Ethnic Diversity, Civil War and Redistribution*

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Abstract

In a game-theoretic framework, we analyse the circumstances under which self-enforcing redistribution and power sharing coalitions can be used to peacefully resolve ethnic conflict. The existence of a pacific equilibrium crucially depends on ethnic diversity (the number of ethnic groups). The risk of civil war is comparatively high at intermediate levels of ethnic diversity. It is low if a society is either very homogeneous or very diverse. Predictions of the model are consistent with evidence on the incidence of civil war.

Keywords: Civil war; ethnic diversity; redistribution; non-cooperative coalition; dynamic game.

JEL classification: H56; J15; K42; N40; N47

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I Introduction

In 2005, the government of the Sudan and southern rebels signed a peace agreement to end the 21-year civil war. The peace treaty stipulates political power sharing and an equal distribution of the revenues from oil extraction (Economist, 2005a). The civil war was sparked by efforts of Arab Muslims in the north to impose Islamic Sharia laws upon the, mostly non-Muslim, black in the south and to expropriate the newly found oil wells there (Lesch, 2003). Observers have stated that the ethnic conflict in the Darfur region of the Sudan can only be resolved if the conflicting parties sign a similar peace treaty (CBS News, 2007). On the Ivory Coast, the death in 1993 of Houpouët-Boigny, president since the independence in 1961, marked the start of ethnic rivalry and unrest (Economist, 2005b). Two features were instrumental in maintaining peace during the late president's long reign: shared political power with his political opponents and heavy redistribution from rich ethnic groups, including his own, to poorer regions (Azam, 1995). Uganda remained essentially peaceful under the regime of Museveni who included representatives from other ethnic groups in his government. However, under his predecessor Obote, who favoured his own ethnic group and its close allies, "Uganda witnesses one of the worst slaughters ever..." (Azam, 1995: 175).

Two observations can be made based on the above examples. First, redistribution and political power sharing can play an important role in curbing ethnic conflict or maintaining peace. Second, political shocks may throw a country into turmoil. Ethnic conflict sometimes seems to be the result of a political failure to reconcile differences rather than some fundamental economic characteristic (Azam, 2001). This paper analyses within a symmetric game-theoretical framework the extent to which conflict-reducing policies can be used to prevent ethnic conflict. The key question we address is: "In a society within which the ruling group has the power to abuse other ethnic groups economically, how does the degree of ethnic diversity (the number of ethnic groups) affect the likelihood of civil war?"

Our main finding is a non-monotonic relationship between ethnic diversity and the likelihood of civil war. Absent redistribution, the probability of conflict is low if society is either homogeneous or there are many conflict groups.[1] Two effects pull in opposite directions. Holding constant the probability that each group rebels, a larger number of ethnic groups means a higher probability of civil war - a *direct effect*. However, the expected amount of resources invested in conflict increases with the number of ethnic groups, making rebellion a less appealing option for each group - a *strategic effect*. Here, the strategic effect dominates when there are two groups or more.

The non-monotonic relationship remains even if we allow for redistribution or power sharing coalitions. With redistribution, the *incumbent* or *ruler* transfers parts of the incumbency rent to the *outsiders* or *non-rulers* in exchange for their passivity. With power sharing coalitions, the groups inside the coalition agree to defend the coalition against attacks. Power sharing coalitions are superior to pure redistributive arrangements in securing pacific outcomes because it is more difficult to topple a coalition of ethnic groups. However, not even the all-inclusive *grand* coalition will always do. No single group can commit to a pacific agreement. Pacific equilibria must be self-enforcing. If the rent for holding office is too high and there are only a few ethnic groups, at least one group will profitably deviate from the grand coali-

[1]Formally, the game we consider is a conflict between a number of exogenously given interest groups; ethnic groups being a leading example. The division could equally well fall along religious lines, class or political conviction. The prevalence of ethnic conflict may, in part, be explained by colonial heritage, as colonies were often organized and segregated along ethnic or tribal lines. See Horowitz (2000) for a study of the logic of ethnic conflict.

tion. Consequently, the likelihood of conflict is at its highest when there are few equally powerful ethnic groups. This prediction is consistent with the empirical findings of Montalvo and Reynal-Querol (2005) of conflict being especially likely in highly polarized societies. Collusive (pacific) equilibria, when they exist, always coexist with non-collusive (war) equilibria. Conflict then arises as a failure to coordinate on the efficient equilibrium and is by no means a disaster that is bound to happen.

This paper adds to the theory of conflicts by investigating how the number of groups contesting power affects the possibility of sustaining self-enforcing pacific policies. Few contributions go beyond the two-antagonist case. Hirshleifer (1995), Esteban and Ray (1999) and Mehlum and Moene (2002) are exceptions. They build on Tullock's (1980) model of rent-seeking and examine how aggregate spending on appropriation is affected by the number of groups. Unlike in the present context, the conflict is taken as given. None of them analyse conflict-reducing policies. The conflict technology used in those papers is less elastic with respect to the number of conflict groups than the one used here, which tends to generate a positive relationship between the number of groups and conflict. Esteban and Ray (1999) obtain results that are in line with ours by effectively removing the direct effect of increasing the number of groups in the standard model.

Garfinkel (1990), Mehlum and Moene (2006) and McBride and Skaperdas (2007) analyse in a two-group model how the value of the contested resource, the cost of warfare and the discount rate affect the possibility of self-enforcing pacific equilibria. Grossman and Noh (1990) and Konrad and Skaperdas (2007) show that one-sided commitment suffices to generate pacific equilibria. Neither ethnic diversity nor coalitions can be analysed within these frameworks.

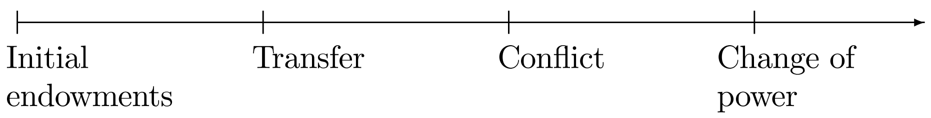
Finally, there is a small literature on coalition formation, considering either fixed sharing rules for peaceful resolution of the conflict (Skaperdas, 1998) or taking the conflict as given (Noh, 2002; Esteban and Sákovics, 2004; Garfinkel, 2004a and b). Given the joint interest of groups in avoiding conflict, we believe it to be relevant to look for coalitions that can sustain peace as an equilibrium outcome, given the short-term incentive for deviation.[2] With a pacific coalition in place, groups are left to haggle over resources upon the threat of war, instead of actually fighting over supremacy in the battlefield.

The rest of this paper is organized as follows. The model is set up in Section II. We derive the main results in Section III, first assuming no transfers and then allowing for redistribution and coalitions. Section IV concludes the paper. All proofs are in the Appendix.

II The Model

We consider a dynamic game between $N+1$ identical ethnic groups. One of them is in power in any given period, the *incumbent* or *ruler*. The $N \geq 1$ other groups are the *outsiders*.

Figure 1:



[2] The literature on endogenous coalition formation, e.g., Bloch (1996) and Ray and Vohra (1999), considers a bargaining situation between potential coalition partners who can sign binding agreements. Equilibrium outcomes of bargaining games with commitment are generally not the same as equilibrium outcomes of games with an infinite time-horizon without commitment, as in the present context. As a consequence, the results obtained in a full commitment setting offer little guidance as to the coalitions that might arise in our setting.

The timing of the stage game is depicted in Figure 1. At the beginning of each period, the ruler receives an incumbency rent θ from holding power. This might be the control of foreign aid distribution or the rents from natural resources like oil fields and diamond mines. The incumbent may choose to share parts of θ with the other ethnic groups. If so, an equal amount $x \geq 0$ is transferred to each group (we allow differentiated transfers at a later stage) at the total cost $X = Nx \leq \theta$. As ethnic groups are often geographically segregated, redistribution can be thought of as regional expenditures on infrastructure, health and schooling financed by the central government.

This is not a democracy: no outsider has any direct saying about the amount x to be distributed. The only way of challenging power is by force. We model this by assuming that outsiders in every period decide simultaneously and non-cooperatively whether to rebel.[3] The decision to rebel is taken after the transfer of that period has been distributed (and consumed). Each belligerent group (including the incumbent who cannot choose whether to fight or not) incurs disutility K. K includes the alternative cost of military expenditures, human sacrifice and so forth associated with war. Finally, the change of power, if any, takes place. Decision-makers are risk-neutral.

Assuming a fixed cost of conflict is standard when the focus of the analysis is the decision whether to engage in conflict (recent examples include Hess and Orphanides, 2001; Levy and Razin, 2004; Jackson and Morelli, 2007). If the marginal returns to conflict are low both at low and high levels of military investment, the question for the involved parties is not so much the marginal amount to invest, but whether to make an all-or-nothing commitment to

[3] Under the simultaneity assumption we avoid assigning arbitrary first-mover or second-mover advantages to groups. When it comes to the assumption that groups move non-cooperatively, this seems at odds with the casual observation that ethnic groups from time to time manage to form coalitions in a rebellion against the government. We study coalitions below.

conflict. The latter approach seems appropriate for the study of large-scale rebellion, such as civil war. Our conflict technology is the limit of a technology with small returns at low and high levels of investment and strong returns at intermediary levels. The fixed cost assumption is convenient and allows us to derive analytical results about how the incidence of civil war and the feasibility of redistribution and coalition formation vary with the degree of ethnic diversity. Some of our results have analogous counterparts in models which treat resource spending as a continuous variable, but we leave an analysis of continuous spending to future research.

We make the standard assumption that the likelihood of any belligerent group winning a conflict depends on the amount of resources it spends on conflict relative to total spending. Suppose that an outsider has decided to engage in conflict, and M other outsiders have made the same decision. As all belligerent groups invest the same amount, the relative amount of resources invested by each ethnic group is $(M+2)^{-1}$, which is then the probability of winning the conflict. The more groups that are involved, the lower is the likelihood of each individual group winning. The number of belligerents is stochastic. An outsider does not know at the time of rebellion how many challengers he is going to face. Outsiders take the decision non-cooperatively and simultaneously, and if everyone is equally likely to participate, the number of belligerents is binomially distributed. Thus, the probability of facing M other outsiders for an ethnic group that has decided to rebel is

$$b(q, M, N-1) = \frac{(N-1)!q^M(1-q)^{N-1-M}}{M!(N-1-M)!}, \tag{1}$$

with q being the (endogenous) probability of rebellion and $N-1$ the total number of other outsiders. The expected probability of winning the conflict equals

$$p(q,N)=\sum_{M=0}^{N-1} b(q,M,N-1)(M+2)^{-1}. \tag{2}$$

This expression can be simplified considerably:

Lemma 1

$$p(q,N)=\frac{(N+1)q-(1-(1-q)^{N+1})}{(N+1)Nq^2}. \tag{3}$$

$p(q,N)$ has the following properties (subscripts denote partial derivatives throughout): (i) $p(q,1)=1/2$; (ii) $p(1,N)=(N+1)^{-1}$; (iii) $\lim_{q\downarrow 0} p(q,N)=1/2$; (iv) $p_q(q,N)<0$ for $N>1$ and $q>0$; (v) $p_N(q,N)<0$ for $q>0$.

Proof. See the Appendix. ■

These results make intuitive sense: the higher is the perceived probability that the other outsiders engage in conflict (the higher is q) or the more ethnic groups that might potentially participate (the higher is N), the more resources are, on average, deployed into conflict and the lower is the expected probability of winning the conflict.

Whenever the players have a decision to make, the *action* they choose is a function of the *history* of the game: the ruler determines the size of the transfer and outsiders randomize between rebelling and remaining peaceful. The history is the vector of observable choices all ethnic groups have made in the past. A player's *strategy* is a plan that assigns to every period which action to take as a function of every conceivable history. We apply the equilibrium concept of *Subgame-Perfection.* A Subgame-Perfect Equilibrium (SPE) is a vector of strategies, one strategy for each player, that has the following property: at no point in time and for no history can any player profitably deviate from the action prescribed by the equilibrium strategy, given that the player expects (i) all other players to play their equilibrium strategies today and (ii) all players, including herself, to play the equilibrium strategy in the future.

To keep the analysis tractable, we restrict our attention to *symmetric and time-invariant* equilibria; the amount of redistribution is constant, and all outsiders rebel with the same probability at every point in time along the equilibrium path.[4]

III Equilibrium Analysis

Consider first the expected value v^O of being an outsider along a symmetric and time-invariant equilibrium path

$$v^O = x + (1-q)\delta v^O + q[p\delta v^I + (1-p)\delta v^O - K]. \tag{4}$$

First, the outsider consumes the equilibrium transfer x in this period. Subsequently, the group remains peaceful with probability $1-q$ and rebels with probability q. In the first event, the group remains an outsider even in the next period, which has the value v^O discounted by $\delta \in (0,1)$. In case of conflict, the belligerent incurs disutility K with certainty, expects to win and gain power with probability $p = p(q, N)$, which has the discounted value δv^I, and to lose and remain an outsider with probability $(1-p)$, which is of value δv^O.

The value v^I of being an incumbent along the same equilibrium path is

[4]Some readers might feel uneasy about ethnic groups playing mixed strategies in a game that is as important as civil war. However, we can transform the game by applying the trick used by Palfrey and Rosenthal (1985) to turn their complete information mixed strategy voting game (Palfrey and Rosenthal, 1983) into an incomplete information pure strategy voting game. Assume that each outsider faces a stochastic cost c of mounting a rebellion, where c is private information to each party. It is common knowledge that c is distributed according to the continuous, cumulative distribution $F(c)$, *i.i.d.* across groups and time. Each group rebels if and only if c drops below a threshold $\bar{c}$. In sequential equilibrium, any player with rebellion cost $\bar{c}$ is indifferent between rebelling or not, given the anticipated probability $q = F(\bar{c})$ of each of the other groups rebelling today and in the future. Generically, each player uses a pure strategy, but behaves as if the other players are using mixed strategies. The results would not change, but the alternative specification would require additional notation, definitions of beliefs, and so on. To keep things simple, we stick to the complete information framework.

$$v^I = \theta - Nx - yK + (1 - Nqp)\delta v^I + Nqp\delta v^O. \tag{5}$$

The ruler keeps $\theta - Nx$ for himself and his group and distributes the rest of the rent to the outsiders. Subsequently, there is war with probability $y = 1 - (1 - q)^N$, in which case the ruler surrenders K to defend himself, survives the ensuing war with probability $1 - Nqp$, valued at δv^I, and is ousted with probability Nqp, valued at δv^O.

Equations (4) and (5) are linear in the two unknowns, v^O and v^I. They can be solved in order to explicitly obtain the equilibrium value functions

$$\begin{aligned} v^O(q,x) &= \frac{\delta pq(\theta - Nx - yK) + (1 - \delta(1 - Npq))(x - qK)}{(1-\delta)(1 - \delta(1 - (N+1)pq))}, \\ v^I(q,x) &= \frac{(1 - \delta(1 - pq))(\theta - Nx - yK) + \delta Npq(x - qK)}{(1-\delta)(1 - \delta(1 - (N+1)pq))}. \end{aligned} \tag{6}$$

Equilibrium without Redistribution

We first describe and analyse the equilibrium of the conflict game for which the incumbent group keeps the entire rent for itself, i.e., $x = 0$. This can be viewed as the benchmark case where the ethnic groups have failed to establish credible redistribution. Let q^{nr} be the equilibrium probability of rebellion by each group (this is shown below to be unique) in non-redistributive equilibrium. Denote by $y^{nr} = 1 - (1 - q^{nr})^N$ the equilibrium probability of war and by $p^{nr} = p(q^{nr}, N)$ the equilibrium expected probability of winning a conflict. Write $v^{Inr} = v^I(q^{nr}, 0)$ and $v^{Onr} = v^O(q^{nr}, 0)$ the value of being the incumbent and the outsider, respectively, along the equilibrium path.

Assume that all outsiders are believed to be unresponsive to all histories of the game. The incumbent can then not affect future actions, notwithstanding how much he redistributes today. Consequently, the ruler maximizes short-term rent and makes no transfers. Next, consider an outsider's incentives.

The outsider rebels with probability $\gamma \in [0,1]$ to maximize

$$V^O(\gamma) = x + \gamma[p^{nr}\delta v^{Inr} + (1-p^{nr})\delta v^{Onr} - K] + (1-\gamma)\delta v^{Onr}.$$

The net benefit of rebellion is

$$V^O_\gamma(\gamma) = p^{nr}\delta[v^{Inr} - v^{Onr}] - K$$

and equals the discounted value of the difference between ruling and being an outsider in the subsequent period times the probability of winning the conflict, less the cost K of rebellion. Observe that the optimal choice of γ is independent of the history of the game under the current set of beliefs; hence, the beliefs are consistent.

Set $x = 0$ and $q = q^{nr}$ in (6), substitute into the expression above and simplify

$$V^O_\gamma(\gamma) = \frac{\delta p^{nr}[\theta - (y^{nr} + Nq^{nr})K] - (1-\delta)K}{1 - \delta(1 - (N+1)p^{nr}q^{nr})}. \quad (7)$$

The denominator is positive; hence, the sign of the marginal incentive for engaging in conflict entirely depends on the sign of the numerator of (7). Through the properties of $p(\cdot, N)$, the numerator is decreasing in q for all values of q such that $V^O_\gamma(\gamma) \geq 0$. The more likely rebellion by other groups is perceived to be in this and future periods, the more resources are expected to be invested into conflict. Thus, the expected likelihood of becoming the ruler and being able to maintain that position is decreasing in q. This makes insurrection a less attractive policy option for each ethnic group, the higher is the estimated likelihood of the other groups rebelling. Decisions are *strategic substitutes*. Note also that if the value of holding office is small [large] as compared to the cost of conflict (θ/K is small [large]), rebellion becomes relatively less [more] appealing. Furthermore, if outsiders are very impatient (δ is small), conflict will never occur since the cost of conflict is realized today and the benefits in the future. In sum:

Proposition 1 *In symmetric and time-invariant SPE without redistribution: (i) there is perpetual peace ($q^{nr} = 0$) if the period benefit of holding office is small as compared to the period cost of engaging in war or if outsiders discount the future heavily ($\delta\theta \leq 2(1-\delta)K$); (ii) there is perpetual civil war ($q^{nr} = 1$) if the discounted period value of holding office outweighs the maximal period disutility of war ($\delta\theta \geq (N+1)K$); (iii) otherwise, each outsider goes to war with probability $q^{nr} \in (0,1)$ implicitly given by*[5]

$$\delta p(q^{nr}, N)[\theta - (1 - (1-q^{nr})^N + Nq^{nr})K] = (1-\delta)K. \tag{8}$$

Proof. See the Appendix. ■

Having characterized the potential equilibria of the game, we move on to the main purpose of this section: to study how the likelihood of conflict varies with the number of ethnic groups. If the incumbent's ability to extract rent is small ($\delta\theta \leq 2(1-\delta)K$), there is never conflict - irrespective of the number of ethnic groups. The country is *politically stable.* Below we consider the other and more interesting case of a *politically unstable* country. To see how the likelihood of conflict in an unstable country depends on the number of ethnic groups, differentiate y^{nr} with respect to N

$$\frac{dy^{nr}}{dN} = -(1-q^{nr})^N \ln(1-q^{nr}) + N(1-q^{nr})^{N-1}\frac{dq^{nr}}{dN}. \tag{9}$$

In an interior equilibrium, an increase in the number of ethnic groups has two effects. Holding fixed the probability that each group rebels, increasing the

[5] Strategies are functions of all possible histories of the game, even off the equilibrium path. There is an infinite number of action profiles that sustain $x = 0$ and q^{nr} as the equilibrium probability of conflict; hence, there is an infinite number of SPE. However, q^{nr} is uniquely defined by the properties of p; hence, there is a unique symmetric and time-invariant equilibrium path in the non-redistributive case. If we restrict the attention to SPE in strategies that are allowed to depend on pay-off relevant state variables only, so-called Markov-Perfect Equilibria (MPE), the SPE with $x = 0$ and probability q^{nr} of rebellion is the unique symmetric MPE. To see this, note from $V^O(\gamma)$ that the incentive to rebel is affected by the prospect of future transfers and the expectations about the actions of other groups. Historic transfers play no part. The forward-looking incumbent realises that no amount of redistribution can affect future choices. Hence, redistribution cannot be part of an equilibrium strategy.

number of ethnic groups leads to an increase in the probability of civil war. This *direct effect* is captured by the first term in (9). However, increasing the number of ethnic groups affects each group's incentive for starting conflict since the probability of winning changes. This *strategic effect* is captured by the second term.

By differentiating (8), one obtains

$$\frac{dq^{nr}}{dN} = \frac{(1-\delta)p_N^{nr} - p^{nr}(q^{nr} - \ln(1-q^{nr})(1-q^{nr})^N)\delta p^{nr}}{Np^{nr}((1-q^{nr})^{N-1}+1)\delta p^{nr} - (1-\delta)p_q^{nr}}. \tag{10}$$

The strategic effect is negative. Varying the number of ethnic groups in this model is like varying the number of firms under Cournot competition. Increasing the number of firms has a direct effect given firm output - lowering prices through increased supply. However, there is a strategic effect working in the opposite direction. Firms reduce output as a response to increased competition. The effect of entry on prices in Cournot equilibrium is ambiguous in general (Amir and Lambson, 2000). Here, the strategic effect dominates the direct effect:

Proposition 2 *Assume that $N \geq 1$. In the politically unstable country without redistribution, the probability of conflict is (weakly) decreasing in the number of ethnic groups along the symmetric, time-invariant equilibrium path.*

Proof. See the Appendix. ∎

By assumption, there is no potential for intra-group conflict in our model. Hence, a perfectly homogeneous society ($N = 0$) would be pacific. Proposition 2 thus holds the implication that the probability of civil war absent redistribution is maximised when there are two ethnic groups.

The mechanisms we have identified through which ethnic diversity affects conflict are by no means particular to this model. Increasing the number of groups while simultaneously holding the conflict intensity in each group constant necessarily leads to more conflict. There will always be an additional strategic effect, as the marginal benefit of investing in conflict for one group depends on the resources deployed into conflict by other groups. If there are N conflict groups and each group spends an amount proportional to $q(N)$ on conflict, total conflict spending - which is proportional to $Nq(N)$ - critically depends on the elasticity $-q'(N)N/q$ of group conflict spending. Typically, group spending is inelastic, i.e., $-q'(N)N/q < 1$ (see, e.g. Tullock, 1980; Hirshleifer, 1995; Mehlum and Moene, 2002); hence, total conflict spending is increasing in ethnic diversity.[6] The "all-or-nothing" conflict technology used in this paper generates more elastic group spending than standard "marginal-spending" technologies deployed elsewhere. Holding the probability of winning the conflict constant at p, the elasticity $-q'(N)N/q = Np_N/qp_q > 1$, which is sufficient to generate a decreasing probability of civil war in this model; see the proof of Proposition 2 in the Appendix.[7]

It is reasonable to think that conflict does not only depend on the size distribution of groups, but also on some measure of how distant the groups consider themselves to be from one another. Esteban and Ray (1994) divide the population into income groups and construct a measure of polarization based on the sizes of these groups and the income differences between them. They argue that social tension and unrest are closely related to polarization. Proposition 2 relates the prevalence of conflict in society to the number of groups competing for resources. As such, our model also comprises a notion

[6] In the standard one-shot Tullock (1980) model, the expected utility of investing x in conflict is $x(x + (N-1)q)^{-1}\delta\theta - Kx$ if the $N-1$ other groups spend q each, the discounted benefit of winning the conflict is $\delta\theta$ and the cost of conflict is linear and equal to Kx. Group spending is equal to $q(N) = \delta\theta(N-1)/N^2K$ in symmetric equilibrium. The corresponding elasticity is $-q'(N)N/q = (N-2)/(N-1) < 1$.

[7] Esteban and Ray (1999) obtain results in line with ours in a model of continuous spending by effectively removing the direct effect of increasing the number of groups.

of distance. Each group places full value on own income and zero value on the income of others. Effectively, the population is divided into those who belong to one's own group and those who do not. Montalvo and Reynal-Querol (2005) use the "belong/do not belong" dichotomy as a substitute for income differences in the Esteban and Ray (1994) measure and construct what they label a discrete polarization measure

$$RQ = 4\sum_{M=1}^{N+1}\pi_M^2(1-\pi_M),$$

where π_M is the fraction of the population belonging to group M, and there is a total of $N+1$ ethnic groups. In our model, all groups are of equal size, i.e., $\pi_M = (N+1)^{-1}$. Therefore, in a symmetric society

$$RQ(N+1) = 4N/(N+1)^2.$$

$RQ(N+1)$ is strictly quasi-concave. It reaches its global minimum at $RQ(1) = 0$, its global maximum at $RQ(2) = 1$ and is strictly decreasing thereafter. In view of these properties, an obvious corollary to Proposition 2 is:

Corollary 1 *The likelihood of civil war is increasing in the RQ measure of ethnic polarization along the symmetric, time-invariant equilibrium path without redistribution.*

The corollary provides a theoretical link between polarization and the incidence of civil war. This prediction finds significant empirical support in Montalvo and Reynal-Querol (2005).

For the other parameters of the model, aggregate behaviour is captured by studying individual behaviour. In the interior equilibrium, we derive

$$\frac{dq^{nr}}{d\theta/K} = \frac{\delta\left(p^{nr}\right)^2}{\delta N\left(p^{nr}\right)^2\left((1-q^{nr})^{N-1}+1\right)p^{nr}-(1-\delta)p_q^{nr}} > 0,$$

$$\frac{dq^{nr}}{d\delta} = \frac{\delta p^{nr}\left(p^{nr}\theta + K\right)}{\delta[\delta N\left(p^{nr}\right)^{2}\left((1-q^{nr})^{N-1}+1\right)K - (1-\delta)p_{q}^{nr}K]} > 0.$$

It immediately follows from this and from $dy^{nr} = N(1-q^{nr})^{N-1}dq^{nr}$ that:

Proposition 3 *In the politically unstable country without redistribution, the probability of conflict is increasing in the rent of holding office θ, decreasing in the cost of conflict K and increasing in the discount rate δ along the symmetric, time-invariant equilibrium path.*

The intuition is straightforward: a higher θ/K makes it more attractive to become a ruler relative to the cost of conflict, which induces outsiders to rebel, as does a high δ, since it implies that the future payoffs to insurrections are given a high weight.

A country can be rich and conflict-ridden if wealth is unequally distributed (high θ but no redistribution) or poor and stable if wealth is equally distributed across ethnic groups (this becomes more evident in the next section). This interpretation is in line with Azam et al. (1999: 19) who list "[g]reat inequality in resource distribution among ethnic groups..." and "[g]reat inequality in the distribution of public expenditure and of taxation..." among their factors of conflict.

We have imposed a great deal of symmetry to make the model analytically tractable. However, the symmetry assumption is not critical to the above results. In the interior symmetric equilibrium, there is a strictly monotone relationship between the parameters of the model and the risk of civil war. By continuity, this monotonicity would prevail even if we allowed the groups to differ somewhat in strength.

We have assumed that an outsider which stays out of conflict altogether escapes its consequences. In a set of Complementary Notes, we instead assume that all groups will be dragged into conflict if at least one group rebels.[8] This alternative specification can be seen as a scenario where full escalation of conflict is unavoidable. The results for the symmetric interior equilibrium remain qualitatively unchanged. The strategic effect still dominates the direct effect and thus, more groups mean a lower risk of civil war. However, there always exists an additional equilibrium with perpetual civil war. If one group anticipates that all other groups will rebel with certainty, war is unavoidable no matter what the group itself decides to do. Hence, rebellion is a best-reply to the others' decision to rebel.

Note that the present and all above papers assume the collective action problem of conflict (Olson, 1965) to be independent of the number of conflict groups. This is a natural assumption if the experiment is to expand the population by adding more groups, because group size then is constant. An alternative experiment would be to hold the population constant and reduce group size (as in Esteban and Ray, 1999). In that case, intra-group distribution rules become important for group performance (Nitzan, 1991). Whether free-riding could sufficiently weaken the strategic effect in this model to overturn Proposition 2 remains an issue for further research.

Redistribution

This part explores the possibility of redistribution as a peaceful resolution to conflict. We assume that opportunistic behaviour either by the ruler or some of the outsiders throws the country into turmoil: the outsiders return to a strategy of non-cooperation (the equilibrium analysed previously). Let a *Pacific Transfer Equilibrium* (PTE) refer to an SPE where civil war does not break out in equilibrium, and deviations from the equilibrium path are

[8] The Complementary Notes are available at http://www.ifn.se/thomast.

punished by reversion to the non-redistributive equilibrium.[9]

How much is the incumbent willing pay for peace? Suppose that the ruler distributes a total of X across the outsiders in each period. If this is sufficient to render all groups perpetually pacific, the value of redistribution to the incumbent is simply $(\theta - X)(1-\delta)^{-1}$. Since cooperation immediately breaks down for any deviation from X, the incumbent optimally keeps the entire rent θ upon deviating. A deviation is unprofitable if and only if $(\theta - X)(1-\delta)^{-1} \geq v^{Inr}$. Incentive compatibility thus creates an upper bound

$$\overline{X}(N) = \theta - (1-\delta)v^{Inr} \tag{11}$$

on the transfers acceptable to the incumbent to preserve peace.

The next question is whether X can be distributed across outsiders in such a way as to maintain peace. The discounted value of being an outsider in a PTE is $x(1-\delta)^{-1}$ if it receives a transfer x every period. The net present value of deviating is

$$\frac{\delta}{2}(v^{Inr} + v^{Onr}) - K - \frac{\delta}{1-\delta}x.$$

The outsider group surrenders K and the net present value $\delta x(1-\delta)^{-1}$ of future redistribution, but wins the conflict with probability $1/2$ since it only has to fight the incumbent. In the next period, cooperation breaks down and the game reverts back to the non-cooperative state analysed previously. The requirement that deviations be unprofitable creates a lower bound

$$\underline{x}^r(N) = (1-\delta)\max\{\frac{1}{2}(v^{Inr} + v^{Onr}) - \frac{K}{\delta}; 0\} \tag{12}$$

[9] This begs the question of how much cooperation can be achieved, i.e., whether optimal punishments exist. Our model is dynamic in the sense that each player's action set is history-dependent. Specifically, the action set depends on whether the player is a ruler or an outsider. Consequently, the results obtained by Abreu (1986) and others cannot be utilised since they apply to infinitely repeated games. We have not been able to verify that a reversion to non-redistributive stationary and symmetric equilibrium does, in fact, constitute an optimal punishment.

on the transfers required by the outsider in order to be willing remain at peace.

Any $X \in [N\underline{x}^r(N), \overline{X}(N)]$ is sufficiently large to keep outsiders in line and constitutes a sufficiently small price for the incumbent to pay for peace. Civil war breaks out for any transfer outside this region, either because the incumbent chooses to grab the rent θ or because power is challenged by one outsider or more. Obviously, a PTE exists if and only if $\overline{X}(N) \geq N\underline{x}^r(N)$.

Consider the effect of the rent for holding office versus the disutility of warfare. From Proposition 3, we know that a high rent relative to the cost of conflict leads to a high risk of rebellion. On the other hand, a high rent also leaves the incumbent a lot of room for redistribution. The incumbent always pockets some of the increased rent for himself. Thus, there exists a point at which the rent from office becomes so high that peace cannot be sustained in equilibrium:

Proposition 4 *When the period value θ of holding office is sufficiently large relative to the disutility K of conflict ($\theta/K > 2(1+\delta)/\delta(1-\delta)$), there exists no PTE.*

Proof. See the Appendix. ■

Consider next what happens as the number of ethnic groups changes. Deviating becomes an increasingly attractive policy option for the incumbent as N increases, since the punishment threat caused by insurrections becomes weaker. A belligerent outsider wins the subsequent conflict with probability 1/2 irrespective of the number of outsiders, since the only other party involved in conflict is the incumbent. Once the conflict has been won, the new ruler is unlikely to be replaced since the probability of subsequent war is low. Hence, each outsider must receive a large period transfer in order to remain peaceful when the number of ethnic groups is large. In sum, the transfer demands of the outsiders and the ruler's willingness to pay are incompatible for N sufficiently high:

Proposition 5 *In the politically unstable country for which the period value of holding office is not too large ($\theta/K \leq 2(1+\delta)/\delta(1-\delta)$), there exists an $N^c \geq 1$ such that a PTE exists if and only if $N \in [1, N^c]$.*

Proof. See the Appendix. ■

The proposition states that pacific equilibria only exist in countries for which the rent to holding office does not overshadow the cost of warfare and where, at the same time, the degree of ethnic diversity is not too large.[10] Allowing for redistribution, it is still the case that the risk of civil war is low when the degree of ethnic diversity is either low or very high. In the first case, the incumbent can use transfers to maintain peace. In the second he cannot, but the risk of conflict is low even in the absence of transfers due to the outsiders' weak incentives for rebelling.

By combining parts (*ii*) and (*iii*) of Proposition 1 with Proposition 5, we prove the existence of multiple equilibria and hence, give an example of the policy failure alluded to by Azam (2001):

Corollary 2 *In politically unstable societies with a limited degree of ethnic diversity ($N \in [1, N^c]$) and with limited possibilities for rent-extraction ($\theta/K \leq 2(1+\delta)/\delta(1-\delta)$), there exist, in addition to conflict equilibria without redistribution, pacific equilibria with redistribution.*

In a polarized society, the ruler may have both the means and the incentives for using transfers to avoid civil war. This does not mean that peace

[10] Another obstacle to reaching a pacific equilibrium is that infinite punishments in the case of deviation may not necessarily be renegotiation-proof. We are grateful to one of the referees for pointing this out.

will necessarily prevail. A country may equally well be caught up in an equilibrium of distrust and conflict for which outsiders rebel in order to gain influence, and the insider takes full advantage of being in power by enriching himself and his peers. Hence, two societies identical in terms of ethnic diversity and wealth may have experienced totally different histories of conflict - one being stable, with political participation and redistribution from the state to all ethnic groups, the other characterized by systematic favourisation of the ethnic group of the current ruler, with political instability and frequent uprisings as the result.[11]

Power Sharing Coalitions

So far, the analysis has assumed each belligerent to act independently of all other groups. However, outsiders may have an incentive to join forces to topple the sitting regime, and the incumbent may have an incentive to invite outsiders to join in a defence coalition. We view the defence coalition as a power sharing agreement between the incumbent and a subset of outsiders. The coalition partners receive favourable treatment in exchange for their promise to assist the incumbent in case of rebellion.

A full investigation of coalition formation is beyond the scope of this paper. Instead, we pose the question: "Under which circumstances can the incumbent form a coalition sufficiently strong to preserve peace?" Assigning the incumbent the right to propose coalitions or power sharing agreements seems natural in this framework. Rebellious coalitions are formed with the promise of *future* rents, whereas the ruler can offer his partners *immediate* gratification through his access to the rent θ. Hence, today the incumbent can replicate any redistribution scheme which will hold a rebellious coalition

[11] The above results are robust to the alternative specification that all groups will be dragged into conflict if at least one group rebels; see the Complementary Notes. The aggregate transfer demands of the outsiders are increasing and the incumbent's willingness to pay is decreasing in the number of ethnic groups.

together tomorrow.

Following Esteban and Sákovics (2004) and Garfinkel (2004a and b), we model coalition formation or power sharing as a coordinated decision among a subset of the players, taking into account the groups' inabilities to make long-term binding agreements. In the power sharing agreement, A outsiders are paid an amount x^A each to help defend the ruler against any attack. The $N - A$ other groups remain outsiders and are paid x each to remain at peace. In accordance with non-cooperative coalition theory, we implement restrictions in terms of stability of the feasible coalition structure. A minimal requirement on stability is that no ethnic group can benefit from a unilateral deviation from the proposed structure. A stronger requirement is that no sub-coalition can profitably be formed. In every period and subsequent to the payment of the transfer, outsiders decide (independently or jointly) whether to attack. Simultaneously, the coalition members decide (jointly or independently) whether to attack or defend the coalition, should it come under attack.[12] To address the issues of stability, we need to ask ourselves what would happen subsequent to a deviation. As it is in everybody's joint interest to avoid war, all ethnic groups agree that it should be avoided if possible. Hence, we assume that the game reverts to the state of non-cooperation following a deviation. Let a *Pacific Coalition Equilibrium* (PCE) refer to a coalition structure where civil war does not break out in equilibrium and where deviations (joint or unilateral) from the equilibrium path are punished by reversion to the non-redistributive equilibrium.

Assume that an A coalition has formed. The non-coalition outsider's net

[12] Due to the simultaneity assumption, we could easily extend the choice set of coalition members to include passive behaviour, i.e., not defending the coalition. Along a pacific equilibrium path, each coalition member is indifferent between remaining passive and defending the coalition.

present value of deviating is

$$\frac{1}{A+2}\delta v^{Inr} + (1 - \frac{1}{A+2})\delta v^{Onr} - K \; - \frac{\delta}{1-\delta}x.$$

A belligerent which takes on the entire coalition by itself surrenders K and the net present value $\delta x(1-\delta)^{-1}$ of future redistribution. It becomes the sole ruler with probability $(A+2)^{-1}$ and loses the conflict with probability $1-(A+2)^{-1}$, the discounted value being δv^{Inr} in the first case and δv^{Onr} in the second. The requirement that deviations be unprofitable creates lower bounds on transfers to non-coalition outsiders and coalition members, respectively

$$\underline{x}(A,N) = (1-\delta)\max\{\frac{v^{Inr} - v^{Onr}}{A+2} + v^{Onr} - \frac{K}{\delta}; 0\},$$
$$\underline{x}^A(A,N) = (1-\delta)\max\{\frac{v^{Inr} - v^{Onr}}{A+1} + v^{Onr} - \frac{K}{\delta}; 0\}.$$

For any transfer below $\underline{x}^A$, the coalition member is better off backstabbing his coalition in the hope of becoming the sole future ruler of the country. Adding up, we get a lower bound

$$\underline{X}(A,N) = A\underline{x}^A(A,N) + (N-A)\underline{x}(A,N) \qquad (13)$$

on the total transfers necessary to uphold peace.

The ruler only cares about the total amount of redistribution as long as the transfers are sufficient to preserve peace. Hence, the upper bound on transfers is still given by $\overline{X}(N)$ defined in (11). Obviously, PCE can be sustained by an A coalition only if $\overline{X}(N) \geq \underline{X}(A,N)$. The ruler would like to minimize transfers. A *minimal-cost* coalition is one that minimizes $\underline{X}(A,N)$ and thus maximises the possibility of having a pacific equilibrium. The ruler faces a trade-off

$$\underline{X}_A(A,N) = \underline{x}^A(A,N) - \underline{x}(A,N) + A\underline{x}^A_A(A,N) + (N-A)\underline{x}_A(A,N)$$

in the choice of A. Coalition members must receive a higher transfer, $\underline{x}^A \geq \underline{x}$, since the coalition member has a competitive advantage over a non-coalition outsider in attacking the coalition. However, large coalitions are stronger than small ones and thus more costly to challenge, $\underline{x}_A^A \leq 0$ and $\underline{x}_A \leq 0$. This deterrence effect is captured by the last two terms in the expression. As it turns out, the deterrence effect dominates the cost effect:

Lemma 2 *The grand coalition ($A = N$) is a minimal-cost coalition.*

Proof. See the Appendix. ■

If $\overline{X}(N) < \underline{X}(N, N)$, there exists no redistribution scheme which can prevent unilateral deviations from the grand coalition. As the grand coalition is a minimal-cost coalition, *no* defence coalition $A < N$ can uphold a pacific equilibrium if the grand coalition cannot. Hence, $\overline{X}(N) \geq \underline{X}(N, N)$ is a necessary condition for the existence of a PCE. But is it sufficient?

If $\overline{X}(N) \geq \underline{X}(N, N)$, the ruler can distribute rent in such a way that no group has an incentive to unilaterally deviate from the grand coalition. However, non-ruling groups might have an incentive to pool their resources in a joint effort to overthrow the government.[13] The question is whether groups have an incentive to form such sub-coalitions. We model the sub-coalition as a coordinated rebellion by J non-ruling groups. As in Esteban and Sákovics (2004) and Garfinkel (2004a and b), the sub-coalition is short-lived: the members of a winning sub-coalition are assumed to fight a winner-take-all battle among themselves for the benefit of becoming the ruler in the

[13] The requirement that an equilibrium be robust to joint deviations is closely related to equilibrium refinements such as *strongness* (Aumann, 1959) or *coalition-proofness* (Bernheim, Peleg and Whinston, 1987). The extension of coalition-proofness to infinite horizon games is non-trivial and has never been made, as far as we know. Rubinstein (1980) develops the concept of Strong Perfect Equilibrium and applies it to infinitely repeated games, but not to dynamic games such as our conflict game.

subsequent period. The net present value of belonging to a deviant sub-coalition is

$$\begin{aligned} & \frac{J}{N+1}\{\frac{1}{J}\delta v^{Inr} + (1-\frac{1}{J})\delta v^{Onr}\} + (1-\frac{J}{N+1})\delta v^{Onr} - K - \frac{\delta}{1-\delta}\underline{x}^A(N,N) \\ = & \frac{1}{N+1}\delta(v^{Inr} - v^{Onr}) + \delta v^{Onr} - K - \frac{\delta}{1-\delta}\underline{x}^A(N,N). \end{aligned}$$

The probability of winning the conflict is equal to the amount of resources the sub-coalition J spends divided by the total amount of resources. It is equal to $J(N+1)^{-1}$ since all groups in the economy are involved in the conflict, either as a member of some rebellious sub-coalition or as a member of the defence coalition. The probability of becoming the leader subsequent to victory is J^{-1}, valued at δv^{Inr}, and the likelihood of failing is $1 - J^{-1}$, which has the discounted value δv^{Onr}. As always, the cost of rebellion is disutility K plus the foregone stream of transfers $\delta\underline{x}^A(N,N)(1-\delta)^{-1}$.

After simplifications, we see that the net present value of belonging to a deviant sub-coalition is independent of its size J. The benefit in terms of joint military strength of belonging to a large sub-coalition is exactly offset by increased rent-sharing inside the coalition. Since a non-ruling group cannot gain more from joining a rebellious sub-coalition than what could be gained from a unilateral attack, $\overline{X}(N) \geq \underline{X}(N,N)$ is also a sufficient condition for the existence of a PCE:

Lemma 3 *If the members of the winning sub-coalition fight a winner take-all competition for leadership, Pacific Coalition Equilibria exist if and only if* $\overline{X}(N) \geq \underline{X}(N,N)$.

Consider now the effect of ethnic diversity on the possibility of sustaining a PCE. The consequences of deviating are small for the ruler in an ethnically diverse society, due to the low probability of subsequent rebellion. The effect on coalition members is ambiguous. As with the incumbent, non-ruling groups are more tempted by the incumbency rent the higher is the degree of ethnic diversity. On the other hand, the deterrence effect of the grand coalition increases in the number of ethnic groups as each individual member becomes relatively weaker. The deterrence effect turns out to be the driving force in the grand coalition:

Proposition 6 *When the period value θ of holding office is sufficiently small relative to the disutility K of conflict ($\theta/K \leq 2(1+\delta)/\delta(1-\delta)$), the PCE can always be sustained by the grand coalition. Otherwise, there exists an $1 < \overline{N} < \infty$ such that the grand coalition can sustain PCE if and only if $N \geq \overline{N}$.*

Proof. See the Appendix. ■

Compare this result to Proposition 5. In situations in which redistribution alone can only be partially successful, i.e., it breaks down when the number of ethnic groups becomes too large, all-inclusive power sharing can be successful in maintaining peace notwithstanding the degree of ethnic diversity. Coalitions are, if sufficiently extensive, superior to pure redistribution in achieving a pacific outcome. However, not even the grand coalition will necessarily do. If the incumbency rent is sufficiently high relative to the cost of conflict and if the number of ethnic groups is sufficiently small, not even the threat of having to fight the grand coalition is sufficient to deter individual groups from rebellion. Since the grand coalition is a minimal-cost coalition, no other coalition or redistribution scheme is able to maintain peace under those circumstances either.

Proposition 6 implies a non-monotonic relationship between ethnic diversity and civil war even when the ruler has the ability to form the powerful grand coalition. Peace can be achieved if the country is completely homogenous or if the country is very heterogenous, but not if incumbency rent is high and the country is divided into a small number of equally powerful groups.

We show in the Complementary Notes that the incumbent does not want to exclude any group due to its being weaker than other groups. Weak groups have little to offer in terms of military strength. However, their weakness also implies a lower compensation for joining. The deterrence effect is always dominant and thus, Lemma 3 is valid even with asymmetric strength.

Esteban and Sákovics (2004) and Garfinkel (2004a and b) show that a free-rider problem arises inside larger coalitions when spending is continuous. In their models, the free-rider problem is sufficiently strong that every group prefers to stand alone. It would be interesting to know whether these results carry over to the present setting, but we leave the issue of continuous spending and pacific coalitions to future research.

IV Conclusion

This paper has studied the circumstances under which self-enforcing redistribution and power sharing coalitions can be used to peacefully resolve ethnic conflict. We have shown that the existence of a pacific equilibrium crucially depends on ethnic diversity (the number of ethnic groups). The risk of civil war is at its highest at intermediate levels of ethnic diversity. The interplay between ethnic diversity and the possibility of sustaining conflict mitigating policies suggests that measures of intergroup-redistribution and power sharing which interact with ethnic diversity be included in the empirical analysis of domestic conflict.

When pacific equilibria do exist, ethnic conflict arises as a failure to coordinate on the pacific outcome and is not due to some fundamental economic characteristic. At the heart of the coordination failure lies the lack of commitment of the ethnic groups. The ruler cannot credibly commit not to appropriate resources, whereas the subjects cannot commit not to rebel against the leader. This points to the importance of shaping institutions capable of securing credible redistribution and power sharing. Hence, our model might lend insights to an expanding empirical literature on the relationship between ethnic diversity, policies, political institutions, and growth (see e.g., Mauro, 1995; Lian and Oneal, 1997; Easterly and Levine, 1997; Burnside and Dollar, 2000; Annett, 2001; Easterly, 2000).

Appendix

Proof of Lemma 1

Multiply and divide each element in the summand of (2) by $M+1$ to obtain

$$p(q,N) = \sum_{M=0}^{N-1} \frac{(N-1)!q^M(1-q)^{N-1-M}}{(M+2)!(N-1-M)!}(M+1). \tag{A1}$$

Define two new variables $S = M+2$ and $B = N+1$ and perform a change of variables on (A1)

$$\begin{aligned} p(q,B-1) &= \sum_{S=2}^{B} \frac{(B-2)!q^{S-2}(1-q)^{B-S}}{S!(B-S)!}(S-1) \\ &= \frac{1}{B(B-1)q^2} \sum_{S=2}^{B} \frac{B!q^S(1-q)^{B-S}}{S!(B-S)!}(S-1) \\ &= \frac{1}{B(B-1)q^2} \sum_{S=2}^{B} b(q,S,B)(S-1). \end{aligned}$$

The second equality follows from multiplying and dividing by $B(B-1)q^2$ in $p(q,B-1)$ and the third equality from the definition of $b(q,\cdot,\cdot)$; see (1). Note that $b(q,1,B)(1-1) = 0$ and $b(q,0,B)(0-1) = -(1-q)^B$. Thus

$$p(q,B-1) = \frac{1}{B(B-1)q^2}[(1-q)^B + \sum_{S=0}^{B} b(q,S,B)(S-1)].$$

Observe that

$$\sum_{S=0}^{B} b(q,S,B)S = Bq \text{ and } \sum_{S=0}^{B} b(q,S,B) = 1$$

by the properties of the binomial distribution. Thus

$$p(q, B-1) = \frac{Bq - [1-(1-q)^B]}{B(B-1)q^2}.$$

Substitute $B = N + 1$ into this expression to obtain (3).

Properties (i) and (ii) follow directly from inserting, respectively, $N = 1$ and $q = 1$ into (3) and simplifying.

Property (iii):

$$\lim_{q\downarrow 0} p(q,N) = \lim_{q\downarrow 0} \frac{1-(1-q)^N}{2Nq} = \lim_{q\downarrow 0} \frac{(1-q)^{N-1}}{2} = \frac{1}{2},$$

where we have applied L'Hôpital's rule twice.

Property (iv):

$$p_q(q,N) = \frac{2[1-(1-q)^{N+1}] - (N+1)[1+(1-q)^N]q}{(N+1)Nq^3}. \tag{A2}$$

The denominator is positive for $q > 0$, hence the sign of $p_q(q,N)$ depends on the sign of the numerator which we define as $A(q)$. $A(q)$ has the following properties

$$A'(q) = (N+1)[N(1-q)^{N-1}q + (1-q)^N - 1], \tag{A3}$$

$$A''(q) = -(N+1)N(N-1)(1-q)^{N-2}q.$$

Since $A''(q) < 0$ for $N > 1$ and $q \in (0,1)$, and $A'(0) = 0$, $A(q)$ is maximized at $q = 0$. $A(0) = 0$, hence $A(q) < 0$ for all $q > 0$ which establishes the result.

Property (v): Differentiate to obtain

$$p_N(q,N) = \frac{2N+1-(N+1)^2q - [2N+1-(N+1)N\ln(1-q)](1-q)^{N+1}}{(N+1)^2N^2q^2},$$

which is negative if the numerator is negative, since the denominator is positive for $q > 0$. Define the numerator as $B(q)$. Note that

$$B'(q) = (N+1)^2[(1 - N\ln(1-q))(1-q)^N - 1], \tag{A4}$$

$$B''(q) = (N+1)^2 N^2 \ln(1-q)(1-q)^{N-1}.$$

$B''(q) < 0$ for all $q \in (0,1)$, which implies $B(q) < 0$ for all $q > 0$ by $B'(0) = 0$ and $B(0) = 0$. ■

Proof of Proposition 1

Define the function

$$H(q,N) = \delta p(q,N)[\theta - (1 - (1-q)^N + Nq)K] - (1-\delta)K. \tag{A5}$$

By definition, the numerator of (7) is equal to $H(q^{nr}, N)$. If it is positive (negative), the outsider will rebel (remain peaceful) with certainty; if it is zero he is indifferent between war and peace. By the properties of $p(\cdot, N)$ (see Lemma 1), it is straightforward to verify that $H_q < 0$ for all $q \in [0,1]$.

Suppose that $\lim_{q\downarrow 0} H(q,N) \leq 0$, or equivalently (recalling from Lemma 1 that $\lim_{q\downarrow 0} p(q,N) = 1/2$), $\delta\theta \leq 2(1-\delta)K$. In this case, the net benefit of conflict is always non-positive and strictly negative as long as the outsider expects the other groups to rebel with positive probability ($q > 0$). No outsider can ever benefit from rebellion. This means that the unique symmetric and time-invariant equilibrium without redistribution in this case is that all groups remain peaceful, $q^{nr} = 0$.

Next, consider the case with $H(1,N) \geq 0$, or equivalently (recalling from Lemma 1 that $p(1,N) = (N+1)^{-1}$), $\delta\theta \geq (N+1)K$. Now the opposite holds. The net benefit of conflict is always non-negative and strictly positive whenever the other outsiders remain peaceful with positive probability ($q <$

1). No outsider can ever benefit from staying peaceful. There is perpetual civil war along the symmetric and time-invariant equilibrium path, $q^{nr} = 1$.

In the intermediate case $\delta\theta \in (2(1-\delta)K, (N+1)K)$, war may or may not break out in equilibrium. $H(0, N) > 0$, $H(1, N) < 0$ and $H_q(q, N) < 0$ yield a unique $q^{nr} \in (0, 1)$ given by (8) such that $H(q^{nr}, N) = 0$. $q > [<]q^{nr}$ cannot be a symmetric equilibrium since each group would prefer to remain peaceful [rebel] in that case. Hence, the unique symmetric and time-invariant mixed strategy equilibrium is the solution to (8). ■

Proof of Proposition 2

We first prove an intermediate result:

Lemma 4 *Assume that $\delta\theta > 2(1-\delta)K$. Then $dy^{nr}/\, dN < 0$ for all $N > \delta\theta K^{-1} - 1$.*

Proof. In this range of the parameter space, $q^{nr} \in (0, 1)$ (see Proposition 1). Substitute (10) into (9) and simplify to obtain (for notational simplicity, superscript nr is dropped)

$$\frac{dy}{dN} = \frac{(1-\delta)(\frac{p_N}{p_q}\frac{N}{q} - 1)qp_q - [\delta Np^2 - (1-\delta)p_q][q + (1-q)\ln(1-q)]}{\delta Np^2(1 + (1-q)^{N-1}) - (1-\delta)p_q}(1-q)^{N-1}.$$

The denominator and the first term in square brackets in the numerator are positive by $p_q < 0$. The second term in square brackets in the numerator is also positive since

$$\frac{d}{dq}(q + (1-q)\ln(1-q)) = -\ln(1-q) > 0,$$
$$\lim_{q\downarrow 0}[q + (1-q)\ln(1-q)] = 0.$$

Thus, $dy/dN < 0$ if $Np_N/qp_q > 1$. In the notation of the proof of Lemma 1, $qp_q = A(q)/(N+1)Nq^2$ and $Np_N = B(q)/(N+1)^2Nq^2$, where $A(q) < 0$ and $B(q) < 0$ for all $q > 0$, see the proof of Lemma 1. Consequently, $Np_N/qp_q > 1$

if and only if $(N+1)A(q) > B(q)$. Using (A3) and (A4) we obtain for all $q > 0$,

$$(N+1)A'(q) - B'(q) = N(N+1)^2(q + (1-q)\ln(1-q))(1-q)^{N-1} > 0,$$

and therefore

$$(N+1)A(q) - B(q) > (N+1)A(0) - B(0) = 0. \blacksquare$$

For $\delta\theta \in (2(1-\delta)K, 2K)$, $N > \delta\theta K^{-1} - 1$ for all $N \geq 1$; hence, the result follows directly from Lemma 4. For $\delta\theta \geq 2K$, $q^{nr} = 1$ and thus, $y^{nr} = 1$ for all $N \in [1, \delta\theta K^{-1} - 1]$ and $dy^{nr}/\, dN < 0$ for all $N > \delta\theta K^{-1} - 1$ by Lemma 4. Hence, y^{nr} is non-increasing in N even in this final case. $\blacksquare$

Proof of Proposition 4

Consider a politically unstable country, $\delta\theta > 2(1-\delta)K$. Define $\overline{R}(N) = \overline{X}(N)/N - \underline{x}^r(N)$. By construction, a PTE exists at N if and only if $\overline{R}(N) \geq 0$. In the proof of this proposition and the next, we utilise the following lemma:

Lemma 5 *Let $\delta\theta > 2(1-\delta)K$. Then, (i) $\overline{R}'(N) < 0$ for all $N \geq 1$; (ii) $\lim_{N\to\infty} \overline{R}(N) < 0$.*

Proof. Part (i): There are two cases to consider. Case (a): $\delta\theta \in (2(1-\delta)K, 2K)$. $\delta\theta < (N+1)K$, hence $q^{nr} \in (0,1)$ for all $N \geq 1$. Rewrite the equilibrium condition (8) as

$$\theta - y^{nr}K = Nq^{nr}K + (1-\delta)K/\delta p^{nr},$$

substitute $Nq^{nr}K + (1-\delta)K/\delta p^{nr}$ for $\theta - y^{nr}K$ in v^{Inr} and v^{Onr}, respectively, and simplify to $v^{Inr} = K/\delta p^{nr}$ and $v^{Onr} = 0$. Plugging these expressions into (11) and (12) gives us

$$\overline{X}(N) = \theta - \frac{(1-\delta)}{\delta}\frac{K}{p^{nr}}, \;\; \underline{x}^r(N) = \frac{(1-\delta)}{\delta}\frac{1-2p^{nr}}{2p^{nr}}K. \qquad \text{(A6)}$$

$\overline{X}'(N) < 0$ and $\underline{x}'(N) > 0$ if $dp^{nr}/dN < 0$. Hence, a sufficient condition for $\overline{R}'(N) < 0$ is $dp^{nr}/dN < 0$.

$$\begin{aligned} \frac{dp}{dN} &= p_N + p_q \frac{dq}{dN} \\ &= \frac{\delta p^2 q p_q (\frac{p_N}{p_q}\frac{N}{q} - 1)(1 + (1-q)^{N-1})}{\delta N p^2((1-q)^{N-1}+1) - (1-\delta)p_q} + \frac{\delta p_q p^2 (q + (1-q)\ln(1-q))(1-q)^{N-1}}{\delta N p^2((1-q)^{N-1}+1) - (1-\delta)p_q}, \end{aligned}$$

where the second equality follows from plugging (10) into the expression and simplifying (superscript nr is omitted for notational simplicity). The first term on the second line is negative since $p_q < 0$ (see Lemma 1) and $Np_N/qp_q > 1$ (see the proof of Proposition 2). The second term on the same line is negative since $p_q < 0$ and $q + (1-q)\ln(1-q) > 0$ (see the proof of Proposition 2). Thus, $dp^{nr}/dN < 0$.

Case (b): $\delta\theta \geq 2K$. In this case $q^{nr} = 1$ for all $N \in [1, \delta\theta/K - 1]$ and $q^{nr} \in (0,1)$ for all $N > \delta\theta/K - 1$. Let $N \in [1, \delta\theta/K - 1]$. By plugging the relevant expressions into the value functions, we get

$$(1-\delta)v^{Inr} = \theta - \frac{N}{N+1}\delta\theta - K \text{ and } (1-\delta)v^{Onr} = \frac{\delta\theta}{N+1} - K,$$

which imply

$$\frac{\overline{X}(N)}{N} = \frac{\delta\theta}{N+1} + \frac{K}{N} \text{ and } \underline{x}^r(N) = \frac{\theta}{2}[1 - \delta\frac{N-1}{N+1}] - \frac{K}{\delta}.$$

Hence,

$$\overline{R}(N) = \frac{K}{N} + \frac{K}{\delta} - \frac{(1-\delta)\theta}{2}, \tag{A7}$$

which is decreasing in N for all $N \in [1, \delta\theta/K - 1]$. As $\overline{R}'(N) < 0$ for all $N > \delta\theta/K - 1$, and $\overline{R}(N)$ is continuous at $\overline{R}(\delta\theta/K - 1)$, it follows that $\overline{R}'(N) < 0$ for all $N \geq 1$.

Part (ii): For $N > \delta\theta/K - 1$, $q^{nr} \in (0,1)$. $q^{nr} \in (0,1)$ and parts (iii) and (iv) of Lemma 1 imply that $p^{nr} < 1/2$ and thus $\underline{x}^r(N) > 0$; see (A6). This, and $\underline{x}^r(N)$ strictly increasing in N for all $N > \delta\theta/K - 1$ imply that $\underline{x}^r(N)$ is bounded away from zero for all N sufficiently large. From (A6) we also have

$$\frac{\overline{X}(N)}{N} = \frac{\theta}{N} - \frac{(1-\delta)}{\delta}\frac{K}{Np^{nr}} < \frac{\theta}{N}$$

for $N > \delta\theta/K - 1$. Thus, $\lim_{N\to\infty}\overline{X}(N)/N \leq 0$, which along with $\underline{x}^r(N)$ bounded away from zero for $N > \delta\theta/K - 1$, completes the proof. ■

We are now ready to complete the proof of the proposition. $\delta\theta > 2K(1+\delta)/(1-\delta) > 2K$ implies $q^{nr} = 1$ for $N = 1$. From (A7), we obtain

$$\overline{R}(1) = \frac{(1-\delta)K}{2}[\frac{2(1+\delta)}{\delta(1-\delta)} - \frac{\theta}{K}] < 0. \tag{A8}$$

$\overline{R}(1) < 0$ combined with part (i) of Lemma 5 implies $\overline{R}(N) < 0$ for all $N \geq 1$. ■

Proof of Proposition 5

We show that $\overline{R}(1) = \overline{X}(1) - \underline{x}^r(1) \geq 0$ in the entire relevant range $\delta\theta \in (2(1-\delta)K, 2(1+\delta)K/(1-\delta)]$, which combined with Lemma 5, implies the existence of a unique $N^c \geq 1$ such that $\overline{R}(N) \geq 0$ if and only if $N \in [1, N^c]$. Note first that $\delta\theta \in (2(1-\delta)K, 2K)$ implies $q^{nr} \in (0,1)$, $p^{nr} = 1/2$ for $N = 1$, see Lemma 1, $\underline{x}^r(1) = 0$ and $\overline{X}(1) = (\delta\theta - 2(1-\delta)K)/\delta > 0$, see (A6). Next, $\delta\theta \in [2K, 2(1+\delta)K/(1-\delta)]$ implies $q^{nr} = 1$ for $N = 1$ and $\overline{R}(1) \geq 0$, see (A8). ■

Proof of Lemma 2

The grand coalition is a minimal-cost coalition whenever $\underline{x}^A(N,N) = 0$. In this case, $\underline{X}(N,N) = 0 \leq \underline{X}(A,N)$ for all $A \in \{0,..,N\}$, where the weak equality follows from non-negativity of $\underline{x}^A(A,N)$ and $\underline{x}(A,N)$. Assume next that $\underline{x}^A(N,N) > 0$. There are two sub-cases. In the first sub-case,

$\underline{x}(A,N) = 0$ for all A. Now,

$$\begin{aligned}\underline{X}_A(A,N) = \underline{x}^A(A,N) + A\underline{x}_A^A(A,N) = &\ (1-\delta)(\frac{v^{Inr} - v^{Onr}}{(A+1)^2} - \frac{K}{\delta} + v^{Onr}) \\ \leq &\ -(1-\delta)\frac{(A^2+A-1)\left(v^{Inr} - v^{Onr}\right)}{(A+1)^2(A+2)},\end{aligned}$$

where the inequality follows from $\underline{x}(A,N) = 0$ being equivalent to $K/\delta - v^{Onr} \geq (v^{Inr} - v^{Onr})/(A+2)$. Next, consider the other sub-case, $\underline{x}(A,N) > 0$. After some manipulations,

$$\underline{X}_A(A,N) = -(1-\delta)\frac{((A^2+A-1)(A+2) + (N-A)(A+1)^2)}{(A+1)^2(A+2)^2}\left(v^{Inr} - v^{Onr}\right).$$

Since $\underline{X}_A(A,N) \leq 0$ for all $A \geq 1$ in both sub-cases, either $A = 0$ or $A = N$ minimizes $\underline{X}(A,N)$. The larger coalition is better since $\underline{X}(0,N) \geq \underline{X}(N,N)$ is equivalent to $\underline{x}(0,N) \geq \underline{x}^A(N,N)$, which is easily verified to hold. ■

Proof of Proposition 6

Define $R(N) = \overline{X}(N) - \underline{X}(N,N)$. Note that $\underline{X}(N,N) = N\underline{x}^A(N,N)$. We consider three parameter ranges.

Range (i) $\delta\theta \in (2(1-\delta)K, 2K)$: In this case, $q^{nr} \in (0,1)$ for all $N \geq 1$, which implies $v^{Inr} = K/\delta p^{nr}$ and $v^{Onr} = 0$, see the proof of Proposition 4. Note that

$$\overline{X}(N) = \theta - (1-\delta)\frac{K}{\delta p^{nr}} = (y^{nr} + Nq^{nr})K > 0,$$

where the third equality follows from substituting in (8) and simplifying. $\underline{X}(N,N) = 0$ since

$$\underline{x}^A(N,N) = (1-\delta)\max\{\frac{K}{\delta}(\frac{1}{p^{nr}(N+1)} - 1); 0\} = 0$$

follows from $p(q,N) \geq (N+1)^{-1}$, see Lemma 1. Hence, $R(N) > 0$ for all $N \geq 1$ in this range

Range (ii) $\delta\theta \in [2K, 2K(1+\delta)/(1-\delta)]$: First, $q^{nr} = 1$ for all $N \in [1, \delta\theta/K - 1]$, which, in turn, implies

$$R(N) = \frac{\delta + N}{\delta}K - \frac{N}{N+1}(1-\delta)\theta,\ R'(N) = \frac{K}{\delta} - \frac{(1-\delta)\theta}{(N+1)^2} \text{ and } R''(N) = \frac{2(1-\delta)\theta}{(N+1)^3}.$$

Convexity of $R(N)$, $R'(1) > 0$ and $R(1) \geq 0$ imply that $R(N) \geq 0$ for all $N \in [1, \delta\theta/K - 1]$ and for all $\delta\theta \in [2K,\ 2K(1+\delta)/(1-\delta)]$. Moreover, $q^{nr} \in (0,1)$ for all $N > \delta\theta/K - 1$. As in (i), $\overline{X}(N) > \underline{X}(N, N)$ whenever $q^{nr} \in (0,1)$. Hence, $R(N) \geq 0$ for all $N \geq 1$ also in this range.

Range (iii) $\delta\theta > 2K(1+\delta)/(1-\delta)$: In this case, $R(1) < 0$. Convexity of $R(N)$ in the range $N \in [1, \delta\theta/K - 1]$ implies the existence of a unique $\overline{N} \in (1, \delta\theta/K - 1)$ such that $R(N) \geq 0$ if and only if $N \geq \overline{N}$. ■

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