A list of Working Papers on the last pages

No. 268, 1990
EQUILIBRIUM IN SEARCH MODELS WITH ADVERSE SELECTION
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November, 1990
Abstract:

This paper examines the problem of nonexistence of equilibrium in a simple search model with asymmetric information. A pure-strategy, symmetric Nash equilibrium fails to exist because adverse selection arising from steady-state considerations causes a nonconcavity in the payoff function.
1. Introduction

In this paper we explore an interesting feature of many equilibrium search models with asymmetric information: pure-strategy, symmetric Nash equilibria fail generally to exist. The problem is one of nonconcavity of the payoff functions caused by adverse selection, similar to the source of nonexistence in the Rothschild-Stiglitz [1976] model of competitive insurance markets. Although the connection between adverse selection and nonexistence is well-understood in the insurance market and signaling contexts, the link has not been appreciated in the search literature.

We present a simple equilibrium search model that illustrates the nonexistence problem. This problem arises in our model because of an informational asymmetry: workers differ according to the disutility of work effort, and these disutilities are workers' private information. In any putative steady-state equilibrium adverse selection occurs, causing the pool of unemployed workers to be biased towards those with higher effort disutilities. This occurs because workers with lower effort disutilities are more likely to have accepted job offers and exited the pool of unemployed. If all firms offer a common wage \( w^* \), this adverse selection implies that any single firm can profit by deviating from \( w^* \). Offering a higher wage increases the probability that the job is accepted, but makes the job less profitable when it is filled. A wage cut reverses the direction of these effects. The effects of a wage increase and a wage decrease on the profitability of a filled job are symmetric, but adverse selection causes the effect on the acceptance probability to be stronger for a wage increase. This creates a
nonconcavity in the firm's payoff function at \( w^* \); consequently a pure-strategy, symmetric Nash equilibrium cannot exist.

The next section presents the search model and the nonexistence result. We then discuss the interpretation of this result, consider its generality, and link it to the equilibrium search literature. Concluding remarks are given in the final section.

2. Nonexistence in a Simple Search Model

The model is set in continuous time. Workers live forever, discounting the future at rate \( r \). A worker can be either employed or unemployed. When employed, a worker puts forth effort at an exogenous rate \( e \). Effort is accounted in efficiency units in the sense that the rates at which output is produced and effort is put forth are equal. When employed at a job paying a wage of \( w \), a worker of type \( \theta \) realizes an instantaneous utility of \( w - \theta e \). The disutility of effort parameter \( \theta \) is distributed across workers according to the distribution function \( F(\theta) \) with support \([0,1]\), and we assume that the corresponding density \( f(\theta) \) is continuous. Jobs end at an exogenous separation rate \( \delta \), and a worker who separates goes back into the unemployment state. Workers without a job realize an instantaneous utility of \( b \), interpreted either as an unemployment benefit or as the value of leisure common to all workers. Job offers come to the unemployed at the exogenous rate \( \alpha \). This offer arrival rate is independent of search effort.

The only decision facing workers is whether to accept or reject any wage offer that might be received. This problem can be analyzed using two value functions: \( V(w,\theta) \), the value to a worker of type \( \theta \) of accepting a job offering
a wage of \( w \), and \( U(\theta) \), the value of unemployment to a worker of type \( \theta \). Using standard dynamic programming techniques, these value functions are:

1. \[
V(w, \theta) = \frac{w - \theta e}{r + \delta} + \frac{\delta}{r + \delta} U(\theta)
\]

2. \[
U(\theta) = \frac{b}{r + \alpha} + \frac{\alpha}{r + \alpha} \text{E}_{\text{max}}[V(w', \theta), U(\theta)],
\]

where the expectation in (2) is taken with respect to the distribution of wage offers \( H(w') \) across all vacancies. A worker accepts an offer of \( w \) iff \( V(w, \theta) \geq U(\theta) \); this is the standard reservation wage criterion for job acceptance.

Whether an offer of \( w \) is accepted depends on the applicant's \( \theta \). Let \( \theta_A(w) \) be defined by:

3. \[
V[w, \theta_A(w)] = U[\theta_A(w)].
\]

Applicants with \( \theta \leq \theta_A(w) \) accept an offer of \( w \); otherwise, the offer is rejected. Note that this critical value \( \theta_A(w) \) depends not only on the wage offer in question, but also on the distribution of wage offers extant in the market since this distribution enters into the determination of \( U(\theta) \).

We model firms as collections of independent jobs, so that firm decision-making can be analyzed on a job-by-job basis. Jobs can be added to or withdrawn from the market at any time, and entry and exit are both costless. A job in the market is either occupied or vacant; new jobs enter the market as vacancies. Whether vacant or occupied, a job in the market incurs fixed costs at the rate \( c \). When occupied, a job generates revenue at the rate \( e \).

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\(^1\)Let time be measured in intervals of length \( \Delta t \). Then:

\[
V(w, \theta) = \frac{1}{1 + r \Delta t} \left( (w - \theta e) \Delta t + \delta \Delta t U(\theta) + [1 - \delta \Delta t] V(w, \theta) + o(\Delta t) \right).
\]

In the first period the worker enjoys a utility of \( [w - \theta e] \Delta t \). At the end of that period the worker has separated with probability \( \delta \Delta t + o(\Delta t) \), in which case he or she obtains the value \( U(\theta) \); with probability \( 1 - \delta \Delta t + o(\Delta t) \) the worker retains the value \( V(w, \theta) \). The discount factor is in "end-of-period" terms. Multiplying both sides by \( 1 + r \Delta t \), canceling common terms, dividing by \( \Delta t \), and taking the limit as \( \Delta t \to 0 \) gives (1). All other value functions in the paper are derived analogously.
The decisions to be made about a job are: (i) should the job be in the market? and (ii) given that the job is in the market as a vacancy and an applicant is met, what wage should be offered? To examine these decisions we develop expressions for $R(w)$, the value of an acceptance at an offer of $w$; $B(w)$, the value of offering a wage of $w$; $B$, the value of meeting an applicant; and $\Pi$, the value of a vacancy.

Given that firms discount the future at the rate $r$ and that separations occur at the rate $\delta$,

$$R(w) = \frac{e^{-w-c}}{r+\delta} + \frac{\delta}{r+\delta}\Pi.$$  

Let $q(w)$ be the probability that a randomly drawn applicant accepts an offer of $w$. This acceptance probability is:

$$q(w) = F_u[\theta_A(w)],$$

where $F_u(\theta)$ is the distribution function of $\theta$ among the unemployed. Note that $F_u(\theta)$ will in general differ from $F(\theta)$, the uncontaminated distribution. The value of a wage offer of $w$ is then:

$$B(w) = q(w)R(w) + [1-q(w)]\Pi = q(w)[\frac{e^{-w-c-r\Pi}}{r+\delta}] + \Pi.$$  

The value of meeting an applicant is then simply:

$$B = \max_w B(w).$$

Finally, suppose that the arrival rate of job applicants is $\lambda$. Then the value of a vacancy is:

$$\Pi = \frac{-c}{r+\lambda} + \frac{\lambda}{r+\lambda}B.$$  

The entry/exit decision for jobs is straightforward. Entry will occur so long as $\Pi > 0$; exit will occur so long as $\Pi < 0$. In equilibrium $\Pi = 0$, so the applicant arrival rate is:

$$\lambda = c/B.$$
The firm chooses a wage to maximize the value, $B(w)$, given in (6). Consideration of the wage-setting decision requires that we examine the acceptance probability $q(w)$ in detail. This in turn requires that we characterize the distribution of $\theta$ among the unemployed, $F_u(\theta)$.

The density function of $\theta$ among the unemployed is:

$$f_u(\theta) = P[\theta = \theta | \text{unemployed}].$$

Therefore, by Bayes Rule:

$$f_u(\theta) = P[\text{unemployed} | \theta = \theta] \cdot P[\theta = \theta] / P[\text{unemployed}].$$

$$= P[\text{unemployed} | \theta = \theta] \cdot f(\theta) / u,$$

where $u$ is the aggregate unemployment rate. We use the steady-state condition for unemployment flows to compute $P[\text{unemployed} | \theta = \theta] = u(\theta)$, i.e., the unemployment rate among workers of type $\theta$. This steady-state condition requires that the flow of type $\theta$ workers out of employment and into unemployment must be balanced by the flow in the reverse direction. The flow from employment to unemployment is $\delta[1 - u(\theta)]$. The flow out of unemployment of workers of type $\theta$ consists of new hires. The offer arrival rate for these workers is $o u(\theta)$. To compute the flow of new hires this offer arrival rate is multiplied by the acceptance probability. Let $q^*(\theta)$ be the probability that a worker of type $\theta$ will find a randomly drawn wage offer acceptable. That is, $q^*(\theta) = P[V(w', \theta) \geq U(\theta)]$, where the probability calculation is now taken relative to the distribution of wage offers across vacancies $H(w')$. The flow of new hires is then $o q^*(\theta) u(\theta)$. Equating this with the flow into unemployment yields:

$$(10) \quad u(\theta) = \delta / [\delta + o q^*(\theta)].$$

The aggregate unemployment rate $u$ is derived by integrating the $\theta$-specific rates against the population density for $\theta$. Inserting the above into the Bayes
Rule expression for \( f_u(\theta) \) gives:
(11) \[ f_u(\theta) = u(\theta)f(\theta)/u - \frac{\delta}{\delta+aq^{*}(\theta)} f(\theta)/u. \]

Finally, the distribution function \( F_u(\theta) \) is derived by integration.

Now we are in a position to consider the possibility of a pure-strategy, symmetric Nash equilibrium, which in this model means that all firms tender a common wage offer \( w^* \) and that no firm has an incentive to deviate from \( w^* \). We will show that such an equilibrium cannot exist.

In order for \( w^* \) to be an equilibrium wage offer it must be the solution to the firm's problem, \( \max_w q(w) \frac{e-w-c}{r+\delta} \), given that all other firms offer \( w \). We show that this is impossible by examining the first derivative of this payoff function. The difficulty arises because \( q(w) \) has a kink at \( w = w^* \); i.e., the derivative, \( q_{w}(w) \), is discontinuous at \( w = w^* \). This discontinuity is such that the firm's payoff function is not concave.

For \( w^* \) to be the common equilibrium wage offer requires that the left- and right-hand side derivatives of the payoff function satisfy:

\[ q_{w}(w) \frac{e-w-c}{r+\delta} - \frac{q(w)}{r+\delta} \geq 0 \quad \text{for } w < w^* \]
\[ q_{w}(w) \frac{e-w-c}{r+\delta} - \frac{q(w)}{r+\delta} \leq 0 \quad \text{for } w > w^* \]

for all feasible \( w \) in the neighborhood of \( w^* \). If both derivatives are positive the firm can increase its profits by raising its wage offer above \( w^* \); if both are negative, the firm can increase its profits by lowering its wage offer below \( w^* \); if the LHS derivative is negative while the RHS derivative is positive, a movement in either direction increases profits. The only case consistent with \( w^* \) as an equilibrium, viz, a nonnegative LHS derivative and a

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2In considering the maximization problem we impose the long-run equilibrium condition \( \Pi = 0 \) in advance. This is simply a notational convenience.
nonpositive RHS derivative, is the one case that cannot occur in our model.

To prove this we need expressions for \( q(w) = F_u[\theta_A(w)] \) and for
\[
d\theta_A(w)
\]
assuming that all other firms are offering \( w^* \). That is, we need to characterize \( \theta_A(w), f_u(\theta), \) and \( F_u(\theta) \) under the common-wage assumption.

To derive \( \theta_A(w) \) we set \( V[w,\theta_A(w)] = U[\theta_A(w)] \), as specified by (3). The value \( U[\theta_A(w)] \) is computed using (2). For \( w \leq w^* \), \( V[w^*,\theta_A(w)] \geq U[\theta_A(w)] \); hence:
\[
U[\theta_A(w)] = \frac{r+\delta}{r+\alpha+\delta} \cdot \frac{b}{r} + \frac{\alpha}{r+\alpha+\delta} \cdot \frac{w^*-\theta_A(w)e}{r}.
\]
If \( w > w^* \), \( V[w^*,\theta_A(w)] < U[\theta_A(w)] \), so \( U[\theta_A(w)] = \frac{b}{r} \). Equating \( V[w,\theta_A(w)] \) and \( U[\theta_A(w)] \) then gives:
\[
\begin{align*}
\theta_A(w) &= \frac{w-b}{e} + \frac{\alpha}{r+\delta} \cdot (\frac{w-w^*}{e}) & w \leq w^* \\
&= \frac{w-b}{e} & w > w^*
\end{align*}
\]
To characterize \( f_u(\theta) \) under the assumption of a common wage offer \( w^* \) we use:
\[
\begin{align*}
q^*(\theta) &= 1 & \theta \leq \theta_A(w^*) \\
&= 0 & \theta > \theta_A(w^*).
\end{align*}
\]
Hence,
\[
\begin{align*}
u(\theta) &= \frac{\delta}{(\delta+\alpha)} & \theta \leq \theta_A(w^*) \\
&= 1 & \theta > \theta_A(w^*),
\end{align*}
\]
and,
\[
\begin{align*}
f_u(\theta) &= \frac{\delta}{\delta+\alpha} \cdot \frac{f(\theta)}{u} & \theta \leq \theta_A(w^*) \\
&= \frac{f(\theta)}{u} & \theta > \theta_A(w^*).
\end{align*}
\]
The distribution function \( F_u(\theta) \) is found by integrating \( f_u(\theta) \).
The LHS and RHS derivatives of the acceptance probability can now be computed:

\[(16) \quad q_w(w) = \frac{\delta}{\delta + \alpha} \frac{f[\theta_A(w)]}{u} \frac{r + \alpha + \delta}{(r + \delta)e} \quad w \leq w^* \]

\[= \frac{f[\theta_A(w)]}{u} \frac{e^{-w}}{e^w} \quad w > w^* \]

and the acceptance probability is:

\[(17) \quad q(w) = \frac{\delta}{\delta + \alpha} \frac{F[\theta_A(w)]}{u} \quad w \leq w^* \]

\[= \frac{\delta}{\delta + \alpha} \frac{F[\theta_A(w^*)]}{u} \frac{F[\theta_A(w)] - F[\theta_A(w^*)]}{u} \quad w > w^* \]

Let \(q_w^-(w^*) = \lim_{\epsilon \to 0} q_w(w - \epsilon)\) and \(q_w^+(w^*) = \lim_{\epsilon \to 0} q_w(w + \epsilon)\). For the pure-strategy equilibrium conditions to hold in a neighborhood of \(w^*\) requires that:

\[q_w^-(w^*)[e^{-w^*} - c] - \frac{q(w^*)}{r + \delta} \geq 0 \quad \text{and} \quad q_w^+(w^*)[e^{-w^*} - c] - \frac{q(w^*)}{r + \delta} \leq 0.\]

However, observe from (16) that both \(q_w^+(w^*)\) and \(q_w^-(w^*)\) are positive. Further, by the continuity of \(f(\theta)\), \(\lim_{\epsilon \to 0} f[\theta_A(w^* - \epsilon)] = \lim_{\epsilon \to 0} f[\theta_A(w^* + \epsilon)] = f(w^* - b/e)\), so, \(q_w^+(w^*) > q_w^-(w^*)\) follows from \((\frac{\delta}{\alpha} - \frac{r + \alpha + \delta}{r + \delta}) < 1\). In addition, we have \([e^{-w^*} - c] > 0; \) if this inequality did not hold, then \(w^*\) could never be a profitable wage offer. Then, \(q_w^+(w^*) > q_w^-(w^*)\) implies that

\[q_w^-(w^*)[e^{-w^*} - c] - \frac{q(w^*)}{r + \delta} < q_w^+(w^*)[e^{-w^*} - c] - \frac{q(w^*)}{r + \delta}.\]

In other words, the required inequalities cannot be satisfied.\(^3\)

\(^3\)The nonexistence argument also holds at the "corners." The lowest possible common wage offer that can be considered in our model is \(w^* = b\). At this wage offer, \(\theta_A(w^*) = 0, q(w^*) = 0,\) and an increase in \(w\) above \(w^*\), the only feasible direction, leads to a higher payoff. The highest possible common wage offer is \(w^* = e - c;\) payoffs for \(w > w^*\) are necessarily negative. At this highest possible common wage offer, the LHS derivative equals \(-q(w^*);\) ie, reducing the wage leads to a higher payoff.
3. Discussion

The model presented in the previous section is relatively simple and straightforward. The advantage of using such a transparent model is that it makes it possible to identify easily the factors responsible for nonexistence. In our model the crucial role of the concavity of the payoff function for the existence of pure strategy equilibrium, as emphasized in Dasgupta and Maskin [1986a,b], can be seen clearly.

In order that a symmetric pure-strategy equilibrium exist, the firm's maximum problem must have a fixed point. That is, there must be at least one $w^*$ that is its own best response. Adverse selection causes nonconcavity of the profit function at any commonly offered $w^*$, which in turn causes the breakdown of the fixed point argument.

The nature of the adverse selection can be seen in Figure 1. Starting with any continuous distribution of effort disutilities across workers, $f(\theta)$, the assumption of a common wage offer $w^*$ causes the distribution of $\theta$ among the unemployed to be discontinuous at $\theta_A(w^*)$. Workers with $\theta \leq \theta_A(w^*)$ are more likely to have accepted a wage offer and exited the pool of unemployed than are workers with $\theta > \theta_A(w^*)$. The fact that a worker is unemployed thus signals a relatively high probability that he or she is unwilling to work at $w^*$ or less. That is, $f_u^+(\theta_A(w^*)) > f_u^-(\theta_A(w^*))$, as shown in Figure 1.

This means that a small decrease in the wage offer below $w^*$ lowers the acceptance probability by less than a similar increase above $w^*$ would raise
Figure 1: Discontinuity in density $f_u(\theta)$
it; i.e., \( q^+_w(w^*) > q^-_w(w^*) \). This asymmetry in the derivative of the acceptance probability is what causes \( B^+_w(w^*) > B^-_w(w^*) \), i.e., the nonconcavity of the firm's payoff function. This is illustrated in Figure 2, which shows the firm's payoff as a function of \( w \) for a range of \( w^* \) offered commonly by all other firms. For \( w^* \) sufficiently low, \( B(w) \) has the characteristic suggested by Figure 2a; the function has a maximum above \( w^* \). For \( w^* \) sufficiently high, \( B(w) \) has a maximum below \( w^* \), as suggested by Figure 2c. For some intermediate value of \( w^* \), Figure 2b applies, and there are maxima to both sides of \( w^* \).

Finally, Figure 3 illustrates the implication for the fixed point argument. At the intermediate value of \( w^* \) shown in Figure 2b, the best-response mapping "jumps over" the \( 45^\circ \)-line. That is, the best-response correspondence is not convex-valued; consequently, Kakutani's Fixed Point Theorem cannot be applied.\(^5\)

Although we have used a specific, simple model to explore nonexistence, the problem is more general. In our model, nonexistence is robust with respect to the form of the utility function, the matching technology, the form of the distribution function, \( F(\cdot) \), etc. The basis for this assertion can be seen in the assumption, the derivative of \( \theta_A(w) \) is asymmetric because \( U(\theta) \) is decreasing in \( \theta \) for \( \theta \leq \theta_A(w^*) \) but constant for \( \theta > \theta_A(w^*) \).

\(^4\)Adverse selection, i.e., \( f_u[\theta^+_A(w^*)] > f_u[\theta^-_A(w^*)] \) ensures that \( q^+_w(w^*) > q^-_w(w^*) \), in spite of an effect that goes in the opposite direction, namely, that \( \frac{d\theta^+_A(w^*)}{dw} < \frac{d\theta^-_A(w^*)}{dw} \). Under the common-wage assumption, the derivative of \( \theta_A(w) \) is asymmetric because \( U(\theta) \) is decreasing in \( \theta \) for \( \theta \leq \theta_A(w^*) \) but constant for \( \theta > \theta_A(w^*) \).

\(^5\)If a symmetric, pure-strategy equilibrium is precluded, an obvious alternative to investigate is a mixed-strategy equilibrium. As discussed in Dasgupta and Maskin [1986a], the results of Glicksberg [1952] on the existence of mixed-strategy equilibria require only that the payoff functions be continuous; in particular, concavity is not required. Another possibility to consider is an asymmetric, pure-strategy equilibrium. Equilibria of the type considered by Wilson [1977] and Riley [1979] in the insurance market and signaling contexts might also be relevant.
Figure 2: Firm's Payoff as a Function of $w$ for various $w^*$

Figure 3
Figure 1. The fundamental source of nonexistence is the discontinuity in the density \( f_u(\theta) \) at \( \theta = \theta_A(w^*) \). This is caused by adverse selection and would not be affected by changes in the details of our model.

More fundamentally, the problem should obtain in any equilibrium search model with adverse selection in which an applicant's decision of whether to accept or reject a wage offer depends on his or her private information. Private information need not be interpreted solely in terms of the disutility of effort.

Note that worker private information by itself is not responsible for the nonexistence problem; adverse selection is also necessary. For example, in Albrecht and Jovanovic [1986] workers have private information, which is match-specific. Since the private information is not about an innate characteristic of the applicant, there is no problem of adverse selection, and a symmetric, pure-strategy equilibrium exists.

The nonexistence problem that we have examined has not, to the best of our knowledge, been appreciated in the equilibrium search literature. The reason is that equilibrium search is most often treated in a "one-shot" framework, so that the adverse selection problem we have identified as the source of the nonconcavity in firms' payoff functions does not come into play. The seminal paper on equilibrium price dispersion by Axell [1977] can be used to illustrate our point. (Rob [1985] uses a similar framework.) In Axell's model consumers differ in their search costs; this consumer heterogeneity is

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6The recent paper by Burdett and Mortensen [1989], which allows for search both on and off the job, is an exception. In their model the possibility that workers may costlessly receive information about job offers while employed rules out a symmetric equilibrium in wage offers. If all firms offer the same wage, then any one firm can profit by slightly exceeding that common level. Any increase above the common offer gives the deviant firm access to other firm's employees.
private information. Each firm sets a price, and consumers search sequentially until each has made a purchase. The analysis is "one-shot" in the sense that all search can be thought of as taking place instantaneously. Firms need not consider that searchers with high reservation prices tend to exit the market early on, leaving more selective searchers to make their purchases in later periods. If search took time in Axell's model, his firms would face a problem akin to that of a dynamic monopolist. Search does take time in our model, but our setup is not "one-shot." Instead, we move to the steady state and investigate the existence of symmetric stationary equilibria in which all firms offer a common wage, which is constant through time. Our point is that adverse selection precludes this possibility as an equilibrium outcome.

The fact that adverse selection rules out the existence of a single-wage equilibrium is a positive development for equilibrium search theory. In much of this literature the presumption is that the Diamond [1971] single-price monopoly equilibrium (or single-wage monopsony equilibrium) is the natural outcome. Dispersion equilibria can be generated by introducing heterogeneity among searchers and/or firms, but these typically coexist with the degenerate outcome. Our nonexistence result can thus be used to rule out the outcome that is contrary to the spirit of the equilibrium search exercise.

4. Conclusion

In this paper we have made two contributions. First, using an equilibrium search model we illustrated and clarified the role of nonconcavity of the payoff functions in the nonexistence of symmetric, pure-strategy equilibria as discussed in Dasgupta and Maskin [1986a,b]. The simplicity of our model makes it possible to understand easily why pure-strategy equilibria may fail to exist in models with adverse selection.
Our second contribution is to the equilibrium search literature. Contrary to the presumption that the symmetric, pure-strategy solution is the natural outcome in these models, either as the unique equilibrium or as an outcome coexisting with dispersion equilibria, we show that when the adverse selection arising from steady-state considerations is taken into account, such an equilibrium does not exist.

References: