Bid Rents under Unemployment Risk: Delayed versus Timeless Uncertainty*

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In this paper, we analyze the impact of uncertainty on the level and properties of the bid rent function. We show that these properties strongly depend upon the way in which uncertainty is introduced into the model (delayed versus timeless uncertainty). We also investigate the implication of income uncertainty for the city equilibrium. We show that under timeless uncertainty, a more generous unemployment benefits program may be in the long run welfare reducing in an urban setting.

1. INTRODUCTION

It is commonly observed that unemployment is unevenly distributed among cities and that unemployment plays a major role in the decision of location in a city. Nevertheless, urban economics has not devoted a lot of attention to this issue and has been more interested in the location of the employed workers. Meanwhile, each individual who resides in a city faces a risk of being unemployed at a certain period of time in his professional life. If he loses his job, will he still reside in the same location and reduce the consumption of other goods or will he change his residential location in the city? This question had been addressed in a monocentric framework by Zenou and Smith [11] and by DeSalvo and Eeckhoudt [2] focusing on different aspects of the problem.

Zenou and Smith [11] describe first the urban configuration with two types of workers, the employed and the unemployed. They show that if these two types of workers have the same housing consumption but differ

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in their monetary transportation cost (employed workers going more often to the CBD than the unemployed workers), the unemployed locate at the periphery of the city, whereas the employed reside at the vicinity of the CBD. Moreover, by giving microfoundations of unemployment through the efficiency wage theory, they show why wages and unemployment rates differ among cities. In their model, at each period, any employed worker can lose his job and therefore change his location to the periphery of the city. The risk of being unemployed is associated with a delocation of the residence because when a worker becomes unemployed, he can no more bid for rents around the CBD since his wage decreases drastically.

DeSalvo and Eeckhoudt [2] also analyze this idea but allow the worker who becomes unemployed to stay in the same location (since the choice of location is made before the realization of the risk is known) while reducing his consumption of the composite good. Actually, they use the so-called delayed or temporal risk model introduced by Drèze and Modigliani [1] which stipulates that an employed worker must decide ex ante his housing consumption and thus his residential location and cannot change his decision ex post, i.e., when the worker knows if he is employed or not. Contrarily to Zenou and Smith [11], they focus solely on the employed workers who face a risk of becoming unemployed. This means that they do not explain where the unemployment stems from and where unemployed versus employed workers locate in the city. On the other hand, DeSalvo and Eeckhoudt [2] deeply analyze the effects of income uncertainty on the urban worker’s behavior. Moreover, Turnbull et al. [9] have extended the analysis of income uncertainty (as well as price uncertainty) to a case with any number of possible states.

The effect of uncertainty on location and consumption as well as bid rent decisions depends upon the timing of uncertainty resolution and upon the flexibility allowed to the agent in adjusting his decisions to the flow of information. Quite naturally an intertemporal model is appropriate to analyze such effects. This path was followed by Leland [5], Sandmo [8], and Drèze and Modigliani [1] who formalized the distinction between delayed and timeless risk. In this paper, as in DeSalvo and Eeckhoudt [2] and in Turnbull et al. [9], we will for the sake of simplicity collapse the two periods into a single one. Yet we will keep the important distinction between delayed and timeless risks. In the delayed version, the agent makes his housing and location as well as his bid rent choices before he knows his employment status. Once this status is revealed, he adjusts his

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1 To the best of our knowledge, it is the only article which explains endogenously the unemployment level within an urban framework.
2 A similar analysis has been done by Papageorgiou and Pines [7] and Ioannides [4] where the risk is due to commuting cost uncertainty.
composite good consumption to it but can no longer change his ("previous") housing and location decisions (nor his bid rent function). In the timeless version, more flexibility is allowed: both housing and consumption decisions are made after the employment status is revealed whereas the bid rent is proposed before the realization of the risk.

In the present article, we deepen the analysis of DeSalvo and Eeckhoudt [2] by restating their model in a bid rent approach (see Fujita [3]) so that the properties of the bid rent function can be analyzed for the first time in an uncertain environment. We show that the flexibility allowed by the timeless model leads to a higher bid rent level. We also show that partly as a consequence of the difference between the bid rents, the two models may sometimes induce rather different comparative statics predictions in term of housing consumption. Finally, we study the city equilibrium and we show that for the delayed risk model an unemployment benefit program is welfare improving whereas it is not always true for the timeless risk model.

The remainder of the paper is organized as follows. In Section 2, we introduce the general notations common to the two models. Their description are analyzed in Section 3. Individual comparative static analysis are carried out in Section 4 for both models. Section 5 focuses on the comparison of the different models, whereas Section 6 analyzes the city equilibrium and its comparative statics properties. Finally, Section 7 concludes this article.

2. GENERAL NOTATIONS

The city is monocentric with a center called the Central Business District (CBD hereafter) where all—identical—firms are located. We assume that our city is a CCA type, i.e., a closed city with absentee landlords. Individuals locate outside of the CBD on a featureless homogeneous plain and travel to the CBD for work and shopping. Each employee earns an income $y$, bears a transport cost $T(k)$ (where $k$ is the distance from the CBD) with $T'(k) > 0$ and $T''(k) \leq 0$, consumes a quantity of land, $q$, and a composite good, $x$, taken as the numéraire. Observe that we consider only monetary transport cost and not time transport cost because in the latter case the income must be included in the transport cost function so that transport cost itself would become risky.

In the standard urban model (certainty case), the individual maximises his increasing and strictly concave (in $q$ and $x$) utility function, $U(x, q)$ under the budget constraint, $\bar{I} = x + p(k)q$, where $\bar{I} = y - T(k)$ is the net income and $p(k)$, the land rent price at distance $k$. Therefore, we have

$$\max_q U(\bar{I} - pq, q).$$  \hspace{1cm} (1)
First-order condition (FOC) yields
\[ U_q - pU_x = 0. \] (2)

Second-order condition (SOC) is satisfied for
\[ U_{qq} - 2pU_{qx} + p^2 U_{xx} < 0. \] (3)

FOC and SOC tell us that for any \( p \) there exists a unique \( q^* \). It is given by
\[ q^* = q(\bar{I}(k), p). \] (4)

The indirect utility function \( V(\cdot) \) is given by:
\[ V(\bar{I}(k), p) = v \] (5)

and the bid rent function is therefore
\[ p = V^{-1}(\bar{I}(k), v) = \phi(\bar{I}(k), v). \] (6)

In our paper, the individual will face income uncertainty so that \( \bar{I} \) will be replaced by a random prospect with a mathematical expectation equal to \( \bar{I} \). To be more specific, each individual has a probability \( \pi \) to become unemployed with a net income equal to \( I_u = y_u - T(k) \) (where \( y_u \) stands for the unemployment benefits paid by the government). If the individual stays employed—an event of probability \((1 - \pi)\) he earns \( y_e > y_u \). The risky prospect is evaluated through a \textit{strictly concave} von Neumann–Morgenstern utility function which is supposed to be continuous and twice differentiable. As indicated before we consider two types of models: the delayed or temporal risk and the \textit{ex ante} timeless models (see Dreze and Modigliani [1] for a discussion of these models in the context of saving decisions).

3. DELAYED VERSUS TIMELESS UNCERTAINTY

3.1. The Delayed Risk Model

Here, knowing his risk of being unemployed, the worker decides today (i.e., while he is still employed), his optimal level of housing consumption, \( q \), and his bid rent. If tomorrow he becomes unemployed, he will not change \( q \) but will reduce the consumption of the composite good, \( x \). Thus, the decision about \( q \) and \( p \) is made \textit{ex ante} and cannot be modified \textit{ex post}. Absentee landlords will give the land to the highest bids today.

Note that we consider here the bid-rent approach, where the worker’s optimal housing decision is made at a fixed distance \( k \). The consumer
solves the following program

$$\max_{q} \left[ \pi U(I_u - pq, q) + (1 - \pi) U(I_e - pq, q) \right]. \quad (7)$$

First-order condition is

$$E_q = \pi (U_q - pU_u) + (1 - \pi) (U_q - pU_e) = 0, \quad (8)$$

where subscripts $u$ and $e$ indicate the value of $I$ at which the subscripted term is to be evaluated and the other subscripts indicate partial direct or cross derivatives with respect to the subscripted variable.

Then, by assuming the normality of the good $q$ (i.e., the marginal rate of substitution of $q$ for $x$ increases with $x$), we have

1. $$(U_q - pU_u)_u < 0 \quad (9a)$$
2. $$(U_q - pU_u)_e > 0 \quad (9b)$$
3. $$(U_{qq} - pU_{ux})_u > 0 \quad (10a)$$
4. $$(U_{qq} - pU_{ux})_e > 0. \quad (10b)$$

Second-order condition is satisfied for

$$E_{qq} = \pi (U_{qq} - 2pU_{q} + p^2U_{xx})_u$$
$$+ (1 - \pi) (U_{qq} - 2pU_{q} + p^2U_{xx})_e < 0. \quad (11)$$

The FOC and SOC tell us that for any $p$, there exists a unique $q^*_d$. It is equal to

$$q^*_d = q_d(\pi, I_u(k), I_r(k), p_d), \quad (12)$$

where $p_d$ is the bid rent under delayed uncertainty.

It is easy to see that

$$q^*_d = q_d \left( \pi, \frac{I_u}{+}, \frac{I_r}{+}, \frac{p_d}{-} \right), \quad (13)$$

where the $+$ and the $-$ signs below each variable indicate the direction of their impact on $q^*_d$.

\footnote{For a rigorous proof, see DeSalvo and Eekhoudt [2, p. 102].}

\footnote{See Appendix 1 for the proof of (13).}
For instance, we notice that an increase in $\pi$ will reduce $q_d^*$. The indirect utility function, $V$, being fixed to a utility level, $v_d$, we have

$$V(\pi, I_u(k), I_e(k), p_d) = v_d$$

(14)

and the bid rent function is

$$p_d = \phi_d(\pi, I_u(k), I_e(k), v_d).$$

(15)

We can now rewrite the optimal housing consumption as

$$q_d^{**} = q_d(\pi, I_u, I_e, p_d(\pi, I_u, I_e, v_d)).$$

(16)

Its comparative statics properties are determined by

$$q_d^{**} = q_d\left(\pi^*, I_u', I_e', v_d\right).$$

(17)

As we can see from (17), in equilibrium, each (exogeneous) variable affects $q_d^{**}$ in two opposite ways. For example, an increase in $\pi$ has a direct negative effect on housing consumption which was already present in (13). However, the increase in $\pi$ also negatively affects $p_d$ and since housing consumption is a normal good the fall in price indirectly stimulates housing consumption. Hence, the net effect is ambiguous.$^5$ This is also true for $I_u$ and $I_e$. In this context, we take into account the fact that the original choice of $q_d^*$ affects the bid rent that in turn influences the (new) housing consumption $q_d^{**}$.

3.2. The ex Ante Timeless Model

Here, contrarily to the previous case, the worker optimally selects both $q$ and $x$ after he knows about his employment status while he has to propose his bid rent before the resolution of uncertainty. In other words absentee landlords offer housing to the highest bids today but allow individuals to change “the number of square meters” of their houses tomorrow. Therefore, $p$ and the associated location is decided ex ante and cannot be changed ex post, whereas $q$ and $x$ can both be adjusted ex post. Formally, in the ex ante timeless model the consumer solves the program

$$\pi\left[\max_q U(I_u - pq, q)\right] + (1 - \pi)\left[\max_q U(I_e - pq, q)\right].$$

(18)

$^5$The direct calculations or even the use of specific forms do not give us any more intuition.
First-order conditions are
\[
(U_q - p U_x)_u = 0 \tag{19a}
\]
\[
(U_q - p U_x)_e = 0. \tag{19b}
\]

Second-order conditions are satisfied for
\[
E_{q_u,q_u} = (U_{qq} - 2p U_{qx} + p^2 U_{xx})_u < 0 \tag{20a}
\]
\[
E_{q_e,q_e} = (U_{qq} - 2p U_{qx} + p^2 U_{xx})_e < 0. \tag{20b}
\]

Here again, FOC and SOC tell us that \( q_u^* \) and \( q_e^* \) are unique for any \( p \).

Hence
\[
q_u^* = q_u(I_u, p_t) \tag{21a}
\]
\[
q_e^* = q_e(I_e, p_t), \tag{21b}
\]

where \( p_t \) stands for the bid rent under timeless uncertainty.

It is easy to see that
\[
q_u^* = q_u \left( I_u, \frac{p_t}{\pi} \right) \tag{22a}
\]
\[
q_e^* = q_e \left( I_e, \frac{p_t}{\pi} \right). \tag{22b}
\]

Now, if we rewrite (21a) and (21b) in terms of indirect utility functions, \( V \), and fix it to a utility level equal to \( \hat{v} \), we obtain
\[
\pi V(I_u, p_t) + (1 - \pi) V(I_e, p_t) = \hat{v}, \tag{23}
\]

where \( V(I_u, p_t) = U(I_u - p_t q_u^*, q_u^*) \) and \( V(I_e, p_t) = U(I_e - p_t q_e^*, q_e^*) \).

The bid rent function is therefore
\[
p_t = \phi_t \left( \pi, I_u(k), I_e(k), \hat{v} \right) \tag{24}
\]

We can now write the two equilibrium housing consumptions in the following way:
\[
q_u^{**} = q_u(I_u, p_t(\pi, I_u, I_e, \hat{v})) \tag{25a}
\]
\[
q_e^{**} = q_e(I_e, p_t(\pi, I_u, I_e, \hat{v})). \tag{25b}
\]

As for the delayed risk model, a change in an exogeneous variable has also an indirect effect on \( q_u^{**} \) or \( q_e^{**} \) through the bid rent adjustment.

Differentiating \( q_u^{**} \) and \( q_e^{**} \) totally with respect to each of the exogeneous
variables, one obtains the following results:

\[
q_u^{**} = q_u \left( \pi, I_u, I_e, v_t \right) \quad (26a)
\]

\[
q_e^{**} = q_e \left( \pi, I_u, I_e, v_t \right). \quad (26b)
\]

If \( \pi \) increases, the worker proposes a low bid rent so that—once he knows his true employment status—he will consume more housing which is available at a low price. In other words, when \( \pi \) increases, the individual commits himself on a very low land price (or cheap area) so that whatever his employment status he can increase his housing consumption. Thus, we have in the timeless model a positive relationship between the equilibrium housing consumptions and \( \pi \) whereas it is ambiguous in the delayed risk model. This is one of the main differences between the two models. Let us stress that in the timeless risk model, each individual decides today the bid rent, or in other words, the location in the city. If for example, the probability to become unemployed is very high, he decides to live in a cheap area (the price per square meter he proposes at each distance from the CBD in order to achieve a certain level of utility is very low and thus the only choice is to live in a cheap area) but when the uncertainty is removed he can have a large housing consumption even if he becomes unemployed. Of course, if he stays employed his housing consumption will still be larger than if he becomes unemployed.

Notice also from (26a) and (26b) that there is a negative relationship between \( q_u^{**} \) and \( I_e \) and \( q_e^{**} \) and \( I_u \), respectively, again because of an indirect effect. So, when \( I_e \) (resp. \( I_u \)) increases, the bid rent raises and thus \( q_u^{**} \) (resp. \( q_e^{**} \)) decreases. In the context of the timeless risk model, it is easy to understand this since when the government announces a more generous unemployment benefits program, people propose a higher bid rent. As a consequence, whatever their employment status, there is a negative pressure on their housing consumption. This negative effect may counteract the positive effect that a more generous \( I_u \) has on the housing consumption of unemployed people. To numerically illustrate our point, we use a Cobb–Douglas utility function \( U(x, q) = x^{1/2}q^{1/2} \) and a linear commuting cost \( (T(k) = t \cdot k) \). By using (A.12a) and (A.12b) in Appendix 2, we have that \( \text{sgn}(\partial q_u^{**}/\partial I_u) = -\text{sgn}(\partial q_e^{**}/\partial I_e) \). Let us now interpret (A.12a) and (A.12b). For \( \pi \leq 1/2 \), increasing \( I_u \) will always raise \( q_u^{**} \), whereas increasing \( I_e \) will always reduce \( q_e^{**} \). On the other hand, if \( \pi > 1/2 \), the net effect depends on the difference between \( I_e \) and \( I_u \).
3.3. Comparison of the Different Models

We are now able to compare the three models: certainty, delayed risk, and timeless risk and to see in which case the bid rent is the greatest. For that purpose, we need to compare these three models for a worker located at a distance \( k \) from the CBD and enjoying for each case a given utility level \( \bar{u} \). The following proposition summarizes our result.

**Proposition 1.** A worker living at a distance \( k \) from the CBD and enjoying a utility level \( \bar{u} \) proposes the following bid rents:

\[
p > p_t > p_d.
\]

**Proof.** First, the fact that \( p_t > p_d \) is a straightforward application of the Marschak's theorem\(^6\) which basically states that loosening a constraint raises the value of the objective function. Second, we want to prove that \( p > p_t \). Since we have assumed that the utility function \( U(\cdot) \) is strictly concave in \( x \) and \( q \) so is the indirect utility function \( V(\cdot) \) in the income \( \bar{I} \) (where \( \bar{I} = \pi I_u + (1 - \pi) I_d \)).\(^7\) Therefore, we have for any given \( p \):

\[
\pi V(I_u(k), p) + (1 - \pi) V(I_d(k), p) < V(\bar{I}, p).
\]

Since we want to keep the utility equal in all models, the only way to establish an equality between utility under certainty \( V(\bar{I}, p) \) and under timeless uncertainty \( (\pi V(I_u, p_t) + (1 - \pi) V(I_d, p_t)) \) is to have \( p_t < p \) since \( V' \) is a strictly decreasing function in \( p \). Q.E.D.

The message of Proposition 1 is twofold: first, uncertainty depresses the bid rent, and second, its impact is more pronounced when less flexibility is allowed.

4. COMPARATIVE STATICS ANALYSIS

Let us now turn to the comparative static properties of the bid rent under uncertainty for an individual. For each type of model (delayed versus timeless), we study how the bid rent reacts to changes in incomes \( (I_u \text{ and } I_d) \), distance to CBD, probability of unemployment and utility level.

\(^6\)The Marschak's theorem may be stated as follows: “Let \( g \) be a function of the decision variable \( d \) and of the random variable \( x \) with density \( f(x) \), then: \( \int d \max_d g(d, x) f(x) \, dx = \max_d \int d g(d, x) f(x) \, dx \).”

\(^7\)We can give here a very quick proof. \( \pi q^* + (1 - \pi) q_d^* \) is affordable at \( I \) with certainty, i.e., \( \pi I_u + (1 - \pi) I_d \geq p(\pi q^* + (1 - \pi) q_d^*) \). But by the strict concavity of \( U(\cdot) \), we have \( U(\pi q^* + (1 - \pi) q_d^*) > \pi U(q^*) + (1 - \pi) U(q_d^*) \). By optimization, \( U(\pi q^* + (1 - \pi) q_d^*) \leq V(\bar{I}, p) \). Hence, the result follows, i.e., \( V(\bar{I}, p) > \pi V(I_u, p) + (1 - \pi) V(I_d, p) \).
4.1. The Delayed Risk Model

4.1.1. Variations of $I_u$ and $I_c$. From the differentiation of (15), we have

$$\frac{d\phi_d}{dI_u} = -\frac{\partial V/\partial I_u}{\partial V/\partial p}$$
$$\frac{d\phi_d}{dI_c} = -\frac{\partial V/\partial I_c}{\partial V/\partial p}.$$  

By applying the Envelope Theorem, we obtain

$$\frac{\partial V}{\partial I_u} = \pi U_i|_u > 0$$  \hspace{1cm} (27)
$$\frac{\partial V}{\partial I_c} = (1 - \pi)U_i|_c > 0$$  \hspace{1cm} (28)

and

$$\frac{\partial V}{\partial p} = -q^*_d [\pi U_i|_u + (1 - \pi)U_i|_c] < 0.$$  

Thus, we finally obtain

$$\frac{d\phi_d}{dI_u} = \frac{\pi U_i|_u}{q^*_d (\pi U_i|_u + (1 - \pi)U_i|_c)} > 0$$  \hspace{1cm} (29)
$$\frac{d\phi_d}{dI_c} = \frac{(1 - \pi)U_i|_c}{q^*_d (\pi U_i|_u + (1 - \pi)U_i|_c)} > 0.$$  \hspace{1cm} (30)

Observe that (29) and (30) are both lower than $1/q^*$ which is the value of the equivalent derivative under certainty.

4.1.2. Variation of $k$. From the differentiation of (15), we obtain

$$\frac{d\phi_d}{dk} = -T'(k) \left[ \frac{\partial p}{\partial I_u} + \frac{\partial p}{\partial I_c} \right] = -\frac{T'(k)}{q^*_d} < 0.$$  \hspace{1cm} (31)

This result is qualitatively the same as in the certainty case (Fujita [3, p. 19]). However, its numerical value is different because in general $q^* \neq q^*_d$.

4.1.3. Variation of $\pi$. From differentiation of (15), we have

$$\frac{d\phi_d}{d\pi} = -\frac{\partial V/\partial \pi}{\partial V/\partial p}.$$
Since by the Envelope Theorem,
\[
\frac{\partial V}{\partial \pi} = U(\cdot)|_u - U(\cdot)|_c < 0
\]  
\[
\frac{d \phi_d}{d \pi} = \frac{U(\cdot)|_u - U(\cdot)|_c}{\pi q^n_u U|_u + (1 - \pi) q^n_c U|_c} < 0.
\]

The bid rent decreases with the probability of unemployment. This intuitive result will be used again in Section 5.

4.1.4. Variation of \(\phi_d\). From the differentiation of (15), we have
\[
\frac{d \phi_d}{d v_d} = \frac{-1}{\pi q^n_q U|_u + (1 - \pi) q^n_c U|_c} < 0.
\]

4.2. The ex Ante Timeless Model

By analogy with the previous section (4.1), the following comparative statics are easily derived by differentiating (24).

4.2.1. Variations of \(I_u\) and \(I_c\).
\[
\frac{d \phi_i}{d I_u} = \frac{\pi U|_u}{\pi q^n_u U|_u + (1 - \pi) q^n_c U|_c} > 0
\]
\[
\frac{d \phi_i}{d I_c} = \frac{(1 - \pi) U|_c}{\pi q^n_u U|_u + (1 - \pi) q^n_c U|_c} > 0.
\]

4.2.2. Variation of \(k\).
\[
\frac{d \phi_i}{d k} = \frac{-T'(k)[\pi U|_u + (1 - \pi) U|_c]}{\pi q^n_u U|_u + (1 - \pi) q^n_c U|_c} < 0.
\]

4.2.3. Variation of \(\pi\).
\[
\frac{d \phi_i}{d \pi} = \frac{V(I_u, p) - V(I_c, p)}{\pi q^n_u U|_u + (1 - \pi) q^n_c U|_c} < 0.
\]

4.2.4. Variation of \(v_i\).
\[
\frac{d \phi_i}{d v_i} = \frac{-1}{\pi q^n_u U|_u + (1 - \pi) q^n_c U|_c} < 0.
\]
All the comparative statics results for the timeless and delayed models are qualitatively the same although their numerical values will in general be different. We can summarize all these results by the following proposition and by Table 1.

**PROPOSITION 2.** In both the delayed and the ex ante timeless risk models, an increase in the income or in the unemployment benefit raises the bid rent proposed by each individual in the city. On the other hand, an increase in the distance from the CBD or in the probability of being unemployed or in the equilibrium utility level decreases the bid rent.

### 5. THE CITY EQUILIBRIUM

Let us turn now to the city equilibrium. Since we have assumed that our city is a CCA-type, i.e., a closed city (Wheaton [10]) with absentee landlords and that land market is competitive, the market land rent curve for both the delayed and the timeless risk models is given by

\begin{align*}
R(x) &= \phi_t(\pi, I_a(k), I_s(k), v_t^*) \quad \text{for } 0 < k \leq \bar{k}_t^*, \quad l = d, t \quad (40a) \\
&= R_A \quad \text{for } k > \bar{k}_t^*, \quad l = d, t, \quad (40b)
\end{align*}

where \( R_A \) is the agricultural rent, \( \bar{k}_t^* \) and \( v_t^* \) (\( l = d, t \)) are, respectively, the equilibrium city fringe and the equilibrium utility level in the city.

Observe that in (40a) and (40b), we have explicitly considered two equilibrium city fringes \( \bar{k}_d^* \) and \( \bar{k}_s^* \). In general, \( \bar{k}_d^* \neq \bar{k}_s^* \) but nothing can be said about their comparison. In particular, Proposition 1 tells us that, at any \( k \), \( p_t(\pi, I_a(k), I_s(k), v) > p_d(\pi, I_a(k), I_s(k), v) \), i.e., at each distance from the CBD and for a fixed level of utility \( v \), bid rents are greater under timeless uncertainty. However, since we consider here a closed city model, the utility levels are endogeneous for each model and the comparison of bid rents as well as of city fringes cannot be made. Let us now turn to the delayed uncertainty model. The conditions to determine the city equilibrium are

\begin{align*}
\phi_d(\pi, I_a(\bar{k}_d^*), I_s(\bar{k}_s^*), v_d^*) &= R_A \quad (41) \\
\int_0^{\bar{k}_s^*} \frac{L(k)}{q_d^*(\pi, I_a(k), I_s(k), v_d^*)} dk &= N, \quad (42)
\end{align*}

### TABLE 1

<table>
<thead>
<tr>
<th>( d\pi )</th>
<th>( dI_a )</th>
<th>( dI_s )</th>
<th>( dk )</th>
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</tbody>
</table>
where \( L(k) \) is the land distribution in the city assumed to be continuous for all \( k \geq 0 \), positive at each \( k > 0 \) and increasing in \( k \). For simplicity, we assume that \( L(k) = 1 \) (linear city). This will not change our results but will avoid some tedious calculations. \( N \) is the number of workers living in the city and \( \bar{k}_d^* \) is the equilibrium city fringe. Here there are two equations with two unknowns, \( v_d^*, \bar{k}_d^* \). It is easy to show that the city equilibrium exists and is unique (see, e.g., Fujita [3, Ch. 3]). We are now able to perform the comparative statics analysis. First, by totally differentiating (41) and (42), we have

\[
\mathbf{A} \left( \frac{d\bar{k}_d^*}{dv_d^*} \right) = \mathbf{B},
\]

where

\[
\mathbf{A} = \begin{pmatrix}
\frac{\partial \phi_d(\bar{k}_d^*)}{\partial \bar{k}_d^*} & \frac{\partial \phi_d(\bar{k}_d^*)}{\partial v_d^*} \\
\frac{1}{q_d^*(\bar{k}_d^*)} - \int_0^{\bar{k}_d^*} \frac{1}{(q_d^*(k))^2} \frac{\partial q_d^*(k)}{\partial v_d^*} \\
\end{pmatrix}
\]

\[
\mathbf{B} = \begin{pmatrix}
-\frac{\partial \phi_d(\bar{k}_d^*)}{\partial \pi} & 0 \\
 \frac{1}{(q_d^*(\bar{k}_d^*))^2} \frac{\partial \phi_d(\bar{k}_d^*)}{\partial \pi} & \int_0^{\bar{k}_d^*} \frac{1}{(q_d^*(k))^2} \frac{\partial q_d^*(k)}{\partial \pi} \\
\int_0^{\bar{k}_d^*} \frac{1}{(q_d^*(k))^2} \frac{\partial q_d^*(k)}{\partial \pi} & 0 \\
\int_0^{\bar{k}_d^*} \frac{1}{(q_d^*(k))^2} \frac{\partial q_d^*(k)}{\partial \pi} & 0 \\
\end{pmatrix}
\]

\[
= \begin{pmatrix}
- & - & - & + & 0 \\
? & ? & ? & 0 & + \\
\end{pmatrix}
\]

In the rest of the section, we use the following notation.

\[
\phi(\bar{k}^*) = \phi_1(\pi, I_u(\bar{k}^*), I_e(\bar{k}^*), v_d^*), \quad l = d, t.
\]
\[ \phi_i(k) = \phi_i(\pi, I_u(k), I_e(k), v_i^n), \quad I = d, t \]
\[ q^*(\overline{k}_d^n) = q^*(\pi, I_u(\overline{k}_d^n), I_e(\overline{k}_d^n), v_i^n), \quad I = d, t \]
\[ q^*(k) = q^*(\pi, I_u(k), I_e(k), v_i^n) \quad I = d, t. \]

It is easily checked that \( \text{det}(A) > 0. \) Moreover, by using the Cramer rule, it is straightforward to show that
\[
\frac{\partial \overline{k}_d^n}{\partial R_A} < 0, \quad \frac{\partial \overline{k}_d^n}{\partial v_i^n} > 0 \tag{44}
\]
\[
\frac{\partial v_i^n}{\partial R_A} < 0, \quad \frac{\partial v_i^n}{\partial N} < 0. \tag{45}
\]

In order to determine the signs of \( \frac{\partial v_i^n}{\partial \pi}, \frac{\partial v_i^n}{\partial I_u}, \frac{\partial v_i^n}{\partial I_e} \), we cannot use simply the Cramer rule. First, observe from (31) that
\[
\frac{1}{q^*} = - \frac{\partial \phi_d}{\partial k} T'(k) \tag{46}
\]
so that replacing in (42), we have
\[
\int_{0}^{\overline{k}_d^n} \frac{\partial \phi_d}{\partial k} T'(k) dk = N. \tag{47}
\]
Moreover, integrating by parts (47) and using (41), we obtain
\[
\frac{R_A}{T'(k)} - \int_{0}^{\overline{k}_d^n} \phi_d(k) X(k) dk = -N, \tag{48}
\]
where \( X(k) = -[T''(k)/T'(k)] > 0. \)

Now, we have to differentiate (48) with respect to \( \pi, I_u, \) and \( I_e. \) Let us start with \( \pi. \) We obtain after some manipulations
\[
\frac{R_A}{T'(\overline{k}_d^n)} \frac{\partial \overline{k}_d^n}{\partial \pi} X(\overline{k}_d^n) - \int_{0}^{\overline{k}_d^n} \left( \frac{\partial \phi_d(k)}{\partial \overline{v}_i^n} \frac{\partial v_i^n}{\partial \pi} + \frac{\partial \phi_d(k)}{\partial \pi} \right) X(k) \frac{T'(k)}{T''(k)} dk
\]
\[ - \frac{\phi_d(\overline{k}_d^n)}{T'(\overline{k}_d^n)} X(\overline{k}_d^n) \frac{\partial \overline{k}_d^n}{\partial \pi} = 0. \tag{49}
\]

By using (41), the first and the third term of the LHS of (49) cancel. It is then easy to show that
\[
\frac{\partial v_i^n}{\partial \pi} = - \left( \int_{0}^{\overline{k}_d^n} \frac{\partial \phi_d(k)}{\partial \pi} X(k) \frac{T'(k)}{T''(k)} dk \right) \left( \int_{0}^{\overline{k}_d^n} \frac{\partial \phi_d(k)}{\partial v_i^n} X(k) \frac{T'(k)}{T''(k)} dk \right)^{-1} < 0 \tag{50}
\]
By the same reasoning, we have

\[
\frac{\partial v^u}{\partial I_u} = -\left( \int_0^x \frac{\partial \phi_a(k)}{\partial I_u} \frac{X(k)}{T'(k)} \, dk \right) \left( \int_0^x \frac{\partial \phi_a(k)}{\partial v^u \partial I_u} \frac{X(k)}{T'(k)} \, dk \right)^{-1} > 0
\]

(51)

\[
\frac{\partial v^e}{\partial I_e} = -\left( \int_0^x \frac{\partial \phi_a(k)}{\partial I_e} \frac{X(k)}{T'(k)} \, dk \right) \left( \int_0^x \frac{\partial \phi_a(k)}{\partial v^e \partial I_e} \frac{X(k)}{T'(k)} \, dk \right)^{-1} > 0.
\]

(52)

In the delayed risk model, the signs of $\partial k^u / \partial \pi$, $\partial k^e / \partial \pi$, $\partial k^u / \partial I_u$, $\partial k^e / \partial I_e$ are ambiguous. Even if we use a specific form such as a Cobb–Douglas utility function and a linear commuting cost, nothing can be said and no intuition is given. The interesting thing is that when $\pi$, $I_u$ or $I_e$ vary, the impact on the equilibrium city fringe is not clear whereas the one on the equilibrium utility level is clear and predictable. This is mainly due to the fact that when one of these exogenous variables varies, the effect on the equilibrium housing consumption is ambiguous and so is the effect on the city size.

Let us turn now to the timeless uncertainty model. The equilibrium conditions (41) and (42) are rewritten as

\[
\phi_t(\pi, I_u(k), I_e(k), v^u) = R_A
\]

(53)

\[
\int_0^\pi \frac{1}{\pi q_{uu}^t(\pi, I_u(k), I_e(k), v^u) + (1 - \pi) q_{ee}^t(\pi, I_u(k), I_e(k), v^u)} \, dk
\]

\[
= N.
\]

(54)

Let us give a justification of (54). Assume that unemployment is uncorrelated across residents, with the probability being $\pi$ for each individual. Then, if the city population is large, the proportion of residents who are unemployed at each $k$ will be indistinguishable from $\pi$ by the law of large numbers. Equation (54) is then the right equilibrium condition where the denominator of the LHS of (54) gives the “actual” land consumption at each distance. Now, if we differentiate (53) and (54), we obtain

\[
A' \begin{pmatrix}
\frac{d \pi}{d I_u} \\
\frac{d \pi}{d I_e} \\
R_A \\
N
\end{pmatrix} = B' \begin{pmatrix}
\frac{d v^u}{d k^u} \\
\frac{d v^e}{d k^e}
\end{pmatrix},
\]

(55)
where

\[ A' = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} + & - \\ - & + \end{pmatrix} \]

with

\[ a_{11} = \frac{\partial \phi_1(\bar{k}_e^*)}{\partial \bar{k}_e^*} < 0; \quad a_{12} = \frac{\partial \phi_1(\bar{k}_e^*)}{\partial \bar{\nu}_i^*} < 0 \]

\[ a_{21} = \frac{1}{\pi q_u^{**}(\bar{k}_e^*) + (1 - \pi) \pi q_e^{**}(\bar{k}_e^*)} > 0; \]

\[ a_{22} = -\int_{\bar{k}_e^*}^{\bar{k}_e^*} \left( \frac{\partial q_u^{**}(k)}{\partial \nu_i^*} + (1 - \pi) \frac{\partial q_e^{**}(k)}{\partial \nu_i^*} \right) \frac{1}{\left[ \pi q_u^{**}(\bar{k}_e^*) + (1 - \pi) \pi q_e^{**}(\bar{k}_e^*) \right]^2} dk < 0 \]

and where

\[ B' = \begin{pmatrix} b_{11} & b_{12} & b_{13} & 1 & 0 \\ b_{21} & b_{22} & b_{23} & 0 & 1 \end{pmatrix} = \begin{pmatrix} + & - & - & + & 0 \\ ? & ? & ? & 0 & + \end{pmatrix} \]

with

\[ b_{11} = \frac{\partial \phi_k(\bar{k}_e^*)}{\partial \nu_i^*} > 0; \quad b_{12} = \frac{\partial \phi_k(\bar{k}_e^*)}{\partial \nu_i^*} < 0; \]

\[ b_{13} = \frac{\partial \phi_k(\bar{k}_e^*)}{\partial \pi} < 0 \]

\[ b_{21} = \int_{0}^{\bar{k}_e^*} \frac{\left\{ q_u^{**}(k) - q_e^{**}(k) + \pi \frac{\partial q_u^{**}(k)}{\partial \nu_i^*} + (1 - \pi) \frac{\partial q_e^{**}(k)}{\partial \nu_i^*} \right\}}{\left[ \pi q_u^{**}(k) + (1 - \pi) q_e^{**}(k) \right]^2} dk \]

\[ b_{22} = \int_{0}^{\bar{k}_e^*} \frac{\left\{ \pi \frac{\partial q_u^{**}(k)}{\partial \pi} + (1 - \pi) \frac{\partial q_e^{**}(k)}{\partial \pi} \right\}}{\left[ \pi q_u^{**}(k) + (1 - \pi) q_e^{**}(k) \right]^2} dk; \]

\[ b_{23} = \int_{0}^{\bar{k}_e^*} \frac{\left\{ \pi \frac{\partial q_u^{**}(k)}{\partial \nu_i^*} + (1 - \pi) \frac{\partial q_e^{**}(k)}{\partial \nu_i^*} \right\}}{\left[ \pi q_u^{**}(k) + (1 - \pi) q_e^{**}(k) \right]^2} dk. \]
It is easily checked that \( \det(A') > 0 \). By using the Cramer rule we have

\[
\frac{\partial \tilde{k}_i^*}{\partial R_A} < 0, \quad \frac{\partial \tilde{k}_i^*}{N} > 0 \\
\frac{\partial v_i^*}{\partial R_A} < 0, \quad \frac{\partial v_i^*}{\partial N} < 0.
\]

But, we cannot sign the other variables. We can use the Cobb–Douglas utility function in order to determine the ambiguous signs (see Appendix 3 for the signs of the elements of the matrices \( A' \) and \( B' \) in the Cobb–Douglas case). By the Cramer rule and with some calculations, we obtain

\[
\frac{\partial \tilde{k}_i^*}{\partial \pi} < 0, \quad \frac{\partial \tilde{k}_i^*}{\partial I_u} > 0, \quad \frac{\partial \tilde{k}_i^*}{\partial I_c} > 0 \\
\frac{\partial v_i^*}{\partial \pi} < 0, \quad \frac{\partial v_i^*}{\partial I_u} > 0, \quad \frac{\partial v_i^*}{\partial I_c} > 0.
\]

However, in the general case, one result in the timeless risk model seems awkward and counter-intuitive: increasing the unemployment benefit may be welfare reducing. More precisely, by using the Cramer rule, one obtains that

\[
\text{sgn} \left( \frac{\partial v_i}{\partial I_u} \right) = \text{sgn} \{a_{11}b_{22} - a_{21}b_{12}\}
\]

or equivalently

\[
\text{sgn} \left( \frac{\partial v_i}{\partial I_u} \right) = \text{sgn} \{D + E\},
\]

where

\[
D = -T'(\overline{k}_i) \left[ \pi U_{1\text{a}}(\overline{k}_i) + (1 - \pi) U_{1\text{b}}(\overline{k}_i) \right] \\
\times \int_{0}^{\overline{k}_i} \pi \frac{\partial q_u^*}{\partial I_u} + (1 - \pi) \frac{\partial q_e^*}{\partial I_u} \\
\quad \times (\pi q_u^* + (1 - \pi) q_e^*)^2 \, dk
\]

\[
E = \frac{\pi U_{1\text{a}}(\overline{k}_i)}{\pi q_u^*(\overline{k}_i) + (1 - \pi) q_e^*(\overline{k}_i)}.
\]
At first glance, it is quite messy but the following conclusion can be drawn from this result. A necessary condition for $\partial v_i/\partial I_u$ to be negative is

$$\pi \cdot \frac{\partial q_u^{**}}{\partial I_u} + (1 - \pi) \cdot \frac{\partial q_e^{**}}{\partial I_u} > 0. \quad (61)$$

If (61) was not true, then $\partial v_i/\partial I_u$ will always be positive since both $D$ and $E$ would be positive.

The following comments are in order. First, since $\partial q_u^{**}/\partial I_u < 0$, obviously we need to have that $\partial q_u^{**}/\partial I_u$ must be positive. This means that when $I_u$ raises, the increase of the bid rent is not enough to reduce the housing consumption, $q_u^{**}$. When we have seen above that it is always the case when $\pi \leq 1/2$ (which it is in general the case in the real world) or if the income difference $I_e - I_u$ is enough large when $\pi > 1/2$. Second, the necessary condition is: $\pi (\partial q_e^{**}/\partial I_u) > (1 - \pi) (\partial q_u^{**}/\partial I_u)$. In other words, when $I_u$ increases, $q_e^{**}$ is reduced through the bid rent and $q_u^{**}$ raises since we have assumed that the direct (positive) effect dominates the indirect (negative) effect of the bid rent. If the first effect dominates the second one, then $\partial v_i^{**}/\partial I_u < 0$. This is true when the indirect (negative) bid rent effect is low for both $q_u^{**}$ and $q_e^{**}$. Observe that this global (negative) effect is possible because of the flexibility allowed by the timeless risk model. Since people can adjust their housing consumption, an increase in the unemployment benefit leads to an ambiguous impact on the utility level because of the indirect effect on the housing consumption through the bid rent. This was not possible in the delayed risk model since people could not adjust ex post their housing consumption.

We can now state the following proposition and summarize it in Table 2.

**Proposition 3.** For the delayed risk model, at the city equilibrium, an increase in $\pi$, $R_A$, or $N$ raises the equilibrium utility level whereas an increase

| Comparative Statics for the City Equilibrium for Both Models |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| $d\pi$ | $dI_u$ | $dI_e$ | $dR_A$ | $dN$ |
| $d\pi^{**}$ | $?$ | $?$ | $?$ | $-$ | $+$ |
| $d\pi$ | $(-)$ | $(+)$ | $(+)$ | $-$ | $+$ |

Note. First line, results for the delayed uncertainty model; second line, results for the timeless uncertainty model. The results in parentheses use a Cobb–Douglas utility function and a linear commuting cost.
in $I_u$ or $I_e$ decreases it. For the timeless risk model, the effects are not clear. Meanwhile, by using a Cobb–Douglas utility function, it is shown that the same effects occur as in the delayed risk model. However, in the general case, the timeless uncertainty model, because of its flexibility, can lead to the fact that increasing the unemployment benefit is welfare reducing.

6. CONCLUSION

In this paper, we have developed two approaches focusing on the timing of uncertainty. In the first one (the delayed uncertainty), the price and the housing demand decisions have to be made before the realization of the risk is known. In the other one (timeless uncertainty), the individual has some flexibility due to the possibility of adjusting housing and composite good consumption to observed income. We have shown in both models that increasing the unemployment benefit or the income raises the bid rent, whereas increasing the probability of being unemployed decreases the bid rent. Moreover, we have also shown that the second approach, because of its flexibility, allows workers to offer higher bid rents if they want to enjoy the same level of utility and to stay in the same location. Lastly, when we study the city equilibrium, one interesting result was that under timeless uncertainty, increasing the unemployment benefits may reduce the total welfare because of the urban setting.

Our model can be extended in at least two different ways. First, it will be interesting to introduce explicitly different categories of workers differentiated by their degree of risk aversion. In this case, the risk of becoming unemployed will not affect the different workers in the same way. Second, in the spirit of Zenou and Smith [11] we could also model explicitly the location of unemployed people versus that of the employed workers. In this case, the two types of agents in the city would have different expected utilities and the government could subsidize housing only for the unemployed people.

APPENDIX 1

Proof of (13)

From the differentiation of (12), we obtain

\[
\frac{\partial q^*_u}{\partial \pi} = -\frac{(U_q - pU_x)_u - (U_q - pU_x)_e}{E_{qq}} < 0 \quad (A.1)
\]

\[
\frac{\partial q^*_d}{\partial I_u} = -\frac{\pi(U_q - pU_x)_u}{E_{qq}} > 0 \quad (A.2)
\]
\[
\frac{\partial q_u^a}{\partial I_u} = -\frac{(1 - \pi)\left(U_{yx} - pU_{xx}\right)}{E_{qq}} > 0 
\]
(A.3)
\[
\frac{\partial q_e^a}{\partial p} = q_e^a\left[\pi(U_{q_y} - pU_{q_x}) + (1 - \pi)(U_{q_x} - pU_{q_y})\right] + \pi U_{q_y} + (1 - \pi)U_{q_x} < 0.
\]
(A.4)

APPENDIX 2

The Cobb–Douglas Case for the Individual Worker in the Timeless Risk Model

In the timeless risk model, the worker solves the following program

\[
\pi\left\{\text{Max}_{q_u}\left(\left(I_u - p \cdot q_u\right)^{1/2}q_u^{1/2}\right)\right\} + (1 - \pi)\left\{\text{Max}_{q_e}\left(\left(I_e - p \cdot q_e\right)^{1/2}q_e^{1/2}\right)\right\},
\]
(A.5)

where \(I_u = y_u - t \cdot k\) and \(I_e = y_e - t \cdot k\).

First-order conditions yield \(^8\)

\[
q_u^a = \frac{I_u}{2p} \quad \text{(A.6a)}
\]
\[
q_e^a = \frac{I_e}{2p} \quad \text{(A.6b)}
\]

It is readily verified that (22a) and (22b) are satisfied. Moreover, the indirect utility functions become

\[
V(I_u, p) = I_u p^{-1/2}(1/2)^{1/2} \quad \text{(A.7a)}
\]
\[
V(I_e, p) = I_e p^{-1/2}(1/2)^{1/2}. \quad \text{(A.7b)}
\]

From (23), we have

\[
\pi I_u p^{-1/2}(1/2)^{1/2} + (1 - \pi) I_e p^{-1/2}(1/2)^{1/2} = v_i \quad \text{(A.8)}
\]

so that the bid rent is expressed as

\[
p_i = \phi_i = \frac{1}{4} \left( \frac{\tilde{I}}{v_i} \right)^2, \quad \text{(A.9)}
\]

\(^8\)It is easy to check that the second order conditions are always satisfied.
where $\bar{I} = \pi I_u + (1 - \pi) I_e$. It is easy to check that all the comparative statics results of Section 4.2 are all verified.

We can now define the two equilibrium housing consumptions, $q_u^{**}$ and $q_e^{**}$. Using (A.9), we obtain

$$q_u^{**} = 2I_u \left( \frac{\bar{I}}{v_i} \right)^{-2} \quad (A.10)$$

$$q_e^{**} = 2I_e \left( \frac{\bar{I}}{v_i} \right)^{-2} \quad (A.11)$$

It is readily verified that all the unambiguous results of (26a) and (26b) hold here (since it is a particular case). Let us therefore focus on the two ambiguous results. In the Cobb–Douglas utility function case, we obtain

$$\frac{\partial q_u^{**}}{\partial I_u} = \frac{\partial q_u^{**}}{\partial I_u} + \frac{\partial q_u^{**}}{\partial p} \frac{\partial p_i}{\partial I_u} = 2I^{-3}v_i^2(1 - \pi)I_e - \pi I_u \quad (A.12a)$$

$$\frac{\partial q_e^{**}}{\partial I_e} = \frac{\partial q_e^{**}}{\partial I_e} + \frac{\partial q_e^{**}}{\partial p} \frac{\partial p_i}{\partial I_e}$$

$$= 2\bar{I}^{-3}v_i^2[\pi I_u - (1 - \pi)I_e] \quad (A.12b)$$

APPENDIX 3

The Cobb–Douglas Case for the City Equilibrium in the Timeless Risk Model

For the Cobb–Douglas utility case in the timeless uncertainty, the city equilibrium conditions (56) and (57) write

$$\frac{1}{4} \bar{I}(\bar{k}_i)^2v_i^{-2} = R_A \quad (A.13)$$

$$\frac{1}{4} \bar{k}_i(2\bar{y} - \bar{k}_i)v_i^{-2} = N_i \quad (A.14)$$

where $\bar{y} = \pi y_u + (1 - \pi)y_e$ and $\bar{I}(\bar{k}_i) = \pi(y_u - \bar{k}_i) + (1 - \pi)(y_e - \bar{k}_i) = \bar{y} - \bar{k}_i$.

In this case, the elements of the matrixes $A'$ and $B'$ write

$$a_{11} = -\frac{1}{2} \bar{I}v^{-1} < 0; \quad a_{21} = \frac{1}{2} \bar{I}v^{-1} > 0$$
\[ a_{12} = -\frac{1}{2} I^2 u^{-3} < 0; \quad a_{22} = -\frac{\bar{I}}{2}(\bar{I} + \bar{y})u^{-3} < 0 \]

\[ b_{11} = \frac{1}{2}(I_e - I_u)\bar{I}u^{-1} > 0; \quad b_{21} = \frac{1}{2}(I_e - I_u)\bar{u}u^{-1} > 0; \]
\[ b_{12} = -\frac{\pi}{2} \bar{I}u^{-1} < 0 \]

\[ b_{22} = -\frac{\pi}{2} \bar{u}u^{-1} < 0; \quad b_{13} = -\frac{1}{2}(1 - \pi)\bar{I}u^{-1} < 0; \]
\[ b_{23} = -\frac{1}{2}(1 - \pi)\bar{u}u^{-1} < 0. \]

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