JOB REALLOCATION, EMPLOYMENT FLUCTUATIONS AND UNEMPLOYMENT*

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Abstract

The purpose of this chapter is twofold. First, it reviews the model of search and matching equilibrium and derives the properties of employment and unemployment equilibrium. Second, it applies the model to the study of employment fluctuations and to the explanation of differences in unemployment rates in industrialized countries.

The search and matching model is built on the assumptions of a time-consuming matching technology that determines the rate of job creation given the unmatched number of workers and jobs; and on a stochastic arrival of idiosyncratic shocks that determines the rate of job destruction given the wage contract between matched firms and workers. The outcome is a model for the flow of new jobs and unemployed workers from inactivity to production (the 'job creation' flow) and one for the flow of workers from employment to unemployment and of jobs out of the market (the 'job destruction' flow). Steady-state equilibrium is at the point where the two flows are equal.

The model is shown to explain well the employment fluctuations observed in the US economy, within the context of a real business cycle model. It is also shown that the large differences in unemployment rates observed in industrialized countries can be attributed to a large extent to differences in policy towards employment protection legislation (which increases the duration of unemployment and reduces the flow into unemployment) and the generosity of the welfare state (which reduces job creation). It is argued that on the whole European countries have been more generous in their unemployment support policies and in their employment protection legislation than the USA. The chapter also surveys other reasons given in the literature for the observed levels in unemployment, including mismatch and real interest rates.

Keywords

*JEL classification: J63, J64, J65, J68, E24, E32, J41*
Introduction

Market economies experience large employment fluctuations and average unemployment rates that are often different from those experienced by apparently similar economies. The search and matching framework provides a convenient lens through which to view explanations of such differences. Our purpose in this chapter is twofold: first, to present the essential concepts that underlie the framework and, second, to use the framework to suggest answers to the questions posed by the data.

Existing employment relationships command monopoly rents because of search and recruiting investments, hiring and firing costs, and other forms of match-specific human capital formation. The surplus that accrues is allocated between the parties to the employment relationship by a wage contract. Given a particular wage rule, employers provide jobs and recruit workers while workers search for employment. At the same time, an existing employer-worker match ends when sufficiently bad news arrives about their expected future. These job creation and job destruction decisions generate worker flows into and out of employment which depend on the current value of the employed stock. When the two flows differ, employment dynamics are set in motion which, under a reasonable set of conditions, lead to a unique steady-state employment level. These properties characterize the equilibrium model of job creation and job destruction applied in the chapter.

The search and matching approach owes its origins to the pioneering works of Stigler (1962), Phelps (1968) and Friedman (1968) and was already at an advanced state when the Phelps et al. (1970) volume was published. The equilibrium analysis of the current vintage of models, however, did not start until the early 1980s, when models by Diamond (1982a,b), Mortensen (1982a,b) and Pissarides (1984a,b) explored the properties of two-sided search and characterized the nature and welfare properties of market equilibrium. Despite a flurry of activity since then, there are still many important questions that are unexplored. One such question is the dynamics of worker movement in and out of the labor force, of which, despite its empirical importance [Clark and Summers (1979), Blanchard and Diamond (1989)] and some attempts to model it by Burdett et al. (1984), Pissarides (1990, Chapter 6) and Andolfatto and Gomme (1996), our knowledge is still scant.

Virtually all search equilibrium models assume an exogenous labor force, which is used to normalize all aggregate quantities, and model either the equilibrium employment or unemployment rate. It is simple enough to superimpose on this structure a neoclassical labor-supply decision, as is done, for example, by Andolfatto (1996) and Merz (1995), but still the worker flow from the labor force to out of the labor force is ignored. Given this restriction, we can interchangeably talk either about employment equilibrium or about unemployment equilibrium. In the latter case, the equilibrium is often referred to as a “natural rate” equilibrium, following Friedman’s introduction of the term in 1968. Indeed, the equilibrium that we shall describe corresponds closely to the one advocated by Friedman (1968) and Phelps (1967, 1968).
We attribute the observed fluctuations in employment (or unemployment) to fluctuations in the natural rate, i.e., we ignore inflation and expectation errors. The driving force in the search and matching models that we describe is virtually without exception a real productivity or reallocation shock [but see Howitt (1988) for an exception]. The reason for this is partly that models with real shocks calibrate the data fairly well but also, more importantly, that the search and matching approach is about the transmission and propagation mechanisms of shocks, not about their origins. It is then more convenient to take the simplest possible shock in this framework, which is a proportional productivity shock, as driving force and concentrate on the dynamics and steady states implied by the model – than dwell on debates of whether employment cycles are due to real or monetary shocks.

Section 1 summarizes the aggregate data for OECD countries. Section 2 contains the core equilibrium matching model with a surplus sharing wage contract. The fundamental determinants of the natural rate of unemployment are reviewed in this section. In Section 3 the model is embedded into a real business cycle model and its consistency with some recent facts on job flows is reviewed. In section 4 we review calibrations of the model and introduce capital accumulation. In section 5 we consider technological innovation and its employment effects. Finally, in Section 6, we return to OECD data and examine the model’s implications for the facts noted in Section 1. How far can the model explain those facts and what remains to be done? We find that although a lot can be explained and the framework of search and matching models is a convenient device for studying those facts, a lot remains for future work.

1. OECD facts

What are the main facts about employment and unemployment that the search and matching approach can help explain? In Figures 1 and 2, labor force and unemployment time series from 1960 to 1995 are illustrated for the USA, Japan and a weighted average of the four largest European economies, Germany, France, Italy and the United Kingdom.

The four European countries are grouped together because their unemployment and labor force experiences have been sufficiently similar to each other. Comparable data can also be found for most of the other members of the European Union, in particular Spain, the Benelux, and Scandinavian countries. The experience of Spain and the Scandinavian countries, however, has been different from that of the big four, essentially for labor-market policy reasons. In Spain, excessive safeguarding of the rights of workers through the legal system created a sharp distinction between insiders and outsiders and led to low aggregate job creation. In the Scandinavian countries large-scale active labor market policies held unemployment artificially low until recently. Since we shall not address issues of labor market policy in any detail, we decided not to aggregate those countries with the four large economies. The experience
of the Benelux countries is sufficiently close to that of France and Germany, to the extent that data from them will not add to the information given here.

Figure 1 shows the sharply contrasting participation experience of the USA on the one hand and Europe and Japan on the other. Whereas in the early 1960s the participation rates in Europe and the USA were essentially the same, since then participation in the USA has been on an upward trend and in Europe on a downward trend. The upward trend in the USA was driven largely by the female participation rate, whereas in Europe, where female participation rates have also increased, the downward trend was driven by early retirements among men and by later school leaving. In Japan the participation rate is uniformly above the European rate but its dynamic behavior since 1960 has been very similar to the European rate. The figure also shows some evidence of cyclical variations in the participation rate: it is these cyclical movements that the search and matching approach could in principle handle, but has so far ignored.

The trend changes, in the USA in particular, are more likely the outcome of lifetime labor supply decisions that are independent of the labor market frictions that underlie the search and matching approach. In Europe, however, much of the decline in labor-force participation has been the result of policy incentives or of private responses to the rise in unemployment ("discouragement"). The policy to encourage early retirement was also largely in response to the rising unemployment, so the fall in participation can be partly attributed to the same factors that increased unemployment during this period. But exogenous labor supply changes have also played an important role, as comparison of Figures 1 and 2 shows. The trend decline in labor force participation
began before the unemployment rise and it was accompanied by a fall in annual hours of work for those that remained in the labor force.

We shall briefly return to the question of participation changes in the final section. In the remainder of this section we look at the behavior of unemployment and at gross job creation and job destruction flows normalized by the labor force. The aim is to point out key features of the data ("stylized facts") that will guide the model presentation in the rest of the chapter.

Figure 2 plots the unemployment rates for the three country groups, as far as possible adjusted to the same (US) definition. Table 1 gives data for more countries for two periods that were approximately in the same cyclical phase. The contrast is clear. Whereas in the USA and Japan unemployment is a cyclical variable without trend, in Europe the biggest changes in unemployment over the last thirty years were due to changes in the average level of unemployment across cycles. This latter feature of the European time series led most who analyzed this problem to conclude that the changes in European unemployment are changes in the "natural rate", not changes in its cyclical component [Layard et al. (1991), Phelps (1994), Blanchard and Katz (1997)]. The approach that we describe in this chapter is motivated by this observation and is especially suitable for the analysis of changes in the natural rate. Inflation, expectations errors and other nominal influences are ignored.

The net changes in employment over the cycle conceal large movements in gross job creation and job destruction, as well as worker turnover for other reasons. Information on this feature of labor markets sheds light on the appropriate flow models that should be used to analyze aggregate labor market changes. This feature of labor markets has been emphasized by Davis and Haltiwanger (1992) in particular, but also by others.
Ch. 18: Job Reallocation, Employment Fluctuations and Unemployment

Table 1
OECD unemployment, 1974–79 to 1986–90\textsuperscript{a,b}

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Europe</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Austria</td>
<td>1.5</td>
<td>81.8</td>
</tr>
<tr>
<td>Belgium</td>
<td>6.3</td>
<td>41.1</td>
</tr>
<tr>
<td>Denmark</td>
<td>5.5</td>
<td>44.7</td>
</tr>
<tr>
<td>Finland</td>
<td>4.4</td>
<td>-2.3</td>
</tr>
<tr>
<td>France</td>
<td>4.5</td>
<td>77.8</td>
</tr>
<tr>
<td>Germany</td>
<td>3.2</td>
<td>61.2</td>
</tr>
<tr>
<td>Ireland</td>
<td>7.6</td>
<td>75.7</td>
</tr>
<tr>
<td>Italy</td>
<td>4.6</td>
<td>51.5</td>
</tr>
<tr>
<td>Netherlands</td>
<td>5.1</td>
<td>54.5</td>
</tr>
<tr>
<td>Norway</td>
<td>1.8</td>
<td>66.5</td>
</tr>
<tr>
<td>Spain</td>
<td>5.3</td>
<td>126.1</td>
</tr>
<tr>
<td>Sweden</td>
<td>1.5</td>
<td>12.5</td>
</tr>
<tr>
<td>Switzerland</td>
<td>1.0</td>
<td>64.2</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>5.1</td>
<td>54.5</td>
</tr>
<tr>
<td>Others</td>
<td></td>
<td></td>
</tr>
<tr>
<td>USA</td>
<td>6.7</td>
<td>-14.4</td>
</tr>
<tr>
<td>Canada</td>
<td>7.2</td>
<td>14.2</td>
</tr>
<tr>
<td>Australia</td>
<td>5.0</td>
<td>36.4</td>
</tr>
<tr>
<td>Japan</td>
<td>1.9</td>
<td>27.4</td>
</tr>
</tbody>
</table>

\textsuperscript{a} Source: Layard et al. (1991), p. 398.
\textsuperscript{b} The table shows the average of annual unemployment in 1974–1979 and the change in the log of this average from 1974–79 to 1986–90.

since then. Recently Contini et al. (1995) have assembled data on job reallocation for several countries. Their summary Table is shown in Table 2 and the results are also summarized in Figure 3.

For most countries the job flow data are calculated from establishment level flows, though for some only firm-level data were available. Annual gross job creation reflects employment change only in the establishments or firms that are new entrants or that have experienced an increase in employment over the period. The job creation rate is defined as the sum of the gross increase in employment expressed as a percentage of the total labor force. Similarly, gross job destruction includes only units that have experienced a decrease in employment and the job destruction rate is equal to the gross decrease in employment as a percentage of the employment level. By definition
Table 2
Net and gross job flows, OECD, late 1980s

<table>
<thead>
<tr>
<th>Country</th>
<th>Period</th>
<th>Job creation</th>
<th>Job destruction</th>
<th>Net job creation</th>
<th>Gross reallocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>UK</td>
<td>1985–1991</td>
<td>8.70</td>
<td>6.60</td>
<td>2.10</td>
<td>15.30</td>
</tr>
<tr>
<td>Germany</td>
<td>1983–1990</td>
<td>9.00</td>
<td>7.50</td>
<td>1.50</td>
<td>16.50</td>
</tr>
<tr>
<td>Finland</td>
<td>1986–1990</td>
<td>10.40</td>
<td>12.00</td>
<td>-1.60</td>
<td>22.40</td>
</tr>
<tr>
<td>Italy</td>
<td>1984–1993</td>
<td>11.90</td>
<td>11.09</td>
<td>0.81</td>
<td>22.99</td>
</tr>
<tr>
<td>USA</td>
<td>1984–1991</td>
<td>13.00</td>
<td>10.40</td>
<td>2.60</td>
<td>23.40</td>
</tr>
<tr>
<td>France</td>
<td>1984–1992</td>
<td>13.90</td>
<td>13.20</td>
<td>0.70</td>
<td>27.10</td>
</tr>
<tr>
<td>Sweden</td>
<td>1985–1992</td>
<td>14.50</td>
<td>14.60</td>
<td>-0.10</td>
<td>29.10</td>
</tr>
<tr>
<td>Denmark</td>
<td>1983–1989</td>
<td>16.00</td>
<td>13.80</td>
<td>2.20</td>
<td>29.80</td>
</tr>
<tr>
<td>New Zealand</td>
<td>1987–1992</td>
<td>15.70</td>
<td>19.80</td>
<td>-4.10</td>
<td>35.50</td>
</tr>
</tbody>
</table>

Source: Contini et al. (1995), Table 3.1 [derived mainly from OECD Employment Outlook (1987, 1994)].

then, the net growth rate in employment is the difference between the job creation rate and the job destruction rate. International comparisons of data of this kind are fraught with difficulties and Contini et al. (1995, p. 18) warn that the numbers for Japan and the United Kingdom are probably understated and for France and New Zealand are overstated. So if anything, the small differences shown in Table 2 are likely to be overstated.

Notwithstanding the statistical problems, the results show that Japan has low gross job creation and job destruction rates, despite high net job creation. The United Kingdom and Germany, also with positive net job creation, have low gross flows. But the rest of the countries have high gross job flows, comparable to those of the USA. There does not seem to be any relation between the volume of gross reallocation and the net employment change, and the USA does not appear unusually turbulent when compared to other countries. These findings are also illustrated in Figure 3.

Some regularities emerge from the international comparison of job creation and job destruction rates. These findings apply to comparisons of economy-wide job creation and job destruction flows but are also consistent with the more detailed analysis of Davis et al. (1996) for US manufacturing flows.

First, the flow data always exclude the public sector, where job reallocation is small. In some European countries the public sector employs a large fraction of the labor force (8% in Japan, 8.5% in the USA, 7.9% in Germany, 11% in the UK, 22.5% in Italy; the highest share in the European Union is in Denmark, 31%).
Second, gross job reallocation is inversely correlated with capital intensity: service jobs create and destroy more jobs than manufacturing does.

Third, smaller and younger establishments create and destroy more jobs than larger and older plants; about one-third of job creation and job destruction is due to plant entry and exit. So in international comparisons countries with a larger fraction of smaller firms (e.g. Italy) are likely to have a larger job reallocation rate than countries with larger firms (e.g. the USA).

Fourth, at the individual level, the main cause of job turnover is idiosyncratic shocks, i.e. shocks that do not appear correlated with common economy-wide or sector-specific shocks, or with other common characteristics across firms. The implication of this fact is that the regularities listed above, as well as the business cycle, explain less than half the variance of gross job creation and job destruction across production units. Aggregate and cyclical shocks explain a small fraction of the variance, about 10 percent. Measurable firm characteristics, such as size and age, explain more, but still less than half.

Fifth, although younger plants are more likely to create and destroy jobs, there is large persistence in job creation and job destruction. The idiosyncratic shocks that cause job reallocation do not reverse shortly after they occur. In both the USA and Italy (the only two countries with comparable data on this issue), about 70 percent of
jobs created in one year are still active the next year and about 55 percent are active two years later. Persistence rates for job destruction are slightly higher.

The cyclical properties of job flows, which is of primary concern in the analysis of employment fluctuations, are not clear-cut in the empirical data so far assembled. A fact that seems to be universal is that job creation and job destruction flows are negatively correlated with each other. Thus, recessions are times when job destruction rates rise and job creation rates fall, and vice versa for expansions. More controversial, but potentially more interesting, is the finding that job destruction is more "volatile", in the sense that even when abstracting from growth, the length of time when job destruction is the dominant flow is shorter than the length of time when job creation is the dominant flow. Since on average over the cycle job destruction and job creation rates must be equal, it follows that job destruction rates must peak at higher values than job creation rates, which are more flat. This asymmetry is consistent with the observation that recessions are on average of shorter duration than booms and has attracted a lot of attention in the empirical literature, where, following Davis and Haltiwanger (1990), it is often reported as a negative correlation between gross job reallocation and net job reallocation. However, the negative correlation, although a strong feature of the US manufacturing data, is not universal. The "asymmetry" of job creation and job destruction rates here is simply taken to mean that the difference between job destruction and job creation when positive is larger and of shorter duration than when it is negative.

One final observation on the international comparison of job flows is of interest. There does not appear to be a significant correlation across countries either between the level of unemployment on the one hand and the gross job reallocation rate on the other or between labor productivity growth and the job reallocation rate. There does seem, however, to be a correlation between the gross job reallocation rate and the rate of long-term unemployment: countries with lower job reallocation rates seem to have, on average, longer unemployment durations [Garibaldi et al. 1997].

Comparative data on worker flows are even less reliable than comparative data on job flows, even though the definition of worker flows can be a lot less ambiguous than the definition of job flows. The gross flow of workers in and out of employment, defined analogously to the gross flow of jobs, is necessarily larger than the job flow. The difference is, however, large. Contini et al. (1995, p. 108) report that in both the USA and the major European economies, the worker flow is about three times as big as the job flow. There is some evidence that worker flows are bigger for the USA than for the European countries or Japan, and also that in the USA there is more movement in and out of unemployment and the labor force. The latter claim, however, may be based on the different kind of question that is often asked about participation in national surveys. Two interesting aggregate facts that have emerged from the study of worker flows, bearing in mind the paucity of the data, are that gross unemployment flows rise in recession and fall in the boom, whereas flows into employment are strongly pro-cyclical and separations mildly pro-cyclical or neutral. Of course, because the stock of unemployment rises in recession as well, the average rate at which workers leave
unemployment goes down, even though the gross number of exits goes up. The finding about employment flows is explained by the fact that in the boom job creation is up and voluntary job-to-job quits are also up, leading to more inflows; whereas in recession quits are sufficiently down but job destruction up giving rise to conflicting influences on separations.

Still, substantial systematic cross-country differences between unemployment inflow and outflow rates do exist, reflecting underlying differences in unemployment incidence and duration between Europe and the USA. In Figure 4, borrowed from Martin (1994), inflow–outflow rate combinations in 1992 are plotted for the OECD countries. These plots show that although the average length of unemployment spells (the inverse of the outflow rate) is much longer in the typical EU country than in the USA, the probability of job loss (to the extent reflected by the inflow rate) is much smaller. Hence, long spells of unemployment rather than more frequent spells is the reason for higher unemployment in the EU relative to the USA.

The contrasting experience of unemployment in the USA and Europe is reflected in contrasting experience in wage growth. The fall in US real earnings at the bottom end of the wage distribution, in contrast to growth in Europe, has been documented by many writers and by the OECD in its official publications [see, e.g. OECD (1994), Chapter 5]. We show in our Figure 5 a feature of wage and unemployment behavior...
that should be explainable within the search and matching framework, though to our knowledge there are as yet no models that claim to explain it fully. We make an attempt to explain it in Section 5.1 [see also Mortensen and Pissarides [1999]]. Thus, for twelve OECD countries with comparable data on wage inequality, there appears to be a close correlation between the percentage change in wage inequality during the 1980s and the percentage rise in unemployment. Wage inequality is measured by the ratio of the earnings of the most educated group in the population to the least educated [usually, university graduates versus early school leavers; see OECD (1994), p. 160-1]. Other measures of inequality, however, give similar results [e.g. OECD (1994), p. 3; the results in Galbraith (1996), are also consistent with our claim, despite his claim to the contrary, if one measures the change in inequality by the change in the Gini coefficient of the wage distribution].

Figure 5 shows that the USA, Canada and Sweden experienced the biggest rises in inequality and the smaller rises in unemployment (fall in the USA). Japan and Australia come next, with moderate rises in both, and the European countries follow, with small rises or falls in inequality but big rises in unemployment. The only country that does not conform to this rule is the United Kingdom, which experienced North-American style increase in inequality and European-style increase in unemployment over the sample of the chart. Recently, however, unemployment in the UK has fallen substantially, giving support to the view that the reforms of the 1980s moved the United
Kingdom closer to a US style economy but had their impact first on inequality and only more recently on unemployment.

2. The equilibrium rate of unemployment

Here we introduce the formalities of the search and matching approach and derive the equations that express the dynamics of the stock of unemployment (or employment). This analysis will point to the variables that need to be explained in order to arrive at an equilibrium characterization of employment flows and unemployment levels. We shall talk explicitly about unemployment, with the solution for employment implied by the assumption of an exogenous labor force.

The search and matching approach to aggregate labor market analysis is based on Pissarides' (1990) model of equilibrium unemployment as extended by Mortensen and Pissarides (1994) to allow for job destruction. The approach interprets unemployment as the consequence of the need to reallocate workers across activities and the fact that the process takes time. The model is founded on two constructs, a matching function that characterizes the search and recruiting process by which new job–worker matches are created and an idiosyncratic productivity shock that captures the reason for resource reallocation across alternative activities. Given these concepts, decisions about the creation of new jobs, about recruiting and search effort, and about the conditions that induce job–worker separations can be formalized.

The job–worker matching process is similar to a production process, in which “employment” is produced as an intermediary production input. The output, the flow of new matches, is produced with search and recruiting efforts supplied by workers and employers respectively. As a simple description, the existence of a market matching function is invoked, an aggregate relation between matching output and the inputs. Under the simplifying assumption that all employers with a vacancy recruit with equal intensity and that only unemployed workers search, also at a given intensity, aggregate matching inputs can be represented simply by the numbers of job vacancies $v$ and of unemployed workers $u$.

Let the function $m(v, u)$ represent the matching rate associated with every possible vacancy and unemployment pair. As in production theory, it is reasonable to suppose

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1 Of course, that at least some unemployment is due to “frictional” factors has always been recognized. Lilien (1982) was among the first to claim that even “cyclical” unemployment was of this kind. Although his results have been criticized, e.g. by Abraham and Katz (1986) and Blanchard and Diamond (1989), the modern approach to unemployment groups all kinds of unemployment into one, as we do here.
that this function is increasing in both arguments but exhibits decreasing marginal products to each input. Constant returns, in the sense that

\[ m(v, u) = m \left( 1, \frac{u}{v} \right) v \equiv q(\theta) v \quad \text{where} \quad \theta = \frac{v}{u}, \tag{2.1} \]

is a convenient additional assumption, one that is consistent with available evidence. The ratio of vacancies to unemployment, \( \theta \), market tightness, is an endogenous variable to be determined.

On average, a job is filled by a worker at the rate \( m(v, u)/u = q(\theta) \) and workers find jobs at rate \( m(v, u)/u = \theta q(\theta) \). By the assumption of a constant returns matching function, \( q(\theta) \) is decreasing and \( \theta q(\theta) \) increasing in \( \theta \). \( \theta q(\theta) \) represents what labor economists call the unemployment spell duration hazard. The duration of unemployment spells is a random exponential variable with expectation equal to the inverse of the hazard, \( 1/\theta q(\theta) \), a decreasing function of market tightness. Analogously, \( q(\theta) \) is the vacancy duration hazard and its inverse, \( 1/q(\theta) \) is the mean duration of vacancies.

As noted above, the most important source of job–worker separations is job destruction attributable to an idiosyncratic shock to match productivity. Because initial decisions regarding location, technology, and/or product line choices embodied in a particular match are irreversible, subsequent innovations and taste changes, not known with certainty at the time of match formation, shock the market value of the product or service provided. For example, the initial decision might involve the choice of locating a productive activity on one of many “islands”. In future, island-specific conditions that affect total match productivity, say the weather, may change. If the news about future profitability implicit in the shock is bad enough, then continuation of the activity on that particular island is no longer profitable. In this case, the worker loses the job.

To model this idea, we assume that the productivity of each job is the mathematical product of two components, \( p \), which is common to all jobs, and \( x \), which is idiosyncratic. The idiosyncratic component takes values in the range \([0, 1] \), it is distributed according to the c.d.f. \( F(x) \) and new shocks arrive at the Poisson rate \( \lambda \). Note that these assumptions satisfy the empirical properties of idiosyncratic job destruction, i.e. the shocks have persistence and they appear to hit the job independently of the aggregate state of the economy (here represented by \( p \)).

Entrepreneurs are unconstrained with respect to initial location, technology and product choice and also have the same information about market conditions. Under the assumption that they know the product that commands the highest productivity, all will create jobs at the highest idiosyncratic productivity, \( x = 1 \). Given this property

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3 As workers are generally happy when an unemployment spell ends, the unemployment hazard seems an ironic label. This unfortunate term is borrowed from statistical duration analysis where the typical spell is that of a “life” that ends as a consequence of some “hazard”, e.g. a heart attack or a failure.
of the model and the assumption that future match product evolves according to a Markov process with persistence, all matches are equally productive initially, until a shock arrives.\footnote{Generalizing the model to realistically allow for productivity heterogeneity across vacancies and for the fact that a random sample of new job–worker matches initially improve in average productivity are still problems at the research frontier.}

Under these assumptions, an existing match starts life with \( x = 1 \) but is eventually destroyed when a new value of \( x \) arrives below some reservation threshold, another endogenous variable denoted as \( R \). Unemployment incidence \( \lambda F(R) \), the average rate of transition from employment to unemployment, increases with the reservation threshold.

As all workers are assumed to participate, the unemployed fraction evolves over time in response to the difference between the flow of workers who transit from employment to unemployment and the flow that transits in the opposite direction, i.e.,

\[
\frac{\dot{u}}{t} = \lambda F(R)(1 - u) - \theta q(\theta) u,
\]

where \( 1 - u \) represents both employment and the employment rate. The steady-state equilibrium unemployment rate is

\[
u = \frac{\lambda F(R)}{\lambda F(R) + \theta q(\theta)}.\tag{2.3}\]

Equivalently, individual unemployment histories are described by a simple two-state Markov chain where the steady-state unemployment rate is also the fraction of time over the long run that the representative participant spends unemployed. It decreases with market tightness and increases with the reservation product, because the unemployment hazard \( \theta q(\theta) \) and the employment hazard \( \lambda F(R) \) are both increasing functions.

### 2.1. Job destruction and job creation conditions

A formal equilibrium model of unemployment requires specification of preferences, expectations, and a wage determination mechanism. We assume that both workers and employers maximize wealth, defined as the expected present value of future net income streams conditional on current information. Forward looking rational expectations are imposed. Several wage determination mechanisms are consistent with the matching approach. Following much of the literature, we shall assume bilateral bargaining as the baseline model.

Given this specification, equilibrium market tightness satisfies the following job creation condition: the expected present value of the future return to hiring a worker equals the expected cost. The hiring decision is implicit in the act of posting a
job vacancy and is taken by an employer. In contrast, the equilibrium reservation product, $R$, reflects the decisions of both parties to continue an existing employment relationship. Individual rationality implies that separation occurs when the forward-looking capital value of continuing the match to either party is less than the capital value of separation. For joint rationality, the sum of the values of continuing the match must be less than the sum of the values of separating, otherwise a redistribution of the pair's future incomes can make both better off. Whether these job destruction conditions also satisfy the requirements of joint optimality depends on the wage mechanism assumed. For a given wage determination mechanism, a search equilibrium is a pair $(R, \theta)$ that simultaneously solves these job creation and job destruction conditions.

For expositional purposes, we invoke the existence of a wage mechanism general enough to accommodate the special cases of interest. A wage contract, formally a pair $(w_0, w(x))$, is composed of a starting wage $w_0 \in \mathbb{R}$ and a continuing wage function $w : X \rightarrow \mathbb{R}$ that obtains after any future shock to match specific productivity. Implicit in this specification is the idea that a worker and an employer negotiate an initial wage when they meet and then subsequently renegotiate in response to new information about the future value of their match\(^5\).

A continuing match has specific productivity $x$ and the worker is paid a wage $w(x)$. Given that the match ends in the future if a new match specific shock $z$ arrives which is less than some reservation threshold $R$, its capital value to an employer, $J(x)$, solves the following asset pricing equation

$$rJ(x) = px - w(x) + \lambda \int_{R}^{1} [J(z) - J(x)] dF(z) + \lambda F(R)[V - pT - J(x)], \quad (2.4)$$

where $r$ represents the risk free interest rate, $V$ is the value of a vacancy, and $pT$ denotes a firing cost borne by the employer, represented as forgone output. We multiply the termination cost by $p$ to show that it is generally more expensive to fire a more skilled worker than a less skilled one. The termination cost is assumed to be a pure tax and not a transfer payment to the worker and to be policy-determined. For example, it may represent the administrative cost of applying for permission to fire, as is the case in many European countries. Of course, $T \geq 0$ and none of the fundamental results are due to a strictly positive $T$.

Condition (2.4), that the return on the capital value of an existing job–worker match to the employer is equal to current profit plus the expected capital gain or loss associated with the possible arrival of a productivity shock, is a continuous-time

\(^5\) Note that contracts of this form are instantly "renegotiated" on the arrival of a new idiosyncratic shock. MacLeod and Malcomson (1993) persuasively argue that the initial wage need not be adjusted until an event occurs that would otherwise yield an inefficient separation. Contracts of this form may well generate more realistic wage dynamics but job creation and job destruction decisions are the same under theirs and our specification. Hence, for the purpose at hand, there is no relevant difference.
Bellman equation. An analogous relationship implicitly defines the asset value of the same match to the worker involved, $W(x)$. Namely,

$$rW(x) = w(x) + \lambda \int_{R}^{1} [W(z) - W(x)] dF(z) + \lambda F(R)[U - W(x)], \quad (2.5)$$

where $U$ is the capital value of unemployment.

Given a match product shock $z$, the employer prefers separation if and only if its value $V$ exceeds the value of continuation $J(z)$. Similarly, the worker will opt for unemployment if and only if its value, $U$, exceeds $W(z)$. Given that both $J(z)$ and $W(z)$ are increasing, separation occurs when a new value of the shock arrives that falls below the reservation threshold

$$R = \max \{R_e, R_w\}, \quad (2.6)$$

where $J(R_e) = V - pT$ and $W(R_w) = U$. Because in the bilateral bargain wealth is transferable between worker and employer, the separation rule should be jointly optimal in the sense that it maximizes their total wealth. The necessary and sufficient condition for joint optimization is that $R = R_e = R_w$ where $J(R) + W(R) = V - pT + U$, a condition that holds only for an appropriately designed wage contract.

Although the idiosyncratic component of a new job match is $x = 1$, the expected profit from a new match will generally be different from $J(1)$, as defined in Equation (2.4), because of the existence of a job creation cost. We therefore introduce the notation $J_0$ for the expected profit of a new match to the employer and write the asset pricing equation for the present value of an unfilled vacancy, $V$, as

$$rV = -pc + q(\theta)[J_0 - V - pC], \quad (2.7)$$

where $pc$ is the recruiting cost flow per vacancy held, and $pC$ is a fixed cost of hiring and training a new worker plus any other match-specific investment required. Here these costs are indexed by the aggregate productivity parameter to reflect the fact that the forgone output that these costs represent is larger when labor is more productive.

The value of unemployment solves

$$rU = b + \theta q(\theta)[W_0 - U], \quad (2.8)$$

where $b$ represents unemployment-contingent income. Crucially for many of the results that hold in matching equilibrium, unemployment-contingent income is independent of employment income or of the aggregate state of the economy.

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6 See Mortensen (1978) for an early analysis of this issue within the search equilibrium framework. For alternative approaches to the modeling of the job destruction flow, see Bertola and Caballero (1994), who model a firm with many employees moving between a high-employment and a low-employment state, and Caballero and Hammour (1994), who analyze the implications of sunk costs and appropriation.
Given an initial wage equal to \( w_0 \), the by now familiar asset pricing relations imply that the initial value of a match to employer and worker respectively satisfy

\[
   rJ_0 = p - w_0 + \lambda \int [J(z) - J_0] \, dF(z) + \lambda F(R)[V - pT - J_0]
\]

and

\[
   rW_0 = w_0 + \lambda \int [W(z) - W_0] \, dF(z) + \lambda F(R)[U - W_0],
\]

where \( J(x) \) and \( W(x) \) represent the values of match continuation defined above.

The job creation condition that we defined earlier is equivalent to a free entry condition for new vacancies. The exploitation of all profitable opportunities from job creation requires that new vacancies are created until the capital value of holding one open is driven to zero, i.e.,

\[
   V = 0 \iff \frac{c}{q(\theta)} + C = \frac{J_0}{p}. \tag{2.11}
\]

As the expected number of periods required to fill a vacancy is \( 1/q(\theta) \), the condition equates the cost of recruiting and hiring a worker to the anticipated discounted future profit stream. The fact that vacancy duration is increasing in market tightness guarantees that free entry will act to equate the two.

### 2.2. Generalized Nash bargaining

The generalized axiomatic Nash bilateral bargaining outcome with “threat point” equal to the option of looking for an alternative match partner is the baseline wage specification assumption found in the literature on search equilibrium\(^7\). Given that the existence of market friction creates quasi-rents for any matched pair, bilateral bargaining after worker and employer meet is the natural starting point for an analysis\(^8\).


\(^8\) Binmore, Rubinstein and Wolinsky (1986), Rubinstein and Wolinsky (1985) and Wolinsky (1987) applied Rubinstein’s strategic model in the search equilibrium framework. The analyses in these papers imply the following: If the worker searches and the employer recruits at the same intensities and if \( \beta \) is interpreted as the probability that the worker makes the wage demand (1 - \( \beta \) is the probability that the employer makes an offer) in each round of the bargaining, then the unique Markov perfect solution to the strategic wage bargaining is the assumed generalized Nash solution. If neither searches but there is a positive probability of an exogenous job destruction shock during negotiations, the solution is again the one assumed but with \( \beta = \frac{1}{2} \). However, if neither seeks an alternative partner while bargaining and there is zero probability of job destruction, the strategic solution divides the joint product of the match \( J_0 - pC + W_0 \) subject to the constraint that both receive at least the option value of searching and recruiting, \( U \) and \( V \), rather than the net surplus, as we assumed. As these bargaining outcomes generate the same job creation and job destruction decisions, we consider only the former case with a \( \beta \) between 0 and 1.
Given the notation introduced above, the starting wage determined by the generalized Nash bargain over the future joint income stream foreseen by worker and employer supports the outcome

\[ w_0 = \arg \max \left\{ \left[ W_0 - U \right]^{\beta} \left[ S_0 - (W_0 - U) \right]^{1-\beta} \right\} \]

subject to the following definition of initial match surplus,

\[ S_0 \equiv J_0 - pC - V + W_0 - U. \] \hspace{1cm} (2.12)

In the language of axiomatic bargaining theory the parameter \( \beta \) represents the worker's relative "bargaining power." Analogously, the continuing wage contract supports the outcome

\[ w(x) = \arg \max \left\{ \left[ W(x) - U \right]^{\beta} \left[ S(x) - (W(x) - U) \right]^{1-\beta} \right\}, \]

where continuing match surplus is defined by

\[ S(x) \equiv W(x) - U + J(x) - V + pT. \] \hspace{1cm} (2.13)

The difference between the initial wage bargain and subsequent renegotiation arises for two reasons. First, hiring costs are "sunk" in the latter case but "on-the-table" in the former. Second, termination costs are not incurred if no match is formed initially but must be paid if an existing match is destroyed.

The solution to these two different optimization problems satisfy the following first-order conditions

\[ \beta(J_0 - V - pC) = (1 - \beta)(W_0 - U) \iff W_0 - U = \beta S_0 \] \hspace{1cm} (2.14)

and

\[ \beta(J(x) - V + pT) = (1 - \beta)(W(x) - U) \iff W(x) - U = \beta S(x). \] \hspace{1cm} (2.15)

As an immediate consequence of Equation (2.15), it follows that the reservation threshold \( R \), defined by Equation (2.6) is jointly rational, i.e., it solves

\[ S(R) = J(R) - V + pT + W(R) - U = 0. \]

As a preliminary step in solving for the match surplus function and the continuing wage contract that supports the bargaining solution, first rewrite Equations (2.4) and (2.5) as follows:

\[ (r + \lambda)(J(x) - V + pT) = px - w(x) - r(V - pT) \]

\[ + \lambda \int_R^1 [J(z) - V + pT] dF(z) \] \hspace{1cm} (2.16)
and
\[(r + \lambda) (W(x) - U) = w(x) - rU + \lambda \int_R^1 [W(z) - U] dF(z). \tag{2.17}\]

By summing these equations, one obtains the following functional equation which the surplus function must solve
\[S(x) = \frac{px - r(U + V - pT) + \lambda \int_R^1 S(z) dF(z)}{r + \lambda}. \tag{2.18}\]

Because \(S(R) = 0\) implies \(\int_R^1 S(z) dF(z) = \int \max(S(z), 0) dF(z)\), the right-hand side satisfies the Blackwell sufficient conditions for a contraction. Furthermore, the solution is linear in \(x\). Hence, the solution can be written as \(S(x) = (x - R)/(r + \lambda)\) where \(R\) is the unique solution to
\[pR + \left(\frac{\lambda}{r + \lambda}\right) p \int_R^1 (z - R) dF(z) = r(U + V - pT). \tag{2.19}\]

The reservation product, \(pR\), plus the option value of continuing the match attributable to the possibility that match product will increase in the future, the left-hand side, equals the flow value of continuation to the pair, the right-hand side of the equation.

As the left- and right-hand sides of Equation (2.16) multiplied by \(1 - \beta\) respectively equal the left- and right-hand sides of Equation (2.17) when multiplied by \(\beta\) given (2.15), the continuing match product specific wage that supports the bargaining outcome is
\[w(x) = rU + \beta \left[ px - r(V - pT) - rU \right]. \tag{2.20}\]

Note that this result is the generalized Nash outcome in a continuous bargain over match output \(px\) given a "threat point" equal to the flow values of continuing the match, namely \((r(V - pT), rU)\).

Analogously, by summing equations (2.9) and (2.10), one obtains
\[(r + \lambda) S_0 = (r + \lambda)(J_0 - V - pC + W_0 - U)
= p - r(U + V) - (r + \lambda)pC - \lambda pT + \lambda \int_R^1 S(z) dF(z) \tag{2.21}
= p(1 - R) - (r + \lambda)p(C + T) = (r + \lambda)(S(x) - pC - pT)\]

by virtue of Equations (2.12) and (2.19). Hence, the free entry conditions (2.11) and the initial surplus division rule (2.14) yield the following equilibrium relationship between market tightness and the reservation product:
\[\frac{pc}{q(\theta)} = J_0 - pC = (1 - \beta) S_0 \tag{2.22}
= (1 - \beta)p \left(\frac{1 - R}{r + \lambda} - C - T\right).\]
The logic of the derivation of the initial wage is similar to that used to obtain the continuing wage function. First, rewrite Equations (2.9) and (2.10) as

$$(r + \lambda) (J_0 - V - pC) = p - w_0 - rV - (r + \lambda)pC - \lambda pT + \lambda \int_R^1 [J(z) - V + pT] dF(z)$$

and

$$(r + \lambda)(W_0 - U) = w_0 - rU + \lambda \int_R^1 [W(z) - U] dF(z).$$

Second, multiply both sides of the first equation by $1 - \beta$, both sides of the second by $\beta$, and then apply Equations (2.14) and (2.15) to obtain

$$w_0 = rU + \beta \left[ p - r(V + U) - (r + \lambda)pC - \lambda pT \right].$$

(2.23)

Note that the initial wage equals the worker’s share of the initial match flow surplus $p - r(V + U + pC)$ less the sum of hiring and firing costs amortized over the initial period prior to the arrival of a subsequent match specific shock $\lambda p(C + T)$. In short, the worker share of both the quasi-rents and match specific investments required to both create and end the match is the market power parameter $\beta$.

To complete the derivation of the equilibrium conditions, we use the fact that the free entry condition (2.22), the surplus sharing rule (2.14), and the value of unemployment equation (2.8) imply that the flow value of unemployment is linear and increasing in market tightness.

$$rU = b + \beta \theta g(\theta) S_0 = b + \left( \frac{pc\beta}{1 - \beta} \right) \theta.$$

By direct substitution into Equations (2.23) and (2.20), the equilibrium wage contract can be written as

$$w_0 = \beta p \left[ 1 + c\theta - (r + \lambda)C - \lambda T \right] + (1 - \beta) b$$

(2.24)

and

$$w(x) = \beta p (x + c\theta + rT) + (1 - \beta) b.$$

(2.25)

Finally, the reservation threshold equation (2.19) becomes

$$p \left( R + \frac{\lambda}{r + \lambda} \int_R^1 (x - R) dF(x) \right) = r(U - pT)$$

$$= b - rpT + \left( \frac{\beta}{1 - \beta} \right) pc\theta.$$

(2.26)

As the left-hand side is increasing in $R$, the equation implicitly defines a positive equilibrium relationship between the reservation product and market tightness, one that reflects the pressure on wages induced by greater market tightness.
An equilibrium solution is any pair \((R^*, \theta^*)\) that solves the job creation condition \((2.22)\) and the job destruction condition \((2.26)\). The associated starting wage \(w_0\), continuing wage function \(w(x)\), and steady-state unemployment rate \(u\) are those specified in Equations \((2.24)\), \((2.25)\), and \((2.3)\). Because the relation defined by the job creation condition \((2.22)\) is downward sloping, as illustrated by the line \(CC\) in Figure 6, while the job destruction condition \((2.26)\) can be represented as the upward sloping line \(DD\), there is a single equilibrium solution to the two equations\(^9\). The equilibrium pair is strictly positive if the product of a new match, \(p\), less the opportunity cost of employment, \(b\), is sufficient to cover recruiting, hiring, and anticipated firing costs.

### 2.3. Fundamental determinants of unemployment

Figure 6 provides insight into how the various parameters of the model affect the steady-state unemployment rate. For this purpose, it is useful to remember that the job creation line \(CC\) reflects the standard dynamic demand requirement that the cost of hiring and training a worker is equal to the expected present value of the future profit attributable to that worker over the life of the job. It is downward sloping because a higher reservation threshold implies a shorter expected life for any new match. The upward slope of the job destruction line \(DD\) reflects the sensitivity of the reservation product threshold to market tightness.

Now it is clear from Equation \((2.22)\) that given \(R\) neither \(p\) nor \(b\) influence equilibrium \(\theta\). Thus, general productivity and the supply price of labor do not shift \(CC\). By dividing Equation \((2.26)\) by \(p\), we find that \(b\) and \(p\) enter the equilibrium conditions as a ratio \(b/p\). Hence, the influence of general productivity and the opportunity cost of employment is due entirely to the fact that the latter is independent of the former. If for whatever reason the opportunity cost of employment \(b\) was proportional to general productivity \([as in the long-run equilibrium model of Phelps (1994), through wealth]

\(^9\) Note in passing that the equilibrium pair is stationary even out of steady state because there is no feedback from current employment to expectations about future match output. This fact is an implication of the linear specification of both agent preferences and production technology and of the absence of memory in the idiosyncratic shock process. A change in any one of these specification assumptions substantially complicates but enriches the model.
accumulation], general productivity changes would not influence the equilibrium rate of unemployment.

Given our specification and the interpretation of the two lines in Figure 6, an increase in the supply price of labor, \( b \), or a fall in general productivity \( p \), shifts the \( DD \) line up but has no direct effect on \( CC \). As a consequence, the equilibrium value of the reservation threshold increases and the equilibrium value of market tightness falls with \( b/p \). Hence, steady-state unemployment increases because both unemployment duration and incidence increase in response.

The other parameters of the model have more complicated effects on equilibrium unemployment and at the analytical level we can only derive unambiguous results for unemployment duration and incidence, but not for the stock of unemployment. Inspection of equations (2.22) and (2.26) shows that the only other parameter that shifts only one of the lines is the job creation cost \( C \). An increase in \( C \) shifts \( CC \) to the left and so implies lower \( R \) and \( \theta \): unemployment duration rises but incidence falls. The intuition behind the result is that higher job creation costs reduce job creation, increasing the duration of unemployment, but also reduce job destruction, to economize on the job creation costs that are incurred if the firm is to re-enter the market. The effect on unemployment is ambiguous.

A similar ambiguity arises from changes in job termination costs. Higher \( T \) shifts the \( CC \) line to the left and the \( DD \) line to the right. Although the effect on \( \theta \) appears ambiguous, a formal differentiation of the equilibrium conditions yields a negative net effect on both \( R \) and \( \theta \). Once again, job destruction falls, because it is now more expensive to fire workers, implying less unemployment incidence. Job creation falls because over its lifetime the job will pay the termination cost with probability 1, implying a longer duration of unemployment.

Other parameters of the model have even more complicated effects on unemployment duration and incidence. The rate of discount, \( r \), and the rate of arrival of shocks, \( \lambda \), both shift the job creation line down, because, in the case of \( r \), future product is discounted more heavily and in the case of \( \lambda \), the expected life of the job falls. But the job destruction line also shifts. Differentiation of the two equilibrium conditions shows that both \( r \) and \( \lambda \) reduce market tightness, and so increase the duration of unemployment. The arrival rate of idiosyncratic shocks also reduces the reservation threshold, reducing the incidence of unemployment but the rate of discount has ambiguous effects on the threshold.

Finally, an increase in the worker’s share of match surplus as reflected in an increase in the “market power” parameter \( \beta \) shift \( CC \) downward but \( DD \) upward in Figure 6. The result is a negative effect on equilibrium market tightness but the sign of the resultant change in the reservation product is indeterminate. Differentiation of the equilibrium conditions shows that the effect of \( \beta \) on \( R \) has the sign of \( \beta - \eta \), where \( \eta \) is the elasticity of the matching function with respect to unemployment. Interestingly, if \( \beta = \eta \) the search externalities are internalized by the wage bargain, and it is a useful benchmark case in simulations with search equilibrium models [Hosios (1990), Pissarides (1990)].
3. Employment fluctuations

The negative co-movement between aggregate measures of vacancies and unemployment, known as the Beveridge curve, has long been an empirical regularity of interest in the literature on labor market dynamics\(^\text{10}\). Generally, high vacancies and low levels of unemployment characterize a "tight" labor market in which workers find jobs quickly and higher wage rates prevail. Time-series observations suggest that job vacancy movements lead unemployment changes both in the sense that drops in job vacancy rates herald downturns in employment and that employment recoveries follow jumps in vacancies. These observations also suggest that fluctuations in derived demand for labor, as reflected in vacancy movements, rather than labor supply shocks are the principal driving force behind cyclical unemployment dynamics.

The empirical work of Davis and Haltiwanger (1990, 1992) and Davis, Haltiwanger and Schuh (1996) has stimulated general interest in the components of net employment change, which they call job creation and job destruction flows. As we saw in Section 1, the job creation and job destruction rates move in opposite directions over the business cycle but are always both large and positive at every level of industry and regional disaggregation. These facts suggest that employment reallocation across economic activities is a significant and continual process that accounts for a large measure of unemployment.

Mortensen and Pissarides (1994), Mortensen (1994b), Cole and Rogerson (1996), and den Haan, Ramey and Watson (1997) claim that an extended version of the equilibrium unemployment model, one that allows for an aggregate shock to labor productivity, can explain the stylized facts of the job creation and job destruction flows that we listed in Section 1. To recall, apart from the negative correlation between them just noted, job destruction is more volatile than job creation (which, at least for US manufacturing, shows up as negative correlation between the sum and difference of the job creation and job destruction flows) and quit rates are procyclical, i.e. there is a positive correlation between quit rates and the difference between job creation and job destruction. The purpose of this section is to present a version of the model that allows for employment fluctuations which can be used to illustrate these claims.

3.1. Stochastic equilibrium

The source of the underlying job reallocation process in the Mortensen–Pissarides model is an idiosyncratic shock which acts as match-specific "news" in the sense that it changes the profit prospect for an individual job on arrival. A general aggregate productivity shock which affects the output of every job by the same proportion is added here. Specifically, we let exogenous jumps in the common component of job

\(^{10}\) For an interesting early treatment, see Hansen (1970). For more recent search-based analyses, see Pissarides (1986) and Blanchard and Diamond (1989).
productivity $p$ induce the cycle. On the argument that recruiting and hiring costs represent forgone output, we continue assuming as well that these costs are indexed by the productivity parameter $p$.

According to real business cycle theory, economic fluctuations are induced by exogenous persistent shocks to aggregate labor productivity. Whether exogenous technical change is the cause or not, labor productivity is procyclical in fact and our model’s implications for wage and employment responses are an implication of that fact whatever its cause. Given sufficient persistence, one would expect these shocks to induce cyclical effects on the market tightness and the reservation idiosyncratic product which are similar to those associated with a permanent change in the level of aggregate productivity.

For the sake of a simple presentation, assume that aggregate productivity fluctuates between a high value $P_h$ and a low value $P_l$, where the continuous time transition rate or frequency is $\eta$. For this specification, the autocorrelation coefficient of the $p$-process given a short time interval of length $\Delta$ is $2e^{-\eta\Delta} - 1$. Indeed,

$$E\{p(t + \Delta) \mid p(t) = p_i\} = e^{-\eta \Delta} p_i + (1 - e^{-\eta \Delta}) p_j \cdot$$

$$= (2e^{-\eta \Delta} - 1) p_i + 2 (1 - e^{-\eta \Delta}) E\{p\},$$

where $1 - e^{-\eta \Delta}$ is the probability of a change in state during the interval $(t, t + \Delta)$, and $E\{p\} = (p_h + p_l)/2$ is the ergodic mean of this symmetric Markov chain. In the case of permanent aggregate productivity, i.e., $\eta = 0$, the equilibrium pair in state $i$, $(R_i, \theta_i)$, solves Equations (2.22) and (2.26) given $p = p_i$. Consequently, $P_h > P_l$ implies $\theta_h > \theta_l$ and $R_l < R_h$ in this case. This fact generalizes but only if the aggregate shock frequency $\eta$ is not too large.

For $\eta > 0$, the equilibrium relationships are more complicated because forward looking agents knowing themselves to be in state $i$ anticipate the effects and likelihood of transiting to state $j$ in the future. Indeed, under generalized Nash bargaining in which the initial wage is set contingent on the aggregate state and the continuing wage is renegotiated in the event of either an aggregate or a match specific shock, the surplus value of a continuing match with idiosyncratic productivity $x$ in aggregate state $i$, $S_i(x)$, and the surplus value of a new match in state $i$, $S_0i$, satisfy the following generalization of Equations (2.18) and (2.21):

$$rS_i(x) = p_ix - (U_i + V_i - p_i T) + \lambda \int_{R_i} [S_i(z) - S_i(x)] \, dF(x) + \eta [S_j(x) - S_i(x)],$$

$$rS_0i = p_ix_0 - (U_i + V_i - p_i T) - (r + \lambda + \eta) p_i (C + T)$$

$$+ \lambda \int_{R_i} [S_i(z) - S_0i] \, dF(z) + \eta [S_0j - S_0i],$$

(3.1)

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11 For example, see Kydland and Prescott (1982) and Lucas (1987).
where the aggregate state contingent values of a vacancy and unemployment solve

\[ rV_i = q(\theta_i)(1 - \beta)S_{0i} + \eta (V_j - V_i) - p_i c, \]
\[ rU_i = b + \theta_i q(\theta_i)\beta S_{0i} + \eta (U_j - U_i). \]  

(3.2)

An equilibrium now is a state contingent reservation threshold and market tightness pair \((R_i, \theta_i)\), one for \(i = l\) and another for \(i = h\), that satisfy the free entry job creation condition and job destruction condition in both states, i.e.,

\[ V_i = 0 \quad \text{and} \quad S_i(R_i) = 0, \quad i \in \{l, h\}. \]

Market tightness is procyclical and market tightness and the reservation product threshold move in opposite directions in response to aggregate shocks if the shock is sufficiently persistent. Formally, a unique equilibrium exists with the property that \(p_h > p_l \Rightarrow R_h < R_l\) for all \(\eta\) while a critical value \(\eta > \hat{\eta} > 0\) exists such that \(\theta_h > (\theta_l)\) as \(\eta < (\eta)\) \(\hat{\eta}\) 12.

Aggregate state contingent equilibrium market tightness is actually lower in the higher aggregate product state for sufficiently large values of the shock frequency because investment in job creation is relatively cheaper when productivity is low and because the present value of the returns to job creation investments are independent of the current aggregate state in the limit as \(\eta\) becomes large. In other words, job creation investment is larger when aggregate productivity is higher only if expected return given high current productivity offsets the cost advantage of investment in the low productivity state, a condition that requires sufficient persistence in the productivity shock.

### 3.2. The Beveridge curve

As just demonstrated, "boom" and "bust" in this simple model are synonymous with the prevalence of the "high" and "low" average labor productivity when the aggregate shock is persistent. Unemployment dynamics in each aggregate state are determined by the law of motion

\[ \dot{u} = \lambda F(R_i)(1 - u) - \theta q(\theta)u. \]  

(3.3)

Hence, the unemployment rate tends toward the lower of the two aggregate state contingent values, represented by

\[ u_i^* = \frac{\lambda F(R_i)}{\lambda F(R_i) + \theta q(\theta)}, \quad i \in \{l, h\}, \]  

(3.4)

during a boom and tends toward the higher value in a bust.

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12 Formal derivations of the value equations, those of (3.1) and (3.2), and proofs can be found in Burdett, Mortensen and Wright (1996).
The observation that actual vacancies and unemployment time series are negatively correlated is consistent with this model under appropriate conditions, a fact illustrated in Figure 7. In the figure, the two rays from the origin, labeled $\theta_l$ and $\theta_h$, represent the vacancy–unemployment ratios in the two aggregate states when $\theta_h > \theta_l$. The negatively sloped curves represent the locus of points along which there is no change over time in the unemployment rate, one for each of the two states. Because the curve for aggregate state $i$ is defined by

$$\frac{\nu q(v/u_i)}{1 - u_i} = \lambda F(R_i),$$

$R_h < R_l$ implies that $u_h < u_l$ for every $v$ as drawn in Figure 7. Finally, the two steady-state vacancy–unemployment pairs lie at the respective intersections of the appropriate curves, labeled $L$ and $H$ in the figure. Provided that the curve along which $\dot{u} = 0$ doesn’t shift in too much when aggregate productivity increases, $u^*_h > u^*_l$ as well as $u^*_h < u^*_l$. However, sufficient persistence, in the form of a low transition frequency, is necessary here. Indeed, the points $L$ and $H$ lie on a common ray when persistence is at the critical value $\eta = \bar{\eta}$ since $\theta_l = \theta_h$ by definition.

### 3.3. Job creation and job destruction flows

In our simple model, the notion of a job is equivalent to that of an establishment, plant, or firm given the linear technology assumption. Consequently, the job creation flow, the employment changes summed across all new and expanding plants over a given period of observation, can be associated with the flow of new matches in the model. Analogously, job destruction, the absolute sum of employment reductions across contracting and dying establishments, is equal to all matches that either experience an idiosyncratic shock that falls below the reservation threshold or were above the
threshold last period but are below it this period. The fact that market tightness and the reservation product move in opposite directions in response to an aggregate productivity shock implies negative co-movements in the two series, as observed. Furthermore, a negative productivity shock induces immediate job destruction while a positive shock results in new job creation only with a lag. This property of the model is consistent with the fact that job destruction "spikes" are observed in the job destruction series for US manufacturing which are not matched by job creation "spurts". As in the OECD data, cyclical job destruction at the onset of recession is completed faster than cyclical job creation at the onset of a boom.

3.4. Quits and worker flows

As the model is constructed so far, aggregate hires are equivalent to job creation and separations equal job destruction. These identities no longer hold when some employed workers quit to take other jobs without intervening unemployment spells. As these so-called job to job flows constitute a significant component of both hires and separations, are procyclical, and represent a worker reallocation process across jobs, their incorporation in the model represents an important extension.

Job to job worker flows can be viewed as the outcome of a decision by some workers to search for vacancies while employed, as in Mortensen (1994b). Given that $\theta_q(\theta)$ represents the rate at which employed as well as unemployed workers find a vacant job, the quit flow representing job to job movement in aggregate state $i \in \{l, h\}$ is

$$Q_i = \theta_q(\theta)(1 - u_i) s_i,$$

where $s_i$ is the fraction of the employed who search and $\theta_i$ is now the ratio of vacancies to searching workers, i.e.

$$\theta_i = \frac{u_i}{u_i + s_i(1 - u_i)}.$$

Once employed, workers have an incentive to move from lower to higher paying jobs. Suppose that employed workers can search only at an extra cost, $\sigma$, interpreted as foregone leisure, a reduction in $b$. As search is jointly optimal for the pair if and only if the expected return, equal to the product of the job-finding rate and the gain in

---

13 These points are discussed in more detail in Mortensen and Pissarides (1994) and Mortensen (1994b).
match surplus realized, exceeds the cost, all workers employed at \( x \) equal to or less than some critical value, denoted as \( Q_i \), will search where\(^{14}\)

\[
\theta q(\theta_i) [S(1) - S_i(Q_i)] = \sigma, \quad i \in \{l, h\}.
\]

(3.5)

Because idiosyncratic productivity is distributed \( F(x) - F(R) \) across jobs, it follows that the fraction of the employed workers who search in aggregate state \( i \) is given by

\[
s_i = F(Q_i) - F(R_i).
\]

(3.6)

Because a quit represents an employment transition for the worker and the loss of a filled job for the employer, the surplus value equation under joint wealth maximization is

\[
rS_i(x) = p_i x - \sigma - r(U_i + V_i - T) + \lambda \int_R^1 [S(z) - S_i(x)] dF(x)
+ \eta [S_i(x) - S_i(x)] + \theta q(\theta_i)(S(1) - S_i(x)) \quad \forall x < Q_i.
\]

(3.7)

Because the worker does not search when \( x \geq Q_i \) and this condition always holds when \( x = 1 \), Equations (3.1) continue to hold in this range. To the extent that market tightness is procyclical, Equation (3.5) implies \( Q_h > Q_l \). Hence, the quit flow is procyclical for two separate reasons. First, because \( Q \) is higher and \( R \) is lower in the high aggregate productivity state, the fraction of employed workers who search is procyclical, i.e., \( s_h > s_l \). Second, because \( \theta h > \theta l \) when the aggregate shock is sufficiently persistent, the rate at which searching workers meet vacancies \( \theta q(\theta) \) is also larger in the high aggregate product state.

Worker reallocation across different activities is represented by both the direct movement from one job to another via quits and by movements through unemployment induced by job destruction and subsequent new job creation. Davis, Haltiwanger and Schuh (1996) estimate that between 30% and 50% of worker reallocation is attributable to the job destruction and creation process. Given the procyclicality of the quit flow and the flow of hires, the sum of job creation and quits is highly procyclical, while the separation flow, the sum of job destruction and quits, is acyclical. Hence, the reallocation of workers across activities is procyclical relative to the more countercyclical reallocation of jobs across activities both in fact and according to the model.

The quit process also interacts with job creation and job destruction in more complicated ways that are not explicitly modeled here. For example, when a worker

\(^{14}\) Although the decision to maximize the sum of the pair’s expected future discounted income by the appropriate choice of the worker’s search effort is individually rational under an appropriate contract, both costless monitoring and enforcement of the contract is generally necessary to overcome problems of dynamic inconsistency. Indeed, otherwise the worker will search if and only if the personal gain exceeds cost, i.e., iff \( W_i(1) - W_i(x) = \beta [S(1) - S_i(x)] > \sigma \) which would imply too few quits.
quits an existing job to take a new one, the employer can choose to search for a replacement. If the decision is not to replace the worker, the quit has induced the destruction of a job with no net change in either the number of jobs or unemployment. If the decision is to declare the job vacant, a new job was created by the original match but there will be no net reduction in unemployment unless the old job vacated is filled by an unemployed worker. Of course, if filled by an employed worker, the employer left by that worker must decide whether or not to seek a replacement. This sequential replacement process by which a new vacancy leads to an eventual hire from the unemployment pool, known in the literature as a vacancy chain, propagates the effects of job creation shocks on unemployment [see Contini and Revelli (1997) and Akerlof, Rose and Yellen (1998)].

Also, quit rates are high in the first several months after the formation of new matches and then decline significantly with match tenure, presumably as a consequence of learning about the initially unknown "quality" of the fit between worker and job. This source of quits is of significant magnitude and it represents the primary form of quits to unemployment. Because this "job shopping" process implies that an unemployed worker typically tries out a sequence of jobs before finding satisfaction, a job destruction shock is likely to be followed by a drawn-out period of higher than normal flow into and out of unemployment. Were the job shopping process incorporated in the model, job reallocation shock effects on worker flows would be prolonged and amplified, features that should also improve the model's fit to the data.

4. Explaining the data

Besides the attempts to use the models that we have described to match the stylized facts of job and worker flows, there have recently been some attempts to calibrate stochastic versions of the models to explain the cyclical behavior of the US economy. These attempts are partly motivated by the emergence of the new data on job flows that need to be explained and partly by the apparent failure of competitive labor market models to match the business cycle facts in the data. In order to explain the business cycle facts the models need to be extended to include capital, an exercise that has attracted some attention recently.

15 There is an extensive labor economics literature on this point initiated by the seminal theoretical development by Jovanovic (1979). See Farber (1994) for a recent analysis of the micro-data evidence on tenure effects on quit rates and the extent to which these are explained by the job shopping hypothesis. Pissarides (1994) explains these facts within a search model with learning on the job.

16 Hall (1995) argues that this effect is apparent in the lag relationships between the time series aggregates.

17 For attempts to estimate structural forms of the matching model see Pissarides (1986) and Yashiv (1997).

18 When used to calibrate the business cycle facts the models are often offered as alternatives and compared with Hansen's (1985) indivisible labor model.
4.1. Explaining job flows data

Cole and Rogerson (1996) conduct an analysis of the extent to which the rudimentary Mortensen–Pissarides model can explain characteristics of the time series observations on employment and job flows in US manufacturing. For this purpose, they construct the following stylized approximation to the continuous time formulation sketched above: Job creation in period $t$, $c_t$, is equal to the matches that form during the observation period and survive to its end. As one can ignore the possibility that a job is both created and destroyed when the observation period is sufficiently short, approximate job creation in period $t$ is

$$c_t = \alpha_{s_{t-1}} (1 - n_{t-1}), \quad 1 - \alpha_{s_t} = e^{-\theta_{s_t} q(\theta_{s_t})},$$

(4.1)

where $n_{t-1} = 1 - u_{t-1}$ is employment at the beginning of the period, $1 - \alpha_t$ is the probability that the representative worker who is unemployed at the beginning of the period is not matched with a job during the period given that aggregate state $i$ prevails, $\theta_{s_t} q(\theta_{s_t})$ is the aggregate state contingent unemployment hazard rate, and $s_t \in \{l, h\}$ is the aggregate state that prevails during period $t$.

Job destruction in period $t$ has two components as already noted. First, the fraction of filled jobs that experience a shock less than the prevailing reservation threshold, which equals $1 - e^{-\lambda F(R_i)}$ given aggregate state $i$ prevails, are destroyed. Second, the fraction of existing jobs that do not experience a shock but have match productivity less than the current reservation threshold are also destroyed. The latter is $G_{t-1}(R_t)$ where $G_{t-1}(x)$ is the fraction of jobs at the beginning of the period that have match productivity less than or equal to $x$. Although this distribution of jobs over productivity is not stationary but instead evolves in response to the history of aggregate shocks, between shock arrivals it converges toward an aggregate state contingent distribution equal to 0 for all $x < R_i$ obviously and $F(x) - F(R_i)/(1 - F(R_i))$ for all values of $R_i < x < 1$. Given sufficient persistence in the aggregate shock (i.e., $\eta$ small enough), Cole and Rogerson argue that these steady-state distributions can be used to approximate $G_{t-1}$. Because $R_h < R_l$ implies that job destruction attributable to a change in the aggregate state only occurs when the transition is from high to low productivity, the following characterization of the job destruction flow holds as an approximation:

$$d_t = (\delta_{s_t} + \phi_t \delta_0) n_{t-1},$$

where

$$\delta_t = 1 - e^{-\lambda F(R_i)};$$

(4.2)

$$\phi_t = \begin{cases} 
1 & \text{if } s_{t-1} = h \text{ and } s_t = l, \\
0 & \text{otherwise,}
\end{cases}$$

$$\delta_0 = \pi_f (F(R_i) - F(R_h)),$$

where

$$\pi_{fh} = \pi_{hl} = \pi = 1 - e^{-\eta}$$

is the probability of an aggregate state transition. Finally, the aggregate employment process $\{n_t\}$ is generated by the following stochastic difference equation defined by the employment adjustment identity

$$n_{t+1} \equiv n_t + c_{t+1} - d_{t+1} = \alpha_{s_t} + (1 - \alpha_{s_t} - \delta_{s_{t+1}} - \phi_{t+1} \delta_0) n_t$$
given the Markov forcing process \( \{s_t\} \) defined on the state space \( \{l, h\} \) and characterized by the symmetric probability of transition \( \pi \).

Obviously, the employment, job creation, and job destruction processes are interrelated and fully characterized by the set of reduced form parameters \( \{\alpha_l, \alpha_h, \delta_l, \delta_h, \delta_0, \pi\} \). The question asked by Cole and Rogerson (1996) is whether an appropriate choice of these parameters will replicate the salient features of the Davis–Haltiwanger–Schuh observations, which they summarize in the following useful way:

(1) **Volatility**: Job creation is roughly four times as volatile as employment, and job destruction is more than six times as volatile.

(2) **Persistence**: The series for job creation, job destruction and employment display strong positive autocorrelation, but the autocorrelation for employment, which is 0.9, is nearly twice that for the other two series.

(3) **Contemporaneous Correlations**: Creation and destruction have a fairly large negative correlation. Destruction is (weakly) negatively correlated with employment, whereas creation is virtually uncorrelated with employment.

(4) **Dynamics**: Creation is negatively correlated with lagged employment, and positively correlated with future employment. The opposite pattern holds for destruction.

To answer their question, Cole and Rogerson simulate the model above for trial parameter values, compute the associated simulation statistics, and then adjust the parameter values to obtain a better match. They conclude that the model can replicate observations in their sense when the probability of finding a job is not too large. Specifically, the model simulation for the parameter set

\[
\{\alpha_l, \alpha_h, \delta_l, \delta_h, \delta_0, \pi\} = \{0.21, 0.30, 0.069, 0.044, 0.01, 0.2\}
\]

generates their preferred results which are not only consistent with their qualitative characterization of the data but are quite close in quantitative terms as well. Given that the two job destruction rates \( \delta_l \) and \( \delta_h \) are set to match the average of 0.055 reported in the Davis–Haltiwanger–Schuh data, one potential problem which Cole and Rogerson emphasize and discuss are the low values of the probabilities of finding employment. To see the significance of the point, simply note that the two state contingent steady-state unemployment rates associated with this parameter set are

\[
u_l = \frac{\delta_l}{\delta_l + \alpha_l} = 0.25, \quad u_h = \frac{\delta_h}{\delta_h + \alpha_h} = 0.13,
\]

two numbers that yield an average unemployment rate of 19%. Nonetheless, the authors argue that these numbers are reasonable given the following observations reported by Blanchard and Diamond (1990): First, although the simple model ignores non-participants, in fact the flow to employment from this stock is roughly equal to the flow from those officially categorized as unemployed. Second, the number of workers classified as out-of-the-labor-force who report they want jobs is also roughly equal to
the number classified as unemployed. Including these individuals in the pool of the unemployed would rationalize the low average value of $\alpha$, especially if these workers search at lower intensities.

4.2. Capital accumulation and shock propagation

Merz (1995) and Andolfatto (1996) each construct different but related syntheses of the neoclassical stochastic growth model and the Pissarides (1990) model of frictional unemployment. The contributions of these authors include a demonstration that the "technology shocks" responsible for business cycles in the real business cycle (RBC) model will also induce negative correlation between vacancies and unemployment, the Beveridge curve, and a positive correlation between flows into and out of unemployment in a version of the model with a labor market characterized by a matching process. However, like the earlier simpler RBC models, the amended models fail to propagate productivity shocks in the manner suggested by the observed persistence in actual output growth rates.

Recently, den Haan, Ramey and Watson (1997) have constructed, calibrated, and simulated a synthesis of the Mortensen and Pissarides (1994) model of job creation and job destruction with the neoclassical stochastic capital accumulation model. As in the Merz and Andolfatto models, job creation is governed by a matching function whose inputs include vacancies and unemployed workers. In addition, a job destruction margin is introduced by supposing that existing job matches experience idiosyncratic productivity shocks orthogonal to the aggregate shock to match productivity as described above. They find that interaction between the household saving decision and the job destruction decision play a key role in propagating aggregate productivity shocks. As a consequence, their synthesis provides an explanation for the observed autocorrelation in output growth rates as well as the correlation patterns observed in job flows with themselves and employment, those matched by Cole and Rogerson (1996).

Den Haan et al. (1997) explicitly formulated the model in discrete time with each period equal to one quarter. Following Merz (1995) and Andolfatto (1996), idiosyncratic variation in labor income attributable to unemployment spells is fully insured through income pooling. Hence, the existence of a representative household can be invoked; one assumed to have additively separable preferences over future consumption streams represented by $\sum \gamma^t u(C_t)$ where $t$ is the time period index, $\gamma$ is the time discount factor, and $u(C)$ is one period utility expressed as a concave function of consumption. A single consumable and durable asset, capital, exists which also serves as a productive input. The sequence of future market returns for holding the asset, denoted $\{r_t\}$, is an endogenous stochastic process. Hence, the optimal consumption plan must satisfy the usual Euler equation

$$u'(C_t) = \gamma E_t\{u'(C_{t+1})(1 - \delta + r_{t+1})\}, \quad (4.3)$$

where the expectation is taken with respect to information available in period $t$ and $\delta$ is the rate of physical capital depreciation.
The surplus value of a new match is another endogenous stochastic process, denoted \( \{S_{t+1}^0\} \). When an unemployed worker and job vacancy meet at the beginning of period \( t+1 \), Nash bargaining takes place. The outcome allocates the share \( \beta S_{t+1}^0 \) to the worker and the remainder \( (1 - \beta)S_{t+1}^0 \) to the employer, where as above \( \beta \) represents worker market power. The anticipated bargaining outcome motivates search and recruiting effort by unemployed workers and employers with vacancies during period \( t \). The flow return to unemployed search is the sum of home production while unemployed, \( b \), and the expected gain attributable to finding a match:

\[
b + \theta_t q(\theta_t) \beta E_t \left\{ \frac{\gamma u'(C_{t+1})}{u'(C_t)} S_{t+1}^0 \right\}. \tag{4.4}
\]

The expected capital gain, the second term, is the product of the probability of finding a job and the expected value of the worker’s share of match surplus given information available in period \( t \) appropriately discounted back to the present by a factor which accounts for any difference in the marginal utility of consumption in the next and the current period. Similarly, free entry of vacancies requires zero profit in the sense that recruiting cost per vacancy posted, \( p_t c \), equals expected return, the product of the probability that the employer finds a match and the employer’s share of its expected discounted surplus value:

\[
p_t c = q(\theta_t)(1 - \beta) E_t \left\{ \frac{\gamma u'(C_{t+1})}{u'(C_t)} S_{t+1}^0 \right\}. \tag{4.5}
\]

The aggregate productivity shock, the process \( \{p_t\} \), is Markov with the transition probability kernel assumed to be common knowledge. For simplicity, den Haan et al. (1997) assume that the match-specific process, represented by \( \{x_t\} \), is i.i.d. with c.d.f. \( F(x) \). Still, the idiosyncratic shock is expected to persist for the duration of the current period. The output of an existing match in period \( t \) is \( p_t x_t f(k_t) \) where \( k_t \) is the amount of capital per worker rented during the period at rate \( r_t \), and \( f(k_t) \), normalized output per worker, is an increasing concave function. Because the option value of continuing the match is zero for the employer and equal to the flow value of search for the worker, \( b + \beta p_t c\theta/(1 - \beta) \) from Equations (4.4) and (4.5), the joint match surplus conditional on idiosyncratic productivity \( x_t \) is

\[
S_t(x_t) = \max_k \left( p_t x_t f(k_t) - r_t k_t - b - \frac{\beta p_t c}{1 - \beta} \theta_t \right) + E_t \left\{ \frac{\gamma u'(C_{t+1})}{u'(C_t)} \max \{S_t(x_{t-1}), 0\} \right\}, \tag{4.6}
\]

where the last term reflects appropriate discounting of next-period surplus and the option to destroy the match next period if need be.

\(^{19}\) Otherwise, the distribution of idiosyncratic productivity across existing matches is a decision relevant state variable. They claim that the model loses no essential property as a consequence of this abstraction.
By implication of the optimal capital choice, the current period demand for rented capital by an existing match characterized by idiosyncratic productivity \( x_t = x \) is

\[
k_t^*(x) = \begin{cases} 
  d \left( \frac{x_t}{x P_t} \right) & \text{if } x \geq R_t, \\
  0 & \text{otherwise},
\end{cases}
\]  

(4.7)

where \( d = (f')^{-1} \) is a decreasing function and \( R_t \) is the reservation value of the idiosyncratic shock. Obviously, the representation reflects the fact that an existing job-worker match is destroyed and no capital is rented if an idiosyncratic shock is realized below the reservation value. The capital rental rate \( r_t \) is determined by the capital market clearing condition which can be written as

\[
K_t = \left[ \int_{R_t}^{1} d \left( \frac{r_t}{x P_t} \right) \ dF(x) \right] N_t,
\]  

(4.8)

where \( (K_t, N_t) \) is the given aggregate capital stock and employment pair as of the beginning of period \( t \).

As the current reservation value \( R_t \) solves \( S_t(R) = 0 \), Equation (4.6) implies

\[
\max_k \left\{ p_t R_t f(k) - r_t k \right\} + E_t \left\{ \frac{\gamma u'(C_{t+1})}{u'(C_t)} \max_k \left\{ S_t(x_{t+1}), 0 \right\} \right\} = b + \frac{\beta p_t c}{1 - \beta} \theta_t.
\]  

(4.9)

Given that \( x_t \sim F(x) \), it follows that expected ex ante match surplus conditional on knowledge of \( (p_t, R_t) \) is

\[
\int \max_k \left\{ S_t(x), 0 \right\} \ dF(x) = \int_{R_t}^{\infty} \left\{ \max_k \left\{ p_t f(k) - r_t k \right\} - \max_k \left\{ p_t R_t f(k) - r_t k \right\} \right\} \ dF(x)
\]  

(4.10)

by Equation (4.6). The fact that \( x_{t+1} \sim F(x) \) as well together with Equation (4.9) and (4.10) imply

\[
\max_k \left\{ p_t R_t f(k) - r_t k \right\} = b + \frac{\beta p_t c}{1 - \beta} \theta_t
\]

\[
- E_t \left\{ \frac{\gamma u'(C_{t+1})}{u'(C_t)} \int_{R_{t+1}}^{1} \left( \max_k \left\{ p_{t+1} f(k) - r_{t+1} k \right\} \right) \ dF(x) \right\},
\]

(4.11)

Finally, because \( x = 1 \) for a new match, \( S_0^0 = S_t(1) \). Hence, Equations (4.6) and (4.10) imply that Equation (4.5) can be written as

\[
p_t c = q(\theta_t)(1 - \beta)
\]

\[
\times E_t \left\{ \frac{\gamma u'(C_{t+1})}{u'(C_t)} \left( \max_k \left\{ p_{t+1} f(k) - r_{t+1} k \right\} - \max_k \left\{ p_{t+1} R_{t+1} f(k) - r_{t+1} k \right\} \right) \right\}.
\]  

(4.12)
Note that Equations (4.11) and (4.12) are generalizations of the job destruction and job creation conditions. Indeed, in the non-stochastic case with linear utility and no capital, these equations are equivalent to Equations (2.26) and (2.22) since Equation (4.3) implies $\gamma = 1/(1 + r_t)$ for all $t$ and the discrete time specification and the assumption that the idiosyncratic shock persists for one period imply that the duration of any shock is unity, i.e., $\lambda = 1$. However, a complete characterization of general equilibrium also requires that the equilibrium conditions of the neoclassical stochastic growth model, Equations (4.3) and (4.8), and the laws of motion hold.

The laws of motion for capital and employment are

$$K_{t+1} = (1 - \delta)K_t + p_t \left[ \int_{R_t}^1 \gamma f \left( g \left( \frac{r_t}{xH_t} \right) \right) \, dx \right] N_t$$

$$N_{t+1} = \theta_t q(\theta_t) (1 - N_t) - C_t$$

respectively. The first equation reflects the effects of job destruction and capital demand decisions made at the beginning of the period on output and the consumption decision while the second reflect the outcomes of current period job creation and destruction decisions. As the information relevant state of the economy is a triple composed of the capital stock, the employment level, and the aggregate shock, a dynamic stationary general equilibrium is a vector function that maps the state variable triple $(N, K, p)$ to the four endogenous variables $(C, r, R, \theta)$; one that solves the Euler equation (4.3), the capital market clearing condition (4.8), the job destruction condition (4.11), and the job creation condition (4.12) under the laws of motion (4.13) and (4.14).

Den Haan et al. (1997) derive the properties of the equilibrium by solving and simulating a particular parameterization of the model numerically. The qualitative properties they report are intuitively suggested by the known implications of the two models married in this synthesis. For example, a positive aggregate shock stimulates current investment in both job creation and physical capital which augment employment and productive capacity in the next period. In the short run, these investments must be financed with an output increase induced by a lower than normal reservation productivity choice and by a reduction in consumption. However, because of the consumption smoothing motive, the limited ability to increase output by increasing utilization through reductions in job destruction, and the complementarity of physical capital and labor, more investment of both types is made in subsequent periods as well, i.e., the shock is propagated.

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20 Following the literature, home production $b$ cannot be used to create capital by assumption. It is simply consumed.
A negative shock has an immediate and sharp negative effect on output along the job destruction margin. Although the effect is cushioned by the reallocation of existing capital to those jobs that continue, rental rates fall on impact in response to the decrease in demand for capital induced by job destruction and will be expected to fall further in the future as a consequence of the persistence in the shock. The result is a reduction in capital formation and job creation which has the effect of reducing output further in the future. Again the consumption smoothing motive interacting with the job creation and destruction process propagates the shock into the future.

As a consequence of the adjustment mechanisms described above, the simulated model implies strong first- and second-order autocorrelation in output growth rates, substantial persistence in the response of physical capital to negative productivity shocks, and a substantial magnification of the effects of productivity shocks on aggregate output. Neither the RBC model nor the augmented model featuring job matching but exogenous job destruction, like those of Merz (1995) and Andolfatto (1996), explain these features of the aggregate time-series data. As in Cole and Rogerson's (1996) reduced form analysis of the Mortensen and Pissarides job creation and destruction model, the calibrated version of the extended model studied by den Haan et al. (1997) also reproduces all the job flow time series stylized facts.

5. Technological progress and job reallocation

Search and matching models have been used to address the old "luddite" question of the influence of technological progress on job flows and unemployment levels. The common view is that new technology destroys jobs. Of course, innovations also generate new job creation. But, the resulting reallocation of workers from the old to new jobs may require an intervening unemployment spell. In this section, we explore the relation between the exogenous rate of technological progress and steady-state employment.

The analysis that follows suggests that the extent to which technical progress is "embodied" is critical. The distinction between embodied and disembodied technology is Solow's. In his original growth model [Solow (1956)], any improvement in technology instantaneously affected the productivity of all factors of production currently employed. But later he introduced the vintage model of embodied technical change in which productive improvements is a property of new capital investment only [Solow (1959)]. In the latter case, to capture the productivity benefits of technical change, older capital vintages must be replaced with the most recent equipment.

Our analysis begins by making the original assumption of disembodied technology. We show that if the rate of interest is independent of the rate of technological progress, faster technological progress leads to more job creation in the steady state. The dominant effect in this case is one of "capitalization". Because the costs of job creation are paid initially, faster technological progress implies a lower effective discount rate on future profits, leading to a higher present discounted value for profits [see Pissarides
D.T. Mortensen and C.A. Pissarides

(1990), Chapter 2]. The effect of faster growth on job destruction is, however, of indeterminate sign. We then consider the vintage model in the sense that "new capital" is assumed to be embodied only in newly created jobs. We show that under the assumption that the same worker cannot be moved from an old job to a new one without intervening unemployment, steady-state unemployment is higher at faster rates of technological progress [as in Aghion and Howitt (1994)]\textsuperscript{21}.

5.1. Disembodied technology

Let $p(t)$ represent the aggregate productivity parameter but now expressed as a function of time $t$. We assume that the rate of technological progress $g$ is constant, exogenous, and less than the rate of time discount, i.e.,

$$\frac{\dot{p}}{p} = g < r.$$ \hfill (5.1)

We treat $r$ as a constant independent of $g$\textsuperscript{22}. The other restrictions made are the same as in the basic model of Section 2.1, with the additional assumption that unemployment income is also a function of time. We assume for simplicity that $b(t) = b(t)p(t)$. This assumption is needed to ensure the existence of a steady-state growth equilibrium and is plausible in a long-run equilibrium when $p(t)$ is an aggregate productivity parameter\textsuperscript{23}.

The job creation and job destruction conditions of Section 2.1 change in an obvious way. Because all parameters in the value expressions (2.4), (2.5), (2.7) and (2.8) are multiplied by $p(t)$, and the wage equation still satisfies either (2.20) or (2.23), there is an equilibrium where all value expressions grow at constant rate $g$. Intuitively, the firm that has a job with value $J(x, t)$ at time $t$, expects to make a capital gain of $dJ(x, t)/dt = \dot{J}(x) = gJ(x)$ on it. The same holds true for the value of a job to the worker, $W(x, t)$, and the value of unemployment, $U(x, t)$, where the capital gain is, respectively, $gW(x)$ and $gU(x)$. But the value of a vacant job, $V(t)$, because it is zero

\textsuperscript{21} Mortensen and Pissarides (1998) consider a more general case of adoption of the new technology at a cost and show that the two cases that we consider here are two limiting cases, the first case approached when the adoption cost tends to zero and the second when the adoption cost tends to infinity. The main result of the paper is that there is a critical level of the adoption cost below which the dominant influences on job creation and job destruction are those described here under disembodied technology and above which the dominant influences are those described under embodied technology. See also Aghion and Howitt (1998, chapter 4) for more analysis of this issue.

\textsuperscript{22} Eriksson (1997) embeds the model in an optimizing (Ramsey) growth model and shows that the restriction that the effective discount rate decline with the rate of growth can be violated by feasible parameter values. He also considers the effects of growth on unemployment in an endogenous growth framework.

\textsuperscript{23} Making $b(t)$ a proportional function of the equilibrium wage rate would not change the results.
by the free entry condition, does not change. It is this asymmetry between $V(t)$ on the one hand and the other asset values on the other that creates the capitalization effect of faster growth.

We do not reproduce all the value expressions with growth but show instead the value of a continuing job to the firm, (2.4):

$$rJ(x, t) = p(t)x - w(x, t) + \lambda \int_0^1 [J(z, t) - J(x, t)] \, dF(z)$$

$$+ \lambda F(R)[V(t) - p(t) T - J(x, t)] + \dot{J}(x, t).$$

The capital gain to the firm is shown as an addition to revenues from continuing the job. Replacing the capital gain by its steady-state value, we get

$$(r - g)J(x, t) = p(t)x - w(x, t) + \lambda \int_0^1 [J(z, t) - J(x, t)] \, dF(z)$$

$$+ \lambda F(R)[V(t) - p(t) T - J(x, t)].$$

The main result of the introduction of growth can be seen from Equation (5.3). Because all value expressions grow at the constant rate $g$, wages will also grow at the constant rate $g$, and so all time-dependent variables in Equation (5.3) can be written as proportional functions of $p(t)$. Letting then $J(x, t) = p(t)J(x)$ and using similar notation for the other time-dependent variables, we can re-write Equation (5.3) in the same form as Equation (2.4), except that the discount rate $r$ is replaced by $r - g$.

It is straightforward to work through the model of Section 2 with the assumption that all time-dependent variables are proportional functions of aggregate productivity and show that there is a solution for the job creation and job destruction flows that replicate the solution shown in Figure 6 but with $r$ replaced by $r - g$. Hence, under the assumption that $r - g$ falls monotonically in $g$, we find that faster disembodied technological progress increases market tightness $\theta$ but has ambiguous effects on the reservation productivity $R$. Therefore, faster growth increases job creation, decreases the duration of unemployment but has ambiguous effects on job destruction and the incidence of unemployment in general. However, much of the literature on the effects of growth on unemployment concentrates on the obsolescence effects of new technology on job destruction (see the next section) and ignores the idiosyncratic reasons for job destruction. This assumption, also adopted in Pissarides (1990, Chapter 2), is justified in the long-run context by the fact that most variations in the job destruction rate in the data are high-frequency, with, at least in the European context where there have been substantial changes in the unemployment rate, virtually a constant job destruction flow across business cycles. This fact justifies a 0, 1 restriction on the support of the distribution of idiosyncratic shocks. In this case, variations in $R$ do not influence the job destruction rate, which is equal to $\lambda$, and so the effect of faster growth is to increase job creation and reduce unemployment.
5.2. Adoption through “creative destruction”

New technology cannot always be adopted by existing jobs. Much of public discussion and a large body of literature deals with the situation where the adoption of new technology requires the creation of new jobs with new capital equipment. This process of implementation is referred to in the literature as “creative destruction”, because old jobs have to be destroyed to release the resources for the creation of new jobs [see Aghion and Howitt (1992, 1994), Grossman and Helpman (1991), and Caballero and Hammour (1994)]. In this section we assume that the process of creative destruction induces a transition of the worker to unemployment and search for a new job. We demonstrate that more rapid technological progress under these assumptions induces more labor reallocation and so higher unemployment because of both lower job creation rate and higher job destruction rate.

In order to emphasize the new element of the model we abstract from idiosyncratic productivity shocks. Instead, heterogeneity in productivity arises because older jobs embody less productive technology and a job is destroyed when the technology embodied becomes obsolete.

Given that current technological improvements affect only productivity in newly created jobs, we need to distinguish between the date at which a job is created, its vintage $v$, and the current date, denoted as $t$. The expected present value of both future profit $J$ and wage income $W$ for a given job–worker match depends on the job’s vintage and the current date. These value functions solve the following asset pricing equations:

$$r J(v, t) = p(v) x - w(v, t) - \delta J(v, t) + J(v, t),$$

$$r W(v, t) = w(v, t) - \delta [W(v, t) - U(t)] + \dot{W}(v, t),$$

where $x$ represents job match productivity, $w(v, t)$ is the wage paid on a job of vintage $v$ at date $t$, $\delta > 0$ represents an exogenous job separation rate, and $U(t)$ is the value of unemployed search at $t$.

The fixed cost of investment in a new job, denoted as $p(t)C$, is incurred when the match forms. The investment is specific to a job, i.e., it is “irreversible” with no outside option value once the match forms. The recruiting costs, $p(t)c$, are modelled as a cost per vacancy posted. New vacancies enter at every date until market tightness is such that the value of creating a vacancy, $V(t)$, is zero, i.e.

$$r V(t) = q(\theta)[J(t, t) - p(t)C] - cp(t) = 0,$$

where $q(\theta)$ is the rate at which vacancies are filled. Similarly, the value of unemployment solves the asset pricing equation

$$r U(t) = p(t) b + \theta q(\theta)[W(t, t) - U(t)] + \dot{U}(t),$$

where $p(t)b$ represents the opportunity cost of employment and where $\theta q(\theta)$ is the rate at which workers find vacancies. As before, recruiting costs, the investment required to
create a match, and the opportunity cost of employment grow at rate \( g \) by assumption to assure the existence of a balanced growth path equilibrium solution to the model.

We assume that the wage bargain divides the surplus value of a continuing match in fixed proportion, i.e.,

\[
\beta J(v, t) = (1 - \beta) [W(v, t) - U(t)],
\]

where \( \beta \) represents the worker’s share\(^24\). Because Equations (5.4) and (5.6) imply

\[
(r + \delta)J(v, t) = p(v) x - w(v, t) + \dot{J}(v, t),
\]

\[
(r + \delta) [W(v, v) - U(t)] = w(v, t) - rU(t) + \dot{W}(v, t),
\]

the wage contract that supports the assumed bargaining outcome (5.7) is

\[
w(v, t) = \beta p(v) x + p(t) \left( (1 - \beta) b + \beta (c\theta + \theta q(\theta) C) \right)
\]

by virtue of the free entry condition (5.5). The first term on the right reflects the worker’s productivity while the second captures the worker’s option value outside the firm. Because the latter grows at the rate of technological progress but the former is stationary, every job becomes obsolete eventually.

By substituting from the wage equation into the first of Equations (5.4), we obtain

\[
(r + \delta)J(v, t) = (1 - \beta) p(v) x - p(t) \left( (1 - \beta) b + \beta (c\theta + \theta q(\theta) C) \right) + \dot{J}(v, t).
\]

Indeed, Equation (5.9) holds only for \( t - v \leq \tau \) where \( \tau \) is the optimal economic lifespan of a job. The employer’s choice of a job’s economic life maximizes its value, i.e.,

\[
J(v, t) = \max_{\tau} \left\{ \int_{t}^{t+\tau} \left[ (1 - \beta) p(v) x - p(s) \left[ (1 - \beta) b + \beta (c\theta + \theta q(\theta) C) \right] \right] e^{-(r+\delta)(s-t)} ds \right\}.
\]

The maximal value of a new job at time \( t \) is the special solution to this equation satisfying the balance growth equation \( J(t, t) = J^0(\theta) p(t) \) where, given the normalization \( p(0) = 1 \),

\[
J^0(\theta) = J(0, 0)
\]

\[
= \max_{\tau} \left\{ \int_{0}^{\tau} \left[ (1 - \beta) x - e^{\delta s} \left[ (1 - \beta) b + \beta (c\theta + \theta q(\theta) C) \right] \right] e^{-(r+\delta)s} ds \right\}.
\]

\(^24\) Here workers do not share the cost of initial investment by accepting a lower starting wage for an initial period of employment as assumed in Section 2. Instead, the initial wage is equal to the continuing wage at initial productivity. Although equilibrium market tightness will be too low relative to a social optimum initially, the qualitative behavior of a model under a jointly efficient wage bargain would be much the same. See Caballero and Hammour (1994, 1996) for more discussion of this issue.
The first-order condition for a positive optimal choice of the economic life of a job equates stationary match product with the rising opportunity cost of continuing an existing match, i.e.

\[(1 - \beta)x - [(1 - \beta)b + \beta(c\theta + \theta q(\theta)C)]e^{\delta r} = 0. \quad (5.12)\]

Since \(J(t, t) = J^0(\theta)p(t)\), the free entry condition (5.5) can be written as

\[c = q(\theta)[J^0(\theta) - C]. \quad (5.13)\]

A search equilibrium is characterized by any market tightness and age at job destruction pair \((\theta^*, \tau^*)\) that solves Equations (5.12) and (5.13).

Because the right-hand side of Equation (5.13) is strictly decreasing in \(\theta\), equilibrium market tightness is unique. Of course, given \(\theta^*\), the associated equilibrium value of the optimal job age at destruction, \(\tau^*\), is the unique solution to Equation (5.12). Since Equations (5.12), (5.13) and (5.11) imply

\[\frac{c}{q(\theta^*)} + C = J^0(\theta^*) = (1 - \beta)x \int_0^{\tau^*} \left[1 - e^{\gamma(s - \tau^*)}\right] e^{-\gamma(\tau^* + s)} ds, \quad (5.14)\]
a necessary but hardly sufficient condition for the existence of a positive equilibrium pair \((\theta^*, \tau^*)\) is that match productivity \(x\) exceed the opportunity cost of employment \(b\). Indeed, given this condition, an economically meaningful equilibrium exists only if both recruiting and creation costs, \(c\) and \(C\), are sufficiently small.

Because the surplus value of a match decreases with the rate of technological progress, \(g\), for every value of market tightness by virtue of Equation (5.11) and the envelope theorem, namely

\[\frac{\partial J^0}{\partial g} = - \int_0^{\tau^*} \left[se^{\gamma s} ((1 - \beta)b + \beta(c\theta + \theta q(\theta)^2 C))\right] e^{-\gamma(\tau^* + s)} ds < 0,\]

and because both the value of a job and the rate at which vacancies are filled decrease with market tightness, the free entry condition (5.13) implies that market tightness falls with the growth rate, i.e.,

\[\frac{\partial \theta^*}{\partial g} = \frac{\frac{\partial J^0}{\partial g}}{\frac{\partial q(\theta^*)^2}{\partial g} + \frac{\partial q(\theta^*)^2}{\partial \theta} \frac{\partial \theta^*}{\partial g}} < 0.\]

Because the left-hand side of (5.14) is decreasing in \(g\) and the right-hand side is increasing in both \(g\) and \(\tau^*\), it follows that the economic life of a new job also falls with the rate of growth, i.e.,

\[\frac{\partial \tau^*}{\partial g} = \frac{\frac{\partial J^0}{\partial g}}{\frac{\partial q(\theta^*)^2}{\partial g} + \frac{\partial q(\theta^*)^2}{\partial \theta} \frac{\partial \theta^*}{\partial g}} < 0.\]

To derive the implications of these facts for unemployment and job flows, first note that job creation at time \(t\) is

\[K(t) = \theta^* q(\theta^*) u(t). \quad (5.15)\]

Job destruction is equal to the flow of jobs that attain the age of optimal obsolescence plus the flow of all jobs that experience exogenous destruction. As the fraction of jobs
of each cohort that survive to age $\tau$ is $e^{-\delta \tau}$ given the exogenous destruction hazard is $\delta$, the job destruction flow at time $t$ is

$$D(t) = e^{-\delta \tau^*} K(t - \tau^*) + \delta[1 - u(t)].$$

Hence, the steady-state unemployment rate that equates job creation and job destruction flows through time is

$$u^* = \frac{\delta}{\delta + (1 - e^{-\delta \tau^*}) \theta^* q(\theta^*)}.$$  

(5.17)

It increases with the rate of embodied technical progress because both market tightness and the economic life of an existing job decline with $g$ and because the unemployment duration hazard $\theta q(\theta)$ is increasing in $\theta$.

Technological progress in this model adversely effects worker flows into and out of employment for two reasons. The first is a restriction that we have imposed on the model, namely, that when a machine is replaced because of obsolescence the worker that was employed on that machine is also replaced. This assumption also underlies the work of Aghion and Howitt (1994) and Caballero and Hammour (1996) and is derived from Schumpeter's notion of "creative destruction". The idea is that when a job is destroyed it is replaced by a technologically more advanced one, with positive effects on factor productivity. The second is a particular assumption about the timing of job creation costs.

The implication of the first restriction for the job destruction flow is straightforward enough: faster technological progress necessitates more job destruction. Job creation also falls in our model when there is faster technological progress because as the life of a job becomes shorter, the expected present value of future profit attributable to a job falls. It may turn out to be surprising that even when the interest rate is independent of growth faster growth does not have a countervailing effect on the present discounted value of profits. Since in the expressions that we have derived for the surplus from a job the effective discount rate is $r - g$, profits are discounted at lower rate. So faster growth has a "capitalization" effect on the profits stream. Our results, however, show that this capitalization effect is dominated by the negative influence on the present value calculation implied by the shorter life of a job.

Aghion and Howitt's (1994) model of the adoption of new technology is essentially the same as the one in this section, yet it has a bigger capitalization effect that is not always dominated by the shorter life of the job. This effect is implied by the assumption that there are job set-up costs that have to be paid before the firm begins the recruiting process. In this case the profit stream is discounted more heavily, since the zero profit restriction requires that the present discounted value of profits at the date the vacant job is created must equal to the set-up costs.

6. OECD unemployment differences

We saw in Section 1 that the unemployment experiences of OECD countries over the last thirty years have been different from each other. This is all the more surprising
because with increasing openness and trade, and with the global oil and material shocks of the 1970s, the shocks affecting OECD countries cannot have been very different in different countries. The different experience of OECD countries is most likely due to a different response of each country to common shocks, due to different market structures, or to differences in policy.

The most frequently discussed contrast in OECD experience is that between the USA and "Europe". Although the contrast is often exaggerated, especially in the more popular discussions, there is some truth in the basic argument, that whereas wages at the lower end of the wage distribution fell in the USA with unemployment remaining the same on average, in most of Europe wages increased but unemployment increased too. We saw in Section 1 that there appears to be a trade off between the increase in wage inequality and the increase in unemployment experienced by OECD countries. Figure 4 shows that over the 1980s the USA experienced a bigger increase in inequality and a smaller increase in unemployment (in fact, a decrease) than the major European countries.

The experience documented in Figure 4 is most likely a response to a heterogeneous aggregate productivity shock that can be decomposed into two parts, one that shifted the productivity distribution to the right and one that widened the range of the distribution for given mean. There has been a long debate in the literature as to whether the second component of the shock, the one that worsened prospects for unskilled workers but improved them for skilled ones, was due to a technology shock, associated for example with computerization, or to a trade shock, associated with the expansion of trade with newly industrialized nations in South East Asia and Latin America. Our analysis, and more generally the search and matching framework, is one that can be used to analyze the consequences of the shocks, whatever their source.

In this section we survey the key influences that have been identified in the literature as the causes of the experience of OECD countries summarized in Section 1. In a discussion of this kind, it is difficult to avoid a discussion of labor market policy, especially if one were to discuss the unemployment experiences of countries like Spain and Sweden and why they have been different from the median European experience\(^2\). The detailed modelling and discussion of labor market policies, however, will take us beyond the scope of our chapter. We mention instead policy influences in passing, using parameters that we already have in our analysis to represent the effects of policy. Two parameters in particular are relevant to our discussion, unemployment income \(b\), which we take also to represent the generosity of the unemployment insurance system, and the firing cost \(T\), which we take to stand for employment protection legislation. The active labor market policies pursued by Sweden, and to a lesser degree by some other

\(^{2}\) There has been a large literature on the unemployment experience of each of these countries. For Spain, see for example, Blanchard et al. (1995), Dolado and Jimeno (1997) and Marimon and Zilibotti (1998). For Sweden, see Calmfors (1995) and Ljungqvist and Sargent (1995). See also Scarpetta (1996) for a cross-country OECD study.
countries, can be shown in the model by reductions in the job creation cost $C$, though this does not do justice to the complexity and sophistication of some of the targeted policies in operation.

6.1. ‘Skill-biased’ technology shocks

As noted above, changes in technology that raise the productivity of skilled workers relative to that of unskilled is one of the explanations given for the recent increase in US wage dispersion. It has been argued that these same shocks may have generated the observed increases in European unemployment [see the OECD Jobs Study (1994), Krugman (1994) and others]. The reason for the different response is a different labor market policy regime. In Europe, where higher level of unemployment compensation, minimum wages, and employment protection restrict accommodation through downward wage adjustment, the response is likely to be higher unemployment, particularly among the unskilled. The purpose of this section is to explore this hypothesis within the equilibrium search and matching framework.

In the Mortensen-Pissarides model, a producing unit is a job–worker match. To capture skill differences across workers, one can simply reinterpret the parameter $p$ as an efficiency unit measure of the worker’s skill. Given two workers in an identical match, the relative product per time period of the second worker is equal to the ratio $P_2/P_1$ where $p_i, i = 1, 2$, represents the “skill” of each.

Let $P$, a set of real numbers, represent the set of skill types and $G : P \rightarrow [0, 1]$ denote the distribution of the labor force population over these types. Given this formalization, a pure skill-biased shock to technology can be interpreted as a mean-preserving increase in the spread of $G$. In this section we argue that such a shock will increase unemployment in the Mortensen–Pissarides framework and that the extent of the increase is likely to be larger for economies with high level of unemployment compensation and stringent employment protection laws.

Given the assumption that skill differences are observable, as say they would be if associated with different levels of education, we can consistently assume that the labor market is segmented along skill lines. Across markets the reservation levels of the idiosyncratic shock and market tightness can differ. In the sequel, let $R(p)$ and $\theta(p)$ characterize equilibrium relationships between these two endogenous variables and worker skill. Obviously, these functions, which satisfy the job creation and job destruction conditions Equations (2.22) and (2.26), and the steady-state Beveridge condition (2.3), determine the equilibrium relationship between unemployment and skill of interest in this section.

The qualitative differences in both market tightness $\theta$ and the reservation value of the idiosyncratic component of match product $R$ at two different skill levels are readily

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26 Acemoglu (1996, 1998) explains changes in unemployment and wage inequality in terms of endogenous technology changes and changes in labor supply.
Table 3
Baseline parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol(s)</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount rate</td>
<td>$r$</td>
<td>0.02 per quarter</td>
</tr>
<tr>
<td>Matching elasticity</td>
<td>$\eta$</td>
<td>0.5</td>
</tr>
<tr>
<td>Recruiting cost</td>
<td>$m(\theta)$</td>
<td>0.3 per worker</td>
</tr>
<tr>
<td>Creation cost</td>
<td>$C$</td>
<td>0.3 per worker</td>
</tr>
<tr>
<td>Productivity shock frequency</td>
<td>$\lambda$</td>
<td>0.1</td>
</tr>
<tr>
<td>Minimum match product</td>
<td>$\gamma$</td>
<td>0.68 per quarter</td>
</tr>
<tr>
<td>Value of leisure</td>
<td>$b$</td>
<td>0.62 per quarter</td>
</tr>
<tr>
<td>Worker's share</td>
<td>$\beta$</td>
<td>0.5</td>
</tr>
<tr>
<td>Firing Cost</td>
<td>$T$</td>
<td>0 per worker</td>
</tr>
</tbody>
</table>

predicted by the model. For a more skilled worker, one characterized by a higher value of general productivity parameter $p$, the relative opportunity cost of employment, the ratio $b/p$, is lower. Given the assumption that hiring and firing costs increase proportionately with $p$, the job destruction relation, $DD$ in Figure 6, is lower given a higher value of $p$ and the job creation relation $CC$ is unaffected by variation in $p$. As a consequence, markets for the more skilled are tighter, unemployment durations are shorter. Furthermore, the reservation value of idiosyncratic productivity is lower in markets for the more skilled and, consequently, the incidence of unemployment is lower. These inferences are consistent with empirical findings to the extent that the level of education is a good indicator of skill.

As the unemployment rate is a positive number by definition, the fact that it declines with the skill parameter $p$ implies that the unemployment–skill profile is convex, at least on average. To the extent that the relationship has this shape, any increase in the mean-preserving spread of the distribution of relative productivity, defined above as a ‘skill-biased’ technology shock, will increase unemployment. This effect can explain the run up in European unemployment rates relative to those in the USA if European labor policies increase the convexity of the unemployment–skill profile. In short, if unemployment compensation and employment protection has a larger relative impact on the unemployment of unskilled workers, then the same ‘skill-biased’ technology shock increases unemployment more in countries with these policies.

To ascertain whether this explanation has force, we calibrate a simple version of the model and then use it to compute the implied unemployment–skill profile for different policy regimes. A matching function of the Cobb–Douglas form is assumed with elasticity with respect to unemployment equal to $\eta$, i.e., $\ln(q(\theta)) = -\eta \ln(\theta)$. The distribution of idiosyncratic shocks is assumed to be uniform on the support $[\gamma, 1]$, i.e., $F(x) = (x - \gamma)/(1 - \gamma)\forall x \in [\gamma, 1]$. The baseline parameters used in the computations are reported in Table 3. Except for value of income while unemployed $b$ and the minimum
match product \( \gamma \), which are chosen so that the steady-state unemployment rate of a worker of average skill \( (p = 1) \) is 6.5% and the average duration of an unemployment spell for such a worker is 3 months, values which reflect experience in the USA over the past twenty years, the parameter values are similar to those assumed and justified in Mortensen (1994b) and Millard and Mortensen (1997).

To obtain parameters that reflect the European experience, we maintain the same values of all parameters except for unemployment income \( b \) and firing cost \( T \) which are chosen to yield the same average unemployment rate but an average spell duration of 9 months. The results, \( b = 0.77 \) and \( T = 1.1 \), are consistent with the fact that unemployment compensation and the implicit cost of employment protection are both substantially higher in Europe than in the USA and the fact that unemployment spells are much longer in Europe.

The computed unemployment–skill profiles for three different policy parameter combinations are illustrated in Figure 8. Specifically, each curve is a plot of the equilibrium unemployment function \( u(b, T, p) \) for value of the skill parameter \( p \). The flattest profile corresponds to low unemployment compensation and no employment protection policy, the base line case of \( (b, T) = (0.62, 0) \). Given a more generous unemployment compensation but still no employment protection, \( (b, T) = (0.77, 0) \), the profile lies everywhere above the original but is substantially more convex, i.e., the steady-state unemployment rate of the less skilled is more responsive to the level of unemployment compensation. Adding employment protection, as illustrated by the solid curve representing the case \( (b, T) = (0.77, 1.1) \), actually lowers the unemployment rate of the more skilled but raises that of the unskilled. In short, employment protection policy induces even more convexity into the unemployment–skill profile.
In sum, a given ‘skill-biased’ technology shock increases unemployment by more when unemployment compensation and the implicit firing cost associated with employment protection policy are higher in the Mortensen–Pissarides model. The magnitudes of the computed differences in unemployment rates across skills suggest that indeed shocks of this form could well explain the secular rise in European unemployment rates relative to the USA in the 1980s.

Returning now briefly to the question of participation, recall that alongside the relative (and absolute) rise in European unemployment there has also been a decline in participation rates. We saw in this section that once the model is reinterpreted as one where there are many submarkets, one for each skill, the policy changes that we have described can be shown to have a bigger impact on the market returns of the lower skills than those of the higher skills. If we now require that participation of skill group p takes place only when the total net return from that group exceeds some fixed cost, any policy or other change that increases unemployment because of the relative decline in the returns from a job match will also increase the threshold participation skill, the one below which no participation takes place. The reasons given here for the rise in European unemployment are ones that reduce the net returns from the participation of low skilled workers and so they are ones that can also explain a fall in the overall participation rate of these groups.

6.2. Mean-preserving shocks to idiosyncratic productivity

A substantial fraction of the increased US wage inequality has also occurred within identifiable skill and education groups. In the Mortensen and Pissarides model earnings dispersion of this form could result as a consequence of greater variation in match specific idiosyncratic productivity. Following Arrow (1965), suppose that the idiosyncratic component of productivity is written as a function of a multiplicative parameter $h$, so

$$x(h) = x + h(x - \bar{x}), \quad (6.1)$$

where $h \geq 0$ is a parameter and $\bar{x}$ is the mean of the distribution. We shall consider the effect of a shift in $h$ on the steady-state equilibrium, evaluated in the neighborhood of the old equilibrium, $h = 0$. In order to make the analysis more meaningful for the question in hand, we assume that $p\bar{x} \geq rU$, i.e. that the reservation wage of the unemployed job seekers is below mean productivity. This ensures that the multiplicative shock reduces the productivity of at least some active low-productivity jobs.

Reworking the job creation and job destruction conditions with $x(h)$ replacing $x$ is straightforward. The job creation condition (2.22) becomes

$$\frac{c}{q(\theta)} = (1 - \beta) \left( \frac{(1 + h)(1 - R)}{r + \lambda} - C - T \right), \quad (6.2)$$

where $\beta$, $r$, $C$, and $T$ are appropriate parameters and $\lambda$ is the firing cost.
whereas the job destruction condition (2.26) becomes

\[ p(1 + h) R - hpx + \frac{p(1 + h)\lambda}{r + \lambda} \int_R^1 (z - R) dF(z) = b - rpT + \frac{\beta}{1 - \beta} \theta. \]  

(6.3)

Equilibrium is still shown by the two lines of Figure 6. Higher \( h \) shifts the DD line up, implying higher reservation productivity at all levels of market tightness, because the productivity of the marginal job is now worse. But higher \( h \) also shifts the CC line to the right, because, for given reservation productivity greater than zero, the benefits from the higher productivity of jobs above the mean outweigh the costs from the lower productivity of jobs below the mean, the tail of which is truncated. Thus, job creation unambiguously goes up at given unemployment stock but the effect of the higher \( h \) on job destruction is ambiguous from the diagram alone. Differentiation of Equations (6.2) and (6.3) with respect to \( h \), however, shows that at \( h = 0 \), the reservation productivity rises unambiguously (see Appendix A). So both job creation and job destruction rise at given unemployment when there is a multiplicative productivity shift.

The effect of this shift on unemployment is ambiguous. On impact, unemployment rises, because job destruction leads job creation, but whether unemployment rises or falls in steady state depends on whether the direct impact on job destruction or job creation dominates. The effect on wage inequality is also ambiguous, because, although the range of productivities falls, the productivity of the marginal job may rise or fall. The impact on the productivity of the marginal job, when evaluated at \( h = 0 \), is given by

\[ \frac{\partial R(h)}{\partial h} = \frac{\partial R}{\partial h} - (\bar{x} - R). \]  

(6.4)

We note, however, the following. If the impact of the multiplicative shock on reservation productivity is large, it is more likely that job destruction will dominate job creation and unemployment will rise in equilibrium, and also that the productivity of the marginal job will rise (or fall less) than otherwise. If, on the other hand, the impact of the multiplicative shock is large on market tightness and small on the reservation productivity, it is more likely that unemployment will eventually fall and the productivity of the marginal job will also fall. Thus, in countries where there are conditions that amplify the impact of multiplicative shocks on job destruction, their consequence is an increase in unemployment associated with a decrease (or small increase) in inequality. In countries where the reverse happens, the consequence of multiplicative shocks is to increase inequality but either reduce or increase unemployment by a smaller amount.

We can identify one factor in our analysis that might play a role in explaining the difference between the experience of Europe and the USA, though the explanation cannot be a complete one. This is the parameter representing labor's bargaining strength, \( \beta \). The Appendix shows that higher \( \beta \) implies lower impact of \( h \) on \( \theta \), though the impact of \( h \) on \( R \) is not likely to depend on \( \beta \) at plausible values of \( \beta \) [more precisely
when $\beta$ is in the neighborhood of the elasticity of the matching function with respect to unemployment, or the elasticity of $q(\theta)$. Therefore, countries with more powerful labor organization when hit by a shock that increases inequality are likely to experience more unemployment, through less job creation, than countries with less powerful labor. It is often asserted that labor is more powerful in Europe than in the USA, either because of more powerful trade unions or because of legislation that favors labor. So this could be one factor behind the different unemployment experience of the two continents. With regard to inequality, however, the model does not have strong predictions.\textsuperscript{27}

\subsection*{6.3. Other influences}

Several other influences on the equilibrium unemployment rate have been investigated in the empirical literature, in search of the elusive explanation for the rise in European unemployment. Virtually all the determinants of the equilibrium rate discussed in Section 3 have been, at one time or another, listed as possible causes of higher unemployment in Europe. This includes, in addition to unemployment income and trade union power discussed above, the real rate of interest, taxes on wages, which reduce the net surplus from a job match, “mismatch”, by which is usually meant more heterogeneity in the labor market and which is represented by a shift of the aggregate matching function, employment protection legislation, which increases the costs of job destruction, and on the positive side “active labor market policies”, which reduce the job creation costs and costs of labor to the firm.

As we saw in Section 2.3, higher real rate of interest reduces market tightness but has ambiguous effects on the reservation productivity. At given unemployment rates job creation falls. In terms of the Beveridge diagram, real interest rates have ambiguous impact on the Beveridge curve but rotate the job creation line down. It has been argued, however, that empirically higher real interest rates have depressed employment in the OECD, i.e. that the job creation effect dominates over the job destruction effect [Phelps (1994)].

Taxes on employment reduce the net surplus from the job, so whether they reduce job creation or not depends on their influence on non-employment income. If non-employment income is not taxed, their effects on the equilibrium of the model is similar to a rise of non-employment income, i.e. they reduce job creation and increase job destruction at given unemployment rate. Taxes, however, may also have distortionary effects if they are not proportional to incomes, a topic that would take us beyond the scope of our chapter.\textsuperscript{28}

\textsuperscript{27} One prediction is that the lowest wage is almost certain to rise when the multiplicative shock arrives, because of the increase in the reservation productivity and in market tightness. Then, it becomes likely that the cross effect of $h$ and $\beta$ on the lowest wage is also positive, so countries with less powerful labor experience more increase in inequality. (These results are not proved here.)

\textsuperscript{28} Pissarides (1998), Mortensen (1994a), and Millard and Mortensen (1997) all study tax effects using search equilibrium models. See also Daveri and Tabellini (1997), who explain the slowdown in growth and rise in unemployment in Europe by tax increases on labor.
Mismatch can arise in our framework in the following sense. The aggregate matching function conceals a lot of heterogeneity in the labor market. It is a convenient modelling device when our interest is in aggregate changes rather than individual employment histories. Out of all the interactions between the many heterogeneous groups in the population, a stable relationship emerges between the job matching rate and the stocks of aggregate unemployment and vacancies. But if conditions are such that the type of workers and jobs available change, either in skill requirement or in location, the aggregate outcome from the interaction between those groups is also likely to change. An increase in mismatch shifts the aggregate matching function down at all levels of vacancies and unemployment.

Mismatch bears some relationship to the more commonly discussed, in the US literature, “sectoral shifts hypothesis”, though it is more general [Lilien (1982)]. It also bears some relationship to the older view of “structural” unemployment, which was thought to be unemployment arising from fast structural change in the economy as a whole. In Europe, mismatch has been proposed by Jackman et al. (1989), Layard et al. (1991) and others as a cause of the rise in European unemployment. The oil, technology and other real shocks of the 1970s and 1980s increased the speed with which unemployed workers needed to adapt to the changing requirements of employers. This led to increased mismatch, which increased unemployment at given vacancies. Although neither the sectoral shifts hypothesis in the USA, nor the mismatch hypothesis in Europe, has had much success in accounting for a large fraction of employment fluctuations, we look here at the implications of the mismatch hypothesis within the search and matching framework.

The argument is that because labor in Europe is less mobile than in the USA, a problem aggravated by the longer durations of unemployment in Europe, the changing requirements of jobs lead to bigger and more prolonged shifts of the aggregate matching function. Mismatch in the formal model is shown as a fall in the productivity of the aggregate matching process, i.e. a downward shift of the transition rate $q(\theta)$ at all values of $\theta$. This shifts the job creation line in Figure 6 down, reducing both market tightness and the reservation productivity. But in addition, mismatch has the implication that for given market tightness, the rate of job matching is lower. This implies, in our model, a shift of the Beveridge curve out, over and above any effects that there might be through job creation and job destruction. It is this additional shift in the Beveridge curve that has attracted most attention in the discussions of mismatch in the search literature.

It is clear that the overall effect of increased mismatch on equilibrium unemployment is uncertain, because of the three interacting effects: less job entry at given

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29 See Jackman et al. (1989) for the United Kingdom, Abraham and Katz (1986) and Blanchard and Diamond (1989) for the USA and Jackman et al. (1990) for an international comparison of Beveridge curve shifts. Andolfatto (1996) incorporates a stochastic shift parameter in the matching function in his calibrations of the search and matching model.
unemployment, less job destruction and less job creation at given vacancies and unemployment. The empirical literature, however, invariably takes the latter effect, shown in the diagrams by the outward shift of the Beveridge curve, as the one that dominates on unemployment. Of course, a sufficient condition for this is that for given $\theta$, the fall in $q(\theta)$ due to the direct effect of increased mismatch dominate the fall in $\lambda F(R)$ due to the indirect effect from the fall in the reservation productivity. But since the job creation line in the Beveridge curve diagram (Figure 7) rotates down when mismatch increases, the effect of increased mismatch on equilibrium job vacancies is thought in the empirical literature to be unimportant. It is this latter property (higher unemployment at given vacancies), which has been a feature of the 1980s rise in European unemployment, that has attracted research in this area.

Countries with more restrictions in job separations are ones that have higher values for the firing cost $T$. We saw that those countries should experience less job creation and job destruction at given unemployment rate, through lower $R$ and $\theta$. The effect on equilibrium unemployment is ambiguous but the effect on job reallocation is negative.

This result might explain why job reallocation rates differ across countries. In an analysis of the data on job reallocations given in Section 1 and the employment protection provisions in different countries as constructed by the OECD, Garibaldi et al. (1997) found a clear relationship between employment protection legislation and job reallocation. Given, however, the Beveridge curve equation that defines equilibrium unemployment, there is no reason to expect a correlation between job reallocation and equilibrium unemployment.

Firing costs might also explain, to some extent, the differences between the job reallocation rates between small and large firms. Usually large firms in Europe are subject to many more restrictions on firing workers, imposed either by legislation or by trade unions. In Italy, where there are severe restrictions on job separations in large firms, many more small firms come into operation and job reallocation rates in those small firms are comparable to those in the USA [see Contini et al. (1995)].

Finally, lower job creation costs lead to more job creation at given unemployment and more job destruction. Once again, the effects on equilibrium unemployment are ambiguous. Many European governments, however, have tried to encourage job creation by giving incentives which reduce job creation costs. One of the criticisms levelled against such policies is that they encourage the creation of “unstable” jobs that do not stay in operation for long periods. This argument is valid in our analysis but still hiring subsidies may be justified, particularly if the worker’s effective share of match-specific investments in training and information are less than their share of continuing match surplus [Mortensen (1996)].

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30 Bertola and Rogerson (1997) claim that job reallocation rates do not differ as much across countries as they should be expected to do, given differences in firing costs. They explain this by the differences in wage inequality that characterizes countries, and which tends to reduce job reallocation rates.
7. Concluding remarks

We have demonstrated that the search and matching approach provides a rich framework for the analysis of aggregate employment fluctuations and of the observed differences in average unemployment rates across countries. Calibrations of the models track the cyclical fluctuations in the job creation and job destruction flows reasonably well. The framework provides a convenient medium for the analysis of policy influences on unemployment, which lie at the heart of the explanations of average unemployment differences across countries. Although there is still no consensus on the causes of the higher unemployment rates in Europe than in the USA, we have shown how policy influences, in particular the unemployment insurance system and employment protection legislation, can contribute to the differences in both unemployment rates and wage inequality.

Wages in the models that we have examined are determined by a fixed rule that shares the economic rents that each employer–worker match creates. Other methods of wage determination are also consistent with our framework and some promising work is being done in this area of research. We discuss some of this work in our companion chapter for the Handbook of Labor Economics. Another promising area of current research is the interaction between technology, capital and labor in markets with frictions. This area of research provides a natural framework for the analysis of hold-up problems and problems of obsolescence and growth. We discussed some work in this area in this chapter but much remains to be done.

Appendix A. Mathematical appendix

A.1. Mean-preserving shifts in productivity

Differentiation of Equation (6.3) with respect to the parameter $h$ and evaluation of the result at $h = 0$ gives

$$\left[1 - \frac{\lambda}{r + \lambda}[1 - F(R)]\right] \frac{\partial R}{\partial h} = (\bar{x} - R) - \frac{\lambda}{r + \lambda} \int_R^1 (z - R) dF(z) - \frac{\beta}{1 - \beta} c \frac{\partial \theta}{\partial h}. \quad (A.1)$$

Differentiation also of (6.2) with respect to $h$ gives

$$\frac{c \eta}{\theta q(\theta)} \frac{\partial \theta}{\partial h} = \frac{1 - \beta}{r + \lambda} \left[1 - R - \frac{\partial R}{\partial h}\right]. \quad (A.2)$$

Substitution of $\partial R/\partial h$ from Equation (A.1) into (A.2) reveals that the sign of $\partial \theta/\partial h$ is the same as the sign of

$$1 - R - \frac{\bar{x} - R - \frac{\lambda}{r + \lambda} \int_R^1 (z - R) dF(z)}{1 - \frac{\lambda}{r + \lambda}[1 - F(R)]}. \quad (A.3)$$
Multiplying out the denominator of Equation (A.3) and collecting terms, we find that the sign of the terms in (A.3) is the same as the sign of

\[ 1 - \tilde{x} - \frac{\lambda}{r + \lambda} \int_R^1 (1 - z) \, dF(z), \]  

which is unambiguously positive since

\[ 1 - \tilde{x} = \int_0^1 (1 - z) \, dF(z). \]

Hence, the effect of higher \( h \) is positive on both \( R \) and \( \theta \).

**A.2. Labor's bargaining strength**

Differentiation of Equation (6.3) with respect to \( \beta \) gives

\[ \left[ 1 - \frac{\lambda}{R + \lambda[1 - F(R)]} \right] \frac{\partial R}{\partial \beta} = \frac{1}{1 - \beta} \left[ \frac{c\theta}{1 - \beta} + \beta c \frac{\partial \theta}{\partial \beta} \right]. \]  

Differentiation of Equation (6.2) gives

\[ \frac{c\eta}{q(\theta)} \frac{\partial \theta}{\partial \beta} = -\frac{c}{q(\theta)(1 - \beta)} - \left(1 - \beta\right) \frac{1}{r + \lambda} \frac{\partial R}{\partial \beta}. \]  

Substitution of \( \partial \theta/\partial \beta \) from Equation (A.7) into (A.6) reveals that the sign of \( \partial R/\partial \beta \) is the same as \( \eta - \beta \). So \( R \) reaches a unique maximum at \( \beta = \eta \), which is also the efficient point, when the search externalities are internalized [see Hosios (1990)].

Although there is no reason why the two parameters should be equal, the usual restriction on \( \beta \) in symmetric bargaining situations is \( \beta = \frac{1}{2} \) and the empirical evidence on \( \eta \) suggests that it is close to 0.5 so the restriction \( \beta = \eta \) is a convenient simplification that may be adopted. We shall do so in the derivations in this Appendix. Under the restriction then that

\[ \frac{\partial R}{\partial \beta} = 0, \]  

the effects of labor's bargaining strength on market tightness become

\[ \frac{\partial \theta}{\partial \beta} = -\frac{\theta}{\eta(1 - \beta)}. \]  

Turning now to the question of the cross partials of \( h \) and \( \beta \) on \( R \) and \( \theta \), i.e. on the response of reservation productivity and market tightness to a multiplicative
productivity shift in countries with different labor bargaining strength, we immediately find from Equation (A.9) that

$$\frac{\partial^2 \theta}{\partial h \partial \beta} = -\frac{1}{\eta(1-\beta)} \frac{\partial \theta}{\partial h} < 0. \quad (A.10)$$

So in countries with more powerful labor, the positive response of market tightness to the productivity shock is smaller. The cross partial of $R$ is calculated by differentiating Equation (A.1) with respect to $\beta$. This shows that the sign of the cross partial $\partial^2 R/\partial h \partial \beta$ is the same as the sign of

$$-\frac{1}{1-\beta} \frac{\partial \theta}{\partial h} - \beta \frac{\partial^2 \theta}{\partial h \partial \beta}. \quad (A.11)$$

Making use of Equations (A.2) and (A.10), we easily find that the sign of Equation (A.11) is the same as

$$1 - \frac{\beta(1-\eta)}{(1-\beta) \eta}, \quad (A.12)$$

i.e., at $\beta = \eta$, it is zero.

References


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