

which may be simplified to

\[ \tau_0 l_0 + \tau_1 l_1 \frac{1}{R} = \phi_0 l_0^u + \phi_1 l_1^u \frac{1}{R}. \]  

(21)

This completes the proof.
Chapter 3: Projections of the reformed Latvian public pension system
Abstract
In the next 50 years, the Latvian labor force is projected to decrease by approximately 0.6 percent per year. The effect, a rapidly increasing old-age dependency ratio would, in most developed countries, constitute a serious threat due to exploding old-age social security expenditures. The recently reformed Latvian public pension system is designed to handle the upcoming difficulties. Special attention has been given in the design to keep the expenditures low relative to the revenues by introducing rules dampening the increase in the pension expenditures. This paper evaluates the performance of the PAYG system. In the light of the pessimistic projection of the Latvian demography, the newly reformed PAYG system performs remarkably well. The expenditure reducing rules introduced have significant effects on the system's financial balance. The reduction in pension expenditures in the next 50 years amounts to more than 120 percent of GDP, measured as a year 2000 present value. The pension reform also includes the launch of a publicly run defined contributions pension system. It is shown that the resulting implicit tax imposed by the public pension system imposes on labor earnings is negative and increasing with age. That is, savings are subsidized in the public pension system. It is also shown that private savings are fully crowded out as individuals try to offset their savings in the pension system. Since individuals are capital constrained, they will have no private assets at all. From a welfare perspective, this suggests the overall contribution rate to the public pension system to be too high.

Keywords: PAYG, pension reform, demography, transition.

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1 Introduction

The reformed Latvian public pension system is a mixture of a PAYG system with individual accounts, notionally defined contributions (NDC), and a pre-funded defined contribution system (DC). This type of system combination has also been introduced in Sweden and Poland. Not only does Latvia’s new pension system share many features with the newly reformed Swedish system, the challenges the new system is intended to solve are also the same, to some extent. Both countries face an ageing population and stagnation in the population growth in the long-run due to lower fertility rates and a low net immigration resulting in rapidly increasing dependency ratios. In the case of Latvia, the population is even projected to fall in the long-run. The dependency ratio and the old-age dependency ratio are plotted in figure 1.

Figure 1: The dependency ratio and the old-age dependency ratio.

1 The dependency ratio is defined as the number of individuals aged 0 to 19 and 62 and above, to the number of individuals in the age group 20 to 61. The old-age dependency ratio is calculated as the ratio of people aged 62 and above to people aged 20 to 61. The figures for 2000-2050 are from Statistics Latvia. The years 2050-2100 are based on my own calculations, see the demography section for details.
From an initial level slightly above 0.3 retirees per individual of working age, the old-age dependency ratio is projected to increase to above 0.6 in 2050 and even higher levels beyond that year. Also taking individuals younger than 20 years into account, the support burden increases so that each individual aged between 20 and 61 must provide for an additional individual besides herself.

More important than the challenges shared by both countries are the specific problems the new Latvian system must overcome. Whereas Sweden has accumulated a large pension fund, attached to the NDC-system to facilitate the financing of pension expenditures in periods with negative primary savings, a Latvian counterpart is still non-existing. The contribution rate to the public system as a whole is 20 percent of the labor earnings. In the year 2000, this entire amount was earmarked for the NDC-system but over a 10-year period, this share is to be changed so that total contributions are equally split between the NDC and the DC-system. The effect on NDC-revenues of such a change is immediate while expenditures only adjust with a long lag. A strained government budget, which can hardly manage to finance eventual deficits within the NDC-system, in combination with a non-existing pension fund, will make it difficult to finance the pension expenditures due to obligations inherited from the old system. Two expenditure-decreasing rules have been introduced. First, instead of increasing with the wage-sum growth rate, the indexation of NDC-pensions follow a reduced wage-sum growth index. Second, NDC-assets belonging to individuals who die before their retirement are not distributed to working generations. Both these rules result in a long-run surplus tendency, as expenditures will grow slower than contributions.

In the light of this pessimistic demographic projection, evaluating the performance of the pay-as-you-go system is a major concern of this paper. In the Latvian case, the importance of a system that is adaptable and adjusts adequately to the demographic and economic development is accentuated because of the limited possibilities of financing possible deficits, at least in the short run. It is also of interest to analyze whether there are any effects of the expenditure reducing rules.
In this paper, a model of the new Latvian public pension system is incorporated in a dynamic partial equilibrium model with overlapping generations. The model is an extension of the Auerbach and Kotlikoff (1987) model and builds on Storesletten (2000) and Sundén (2002). The economy is small and open with an exogenously determined interest rate. Furthermore, it is assumed to be in transition so that an exogenous process determines the labor augmenting technological progress, where the domestic productivity level converges to the world market level over time. The results are based on the policy experiment of changing the tax rate on labor earnings and pension income from the year 1999 to a level such that the new tax rate balances the intertemporal budget of the public sector, and secures a public sector debt to output ratio that does not accelerate over time.

2 Model

The model used is an overlapping generations model with life-cycle uncertainty, where agents are identical, except their age. All individuals live for a maximum of $H + 1$ years. During the life-cycle, an individual is classified as a child, a worker or a retiree. In the first $h^w$ years, from the age 0 to $h^w - 1$, the agent is considered as a child with no economic activity at all and fully financed by her parents. At age $h^w$, all individuals start working in exchange for wages. They work until age $h^p - 1$ and at age $h^p$, they retire and receive old-age social security benefits until age $H$ at most. No individual reaches the age $H + 1$. In their economically active life, they also receive a government transfer. Consumption is smoothed over the life-cycle by investing in government bonds or physical capital. The economy is small and open where the rate of return to investment is given by an exogenously determined world market rate of return. An exogenously determined process gives the labor augmenting technological progress, where the domestic productivity level converges to the world market level over time. The public sector includes the government and a public pension system. The government taxes consumption, labor income, asset income and
pensions in order to finance transfers and government consumption. The public pension system consists of both a PAYG pension system, with an attached pension fund, as well as a pre-funded system. The budgets of the systems are assumed to be separate from the government budget and from each other. The demographic development is assumed to converge to a stationary state, to be followed by an economic steady-state. For any variable x, a subscript denotes age and an argument in parenthesis calendar time.

2.1 Demographics

Let \( N_h(t) \) be the number of individuals of age \( h \) in period \( t \). In each period, the number of newborn is given by \( N_0(t) \). All individuals are identical and face an exogenously given survival probability from one period to another. The unconditional survival probability from the birth in period \( t-h \) to age \( h \) is given by \( s_h(t) \), where \( s_0(t) = 1 \) \( \forall t \). Since the maximum number of years alive is \( H+1 \), \( s_{H+1}(t) = 0 \) \( \forall t \). The total population is given by \( N(t) = \sum_{h=0}^{H} N_h(t) \). The labor force in period \( t \), denoted \( N^L(t) \), is the sum of the population between age \( h^w \) and age \( h^p - 1 \), and given by \( N^L(t) = \sum_{h=h^w}^{h^p-1} N_h(t) \).

2.2 Production and factor prices

Production is given by a standard CRS Cobb-Douglas function

\[
Y(t) = K(t-1)\alpha [z(t)L(t)]^{1-\alpha},
\]

where \( \alpha \) is the income share of capital. The labor argument is given by

\[
L(t) = \sum_{h=h^w}^{h^p-1} \epsilon_h (1 - l_h(t)) N_h(t),
\]

where \( \epsilon_h(t) \) is an age-efficiency index and \( 1 - l_h(t) \) an age \( h \) individual’s labor supply at time \( t \). The labor augmenting technological progress is given by

\[
z(t) = (1 + \gamma)^{t-s} z(s) \left[ (1 - v)^{t-s} + \left( 1 - (1 - v)^{t-s} \right) z^W(s) / z(s) \right].
\]

Disregarding the term in brackets, the technology level, \( z(t) \), in expression (3) is determined by an exogenous growth rate, \( \gamma \), and its initial level, \( z(s) \). This
term represents the long-run underlying growth in the economy and the world. The term in brackets represents the convergence in the technology level towards the world market level. Here, $z^W(s)/z(s)$ is the relative level of the world market labor augmenting technology as a share of the Latvia level in period $s$, and $v$ the convergence speed of the economy. As $t$ grows large, the term within brackets approaches $z^W(s)/z(s)$ and the technology level $(1 + \gamma)^{t-s}z^W(s)$, which is the world market level at $t$.

First-order conditions under the assumption of markets being competitive are given by

$$r(t) = \alpha \left[ \frac{K(t-1)}{z(t)L(t)} \right]^{\alpha-1} - \delta \quad (4)$$

$$w(t) = (1 - \alpha)z(t)\left[ \frac{K(t-1)}{z(t)L(t)} \right]^{\alpha} \cdot \quad (5)$$

Here, $\delta$ is the physical depreciation rate of capital. The economy is assumed to be small and open where the interest rate is given by the exogenously determined world market rate of return.

2.3 Agents

The instantaneous utility function for an age $h$ individual in period $t$ is given by a CES class with unit elasticity of substitution between consumption and leisure

$$u(c_h(t),l_h(t)) = \left\{ c_h(t)^{\rho} [l_h(t)]^{1-\rho} \right\}^{1-1/\theta}. \quad (6)$$

Here, $c$ represents consumption, $l$ leisure, where $0 \leq l \leq 1$, and $\theta$ the intertemporal elasticity of substitution. The expected lifetime utility is additively separable

2The assumption of an exogenously determined interest rate in conjunction with a Cobb-Douglas production implies, from equation (4), that the capital to output ratio in the economy is exogenously determined by

$$\frac{K_{t-1}}{Y_t} = \frac{\alpha}{r(t) + \delta}.$$ 
Furthermore, from equation (5), the wage rate can be rewritten as

$$w(t) = (1 - \alpha)z(t)\left[ \frac{\alpha}{r(t) + \delta} \right]^{\alpha/(1-\alpha)}.$$ 

With both $r(t)$ and $z(t)$ being exogenously determined, the wage rate will also be exogenous.
and given by
\[ U = \sum_{h=h^w}^{H} \beta^{h-h^w} u(c_h(t), l_h(t)) s_h(t), \tag{7} \]
where \( \beta \) is the subjective discount factor and \( s_h(t) \) the unconditional survival probability for an age \( h \) individual of being alive at time \( t \). Observe that the sum starts from the first economically active age, \( h^w \), and not from age 0. Individuals have no bequest motives but may leave accidental bequests. They are capital constrained and may not have negative asset holdings, that is \( a_h(t) = 0 \) \( \forall h, \forall t \). The recursive budget constraint is given by
\[ c_h(t) + a_h(t) \]
\[ = [1 + r(t)] a_{h-1} (t - 1) + w(t) \epsilon_h(t) [1 - l_h(t)] + \text{transfer}(t) + \text{bequest}_h(t) + p_h(t) - \tau_h^p(t) - \tau_h^{ndc}(t) - \tau_h^{dc}(t). \tag{8} \]
The transfer received in each period is denoted by \( \text{transfer}(t) \) and is the same irrespective of age. Accidental bequests in the period are denoted \( \text{bequest}_h(t) \). The public pension is given by \( p_h(t) \), where \( p_h(t) = 0 \) \( \forall h < h^p \). Total taxes paid are represented by \( \tau_h^p(t) \) and given by
\[ \tau_h^p(t) = [\tau^e + \tau^p] \omega_h(t) \epsilon_h(t) [1 - l_h(t)] + \tau^e c_h(t) + \tau^e p_h(t) + \tau^a r(t) a_{h-1}(t - 1). \tag{9} \]
Here, \( \tau^e \) is the labor earnings tax rate, which is also the tax rate on pension earnings. The payroll tax rate is denoted by \( \tau^p \), the consumption tax rate by \( \tau^c \) and the tax rate on asset income by \( \tau^a \). The contribution to the respective part of the public pension system is given by \( \tau_{h}^{ndc}(t) \) for the NDC contribution and by \( \tau_{h}^{dc}(t) \) for the DC contribution and calculated according to
\[ \tau_{h}^{ndc}(t) = \tau_{h}^{ndc}(t) \omega_h(t) \epsilon_h(t) [1 - l_h(t)], \tag{10} \]
\[ \tau_{h}^{dc}(t) = \tau_{h}^{dc}(t) \omega_h(t) \epsilon_h(t) [1 - l_h(t)]. \tag{11} \]
Observe here that the contribution rates are time dependent, since the legislation passed stipulates a change in the contribution rates over time. Labor income
is defined as \( w(t) \epsilon_h(t)(1 - l_h(t)) \), whereas labor earnings are defined as labor income after payroll taxes and pension contributions have been paid. This base is used as the tax base for the labor earnings tax rate and the contribution tax rates and is given by \( \omega_h(t) = w(t)(1 - l_h(t)) \). Here, the wage rate after payroll taxes and contributions is calculated according to

\[
\omega_h(t) = \frac{w(t)}{1 + \tau_p + \tau^\text{ndc}(t) + \tau^\text{dc}(t) - 0.09}.
\]  

(12)

In each period, total accidental bequests in the economy are divided among individuals in the age group \( h_b \). The bequests to an individual in this age group are denoted \( \text{bequest}_{h_b}(t) \) and given by

\[
\text{bequest}_{h_b}(t) = \sum_{j=h_w}^{H} \frac{a_{j-1}(t-1)N_{j-1}(t-1)[1 + [1 - \tau^a]r(t)][1 - \frac{s_j(t)}{s_{j-1}(t-1)}]}{N_{h_b}(t)}.
\]  

(13)

Here, the \( a_{j-1}(t-1)N_{j-1}(t-1) \) is the amount of assets for the age group \( j-1 \) in period \( t-1 \), multiplied by \( 1 + [1 - \tau^a]r(t) \) to obtain the period \( t \) asset value after the asset income tax has been deducted. \( 1 - s_j(t)/s_{j-1}(t-1) \) gives the relative share of the number of non-survivors between period \( t-1 \) and \( t \) for age group \( j \). Summing over all relevant ages gives the total amount of assets that should be redistributed to \( N_{h_b}(t) \) individuals in the age group \( h_b \).

### 2.4 The pension system and the government

The pension system and the government are divided into three parts. The fully funded defined contributions system, the pay-as-you-go notional defined contributions system and the government. The public sector is defined as the compound of the pay-as-you-go system and the government. The model here builds on Sundén (2002).

\[\text{Not all contributions are paid by the employer. 9 percentage units are paid by the employee.}\]
2.4.1 The Pension System

Suppressing the subscript for age and the time argument, the fixed pension annuity for the defined contribution system is calculated as,

\[ p^{dc} = \frac{a^{dc}}{\lambda^{dc}}, \]  

(14)

where \( a^{dc} \) is the accumulated assets at retirement. The contributions made are accrued on the individuals' defined contributions account during their working life. The accrual rate follows the market rate of return. Asset returns are not subject to income taxation, but the pension is taxed at the same rate as labor earnings. The denominator in expression (14), \( \lambda^{dc} \), is life expectancy upon retirement, adjusted for future interest rates. This factor converts the assets upon retirement into a fixed pension annuity in real terms.

The entry pension in the NDC-system is calculated in the same way as in the DC-system, but the pension is variable over time since it is indexed in real terms. The exact formulation of the indexation is given in the next section. The denominator here, \( \lambda^{dc} \), only represents the life-expectancy at retirement.

Following Sundén (2002) the implicit tax rate in the NDC-system, \( \phi_{h}^{ndc}(t) \), for an individual of age \( h \) in period \( t \), where \( 20 \leq h \leq 62 \), is defined as

\[ \phi_{h}^{ndc}(t) = \pi^{ndc} - \pi_{h}^{ndc}(t). \]  

(15)

Here, \( \pi_{h}^{ndc}(t) \) represents the part of the contribution made at age \( h \) in period \( t \) that is viewed as savings. This is calculated as the present value of the pension payments as a share of the individual's earnings at \( t \), in which the contribution result. Subtracting this savings rate from the contribution rate yields the implicit tax rate. The counterpart of the defined contribution system is

\[ \phi_{h}^{dc}(t) = \tau^{dc} - \pi_{h}^{dc}(t). \]  

(16)

A pension fund is linked to the NDC-system to accommodate yearly imbalances in the cash flow. The budget constraint of the NDC-system can be written as

\[ B(t) + \sum_{h=h_{P}}^{H} p_{h}^{ndc} N_{h}(t) = (1 + r(t)) B(t - 1) + \sum_{h=h_{P}}^{h_{P}-1} \tau_{h}^{ndc}(t) N_{h}(t). \]  

(17)
Here, $B(t)$ represents the end of period pension fund and $p_h^{ndc}(t)$ the pension payments to age $h$ individuals. The budget of the pay-as-you-go system is separated from the governmental budget purely for bookkeeping purposes. Neither of them need to balance over time in themselves, but in the end the two together must.

### 2.4.2 Government

The government budget constraint at $t$ is

$$G(t) + T(t) + (1 + r(t))D(t-1) = D(t) + \sum_{h=hW}^{H} \gamma_h^q(t)N_h(t), \quad (18)$$

where $D(t)$ is government debt at the end of period $t$. Government consumption is denoted by $G(t)$.

Transfers at $t$ are determined according to

$$T(t) = tr(t) \sum_{h=hW}^{H} N_h(t) = y(t) tr \sum_{h=hW}^{H} N_h(t), \quad (19)$$

where $tr(t)$ is the transfer per recipient and $tr$ the assumed constant transfer per recipient as a share of GDP per capita. Observe that no individuals below age $hW$ receive any transfers. The above specification of $T(t)$ implies the growth in total transfers to be determined by the growth in output per capita as well as demographic changes.

### 2.4.3 Balancing the public sector budget over time

The public sector is defined as the compound of the NDC-system budget and the government budget. Combining the budget constraint for the pension system in expression (17) with that for the government in expression (18), an intertemporal budget constraint for the public sector is obtained

$$B(t) - D(t) + \sum_{h=hP}^{H} p_h^{ndc}(t)N_h(t) + G(t) + T(t)$$

$$= (1 + r(t))(B(t-1) - D(t-1)) + \sum_{h=hW}^{H} (\gamma_h^q(t) + \gamma_h^{ndc}(t))N_h(t). \quad (20)$$

72
The financial balance of the public sector as a whole is achieved by finding the tax on labor and pension earnings, \( \tau^e \), which balances equation (20) intertemporally so that the consolidated debt to output ratio, \( \frac{B(t) - D(t)}{Y(t)} \), is non-accelerating over time. This tax is valid for the whole simulation period, implying that all results are based on the tax policy experiment of changing the labor income tax at the beginning of the simulation.

3 Parameterization and calibration

3.1 Demography

The Social Ministry of Latvia has provided the demographic projection used. This projection ends in the year the 2050; until that time the population size and the mortality risks are taken directly from the projection. The cohort size is extrapolated beyond year 2050 by setting the mortality risks in each period after 2050 to their respective averages during the period 2041-2050. The number of newborn in each period after 2050 is set to 18 862. Mortality risks and the number of newborn are thus constant after 2050 and, as a consequence, the population will converge to a stationary state over time. It should be noted here that this demographic projection is very negative, almost extreme. For instance, from 1998 to 2050, the total population is projected to fall to 82 percent of its size in 1998 and 61.4 percent in the stationary state. The same figures for the labor force are 69.9 percent and 55.7 percent, which suggests that the underlying mortality risks used in the projection as well as the development of the number of newborn are questionable, which implies that only qualitative results can be drawn from the model. On the other hand, such a negative demographic scenario may very well serve as an extreme test of how well the NDC-system handles such a development.

Individuals start to work at the age \( h^w = 20 \). Accidental bequest are received at the age \( h^b = 45 \). The retirement age is legislated to be increased over time to the age 62, for both men and women. I will assume \( h^p = 62 \) to be the retirement
age from the beginning of the simulation. The highest possible age is \( H = 100 \).

### 3.2 The pension system and the government

The asset income tax rate, \( \tau^a \), is set to 25.0 percent, which is its legislated value. The consumption tax rate, \( \tau^c \), is calculated to 20.3 percent as an average of the indirect tax for the years 1998-2000. The indirect tax here consists of the VAT, and the excise and customs taxes. The total payroll tax is 33.0 percent, of which 20.0 percentage units are contributions to the public pension system, leaving an effective payroll tax rate, \( \tau^{pt} \), of 13.0 percent. The labor earnings tax rate, \( \tau^e \), is legislated to 25.0 percent but is endogenous in the model to balance the long-run budget of the public sector. The transfer to individuals, excluding early retirement and old-age pension expenditures, is set to \( tr = 12.0 \) percent of the GDP per capita, based on the average for the years 1999 and 2000. Included here are also the disability pension, the survivors’ pension and other pension related expenditures legislated to be paid over the NDC budget. By excluding these expenditures from the NDC-budget, it is isolated to handle pure old-age pension expenditures linked to previous earnings. This makes it easier to analyze the more fundamental dynamic features of the NDC-system.\(^4\)

The government consumption as a share of GDP is set to \( G(t)/Y(t) = 24.2 \) percent based on the average for the years 1998-2000.

The contribution rates to the respective parts of the public pension system are given in table 1. Individuals born in 1951 and earlier are only covered by the NDC-scheme. Their pensions assets accrued within the old system are converted to NDC-assets upon retirement, and are not subject to earnings taxation. Individuals born in the period 1952-1970 have the choice of participating in the defined contributions scheme. They may choose to let all their contributions go into the PAYG system. I will assume that all individuals will participate in the DC-scheme, since these assets will have a higher long-run accrual rate than the

\(^4\)In total, the transfers shifted to the governmental budget constitute approximately 2 percent of GDP in the year 2000. In order to get a proper picture of the actual development of the NDC - budget, these expenditures must be added to the old-age pension expenditures.
NDC-assets and consequently, would be the rational choice.

Table 1: Contribution rates to the NDC- and DC-systems.

<table>
<thead>
<tr>
<th>Year</th>
<th>NDC</th>
<th>DC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999</td>
<td>20.0%</td>
<td>0.0%</td>
</tr>
<tr>
<td>2000</td>
<td>20.0%</td>
<td>0.0%</td>
</tr>
<tr>
<td>2001*</td>
<td>19.0%</td>
<td>1.0%</td>
</tr>
<tr>
<td>2002</td>
<td>18.0%</td>
<td>2.0%</td>
</tr>
<tr>
<td>2003</td>
<td>18.0%</td>
<td>2.0%</td>
</tr>
<tr>
<td>2004</td>
<td>18.0%</td>
<td>2.0%</td>
</tr>
<tr>
<td>2005</td>
<td>18.0%</td>
<td>2.0%</td>
</tr>
<tr>
<td>2006</td>
<td>18.0%</td>
<td>2.0%</td>
</tr>
<tr>
<td>2007*</td>
<td>17.0%</td>
<td>3.0%</td>
</tr>
<tr>
<td>2008</td>
<td>14.0%</td>
<td>6.0%</td>
</tr>
<tr>
<td>2009</td>
<td>11.5%</td>
<td>8.5%</td>
</tr>
<tr>
<td>2010</td>
<td>10.5%</td>
<td>9.5%</td>
</tr>
<tr>
<td>2011</td>
<td>10.0%</td>
<td>10.0%</td>
</tr>
</tbody>
</table>

* Changes are legislated to take place mid-year, from e.g. 20 to 18 percent. When this occurs, the average rate for that year is used.

The per capita pension levels of current retirees at the beginning of the simulation are calculated from official data. This also includes early retirement expenditures for individuals younger than 62, paid over the NDC-budget. These expenditures are phased out as the generations reach the age of 62.

The gains of inheritance are defined as the capital of individuals who die during the period. In a defined contributions system, this capital is usually distributed among the other insured and consequently, amounts to an additional gain for the surviving individuals. In the Latvian system, these gains are instead transferred to the government budget. The gains of inheritance in the NDC-system cannot be transferred in the same way since they are based on notional capital. Instead, they are used to generate a surplus tendency within the NDC-system by not being distributed among the insured at all. In the steady-state, this would imply a yearly surplus for the NDC-system that would last forever. To avoid this problem, I assume that the gains of inheritance will start to be distributed from the year 2050 thus ensuring that the NDC-budget will be
period-wise budget balanced in the steady-state.

The NDC-assets are subject to an indexation based on the wage-sum growth rate. But to create a surplus tendency, the legislators have introduced a reduced pension indexation. They are thus indexed with a quarter of the wage-sum growth from 2002-2009; up to 2002 there is no real indexation at all. From 2010, the pensions are to be indexed by a half the wage-sum growth, which will create a strong surplus tendency in the steady-state. To avoid this, I assume the pensions to be indexed with the full wage-sum growth from 2050 for the system to balance in the steady-state.

3.3 Preferences, technology and age-efficiency

The intertemporal elasticity of substitution, $\theta$, is set to 0.5 and the parameter $\rho$ is set to 0.25. The time preference parameter $\beta$ is set to 0.98. Capital is assumed to depreciate by 10.0 percent per year, $\delta = 0.10$. The world-market growth rate of labor augmenting technological progress, $\gamma$, is kept constant at a level of 1.5 percent per annum. The results for 1 and 2 percent growth rates are also reported. The speed of convergence is set to 2 percent per annum, based on Barro and Sala-I-Martin (1995). The results for 1 and 4 percent are also reported. A 2 percent convergence speed per annum corresponds to a reduction in the productivity level gap by half in 28 years. For the 1 percent and 4 percent convergence assumptions, the distance is halved in 55 and 15 years, respectively. Due to lack of data, the age-efficiency vector is approximated by a Swedish age efficiency index.

3.4 Initial conditions and factor prices

The initial population is the 1998 distribution of natives. Whereas the initial government debt level is $D/Y = 13.0$ percent of GDP, the NDC-system is assumed to start with no assets in the pension fund, $B/Y = 0$. The initial distribution of NDC-assets, from 1998, has been provided and calculated by the Social Ministry of Latvia. Household wealth at the outset of the simulation
is assumed to be low. I will use a wealth to output ratio of 1.5 as an initial condition, but results for an output ratio of 2 will also be included.

The initial wealth distribution is set to the steady-state distribution outcome for a simulation with no defined contributions system and where all pension contributions finance the NDC-system. The labor income tax rate is here calibrated to finance a debt to output level of 13 percent, which is the 1998 debt level.

The interest rate is set to a constant 5 per cent per annum. The net rate after tax is then 3.75 percent. An interest rate of 4 percent and 6 percent per annum, is also used. The initial relative productivity level, \( z_W(s)/z(s) \), is set to 7. This figure has been approximated by the relative size of GDP per individual in the age span 15-64 for the OECD and Latvia. It turns out that the Latvian level is approximately 14 percent of the OECD level.\(^5\)

4 Results

This section contains a description of the main scenario, where the results for the steady-state as well as the transition are discussed. At the end of the section, the robustness of the results and their sensitivity to certain key parameters are discussed in the light of some alternative scenarios.

4.1 The main scenario

The public debt in the final steady-state is 93.7 percent of GDP, as compared to the initial level of 13.0 percent. For this debt not to accelerate over time, the tax on labor earnings and pensions, \( \tau_e \), must be 22.0 percent. Using labor income as the tax base, the total tax rate on labor income is 35.5 percent, adding 16.1 percent in pension contributions. In year 2000 present value, the size of the NDC-pension fund in the steady-state amounts to 103.7 percent. Since there is no initial pension fund at the beginning of the simulation this, value represents the sum of the discounted primary savings of the system. The high value indi-

\(^5\)The figure when comparing the wage rates is of the same size.
icates some sort of long-run distortion between revenues and expenditures. As described later, this is, to a large extent, explained by the expenditure reducing rules of the system.

In the model, the NDC-system only finances old-age pensions. Expenditures for the disability pension and the survivors’ pension and other pension related expenditures are assumed to be financed over the governmental budget. In the new legislation passed, these expenditures are to be included in the NDC-budget. The year 2000 present value of these expenditures for the period 2000-2050 is 100.3 percent of GDP. Consequently, almost the entire surplus generated in the NDC-system is used to finance expenses not related to old-age social security.

4.1.1 The steady-state

Government transfers in the steady-state amount to 6.8 percent of GDP, which is the same as the initial level. Total pension payments are 20.7 percent of GDP, where 15.3 percent are defined contributions. Thus, on the contribution side, the defined contribution part constitutes 50.0 percent of total pension contributions, whereas on the expenditure side, it constitutes almost three quarters of the pension. These figures are based on a difference in the annual rate of return between the two pension systems of 4.5 percent in the steady-state (5 percent in the DC-system and 1.5 percent in the NDC-system).

In figure 2, the implicit tax rates on labor earnings generated by the NDC and DC-systems are plotted together with the total. The effect of the high contribution rate on the defined contribution system is clearly shown as the total implicit tax rate is found to be negative over the whole life-cycle. Savings in the DC-system are subsidized as compared to private savings, since assets in the DC-system are not subject to asset income taxation. This results in an implicit tax of −11.7 percent of labor earnings for a 20 year old. The tax rate will increase with age and at 61 it is −3.3 percent. The implicit tax in the NDC-system is positive and 5.6 percent for a 20 year old, whereas it is only 0.2 percent for a 61 year old, as compared to a contribution rate of 10.0 percent.
In total, this results in a negative implicit tax for the public pension system as a whole.

The negative implicit tax means that the present value of the public pension generated by contribution is higher than the contribution itself. Thus, from a wealth point of view, the individual prefers savings in the public pension system to private savings at the going market rate. But wealth accumulated in the pension system and private life-cycle savings are not interchangable. Pension wealth is a discounted sum of yearly pension payments and can, as such, not be used to smooth consumption. This is not a problem as long as the individual can attain her optimal level of total asset holdings during the life-cycle by offsetting the forced savings in the public system by adjusting the level of private savings. But since individuals are capital constrained, they can only offset their forced savings up to a certain point, i.e no negative private assets are allowed. In the steady-state individuals accumulate no private assets at all and will be constrained to a zero level of private assets during the whole life-cycle. Their entire wealth at any given age exists as future pension annuities. Unless the
forced savings in the public system coincide with the individual's optimal level, the public system forces the individual to save more than that level. Thus, even though saving within the public system seems like a good deal, the downside is a substantial overprovision of pension, thereby limiting the scope of consumption smoothing. From a welfare perspective, this suggests the total contribution rate to the public pension system to be too high. It is difficult to estimate the negative welfare effect of the overprovision of pensions. A sufficiently large reduction in the contribution rate would imply the capital constraint not to be binding over the entire life cycle, but this is not the only effect. The implicit tax rate would also change. In the case of proportional reduction in both the NDC and the DC-contribution rate, the total implicit tax rate would be less negative. That is, savings in the public pension system would be less subsidized and consequently, the individual would be worse off. Furthermore, with a change in individual behavior, the labor earnings tax rate balancing the public sector budget would also change, which has an ambiguous effect on welfare.

4.1.2 The transition

Figure 3 shows some selected growth rates. The rate of return on assets in the NDC-system is the growth rate in the wage-sum which, in turn, coincides with the GDP growth due to the use of Cobb-Douglas technology with a fixed capital income share. Furthermore, in the short run, the change in pension expenditures can be approximated by the indexation of pensions. Thus, by comparing the rate of return on NDC-assets to the indexation of pensions, it can be deduced that the NDC-pension expenditures as a share of GDP will initially fall since output grows faster than pension expenditures. Up to 2049, the returns on NDC-assets are higher than the indexation of pensions, which indicates that the legislated reduced indexation of pensions constitutes a significant factor for reducing the pension expenditures as a share of GDP. From 2050, the indexation of pensions follows the wage-sum growth rate.
The rate of return on assets in the NDC-system is high in the first 15 years, which is largely explained by the high growth rate in the per capita wage following from the high productivity growth in this period. To a limited degree, the labor supply also increases which further adds to the rate of return on NDC assets. From the labor force growth rate, it can be seen that the size of the labor force is falling in almost the entire simulation period; on average, it falls by 0.5 percent per annum from 1998 to its stationary state.

The government primary savings are depicted in figure 4. On the expenditure side, government consumption is fixed to 24.2 percent of GDP and transfers only change marginally. This implies that basically all dynamics in the government primary savings are due to changes in the revenues. Taxes on labor and pension earnings increase from 13.1 percent of GDP to 16.5 percent over a 50-year period, driven by an increase in the pension expenditures as a share of GDP. The initial fall in the primary savings is due to a fall in consumption tax revenues. The primary savings level out at 2.6 percent of GDP, this surplus is used for debt service.
The primary savings of the NDC-system are shown in figure 5, the expenditures and the contributions to the system are also included.

Observe here that the pension expenditures only include old-age pension
payments and also some early retirement expenditures, inherited from the old system. The early retirement expenditures are phased out as the generations receiving them turn 62 and instead receive old-age pension. The pension expenditures do not include expenditures for disability pension, survival pension and other pension expenditures. These are here financed through the government budget.

The revenues fall from about 11.5 percent of GDP to 5.4 percent, due to the reduction in of the contribution rate to the NDC-system. As discussed earlier, the expenditures initially fall as a share of GDP, due to the relatively high growth rate of GDP as compared with the indexation of pensions. With the exception of the first 15 years, the primary savings of the NDC-system are lower than 1 percent of GDP in absolute terms. In the light of the exceptional decline in the labor force and the population, this indicates a good adjustability to the demographic development.

The assets of the public system are plotted in figure 6. Disregarding an initial period of increase, the public sector assets decrease in the long-run and stabilize on a level of −93.8 percent of GDP. This decrease is due to the negative primary savings in the government budget.

Figure 6: Public sector assets. Share of GDP.
Figure 7 depicts the primary savings of the DC-system.

The increase at the beginning of the simulation is due to the increase in the contribution rate up to the year 2010. The primary savings continue to increase until 2013, when the DC-pensions are paid for the first time. It takes about 75 years for the system to mature. At that point in time the DC-assets to output ratio will be slightly below 3. The effect of the high contribution rate to the public system is illustrated in figure 8 where the distribution of the household assets is shown. The public pension system crowds out private life-cycle savings. The individuals' wealth at any given age is locked into pension annuities. The private assets fall drastically during the first decade of the simulation, when individuals adjust to the new rules by consuming their wealth. By 2017, the last of the privately owned assets will have been consumed. It should be noted here that, the implicit assets from the NDC-system should be added on top of the assets in the defined contribution system. During the period 2035-2087, the private assets are positive, which coincides with a period of relatively low rates.
of return in the NDC-system. The lower rate of return reduces the value of the implicit assets and individuals thus accumulate private assets to smooth their consumption.

4.2 Robustness and sensitivity

In order to see how robust and sensitive the results in the main scenario are to the parameter assumptions, the results from a bundle of alternative scenarios where some of the key parameters have been altered are presented below. Table 2 summarizes the results from the different scenarios. The name of each scenario indicates the change from the main scenario. Observe here that the year 2000 present value of the steady-state NDC-pension fund coincides with the discounted present value of the primary savings in the NDC-system, and as such, measures the discrepancy between revenues and expenditures in the NDC-system.

The NDC-system seems to be fairly robust to the interest rate assumptions as well as to the choice of growth rate in the labor-augmenting technology.
Assuming an interest rate of 4 percent results in a 9.3 percentage units higher year 2000 present value of the steady-state pension fund. Reducing the interest rate to 4 percent does not result in any increase in private assets; individuals try to fully offset public savings by having no private assets at all. An interest rate of 6 instead of 5 percent gives a present value that is 6.8 percentage units lower. Changing the interest rate has a direct effect on the DC-system. A one-percentage unit increase (decrease) in the interest rate increases (decreases) the DC-expenditures as a share of GDP by approximately 5 percentage units.

Table 2: Comparing different scenarios.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Earnings tax</th>
<th>Year 2000 present value of the steady-state buffer fund.</th>
<th>DC expenditures</th>
<th>Foreign investment.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Share of GDP</td>
<td>Share of GDP</td>
<td>Share of GDP</td>
<td>Share of GDP</td>
</tr>
<tr>
<td>Main</td>
<td>22.0%</td>
<td>103.7%</td>
<td>15.3%</td>
<td>13.0%</td>
</tr>
<tr>
<td>Interest rate = 4</td>
<td>22.4%</td>
<td>113.0%</td>
<td>11.1%</td>
<td>-46.7%</td>
</tr>
<tr>
<td>Interest rate = 6</td>
<td>21.6%</td>
<td>96.9%</td>
<td>21.1%</td>
<td>64.1%</td>
</tr>
<tr>
<td>Growth rate = 1</td>
<td>21.8%</td>
<td>91.6%</td>
<td>18.3%</td>
<td>32.3%</td>
</tr>
<tr>
<td>Growth rate = 2</td>
<td>22.1%</td>
<td>114.9%</td>
<td>12.8%</td>
<td>13.5%</td>
</tr>
<tr>
<td>Convergence speed = 1</td>
<td>21.3%</td>
<td>63.0%</td>
<td>15.3%</td>
<td>0.0%</td>
</tr>
<tr>
<td>Convergence speed = 4</td>
<td>22.2%</td>
<td>158.6%</td>
<td>15.3%</td>
<td>16.0%</td>
</tr>
<tr>
<td>Initial wealth to output ratio = 2</td>
<td>21.7%</td>
<td>103.3%</td>
<td>15.3%</td>
<td>6.1%</td>
</tr>
<tr>
<td>No expenditure reducing legislations</td>
<td>24.3%</td>
<td>-17.8%</td>
<td>15.3%</td>
<td>53.0%</td>
</tr>
</tbody>
</table>

The changes in the present value when altering the growth rate are largely due to the reduced indexation of pensions. The lower the growth rate, the smaller the gain from reducing the indexation. This is especially evident when viewing the results where the convergence speed has been altered. A rapid convergence towards the world market technology level implies relatively high growth rates during the first 50 years of the simulation and consequently, relatively large gains from reducing the indexation of pensions. A 4 percent conver-
gence speed implies a pension fund present value, as a share of GDP, which is 95.6 percentage units higher than in the 1 percent convergence speed scenario.

The level of initial wealth has a limited effect on the results. This can be explained by the very quick adjustment at the beginning of the simulation of the private assets to the rules of the new pension system. Irrespective of its size, most of the initial wealth is rapidly consumed.

In one scenario, there are no expenditure reducing rules. That is, the indexation of pensions follows the wage-sum growth during the whole simulation period and the gains of inheritance are distributed from the beginning of the simulation. Comparing the results from this scenario with the main scenario, it is possible to approximate how large the gains from the expenditure reducing rules are. The present value of the steady-state pension fund in this alternative scenario is –17.8 percent of GDP, which is 121.5 percentage units lower than in the main scenario. This suggests that the legislated rules have a considerable effect on the expenditures and significantly improve the financial balance of the NDC-system. The reduction in pension expenditures is also sufficiently large to affect the public sector budget, resulting in an earnings tax that is 2.3 percentage units lower with than without the rules.

5 Conclusion

The newly reformed Latvian pay-as-you-go system seems to adjust well to the demographic development. In the light of the demographic projection used, with an exceptional decline in the labor force and the population in the long-run, the system must be said to perform remarkably well. The expenditure decreasing legislations, the reduced indexation of pensions and the non-distribution of the gains of inheritance, have a very strong effects. The reduction in expenditures, expressed in year 2000 present value terms, is 121.5 percent of GDP. The year 2000 present value of the pension fund in the steady-state is 103.7 percent of GDP. Thus, most of the reduction in expenditures is accumulated in the pension fund, which suggests that the expenditure reducing rules are too ample
when it comes to handling the balance of the system when only old-age pension expenditures are included. It should be noted that the NDC-system is legis­lated to finance expenditures outside old-age social security, which are financed over the governmental budget in the model used here. The year 2000 present value of these expenditures for the period 2000-2050 is 100.3 percent of GDP. Consequently, almost the entire surplus generated in the NDC-system by the expenditure reducing rules is used to finance expenses not related to old-age social security.

The result does not seem robust to the choice of convergence speed. A lower convergence speed significantly reduces the year 2000 present value of the steady-state pension fund, since a lower convergence speed reduces the wage-sum growth and consequently, reduces the gains from having a reduced indexation of pensions.

The high contribution rate to the DC-system results in a negative and increasing implicit tax for the entire life cycle in the defined contributions system. The implicit tax in the NDC-system is positive and decreasing. For the public system as a whole, the implicit tax is negative and increasing over the life cycle, that is, saving in the public pension system is subsidized as compared to saving at the going market rate.

In the steady-state, and during most of the transition period, individuals accumulate no private assets at all and are constrained to zero private asset holdings in the whole life cycle. Their entire wealth at any given age is contained in future pension annuities. Unless the forced savings in the public system coincide with the individual’s optimal level, the public system forces her to save more than that optimal level. Thus, even though saving within the public system seems like a good deal due to the negative implicit tax, the downside is that this results in a substantial overprovision of pension, which limits the scope of consumption smoothing. From a welfare perspective, this suggest that the total contribution rate to the public pension system is too high. Further research includes a more elaborate model with bequest motives, age dependent transfers, a flexible retirement age and intragenerational heterogeneity.
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