Abstract

A labor market model is developed where there are search frictions in the formal sector whereas the informal sector is competitive. We show that there exists a unique steady-state equilibrium in this dual economy. We then consider different policies. We find that reducing the unemployment benefit in the formal sector or the firms’ entry cost in this sector induces more job creation but has an ambiguous effect on employment, unemployment and wages in both sectors. On the contrary, an employment/wage subsidy or a hiring subsidy policy has a clear positive effect on formal employment and wages in both sectors while a negative effect on informal unemployment.

Key words: Job matching, informal sector, policies, Todaro paradox.

JEL Classification: D83, J41, J64, O17.
1. Introduction

The informal sector is a pervasive and persistent economic feature of most developing economies, contributing significantly to employment creation, production, and income generation. Recent estimates of the size of the informal sector in developing countries in terms of its share of non-agricultural employment range roughly between one-fifth and four-fifths. In terms of its contribution to GDP, the informal sector accounts for between 25% and 40% of annual output in developing countries in Asia and Africa.¹

Some researchers in this area have tried to understand how an informal sector emerges. The usual argument put forward is that firms and workers join the informal sector in order to avoid taxation or any formal regulation from the government (see e.g. Rauch, 1991, or Loayza, 1996). Other researchers have focus on the implications of the existence of the informal sector on the economy. In particular, they have studied how the informal sector generates externalities (both positive and negative) for the formal economy (see e.g Marcoullier and Young, 1995, Dessy and Pallange, 2003, Fugazza and Jacques, 2003).

In the present paper, we focus on the labor-market aspects of the informal sector, especially the job search process of workers, and evaluate different policies aiming at reducing unemployment. What we show is that, even if the informal sector is unregulated and cannot be directly targeted by a government’s policy, the latter affects indirectly the wage, the employment and thus the size of the informal sector.

Our paper is related to the literature on rural-urban migration, initiated by the two seminal papers of Todaro (1969) and Harris and Todaro (1970). One of the main issues raised in this literature is that creating urban jobs may increase rather than decrease urban unemployment because of the induced negative effect on rural migration, which may outweigh the positive effect of creating jobs (Todaro, 1976). This is referred to as the Todaro paradox.² Even though, it is not the main issue of this paper, we also investigate the Todaro paradox in the context of formal and informal sectors.

We consider a search-matching model. There is a tradition of search models in the

¹For more empirical evidence and literature surveys on this issue, see Schneider and Enste (2000) and Schneider (2005).

migration literature. The early models were using the old search approach where only one side of the market (the workers) was modeled (see e.g. Fields, 1975, 1989, Banerjee, 1984, Mohtadi, 1989). There is also a more recent literature, which incorporates the search-matching approach a la Pissarides-Mortensen (Mortensen and Pissarides, 1999; Pissarides, 2000) in a Harris-Todaro model (see Coulson et al., 2001, Ortega, 2000, Sato, 2004, Laing et al., 2005). This is what we are using here and we believe that this is one of the few papers that uses a general equilibrium search-matching approach to deal with policies issues in developing countries.

To be more precise, we develop a model where there are search frictions in the formal sector whereas the informal sector is competitive. In the formal sector, the wage is determined by a bargaining between a worker and a firm and because of search frictions unemployment emerges in equilibrium. In the informal sector, wages are paid at the marginal productivity of workers and there is full employment. The informal sector is fully accessible for everybody while there is an entry cost both for firms and workers in the formal sector. Workers trade off the costs and benefits of the two sectors and optimally decide in which sector they want to work. We characterize the steady-state equilibrium of the economy and show that the equilibrium exists and is unique but not efficient because of search externalities. We then consider different policies. We find that reducing the unemployment benefit in the formal sector or the firms’ entry cost in this sector induces more job creation but has an ambiguous effect on employment, unemployment and wages in both sectors. However, under some condition, especially if the entry-cost elasticity of job creation is high enough, we show that this unemployment-benefit policy decreases employment in the informal sector and increases the informal wage and the employment in the formal sector. An employment/wage subsidy or a hiring subsidy policy has on the contrary a clear positive effect on formal employment and wages in both sectors while a negative effect on informal unemployment. Interestingly, in all the four policies considered, one can always find conditions such that a Todaro paradox exists, i.e. an increase or decrease in a policy instrument can increase both employment and unemployment.

2. Model and notations

There are two sectors in the economy: formal and informal. Everybody can obtain a job in the informal sector and it is thus assumed that the wage in the informal sector is flexible
enough to guarantee that there is full-employment; this wage is denoted by $w_L$. In the formal sector, there are search frictions that prevent workers and firms to find instantaneously a match and thus unemployment will prevail in equilibrium. There is a continuum of ex ante identical workers whose mass is $N$. Among the $N$ workers, $N^F$ and $N^I$ are in the formal and informal sector, respectively ($F$ and $I$ stand for formal and informal). We have $N = N^F + N^I$, and

$$N^F = L^F + U^F$$
$$N^I = L^I$$

where $L^S$ and $U^S$ is the total employment and unemployment levels in sector $S = F, I$ in this economy. Since there is no unemployment in the informal sector, $U^F$ is also the unemployment level in the economy. Thus, by combining these two equations, we obtain:

$$U^F = N - L^F - L^I$$ (2.1)

In the informal sector, we have the following production function:

$$y^I = F(L^I) \quad \text{with } F'(L^I) > 0 \text{ and } F''(L^I) \leq 0$$ (2.2)

We also assume that the Inada conditions hold, that is $\lim_{L^I \to 0} F'(L^I) = +\infty$ and $\lim_{L^I \to +\infty} F'(L^I) = 0$. The price of the good is taken as a numeraire and, without loss of generality, normalized to 1. As stated above, in the informal sector, jobs are mainly menial and wages are flexible and equal to marginal product, so that there is no unemployment. We thus have:

$$w^I_L = F'(L^I)$$ (2.3)

In the formal sector, each worker produces $y^F$ and it is assumed that $\forall L^I > 0, y^F > y^I$. There are search frictions and we use the standard search matching framework (Mortensen and Pissarides, 1999, and Pissarides, 2000) to model these frictions. The starting point is the following matching function

$$M(U^F, V^F)$$

where $V^F$ is the total number of vacancies in the formal sector. This matching function captures the frictions that search behaviors of both firms and workers imply. It is assumed
that $M(.)$ is increasing in its arguments, concave and homogeneous of degree 1. Thus, the rate at which vacancies are filled is $M(U^F, V^F)/V^F = M(1/\theta^F, 1) \equiv q(\theta^F)$, where

$$\theta^F = \frac{V^F}{U^F}$$

is the labor market tightness in the formal sector and $q(\theta^F)$ is a Poisson intensity. Similarly, the rate at which an unemployed worker leaves unemployment (job acquisition rate) is now given by

$$a^F = \frac{M(U^F, V^F)}{U^F} \equiv \theta^F q(\theta^F)$$

Time is continuous and workers live forever. We assume that changes in employment status are governed by a Poisson process in which $a^F$ is the (endogenous) job acquisition rate in the formal sector and $\delta$ the (exogenous) destruction rate. Let us denote by $r$ the common discount rate of all workers. Then, the standard steady-state Bellman equations for the employed and unemployed workers in the formal sector are respectively given by:

$$r I_L^F = w^F_L - \delta (I_L^F - I_U^F)$$

$$r I_U^F = w^F_U + \theta^F q(\theta^F) (I_L^F - I_U^F)$$

where $w^F_L$ and $w^F_U$ are the wage and the unemployment benefit in the formal sector (subscripts $L$ and $U$ stand respectively for employed and unemployed). By combining (2.6) and (2.7), we obtain:

$$I_L^F - I_U^F = \frac{w^F_L - w^F_U}{r + \delta + \theta^F q(\theta^F)}$$

For firms with a filled job (subscript $J$) and a vacancy (subscript $V$), we have the following steady-state Bellman equations:

$$r I_J = y^F - w^F_L - \delta (I_J - I_V)$$

$$r I_V = -\gamma + q(\theta^F) (I_J - I_V)$$

where $\gamma$ is the search cost for the firm and $y^F$ is the product of the match. Because of free entry, $I_V = 0$. From (2.10) and using $I_V = 0$, the value of a job is now equal to:

$$I_J = \frac{\gamma}{q(\theta^F)}$$

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3For simplicity, it is assumed that each firm only hires one worker.
Firms enter the labor market until the expected benefit $I_J$ is equal the expected cost $\gamma/q(\theta^F)$ (remember that, in a Poisson process, the inverse of the exist rate $q(\theta^F)$ expresses the average duration of a vacant job). Finally, plugging (2.11) into (2.9) and using $I_V = 0$, we obtain the following decreasing relation between labor market tightness and wages in equilibrium:

$$\frac{\gamma}{q(\theta^F)} = \frac{y^F - w_L^F}{r + \delta}$$

(2.12)

In words, the value of a job is equal to the expected search cost, i.e. the cost per unit of time multiplied by the average duration of search for the firm. So, firms’ job creation is endogenous and is determined by (2.12).

Let us now determined the wage. At each period, the total intertemporal surplus is shared through a generalized Nash-bargaining process between the firm and the worker. The total surplus is the sum of the surplus of the workers, $I^F_L - I^F_U$, and the surplus of the firms $I_J - I_V$. At each period, the wage is determined by:

$$w^F_L = \arg \max_{w^F_L} (I^F_L - I^F_U)^\beta (I_J - I_V)^{1-\beta}$$

(2.13)

where $0 \leq \beta \leq 1$ is the bargaining power of workers. First order condition gives:

$$\frac{\beta}{1-\beta} \left( \frac{\partial I^F_L}{\partial w^F_L} - \frac{\partial I^F_U}{\partial w^F_L} \right) I_J + (I^F_L - I^F_U) \frac{\partial I_J}{\partial w^F_L} = 0$$

(2.14)

Since the wage is negotiated at each period, $I_U$ does not depend on the current wage $w^F_L$ and so $\frac{\partial I^F_U}{\partial w^F_L} = 0$. Since by (2.6), $\frac{\partial I^F_L}{\partial w^F_L} = 1/(r + \delta)$, by (2.11), $I_J = \gamma/q(\theta^F)$ and by (2.9), $\frac{\partial I_J}{\partial w^F_L} = -1/(r + \delta)$, equation (2.14) can be written as:

$$I^F_L - I^F_U = \frac{\beta}{1-\beta} \frac{\gamma}{q(\theta^F)}$$

(2.15)

Then, using (2.8) and (2.12), we finally obtain the following wage:

$$w^F_L = (1 - \beta) w^F_U + \beta (y^F + \gamma \theta^F)$$

(2.16)

This is the wage-setting curve (a relation between wages and the state of the labor market, here $\theta^F$) that replaces, in search-matching models, the traditional labor-supply curve.
3. Steady-state equilibrium

In steady-state, flows in and out unemployment have to be equal and we obtain the following relationship in cities:

\[ L^F = \frac{\theta^F q(\theta^F)}{\delta + \theta^F q(\theta^F)} (N - L^I) \]  

(3.1)

We assume that a worker in the informal sector cannot search directly a job in the formal sector but must first be unemployed in the formal sector. One way to justify this assumption is the fact that, in developing countries, most jobs (at least for the uneducated) are found through word-of-mouth communication and social networks (see, e.g. Wahba and Zenou, 2005). So one has first to be in the formal sector to gather information about jobs. Another way is that formal and informal sectors are usually not located in the same part of the city. So one has first to move to the location where formal jobs are, and then, while unemployed search for a formal job. In some sense, the informal sector plays a buffer role in the transitional stage of a search for a formal sector job.

The equilibrium migration condition can thus be written as:

\[ r I^F = \int_{0}^{+\infty} w^F_L e^{-rt} dt = \frac{w^L_I}{r} \]  

(3.2)

Using (2.7), (2.8), and (2.3), this equality is equivalent to:

\[ \frac{(r + \delta) w^F_U + \theta^F q(\theta^F) w^F_L}{r + \delta + \theta^F q(\theta^F)} = \frac{F'(L^I)}{r} \]  

(3.3)

**Definition 1.** A Harris-Todaro equilibrium with urban search externalities and bargained wages is a 7-tuple \((w^F_L, \theta^F, w^I_L, L^F, U^F, V^F, L^I)\) such that (2.16), (2.12), (2.3), (3.1), (2.1), (2.4) and (3.3) are satisfied.

Here is the way the equilibrium is calculated. The system is recursive. First, by combining (2.16) and (2.12), we obtain a unique \(\theta^F\) that is only function of parameters and given by:

\[ (1 - \beta) (y^F - w^F_U) - \beta \gamma \theta^F = \frac{\gamma (r + \delta)}{q(\theta^F)} \]  

(3.4)

Second, by combining (2.16) and (3.3), we obtain:

\[ \frac{(r + \delta) w^F_U + \theta^F q(\theta^F) [(1 - \beta) w^F_L + \beta (y^F + \gamma \theta^F)]}{r + \delta + \theta^F q(\theta^F)} = \frac{F'(L^I)}{r} \]  

(3.5)
which using $\theta^{F*}$ gives a unique $L^{I*}$ as a function of parameters only. Furthermore, by plugging $\theta^{F*}$ and $L^{I*}$ in (3.1), we obtain a unique $L^{F*}$. Figure 1 illustrates the way the equilibrium is calculated.

\[ \text{[Insert Figure 1 here]} \]

Finally, by plugging $L^{F*}$ and $L^{I*}$ in (2.3) and (2.1), we obtain respectively $w^{I*}_L$ and $U^{F*}$ and by plugging $\theta^{F*}$ in (2.16), we obtain $w^{F*}_L$. Also, using the values of $\theta^{F*}$ and $U^{F*}$ in (2.4), we obtain the equilibrium number of vacancies in cities, $V^{F*}$.

There exists thus a unique steady-equilibrium and it is not efficient because of search externalities (as in Pissarides, 2000). So we would like now to consider different policies that aim at reducing unemployment.

4. Policies

The policies that we consider are implemented in the formal sector. We do not focus on how to finance these policies but rather, given that a certain amount of money is to be spent (say for example some funds from an international organization), which policy has an effect on employment and unemployment and which mechanism prevails.

4.1. Reducing unemployment benefits

We consider a first simple policy where the government reduces the unemployment benefit in the formal sector. We have the following result:4

**Proposition 1.** In an Harris-Todaro model with urban search externalities and bargained wages, a decrease in unemployment benefits $w^{F}_U$ leads to:

(i) an increase in job creation in the formal sector $\theta^{F}$;

(ii) a decrease in employment in the informal sector $L^{I}$ and an increase in the informal wage $w^{I}_L$ if

$$\frac{1}{\beta} \left[ 1 + \frac{y^{F} - w^{F}_L}{\gamma \theta^{F}} \right] + \gamma \frac{\partial \theta^{F}}{\partial w^{F}_U} < 1$$

(4.1)

holds. Otherwise, the effect is ambiguous.

4The proofs of all propositions can be found in the Appendix.
(iii) an increase in employment in the formal sector $L^F$ if (4.1) holds. Otherwise, the effect is ambiguous.

(iv) has an ambiguous effect on the unemployment level in the economy $U^F$.

(v) has an ambiguous effect on wages $w^F_L$.

When the government reduces the unemployment benefit, it has a direct positive effect on job creation in the formal sector $\theta^F$. Indeed, firms holding a vacant job are forward looking and thus there are more of them that enter the labor market because they anticipate that it will be less costly in terms of wages to hire a worker. Because $\theta^F$ increases following a decrease in unemployment benefits, the *equilibrium wage* will not necessarily decrease because of the indirect effect of $\theta^F$. Indeed, since $\theta^F$ increases, it becomes easier to find a job for workers and thus the latter can increase their wages during the bargaining process because their outside option is better. It is easy to verify that:

$$
\frac{\partial w^F_L}{\partial w^F_U} = \frac{1-\beta}{\beta} + \beta \gamma \frac{\partial \theta^F}{\partial w^F_U}
$$

Concerning the effect of $w^F_U$ on both formal and informal employment, the intuition is as follows. There are again two effects. Remember that equation (3.5) was determining the mobility between the two sectors, and it was given by $r I^F_U = w^I_L / r$. So, when $w^F_U$ decreases, there is a *direct positive (resp. negative) effect* on $L^I$ (resp. $L^F$) since $I^F_U$, the lifetime expected value of being unemployed in the formal sector, decreases, and thus more individuals are willing to work in the informal sector. There is also an *indirect negative (resp. positive) effect* on $L^I$ (resp. $L^F$) since the increase in job creation $\theta^F$ yields a higher value of $I^F_U$ become it becomes easier to find a job. As a result, the net effect is ambiguous. However, if (4.1) holds, which expresses the fact that the indirect effect is much stronger than the direct effect, then $L^I$ decreases and $L^F$ increases following a decrease in the unemployment benefit. Of course, because wages in the informal sector are competitive and equal marginal productivity, an increase in the unemployment benefit will have the opposite effect on $w^I_L$. The effect of $w^F_U$ on the unemployment $U^F$ is ambiguous even if (4.1) holds because the effects go in the opposite directions. Observe that here, there is a possibility for a Todaro paradox, that is a decrease in unemployment benefit can increase both urban employment and unemployment if both condition (4.1) and
\[
\frac{\partial L^F}{\partial w^F_U} > \frac{\partial L^I}{\partial w^F_U}
\]  
(4.2)

hold.

4.2. Reducing firms’ entry costs

Another interesting policy to be considered is to reduce the entry cost \( \gamma \) for firms in the formal sector. Define \( \eta \equiv -\frac{\partial \theta^F}{\partial \gamma} \frac{\gamma}{\theta^F} > 0 \), which is entry-cost elasticity of job creation. We have the following result.

**Proposition 2.** In an Harris-Todaro model with urban search externalities and bargained wages, a decrease in \( \gamma \), the firms’ entry cost in the formal sector, leads to:

(i) an increase in job creation in the formal sector \( \theta^F \);

(ii) a decrease in employment in the informal sector \( L^I \) and an increase in the informal wage \( w^I_L \) if \( \eta > 1 \). Otherwise, the effect is ambiguous.

(iii) an increase in employment in the formal sector \( L^F \) if \( \eta > 1 \). Otherwise, the effect is ambiguous.

(iv) an ambiguous effect on the unemployment level in the economy \( U^F \).

(v) an ambiguous effect on wages \( w^F_L \).

As with the unemployment benefit, when the government reduces firms’ entry cost in the formal sector, more firms holding a vacant job enter the labor market and therefore more jobs are created (\( \theta^F \) increases). The effect on the *equilibrium wage* is however ambiguous because of the indirect negative effect of \( \theta^F \). Furthermore, when \( \gamma \) increases, contrary to the unemployment benefit policy, there is *no* direct effect on \( L^I \) or \( L^F \) since \( I^F_U \), the lifetime expected value of being unemployed in the formal sector, is not directly affected by \( \gamma \). There is however an *indirect effect* through the increase of the job creation \( \theta^F \), which affects both the rate at which workers find a job in the formal sector and their wage in case of a match. The net effect is ambiguous because, as we have seen above, the effect on wages is ambiguous. If however \( \eta > 1 \), which means that (in absolute value) the effect of \( \gamma \) on \( \theta^F \) is important,
then quite naturally, a decrease in the entry cost increases employment in the formal sector and decreases it in the informal sector.

Overall, the effect of $\gamma$ on the different equilibrium values is similar to that of $w^F_U$, with the difference that $\gamma$ does not affect directly $I^F_U$, the mobility decision between the two sectors, but indirectly through job creation.

### 4.3. Employment/wage subsidies

We now consider a policy where employment is subsidized at a rate $S^F > 0$ per job and the employment subsidy $S^F$ is paid to firms throughout the duration of the job. In that case, equation (2.9) is changed and becomes

$$rI_J = y^F - w^F_L + S^F - \delta(I_J - I_V)$$

Observe that it is the firm who receives the subsidy and not the worker so that

$$rI_L = w^F_L - \delta(I_L - I_U)$$

It is clear here that $S^F$ can be interpreted as either an employment or a wage subsidy. Let us now solve the model with the subsidy. The value of a job is still given by

$$I_J = \frac{\gamma}{q(\theta^F)}$$

but the job creation equation is modified and now given by:

$$\frac{\gamma}{q(\theta^F)} = \frac{y^F - w^F_L + S^F}{r + \delta}$$

It is easy to verify that the wage is now equal to:

$$w^F_L = (1 - \beta) w^F_U + \beta (y^F + S^F + \gamma \theta^F)$$

The steady-state equilibrium is now defined by

$$(1 - \beta) (y^F - w^F_U + S^F) - \beta (y^F + \gamma \theta^F) = \frac{\gamma (r + \delta)}{q(\theta^F)}$$

which implicitly determines $\theta^F$. Furthermore, $L^I$ is given by:

$$\frac{(r + \delta) w^F_U + \theta^F q(\theta^F) [(1 - \beta) w^F_U + \beta (y^F + S^F + \gamma \theta^F)]}{r + \delta + \theta^F q(\theta^F)} = \frac{F'(L^I)}{r}$$

Finally, the employment $L^F$ and unemployment $U^F$ in the formal sector are defined as before by equations (3.1) and (2.1), respectively. We have the following result:
Proposition 3. In an Harris-Todaro model with urban search externalities and bargained wages, an increase in the employment/wage subsidy $S^F$ in the formal sector, leads to:

(i) an increase in job creation in the formal sector $\theta^F$;

(ii) a decrease in employment in the informal sector $L^I$ and an increase in the informal wage $w^I_L$.

(iii) a increase in employment in the formal sector $L^F$.

(iv) an ambiguous effect on the unemployment level in the economy $U^F$.

(v) an increase in wages $w^F_L$.

The employment/wage subsidy policy is quite different from the previous ones. First, it has a direct positive effect on job creation since forward-looking firms are more willing to create new jobs because the cost of hiring a worker is lower. Second, contrary to the previous policies, the effect of $S^F$ on wages $w^F_L$ is unambiguously positive. Indeed, when $S^F$ increases, there is a direct positive effect on wages since workers have a better outside option given that the value of a job is less costly for a firm. There is also an indirect positive effect since increasing $S^F$ increases job creation $\theta^F$, which means that workers find more easily a job, and thus their outside option in the bargaining process is increased. Third, since wages and job creation increase following an increase in the subsidy, both the surplus $I^F_U - I^F_U$ of being employed and $\theta^F q(\theta^F)$ the rate at which workers find a job increase, which imply that $I^F_U$, the lifetime expected value of being unemployed in the formal sector, increases. This means that workers are more willing to work in the formal sector than in the informal sector, and thus $L^F$ increases while $L^I$ decreases. Finally, the effect of $S^F$ on the unemployment $U^F$ is ambiguous because of the opposite effects on $L^F$ and $L^I$.

4.4. Hiring subsidies

We now consider a different policy that consists in giving to firms a hiring subsidy $H^F > 0$ when a worker is hired. In that case, equation (2.10) that gives the value of a vacant job is now equal to:

$$rI_V = -\gamma + q(\theta^F)(I_J + H^F - I_V)$$

(4.8)
Contrary to the previous policy of employment/wage subsidy, here the hiring subsidy $H^F$ is not paid to firms throughout the duration of the job but only once when they hire a new worker. So after the worker is hired, the benefit to the firm from continuing of hiring the worker is only $I_J$ and not $I_J + H^F$, since no further subsidies are received. As a result, $I_J$ is still given by (2.9) and the value of a job following the free-entry condition $I_V = 0$ is now equal to:

$$I_J = \frac{\gamma}{q(\theta^F)} - H^F$$

(4.9)

Using this value, we obtain the following decreasing relation between labor market tightness and wages in equilibrium:

$$\frac{\gamma}{q(\theta^F)} - H^F = \frac{y^F - w^F_L}{r + \delta}$$

(4.10)

The wage will also be modified since the value of a filled job has changed. The wage solution is now given by:

$$w^F_L = \max_{w^F_L} (I^F_L - I^F_U) \beta (I_J + H^F - I_V)^{1-\beta}$$

Solving this program, we obtain:

$$w^F_L = (1 - \beta) w^F_U + \beta (y^F + \gamma \theta^F)$$

which is (2.16), that is the wage without the hiring subsidy policy. This is not surprising because the subsidy has already been received by the firm before it negotiates with the worker.

Plugging this wage into (4.10) gives the equilibrium job creation $\theta^F$, which is implicitly defined as:

$$(1 - \beta) (y^F - w^F_U) - \beta \gamma \theta^F = \left[ \frac{\gamma}{q(\theta^F)} - H^F \right] (r + \delta)$$

(4.11)

The other equilibrium values are determined by exactly the same equations as in the case with no policy, i.e. $L^I$, $L^F$, and $U^F$ are defined by (3.5), (3.1), and (2.1).

We have the following result:

**Proposition 4.** In an Harris-Todaro model with urban search externalities and bargained wages, an increase in the hiring subsidy $H^F$ in the formal sector, leads to:

(i) an increase in job creation in the formal sector $\theta^F$;
(ii) a decrease in employment in the informal sector $L^I$ and an increase in the informal wage $w^I_L$.

(iii) a increase in employment in the formal sector $L^F$.

(iv) an ambiguous effect on the unemployment level in the economy $U^F$.

(v) no effect on wages $w^F_L$.

These results are comparable to that of Proposition 3 where an employment-wage subsidy policy was implemented. Indeed, increasing the hiring subsidy induces firms to hire more workers, which reduces the employment in the informal sector because the latter is less attractive. Now the new aspect here is that this policy has no effect on wages because the hiring subsidy is paid only once and thus cannot be used in the negotiation while the employment-wage subsidy policy is paid to firms throughout the duration of the job.

5. Concluding remarks

In this paper, we consider different policies in a dual labor market where the formal sector is characterized by search frictions and wage bargaining while the informal sector is competitive. We show that there exists a unique steady-state equilibrium in this dual economy. We then consider different policies. We find that reducing the unemployment benefit in the formal sector or the firms’ entry cost in this sector induces more job creation but has an ambiguous effect on employment, unemployment and wages in both sector. On the contrary, an employment-wage subsidy or a hiring subsidy policy has a clear positive effect on formal employment and wages in both sectors while a negative effect on informal unemployment. Interestingly, in all policies, a Todaro paradox can exist under some condition of the parameters. This means that an increase or decrease of a policy variable can lead to an increase in the equilibrium values of both urban employment and unemployment.

We believe that this paper gives some answers to important questions about mobility between formal and informal sectors in developing countries. In particular, even if the informal sector is unregulated and cannot be directly targeted by a government’s policy, the latter affects indirectly the wage, the employment and thus the size of the informal sector. Thus, any policy implemented, especially in cities where the informal sector is large, should
take into account not only the direct effect on the formal sector, but also the induced effect on the informal sector.

References


6. Appendix: Proof of Propositions

6.1. Proof of Proposition 1

Effect of \( w^F_U \) on \( \theta^F \)

By differentiating (3.4), it is easy to verify that

\[
\frac{\partial \theta^F}{\partial w^F_U} = \frac{(1 - \beta) q(\theta^F)}{(1 - \beta) (y^F - w^F_F) q'(\theta^F) - \beta \gamma \frac{\partial [\theta^F q(\theta^F)]}{\partial \theta^F}} < 0 \quad (6.1)
\]

Effect of \( w^F_U \) on \( L^I \)

By differentiating (3.5), we have:

\[
\frac{\partial L^I}{\partial \theta^F} = \frac{\frac{\partial [\theta^F q(\theta^F)]}{\partial \theta^F} \left[ w^F_L - \frac{F'(L^I)}{r} \right] + \beta \gamma q(\theta^F)}{F''(L^I) \left[ r + \delta + \theta^F q(\theta^F) \right]} < 0 \quad (6.2)
\]

Now, by differentiating (3.5) and using the fact that \( w^F_L = (1 - \beta) w^F_U + \beta (y^F + \gamma \theta^F) \), we obtain:

\[
\frac{\partial L^I}{\partial w^F_U} = \frac{\frac{\partial [\theta^F q(\theta^F)]}{\partial \theta^F} \left[ w^F_L - \frac{F'(L^I)}{r} \right] + \theta^F q(\theta^F) \left[ 1 - \beta + \beta \gamma \frac{\partial \theta^F}{\partial w^F_U} \right]}{F''(L^I) \left[ r + \delta + \theta^F q(\theta^F) \right]}.
\]

Therefore,

\[
\text{sgn} \frac{\partial L^I}{\partial w^F_U} = -\text{sgn} \left\{ r + \delta + \frac{\partial [\theta^F q(\theta^F)]}{\partial \theta^F} \frac{\partial \theta^F}{\partial w^F_U} \left[ w^F_L - \frac{F'(L^I)}{r} \right] + \theta^F q(\theta^F) \left[ 1 - \beta + \beta \gamma \frac{\partial \theta^F}{\partial w^F_U} \right] \right\}
\]

As a result, if (4.1) holds, then

\[
\frac{\partial L^I}{\partial w^F_U} > 0
\]

Otherwise, the sign of \( \frac{\partial L^I}{\partial w^F_U} \) is indeterminate.

Effect of \( w^F_U \) on \( w^F_L \)

Using (2.3), we have

\[
\frac{\partial w^F_L}{\partial w^F_U} = \frac{\partial w^F_L}{\partial L^I} \frac{\partial L^I}{\partial w^F_U}
\]
which implies that

\[ \text{sgn} \left[ \frac{\partial w^I_L}{\partial w^F_U} \right] = -\text{sgn} \left[ \frac{\partial L^I}{\partial w^F_U} \right] \]

**Effect of** \( w^F_U \) **on** \( L^F \)

\[ L^F(\theta^F, L^I) = \frac{\theta^F q(\theta^F)}{\delta + \theta^F q(\theta^F)} (N - L^I) \]

By differentiating (3.1), we have:

\[ \frac{\partial L^F}{\partial w^F_U} = \frac{\partial \left[ \theta^F q(\theta^F) \right]}{\partial \theta^F} \frac{\partial \theta^F}{\partial w^F_U} \frac{\delta}{\delta + \theta^F q(\theta^F)} \left( N - L^I \right) - \frac{\theta^F q(\theta^F)}{\delta + \theta^F q(\theta^F)} \frac{\partial L^I}{\partial w^F_U} \]

Using (6.1), we have the following. If (4.1) holds, then

\[ \frac{\partial L^F}{\partial w^F_U} < 0 \]

Otherwise the sign of \( \frac{\partial L^F}{\partial w^F_U} \) is indeterminate.

**Effect of** \( w^F_U \) **on** \( U^F \)

Differentiating (2.1) leads to:

\[ \frac{\partial U^F}{\partial w^F_U} = -\frac{\partial L^F}{\partial w^F_U} - \frac{\partial L^I}{\partial w^F_U} \quad \text{(6.3)} \]

Thus the impact on \( U^F \) is indeterminate.

**Effect of** \( w^F_U \) **on** \( w^F_L \)

Differentiating (2.16) yields:

\[ \frac{\partial w^F_L}{\partial w^F_U} = 1 - \beta + \beta \gamma \frac{\partial \theta^F}{\partial w^F_U} \]

which is ambiguous because of (6.1).

**6.2. Proof of Proposition 2**

**Effect of** \( \gamma \) **on** \( \theta^F \)

Differentiating (3.4) yields:

\[ \frac{\partial \theta^F}{\partial \gamma} = \frac{\beta \theta^F q(\theta^F) + r + \delta}{(1 - \beta) (y^F - w^F_U) q'(\theta^F) - \beta \gamma \frac{\partial [\theta^F q(\theta^F)]}{\partial \theta^F}} < 0 \]
Effect of $\gamma$ on $L^I$

Differentiating (3.5) yields:

$$\frac{\partial L^I}{\partial \theta^F} = \frac{\partial [\theta^F q(\theta^F)]}{\partial \theta^F} \left[ w_L - \frac{F'(L^I)}{r} \right] + \beta \gamma \theta^F q(\theta^F) \left[ \theta^F + \gamma \frac{\partial \theta^F}{\partial \gamma} \right] r < 0$$

Again, differentiating (3.5) leads to

$$\frac{\partial L^I}{\partial \gamma} = \frac{\partial [\theta^F q(\theta^F)]}{\partial \theta^F} \frac{\partial \theta^F}{\partial \gamma} \left[ w_L - \frac{F'(L^I)}{r} \right] + \beta \theta^F q(\theta^F) \left[ \theta^F + \gamma \frac{\partial \theta^F}{\partial \gamma} \right] r$$

Thus

$$\text{sgn} \frac{\partial L^I}{\partial \gamma} = -\text{sgn} \left\{ \frac{\partial [\theta^F q(\theta^F)]}{\partial \theta^F} \frac{\partial \theta^F}{\partial \gamma} \left[ w_L - \frac{F'(L^I)}{r} \right] + \beta \theta^F q(\theta^F) \left[ \theta^F + \gamma \frac{\partial \theta^F}{\partial \gamma} \right] \right\}$$

Under which condition

$$\frac{\partial [\theta^F q(\theta^F)]}{\partial \theta^F} \frac{\partial \theta^F}{\partial \gamma} \frac{1}{\theta^F} \left[ w_L - \frac{F'(L^I)}{r} \right] + \beta \theta^F q(\theta^F) \left[ 1 + \frac{\partial \theta^F}{\partial \gamma} \gamma \theta^F \right] < 0$$

Define $\eta = -\frac{\partial \theta^F}{\partial \gamma} \frac{\gamma \theta^F}{\theta^F} > 0$. Then this inequality is equivalent to:

$$\frac{\partial [\theta^F q(\theta^F)]}{\partial \theta^F} \frac{\partial \theta^F}{\partial \gamma} \frac{1}{\theta^F} \left[ w_L - \frac{F'(L^I)}{r} \right] + \beta \theta^F q(\theta^F) \left( 1 - \eta \right) < 0$$

Therefore, if $\eta > 1$, then

$$\frac{\partial L^I}{\partial \gamma} > 0$$

If $\eta < 1$, the sign of $\frac{\partial L^I}{\partial \gamma}$ becomes indeterminate.

Effect of $\gamma$ on $w_L^I$

Using (2.3), we have

$$\frac{\partial w_L^I}{\partial \gamma} = \frac{\partial w_L^I}{\partial L^I} \frac{\partial L^I}{\partial \gamma}$$

which implies that

$$\text{sgn} \left[ \frac{\partial w_L^I}{\partial \gamma} \right] = -\text{sgn} \left[ \frac{\partial L^I}{\partial \gamma} \right]$$

Effect of $\gamma$ on $L^F$

Differentiating (3.1) yields:
\[
\frac{\partial L^F}{\partial \gamma} = \frac{\partial}{\partial \gamma} \left[ \theta^F q(\theta^F) \right] \frac{\partial \theta^F}{\partial \gamma} \delta (N - L^I) - \frac{\theta^F q(\theta^F)}{\delta + \theta^F q(\theta^F)} \frac{\partial L^I}{\partial \gamma}
\]

Again, if \( \eta > 1 \), then \( \frac{\partial L^I}{\partial \gamma} > 0 \), and thus since \( \frac{\partial \theta^F}{\partial \gamma} < 0 \), we have

\[
\frac{\partial L^F}{\partial \gamma} < 0
\]

If \( \eta < 1 \), the sign of \( \frac{\partial L^F}{\partial \gamma} \) is indeterminate.

**Effect of \( \gamma \) on \( U^F \)**

Differentiating (2.1) leads to:

\[
\frac{\partial U^F}{\partial \gamma} = -\frac{\partial L^F}{\partial \gamma} - \frac{\partial L^I}{\partial \gamma}
\]

If \( \eta > 1 \), then \( \frac{\partial L^I}{\partial \gamma} > 0 \) and \( \frac{\partial L^F}{\partial \gamma} < 0 \), thus the impact on \( U^F \) is indeterminate.

**Effect of \( \gamma \) on \( w^F \)**

\[
\frac{\partial w^F}{\partial \gamma} = \beta \left[ \theta^F + \gamma \frac{\partial \theta^F}{\partial \gamma} \right]
\]

which has an ambiguous sign.

### 6.3. Proof of Proposition 3

**Effect of \( S^F \) on \( \theta^F \)**

Differentiating (4.6) gives:

\[
\frac{\partial \theta^F}{\partial S^F} = -\frac{(1 - \beta) q(\theta^F)}{(1 - \beta) (y^F - w^F_U - S^F) q'(\theta^F) - \beta \gamma \frac{\partial \theta^F q(\theta^F)}{\partial \theta^F}} > 0
\]

**Effect of \( S^F \) on \( L^I \)**

Differentiating (4.7) yields:

\[
\frac{\partial L^I}{\partial S^F} = \frac{\partial \left[ \theta^F q(\theta^F) \right]}{\partial \theta^F} \frac{\partial \theta^F}{\partial S^F} \left[ (1 - \beta) w^F_U + \beta \left( y^F + S^F + \gamma \theta^F - \frac{F'(L^I)}{r} \right) \right] + \beta \theta^F q(\theta^F) \left[ 1 + \gamma \frac{\partial \theta^F}{\partial S^F} \right] r
\]
Since \( w_L^F = (1 - \beta) w_U^F + \beta (y^F + S^F + \gamma \theta^F) \), and by assumption \( w_L^F > w_I^F \), then

\[
\frac{\partial L_I}{\partial S^F} = \frac{\delta \left[ \theta^F q(\theta^F) \right] \partial \theta^F}{\partial \theta^F} \left[ \frac{w_L^F - F'(L_I)}{r} \right] + \beta \theta^F q(\theta^F) \left[ 1 + \gamma \frac{\partial \theta^F}{\partial S^F} \right] r < 0
\]

**Effect of \( S^F \) on \( w_L^I \)**

Using (2.3), we have

\[
\frac{\partial w_L^I}{\partial S^F} = \frac{\partial w_L^I}{\partial L^I} \frac{\partial L^I}{\partial S^F}
\]

which implies that

\[
\frac{\partial w_L^I}{\partial S^F} > 0
\]

**Effect of \( S^F \) on \( L^F \)**

Differentiating (3.1) leads to:

\[
\frac{\partial L^F}{\partial S^F} = \frac{\partial \left[ \theta^F q(\theta^F) \right] \partial \theta^F}{\partial \theta^F} \frac{\partial \theta^F}{\partial S^F} \delta (N - L^I) - \frac{\theta^F q(\theta^F)}{\delta + \theta^F q(\theta^F) \partial S^F} > 0
\]

**Effect of \( S^F \) on \( U^F \)**

Differentiating (2.1) gives:

\[
\frac{\partial U^F}{\partial S^F} = -\frac{\partial L^F}{\partial S^F} - \frac{\partial L_I^F}{\partial S^F}
\]

which is indeterminate.

**Effect of \( S^F \) on \( w_L^F \)**

Differentiating (4.5), we have:

\[
\frac{\partial w_L^F}{\partial S^F} = \beta \left( 1 + \gamma \frac{\partial \theta^F}{\partial S^F} \right) > 0
\]

\[ \square \]

6.4. Proof of Proposition 4

**Effect of \( H^F \) on \( \theta^F \)**

Differentiating (4.11) gives:

\[
\frac{\partial \theta^F}{\partial H^F} = -\frac{(r + \delta) q(\theta^F)}{(1 - \beta) [y^F - w_U^F + H^F (r + \delta)] q'(\theta^F) - \beta \gamma \frac{\partial \theta^F q(\theta^F)}{\partial S^F}} > 0
\]
Effect of $H^F$ on $L^I$

Differentiating (3.5) yields:

$$\frac{\partial L^I}{\partial H^F} = \frac{\delta [\theta^F q(\theta^F)]}{\partial \theta^F} \frac{\partial \theta^F}{\partial H^F} \left[ (1 - \beta) w^F_U + \beta (y^F + \gamma \theta^F) - \frac{F'(L^I)}{r} \right] + \beta \gamma \theta^F q(\theta^F) \frac{\partial \theta^F}{\partial H^F}$$

Since $w^F_L = (1 - \beta) w^F_U + \beta (y^F + \gamma \theta^F)$, and by assumption $w^F_L > w^I$, then

$$\frac{\partial L^I}{\partial H^F} = \frac{\delta [\theta^F q(\theta^F)]}{\partial \theta^F} \frac{\partial \theta^F}{\partial H^F} \left[ w^F_L - \frac{F'(L^I)}{r} \right] + \beta \theta^F q(\theta^F) \frac{\partial \theta^F}{\partial H^F} r < 0$$

Effect of $H^F$ on $w^I_L$

Using (2.3), we have

$$\frac{\partial w^I_L}{\partial H^F} = \frac{\partial w^I_L}{\partial L^I} \frac{\partial L^I}{\partial H^F}$$

which implies that

$$\frac{\partial w^I_L}{\partial H^F} > 0$$

Effect of $H^F$ on $L^F$

Differentiating (3.1) leads to:

$$\frac{\partial L^F}{\partial H^F} = \delta \left[ \frac{\partial \theta^F q(\theta^F)}{\partial \theta^F} \frac{\partial \theta^F}{\partial H^F} \right] \delta (N - L^I) - \frac{\theta^F q(\theta^F)}{\delta + \theta^F q(\theta^F)} \frac{\partial L^I}{\partial H^F} > 0$$

Effect of $H^F$ on $U^F$

Differentiating (2.1) gives:

$$\frac{\partial U^F}{\partial H^F} = - \frac{\partial L^F}{\partial H^F} - \frac{\partial L^I}{\partial H^F}$$

which is indeterminate.

Effect of $H^F$ on $w^F_L$

Differentiating (2.16), we have:

$$\frac{\partial w^F_L}{\partial H^F} = 0$$
Figure 1: Harris-Todaro equilibrium with search externalities