Search in Cities*

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Abstract: The aim of this paper is to expose the recent developments of urban search models, which incorporate a land market into a search matching framework. Using these models, we will be able to explain why unemployment rates and wages vary within a city, how city structure affects workers’ labor-market outcomes, how unemployment benefits and the job-destruction rate affect the growth of cities, why workers living far away from job centers search less intensively and experience higher unemployment rates than those residing closer to jobs, and why, within the same city, ethnic minorities experience higher unemployment rates than workers from the majority group.

Keywords: job search, urban land use, search intensity, spatial mismatch.

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1 Introduction

Why unemployment rates vary within a city and, in particular, why they are so high in some specific areas? Why there is so much variation in wages within a city? Does city structure affect the labor-market outcomes of workers? Does space create inefficiencies? Do unemployment benefits and the job-destruction rate affect the growth of cities? Why workers living far away from jobs search less intensively and experience higher unemployment rates than those residing closer to jobs? Why in the same city ethnic minorities experience higher unemployment rates than workers from the majority group?\(^1\)

With standard labor economics or urban economics, one cannot answer to these important questions. Indeed, labor economists traditionally do not directly incorporate space into their studies (see e.g. Layard et al., 1991; Pissarides, 2000; Cahuc and Zylberberg, 2004). Similarly, in urban economics, it is mainly assumed perfect competition in the labor market and the issue of urban unemployment is not even discussed (see, in particular, Fujita, 1989; Fujita et al. 1999; Fujita and Thisse, 2002).

One thus needs a theory that brings together labor and urban aspects to be able to give some answers to the above questions. Search models solve for the steady-state unemployment rate.\(^2\) Land use models tell us about the location of people in space. When the two are put together, we learn on where unemployed and employed workers are located and how their search activity is influenced by the spatial nature of cities.

The aim of this paper is to expose the recent developments of urban search models. In all these models, a land market is explicitly introduced in a search matching framework. The link between the land and the labor market is realized through the average search intensity of unemployed workers. Indeed, the latter depends on the location of the unemployed workers in the city, which is endogenously determined in the land-use equilibrium. The location of workers, in turn, depends on the outcomes in the labor market.

We first develop a model (section 2) where workers’ search intensity is a negative function of distance to jobs, capturing the fact that workers residing further away have worse job information than those living closer to jobs. Because workers trade off access costs in terms of commuting with access costs in terms of job information, we obtain two different equilibrium urban structures: either the unemployed workers reside close to or far away from jobs. With


\(^2\)There is a vast literature on search and matching theory that emphasizes the importance of flows and search frictions in the labor markets (see the literature surveys by Mortensen and Pissarides, 1999; Pissarides, 2000; Rogerson et al., 2005; Yashiv, 2007).
this simple model, we are able to explain why unemployment rates vary within a city and how city structure affects the labor-market outcomes of workers. Indeed, because spatial frictions amplify search frictions, workers residing far away from jobs experience higher unemployment rates than those living closer to jobs. As a result, different unemployment rates will emerge within the same city. Also, if we compare two cities with different structures, then the one where the unemployed workers reside close to jobs has a lower aggregate unemployment rate than the city where the unemployed workers are far away from jobs, even though the total welfare is nearly the same in the two cities. We are also able to show that space does not create additional inefficiencies beyond the ones created by search frictions. Indeed, the condition that guarantees that the market solution is Pareto optimal is the same in the spatial and nonspatial models. The allocation of workers in space is thus optimal, a result already proved in previous research (Fujita, 1989).

Because most of the previous results hinge on the assumption that search intensity decreases with distance to jobs, we then develop a model (section 3) where this relationship is derived rather than assumed. We show that the unemployed workers search less, the further away they reside from jobs. There is indeed a fundamental trade-off between short-run and long-run benefits of various location choices for the unemployed workers. Locations near jobs are costly in the short run (both in terms of high land rents and low housing consumption), but allow higher search intensities, which, in turn, increase the long-run prospects of reemployment. Conversely, locations far from jobs are more desirable in the short run (low land rents and high housing consumption) but allow only infrequent trips to jobs and hence reduce the long-run prospects of reemployment. Therefore, for workers residing further away from the job center, it is optimal to spend the minimal search effort whereas workers residing close to jobs provide high search effort. In this model, a new urban structure emerges: a core-periphery one where the unemployed workers reside close and far away from jobs whereas the employed workers live in between them. As a result, we now better understand why remote workers spend less effort in finding a job and why long-term unemployed workers are those who reside far away from jobs.

In section 4, we consider another important aspect of urban search models: the fact that workers are not always perfectly mobile. We assume that workers are stuck in their location and thus do not relocate when their employment status changes. We are able to show that wages are now increasing with distance to jobs, which is consistent with what is observed in most cities. Firms must pay higher wages to remote workers in order to compensate them for the extra commuting cost incurred compared to the situation when they were unemployed.

In section 5, we use the different models described above to explain why ethnic minori-
ties and whites experience different labor-market outcomes within the same city. If ethnic minority workers reside far away from jobs (because they are discriminated against in the housing market or because they want to live together even if it implies remote locations), then, as observed in most cities, they are stuck in locations badly connected to job centers, search less intensively, have a lower expected wage, and experience a higher unemployment rate than whites.

Section 6 exposes different extensions of the previous models putting forward the role of workers’ and firms’ heterogeneity in the matching process and how non-monocentric cities affect the location of workers. Finally, section 7 concludes and we discuss some possible avenues of research in the future.

2 The benchmark model

There is a continuum of ex ante identical workers whose mass is $N$ and a continuum of $M$ identical firms. Among the $N$ workers, $L$ are employed and $U$ unemployed, so that $N = L + U$. The workers are uniformly distributed along a linear, closed and monocentric city. All land is owned by absentee landlords and all firms are exogenously located in the Central Business District (CBD hereafter) and consume no space. Workers are assumed to be infinitely lived, risk neutral and decide their optimal place of residence between the CBD and the city fringe. There are no relocation costs, either in terms of time or money.

Each individual is identified with one unit of labor. Each employed worker goes to the CBD to work and incurs a fixed monetary commuting cost $\tau_L$ per unit of distance. When living at a distance $x$ from the CBD, he/she also pays a land rent $R(x)$, consumes $h_L = 1$ unity of land and $z_L$ unities of the non-spatial composite good (which is taken as the numeraire so that its price is normalized to 1), and earns a wage $w_L$. There is no on-the-job search. The budget constraint of an employed worker is thus given by:

$$ R(x) + \tau_L x + z_L = w_L $$

Because of risk neutrality, we assume that preferences of all workers (including the unemployed) are described by $\Omega(z_L) = z_L$, so that the instantaneous (indirect) utility of an employed worker located at a distance $x$ from the CBD is equal to:

$$ W_L(x) = w_L - \tau_L x - R(x) $$

$^3$The subscript $L$ refers to the employed workers whereas the subscript $U$ refers to the unemployed workers.
Concerning the unemployed workers, they commute also to the CBD and but occur a different commuting cost per unit of distance, denoted by $\tau_U$. We assume that $\tau_U < \tau_L$. The instantaneous (indirect) utility of an unemployed worker residing at a distance $x$ from the CBD can be written as:

$$W_U(x) = w_U - \tau_U x - R(x)$$

where $w_U$ is the unemployment benefit. We assume that $w_U$ is exogenously financed by taxpayers who reside elsewhere (for example absentee landlords).\(^4\)

Depending on their residential location, the unemployed workers can provide different levels of search intensities. If $s(x)$ is a measure of search intensity, then it is assumed that $s'(x) < 0$. Indeed, there are strong evidences showing that distance to jobs has a negative impact on search behavior. Several empirical studies have shown that distance to jobs deteriorates the information one has on job opportunities, and that job accessibility is crucial to obtain a job (see, e.g., Rogers, 1997, Ihlanfeldt, 1997, Turner, 1997, Stoll, 1999). For example, Ihlanfeldt (1997) has showed that Atlanta’s inner-city residents are less able to identify the location of suburban employment centers than suburbanites, which implies that they have less information on these jobs. Turner (1997) has documented that, in Detroit’s suburbs, firms that resort to local recruitment methods have very few inner-city black applicants. Following Wasmer and Zenou (2002, 2006), the search intensity $s(x)$ is assumed to have the following linear form:

$$s(x) = s_0 - s_a x$$

where $s_0 > 0$ and $s_a > 0$. In this formulation, $s_a$ measures the marginal loss of information following an increase in distance to jobs. In order for the search intensity to be always positive, we impose that $s_0 > s_a N$. The linearity assumption will be very useful in the urban land use analysis. So, in this formulation, workers residing further away from jobs search less intensively because of the loss of information. Observe that the unemployed workers do not need to commute to the CBD to gather information about jobs. Indeed, it is assumed that the unemployed workers gather information either locally (like e.g. help-wanted signs on firms’ windows or local employment agencies) or through newspapers. As a result, $\bar{s}$, the aggregate search efficiency in the city, will depend on the average location $\bar{x}_U$ of the unemployed workers. It is given by:

$$\bar{s} = s_0 - s_a \bar{x}_U$$

\(^4\)In search-matching models, $w_U$ is often interpreted as the utility of leisure.
Let us now describe the labor market. A firm is a unit of production that can either be filled by a worker whose productivity is $y$ or be unfilled and thus unproductive. In order to find a worker, a firm posts a vacancy. A vacancy can be filled according to a random Poisson process. Similarly, workers searching for a job will find one according to a random Poisson process. In aggregate, these processes imply that there is a number of matches per unit of time between the two sides of the market that are determined by the following matching function:\footnote{This matching function is written under the assumption that the city is monocentric, i.e. all firms are located in one fixed location.} \( M(\overline{s}U, V) \), where \( \overline{s} \) is the average search efficiency of the unemployed workers defined in (5), and \( U \) and \( V \) are the total number of unemployed workers and vacancies. As in the standard search-matching models (Mortensen and Pissarides, 1999, and Pissarides, 2000), we assume that \( M(.) \) is increasing in both its arguments, concave and homogeneous of degree 1 (or equivalently has constant return to scale). Thus, the rate at which vacancies are filled is \( M(\overline{s}U, V) / V \). By constant returns to scale, this rate can be written as: \( M(1/\theta, 1) \equiv q(\theta) \), where \( \theta = V / (\overline{s}U) \) is a measure of labor market tightness in efficiency units and \( q(\theta) \) is a Poisson intensity. Similarly, the rate at which an unemployed worker with search intensity \( s(x) \) leaves unemployment is given by:

\[
a(x) = \frac{s(x) M(\overline{s}U, V)}{\overline{s}} \equiv s(x)\theta q(\theta) \tag{6}
\]

which depends on his/her residential location \( x \) and on \( \overline{s} \). Here, when workers search more actively for jobs, their chance to leave unemployment increases. Finally, the rate at which jobs are destroyed is exogenous and denoted by \( \delta \).

A steady-state equilibrium requires solving simultaneously an urban land use equilibrium and a labor market equilibrium. It is convenient to present first the former and then the latter.

\subsection*{2.1 Urban land use equilibrium}

Let us write the expected lifetime utilities of the employed and unemployed workers. In steady state, they are given by:

\[
\begin{align*}
    rI_L(x) &= w_L - \tau_L x - R(x) - \delta (I_L - I_U) \\
    rI_U(x) &= w_U - \tau_U x - R(x) + a(x) (I_L - I_U)
\end{align*}
\]

Since there are no relocation costs, the urban equilibrium is such that all employed and unemployed workers enjoy the same utility level, i.e. \( rI_L(x) = rI_U(x) = rI_L \), \( \forall x \), and \( rI_U(x) = rI_U \).
∀x. Workers’ bid rents\(^6\) can thus be written as:

\[
\Psi_L(x, I_U, I_L) = w_L - \tau_L x + \delta I_U - (r + \delta) I_L
\]

(7)

\[
\Psi_U(x, I_U, I_L) = w_U - \tau_U x + a(x) I_L - [r + a(x)] I_U
\]

(8)

These bid rents are linear (because \(s(x)\) is linear in \(x\)) and decreasing in \(x\). However, no clear urban pattern emerges because workers are trading off commuting costs and access to jobs. We have the following result:\(^7\)

**Proposition 1**

(i) If

\[
\tau_L - \tau_U < s_a \theta^1 q(\theta^1)(I_L^1 - I_U^1)
\]

(9)

we have equilibrium 1 in which the unemployed workers live close to jobs.

(ii) If

\[
\tau_L - \tau_U > s_a \theta^2 q(\theta^2)(I_L^2 - I_U^2)
\]

(10)

equilibrium 2 prevails in which the employed live close to jobs.

The unemployed workers would like to live close to jobs because it reduces both their commuting costs and unemployment duration (which is the inverse of \(a(x)\)). For the employed workers, it is only their commuting costs that drive their choice. Condition (9) states that if the differential in commuting costs between the employed and unemployed workers is lower than the marginal expected return from search for the unemployed workers, then the latter will bid away the employed workers and occupy the center of the city. If, on the contrary, condition (10) holds, then the employed workers will live close to jobs while the unemployed workers will reside at the outskirts of the city. Observe that these conditions depend on the endogenous variables \(\theta^1\) and \(\theta^2\), which will be determined at the labor market equilibrium. Given that conditions (9) and (10) hold, we have the following definitions:

\(^6\)The bid rent is a standard concept in urban economics. It indicates the maximum land rent that a worker located at a distance \(x\) from the CBD is ready to pay in order to achieve a utility level. See Fujita (1989) for a formal definition.

\(^7\)Superscripts 1 and 2 refer to equilibrium 1 and equilibrium 2.
Definition 1 An urban land use equilibrium $k = 1, 2$ with no relocation costs, fixed-housing consumption and endogenous search intensity is a 5-tuple $(I_L^k, I_U^k, x_b^{k*}, x_f^{k*}, R^k(x))$ such that:

$$\Psi_U(x_b^{k*}, I_U^k, I_L^k) = \Psi_L(x_b^{k*}, I_U^k, I_L^k) \quad k = 1, 2$$  (11)

$$\Psi_L(x_f^{1*}, I_U^k, I_L^k) = 0 \quad \text{or} \quad \Psi_U(x_f^{2*}, I_U^k, I_L^k) = 0$$  (12)

$$x_b^{1*} = Nu^1 \quad \text{or} \quad x_b^{2*} = N(1 - u^2)$$  (13)

$$x_f^{k*} = N \quad k = 1, 2$$  (14)

$$R^k(x) = \max \{\Psi_L(x, I_U^k, I_L^k), \Psi_U(x, I_U^k, I_L^k), 0\} \quad \text{at each } x \in (0, x_f^{k*}]$$  (15)

Equations (11) and (12) reflect the equilibrium conditions in the land market and guarantee that the land rent is continuous everywhere in the city. Equation (11) states that, in the land market, at the frontier $x_b^{k*}$, the bid rent of the employed workers is equal to that of the unemployed workers. Equation (12), in turn, says that, at the city fringe, the bid rent of the workers located at the outskirts of the city has to be equal to the agricultural land, which price has been normalized to zero. Equations (13) and (14) give the two population constraints. Finally, equation (15) defines the equilibrium land rent as the upper envelope of the bid rent curves of all workers and the agricultural rent line. The two equilibria 1 and 2 are illustrated in Figures 1 and 2. The average search intensity in equilibrium $k$ is equal to:

$$\bar{s}^k = s_0 - s_a \bar{x}_U^k \quad k = 1, 2$$  (16)

where $\bar{x}_U^k$ is the average location of the unemployed workers in equilibrium $k$. It is easy to verify that $\bar{x}_U^1 = Nu^1/2$ and $\bar{x}_U^2 = N(1 - u^2/2)$. Since $\bar{x}_U^1 < \bar{x}_U^2$ (this is always true because $u^1 + u^2 < 2$), the average search efficiency in urban equilibrium 1 is always higher than in urban equilibrium 2, i.e., $\bar{s}^1 > \bar{s}^2$. Indeed, in the integrated city (equilibrium 1), the unemployed workers reside closer to the CBD than in the segregated city (equilibrium 2), and thus the rate at which they leave unemployment is higher.

[Insert Figures 1 and 2 here]
2.2 Steady-state equilibrium

The unemployment rate is defined by \( u^k = U^k/N \). We have:

**Definition 2** A steady-state labor market equilibrium \( k = 1, 2 \) is a triple \((w^*_k, \theta^k, u^*_k)\) such that, given the matching technology \( M(\cdot) \), all agents (workers and firms) maximize their respective objective function, i.e. this triple is determined by a wage-setting mechanism, a free-entry condition for firms, and a steady-state condition on flows.

As it will become clear below, this definition of the steady-state labor equilibrium is related to that of the urban equilibrium (Definition 1) through the average search intensity \( \bar{s}^k \). Indeed, the equilibrium values of \( w^*_k, \theta^k \) and \( u^*_k \) depend on the urban configuration considered (equilibrium 1 or 2) through \( \bar{s}^k \), and the urban configuration that will prevail in equilibrium, in turn, depends on the equilibrium labor variables (see (9) and (10)). Let us denote by \( I^k_F \) and \( I^k_V \) the expected lifetime utility of a filled job and a vacancy. If \( c \) is the search cost per unit of time and \( y \) is the product of a match, then, in steady-state, \( I^k_F \) and \( I^k_V \) can be written as:

\[
\begin{align*}
 rI^k_F &= y - w^k_L - \delta(I^k_F - I^k_V) \quad (17) \\
 rI^k_V &= -c + q(\theta^k)(I^k_F - I^k_V) \quad (18)
\end{align*}
\]

We assume that firms post vacancies up to a point where \( I^k_V = 0 \). Using (17) and (18), we obtain, for each \( k = 1, 2 \), the following decreasing relationship between labor-market tightness and wages:

\[
\frac{y - w^k_L}{r + \delta} = \frac{c}{q(\theta^k)} \quad (19)
\]

In words, the value of a job is equal to the expected search cost, i.e. the cost per unit of time multiplied by the average duration of a vacant job. So job creation \( \theta^k \) is endogenous and determined by (19).

At each period, the total intertemporal surplus is shared through a generalized Nash-bargaining process between the firm and the worker. The total surplus is the sum of the surplus of the workers, \( I^k_L - I^k_U \), and the surplus of the firms, \( I^k_F - I^k_V \). As a result, at each period, the wage is determined by:

\[
w^k_L = \arg \max_{w^k_L} (I^k_L - I^k_U)^\beta (I^k_F - I^k_V)^{1-\beta}
\]
where $0 \leq \beta \leq 1$ is the bargaining power of workers. By solving this program, Wasmer and Zenou (2002) have showed that:

$$w_k^L = (1 - \beta) \left[ w_U + (\tau_L - \tau_U)x^k_b \right] + \beta \left[ y + (s_0 - s_u x^k_b) \theta^k c \right]$$  \hspace{1cm} (20)

The first part of (20), $(1 - \beta) \left[ w_U + (\tau_L - \tau_U)x^k_b \right]$, is what firms must pay to induce workers to accept the job offer: firms must pay the unemployment benefit and exactly compensate the transportation cost difference between employment and unemployment. This is referred to as the compensation effect. The second part $\beta \left[ y + (s_0 - s_u x^k_b) \theta^k c \right]$ is the part of the surplus obtained by workers. This is referred to as the outside option effect. The first effect is a pure spatial cost since $(\tau_L - \tau_U)x^k_b$ represents the space cost differential between the employed and unemployed workers paying the same bid rent (i.e. at $x = x^k_b$) whereas the second effect is a mixed of labor and spatial costs. Observe that $\theta^k$ has a positive impact on $w_k^L$, implying in particular that unemployment negatively affects wages.

In search-matching models, the wage-setting curve $WS$ (a relationship between wages and the state of the labor market, here $\theta$) replaces the traditional labor-supply curve. Equations (20) give two wage-setting curves, $WS^1$ and $WS^2$, which are both positively sloped in the $(\theta, w_L)$ space. One can show that $WS^1$ is steeper than $WS^2$ but the intercept of $WS^1$ has a lower value (see Figures 3 and 4). $WS_1$ is indeed steeper than $WS_2$ because the slope of these curves represents the ability of workers to exert wage pressures when the labor market is tighter. As a result, for a given labor market tightness, a more efficient labor market, i.e. $s^1 > s^2$, leads to a higher wage pressure per unit of $\theta^k$, and thus a steeper $WS^1$ curve. The intercept is, however, higher in equilibrium 2 because, when $\theta^k = 0$, the outside option effect vanishes and only the compensation effect remains. Since the same marginal worker is further away in equilibrium 2, he/she must have a higher compensation for his/her transportation costs.

Let us now close the model. Since each job is destroyed according to a Poisson process with arrival rate $\delta$, the number of workers who enter unemployment is $\delta (1 - u^k_t)$ while those who leave unemployment are equal to: $s^k \theta^k_t q(\theta^k_t) u^k_t$. The evolution of the unemployment rate is thus given by the difference between these two flows,

$$\dot{u} = \delta (1 - u_t) - s^k \theta^k_t q(\theta^k_t) u^k_t$$  \hspace{1cm} (21)

where $\dot{u} = du_t/dt$ is the variation of unemployment with respect to time $t$. In steady state, the rate of unemployment is constant and therefore these two flows are equal. We have:

$$u^{k*} = \frac{\delta}{\delta + s^k \theta^{k*} q(\theta^{k*})}$$  \hspace{1cm} (22)
Equations (22) define in the \((u,V)\) space a downward sloping curve referred to as the Beveridge curves. The interesting feature of these Beveridge curves is that they are indexed by \(\pi^k\), which depend on the spatial dispersion of the unemployed workers in equilibrium \(k = 1, 2\). It can be shown that, in the \((\theta,w_L)\) space, the Beveridge curve in urban equilibrium 2 is always above the Beveridge curve in urban equilibrium 1. Indeed, a lower \(s_a\) is associated with an outward shift of the Beveridge curve because more vacancies are needed to maintain the steady-state level of unemployment. If \(s_a\) increases or \(s_0\) decreases, the Beveridge curve is shifted away from the origin meaning that the labor market is less efficient. The same would arise if the city size \(N\) would increase because the unemployed workers would be on average further away. By combining (19) and (20), we obtain the following equation that determines \(\theta^k_s\):

\[
y - w_U = \frac{c}{q(\theta^k)} \left[ \delta + r + \theta^k q(\theta^k) \left( s_0 - s_a x_b^k \right) \beta \right] + x_b^k (\tau_L - \tau_U)
\]

(23)

Denote by \(\hat{\theta} = \frac{1 - \beta (\tau_L - \tau_U)}{\beta s_a c}\)

the labor-market tightness that is implicitly defined by the intersection between the two wage-setting curves \(WS_1\) and \(WS_2\) (see Figures 3 and 4). We have the following result (proved by Wasmer and Zenou, 2002):

**Proposition 2** There exists a unique and stable steady-state equilibrium \((I^k_L, I^k_U, x^k_b, x^k_f, R^k(x), w^k_L, \theta^k_s, u^k)\), \(k = 1, 2\), and only the two following cases are possible:

(i) If \(\hat{\theta} < \theta^{1*} < \theta^{2*}\), urban equilibrium 1 prevails;

(ii) If \(\theta^{2*} < \theta^{1*} < \hat{\theta}\), urban equilibrium 2 prevails.

These two cases, (i) and (ii), are plotted in Figures 3 and 4. Inequalities (i) and (ii) replace conditions (9) and (10). Observe that multiple urban equilibria can never exist (i.e. having both a segregated and an integrated city) because, according to Proposition 2, the condition for multiple equilibria is \(\theta^{2*} < \hat{\theta} < \theta^{1*}\) and it can never be verified since the intersection of the labor demand curve with the wage setting curve never satisfies this condition. This is because \(\hat{\theta}\), initially defined as the intersection point between the two wage-setting curves, is also the critical value that determines which urban equilibrium prevails.

Observe that the uniqueness of equilibrium is a direct consequence of the strong linearity assumptions, especially of the search effort function (see (4)). In a more general model, we
would not necessarily expect that the equilibrium is unique. Observe also that, in steady state, people are moving constantly within the city: some of the employed are losing jobs, and some of the unemployed are finding jobs, but the numbers are balanced. So people are exchanging places in the employed and unemployed residential zones.

### 2.3 Welfare and efficiency

Let us now proceed with the welfare analysis. The social welfare function is given by the sum of the utilities of the employed and the unemployed workers, the production of the firms net of search costs, and the land rents received by the (absentee) landlords. Being pure transfers, wages $w^k_L$ as well as land rents $R^k(x)$ are excluded from the social welfare function. The latter is therefore given by:

$$ W^k = \int_0^{+\infty} e^{-rt} \left\{ \int_{\text{employed}} (y - \tau_L z) dz + \int_{\text{unemployed}} (w_U - \tau_U z) dz - c \theta_t^{kk} u_t^k \right\} dt $$

The first issue we address is the respective efficiency of the two types of land market configurations. Since there are no multiple equilibria in the land market, we cannot Pareto-rank the equilibria and it is therefore difficult to compare them. Nevertheless, we are able to investigate what happens to the welfare difference in the range of the parameters around the frontier separating the two types of land market equilibria. We then focus on the efficiency and welfare analyses of each equilibrium.

**Comparison of welfare between cities** We first investigate the welfare and unemployment differences between the two cities. For that, we need to determine the part of unemployment that is only due to spatial frictions. Let us normalize the total population to 1. Since the analysis for equilibrium 1 is nearly identical, we focus only on equilibrium 2 where the unemployed workers live far away from jobs. For simplicity, wages are assumed to be exogenous. In that case, $\theta^2$ is constant and determined by (19). By using (22), the unemployment rate is given by:

$$ u^2 = \frac{\delta}{\delta + \theta^2 q(\theta^2) [s_0 - s_a(1 - u^2/2)]} $$

Let us further define:

$$ u^2_0 = \frac{\delta}{\delta + \theta^2 q(\theta^2) s_0} $$

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8The extension to bargained wages is straightforward but much more cumbersome. See Wasmer and Zenou (2002, 2006).
the part of unemployment that is independent of spatial frictions, i.e. \( s_a = 0 \). By a Taylor first-order expansion for small \( s_a/s_0 \), we easily obtain:

\[
  u^2* = u^2_0 \left[ 1 + \frac{s_a}{s_0} (1 - u^2_0) (1 - u^2_0/2) \right] = u^2_0 + u^2_\sigma
\]

(27)

where \( u^2_\sigma \equiv u^2_0 \left[ s_a (1 - u^2_0) (1 - u^2_0/2) / s_0 \right] \) is the unemployment rate that is only due to spatial frictions while \( u^2_0 \) is defined by (26). Observe that \( u^2_\sigma \) is increasing in \( s_a/s_0 \), the parameter representing the loss of information per unit distance, and null when \( s_a = 0 \).

To see how the welfare and the unemployment rate vary across cities, let us proceed to a simple numerical resolution of the model. We use the following Cobb-Douglas function for the matching function: \( M(\pi^k u^k, V^k) = \sqrt{\pi^k u^k V^k} \). This implies that \( q(\theta^k) = 1/\sqrt{\theta^k} \), \( \theta^k q(\theta^k) = \sqrt{\theta^k} \) and, whatever the prevailing urban equilibrium, the elasticity of the matching rate, defined as \( \eta(\theta^k) = -q'(\theta^k)\theta^k/q(\theta^k) \), is equal to 0.5. The values of the parameters (in yearly terms) are the following: the output \( y \) is normalized to unity. The relative bargaining power of workers is equal to \( \eta(\theta^k) \), i.e. \( \beta = \eta(\theta^k) = 0.5 \). Unemployment benefits \( w_U \) have a value of 0.3 and the costs of maintaining a vacancy \( c \) are equal to 0.3 per unit of time. Commuting costs \( \tau_L \) are equal to 0.4 for the employed workers, and \( \tau_U = 0.1 \) for the unemployed workers. The discount rate is such that: \( r = 0.05 \), whereas the job destruction rate is set to: \( \delta = 0.1 \), which means that jobs last on average ten years. Finally, \( s_0 \) is normalized to 1, implying that \( 0 \leq s_a \leq 1, \forall x \in [0, N] \).

The results are displayed in Table 1, where we have chosen to vary a key parameter \( s_a \), the loss of information per unit of distance. This parameter \( s_a \) varies from a very large value 1 (where city 1 is the prevailing equilibrium) to a very small value 0.1 (where city 2 is the prevailing equilibrium). The cut-off point is equal to \( s_a = 0.522 \). The sign ‘−’ indicates the ‘limit to the left’, whereas the sign ‘+’ indicates the ‘limit to the right’.

The first interesting result is that, when we switch from an integrated city (equilibrium 1) to a segregated city (equilibrium 2), for values very close to the cut-off point \( s_a = 0.522 \), the unemployment rate \( u^{k*} \) nearly doubles (from 6.85% to 12.4%). It should however be clear that this result is mainly due to the spatial part of unemployment \( u^k_\sigma \) since the non-spatial one \( u^k_0 \) is not at all affected by this increase. Indeed, when we switch from equilibrium 1, where the unemployed workers are close to jobs and are very efficient in their job search (\( \pi^1 = 0.982 \)), to equilibrium 2, where the unemployed reside far away from jobs and are on average less active (\( \pi^2 = 0.511 \)), the spatial part of the unemployment rate changes values from 0.12% to 5.65%. Another way to see this is to consider column 6 (\( u^k_\sigma/u^{k*} \)): the part of unemployment due to space increases from 2% to 46%. So the main effect from switching from one equilibrium to the other is that search frictions are amplified by space.
and consequently unemployment rates sharply increase. So here the spatial access to jobs is crucial to understand the formation of unemployment in cities.

The last column of the table shows the value of the welfare $W^k$ when $s_a$ varies. The result is very striking: even though unemployment rates are higher in equilibrium 1 than in equilibrium 2, this does not imply that the general welfare of the economy is higher. Indeed, even though the unemployed workers are better off in equilibrium 1 (lower unemployment spells and lower commuting costs), the employed workers can be in fact worse off because of much higher commuting costs in equilibrium 1. In Table 1, it is interesting to see that, at the vicinity of $s_a = 0.522$, switching from equilibrium 1 to equilibrium 2 does not involve much change in the welfare level (from 0.714 to 0.712). Of course, this is just a numerical example and we do not have a general result. However, we are confident that it can be generalized since the same qualitative results have been obtained in the numerous simulations performed by us.

<table>
<thead>
<tr>
<th>$s_a$</th>
<th>City</th>
<th>$u^k$ (%)</th>
<th>$u_0^k$ (%)</th>
<th>$u_*^k$ (%)</th>
<th>$u_*^k / u^k$</th>
<th>$\theta^k$</th>
<th>$\beta^k$</th>
<th>$\pi^k$</th>
<th>Welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>6.86</td>
<td>6.64</td>
<td>0.22</td>
<td>0.03</td>
<td>1.98</td>
<td>0.069</td>
<td>0.966</td>
<td>0.715</td>
</tr>
<tr>
<td>0.75</td>
<td>1</td>
<td>6.85</td>
<td>6.69</td>
<td>0.16</td>
<td>0.02</td>
<td>1.95</td>
<td>0.069</td>
<td>0.974</td>
<td>0.714</td>
</tr>
<tr>
<td>0.6</td>
<td>1</td>
<td>6.85</td>
<td>6.72</td>
<td>0.13</td>
<td>0.02</td>
<td>1.93</td>
<td>0.069</td>
<td>0.979</td>
<td>0.714</td>
</tr>
<tr>
<td>0.55</td>
<td>1</td>
<td>6.85</td>
<td>6.73</td>
<td>0.12</td>
<td>0.02</td>
<td>1.92</td>
<td>0.069</td>
<td>0.981</td>
<td>0.714</td>
</tr>
<tr>
<td>0.525</td>
<td>1</td>
<td>6.85</td>
<td>6.73</td>
<td>0.15</td>
<td>0.02</td>
<td>1.92</td>
<td>0.069</td>
<td>0.982</td>
<td>0.714</td>
</tr>
<tr>
<td>0.522+</td>
<td>1</td>
<td>6.85</td>
<td>6.73</td>
<td>0.12</td>
<td>0.02</td>
<td>1.92</td>
<td>0.069</td>
<td>0.982</td>
<td>0.714</td>
</tr>
<tr>
<td>0.522−</td>
<td>2</td>
<td>12.4</td>
<td>6.73</td>
<td>5.65</td>
<td>0.46</td>
<td>1.92</td>
<td>0.876</td>
<td>0.511</td>
<td>0.712</td>
</tr>
<tr>
<td>0.5</td>
<td>2</td>
<td>12.4</td>
<td>6.74</td>
<td>5.63</td>
<td>0.45</td>
<td>1.91</td>
<td>0.876</td>
<td>0.512</td>
<td>0.712</td>
</tr>
<tr>
<td>0.25</td>
<td>2</td>
<td>12.1</td>
<td>6.82</td>
<td>5.31</td>
<td>0.44</td>
<td>1.86</td>
<td>0.879</td>
<td>0.530</td>
<td>0.713</td>
</tr>
<tr>
<td>0.1</td>
<td>2</td>
<td>9.15</td>
<td>8.35</td>
<td>0.80</td>
<td>0.09</td>
<td>1.20</td>
<td>0.909</td>
<td>0.905</td>
<td>0.732</td>
</tr>
</tbody>
</table>

**Welfare within each city** The shape of the city has thus apparently a little impact on welfare since, in the segregated city, what is lost from lower search efficiency is gained through lower commuting costs. We now investigate the issue of the optimality of the decentralized equilibrium within each land market equilibrium. The social planner chooses $\theta^k$ and $u^k$ that maximize (24) under the constraint (21). In this problem, the control variable is $\theta_k$ and the
state variable is $u_k$. The solution to this problem is given by (Wasmer and Zenou, 2002):

$$y - w_U = \frac{e}{q(\theta)} \left[ r + \delta + \theta^k q(\theta^k) \left( s_0 - s a x_b^k \right) \frac{\eta(\theta^k)}{1 - \eta(\theta^k)} \right] + x_b^k (\tau_L - \tau_U) \quad (28)$$

where $\eta(\theta^k) = -q'(\theta^k)q(\theta^k)/q(\theta^k)$ is the elasticity of the matching function in equilibrium $k = 1, 2$. In order to see if the private and social solutions coincide, we compare (23) and (28). We have the following result.

**Proposition 3** In a spatial search-matching model with endogenous search intensity, for each equilibrium $k = 1, 2$, the market equilibrium is efficient if and only if:

$$\eta(\theta^k) = \beta \quad (29)$$

Condition (29) is referred to as the Hosios-Pissarides condition and is exactly the same as in the standard matching model without space (Pissarides, 2000, chap. 8, Hosios, 1990). Indeed, in this model, market failures are only caused by search externalities, and the land market is thus efficient (Fujita, 1989). There are in fact two types of search externalities that must be considered: negative intra-group externalities (when more workers are looking for a job, the workers’ job-acquisition rate is reduced while when more firms are seeking to fill their vacancies, the firms’ vacancy-filling rate is reduced) and positive inter-group externalities (when more workers are looking for a job, the firms’ vacancy-filling rate is increased while when more firms are seeking to fill their vacancies, the workers’ job-acquisition rate is increased). For a class of related search-matching models, Hosios (1990) and Pissarides (2000) have established that these two externalities just offset one another in the sense that search equilibrium is socially efficient if and only if the matching function is homogenous of degree one and the worker’s share of surplus $\beta$ is equal to $\eta(\theta^k)$, the elasticity of the matching function. Of course, there is no reason for $\beta$ to be equal to $\eta(\theta^k)$ since these two variables are not related at all and, therefore, the search-matching equilibrium is in general inefficient. In particular, when $\beta$ is larger than $\eta(\theta^k)$, there is too much unemployment, creating congestion in the matching process for the unemployed workers. When $\beta$ is lower than $\eta(\theta^k)$, there is too little unemployment, creating congestion for firms.

In our present model, we have exactly the same search externalities (intra- and inter-group externalities). The spatial dimension does not entail any inefficiency and this is why the Hosios-Pissarides condition still holds in each equilibrium $k = 1, 2$, i.e. $\beta = \eta(\theta^k)$. 
3 Endogenous search intensity and housing consumption

The previous model can explain some of the stylized facts described in the introduction. In particular, we now understand better why unemployment rates vary within a city. It is because workers residing at different locations face different search and spatial frictions. We have also seen how city structure affects labor-market outcomes. Table 1 has illustrated how spatial frictions amplify search frictions and lead to different aggregate unemployment rates for different city structures.

One of the crucial assumptions of the previous model was that workers’ search intensity was a negative function of distance to jobs (see (5)). We have now understood the consequences of this assumption on workers’ labor market outcomes. However, we need to investigate if this is a reasonable assumption and how it can be derived from workers’ behavior. This is what is done in the present section. Based on Smith and Zenou (2003a), we would like to derive the search behavior of workers and show under which condition their search intensity depends on distance to jobs.

3.1 The model

We assume that all workers have identical preferences among consumptions bundles of land (housing), $h_j$, and composite good, $z_j$, for $j = L, U$, representable by a log-linear utility:

$$\Gamma(h_j, z_j) = h_j^\alpha z_j^\omega$$

(30)

with $\alpha, \omega > 0$, where it is also assumed that $\alpha + \omega < 1$. Compared to the previous section, $s$ is now included in the commuting costs, which implies that workers obtain job information by commuting to the CBD. In the previous section, $s$ was just affecting the job acquisition rate so that all information was obtained locally without the need to commute to the CBD. We have now: $\tau_L = \tau$ and $\tau_U = s\tau$, where, for example, searching every other day ($s = 1/2$) would yield an average daily travel cost of $\tau x/2$. By maximizing (30) under the budget constraint for each type of worker $j = L, U$, we obtain the following land demand for employed workers at $x$:

$$h_L(x) = \left( \frac{\alpha}{\alpha + \beta} \right) \left[ \frac{w_L - \tau x}{R(x)} \right]$$

(31)

In the previous section, $h_L = h_U = 1$. 

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and the following land demand for unemployed workers at $x$:

$$h_U(x) = \left( \frac{\alpha}{\alpha + \beta} \right) \left[ \frac{w_U - s\tau x}{R(x)} \right]$$

(32)

By plugging these values into (30), we can derive the following indirect utility:

$$W_L(x) = \chi (w_L - \tau x)^{\alpha + \omega} R(x)^{-\alpha}$$

(33)

for each employed worker at $x$, where $\chi = [\alpha/(\alpha + \omega)]^\alpha [\omega/(\alpha + \omega)]^\omega$ and the following indirect utility

$$W_U(x) = \chi (w_U - s\tau x)^{\alpha + \omega} R(x)^{-\alpha}$$

(34)

for each unemployed worker at $x$. The different Bellman equations for employed and unemployed workers are thus given by:

$$r I_L = W_L(x) - \delta [I_L - I_U(x)]$$

(35)

$$r I_U(x) = W_U(x) + s \theta q(\theta) [I_L - I_U(x)]$$

(36)

Since there are no relocation costs, at any urban equilibrium it has to be that $r I_L = r \bar{T}_L, \forall x,$ and $r I_U(x) = r \bar{T}_U, \forall x$. We have the following result (proved by Smith and Zenou, 2003a):

**Proposition 4**

(i) At each location $x$, there is a unique search intensity $s$ that maximizes (36).

(ii) For any prevailing job acquisition rate, $\theta q(\theta)$, and constant lifetime values, $\bar{T}_L, \bar{T}_U$, the optimal search intensity function, $s(x)$, for unemployed workers is given for each location, $x$, by

$$s(x) = \begin{cases} 
1 & \text{for } x \leq x(1) \\
\frac{(\alpha + \omega)}{[1-(\alpha + \omega)]} \left[ \frac{w_U}{(\alpha + \omega)\tau x} - \frac{s \bar{T}_U}{\theta q(\theta)(\bar{T}_L - \bar{T}_U)} \right] & \text{for } x(1) < x < x(s_0) \\
s_0 & \text{for } x \geq x(s_0)
\end{cases}$$

(37)

where

$$x(s) = \frac{w_U}{\tau} \cdot \frac{\theta q(\theta) (\bar{T}_L - \bar{T}_U)}{s [1 - (\alpha + \omega)] \theta q(\theta) (\bar{T}_L - \bar{T}_U) + (\alpha + \omega) r \bar{T}_U}$$

(38)

which is the unique inverse function of (37).
It is optimal for workers to have constant search intensities close and far away from the CBD. Figure 5 describes the way \( s(x) \) looks like. There is a non-linear decreasing relationship between the residential distance to jobs of the unemployed workers and their search intensity \( s \). In fact, individuals living sufficiently close to jobs search every day, \( s = 1 \), whereas those residing far away provide a minimum search intensity, \( s = s_0 \). Workers living in between these two areas experience a decrease in their search intensity from \( s = 1 \) to \( s = s_0 \). The intuition runs as follows. There is a fundamental trade-off between short-run and long-run benefits of various location choices for the unemployed workers. Indeed, locations near jobs are costly in the short run (both in terms of high rents and low housing consumption), but allow higher search intensities, which, in turn, increase the long-run prospects of reemployment. Conversely, locations far from jobs are more desirable in the short run (low rents and high housing consumption) but allow only infrequent trips to jobs and hence reduce the long-run prospects of reemployment. Therefore, for workers residing further away from the CBD, it is optimal to spend the minimal search effort whereas workers residing close to jobs provide high search effort. Compared to the previous section, we have endogeneized the relationship between \( s \) and \( x \) and given a mechanism that explains how search intensity decreases with distance to jobs.

\[ \text{[Insert Figure 5 here]} \]

For interior \( s^*(x) \), that is search intensity for workers living between \( x(1) \) and \( x(s_0) \), it is easy to verify that it negatively varies with commuting costs \( \tau \), the unemployment benefits \( w_U \), and the lifetime value of the unemployed \( T_U \), and positively with the hiring probability \( \theta q(\theta) \) and the lifetime value of the employed \( T_L \). The intuition is straightforward since when \( \tau \), \( w_U \) or \( T_U \) is high and when \( \theta q(\theta) \) or \( T_L \) is low, then workers reduce their search effort since either costs of searching are too high or the rewards of searching are too low. Of course these results hold for a given \( \theta \) and are not necessarily true in equilibrium since \( \theta \) will be itself a function of the different parameters like e.g. \( \tau \) and \( w_U \).

### 3.2 The different urban land use equilibria

Given this function \( s(x) \), let us know determine the residential locations of all unemployed and employed workers in the city. The basic trade off for the employed workers is now between commuting costs and housing consumption whereas for the unemployed workers, both commuting/search costs, housing consumption and search intensity matter. As usual, in order to determine the urban land use equilibrium, we have to define the bid rent function for each group of workers.
Given the utilities and lifetime values above, we now define the equilibrium bid-rents. It follows from (35), that the bid rent function for the employed workers at each location $x$, is equal to:

$$\Psi_L(x, T_U, T_L) = \left[ \frac{\chi(w_L - r x)^{\alpha + \omega}}{r T_L + \delta (T_L - T_U)} \right]^{1/\alpha}$$

(39)

As usual, this bid rent is decreasing in $x$ but is not anymore linear, which implies that more than two urban configurations can emerge. The bid rent function for unemployed workers is considerably more complex since it depends on the optimal search intensity level at each location. Using (36), we obtain the following bid rent function for the unemployed workers at each location $x$:

$$\Psi_U(x, T_U, T_L) = \left[ \frac{\chi[w_U - s(x) r x]^{\alpha + \omega}}{r T_U - s(x) \theta \phi(\theta) (T_L - T_U)} \right]^{1/\alpha}$$

(40)

where $s(x)$ is given by (37). It should be clear that the bid rents are calculated such that the lifetime utilities of both the employed and the unemployed workers, respectively, $T_U$ and $T_L$, are spatially invariant. Compare, for example, an unemployed worker residing close to jobs and another unemployed worker living far away from jobs. The former has a lower search (commuting) cost and a higher chance of finding a job but consumes less land whereas the latter has a higher search (commuting) cost and a lower chance of finding a job but consumes more land. The bid rent defined by (40) exactly compensates for these differences by ensuring that these two workers obtain the same lifetime utility $T_U$. This is however not true for the current utility of the unemployed workers $W_U(x)$ because, as can be seen by (34), the land rent does not compensate for $s(x)$. In fact, the unemployed workers residing close to jobs have a lower current utility than the ones living far away from jobs because they provide more search intensity (indeed, using (34), it is easy to see that $W'_U(x) > 0$). However, because they are searching more actively for jobs, they have a higher chance of obtaining a job, and thus, in the long-run, they compensate the short-run disadvantage, so that all unemployed workers obtain the same $T_U$.

With the non-linear bid rents defined by (39) and (40), different urban configurations can emerge. Indeed, the land market being perfectly competitive, all workers propose different bid rents at different locations and (absentee) landlords allocate land to the highest bids. So, depending on the different steepness of the bid rent functions (as captured by their slopes), at each location, the employed workers can outbid the unemployed workers or can be outbidden by the unemployed workers. An example of the equilibrium bid rent function is displayed in
Figure 6. In particular, this figure illustrates a case where unemployed workers occupy both a central core of locations and a peripheral ring of locations, separated by an intermediate ring of employed workers. Other urban configurations may also emerge. For example, the unemployed can occupy the core of the city and the employed the suburbs. The reverse pattern may also prevail.

[Insert Figure 6 here]

Since we want to focus on interesting urban configurations in which the unemployed workers can outbid the employed workers for peripheral land in equilibrium, we shall assume

$$w_L < \frac{w_U}{s_0}$$  \hspace{1cm} (41)

**Proposition 5** In equilibrium there are exactly three possible locational patterns:

(i) A central core of unemployed surrounded by a peripheral ring of employed,

(ii) A central core of employed surrounded by a peripheral ring of unemployed,

(iii) Both a central core and peripheral ring of unemployed separated by an intermediate ring of employed.

This proposition, proved by Smith and Zenou (2003b), shows that, in a framework where workers’ search intensity is endogenously chosen, different urban equilibrium configurations can emerge. In the first one (i), referred to as the *Integrated Equilibrium* (Equilibrium 1 depicted in Figure 1), the unemployed workers reside close to the CBD, have high search intensities and experience short unemployment spells. In the second one (ii), referred to as the *Segregated Equilibrium* (Equilibrium 2 depicted in Figure 2), the employed workers occupy the core of the city and bid away the unemployed workers in the suburbs. In that case, the latter tend to stay unemployed for a longer time since their search intensity is quite low. Finally, the third case (iii), referred to as the *Core-Periphery Equilibrium* (Equilibrium 3 depicted in Figure 6), is when there are two categories of unemployed workers: those who are short-run unemployed reside close to jobs while those who are long-term unemployed live at the periphery of the city.

For each equilibrium $k = 1, 2, 3$, we can calculate the steady-state labor equilibrium. The key variable that differs between the different equilibria is $\bar{s}^k$ since its value depends on the average location of the unemployed workers.
3.3 Discussion and stylized facts

This model can explain some of the stylized facts mentioned in the introduction. First, it is even clearer here how city structure affects workers’ labor-market outcomes. We have three different city structures, described by Figures 1, 2, and 6, which imply very different labor-market experiences for workers. We also better understand why workers living far away from jobs search less intensively than those residing closer to jobs. It is simply because they have less incentives since their land rent and housing consumption are low. There is also a new implication generated by this model: how labor-market variables impact on the growth of cities. Indeed, because housing consumption is endogenous, the city size is not anymore measured by $N$, the number of people living in the city, but has to include also the population density (which is the inverse of housing consumption).

To illustrate this issue, take equilibrium 2, where the unemployed workers reside far away from jobs, and, for simplicity, assume that $x^*_b \geq x(s_0)$, so that all the unemployed workers provide a search intensity equals to $s(x) = s_0$, $\forall x$. The definition of the urban equilibrium is as in Definition 1, where the agricultural land is normalized to one instead of zero, and the population density is now equal to $1/h_L(T^*_L, T^*_U)$ and $1/h_U(T^*_L, T^*_U)$ for the employed and unemployed workers, respectively. Observe that, in this definition, $\Psi_U(x^*_b, T^*_U, T^*_L)$ and $\Psi_L(x^*_b, T^*_U, T^*_L)$ are determined by (39) and (40), $h_L(T^*_L, T^*_U)$ and $h_U(T^*_L, T^*_U)$ by (31) and (32) where $R(x)$ has been replaced by the bid rents defined in (39) and (40). One can now close the model exactly as in the previous section. Therefore, the equilibrium labor-market variables $\theta^*$, $w^*_L$, and $u^*$ are still determined by equations (19), (20), and (22) with $\tau_L = \tau$, $\tau_U = s_0\tau$, $s_0 - saxb = s_0$, and $\bar{\tau} = s_0$.

Because the model gets quite complicated, let us run some simple numerical simulations. We use the same assumptions and numerical values as in Section 2.3 with some differences due to the specificity of the model. First, a stated above, search intensity is now independent of distance and fixed to $s_0 = 0.1$, which means that the unemployed workers have a very low search intensity (if the employed workers search everyday, then the unemployed workers search every ten days) because they all reside far away from jobs. As a result, by still assuming that the commuting costs $\tau$ are equal to 0.4, we have: $\tau_L = \tau = 0.4$ and $\tau_U = s_0\tau = 0.04$. Second, there are some new parameters, which are set as follows: $\alpha = \omega = 1/4$. We obtain the following equilibrium values:

$$\theta^* = 10.46; \ u^* = 0.24; \ w^*_L = 0.85; \ x^*_b = 0.26; \ x^*_f = 0.30$$

Not surprisingly, the unemployment rate is quite high (24 percent) because the unemployed
workers, who reside far away from jobs, do not search very intensively. The wage is relatively high (85 percent of the productivity) and the replacement ratio, \( w_U/w^*_L \), relatively low (35 percent). The employed workers, who live between \( x = 0 \) and \( x = x^*_b \), occupy 87 percent of the city. Indeed, even though 24 percent of the workers are unemployed, they occupy a smaller part of the city because their housing consumption is much smaller than that of the employed workers. This is mainly because the wage \( w^*_L \) is very high compared to the unemployed benefit.

Let us now examine how the labor market variables affect the size of the city, i.e. \( x^*_f \). Figure 7 reports some of the simulation results (not all). First, when the unemployment benefits \( w_U \) increase, the size of the city also increases (Figure 7a), even if the effect on \( x^*_b \) is extremely small. Indeed, an increase in \( w_U \) raises the unemployment rate \( u^* \) because firms create less jobs (\( \theta^* \) decreases) and, as a result, the housing consumption of the unemployed workers increases. Thus, even though the employment area is not affected (the wage and housing consumption of the employed workers increase but there are less of them), the unemployment area sharply increases, which, in turn, raises the size of the city. On the contrary, an increase in the firms’ entry cost \( c \) or the job-destruction rate \( \delta \), reduces both the size of the city (Figure 7a) and the employment area. Indeed, wages and job creation are reduced while unemployment increases, which mean that there is less competition in the land market and lower housing consumption for all workers. If we now look at the effect of search intensity \( s = s_0 \) on \( x^*_f \) (Figure 7b), we have exactly the opposite results, i.e. both \( x^*_b \) and \( x^*_f \) increase because of lower unemployment and higher wages. However, this increase is not very important because job creation decreases since it becomes more expensive to hire workers. Finally, it is interesting to examine how commuting costs \( \tau \) (which is a spatial variable) affect the growth of cities (Figure 7b). We find that both \( x^*_b \) and \( x^*_f \) decrease because of higher competition in the land market (the access to the CBD becomes more valuable), unemployment is increased and job creation reduced despite the increase in wages (firms need to compensate more workers for their commuting costs).

\[\text{[Insert Figure 7 here]}\]

Note that the unemployment rate is much higher than the one displayed in Table 1 because here \( s(x) = s_0 = 0.1, \forall x \), while, in Table 1, we had: \( s(x) = 1 - s_\alpha x \) with \( s_\alpha \in [0.1, 1] \).
4 High relocation costs

The models developed so far had no relocation costs. Although this assumption is quite frequent in urban economics, its relevance may depend on the nature of the labor market studied. Indeed, when unemployment and employment spells are short (i.e. a U.S. style labor market), it is not necessarily appealing: low-income households do not necessarily change their residential location as soon as they change their employment status. However, in a European context, long spells of employment and unemployment make it more likely that relocation and labor transitions coincide, in which case our benchmark assumption of zero mobility costs is relevant.

Let us therefore develop a model in which workers have very high-relocation costs so that they stay where they live when they change their employment status. As it is well-known in a Poisson process, the steady-state unemployment rate $u(x)$ and employment rate $1 - u(x)$ correspond to the fractions of time a worker remains unemployed and employed over his/her infinite lifetime. In the present model, each individual located in $x$ will spend $u(x)$ fraction of his/her time unemployed, where

$$u(x) = \frac{\delta}{\delta + s(x) \theta q(\theta)}$$

(42)

where, as in section 2, we assume that search intensity is a negative function of distance to jobs and given by (4). We are thus able to calculate the expected utility of workers. To do so, we assume perfect capital markets with a zero interest rate,\footnote{When there is a zero interest rate, workers have no intrinsic preference for the present so that they only care about the fraction of time they spend employed and unemployed. Therefore, the expected utilities are not state dependent.} i.e. $r \to 0$. With perfect capital markets, workers are able to smooth their disposable income over time so that at any moment in time, the disposable income of a worker is equal to his/her average net income over the job cycle. Therefore, the expected utility of a worker residing in $x$ is given by:

$$EW(x) = [1 - u(x)] W_L(x) + u(x) W_U(x)$$

$$= w_L(x) - \tau x - R(x) - \frac{\delta \{w_L(x) - w_U - [1 - s(x)] \tau x\}}{\delta + s(x) \theta q(\theta)}$$

(43)

where, as in the previous section, $\tau_L = \tau$ and $\tau_U = s(x) \tau$. Observe that, because workers are able to smooth their income over time, a worker’s residential location remains fixed as he/she enters and leaves unemployment. In other words, after a change in his/her employment status, the worker does not relocate.
Similarly, with zero interest rate, the expected profit of a firm $E\Pi$ is equal to:

$$E\Pi(x) = (1 - v)(y - w_L) + v(-c)$$

$$= y - w_L - \frac{\delta(y - w_L + c)}{\delta + q(\theta)}$$

(44)

where $v$, the vacancy rate, corresponds to the fraction of time a firm holds a vacancy over its lifetime. We can now determine the wage using the same Nash-bargaining rule as in the models of the two previous sections. We obtain (Zenou, 2007a):

$$w^*_L(x) = \beta(y + c) + (1 - \beta)\{w_U + [1 - s(x)]\tau x\}$$

$$= \beta(y + c) + (1 - \beta)\left[w_U + (1 - s_0)\tau x + s_0\tau x^2\right]$$

(45)

The spatial aspect of the wage is here captured by $[1 - s(x)]\tau x$, which implies that wages increase with distance to jobs and commuting costs. Indeed, $[1 - s(x)]\tau x$ is what firms must pay to induce workers to accept the job offer since they must exactly compensate the transportation cost difference between the employed and unemployed workers at each location $x$. Contrary to the previous models, the wage is now increasing with distance to jobs because workers are not perfectly mobile and are stuck in their location all their life.

The fact that firms offer transport-related fringe benefits, which include monetary and nonmonetary transport benefits (company cars, travel benefits, subsidized travel, etc.), is a common practice (Barber, 1998). For example, using information on firms’ recruitment strategy in the United Kingdom, van Ommeren et al. (2006) show that workers’ journey-to-work time induces firms to offer transport benefits to job applicants. Other evidence on the fact that firms reimburse workers’ commuting costs are documented by Rouwendal and Van Ommeren (2007) for the case of the Netherlands.

The fact that wages increase with distance to jobs is also a well-established empirical fact. For example, Manning (2003) using British data (the Labour force Survey for 1993-2001 and the British Household Panel Survey for 1991-2000) shows that an extra hour of commuting each day is associated, on average, with an increase in wages of 27 log points. This is even truer for highly educated workers since those with more education and in the higher-status occupations are more likely to have both high wages and a long commute. These results are consistent with the ones found in the United States. For instance, Madden (1985) uses the PSID to investigate how wages vary with distance to the CBD. She finds that, for all workers who changed job, there is a positive relationship between wage change and change in commute. Zax (1991), who uses data from a single company and regresses wages on commutes, also finds a positive relationship.
The exact mechanism behind these empirical results is however unclear. Our model stipulates that the positive relationship between wages and distance to jobs is due to the fact that firms need to compensate workers with longer commutes. This is obviously not the only explanation. For example, if one takes the viewpoint of workers, then it is likely that high-wage workers choose residential locations that are more distant from their workplace than lower-wage workers in return for greater housing consumption. The causality is not obvious. Do workers locate first and then are compensated by firms for their long commutes? Or do workers select themselves by income by locating far away from jobs? The answer to this question is clearly important and only an empirical study based on the present model could provide the answer.

Our model can then easily be closed as before by adding a condition on flows and a free-entry condition (for details, see Zenou, 2007a). This model can thus explain why wages vary within cities, even if workers are identical.

5 Discussion: The spatial mismatch hypothesis

The different models developed in this article can also provide some mechanisms explaining why black workers, who tend to live far away from jobs, experience high unemployment rate. This is known as the “spatial mismatch hypothesis”. Indeed, first formulated by Kain (1968), the spatial mismatch hypothesis states that, residing in urban segregated areas distant from and poorly connected to major centres of employment growth, black workers face strong geographic barriers to finding and keeping well-paid jobs. In the US context, where jobs have been decentralized and blacks have stayed in the central parts of cities, the main conclusion of the spatial mismatch hypothesis is that distance to jobs is the main cause of high unemployment rates. Since Kain’s study, hundreds of others have been conducted trying to test the spatial mismatch hypothesis (see, in particular, the literature survey by Ihlafeldt and Sjoquist, 1998). The usual approach is to relate a measure of labor-market outcomes, typically employment or earnings, to another measure of job access, typically some index that captures the distance between residences and centres of employment. The general conclusions are: (a) poor job access indeed worsens labour-market outcomes, (b) black and Hispanic workers have worse access to jobs than white workers, and (c) racial differences in job access can explain between one-third and one-half of racial differences in employment.

Despite this huge empirical literature, few theoretical models have been proposed (for a survey on the theoretical literature, see Zenou, 2006 and Gobillon et al., 2007). The models developed in this paper provide the two following mechanisms of why distance to jobs can
have adverse labor-market outcomes for ethnic minorities:

(i) Workers’ job search efficiency may decrease with distance to jobs. Indeed, in section 2, we have shown that, for a given search effort level, workers who live far away from jobs have less chance to find a job because they obtain poor information on distant job opportunities. Observe that the social isolation of inner-city neighborhoods can also impact on local social interactions and thus explain the lack of good job contacts. Indeed, since high-poverty neighborhoods are usually poorly connected to job centers and characterized by low employment rates, then black workers who live in these neighborhoods may have few friends who are employed and can help them finding a job (Topa, 2001; Selod and Zenou, 2006).

(ii) Workers residing far away from jobs may not search intensively. Indeed, in section 3, we have seen that housing prices decrease with distance to jobs, and thus distant workers feel less pressured to search for a job in order to pay their rent.

It should be clear that, using mechanisms (i) or (ii), one can easily understand why, in the same city, ethnic minorities experience higher unemployment rates than workers from the majority group. Indeed, if for some reason ethnic minorities reside far away from jobs (because they are discriminated against in the housing market or because they want to live together), then they will search less intensively because either they have less information about jobs or it is optimal to do so. As a result, they will leave unemployment at a slower rate and thus experience higher unemployment rates.

By highlighting these two mechanisms, we have learned how distance to jobs affects the labor-market outcomes of ethnic minorities. This is important because it leads to interesting testable predictions. Indeed, instead of testing how some measure of job accessibility affects workers’ unemployment rate, we can here see if job information decreases with distance to jobs and if search intensity is affected by housing prices and housing consumption.

There are some tests of the spatial mismatch hypothesis that confirm the two mechanisms highlighted above. Concerning (i), Rogers (1997) and Immergluk (1998) argue that, for informational reasons, the workers who reside close to jobs remain unemployed for a shorter period of time. Stoll and Raphael (2000) show that whites have a better job-search quality than blacks because they search in areas where employment growth is higher and that the difference in spatial job search quality between whites and blacks explains nearly 40% of the difference in their employment rates. Holzer and Reaser (2000) show that less educated black males (who search less in the suburbs) are less likely to be hired in the suburbs. They attribute this result to low information flows (but also to higher costs of applying). Concerning (ii), using English sub-regional data, Patacchini and Zenou (2005, 2006) empirically
confirm that living in areas where housing prices are higher induces workers to search more for a job: a one-standard deviation increase in housing prices raises search intensity by about one third of a standard deviation.

However, in these empirical applications, measuring search intensity is a difficult task. For example, in the studies by Patacchini and Zenou (2005, 2006), the geographical areas are quite large and search intensity is an aggregate variable measured by the ratio between active job seekers and “potential” active job seekers (i.e. non-employed workers unavailable for work for no valid reason). Ideally, one needs to measure search intensity at the individual level, by having the number of hours spent looking for a job. So, in the future, an interesting test based on mechanism (ii) could be performed at the individual level to see if indeed the search activities of the unemployed workers are affected by housing prices and housing consumption.

6 Other contributions

The results of the models exposed so far hinge on different assumptions that we would like to relax now.

Skill heterogeneity. It was assumed throughout this paper that workers were identical in terms of skills. Sato (2001, 2004) develops a model similar to that of section 2, but assumes that workers are ex ante heterogenous in terms of skills. Then, when a match occurs, workers pay all the training costs, so that, ex post, every worker-firm pair attains the same level of productivity, and all employed workers receive the same wage. Contrary to the model of section 2, Sato (2001, 2004) shows there are two sources of inefficiencies. First, as before, there is a job-creation inefficiency where, depending on the value of workers’ bargaining power, firms create too many or too little jobs compared to the efficiency solution. Second, and this is new, there is another source of inefficiency due training costs. Indeed, at the market solution, workers tend to refuse socially beneficial jobs because, while workers bear all the training costs, the revenues from production are divided through a bargaining between firms and workers.

One of the drawbacks of these papers is that, ex post, workers are identical and obtain the same wage. As the result, the urban land use equilibrium is as before and these models cannot explain the location of workers with different skills/incomes. Borrowing tools from the product differentiation literature, especially from Salop (1979), Kim (1990, 1991) and Brueckner et al. (2002) relax this assumption and consider an urban matching model were
workers and firms are ex ante and ex post heterogenous. Kim (1990, 1991) shows that, because of increasing returns to scale and specialized production methods, as the number of workers in the market increases, the average match between heterogeneous job requirements of firms and heterogeneous skill characteristics of workers improves. Thus, training costs decrease and net productivity increases as the size of the market increases. As a result, the wage rate increases with the size of the city, because the larger the city, the better the match between the diverse job requirements and the diverse labor pool.12 Focussing on a different issue, Brueckner et al. (2002) show how heterogeneity in the skill space is mirrored in the residential-location choices of workers, drawing a connection between outcomes in the land and labor markets. In particular, low-skill workers have long commute trips, which yield a low wage net of training and commuting costs. Low skill workers are therefore distant from firms in both the skill and urban spaces. Because such workers thus live on the urban periphery, this model provides a rationale for the existence of socio-economic ghettos, occupied by workers who are socially and physically distant from their employers (Akerlof, 1997). This twofold segregation is found in a number of European cities, where high-income residents reside near the center and lower income workers live on the outskirts of the city.

**Polycentric cities** All models in this paper used the monocentric city paradigm. A question naturally arises. Would polycentric city configurations, or mixed city configurations (where firms and consumers co-habit locations), overturn the main location results in the paper? There are in fact very few papers that have a complete analysis of land and labor markets in a non-monocentric city. Exceptions include Smith and Zenou (1997), Brueckner and Zenou (2003), Coulson et al. (2001), where the latter uses a search-matching model whereas the others develop an efficiency wage model. Smith and Zenou (1997) and Brueckner and Zenou (2003) show that, apart from the CBD, a second employment center can emerge in equilibrium. In that case, under some condition, there can exist a core-periphery equilibrium where the employed workers live close to jobs whereas the unemployed workers reside further away from jobs. In Coulson et al. (2001), a land market is not explicitly introduced, and it is shown that, by assuming a higher entry cost for firms in the CBD, the unemployment rate is higher for central city residents than for suburban residents, and workers in the suburbs earn a higher wage than in the CBD. These facts are compatible with the spatial mismatch hypothesis since black workers are over-represented in the central part of cities.

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12See also Helsley and Strange (1990) and Wheeler (2001) for similar models showing that urban agglomeration enhances productivity by facilitating the firm-worker matching process.
7 Concluding remarks

In this article, we have exposed different models that improve our understanding of urban spatial structures and commuting patterns and show how they can be useful in explaining the stylized facts mentioned in the introduction. This literature on urban search models is however very recent and there are some open questions that need to be answered in the future. First, in none of these models, the employed workers were allow to search for jobs (i.e. on-the-job search) to improve their current wages. This is an important feature of labor markets, especially in Europe, where plenty of workers find a job while already employed (Cahuc and Zylberberg, 2004). As shown by Burdett and Mortensen (1998), including on-the-job search has important consequences on the wage distribution and, as a result, on the location of workers in cities. Second, very few models have dealt with the issue of non-monocentric cities. Most cities in the world tend to be more and more decentralized (White, 1999), which means that jobs and workers tend to be located at the outskirts of the city. More models need to be written to better understand these issues. We have seen that, within a monocentric city, the urban spatial structure strongly affects workers’ labor market outcomes. What about polycentric cities? Are labor-market outcomes very different between a monocentric city like e.g. Paris and a very decentralized city like e.g. Los Angeles? Finally, it has been assumed throughout that only one transport mode was available. It is well-documented that, both in Europe and the US, ethnic minorities tend to mostly use public transportation while whites take their car to commute to their workplace (Patacchini and Zenou, 2005; Raphael and Stoll, 2001). This has certainly important consequences on the search behavior of workers and their labor-market outcomes. For example, it may well be that ethnic minorities may refuse jobs that involve too long commutes. This will limit their horizon of search and can have dramatic consequences in the labor market.13 Also, if the city is non-monocentric, commuting patterns become more complex and the interaction between urban and labor markets even more crucial.

References


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13Without introducing a land market, Zenou (2007b) has developed a model along these lines.


Figure 1. Urban equilibrium 1 (The integrated city)
Figure 2. Urban equilibrium 2 (The segregated city)
Figure 3. Steady-state labor equilibrium 1
Figure 4: Steady-state labor equilibrium 2
Figure 5. Optimal search intensities
Figure 6. The core-periphery equilibrium

\[ R(x) \]

\[ \Psi_U(x, \bar{I}_U, \bar{I}_L) \]

\[ \Psi_L(x, \bar{I}_U, \bar{I}_L) \]

0  Employed  Long-term unemployed  \( x_f = N \)

Short-term unemployed
Figure 7a. Impact of labor-market variables on the growth of cities
Figure 7b. Impact of search intensity and commuting costs on the growth of cities