Strategic Investment and Market Integration

by Mattias Ganslandt
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Abstract

The competitive effect of international market integration in industries with imperfect competition is of great policy interest. This paper focuses on the link between monopolization and market segmentation. It presents a model of multi-market entry deterrence with or without market commitments. We derive sufficient conditions for entry deterrence with productive capacity in the multi-market game. It is shown that to deter entry in the multi-market game, the first-mover installs productions capacity which is strictly larger than the capacity needed to deter entry, if it is possible to assign parts of the capacity to specific markets. Market integration for production capacity may, thus, have a pro-competitive effect in international markets.

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1 Introduction

Established firms can restrict or prevent competition, due to first-mover advantages.\(^1\) Despite the fact that most industrialized countries have regulations against monopolization, recent empirical evidence suggests that entry deterrence is common business practice.\(^2\)

This paper considers a model where established firms can invest in capacity to the extent that entry by other firms is deterred. While previous studies have characterized entry deterrence in a single market, this paper analyzes entry deterrence in a multi-market game.\(^3\)

The crucial condition for strategic entry deterrence is that the incumbent can make early decisions, in order to restrict its future freedom of action. While this might be possible in the single-market game, the conditions may change when firms compete in many markets. Even if the cost of capacity is sunk, the multi-market incumbent can redistribute some of its capacity from markets with competition, to markets without. Thus, the firm maintains some degrees of freedom when acting in more than one market.

This paper is therefore based on two sets of questions: What is the scope for an incumbent to exploit its first-mover advantage in a multi-market game? Does an incumbent firm have an incentive to make a commitment to a specific market, in order to prevent competition in that market, even if such commitment is costly?

If multi-market competition facilitates entry-deterrence, it should be expected that integrated markets are more concentrated than segmented markets. On the other hand, if the opposite holds and multi-market competition obstructs the incumbent’s possibilities to restrict competition,


\(^2\)For a summary of different features of national competition laws in industrial countries, see OECD (1996), and for empirical evidence on strategic entry deterrence, see Smiley (1988), Bunch and Smiley (1992), and Allen et al (1995). It should be noted, however, that American case law has placed a heavy burden on plaintiffs to prove that a capacity expansion is clearly meant to hurt competitors and harm competition, which would be the case if such conduct were to be considered illegal (see Dobson et al, 1994).

\(^3\)Entry deterrence through capacity investment in single market games was first analyzed in Spence (1977) and Dixit (1980).
integrated markets should be expected to be less concentrated. Hence, the issue of market-linkages is important for any theory of market integration.

This paper considers a market situation described as a multi-stage game, where the incumbent first selects a global capacity, then competes with a number of entrants determined at the local level. In this respect, this model differs from most previous studies of multi-market interaction, where it is often assumed that firms are allowed to make decisions at the multi-market level exclusively, referred to as the integrated market hypothesis, or at the local level, referred to as the segmented market hypothesis.4

In the model presented in this paper, each firm is assumed to exhibit a symmetric Leontief technology with a fixed unit-cost of production. Furthermore, demand is considered to be independent between markets and firms compete in strategic substitutes in the last stage of the game. The incumbent firm is free to redistribute its global capacity between different markets. Hence, there is a strategic link between different markets.5

The possibility to redistribute global capacity between markets makes entry-deterrence more difficult and more costly than in a single-market game. To deter entry, the multi-market firm must install capacity beyond the level required in a single-market game.6 Interestingly, the per-market capacity installed to deter entry can be strictly larger than the largest subgame-perfect investment in the single-market game. However, no capacity will be left idle in equilibrium.7

4Venables (1990) and Ben-Zvi and Helpman (1992) are two exceptions. In their models, capacity decisions are made on an integrated basis and other decisions, e.g. price and sales decisions, on a national basis. The model in this paper closely resembles Venables’ as well as Ben-Zvi and Helpman’s models in its attempt to analyze the importance of investment when capacity can be used on a multi-market level, while sales decisions are taken on a local basis.

5See Witteloostuijn and Wegberg (1992), for an extensive summary on multi-market competition models where existing firms are potential entrants. In particular, Bulow, Geanakoplos and Klemperer (1985) present a multi-market model relating to our analysis. They study a multi-market game where two firms compete in one market, but where one of the firms is a monopolist in a second market. If the two markets exhibit joint economies, then a positive shock in one market has positive effects on entry deterrence in the other market, provided that the products are strategic substitutes or strategic complements. In our model, however, the unit-cost is fixed and Bulow, Geanakoplos and Klemperer’s analysis does not apply.

6This paper is not concerned with the relative profitability of entry deterrence and accommodation. In Ganslandt (1997), it has been shown that entry deterrence is profitable, if sufficient conditions are satisfied.

7It should be noted that these results do generally not hold. In a similar two-firm, two-stage game with iso-elastic
In an extension of the model, it is demonstrated that the results also hold for strategic comple-
ments, if sufficient conditions apply. It is concluded that in many reasonable cases, the incumbent
is obliged to install extra capacity in order to deter entry in the multi-market game.

If the capacity that would deter entry is beyond the monopoly output, the multi-market incum-
bent has an incentive to induce market segmentation. In particular, the incumbent may induce
market segmentation through bundling of products and services. Firms can bundle their tradable
products with locally produced and consumed nontradables. If the product cannot be used with-
out local services, the capacity is assigned to the local market, provided that the marginal cost of
expanding the local capacity of services in other markets is sufficiently high. In this respect, these
results relate to Horn and Shy (1996), where market segmentation is endogenously determined
through bundling of tradables with nontradables.

The paper is organized as follows. Section 2 introduces four versions of the multi-market
game. Section 3 is devoted to the first version of the game, which is similar to Selten’s (1978)
chain store game. In this version, a multi-market firm competes sequentially with several potential
entrants in distinct markets. Section 4 studies the second version of the multi-market game, where
the incumbent competes with \( n \) firms simultaneously, after the capacity choice has been made.
Section 5 deals with the third version, where the multi-market firm competes with a second large
player, which is a potential entrant in all \( n \) markets. Section 6 introduces market commitments and
analyzes under what circumstances the incumbent will serve markets from a single multi-market
plant as opposed to many local plants. Section 7 shows that our main result holds if firms compete
in strategic complements, if sufficient conditions apply. Section 8 illustrates three applications and
section 9 concludes.

2 Multi-Market Entry Deterrence

Four versions of a multi-market game are considered. In the first three versions, production capacity
is assumed to be used at the multi-market level. The incumbent is not allowed to assign parts of its
demand, the incumbent will hold excess capacity which is idle and will be utilized only in the event of entry. This
result is easily shown in a simple model, originally set up by Bulow, Geanakopolos and Klemperer (1985). Similar
results with multiple incumbent firms are shown by Barham and Ware (1993).
total capacity to local markets. Instead, capacity can be redistributed between different markets, without additional costs. The first three cases differ with respect to potential competition and timing.

In the first version of the multi-market game, analyzed in section 3, an incumbent meets sequential competition from local entrants. The sequential structure is plausible when firms independently try to specify a certain product. They consider entry as soon as the product specification is correct and they have raised enough money for local production. This first happens to firm one, then to firm two etc. In this first version of the game, it is assumed that potential entrants only consider local entry. One rational for this assumption is that the firm has to succeed in its domestic market before it can raise money for multi-market expansion.

In the second version of the multi-market game, analyzed in section 4, the incumbent faces simultaneous competition from local entrants. The simultaneous structure arises when the incumbent owns a global patent expiring at the same time in all local markets. In this case, local competitors already have a correct specification of the product. As soon as the patent expires, they immediately consider entry in the local market. In the second version of the game, the assumption that potential competitors only consider local entry is maintained.

In the third version of the multi-market game, analyzed in section 5, the incumbent faces simultaneous competition from a single multi-market competitor in all markets. This market structure is plausible if the first competitor to finish the process of product specification immediately considers a multi-market strategy, or if a global patent expires in all markets simultaneously and the potential entrant can raise enough money for multi-market entry.

After the analysis of the first three versions of the multi-market game, the assumptions about the incumbent’s possibilities to restrict competition are changed. In the fourth version of the game, analyzed in section 6, the incumbent is allowed to assign parts of its capacity to local markets. The choice of a certain production organization is a trade-off between the cost of entry-deterrence with the multi-market capacity and the cost of market assignments.
3 Sequential Competition from Local Entrants

A multi-market firm, type $m$, has advertised its product and now meets demand for its product in $n$ markets, numbered 1 to $n$. In each market, there is a potential entrant, type $e$, who might raise enough funding from creditors to establish a firm in market $t$, selling the same product as the multi-market enterprise.

Entry in a local market is associated with a fixed cost $A$, which can be considered an advertising cost, that makes consumers in the local market aware of the entrant. Advertising makes all consumers in the market aware of the firm and its products, but does not affect aggregate demand for the homogenous goods. There is no personal arbitrage, since consumers are only aware of firms advertising in their home market. Accordingly, prices need not be internationally equalized.

In the first version of the multi-market game, we focus on a situation where each potential competitor considers advertising in a single market only and, consequently, intends to remain local. At the beginning of the game none of the potential entrants has a sufficiently correct specification for starting production. But as time passes, one after another, they finish the process of specification and raise enough credit to enter the local market. This will first happen to entrant 1, then to entrant 2, etc. As soon as a player has specified the product correctly, he must decide to enter or stay out of the market. If he decides to stay out, he is no longer a potential competitor.\footnote{This assumption is made to simplify the analysis. It is not restrictive. Indeed, it can be shown that a potential entrant will not benefit from delaying its entry decision.}

If a local firm enters a market, the incumbent and the entrant choose outputs simultaneously and the market clears as a duopoly. If the potential entrant stays out, monopoly will prevail.

After this description of the market situation in the first version of the multi-market game, we turn to a formal specification of the model. The game, $\Gamma^1_n$, has $n + 1$ players, player $m$ and player $1, ..., n\ (n \geq 1)$. There are $n$ separate markets, labelled $1, ..., n$. The game is played over a sequence of periods $0, ..., n$. In period 0, the incumbent, player $m$, must choose a pre-entry capacity $k$, which is immediately announced to all players. At the beginning of period $t = 1, ..., n$, player $t$ decides to enter or stay out of market $t$. Player $t$’s decision is announced to all players. If player $t$ decides to enter, player $m$ and player $t$ will choose $x^m_t$ and $x^e_t$ simultaneously, where subscripts refer to
markets and superscripts to firm-type. If player \( t \) decides to stay out of market \( t \) monopoly will prevail in that market. The output decision is immediately announced to all players. At the end of period \( t \), the market clears and payoffs are distributed to player \( m \) and player \( t \). Next, for \( t = 1, \ldots, n-1 \) period \( t+1 \) begins and is played according to the same rules. The game ends after period \( n \).

Player \( m \)'s payoff is the sum of \( n \) partial payoffs for \( t = 1, \ldots, n \). Player \( m \)'s revenue in market \( t \) is \( v(x^m_t, x^e_t) \). The cost of capital is additive and the marginal cost is \( c > 0 \). The objective of player \( m \) is to maximize its total payoff:

\[
\pi^m(k, x^m_1, \ldots, x^m_n, x^e_1, \ldots, x^e_n) = \sum_{t=1}^{n} v(x^m_t, x^e_t) - ck
\]  

and it is required that \( x^m_1 + \ldots + x^m_n \leq k \). Setting up a firm, i.e. entering market \( t \), is associated with a fixed cost \( A > 0 \) for player \( t \). Player \( t \)'s revenue is \( v(x^e_t, x^m_t) \). Marginal capital cost is \( c > 0 \) and additive. The objective of player \( t = 1, \ldots, n \) is to maximize its payoff:

\[
\pi^e(x^e_t, x^m_t) = \begin{cases} 
  v(x^e_t, x^m_t) - cx^e_t - A & \text{if it enters} \\
  0 & \text{if it stays out}
\end{cases}
\]  

Next, we introduce some notation before proceeding with the analysis. I will define strategic substitutes, introduce a necessary and sufficient condition on entry-deterrence and define the deterrence level.

We shall call \( x^i_t \) a strategic substitute for \( x^j_t \), if the partial cross-derivative of the profit function with respect to the strategic variables is strictly negative. Strategic substitutes imply that when a firm has a more aggressive strategy, the optimal response of the other firm is to play less aggressively. The condition that \( x^i_t \) is a strategic substitute for \( x^j_t \) is referred to as \( S \):

\[
(S) \quad \frac{\partial^2 \pi(x^i_t, x^j_t)}{\partial x^i_t \partial x^j_t} < 0
\]

Second, a best-reply function with a non-binding capacity restriction on player \( m \) in the one-period game \( \Gamma^1_1 \), denoted \( \beta^m(x^1_t) \), is introduced. Correspondingly, the entrant’s best reply function
is denoted $\beta^e (x^m_1)$. The best reply functions $\beta^m (x^e_1)$ and $\beta^e (x^m_1)$ are implicitly defined by

$$\frac{\partial v (\beta^m (x^e_1), x^e_1)}{\partial x^m_1} = 0, \quad \frac{\partial v (x^m_1, \beta^e (x^m_1))}{\partial x^e_1} - c = 0 \tag{3}$$

If a potential competitor decides to enter in period 1, this gives the following Nash equilibrium, when the capacity constraint is non-binding for the incumbent: $\{x^m_1, x^e_1\}$, where $x^m_1 = \beta^m (x^e_1)$, $x^e_1 = \beta^e (x^m_1)$. If $k \leq x^m_1$, the incumbent will use the entire capacity, but with $k > x^m_1$ some capacity will be left idle. The unique Nash equilibrium in the subgame with entry is $\{\hat{x}^m_1 (k), \hat{x}^e_1\}$ where $\hat{x}^m_1 (k) = \min \{k, x^m_1\}$ and $\hat{x}^e_1 = \beta^e (\hat{x}^m_1 (k))$.

In the second subgame in the second stage, with no entry, we obtain the following Nash equilibrium, when the capacity constraint is non-binding for the incumbent: $\{x^m_1, 0\}$, where $x^m_1 = \beta^m (0)$. Thus, $x^m_1$ is the monopoly level the incumbent would choose, if the cost of capacity was sunk and capacity did not restrict output.

When the firms compete in strategic substitutes, the potential entrant’s profit is decreasing in the incumbent’s output. However, the incumbent does not choose an output above the limit $x^m_1$, if the potential competitor enters the local market. Thus, under condition (S), it is a necessary condition for entry deterrence that the profit of the potential entrant is non-positive in a Nash equilibrium with a non-binding capacity restriction for the incumbent. This condition will be denoted D:

$$(D) \quad v (x^m_1, x^m_1) - c x^m_1 - A \leq 0$$

If the necessary deterrence condition D is satisfied, condition S is a sufficient condition for entry deterrence. However, it can easily be shown that S is not a necessary condition for the result. In particular, the result can hold, even if the strategic variables are strategic complements.

If D is satisfied and player $t$ would earn a positive profit as a monopoly it follows from the Theorem of Intermediate Values that the profit of the potential entrant must be equal to zero at some positive level of output by the incumbent. This deterrence level will be denoted $\bar{x}$ and defined:

$$\pi (\beta^e (\bar{x}), \bar{x}) - c \beta^e (\bar{x}) - A = 0 \tag{4}$$

Thus, if the established firm successfully commits to an output $\hat{x}$, it deters entry. It is also
assumed that $\hat{x}$ is above the output level of a natural monopoly. In other words, the entry-deterring incumbent in our model is operating beyond the scale of operation it would choose, if it did not face potential entry.

Next, three results from the first version of the multi-market game, $\Gamma^1_n$, can be shown. First, D is a sufficient condition on entry deterrence in the multi-market game. Second, if firms compete in strategic substitutes, then D is not only a sufficient, but a necessary, condition for entry deterrence. Third, if both conditions S and D are satisfied, the incumbent installs strictly more than $n \cdot \hat{x}$ to deter entry in $\Gamma^1_n$.

If D is satisfied the local entrant does not earn a positive profit in $\{\bar{x}_m^t, \bar{x}_e^t\}$, and would thus stay out of the local market. To see that D is a sufficient condition for entry deterrence, assume that the incumbent has installed more capacity in period 0 than he will ever use. Thus, every market can be treated independently and the unique Nash equilibrium in every market $t$ is $\{\bar{x}_m^t, \bar{x}_e^t\}$, and entry deterrence is thus possible.

**Proposition 1** If D is satisfied, then entry deterrence is possible in $\Gamma^1_n$.

**Proof.** ($D \Rightarrow$ entry deterrence is possible). Let the pre-commitment capacity be very large. The capacity constraint is not binding in any subgame. The objective of the incumbent is to maximize its profit with respect to $x_m^t$, for all $t$,

$$\frac{\partial v(x_m^t, x_e^t)}{\partial x_m^t} = 0 \quad \forall t.$$  

This problem is additively independent and each market can be considered as a separate one-market game $\Gamma^1_1$. If the capacity constraint is not binding, the unique Nash equilibrium with entry is $\{\bar{x}_m^t, \bar{x}_e^t\}$, where $\bar{x}_m^t = \beta^0 (\bar{x}_e^t)$, $\bar{x}_e^t = \beta^t (\bar{x}_m^t)$. Since $v (\bar{x}_e^t, \bar{x}_m^t) - c\bar{x}_e^t - A \leq 0$, player $t$ will choose to stay out and monopoly prevails. 

Next, we will show that, the deterrence condition (D) is not only a sufficient, but also a necessary condition on entry deterrence, if firms compete in strategic substitutes. Strategic substitutes (S) imply that the profit of a potential entrant is monotonically decreasing in the incumbent output. If $k$ does not restrict output, then $\{\bar{x}_m^t, \bar{x}_e^t\}$ is the unique Nash equilibrium with entry in market $t$. Furthermore, $\bar{x}_m^t$ is the highest output the incumbent will select with any capacity $k$. Hence,
if the potential competitor earns a positive profit in \( \{ x^m_t, \bar{x} \} \), the same will hold in any Nash equilibrium in the post-entry game. Thus it enters market \( t \) and entry deterrence is not possible.

**Proposition 2** If condition \( S \) is satisfied and condition \( D \) is violated, then entry deterrence is not possible in \( \Gamma^1_n \).

**Proof.** \((S \text{ and } D \Rightarrow \text{entry deterrence})\). First, note that \( x^m_t \) is player \( m \)'s highest output level in a subgame with entry in market \( t \). From \( (S) \), \( \pi (x^c_t, x^m_t) \) is monotonically decreasing in \( x^m_t \) and reaches its minimum at \( x^m_t \). If \( -D \), i.e. \( v(x^c_t, x^m_t) - cx^c_t - A > 0 \), player \( t \) could ensure a positive profit, if entering market \( t \). ■

After these two qualitative results, a more precise result can be established, characterizing the disadvantage of multi-market competition on entry-deterrence. If firms compete in strategic substitutes and the necessary deterrence condition is satisfied, the incumbent must install \( k > n\bar{x} \) to deter entry in the \( n \)-market game \( \Gamma^1_n \).

Consider for instance the two-market game. Why is twice the deterrence level, \( \bar{x} \), not enough to deter entry in two markets? The main reason is that if one potential competitor enters and the other stays out, the incumbent has an incentive to redistribute capacity to the monopoly market.

In the last period, the remaining capacity is \( k - x^m_1 \). If condition \( D \) is satisfied, \( k - x^m_1 \geq \bar{x} \) will deter entry. Working backwards to period 1, there are two subgames. If player 1 stays out, the incumbent will split the capacity equally in both markets. If the potential competitor enters, the marginal incentive to use capacity in market 1 and 2 must be equal:

\[
\frac{\partial \pi}{\partial x^m_T} (x^m_1, x^c_1) = \frac{\partial \pi}{\partial x^m_T} (k - x^m_1, 0) \quad (6)
\]

It follows from strategic substitutes that \( k - x^m_1 > x^m_1 \). Thus, if \( k = 2\bar{x} \), then \( x^m_1 < \bar{x} \) and entry is not deterred in the first market. More specifically,

**Proposition 3** If \( D \) and \( S \) are satisfied in the first version of the \( n \)-market game, \( \Gamma^1_n \), then the multi-market incumbent installs capacity \( n\bar{x} < k^1_n \leq \bar{x} + (n - 1) x^0 \) to deter entry.

**Proof.** Appendix A ■
4 Simultaneous Competition from Local Entrants

Consider a market situation similar to the first version of the multi-market game. In this version, the incumbent owns a global patent expiring at the same time in all markets and potential competitors can enter the local markets simultaneously. If a potential competitor challenges the established firm in a local market, the incumbent and the entrant choose outputs simultaneously and the market will clear as duopoly. If the potential entrant stays out, monopoly will prevail.

The rules of the second version of the multi-market game are defined as follows. The game, $\Gamma^2_n$, has $n+1$ players, player $m$ and player $1, \ldots, n$ ($n \geq 1$). The game is played over two periods. In the first period, the incumbent must choose a pre-entry capacity, $k$. At the beginning of the second period, player $t = 1, \ldots, n$ must simultaneously decide to enter or stay out of market $t$. Player $t$’s decision is immediately announced to all other players. If player $t$ decides to enter market $t$, then the incumbent and the entrant choose $x^m_t$ and $x^e_t$ simultaneously. At the end of the second period, all markets clear and payoffs are distributed to the incumbent and players $1, \ldots, n$. Player $m$’s payoff is given by eq. (1) and player $t$’s payoff by eq. (2).

The analysis in the second version of the multi-market game is similar to the analysis in the first version. If players compete in strategic substitutes and the necessary deterrence condition is satisfied, entry can also be deterred in the second version of the game. To deter entry, the established firm must install $k > n\tilde{x}$ in the $n$-market game.

Consider, for instance, the two-market case. There are four subgames in the last stage of the two-market game. In two of the four subgames, one potential competitor enters, and the other stays out. To see why twice the single market deterrence capacity does not suffice, consider the profit maximizing conditions when $k = 2\tilde{x}$:

$$\frac{\partial \pi}{\partial x^m_1} (x^m_1, \beta^e (x^m_1)) = \frac{\partial \pi}{\partial x^e_2} (2\tilde{x} - x^m_1, 0)$$

(7)

Strategic substitutes imply that the output in the duopoly market is strictly lower than the deterrence level, i.e. $x^m_1 < \tilde{x}$. Thus, entry would not be deterred.

**Proposition 4** If $D$ and $S$ are satisfied in the second version of the $n$-market game, $\Gamma^2_n$, then the incumbent installs capacity $n\tilde{x} < \kappa^2_n \leq \tilde{x} + (n - 1)\pi^0$ to deter entry.
Proof. Appendix B ■

The first and the second version of the multi-market game differ in one important respect. If the incumbent installed enough capacity to deter simultaneous entry by all potential competitors but not enough to deter unilateral entry by one potential competitor, then the potential entrants would face a coordination problem in the second version of the game. This coordination problem does not occur in the first version where player 1 enters and player 2 stays out. In the second version, both potential competitors wish to enter if they are the only entrant, but not otherwise. 9

The coordination problem in the second version of the game remains unsolved, since both Nash equilibria are strict. This problem will not be further dealt with, since we are mainly interested in the conditions on entry deterrence. In a real market situation, however, the coordination problem may affect the entrants’ decisions and, possibly, facilitate entry-deterrence.

5 Competition from a Multi-Market Entrant

Once more, a multi-market firm has advertised and meets demand for its product in \( n \) markets. In the third version of the multi-market game, a single potential competitor, another multi-market company, considers entry in all markets selling the same product as the established firm. The incumbent’s global patent expires at the same time in all markets and the potential competitor may enter all local markets simultaneously. Entry in each market is associated with a fixed sunk cost, which can be considered an advertising cost. The multi-market entrant remains unknown in all markets where it does not advertise. If the second multi-market firm enters the local market, the incumbent and the entrant choose their output simultaneously and the market will clear as a duopoly.

The rules of the third version of the game are defined as follows. The game, \( \Gamma^3 \), has two players, called player \( m \) and player \( e \). The game is played over a sequence of two periods. In the first period, the established firm must choose a pre-entry capacity, \( k \). At the beginning of the second period, the potential competitor must decide to enter or stay out in \( n \) separate markets called \( t = 1, \ldots, n \). Player \( e \)'s decision is immediately announced to player \( m \). If player \( e \) decides

\[9\text{This is a version of the "chicken" game.}\]
to enter market $t$, the players will choose $x_t^m$ and $x_t^e$ simultaneously. If player $e$ decides to stay out, monopoly will prevail in that market. At the end of the second period, all markets clear and payoffs are distributed to player $m$ and player $e$.

The incumbent’s payoff is given by eq. (1). Entry in market $t$ is associated with a market-specific fixed cost $A > 0$ for player $e$. Let $E$ be the set of all markets that player $e$ will enter. Player $e$’s partial revenue, in a market it enters, is $v(x_t^e, x_t^m)$. The per-unit capital cost is $c > 0$.

The objective of player $e$ is to maximize its total payoff:

$$
\pi^e (x_1^e, \ldots, x_n^e, x_1^m, \ldots, x_n^m) = \sum_{t \in E} (v(x_t^e, x_t^m) - cx_t^e - A)
$$

Inequality D is also a sufficient condition on entry deterrence in the third version of the multi-market game. If the incumbent invests in a sufficiently large capacity, which makes the capacity constraint non-binding in every subgame, the optimal output in every market can be independently determined. The potential competitor chooses its optimal strategy in each market separately, and the best reply functions in all markets are identical. The unique Nash-equilibrium output in every market is $\{x_t^m, x_t^e\}$. Thus, player $e$’s partial revenue does not cover the fixed and variable costs in any market and the total payoff is negative.

In fact, the strategic interaction in the second and third versions of the multi-market game is identical, except for the coordination problem in the second version of the game. Two factors make the strategic decisions in the two games identical with respect to entry deterrence. First, the strategic variables $x_1^e, \ldots, x_n^e$ are independent to the entrant in the third version of the multi-market game and it will choose its optimal strategy in each market separately. Thus, player $e$’s best reply function in market $t$ is identical to player $t$’s best reply function in the second version of the multi-market game.

Second, since the fixed cost $A$ is the same in all markets, the revenue in each market the potential competitor enters must cover the variable and fixed costs. Player $e$ would only enter a market where the expected payoff is positive, which exactly resembles the condition on entry for a local competitor in $\Gamma_n^2$. The analysis of the second version of the game therefore also applies to the third version. Player $m$ must install $k > n \bar{x}$ to deter entry in the $n$-market game $\Gamma_n^3$. 

13
**Proposition 5** If $D$ and $S$ are satisfied in the third version of the two-market game, $\Gamma^3_n$, then the incumbent installs capacity $n\tilde{x} < k^3_n \leq \tilde{x} + (n - 1)\overline{x}^m$ to deter entry.

**Proof.** Appendix B

In the previous sections, the difficulties of entry deterrence in the first, second and third versions of the multi-market game have been characterized. It takes more capacity than $n$ times the deterrence level $\tilde{x}$ to deter entry of many potential competitors in a sequential or simultaneous market structure. More specifically, the established firm installs exactly the same capacity to deter entry in $\Gamma^1_n$, $\Gamma^2_n$ and $\Gamma^3_n$. Thus, the unique optimal deterring capacity is independent of the market situation, as described in the first, second and third versions of the multi-market game.

**Proposition 6** If conditions $D$ and $S$ are satisfied, the global capacity required to deter entry in the $n$-market game is independent of the timing of the game, i.e. sequential or simultaneous entry of potential competitors, and the size of the potential entrant.

**Proof.** Appendix C

This proposition is interesting for two reasons. First, it might be difficult for the incumbent to obtain information about potential entrants *ex ante*, but our results suggest that such information might not be necessary. The result implies that an incumbent does not need information about the timing and the number of potential entrants to determine its entry-deterring strategy. The results of the model apply to several different situations, for example both to a situation with one large competitor and to a situation with competition from a series of local competitors.

Not surprisingly, it also follows that the difference between the single-market game and the multi-market game increases with the number of markets in the multi-market game.

If condition D holds with equality, the entry-deterring capacity per market in an $n$-market game increases in the number of markets and converges to $\overline{x}^n$, as $n$ goes to infinity.

The intuition for this result is that unilateral entry in a single market is harder to deter as partial exit to the remaining $n - 1$ markets becomes increasingly attractive. As the number of monopoly-markets increases, the alternative to fight entry in a single market looks less and less attractive, in comparison to using the capacity in the remaining monopoly markets. It should,
however, be noted that per-market profits are less affected by unilateral entry in a single market, if the number of markets is large.

6 Market Commitments

In this section, I extend the analysis and let the incumbent first determine the organization of its production, either with a global capacity, referred to as the global strategy, or with a combination of a global capacity and local capacities that can be used in specific markets only, referred to as the local strategy. The local strategy can be regarded as a vertically integrated production process, where the production process is split into two vertical stages. It will be shown that if sufficient conditions apply, then local capacities can be assigned to local markets and successfully deter entry.

We study a three-stage game similar to the two-stage game in the previous sections. In the first stage, the multi-market firm can choose a global or a local strategy. The local strategy, i.e. assigning a local capacity to each local market, is associated with an extra fixed cost $G$ in each market.

We can now describe the rules of the fourth version of the game. The game, $\Gamma^4_n$, has two players, player $m$ and player $e$. The game is played over a sequence of three stages. In the first stage, the incumbent must begin by choosing a local or global strategy. In the second stage, the incumbent must choose local capacities in each market, $k_t$, and a multi-market capacity, $k$. Unlike the global capacity, it is assumed that local capacities can be increased in the third stage. All decisions of the established firm is immediately announced to the potential competitor. At the beginning of the third stage, player $e$ must decide to enter or stay out in $n$ separate markets called $t = 1, ..., n$. Player $e$’s decision is announced to the incumbent. If player $e$ decides to enter market $t$, player $m$ and player $e$ will choose $x^m_t$ and $x^e_t$ simultaneously. Finally, all markets clear and payoffs are distributed to player $m$ and player $e$.

If the incumbent chooses a local strategy, the unit-cost of local capacity is $c_1 > 0$, and the unit-cost of multi-market capacity is $c_2 > 0$. Moreover, each local assignment is associated with a fixed cost $G > 0$. If the incumbent chooses a global strategy, the cost of capacity is $c$. For
simplicity, we assume that the total unit-cost is independent of the strategy, i.e. \( c_1 + c_2 = c \). The incumbent’s payoff is given by:

\[
\pi^m(x^m_t, x^e_t) = \begin{cases} 
\sum_{i=1}^{n} [v(x^m_i, x^e_t) - c_k] & \text{global} \\
\sum_{i=1}^{n} [v(x^m_i, x^e_t) - c_1 q_t] + c_2 k - nG & \text{local}
\end{cases}
\tag{9}
\]

where \( q_t = \max \{x^m_i, k_t\} \). The potential competitor must incur a market-specific fixed cost \( A > 0 \) to enter market \( t \). Let \( E \) be the set of all markets that player \( e \) will enter. Player \( e \)’s revenue is \( v(x^e_t, x^m_t) \). The marginal capital cost is \( c > 0 \) and additive. The objective of player \( e \) is to maximize its payoff given by eq. (8).

We shall call \( k_t \) a market commitment, if this part of the total capacity in a multi-market firm is assigned to market \( t \) and cannot profitably be used for production of goods sold in other local markets. A sufficient condition for market commitments is that the marginal cost to increase local capacity is larger than the marginal incentive to increase the output in a monopoly market at the deterring level \( \tilde{x} \). We refer to this condition as (C). More precisely,

\[
(C) \quad c_1 > \frac{\partial v}{\partial x^m_1}(\tilde{x}, 0)
\]

Condition C simply guarantees that it is not profitable for player \( m \) to redistribute capacity to a monopoly market, if entry occurs in other markets. If condition C is satisfied and condition D is satisfied with equality, it is sufficient for player \( m \) to install a local capacity equal to the deterrence level \( k_t = \tilde{x} \) and a multi-market capacity \( k = n\tilde{x} \), to deter entry.

**Proposition 7** If conditions C, D and S are satisfied in the fourth version of the n-market game, \( \Gamma^4_n \), local capacities \( \bar{k}_t = \tilde{x} \) and global capacity \( \bar{k}^4_n = n\tilde{x} \) is sufficient to deter entry.

**Proof.** Entry deterrence is possible in \( \Gamma^4_n \), due to (D). Player \( m \) will choose a local strategy and installs capacity \( \bar{k}^4_n = n\tilde{x} \) and \( \bar{k}_t = \tilde{x} \) for \( t = 1,..,n \). If player \( e \) enters all markets, symmetric incentives imply that \( x^e_t = \tilde{x} \) and D implies that the profit of player \( e \) is not positive. If player \( e \) enters one market (w.l.o.g. market 1) and stays out of all other markets, the following inequality must hold for the incumbent to deter entry

\[
\frac{\partial v}{\partial x^m_1}(\tilde{x}, \beta^e(\tilde{x})) + \left(c_1 - \frac{\partial v}{\partial x^m_1}(\tilde{x}, 0)\right) \geq 0
\tag{10}
\]

16
for \( t = 2, \ldots, n \). The first part of the LHS is equal to zero and from (C), the second part is positive. Thus the inequality holds. Equal parts of the total capacity should be assigned to each market, i.e. \( \frac{k_t}{n} \). \( \tilde{x} \) deters entry in market \( t \), hence \( n\tilde{x} \) is enough to deter entry in all markets. 

The incumbent installs strictly less capacity with market commitments compared to the capacity needed to deter entry, if the capacity is not assigned to specific markets. The difference in the established firm’s profit, if \( C \) is satisfied in \( \Gamma^4_n \), between the local and the global strategy is called the commitment premium, denoted \( \Delta \pi \). Working backwards, the multi-market firm will choose a local strategy if the commitment premium minus the cost of assignment is positive.

**Proposition 8** If \( C \) is satisfied in \( \Gamma^4_n \) the multi-market firm will choose a local strategy to deter entry i.f.f. \( \Delta \pi - nG > 0 \).

**Proof.** Follows immediately from the definition of the commitment premium and the cost of a local strategy. 

It follows from this proposition that a local strategy is more likely, the lower the assignment cost. Thus, the organization of production within the multi-market firm is primarily determined by the relationship between economies of scale at the local level and the commitment premium.

Another important issue is what factors determine the incumbent’s opportunities to make market commitments. These factors can be exogenous, e.g. different national standards or trade regulations. A more interesting case, however, is when the incumbent chooses to induce market segmentation endogenously.

First, firms can bundle their tradable products with locally produced and consumed nontradables, e.g. services. If the product cannot be used without local services, the capacity is assigned to the local market provided that the marginal cost to expand the service capacity is sufficiently high. In this case, a global strategy would correspond to the manufacturing of a sophisticated product, which can be used without services. A local strategy, on the other hand, would be to produce a less sophisticated product which must be consumed with some local support or services.

Second, strategic market segmentation can occur in a horizontally differentiated product space. If consumers in the local markets have preferences for local products, capacities can be assigned to
the domestic market. The local strategy is manufacturing of goods adapted to local preferences, i.e. products which can be used by consumers in a specific market only, and the global strategy is production of a standardized good, which can be used by consumers in all markets. If the cost of adjusting the adapted products in the post-entry game is sufficiently high, the local strategy can successfully deter entry.

Third, market commitment can be induced by network lock-ins. The producer can introduce local standards, which assign capacities to a specific market. In this case, a global strategy is a standard common to all markets.

Thus, the model of endogenously determined multi-market production potentially applies to many different market conditions.

7 Price Competition in Differentiated Goods

Having shown that multi-market competition obstructs the incumbent’s possibilities to deter entry if firms compete in strategic substitutes, we will now show that strategic complements give the same result, if sufficient conditions apply.

An incumbent commit to a global capacity for two markets in the first stage. A potential entrant in each market, called player $t$, observes the incumbent’s capacity and then chooses to enter or stay out. If player $t$ enters market $t$, the incumbent and the entrant both choose prices for their respective variety of the differentiated good.

We use the Shubik (1980) system of demand functions where the demand for variety $i$ in market $t$ is given by

$$x_i^t = \frac{1}{n} \left[ a - b \left( p_i^t + g (p_i^t - \bar{p}_t) \right) \right],$$

where $n$ is the total number of active firms in the local market, $\bar{p}_t$ is the average price in the local market and $g$ is a measure of substitutability between products. Assume that the parameters of the model satisfy some restrictions, $a \geq b \geq c$, and that the degree of substitutability is not too large, $g \leq 2$.

Consider a situation where entry deterrence is possible in the single-market game and the entrant makes zero profit in a subgame with a nonbinding capacity constraint for the incumbent.
It can be shown that twice the capacity needed to deter entry in a single market game does not suffice to deter entry in the multi-market game. For this purpose, let \( k \) be exactly twice the capacity needed to deter entry in a single market game. Capacity \( k/2 \) in a market without entry results in a price which is strictly higher than the price the incumbent would set as a monopolist, if the capacity constraint was not binding. If unilateral entry in market 1 occurs, profit maximization under the binding capacity constraint requires

\[
\frac{d\pi_1^m(p_1^m, p_e^1)}{dp_1^m} = \frac{d\pi_2^m(p_2^m)}{dp_2^m}. \tag{12}
\]

It is not satisfied, however, if the capacity is evenly distributed between the markets. In this case, the RHS is strictly negative and the incumbent will increase its profit by setting a lower price in its monopoly market and move some productive capacity to this market.\(^{10}\) Accordingly, the resulting price in market 1 is higher. But firms compete in strategic complements and a price increase by the incumbent is followed by a price increase by the entrant, which increases the profit of the entrant in equilibrium and, therefore, entry is not deterred. Hence, as in the case of strategic substitutes, the multi-market incumbent must install more capacity to deter entry in the multi-market game.

8 Applications

(i) Franchising and Strategic Delegation

Franchising is a long-term vertical contract between a franchisor (the incumbent) and a franchisee. Through the contract, the franchisor collects revenues from a franchise fee as well as from the wholesale markup. The contract allows the incumbent to strategically design the terms of the contract in order to overcome its own incentives in the future.\(^{11}\) Hadfield (1991) shows that in a model of horizontal product differentiation, strategically designed franchise contracts can deter entry.

Following Hadfield (1991), we can analyze market commitments through strategic delegation in our model. Consider a franchise contract which is a standard-form, long-term-duration contract

\(^{10}\)It can be shown that the equality is \((1/2)^2d - (1/2)(2 + g)bp_e^m + (1/4)bgp_e^1 = a - 2bp_{e}^m\)

\(^{11}\)This idea of strategic delegation was first suggested by Schelling (1980).
designed by the incumbent and offered to potential franchisees. The contract consists of a franchise fee, $F$, a wholesale price scheme, $w(x_t)$, and an exclusive territory, $t$. The contract obliges a franchisee to sell the product to customers in its own market only, i.e. exporting the product to another territory is either prohibited or associated with an additional fee, $c$. The contract also specifies that violations of the contract are associated with damages, $V$.

The incumbent can then design a contract with the following terms; the wholesale price is zero up to a quantity equal to one $n$:th of the incumbent’s global capacity and infinite thereafter, the exclusive territory is a local market, $t$, and the franchise fee is the expected revenue for a monopolist in that market, minus the assignment cost, i.e. $F = v(k/n, 0) - G$.

Under this contract, the independent franchisee in each market has an incentive to produce and sell its full capacity and entry is successfully deterred. The market assignment cost is identical to the profit of the franchisee and it is determined by the relative bargaining power of the franchisee and the incumbent. Hence, the profitability of franchising for the manufacturing firm is determined by the outcome of the bargaining between the franchisor and the franchisees.

(ii) Strategic Investment and Multinational Production

Multinational production and strategic foreign direct investment constitute another natural application of the model.\(^{12}\)

Consider a modified version of the game. The incumbent firm has incurred the market-specific fixed costs and meet demand for its product in all markets. In the first stage, the incumbent has two options: either to concentrate production in a single plant, i.e. an export strategy, or to install local plants, i.e. a multinational strategy. If it is choosing the former strategy, the incumbent must choose a global pre-entry capacity, whereas, if it is choosing the latter strategy, the incumbent must choose a global capacity and local capacities assigned to each of the plants. In the second stage, a potential competitor considers entry in the local markets. If it enters, it must also decide whether

\(^{12}\)In models with variable trade costs, Smith (1987) and Horstmann and Markusen (1987), show that an incumbent has an incentive to make a foreign direct investment to deter entry. Multinational production reduces variable costs and makes the incumbent more aggressive. A more aggressive play will reduce the revenues of potential entrants and, thus, entry is deterred. If monopoly rents outweigh any costs associated with installing an additional plant, the first-mover would choose this strategy.
to establish one or several plants.

In this game a multi-market incumbent can choose a multinational or export strategy to deter entry. The multinational strategy requires less total capacity, while the export strategy requires fewer plants. For some parameter values the multinational strategy is a more profitable strategy to deter entry, for other values the export strategy is more profitable.

However, if the firms must incur a firm-specific cost, \( F \), as well as plant-specific costs, \( G \), the current specification adds a new dimension to the problem. The firm-specific cost results in economies of scale at the firm level and the plant-specific cost in economies of scale at the plant level. An entrant can use these assets in all markets, which makes single-market entry less profitable compared to multi-market entry. Hence, single-market entry can be a strictly dominated strategy. But this is not the case in all situations. If scale-economies at the firm and plant level are not too large, the potential entrant will consider single-market entry rather than multi-market entry.

(iii) Mergers

An international merger is a union of assets from two firms previously active in two distinct geographic markets. The multi-market model in this paper can be used for analyzing the effect of these types of mergers.

Consider a situation where two firms have separately entered two local markets and successfully deterred further entry. Each firm is active in one market only. Local production is associated with a fixed cost, \( G \). If the firms choose to merge, they will reduce their fixed costs. If capacity can be used in all markets, the merged firm is obliged to install more capacity and expand its output to successfully deter entry in the post-merger equilibrium. If the firm cannot expand its capacity to deter entry, the result is local entry in one of the markets. In both cases, production is expanded and the monopoly distortion is reduced. Hence, the merger is clearly pro-competitive.

9 Conclusions

Multi-market competition without market commitment makes the incumbent’s possibilities to exploit first-mover advantages more difficult. A firm’s opportunity in one market influences its
possibility to successfully commit to its optimal strategy in a second market. The incumbent must install a higher level of global capacity to successfully deter entry in all markets. If exogenous or endogenous factors allow the incumbent to assign parts of its capacity to local markets, multi-market production can be profitable, even under increasing returns to scale at the global level. The results suggest that local investments can be regarded as market commitments, in order to restrict or prevent competition in specific markets.
References


Appendix A. Sequential Entry

Proof. Step 1. Start in period $n$. Let the remaining capacity be $k_n = k_{n-1} - x_{n-1}^m$. There are two subgames; either player $n$ enters or stays out of market $n$. In the subgame with entry, the unique Nash equilibrium is $\{\tilde{x}_n^m (k_n), x_n^m\}$, where $\tilde{x}_n^m (k_n) = \min \{k_n, \pi^m\}$ and $x_n^m (k_n) = \beta_e (\tilde{x}_n^m (k_n))$. If player $n$ decides to stay out of market $n$, we have the following limit Nash equilibrium $\{\pi^m, 0\}$, where $\frac{\partial \pi_e}{\partial x_n} (\pi^m, 0) = 0$. The unique Nash equilibrium in the subgame with no entry is $\{\tilde{x}_n^m (k_n), 0\}$, where $\tilde{x}_n^m (k_n) = \min \{k_n, \pi^m\}$. From S, it follows that $\pi^m > \pi^m$.

Step 2. Player $n$ would enter if $k_n < \tilde{x}$ and stay out as long as $k_n \geq \tilde{x}$. To deter entry, player $m$ would need $k_n \geq \tilde{x}$. Now, assume that enough unused capacity remains to deter entry. Rewrite the equilibrium output of player $m$ in period $n$ as a function of $k_{n-1}$ and $x_{n-1}^m$, i.e. $x_n^m (k_{n-1}, x_{n-1}^m) = \min \{k_{n-1} - x_{n-1}^m, \pi^m\}$. States in period $n - 1$, we have two subgames; either player $n - 1$ enters or stays out of market $n - 1$. First, capacity $k_{n-1}$ would ensure a successful commitment by player $m$ in market $n - 1$ to an output $\tilde{x}_{n-1}^m$, if and only if:

$$\frac{\partial v}{\partial x_{n-1}^m} (\tilde{x}_{n-1}, \beta^1 (\tilde{x}_{n-1})) + \frac{\partial x_n^m}{\partial x_{n-1}^m} \frac{\partial v}{\partial x_n^m} (x_n^m (k, \tilde{x}_{n-1}), 0) \geq 0 \quad (13)$$

Now, $\frac{\partial x_n^m}{\partial x_{n-1}^m} = -1$ if $k \leq \pi^m + \tilde{x}$ and $\frac{\partial x_n^m}{\partial x_{n-1}^m} (k, \tilde{x}_{n-1}) = 0$ if $k > \pi^m + \tilde{x}_{n-1}$. To deter entry, player $m$ has to commit to $\tilde{x}$ in the subgame with entry. The following inequality must be satisfied:

$$\frac{\partial v}{\partial x_{n-1}^m} (\tilde{x}, \beta^m (\tilde{x})) = \frac{\partial v}{\partial x_n^m} (x_n^m, 0) \quad (14)$$

If $x_{n-1}^m > 0$, it follows from (S) that $x_n^m > \tilde{x} \Rightarrow k_{n-1} > 2\tilde{x}$. If (D) holds with equality, i.e. $\tilde{x} = \pi^m$, then the LHS of equality [14] is equal to zero and the equality is satisfied if and only if $k_{n-1} - \tilde{x} = \pi^m \Rightarrow k_{n-1} = \tilde{x} + \pi^m$.

Step 3. Working backwards to period $n - 2$, we have two subgames; either player $n - 2$ enters or stays out of market $n - 2$. First, capacity $k_{n-2}$ would deter entry if:

$$\frac{\partial v}{\partial x_{n-2}^m} (\tilde{x}, \beta^m (\tilde{x})) = \frac{\partial v}{\partial x_{n-1}^m} (x_{n-1}^m, 0) = \frac{\partial v}{\partial x_n^m} (x_n^m, 0) \quad (15)$$

26
From (S), we have $x_n^m = x_{n-1}^m > \bar{x} \Rightarrow k_{n-2} > 3\bar{x}$. If (D) holds with equality, i.e. $\bar{x} = \bar{x}^m$, then the LHS of equality [14] equals to zero and the equality is satisfied if and only if $k_{n-2} - \bar{x} = 2\bar{x}^m \Rightarrow k_{n-1} = \bar{x} + 2\bar{x}^m$. Work in the same way inductively to period 1. In period 1, we have $x_1^m = x_{n-1}^m = .. = x_2 > \bar{x} \Rightarrow k > n\bar{x}$ and as (D) holds with equality $k_{n-1} = \bar{x} + (n-1)\bar{x}^m$. The entry deterring capacity $k^1$ is implicitly defined by

$$\frac{\partial v}{\partial x_1^m}(\bar{x}, \beta^1(\bar{x})) = \frac{\partial v}{\partial x_t^m}\left(\frac{k_1^1 - \bar{x}}{n-1}, 0\right) \quad \text{for } t = 2, .., n$$

(16)

and we conclude that $k^1 \in (n\bar{x}, \bar{x} + (n-1)\bar{x}^m]$.

**Step 5.** In a subgame without entry

$$\frac{\partial v}{\partial x_1^m}(x_1^m, 0) - \frac{\partial v}{\partial x_t^m}\left(\frac{k_1^1 - x_t^m}{n-1}, 0\right) = 0$$

(17)

and $x_t^m = k/n < \bar{x}^m$ for all $t = 1, .., n$. Hence, the entire capacity will be used in an equilibrium without entry. No capacity is left idle.

**Step 6.** Working backward to period 0, the incumbent would install $k^1$ to deter entry in all markets.

\[\square\]
Appendix B. Simultaneous Entry

This proof is valid for the main result in the second and third versions of the multi-market game.

**Proof.**  Step 1. Begin in stage two. The objective of player $m$ in the second stage is to solve the following program:

$$\max \ v(x_1^m, x_2^m) + v(x_2^m, x_3^m) + \ldots + v(x_n^m, x_1^m)$$

s.t. $x_1^m + x_2^m + \ldots + x_n^m \leq k$

If $x_1^m + x_2^m + \ldots + x_n^m < k$, then $\partial v(x_i^m, x_j^m) / \partial x_i^m = 0$ for $t = 1, \ldots, n$. If $x_1^m + x_2^m = k$, then

$$\partial v(x_1^m, x_2^m) / \partial x_1^m = \partial v(x_2^m, x_3^m) / \partial x_2^m = \ldots = \partial v(x_n^m, x_1^m) / \partial x_n^m.$$

**Step 2.** In the last stage there are $2^n$ subgames. First, if entry does not occur in any market and $k > n\bar{x}$, then $\partial v(x_1^m, 0) / \partial x_1^m = 0$ for all $t = 1, \ldots, n \Rightarrow x_1^m = \bar{x}$ for all $t$. If $k \leq n\bar{x}$, then

$$\partial v(x_1^m, x_2^m) / \partial x_1^m = \partial v(x_2^m, x_3^m) / \partial x_2^m = \ldots = \partial v(x_n^m, x_1^m) / \partial x_n^m \Rightarrow x_1^m = k / n$$

for all $t$.

**Step 3.** Second, if one player enters (w.l.o.g. player 1) and $k > \bar{x} + (n-1)\bar{x}$, then $\partial v(x_1^m, x_1^m) / \partial x_1^m = 0$ and $\partial v(x_1^m, 0) / \partial x_1^m = 0 \Rightarrow x_1^m = \bar{x}$ and $x_t^m = \bar{x}$ for $t = 2, \ldots, n$. If $k \leq \bar{x} + (n-1)\bar{x}$, then from (S) $\partial v(x_1^m, x_1^m) / \partial x_1^m = \partial v(x_1^m, 0) / \partial x_1^m$ for $t = 2, \ldots, n \Rightarrow x_t^m < k/n$ and $x_t^m > k/n$. To deter the entry of a single entrant while $n - 1$ players stays out, the incumbent must install

$$\frac{\partial v(\bar{x}_1^m, \bar{x})}{\partial x_1^m} = \frac{\partial v(\frac{k}{n-1}, \bar{x})}{\partial x_1^m}$$

(18)

and from (S) $k > n\bar{x}$.

**Step 4.** Next, if capacity $k$ deters the entry of a single entrant, $k$ deters the entry of more than one player, which is shown with induction. Assume $k$ deters the entry of $t$ players. Then

$$\partial v(\bar{x}_i^m, \bar{x}) / \partial x_i^m - \partial v \left( \frac{(k-t\bar{x}_i^m)}{(n-t)}, 0 \right) / \partial x_j^m \geq 0$$

(19)

where entry occurs in $i$ and no entry occurs in market $j$. If $t+1$ players enter, deterrence is credible if

$$\partial v(\bar{x}_i^m, \bar{x}) / \partial x_i^m - \partial v \left( \frac{(k-(t+1)\bar{x}_i^m)}{(n-t-1)}, 0 \right) / \partial x_j^m \geq 0$$

(20)

where entry occurs in $i$ and no entry occurs in market $j$. The last inequality holds as long as $k > n\bar{x}$. Hence, we have shown that if capacity $k$ deters the entry of a single entrant, then $k$ deters the entry of more than one entrant.
Step 5. If (D) holds with equality, i.e. \( \tilde{x} = \bar{x}^m \), then
\[
\frac{\partial \pi (\tilde{x}^m, \tilde{x}^c)}{\partial x^m_j} = \frac{\partial \pi}{\partial x^m_j} \left( \frac{k - \tilde{x}}{n - 1}, 0 \right)
\]
and the LHS is zero and, therefore, the entry-deterring capacity is \( k = \bar{x}^m + (n - 1) \bar{x}^m \).

Step 6. Working backwards to the first stage. Now, the incumbent capacity is \( \bar{x}^2 \in (n\tilde{x}, \bar{x}^m + (n - 1) \bar{x}^m) \), where \( \bar{x}^2 \) is determined by equation (18). ■
Appendix C. Equivalence

Proof. $k_1^n, k_2^n$ and $k_3^n$ are all implicitly defined by

$$\frac{\partial v}{\partial x^T_t}(\bar{x}, \beta^e(\bar{x})) - \frac{\partial v}{\partial x^T_t}(\frac{k - \bar{x}}{n - 1}, 0) = 0$$

for $t = 2, ..., n$. Hence, the implicit conditions are identical for all three versions of the multi-market game and the entry-deterring capacity is the same. ■