OPTIMAL UTILITARIAN TAXATION AND HORIZONTAL EQUITY

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Abstract

We impose a horizontal equity restriction on the problem of finding the optimal utilitarian tax mix. The horizontal equity constraint requires that individuals with the same ability have to pay the same amount of taxes regardless of their preferences for leisure. Contrary to normal findings, we find that a good that is complementary to leisure can be encouraged by the tax system and that a good that normally should be discouraged by the tax system can be subsidized even if the economy is composed of only two private commodities plus leisure. Also, the marginal effective tax rate can be different from zero at the top of the ability distribution when the tax mix obeys the horizontal equity constraint.

1. Introduction

Nozick (1974) asks why somebody who prefers looking at the sunset should pay less taxes than somebody who has to earn money in order to attain his pleasures. This question is important not only on moral ground but also because a tax system will be replaced if enough citizens object to it.
Equity questions of this kind have largely been ignored in the optimal taxation literature, where one of the standard assumptions is that all individuals have the same preferences. Yet philosophers and social choice scholars investigate the importance of social arrangements in which individuals are held responsible for certain inequalities. In particular, it is often argued that an individual ought to bear the consequences of his actions. This line of reasoning, much of which originates from Dworkin (1981a, b), is especially relevant for optimal income taxation if the utility of leisure is heterogeneous across individuals. In such case, the government may not want to redistribute wealth on the basis of income differences that are due to differences in tastes. However, since it is generally assumed that the government can only observe the income of an individual, it is impossible to find an income tax scheme that only compensates for differences in abilities. Indeed, in the public debate it is frequently pointed out that transfers to low skilled but hard-working persons are also benefiting more highly skilled but epicurean individuals. In the eyes of the government, they are alike since their pretax incomes are similar. In this paper, we investigate if and how the government can escape this dilemma. The possible solution involves linear commodity taxation and non-linear income taxation.

Related to the principle of responsibility for certain inequalities is the horizontal equity (hereafter HE) principle, which calls for the equal treatment of equals. One prominent interpretation of the HE principle is that if two individuals differ only in tastes, then the government ought to treat them identically. The literature contains several suggestions of the status and definition of HE. Musgrave (1959: 160) argues that within the ability-to-pay approach to taxation, “horizontal and vertical equity are but different sides of the same coin.” However, there are several reasons for taxing people with the same ability differently. Besides the lack of information problem, Stiglitz (1982) demonstrates that the HE requirement does not follow from the maximization of a traditional utilitarian or more general social welfare function (which does not consider relations between individual

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1 Possible exceptions are provided by Cuff (2000) and Boadway et al. (2002) for the finite case, while Tarkiaimen and Tuomala (1999) develop a computational approach to tackle the problem of a two-dimensional population in the continuous case. All the quoted authors neglect the problem of the optimal structure of commodity taxation and work with models where leisure is additive separable from other consumption goods. Sandmo (1993) examines the utilitarian case for a linear income tax under the assumption that differences in earnings are explained by differences in preferences over work and consumption; he also has a brief section in which both market abilities and preferences for leisure are allowed to vary.

2 See Fleurbaey and Maniquet (2002) for a review.

3 In this approach, vertical equity is the principle that the greater a person’s ability to pay, the more tax should he hand over to the government.
outcomes) and, more strongly, that it may also be inconsistent with Pareto optimality.\(^4\)

In view of this likely conflict between horizontal and vertical equity, it is often argued that horizontal should take precedence. Atkinson and Stiglitz (1976, 1980) suggest the imposition of a HE constraint on the maximization of a social welfare function. Feldstein (1976) instead suggests to balance the fulfillment of HE against the utilitarian principle of welfare maximization. Regarding the definition of HE, the proposed measures of this principle are as a rule either based on tax payments or on utilities.

In this paper, we stay close to the interpretation of the concept given by Bossert (1995) in terms of “equal transfers for equal circumstances,”\(^5\) and require that individuals of the same ability must pay the same amount of taxes irrespective of their preferences.\(^6\) This can be justified by the observation that people with the same ability share the same opportunity set, and while differences in this set can in some moral sense be deemed “irrelevant” and therefore call for compensation, differences in preferences may be regarded as “morally relevant,” suggesting that compensation is ruled out for such differences. According to this reasoning the individuals are fully responsible for the way that their preferences affect their actions.

Our approach is to introduce the principle of equal transfers for equal circumstances as a constraint on the maximization of a utilitarian social welfare function. Although we have to admit that the choice of a tax-based rather than a utility-based measure is to some extent arbitrary, it is simple and also sufficient for focusing on the moral difficulties raised by the fact that the government cannot observe differences in abilities or preferences but can observe differences in income.\(^7\)

The concern for HE modifies the rule for optimal commodity taxation. Contrary to normal findings, a good that is complementary to leisure need not be discouraged by the tax system and, perhaps more peculiar, a good that should be discouraged by the tax system in the absence of the HE condition need not be taxed, even if the economy is composed of only two private

\(^4\)Kaplow and Shavell (2001) and Fleurbaey et al. (2003) show that the Pareto principle restricts the use of non-welfarist methods of policy assessment, such as the HE principle. (The term “non-welfarist” refers to any conception of social welfare that gives weight to factors other than the satisfaction of the individuals’ preferences.)

\(^5\)This terminology comes from the division of the sources of individual outcomes into wills, resources, and circumstances. According to this division, the individual is responsible for his wills, whereas the circumstances are factors outside his control. Differences in circumstances can be compensated by reallocating the resources.

\(^6\)A similar argument was put forward by Allingham (1975).

\(^7\)The policies analyzed in this paper differ from policies that can be derived from conceptions of justice based on equality of opportunity. In contrast to what is suggested by Roemer (1998, 2002) the government in our model does not seek to equalize outcomes for comparable people with different abilities.
commodities plus leisure. We derive effective marginal tax rates for individuals with different characteristics and compare them with the tax rates derived in an ordinary optimal taxation model. In this case we find that the popular endpoint result of no distortion at the top of the skill distribution can be violated.

2. The Model

We consider an economy with three goods (two private consumption goods \( c \) and \( z \) plus leisure), and three types of individuals. The individuals are characterized by their skill or ability \((w^H, w^L)\) (reflected, by assumption of perfect competition, in the unitary wage rate they are paid) and by their taste for leisure \((\alpha^H, \alpha^L)\), where superscript \( H \) (\( L \)) denotes a high (low) ability or taste for leisure. There are \( \pi^1 \) low skilled, low taste for leisure individuals (type 1 with \( w^L, \alpha^L \)), \( \pi^2 \) high skilled, high taste for leisure individuals (type 2 with \( w^H, \alpha^H \)), and \( \pi^3 \) high skilled, low taste for leisure individuals (type 3 with \( w^H, \alpha^L \)). Preferences are represented by the utility function \( u(c, z, \alpha^l) \), where \( \alpha^l \) is the preference parameter of an individual of type \( i \) and \( l \) is the supply of labor. Both the market ability and taste for leisure parameters are private information. The government knows the joint distribution of \( w, \alpha \) in the population and observes the actual pretax income of each individual but is not able to observe the hours of work of any particular individual. This rules out first-best personalized lump-sum taxes. The government is then forced to design an “anonymous” tax system that does not discriminate among individuals and is thwarted by a set of self-selection (incentive-compatibility) constraints requiring that each individual is better off with the bundle of goods intended for him than with any other bundle.

Production is linear and uses labor as the only input; units are chosen to make all producer prices equal to one and good \( z \) is chosen as numérateur and set untaxed. Consumer prices are represented by the vector \((1 + t, 1) = (q, 1)\). In addition to the commodity tax, the individuals also have to pay a non-linear tax \( T(Y) \) on income \( Y \). Thus, disposable income \( B \) equals \( Y - T(Y) \) and the total tax liability is \( \tau(Y) = T(Y) + tc \). The individuals’ behavior can be formalized in the following manner, breaking their optimization problem into two stages. At the first stage the individuals take labor supply as given and allocate a fixed amount of expenditure over the consumption goods. Forming the Lagrangian \( \Lambda = u(c, z, \alpha l) - \varphi(qc + z - B) \) we can derive the first-order conditions \( \frac{\partial u(c, z, \alpha l)}{\partial c} - \varphi q = 0 \), \( \frac{\partial u(c, z, \alpha l)}{\partial z} - \varphi = 0 \) and \( qc + z - B = 0 \). These conditions define a special kind of demand functions, the so-called “conditional” demand functions \( c(q, B, \alpha l) \) and \( z(q, B, \alpha l) \), that we will assume to be

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8In the standard optimal taxation problem with two private commodities plus leisure, the concept of discouragement (encouragement) becomes equivalent to “being taxed” (“being subsidized”).
differentiable. The corresponding conditional indirect utility function \( V(q, B, \alpha l) \), defined as \( \max_{c, z} \{ u(c, z, \alpha l) \mid qc + z = B \} \), may then be written as \( V(q, B, \alpha l) = u(c(q, B, \alpha l), z(q, B, \alpha l), \alpha l) \). At the second stage the individuals choose hours of work to maximize \( V(q, B, \alpha l) \) subject to the link between pre- and posttax earnings implied by the direct tax schedule. If the tax function were differentiable, the first-order conditions of this maximization problem could be manipulated to derive

\[
-\alpha \frac{V_3}{V_B} = w (1 - T'),
\]

where \( T' = \frac{dT}{dY} \) is the marginal income tax rate and subscripts on \( V \) denote partial derivatives (in particular \( V_3 \) denotes the partial derivative with respect to the third argument, namely \( \alpha l = \frac{\alpha}{w} Y \)). However, with a finite group of individuals the optimal tax function is non-differentiable precisely at the points at which the schedule is actually observed. In consequence, marginal income tax rates are not well defined at an optimal allocation. Nonetheless, since the utility functions are differentiable, it is possible to define implicit marginal tax rates using the marginal rates of substitution. We will use what has become standard terminology in the optimal taxation literature and refer to

\[
1 + \frac{\alpha}{w} \frac{V_3}{V_B}
\]

as the (implicit or shadow) marginal income tax rate.\(^9\)

The indirect utility function has the following properties: \( V_B < 0, V_B > 0, V_3 < 0 \). In order to satisfy the single-crossing condition (indifference curves cross only once) we will also assume that, \( q \) being held fixed, \( V_{33} < 0 \) (labor is annoying at increasing rates) and \( V_{BS} < 0 \) (an increase in private consumption is valued more, the less “experienced hours” (\( \alpha l \) the person is working). These are sufficient assumptions for the normality of private consumption (i.e., disposable income).\(^10\) We will also assume normality of “experienced” leisure (i.e., leisure time multiplied by the individual’s value of \( \alpha \)). This is done in order to ensure that a utilitarian government would like to redistribute consumption in an “egalitarian” way, namely from those who are better off (have higher utility) toward those who are worse off (have lower utility).\(^11\)

Compared with the related models developed by Cuff (2000) and Boadway et al. (2002), the distinguishing feature of our model is the introduction of an additional taxable commodity. Cuff uses a model with three types of individuals and a two-goods economy (private consumption plus leisure), where high-skilled individuals have low taste for leisure, while there are low-skilled individuals both with high and with low taste for leisure. The individuals’ utility functions are quasi-linear and affine in consumption. Boadway et al.

\(^9\)It can be shown that there exist optimal tax functions for which \( 1 + \frac{\alpha}{w} \frac{V_3}{V_B} \) is the left-handed derivative of the tax function at \( Y = w l \).

\(^10\)A formal proof of this result can be found in Jordahl and Micheletto (2002).

\(^11\)This consequence will be taken into account when we present the government’s maximization problem and write the relevant self-selection constraints.
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(2002) use a model with four types of individuals (and the same two-goods economy), in which low-skilled individuals with low taste for leisure are indiscernible from high-skilled individuals with high taste for leisure. The individuals’ utility functions are quasi-linear, but in their case affine in labor.

Dealing with a two-goods economy, the quoted papers are confined to studying the shape of the optimal income tax schedule and cannot examine the potential role of commodity taxation. Notice, however, that, even if individual utility functions are not separable between leisure and other goods, commodity taxes cannot be employed to screen between the low skilled, low taste for leisure type 1 individuals and the high skilled, high taste for leisure individuals of type 2, as long as \( \frac{\alpha_L}{w_L} = \frac{\alpha_H}{w_H} \). Relaxing this assumption, two cases become possible: (i) \( \frac{\alpha_L}{w_L} < \frac{\alpha_H}{w_H} \) or (ii) \( \frac{\alpha_L}{w_L} > \frac{\alpha_H}{w_H} \). Suppose first that \( \frac{\alpha_L}{w_L} < \frac{\alpha_H}{w_H} \); then we face the following chain of inequalities: \( \frac{\alpha_L}{w_H} < \frac{\alpha_L}{w_L} < \frac{\alpha_H}{w_H} \). This means that at every point in \((Y, B)\)-space, the slope of the indifference curve of a low skilled, hard working type 1 individual is shallower than the curve of a high-skilled, epicurean type 2 individual, and that for this pair of individuals the ranking of the slopes of indifference is reversed compared with models where individuals differ only in \( w \). If instead \( \frac{\alpha_L}{w_L} > \frac{\alpha_H}{w_H} \), then the chain of inequalities is \( \frac{\alpha_L}{w_H} < \frac{\alpha_H}{w_H} < \frac{\alpha_L}{w_L} \). This case reflects more closely the standard one since none of the two types with high ability have indifference curves in \((Y, B)\)-space that are steeper than the ones of the low-skilled type. In this case the ordinary ranking of the indifference curves persists. Since this is probably the more realistic setting, we will only present the solution of the model under case (ii).12

Before proceeding with the formal analysis, two remarks are needed. The first concerns the use of two parameters to characterize individuals. Although our individuals differ in both innate labor productivity and taste for leisure, their preferences over consumption and income can be summarized by a single parameter, the type aggregator ratio \( \frac{\alpha}{w} \).13

The second remark is the following. In standard optimal taxation models, the inability to observe the types of the individuals raises a familiar problem. The government wishes to redistribute resources from high-skilled to low-skilled individuals14 but, not knowing who is who, all it can do is to tax higher incomes more heavily. This may create an incentive for high-skilled

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12Had we instead developed case i) in which \( < \frac{\alpha_H}{w_H} \), the particular conclusions reached would obviously need modification, but the method of analysis would not. In neither case do the self-selection constraints imply that we must have bunching between individuals with different skill levels.

13Due to the complexity of optimal taxation problems with more than one unknown characteristic, type aggregators are quite commonly used to reduce a multidimensional screening problem to a one-dimensional problem. For some recent examples, see Cuff (2000), Boadway et al. (2002), and Brett and Weymark (2003).

14This happens because utility normally increases with the wage rate.
individuals to reduce their labor supply and “mimic” low-skilled individuals. Having imposed the single-crossing condition, the binding self-selection constraint thwarting the government’s attempts to redistribute income runs downward from high-skilled (high earning) individuals toward low-skilled (low earning) individuals. In a finite-class economy this is generalized by saying that an optimal allocation results in a simple monotonic chain to the left (Guesnerie and Seade 1982), which means that each pair of successive bundles is L-linked by a downward binding incentive-compatibility constraint. However, as long as individuals differ both in market ability and in preferences for leisure, this is no longer necessarily true even if (as in our case) the single-crossing condition still holds. We cannot tell a priori which one of the pair of self-selection constraints is going to be binding. Both constraints could even be binding at the same time.

3. The Optimal Tax Mix

In this section, we derive the optimal tax mix for a utilitarian government. To get a benchmark case, we will first quickly present the results that are obtained without imposing the HE constraint. Then, we present the results of the full model including this constraint.

3.1. Without the HE Constraint

When the (utilitarian) government neglects questions raised by the heterogeneity in tastes, its problem is the following:

\[
\max_{B^1, B^2, B^3, Y^1, Y^2, Y^3, t} \pi^1 V\left(q, B^1, \alpha^L \frac{wL Y^1}{wL}ight) + \pi^2 V\left(q, B^2, \alpha^H \frac{wH Y^2}{wH}ight) + \pi^3 V\left(q, B^3, \alpha^L \frac{wL Y^3}{wL}\right),
\]

subject to the budget constraint.

15 Using the terminology of Guesnerie and Seade (1982), a corner (or chosen bundle) is linked to another if they both belong to the optimal set of some individual \( h \), or equivalently if there is an indifference curve of \( h \), which passes through both corners and is the highest \( h \) can reach on his budget set. Individual \( h \) is said to link these corners. A corner \( C_i \) is W-linked (W for winner) if some \( h \) links \( C_i \) to some other corner \( C_j \), and is allocated \( C_i \). A corner \( C_i \) is L-linked (L for loser) if some \( h \) links \( C_i \) to \( C_j \), and is allocated \( C_j \).

16 In accordance with standard practice in the optimal taxation literature, we will simply assume that a solution exists and characterize the optimal tax mix conditional on this assumption.

17 To simplify the presentation, non-negativity constraints are omitted in all the optimization problems that follow.

18 We assume that taxation serves a merely redistributive purpose.
\[ \pi^1(Y^1 - B^1 + tc^1) + \pi^2(Y^2 - B^2 + tc^2) + \pi^3(Y^3 - B^3 + tc^3) \geq 0, \quad (\gamma) \]

and the following self-selection constraints:\(^{19}\)

\[ V(q, B^2, \frac{\alpha^H}{w_H} Y^2) \geq V(q, B^1, \frac{\alpha^H}{w_H} Y^1), \quad (\lambda_2^d) \]

\[ V(q, B^3, \frac{\alpha^L}{w_H} Y^3) \geq V(q, B^2, \frac{\alpha^L}{w_H} Y^2), \quad (\lambda_3^d) \]

where the subscripts on the Lagrange multipliers indicate the type of the potential mimicker, and the superscripts indicate the direction of the incentive-compatibility constraint: “u” for upward and “d” for downward (according to the ranking given by the slopes of the indifference curves). Since single-crossing holds, we only need to take the self-selection constraints linking pairs of adjacent individuals into account.

For brevity, we will often omit arguments and use a superscript \(i\) on \(V\) to denote that an individual of type \(i\) is evaluating his own bundle and a superscript \(i(j)\) to denote that an individual of type \(i\) is evaluating the bundle designed for an individual of type \(j\). Similarly, \(c_{i(j)}\) will indicate the demand for good \(c\) of an individual of type \(i\) mimicking an individual of type \(j\).

### 3.1.1. The Indirect tax structure

Denoting by a “tilde” an income-compensated variable, the optimal commodity tax rate \(t\) is implicitly given (see Appendix A) by

\[ t \sum_{i=1}^3 \pi^i \frac{\partial \tilde{c}^i}{\partial q} = \frac{1}{\gamma} \left[ \lambda_2^d V_B^{2(1)} (c^1 - c^{2(1)}) + \lambda_3^d V_B^{3(2)} \left( \frac{c^2}{A} - c^{3(2)} \right) \right]. \quad (1) \]

Notice that in Equation (1) the term \(A\) inside brackets (which refers to individuals that are both high skilled) is non-zero as long as \(c_{3}^i\), the derivative of the demand of individuals of type \(i\) for commodity \(c\) with respect to the third argument in the individual utility function, is different from zero. The consumption of the taxed commodity is positively related to experienced labor if \(c_3 > 0\) and negatively related to experienced labor if \(c_3 < 0\).

Without the HE constraint the sign of the right-hand side (R.H.S.) of (1), and, therefore, of \(t\), is unambiguously determined once the relation between the taxed commodity and leisure is known.

### 3.1.2. The Marginal Effective Tax Rates

Now consider the marginal effective tax rate (METR). Since there are only two commodities and one is chosen as \(numénaire\) and set untaxed, the effective

\(^{19}\)When writing the self-selection constraints, we are implicitly exploiting the circumstance that the utilitarian solution belongs to the family of “normal cases,” i.e., entails redistribution from high-skilled to low-skilled individuals.
tax liability is defined as \( \tau(Y) = T(Y) + tc[q, Y - T(Y), \frac{\alpha}{w}Y] \). For a differentiable tax function, the METR is given by

\[
\tau' = T' + t \left[ \frac{\partial c}{\partial B} (1 - T') + c_3 \frac{\alpha}{w} \right].
\]

(2)

As we know from the discussion in Section 2, the formula given in (2) applies more generally if the implicit marginal income tax rate \( 1 + \frac{\alpha V_3}{w V_B} \) is substituted for \( T' \). Thus,

\[
\tau' = 1 + tc_3 \frac{\alpha}{w} + \frac{\alpha V_3}{w V_B} \left( 1 - \frac{\partial c}{\partial B} \right).
\]

(3)

As expected, Appendix B shows that the METR is positive for individuals of types 1 and 2, but zero for individuals of type 3.

### 3.2. With the HE Constraint

Since we defined HE as the requirement that individuals with the same skill level must pay the same amount of taxes, the constraint on the government’s problem takes the form

\[
Y^3 = t(c^2 - c^3) + Y^2 - B^2 + B^3.
\]

(4)

Substituting (4) into the indirect utility function of type 3 individuals, the government’s problem becomes

\[
\max_{B^1, B^2, B^3, Y^1, Y^2, t} \pi^1 V \left( q, B^1, \frac{\alpha}{w L} Y^1 \right) + \pi^2 V \left( q, B^2, \frac{\alpha}{w H} Y^2 \right) + \pi^3 V \left( q, B^3, \frac{\alpha}{w H} [t(c^2 - c^3) + Y^2 - B^2 + B^3] \right),
\]

subject to the budget constraint

\[
\pi^1 (Y^1 - B^1 + tc^1) + \pi^2 (Y^2 - B^2 + tc^2) + \pi^3 (Y^2 - B^2 + tc^2) \geq 0, \quad (\gamma)
\]

and the following self-selection constraints:20

\[
V \left( q, B^1, \frac{\alpha}{w L} Y^1 \right) \geq V \left( q, B^2, \frac{\alpha}{w L} Y^2 \right), \quad (\lambda_1)
\]

\[
V \left( q, B^2, \frac{\alpha}{w H} Y^2 \right) \geq V \left( q, B^1, \frac{\alpha}{w H} Y^1 \right), \quad (\lambda_2)
\]

\[
V \left( q, B^3, \frac{\alpha}{w H} [t(c^2 - c^3) + Y^2 - B^2 + B^3] \right) \geq V \left( q, B^2, \frac{\alpha}{w H} Y^2 \right), \quad (\lambda_3)
\]

20Since single-crossing holds, we only need to take the self-selection constraints linking pairs of adjacent individuals into account.
\[
V\left(q, B^3, \frac{\alpha^L}{w_H} \left[ t(c^2 - c^3) + Y^2 - B^2 + B^3 \right] \right) \geq V\left(q, B^2, \frac{\alpha^L}{w_H} Y^2 \right). \quad (\lambda^d_3)
\]

Notice that every variation in \(B^2, B^3, Y^2,\) and \(t\) must be accompanied by a proper variation in \(Y^3,\) the pretax income of type 3 individuals, in order to match the HE requirement. Differentiating the HE constraint (4) we get the following results:

\[
\frac{dY^3}{dB^2} = \frac{t \frac{\partial c^2}{\partial B^2} - 1}{1 + tc^3 \frac{\alpha^L}{w_H}}, \quad (5)
\]

\[
\frac{dY^3}{dB^3} = \frac{1 - t \frac{\partial c^3}{\partial B^3}}{1 + tc^3 \frac{\alpha^L}{w_H}}, \quad (6)
\]

\[
\frac{dY^3}{dY^2} = \frac{1 + tc^3 \frac{\alpha^H}{w_H}}{1 + tc^3 \frac{\alpha^L}{w_H}}, \quad (7)
\]

\[
\frac{dY^3}{dq} = \frac{c^2 - c^3 + t \left( \frac{\partial c^2}{\partial q} - \frac{\partial c^3}{\partial q} \right)}{1 + tc^3 \frac{\alpha^L}{w_H}}. \quad (8)
\]

Considering the “normal” case with redistribution from high-skilled to low-skilled individuals, we can show that an optimum is not compatible with the Lagrange multiplier \(\lambda^u_1\) being different from zero. The argument is very close to that provided by Brito et al. (1990) (in their proposition 1) that, at any efficient allocation, individuals of one type will always view the bundles chosen by individuals of other types who have a larger total tax liability, as strictly inferior to their own. To show that a similar result holds in our model, suppose that there exists a solution to the government’s problem such that the constraint \(\lambda^u_1\) is binding. Then, the government could improve upon the suggested allocation by simply letting the low-skilled individuals reach the point intended for those whom they are willing to mimic. Since redistribution is assumed to go from high-skilled to low-skilled individuals, this would imply that the low-skilled individuals switch from a bundle where the total tax liability is negative (they are net receivers) to a bundle where it is positive. Leaving the value of the maximand of the government’s problem unaffected (since the low-skilled individuals were supposed to be indifferent between the two points), such a switch would be socially desirable since it weakens the budget
constraint of the government without tightening the incentive-compatibility constraints.

On the other hand it is not possible to avoid taking the other self-selection constraints into account. Contrary to the case without the HE restriction, we cannot disregard the constraint $\lambda^u_2$. This is because individuals belonging to types 2 and 3 are adjacent and, since we require them to pay the same total tax liability, it is possible that either of these types would like to mimic the other. Notice, however, that the HE constraint rules out bunching, which would mean that they earn the same gross income and have the same income tax liability. In order to pay the same total amount of taxes, they would have to pay the same amount of commodity taxes and this can only happen if $c_3 = 0$. It follows that, apart from this special case, the constraints $\lambda^u_2$ and $\lambda^d_3$ cannot be binding at the same time.

### 3.2.1. The Indirect Tax Structure

The optimal commodity tax rate $t$ is implicitly given (see Appendix C) by

$$\gamma t \left\{ \pi^1 \frac{\partial c^1}{\partial q} + \left( \pi^2 + \pi^3 \right) \frac{\partial c^2}{\partial q} \right\} + \left( \pi^3 + \lambda^d_3 \right) V^3 \frac{\alpha^L}{w^H} \frac{\partial c^2}{\partial q} - \frac{\partial \tilde{c}^3}{\partial q} + \frac{\partial \tilde{c}^3}{\partial q} \right\} - \lambda^u_2 V^2(3) \frac{\alpha^H}{w^H} \frac{t}{1 + tc^3} \frac{\alpha^L}{w^H} \left( \frac{\partial \tilde{c}^2}{\partial q} - \frac{\partial \tilde{c}^3}{\partial q} \right) = \lambda^d_2 V^2(1) \left( c^1 - c^2(1) \right) + \lambda^u_2 V^2(3) \left( c^3 - c^2(3) \right) + \lambda^d_3 V^3(2) \left( c^2 - c^3(2) \right) \right).$$

(9)

In a framework, where a non-linear income tax is in place, we know that the standard formula for optimal commodity taxation balances the gains of weakening the self-selection constraints against the effects on revenue from a marginal (compensated) change in the commodity tax rate. Comparing (9) and (1) we see that the main differences are confined to the left-hand side (L.H.S). The pattern of binding self-selection constraints could also differ (but not the number since, as we previously noticed, the constraints $\lambda^u_2$ and $\lambda^d_3$ in (9) cannot be binding at the same time).

21See for instance Edwards et al. (1994).

22The second term on the L.H.S. of (9) provides a social evaluation of the effect of a compensated increase in the commodity tax rate on the indirect utility of individuals of type 3. The last term on the L.H.S. of (9) evaluates a
compensated increase in \( t \) in terms of the effect on the (eventually) binding incentive-compatibility constraint requiring that individuals of type 2 must not be tempted to mimic individuals of type 3.

The second and third term on the L.H.S. of (9) can be understood by analytically investigating a hypothetical tax reform. Consider the effects of a small increase \( dq \) in the commodity tax rate accompanied by reductions \( dT_i = -c_i dq < 0, \ i = 1, 2, 3 \), in the income tax liabilities of the three types of individuals at their original earnings. This reform has no effect on the welfare of individuals of types 1 or 2 since, \( Y_i \) being held fixed, by use of Roy’s identity we get

\[
\frac{dV_i}{dq} = \frac{V_i}{q} dq + V_i B_i \frac{dB_i}{dq} = -V_i (c_i dq + dT_i) = -V_i (c_i - c_i) dq = 0, \ i = 1, 2.
\]

The net effect of this “compensated” reform on the utility of the individuals of type 3 is

\[
dV^3 = \frac{V_3^3 dq + V_3^3 dB^3 + V_3^3 \alpha L \frac{B^3}{wH}}{1 + t c_3 \alpha L \frac{B^3}{wH}} \left[ e^2 - c^3 + t \left( \frac{\partial e^2}{\partial q} - \frac{\partial e^3}{\partial q} \right) \right] dq
\]

Substituting \( dB^2 = c^2 dq \) and \( dB^3 = c^3 dq \) into the previous equation, and making use of Roy’s identity and Slutsky equation, gives

\[
dV^3 = \frac{V_3^3 \alpha L \frac{B^3}{wH} \left( \frac{\partial e^2}{\partial q} - \frac{\partial e^3}{\partial q} \right) - t}{1 + t c_3 \alpha L \frac{B^3}{wH}} dq.
\]

Thus, a tax reform that normally would not affect the utility of individuals of type 3 might do so when we impose the HE constraint. On the other hand, if the reform affects the utility of individuals of type 3, it will also affect the well being that individuals of other types can obtain by mimicking the type 3 individuals. This is relevant since we cannot rule out the possibility that individuals of type 2 would like to misrepresent their true type and choose the bundle intended for individuals of type 3. Therefore, if \( \lambda_2^u \neq 0 \), the indirect
utility of an individual of type 2 who is mimicking an individual of type 3 is not only affected by the different way in which a mimicker spends income across goods \((V^2_3(c^3 - c^{2(3)}))\), but also by the change in labor supply required to be recognized as an individual of type 3.

Denoting by \(d\tilde{V}^3\) and \(d\tilde{V}^{2(3)}\) (where the “tildes” help to remember that the effects are produced by a “compensated” marginal variation in \(t\)) the quantities \(dV^3_dq\) and \(dV^{2(3)}_dq\) provided by (10) and (11), we can rewrite (9) as

\[
\gamma t\left[\pi_1 \frac{\partial \tilde{c}^1}{\partial q} + (\pi^2 + \pi^3) \frac{\partial \tilde{c}^2}{\partial q}\right] + (\pi^3 + \lambda^d_3) d\tilde{V}^3 - \lambda^u_2 d\tilde{V}^{2(3)} =
\]

\[
= \lambda^d_2 V^{2(1)}_B (c^1 - c^{2(1)}) + \lambda^u_2 V^{2(3)}_B (c^3 - c^{2(3)}) + \lambda^d_3 V^{3(2)}_B (c^2 - c^{3(2)}). \tag{12}
\]

More insights into this modified commodity taxation rule can be gained by looking at sufficient conditions that make (9) collapse into (1). These require that, together with \(\lambda^u_2 = 0\), one of the following holds:

\[
\frac{\partial \tilde{c}^2}{\partial q} = \frac{\partial \tilde{c}^3}{\partial q}, \tag{13}
\]

\[
(\pi^3 + \lambda^d_3) V^3_3 \frac{\alpha^L}{wH} = \gamma \pi^3 \left(1 + tc^3_3 \frac{\alpha^L}{wH}\right). \tag{14}
\]

For \(\lambda^u_2 = 0\) condition (13) tells us that, if the value of the derivative of the Hicksian demand is the same for individuals of types 2 and 3, the requirement of HE does not alter the traditional rule governing optimal commodity taxation.

The L.H.S. of (14) represents the costs of raising an additional unit of revenue by increasing the income tax liability of individuals of type 3. Holding their disposable income constant, a marginal increase in the gross income of these individuals has a total direct negative impact on their indirect utility measured by \(\pi^3 V^3_3 \frac{\alpha^L}{wH}\) (since there are \(\pi^3\) individuals of type 3), and as such it affects the objective function of the government negatively. Moreover, since this policy change also tightens the self-selection constraint that prevents individuals of type 3 from mimicking individuals of type 1, there is another social cost captured by \(\lambda^d_3 V^3_3 \frac{\alpha^L}{wH}\).

The R.H.S. of (14) represents the benefits of this policy measure: the change raises additional funds and, when evaluated at the shadow price for public funds, the social value of this increase is \(\gamma \pi^3 \left(1 + tc^3_3 \frac{\alpha^L}{wH}\right)\).

Condition (14), therefore, says that we are also back to the standard formula for optimal commodity taxation if the social benefits of a marginal increase in the gross income \(Y^3\) are exactly offset by its social costs.

The popular prescription in the literature on optimal taxation recommends that goods complementary to labor should be encouraged while goods complementary to leisure should be discouraged by the commodity tax system
(where “encouraged” and “discouraged” are both intended in the Mirrleesian sense\textsuperscript{23}). In the standard\textsuperscript{25} counterpart of (1) with many types of individuals and many commodities, the terms on the R.H.S. provide a social evaluation of the gains, in terms of relaxing the binding incentive-compatibility constraints, from a marginal (compensated) increase in one of the commodity tax rates. In that case the prescription to tax (heavier) the commodities that are complementary to leisure is due to the fact that the R.H.S. is negative (positive) when the commodity, whose price is marginally increased, is complementary to leisure (labor).

To find two novel properties of the modified rule, note that four cases are possible in Equation (12)

Case 1: $R.H.S. > 0$, $t > 0$,

\begin{align*}
\gamma \pi^1 \frac{\partial \tilde{c}_1}{\partial q} + \gamma \frac{\partial \tilde{c}_2}{\partial q} \sum_{i=2}^{3} \pi^i \\
+ \left( \pi^3 + \lambda^d_3 \right) \frac{d\tilde{V}^3}{t} - \lambda^u_2 \frac{d\hat{V}^{(3)}}{t} > 0;
\end{align*}

Case 2: $R.H.S. > 0$, $t < 0$,

\begin{align*}
\gamma \pi^1 \frac{\partial \tilde{c}_1}{\partial q} + \gamma \frac{\partial \tilde{c}_2}{\partial q} \sum_{i=2}^{3} \pi^i \\
+ \left( \pi^3 + \lambda^d_3 \right) \frac{d\tilde{V}^3}{t} - \lambda^u_2 \frac{d\hat{V}^{(3)}}{t} < 0;
\end{align*}

Case 3: $R.H.S. < 0$, $t > 0$,

\begin{align*}
\gamma \pi^1 \frac{\partial \tilde{c}_1}{\partial q} + \gamma \frac{\partial \tilde{c}_2}{\partial q} \sum_{i=2}^{3} \pi^i \\
+ \left( \pi^3 + \lambda^d_3 \right) \frac{d\tilde{V}^3}{t} - \lambda^u_2 \frac{d\hat{V}^{(3)}}{t} < 0;
\end{align*}

Case 4: $R.H.S. < 0$, $t < 0$,

\begin{align*}
\gamma \pi^1 \frac{\partial \tilde{c}_1}{\partial q} + \gamma \frac{\partial \tilde{c}_2}{\partial q} \sum_{i=2}^{3} \pi^i \\
+ \left( \pi^3 + \lambda^d_3 \right) \frac{d\tilde{V}^3}{t} - \lambda^u_2 \frac{d\hat{V}^{(3)}}{t} > 0.
\end{align*}

There are basically two reasons explaining the possibility of deviations from ordinary tax prescriptions. On one hand, we already pointed out that

\textsuperscript{23}In a general context where there are $n$ commodities and $m$ individuals, the index of discouragement of commodity $i$ is defined by Mirrlees (1976) as $d_i = \sum_{h=1}^{m} \sum_{j=1}^{n} (\partial \tilde{x}_h^i / \partial q_j) t_i (\sum_{h=1}^{m} x_h^i)^{-1}$, where $q$ and $t$ denote consumer prices and commodity tax rates, $x_h^i$ is the demand for commodity $i$ by individual $h$ and a “tilde” denotes Hicksian demand. The index is an approximate measure of the change in compensated demand due to the tax system; positive values of the index mean that the commodity is encouraged by the indirect tax system, while negative values correspond to discouragement.

\textsuperscript{24}See again Edwards et al. (1994).

\textsuperscript{25}Standard is here meant to describe a situation where wages are exogenous and individuals differ only with respect to their skills.
we can no longer be sure that the budget set will result in a simple monotonic chain to the left. It can well be the case that $\lambda_u^2 > 0$ and $\lambda_d^3 = 0$. This accounts for the fact that the R.H.S. of (12) could be positive (negative) even when commodity $c$ is complementary to leisure (labor). If commodity $c$ is complementary to labor we have that $c^1 > c^{2(1)}$, $c^3 < c^{2(3)}$, and $c^2 > c^{3(2)}$. Thus, if $\lambda_d^3 = 0$ and $\lambda_u^2 > 0$, case 3 given above encompasses a “non-ordinary” subcase with a commodity complementary to labor which in spite of this is taxed, besides the standard case of a subsidized commodity which is complementary to labor. If commodity $c$ is instead complementary to leisure we have that $c^1 < c^{2(1)}$, $c^3 > c^{2(3)}$, and $c^2 < c^{3(2)}$. Thus, if $\lambda_d^3 = 0$ and $\lambda_u^2 > 0$, case 2 encompasses a “non-ordinary” subcase with a commodity complementary to leisure which in spite of this is subsidized. These two “non-ordinary” subcases demonstrate the first novel property of our modified tax rule: that a good that is complementary to leisure may actually be encouraged and a good complementary to labor be discouraged by the indirect tax system.

On the other hand, whereas in the standard model with two private consumption goods the factor by which $t$ is multiplied is always negative (because of the concavity of the expenditure function), here it is not possible to rule out the possibility that, due to the presence of two additional factors ($d\tilde{V}^3$ and $d\tilde{V}^{2(3)}$), it turns out to be positive. If this happens, then we would have the “anomalous” result that a commodity that should be encouraged according to the R.H.S. of (12), should actually be taxed, whereas a commodity that should be discouraged according to the R.H.S. of (12), should actually be subsidized. To see this, consider in case 1 the subcase of a commodity complementary to labor that should be encouraged, according to the R.H.S. of Equation (12); nevertheless, due to a positive value of $(\pi^3 + \lambda_d^3) \frac{d\tilde{V}^3}{t} - \lambda_u^2 \frac{d\tilde{V}^{2(3)}}{t}$ which is greater than the absolute value of $\gamma \pi^1 \frac{\partial \tilde{c}^2}{\partial q} + \gamma \frac{\partial \tilde{c}^3}{\partial q} + \sum_{i=2}^{3} \pi^i$, the commodity is taxed. Similarly, consider in case 4 the subcase of a commodity complementary to leisure that should be discouraged according to the R.H.S. of Equation (12). Also in this situation, HE can be upheld if the factor that multiplies $t$ on the L.H.S. of Equation (12) is positive and the commodity is subsidized. However, while in the former subcase this requires a high and positive value of $(\pi^3 + \lambda_d^3) d\tilde{V}^3 - \lambda_u^2 d\tilde{V}^{2(3)}$ (since we are looking conditions compatible with a tax), in the latter subcase this requirement means a high and negative value of the aforesaid term (since we are looking for conditions compatible with a subsidy). Let us look more closely at these two “anomalous” outcomes. For this aim, consider the case when $\lambda_d^3 = 0$ and one of the two following conditions holds:

(i) $t > 0$, 
\[ \left| \frac{\partial \tilde{c}^2}{\partial q} \right| > \left| \frac{\partial \tilde{c}^3}{\partial q} \right| \quad \text{and} \quad 1 + tc^3_3 \frac{\alpha_L}{wH} > 0; \]

(ii) $t < 0$, 
\[ \left| \frac{\partial \tilde{c}^2}{\partial q} \right| > \left| \frac{\partial \tilde{c}^5}{\partial q} \right| \quad \text{and} \quad 1 + tc^3_3 \frac{\alpha_L}{wH} > 0. \]
Assuming \( c \) to be a commodity complementary to labor, condition (i) provides an example of the possibility of the former “anomalous” case. In this case a (compensated) decrease in the positive value of the excise would be beneficial in terms of (compensated) revenue and weakening of the self-selection constraints. However, a marginal cut in the commodity tax rate lowers the total tax payment of an individual of type 3 more than it does for an individual of type 2; the HE constraint then requires an additional increase in \( Y^3 \) which has the (damaging) effect to lower the utility of type 3 individuals. If this cost outweighs the other benefits, the reduction in \( t \) is not implemented.

Assuming \( c \) to be a commodity complementary to leisure, condition (ii) instead provides an example of the possibility of the latter “anomalous” case. In this case, a (compensated) reduction of the subsidy would be beneficial in terms of (compensated) revenue and weakening of the self-selection constraints. However, for the same reason as above, the HE constraint requires that this reform should be performed together with an additional increase in \( Y^3 \). This increase in income tax payment reduces the utility of individuals of type 3 and if this welfare cost more than offsets the other benefits, the subsidy will not be cut down.

### 3.2.2. The Marginal Effective Tax Rates

We now turn to the problem of evaluating the METR faced by the individuals at the optimal allocation. Using (3) we can characterize the METR in the following proposition.

**PROPOSITION 1:** Under the assumption \( \frac{w_H}{w_L} < \frac{\alpha_L}{\alpha_H} \), the constrained utilitarian optimum with redistribution from high-skilled to low-skilled individuals is characterized by

(a) a positive METR faced by individuals of type 1 (low skilled, low taste for leisure);

(b) a zero METR faced by either individuals of type 2 (high skilled, high taste for leisure) or type 3 (high skilled, low taste for leisure); and a METR different from zero for the other type.

**Proof:** See Appendix D. ■

In Appendix D, we show that the METR faced by type 2 and type 3 are, respectively,

\[
\tau_2' = \frac{\lambda_3 d V_B^{3(2)} \left( V_B^{3(2)} \frac{\alpha_L}{V_B^{3(2)}} - \frac{V_3^2}{V_2^2} \alpha_H \right)}{\gamma w^H (\pi^2 + \pi^3) + \frac{1}{1 + t c_3^3 \alpha_L \alpha_H / w^H} \left[ (\pi^3 + \lambda_3 d) V_3^2 \alpha_L - \lambda_2 w^H V_3^{2(3)} \alpha_H \right]},
\]

(15)
\[ \tau_3' = \frac{\lambda_3^u V_B^{2(3)} \alpha^H}{\left(\pi^3 + \lambda_3^d\right) V_B^3} \left(1 + \frac{V_B^{2(3)}}{w^H} \frac{1 - t \frac{\partial \ell^3}{\partial B^3}}{1 + t \ell^3 \frac{\alpha L}{wH}}\right) \left(1 + t \ell^3 \frac{\alpha L}{wH}\right). \]  

The positive METR faced by low-skilled individuals is not surprising if we notice that the corner intended for them is L-linked with another corner (the one intended for individuals of type 2) by a downward incentive-compatibility constraint. In such a circumstance, this distortion is the standard one which makes it possible to relax the binding constraint.

Now consider the high-skilled individuals. We have already noticed that \( \lambda_3^u \) and \( \lambda_3^d \) cannot be binding at the same time. Suppose first that \( \lambda_3^u \neq 0 \) while the other self-selection constraint is slack (\( \lambda_3^d = 0 \)). From (15) we get \( \tau_2' = 0 \): the corner intended for individuals of type 2 is not L-linked to any other corner and they should therefore be “on average” undistorted at the margin. From (16), instead, we get \( \tau_3' \neq 0 \), in accordance with the rule prescribing that an individual should be “on average” distorted at the margin when the bundle intended for him is L-linked to another bundle by individuals of a different type. However, whereas we would have expected a marginal subsidy since the self-selection constraint is binding upwards the sign of the METR is ambiguous. Denoting by \( \frac{dV^{2(3)}}{dB^3} \) the term \( V_B^{2(3)} (1 + \frac{V_B^{2(3)}}{w^H} \frac{1 - t \frac{\partial \ell^3}{\partial B^3}}{1 + t \ell^3 \frac{\alpha L}{wH}}) \), we have that the sign of the METR is determined by the sign of the product \( 1 + t \ell^3 \frac{\alpha L}{wH} \cdot \frac{dV^{2(3)}}{dB^3} \). Therefore, \( \tau_3' > 0 \) when one of the following conditions holds:

- (iii) \( 1 + t \ell^3 \frac{\alpha L}{wH} > 0 \) and \( \frac{dV^{2(3)}}{dB^3} > 0 \);
- (iv) \( 1 + t \ell^3 \frac{\alpha L}{wH} < 0 \) and \( \frac{dV^{2(3)}}{dB^3} < 0 \).

Differentiating the budget constraint of an individual of type 3 \((q c^3 + z^3 = B^3)\) with respect to disposable income \( B \), it can be shown that normality of commodities \( c \) and \( z \) implies \( 1 - t \frac{\partial c^3}{\partial B^3} > 0 \). Then, condition (iii) implies \( \frac{dB^3}{dY^3} = \frac{1+t \ell^3 \frac{\alpha L}{wH}}{1-t \frac{\partial c^3}{\partial B^3}} > 0 \). This means that in order to satisfy the HE constraint, a marginal increase in the labor supply of type 3 individuals (in this case the ones being potentially mimicked) would require an increase in their disposable income. However, we also have that \( \frac{dV^{2(3)}}{dB^3} > 0 \), which means that the mimickers would profit by the change. To prevent this and weaken the binding self-selection constraint it is useful to let the individuals of type 3 face a positive distortion at the margin, discouraging them from increasing their supply of labor.

On the other hand, condition (iv) implies \( \frac{dB^3}{dY^3} = \frac{1+t \ell^3 \frac{\alpha L}{wH}}{1-t \frac{\partial c^3}{\partial B^3}} < 0 \): a marginal increase in the labor supply of individuals of type 3 would require a reduction
in their disposable income in order to keep the HE constraint satisfied. Since \( \frac{dV_2(3)}{dB} < 0 \), the mimickers would profit by the change in this case as well. Once again, to prevent this and weaken the binding self-selection constraint, a positive METR, discouraging individuals of type 3 from increasing their labor supply, is recommended.

Suppose now that \( \lambda_3^d \neq 0 \) whereas \( \lambda_2^u = 0 \). From (16) we get \( \tau'_3 = 0 \): the corner intended for type 3 individuals is not L-linked to any other corner and they are “on average” undistorted at the margin. From (15), instead, we get \( \tau'_2 \neq 0 \). Considering the pattern of the binding self-selection constraints (only downward), we would expect that the sign of \( \tau'_2 \) were positive since this is the standard prescription for weakening the binding constraints in such a case. However, with \( \lambda_2^u = 0 \), the standard formula for the METR faced by individuals of type 2 is amended by the additional term \( (1 + \tau_3^d \alpha^L) V_3^3 \alpha^L \) in the denominator. Since when \( \lambda_2^u = 0 \), a necessary condition for the existence of an optimum (given that commodities \( c \) and \( z \) are normal) is \( 1 + \tau_3^d \alpha^L > 0 \) (see Appendix C, Equation (C3)), this additional term will be negative. Thus, the sign of the denominator in (15) becomes ambiguous and the fact that the numerator is positive is not sufficient to decide the sign of \( \tau'_2 \). A sufficiently high absolute value of \( (1 + \tau_3^d \alpha^L)^{-1} (\pi_3^3 + \lambda_3^d) V_3^3 \alpha^L \) would entail \( \tau'_2 < 0 \).

The reason is once again related to the requirement of HE. The sign of the METR faced by individuals of type 2 determines the sign of the variation in their total tax liability as they increase their labor supply. This in turn means that, if individuals of type 2 should marginally increase their labor supply, it would be necessary to adjust \( Y_3^3 \) to restore the condition of equal total tax payments. This would obviously affect the indirect utility of individuals of type 3 in a way that the government must take into account. Suppose that individuals of type 2 choose to marginally increase their gross income \( Y_2^2 \). In order to be induced to do so, their disposable income has to be increased by their marginal valuation of foregone leisure \( -\alpha^H \frac{V_2^3}{\omega^H \beta^H} \). From Equation (7) and (5) the variation in \( Y_3^3 \) needed to keep constraint (4) satisfied is

\[
dY_3^3 = \frac{-\alpha^H \frac{V_2^3}{\omega^H \beta^H} \left( \frac{\partial c^2}{\partial B^2} - 1 \right) + 1 + \tau_3^d \alpha^L}{1 + \tau_3^d \alpha^L} = \frac{\tau'_2}{1 + \tau_3^d \alpha^L}.
\]

If \( 1 + \tau_3^d \alpha^L > 0 \), \( \tau'_2 < (>) 0 \) becomes a more (less) attractive policy option since it implies \( dY_3^3 < (>) 0 \), which is welfare improving (damaging) for individuals of type 3 since it means more (less) leisure.
4. Concluding Remarks

Our aim has not been to make an ethical case for the HE principle. As argued by Kaplow (1995), although HE is intuitively appealing, there is need for studies that try both to justify this principle and to derive a precise measure of equity from the justification. Our aim has rather been to investigate how the preferred tax mix might change if we were to take HE seriously.

The investigation makes clear that the HE principle may seriously affect the incentives for income and commodity taxation. The basic intuition and also the more well-established policy implications from models with heterogeneity in ability can turn out to be very misleading if we consider a more realistic setting that allows heterogeneity both in preferences and in abilities. Contrary to normal findings, we find that a good that is complementary to leisure need not be discouraged by the tax system, and that a good that is normally expected to be discouraged need not be taxed even if the consumption space consists of only two private goods and leisure. As for marginal tax rates, the direction of redistribution is crucial, as expected, but the introduction of the HE restriction complicates matters here as well. It is for instance possible to have for the high ability, hard-working type a positive marginal tax even though the self-selection constraint relating him to the high ability, epicurean type is binding upward.

Before concluding, we note some possible caveats. Since the individuals are held responsible for their preferences, a higher taste for leisure can in our model be interpreted as laziness. Obviously there are several alternative interpretations. Cuff (2000) discusses a high taste for leisure as a kind of disability. That interpretation might lead to very different implications. Whereas it is intuitive to argue that compensation for laziness should be ruled out, it is—at least in the framework of responsibility and compensation—less obvious that disabled people should not receive any compensation for the disability. The discussion clearly touches upon the concept of free will and whether preferences are to be treated as given or as acquired. Such questions cannot be addressed in passing; we confine ourselves to saying that besides the benefits associated with a focus on one of the polar cases, our findings are also relevant as long as the taste for leisure among some individuals is to some extent interpreted more as laziness than as a disability.

Moreover, the economic analysis of the family (Becker 1991, Cigno 1991) can be used to explain why certain individuals are apt to work longer hours. If production of certain goods and services can take place at home, individuals who are relatively more productive at home than at work will act as if they had a greater taste for leisure. Home production could mean the producing of substitutes to services available on the market (Kleven et al. 2000), or it could be child rearing (Balestrino et al. 2002). Apart from explaining why the labor market has a greater appeal to certain individuals, these studies also narrow down the set of goods that are candidates for relatively higher taxes. More than 50 years ago, Corlett and Hague (1953) suggested that efficiency
could be improved by increasing taxes on goods that are complementary to leisure. Yet their suggestion has not come to much since the relation between most goods and leisure is wrapped in mystery. This lack of information is, of course, just as problematic in our model. Therefore, models with home production provide promising inputs to extensions of our model which aim at more practically oriented policy implications.

Although our model is very simple and stylized, we hope that we have called attention to the relevance of the HE principle and some potential consequences for tax policy, once we relax the traditional assumption of homogeneity in individuals’ preferences. Without doubt there are prospects for more research in this relatively unexplored area of tax theory.

Appendix A: Derivation of Equation (1)

The f.o.c. for $B^1$, $B^2$, $B^3$, and $t$ are respectively given by

$$-\pi^1 V^1_B = \pi^1 \gamma \left( t \frac{\partial c^1}{\partial B^1} - 1 \right) - \lambda^d \gamma \frac{V^{2(1)}}{B^1}; \quad (A1)$$

$$-(\pi^2 + \lambda^d q) V^2_B = \gamma \pi^2 \left( t \frac{\partial c^2}{\partial B^2} - 1 \right) - \lambda^d \gamma \frac{V^{3(2)}}{B^2}; \quad (A2)$$

$$-\left( \pi^3 + \lambda^d \gamma \right) V^3_B = \pi^3 \gamma \left( t \frac{\partial c^3}{\partial B^3} - 1 \right); \quad (A3)$$

$$\pi^1 V^1_q + \pi^2 V^2_q + \pi^3 V^3_q + \lambda^d (V^2_q - V^{2(1)}_q) + \lambda^d \gamma (V^3_q - V^{3(2)}_q) + \gamma (\pi^1 c^1 + \pi^2 c^2)$$

$$+ \gamma \left[ \pi^3 c^3 + t \left( \frac{\partial c^1}{\partial q} \pi^1 + \frac{\partial c^2}{\partial q} \pi^2 + \frac{\partial c^3}{\partial q} \pi^3 \right) \right] = 0. \quad (A4)$$

Applying Roy’s identity, Equation (A4) becomes

$$-\pi^1 V^1_B - \pi^2 c^2 V^2_B - \pi^3 c^3 V^3_B - \lambda^d c^2 V^2_B - \lambda^d c^3 V^3_B + \lambda^d c^{(1)} V^{2(1)}_B$$

$$+ \lambda^d c^{(2)} V^{3(2)}_B + \gamma \left[ \pi^1 c^1 + \pi^2 c^2 + \pi^3 c^3 + t \left( \frac{\partial c^1}{\partial q} \pi^1 + \frac{\partial c^2}{\partial q} \pi^2 + \frac{\partial c^3}{\partial q} \pi^3 \right) \right] = 0. \quad (A5)$$

Using (A1), (A2), and (A3), we can rewrite (A5) as

$$\gamma \pi^1 c^1 \left( t \frac{\partial c^1}{\partial B^1} - 1 \right) + \gamma \left[ c^2 \pi^2 \left( t \frac{\partial c^2}{\partial B^2} - 1 \right) + c^3 \pi^3 \left( t \frac{\partial c^3}{\partial B^3} - 1 \right) \right]$$

$$- c^1 \lambda^d V^{2(1)}_B - c^2 \lambda^d V^{3(2)}_B + \lambda^d c^{(1)} V^{2(1)}_B + \lambda^d c^{(2)} V^{3(2)}_B$$

$$+ \gamma \left( \pi^1 c^1 + \pi^2 c^2 + \pi^3 c^3 + t \frac{\partial c^1}{\partial q} \pi^1 \right) + \gamma t \left( \frac{\partial c^2}{\partial q} \pi^2 + \frac{\partial c^3}{\partial q} \pi^3 \right) = 0. \quad (A6)$$
Equation (1) is obtained using the Slutsky decomposition in (A6) and simplifying terms.

**Appendix B: The Marginal Effective Tax Rates**

The f.o.c. for \( Y_1, Y_2, \) and \( Y_3 \) are respectively given by

\[
\pi^1 V^1_3 \frac{\alpha^L}{w^L} = \lambda^d_2 V^2_3 \frac{\alpha^H}{w^H} - \pi^1 y \left( 1 + tc^1_3 \frac{\alpha^L}{w^L} \right); \tag{B1}
\]
\[
\left( \pi^2 + \lambda^d_2 \right) V^2_3 \frac{\alpha^H}{w^H} = \lambda^d_3 V^3_3 \frac{\alpha^L}{w^L} - \pi^2 y \left( 1 + tc^2_3 \frac{\alpha^H}{w^H} \right); \tag{B2}
\]
\[
\left( \pi^3 + \lambda^d_3 \right) V^3_3 \frac{\alpha^L}{w^H} = -\pi^3 y \left( 1 + tc^3_3 \frac{\alpha^L}{w^L} \right). \tag{B3}
\]

To find the METR faced by type 1, divide (B1) by (A1) and multiply the result by \( \pi^1 y \left( \frac{\partial c^1_1}{\partial B^1} - 1 \right) - \lambda^d_2 V^2(1)_B \). This gives

\[
\frac{\alpha^L}{w^L} V^1_3 \left[ \lambda^d_2 V^2(1)_B - \pi^1 y \left( t \frac{\partial c^1_1}{\partial B^1} - 1 \right) \right] = \lambda^d_2 V^2(1)_B \frac{\alpha^H}{w^H} - \pi^1 y \left( 1 + tc^1_3 \frac{\alpha^L}{w^L} \right).
\]

Using notation \( \overline{w^1} = \frac{w^L}{\alpha^L}, \overline{w^2} = \frac{w^H}{\alpha^H}, \overline{w^3} = \frac{w^L}{\alpha^L}, \) and \( \Omega^{1,2} = \frac{\overline{w^1}}{\overline{w^2}} \) we get

\[
\tau^1_1 = \frac{\lambda^d_2 V^2(1)_B}{\gamma \pi^1} \frac{1}{\overline{w^1}} \left( \frac{V^2(1)_B}{V^2_B} \Omega^{1,2} - \frac{V^1_3}{V^1_B} \right).
\]

Since single-crossing holds and \( \frac{\alpha^L}{w^L} > \frac{\alpha^H}{w^H} \) implies that \( \Omega^{1,2} < 1 \), the METR faced by type 1 is positive.

Similarly, for the METR faced by type 2, divide (B2) by (A2) and multiply the result by \( \gamma \pi^2 y \left( \frac{\partial c^2_2}{\partial B^2} - 1 \right) \). This gives

\[
\frac{\alpha^H}{w^H} V^2_3 \left[ -\gamma \pi^2 \left( t \frac{\partial c^2_2}{\partial B^2} - 1 \right) + \frac{\alpha^H}{w^H} \right] = -\gamma \pi^2 \left( 1 + tc^2_3 \frac{\alpha^H}{w^H} \right) + \lambda^d_3 V^3(2)_B \frac{\alpha^L}{w^H},
\]

which can be rewritten as

\[
\gamma \pi^2 \left[ 1 + tc^3_3 \frac{\alpha^H}{w^H} + \frac{\alpha^H}{w^H} \frac{V^2_3}{V^3_B} \left( 1 - t \frac{\partial c^2_2}{\partial B^2} \right) \right] = \lambda^d_3 V^3(2)_B \left( \frac{V^3(2)_B}{V^3_B} \frac{\alpha^L}{w^H} - \frac{V^2_3}{V^2_B} \frac{\alpha^H}{w^H} \right).
\]

Using the definition of METR \( (\tau^1 = 1 + tc^1_3 \frac{\alpha^L}{w^L} + \frac{\alpha^L}{w^L} \frac{V^1_3}{V^1_B} (1 - t \frac{\partial c^1_1}{\partial B^1}) ) \) and rearranging, we get

\[
\tau^2 = \frac{\lambda^d_3 V^3_B}{\gamma \pi^2 w^H} \left( \frac{V^3(2)_B}{V^3_B} \frac{\alpha^L}{w^H} - \frac{V^2_3}{V^2_B} \frac{\alpha^H}{w^H} \right),
\]

which again gives a positive value for \( \tau^2_2 \).
Finally, to obtain the METR faced by type 3, divide (B3) by (A3) and multiply the result by $\pi^3 \gamma \left( \frac{\pi^2}{\pi^2 + \pi^3} \right)$. This gives $\frac{\pi^3 \gamma (1 + t + \pi^3)}{\pi^2 (\pi^2 + \pi^3)}$. Using the definition of METR (Equation (3)) and rearranging terms, we get $\tau_3' = 0$.

Appendix C: Derivation of Equation (9)

The f.o.c. for $B^1$, $B^2$, $B^3$, and $t$ are, respectively,

$$-\pi^1 V^1_B = \pi^1 \gamma \left( t \frac{\partial c}{\partial B^1} - 1 \right) - \lambda_2^d V^{2(1)}_B;$$ (C1)

$$-(\pi^2 + \lambda_2^d + \lambda_2^c) V^2_B = (\pi^3 + \lambda_2^d) V^3_B \frac{\alpha^L}{\partial H} \frac{1}{1 + t c_3^3 \alpha^L} - \gamma (\pi^2 + \pi^3)$$

$$+ \gamma (\pi^2 + \pi^3) t \frac{\partial c}{\partial B^2} - \lambda_2^d V^{2(2)}_B - \lambda_2^c V^{2(3)}_B \frac{\alpha^H}{\partial H} \frac{1}{1 + t c_3^3 \alpha^L};$$ (C2)

$$-(\pi^3 + \lambda_2^d) V^3_B = -(\pi^3 + \lambda_2^d) V^3_B \frac{\alpha^L}{\partial H} \frac{1}{1 + t c_3^3 \alpha^L} + \lambda_2^d V^{2(3)}_B \frac{\alpha^H}{\partial H} \frac{1}{1 + t c_3^3 \alpha^L}$$

$$- \lambda_2^u \left( V^{2(3)}_B + \frac{V^{2(3)}_B \alpha^H}{\partial H} \frac{1}{1 + t c_3^3 \alpha^L} \right);$$ (C3)

$$\pi^1 V^1_q + \pi^2 V^2_q + \pi^3 \left\{ V^3_q + V^3_q \frac{\alpha^L}{\partial H} \frac{1}{1 + t c_3^3 \alpha^L} \right\}^{-1} \left[ c^2 - c^3 + t \left( \frac{\partial c^2}{\partial q} - \frac{\partial c^3}{\partial q} \right) \right]$$

$$+ \gamma \left[ \pi^1 c^1 + (\pi^2 + \pi^3)^2 + t \left( \frac{\partial c^1}{\partial q} \pi^1 + \frac{\partial c^2}{\partial q} (\pi^2 + \pi^3) \right) \right] + \lambda_2^d \left( V^2_q - V_q^{2(1)} \right)$$

$$+ \lambda_2^u V^2_q - \lambda_2^u \left\{ V^{2(3)}_q + V^{2(3)}_q \frac{\alpha^H}{\partial H} \frac{1}{1 + t c_3^3 \alpha^L} \right\}^{-1} \left[ c^2 - c^3 + t \left( \frac{\partial c^2}{\partial q} - \frac{\partial c^3}{\partial q} \right) \right]$$

$$+ \lambda_3^d V^3_q - \lambda_3^d V^{2(2)}_q + \lambda_3^d V^{2(3)}_q \frac{\alpha^L}{\partial H} \frac{1}{1 + t c_3^3 \alpha^L} \left[ c^2 - c^3 + t \left( \frac{\partial c^2}{\partial q} - \frac{\partial c^3}{\partial q} \right) \right] = 0.$$ (C4)
Applying Roy’s identity, f.o.c. \( t \) becomes

\[
-\pi^1 c^1 V^1_B - \pi^2 c^2 V^2_B - \pi^3 c^3 V^3_B - \lambda^d_2 c^2 V^2_B - \lambda^u_2 c^2 V^2_B \\
-\lambda^d_3 c^3 V^3_B + \gamma \left( \pi^1 c^1 + \pi^2 c^2 \right) + \gamma t \\
\times \left[ \frac{\partial c^1}{\partial q} \pi^1 + \frac{\partial c^2}{\partial q} \left( \pi^2 + \pi^3 \right) \right] + \left( \pi^3 + \lambda^d_3 \right) V^3_B \frac{\alpha^L}{w^H} \right]
\]

\[
\times \left[ c^2 - c^3 + t \left( \frac{\partial c^2}{\partial q} - \frac{\partial c^3}{\partial q} \right) \right] \frac{1}{1 + t c^3_3 \frac{\alpha^L}{w^H}}
\]

\[
-\lambda^u_2 V^2(3) \frac{\alpha^H}{w^H} \left[ c^2 - c^3 + t \left( \frac{\partial c^2}{\partial q} - \frac{\partial c^3}{\partial q} \right) \right] \frac{1}{1 + t c^3_3 \frac{\alpha^L}{w^H}}
\]

\[
+\lambda^d_3 c^3(2) V^3_B + \gamma \pi^3 c^2 = 0. \tag{C5}
\]

Using (C1), (C2), and (C3), we can rewrite (C5) as

\[
\gamma \pi^1 c^1 \left( \frac{t \frac{\partial c^1}{\partial B^1}}{1} - 1 \right) - c^1 \lambda^d_2 V^2(1) + \left( \pi^3 + \lambda^d_3 \right) c^2 V^3_B \frac{\alpha^L}{w^H} \right]
\]

\[
\times \left( \frac{t \frac{\partial c^2}{\partial B^2}}{1} - 1 \right) \frac{1}{1 + t c^3_3 \frac{\alpha^L}{w^H}} + \gamma c^2 \left( \pi^2 + \pi^3 \right) \left( \frac{t \frac{\partial c^2}{\partial B^2}}{1} - 1 \right)
\]

\[
-\lambda^d_3 c^3 V^3_B + c^3 \lambda^u_2 V^3(3) \frac{\alpha^H}{w^H} \left( \frac{t \frac{\partial c^2}{\partial B^2}}{1} - 1 \right) \frac{1}{1 + t c^3_3 \frac{\alpha^L}{w^H}}
\]

\[
+\left( \pi^3 + \lambda^d_3 \right) c^3 V^3_B \frac{\alpha^L}{w^H} \left( 1 - t \frac{\partial c^3}{\partial B^3} \right) \frac{1}{1 + t c^3_3 \frac{\alpha^L}{w^H}}
\]

\[
-\lambda^u_2 \left( V^2(3) + V^3(3) \frac{\alpha^H}{w^H} \frac{1}{1 + t c^3_3 \frac{\alpha^L}{w^H}} \right) + c^3 \lambda^u_2 V^3(3)
\]

\[
\times \frac{\alpha^H}{w^H} \frac{1}{1 + t c^3_3 \frac{\alpha^L}{w^H}} + \gamma \left[ \pi^1 c^1 + \left( \pi^2 + \pi^3 \right) c^2 \right.
\]

\[
+ t \left( \frac{\partial c^1}{\partial q} \pi^1 + \frac{\partial c^2}{\partial q} \left( \pi^2 + \pi^3 \right) \right] + \lambda^d_2 c^2(1) V^2(1)
\]
\[ + \lambda_2^u c_{2(3)}^2 V_B^2 - \lambda_2^u V_3^2(3) \frac{\alpha_H^L}{w^H} \left[ c^2 - c^3 \right] \]

\[ + t \left( \frac{\partial c^2}{\partial q} - \frac{\partial c^3}{\partial q} \right) \frac{1}{1 + tc_3^3 \frac{\alpha_H^L}{w^H}} + \lambda_3^d c_{3(2)}^3 V_B^3(2) \]

\[ + \left( \pi_3 + \lambda_3^d \right) V_3^3 \frac{\alpha_L}{w^H} \left[ c^2 - c^3 + t \left( \frac{\partial c^2}{\partial q} - \frac{\partial c^3}{\partial q} \right) \right] \frac{1}{1 + tc_3^3 \frac{\alpha_H^L}{w^H}} = 0. \]

Equation (9) is obtained using the Slutsky decomposition and simplifying terms.

**Appendix D: Proof of Proposition 1**

Considering the “normal” case when redistribution is directed toward the low-skilled individuals and \( \lambda_3^u = 0 \), the f.o.c. of the government’s problem w.r.t. gross incomes \( Y^1 \) and \( Y^2 \) are, respectively,

\[ \pi^1 V_3^1 \frac{\alpha_L}{w^L} = \lambda_2^d V_3^2(1) \frac{\alpha_H}{w^H} - \pi^1 \gamma \left( 1 + tc_3^3 \frac{\alpha_L}{w^L} \right); \quad (D1) \]

\[ \pi^2 V_3^2 \frac{\alpha_H}{w^H} + \pi^3 V_3^3 \frac{\alpha_L}{w^H} \frac{dY^3}{dY^2} + \gamma \left( \pi^2 + \pi^3 \right) \left( 1 + tc_3^2 \frac{\alpha_H}{w^H} \right) + \lambda_2^d V_3^2 \frac{\alpha_H}{w^H} \]

\[ + \lambda_2^u V_3^2(3) \frac{\alpha_H}{w^H} \frac{dY^3}{dY^2} + \lambda_3^d V_3^3 \frac{\alpha_L}{w^H} \frac{dY^3}{dY^2} - \lambda_3^d V_3^3(2) \frac{\alpha_L}{w^H} = 0. \quad (D2) \]

Making use of Equation (7), Equation (D2) becomes

\[ \left( \pi^2 + \lambda_2^u + \lambda_2^d \right) V_3^2 \frac{\alpha_H}{w^H} = - \left( \pi^3 + \lambda_3^d \right) V_3^3 \frac{\alpha_L}{w^H} \frac{1 + tc_3^3 \frac{\alpha_H}{w^H}}{1 + tc_3^3 \frac{\alpha_L}{w^H}} - \gamma \left( \pi^2 + \pi^3 \right) \]

\[ \times \frac{tc_3^3 \frac{\alpha_H}{w^H} - \gamma \left( \pi^2 + \pi^3 \right) + \lambda_2^d V_3^2(3) \frac{\alpha_H}{w^H} \frac{1 + tc_3^2 \frac{\alpha_H}{w^H}}{1 + tc_3^3 \frac{\alpha_L}{w^H}} + \lambda_3^d V_3^3(2) \frac{\alpha_L}{w^H}. \quad (D3) \]

For (a), notice that (D1) = (B1) and (C1) = (A1) and then follow the argument put forward in Appendix B to show that the METR faced by type 1 is positive.

For (b), we first need expressions for the METR faced by types 2 and 3. Starting with type 2, we divide (D3) by (C2) and multiply
the result by \((\pi^3 + \lambda_3^d) V_B^3 \alpha^L \frac{\partial c^2}{\partial B^2} (t \frac{\partial c^2}{\partial B^2} - 1) \frac{1}{1 + t c^3} \alpha^L \) + \(\gamma (\pi^2 + \pi^3) (t \frac{\partial c^2}{\partial B^2} - 1) - \lambda_3^d V_B^{3(2)} - \lambda_2^u V_3^{2(3)} \alpha^L \frac{\partial c^2}{\partial B^2} (t \frac{\partial c^2}{\partial B^2} - 1) \frac{1}{1 + t c^3} \alpha^L \). This gives

\[
\frac{\alpha^H V_B^2}{w^H V_B^2} \left[ - (\pi^3 + \lambda_3^d) V_3^3 \alpha^L \frac{\partial c^2}{\partial B^2} (t \frac{\partial c^2}{\partial B^2} - 1) \frac{1}{1 + t c^3} \alpha^L \right] + \gamma (\pi^2 + \pi^3) + \lambda_2^u V_3^{2(3)}
\]

\[
\times \left[ \frac{\alpha^H V_B^2}{w^H V_B^2} \frac{\partial c^2}{\partial B^2} \frac{1}{1 + t c^3} \alpha^L \right] - \gamma (\pi^2 + \pi^3) \frac{\lambda_3^d V_3^{3(2)} \alpha^L}{w^H} \frac{1}{1 + t c^3} \alpha^L
\]

\[
+ \lambda_3^d V_3^{3(2)} \frac{\alpha^L}{w^H} - \gamma (\pi^2 + \pi^3) t c^3 \frac{\alpha^H}{w^H}.
\]

Manipulating this expression we get

\[
\left[ \frac{1 + t c^3 \alpha^H}{w^H} + \frac{\alpha^H V_B^2}{w^H V_B^2} \left( 1 - t \frac{\partial c^2}{\partial B^2} \right) \right] \times \left[ \frac{(\pi^3 + \lambda_3^d) V_3^3 \alpha^L}{w^H} - \lambda_2^u V_3^{2(3)} \frac{\alpha^H}{w^H} + \gamma (\pi^2 + \pi^3) \right]
\]

\[
= \lambda_3^d V_B^{3(2)} \left( \frac{V_3^{3(2)} \alpha^L}{w^H} - \frac{V_B^2 \alpha^H}{V_B^2 \alpha^H} \right).
\]

Using Equation (3) and rearranging, we get

\[
\tau'_2 = \frac{\lambda_3^d V_B^{3(2)} \left( \frac{V_3^{3(2)} \alpha^L}{V_B^{3(2)}} \alpha^L - \frac{V_B^2 \alpha^H}{V_B^2 \alpha^H} \right)}{\gamma w^H (\pi^2 + \pi^3) + \frac{1}{1 + t c^3} \left[ (\pi^3 + \lambda_3^d) V_3^3 \alpha^L - \lambda_2^u V_3^{2(3)} \alpha^H \right]} .
\](D4)
To obtain the METR faced by type 3, note that from Equation (C3) we have that

\[
-1 = \frac{V_3^3 \alpha^L}{V_B^3 w^H} \left( 1 - t \frac{\partial c^3}{\partial B^3} \right) - \frac{\lambda^u_2}{\lambda^d_2} \left( \frac{V_B^{2(3)}}{w^H} \left( 1 - t \frac{\partial c^3}{\partial B^3} \right) \right).
\]

Multiplying by \(1 + tc^3_3 \alpha^L\) and rearranging terms gives

\[
\frac{V_3^3 \alpha^L}{V_B^3 w^H} \left( 1 - t \frac{\partial c^3}{\partial B^3} \right) + 1 + tc^3_3 \alpha^L = \frac{\lambda^u_2}{\lambda^d_2} \frac{V_B^3}{V_3^v(3)}
\]

\[
\times \left( \frac{V_B^{2(3)}}{w^H} + \frac{V_3^{2(3)}}{w^H} \left( 1 - t \frac{\partial c^3}{\partial B^3} \right) \right) + tc^3_3 \alpha^L \frac{\lambda^u_2}{\lambda^d_2} \frac{V_B^3}{V_3^v(3)}
\]

\[
\times \left( \frac{V_B^{2(3)}}{w^H} + \frac{V_3^{2(3)}}{w^H} \left( 1 - t \frac{\partial c^3}{\partial B^3} \right) \right).
\]

(D5)

Using Equation (3), Equation (D5) can be written

\[
\tau'_3 = \frac{\lambda^u_2 V_B^{2(3)}}{(\lambda^d_3 + \lambda^u_3) V_B^3} \left( 1 + \frac{V_3^{2(3)}}{V_B^{2(3)}} \frac{\alpha^H}{w^H} \left( 1 - t \frac{\partial c^3}{\partial B^3} \right) \right) \left( 1 + tc^3_3 \alpha^L \frac{\lambda^u_2}{\lambda^d_2} \right).
\]

(D6)

Since we already noticed that it is not possible that \(\lambda^d_3\) and \(\lambda^u_3\), which enter the expressions for \(\tau'_2\) and \(\tau'_3\) multiplicatively, are both binding or slack at the same time, part (b) of the Proposition follows from Equations (D4) and (D6).

References


