Peers and Culture

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Abstract

We analyze the evolution of cultural traits when parents purposefully invest resources in order to socialize their children to the cultural traits that maximize child lifetime utility. We assume that children are not passive in their adoption of traits from peers. Instead they are guided by an evaluation of the merit of traits. We show that such evaluation is likely to render this process of "oblique transmission" biased. We then show that when transmission of traits from society is biased or frequency dependent, cultural diversity is sustainable even when all parents strive to transmit the same trait. We also show that demand for cultural pluralism on the part of parent does not guarantee cultural diversity. Journal of Economic Literature: Classification numbers: D10, I20,J13.

Keywords: Peer groups, Cultural transmission, cultural diversity, oblique transmission.

1 Introduction

Families have large influence over their children. They are the first role models and they choose schools and neighborhoods. But parents are not everything. In particular, they cannot control how the young individual learns and adopts values though interactions outside with peers. Economic models of cultural transmission, Bisin and Verdier (2000a,2000b) and Hauk and Sáez-Martí (2002) recognize that parents try to influence the cultural traits of their children through active nurturing efforts because parents believe cultural traits and values have important implications for the future wellbeing of their offspring.

The main insight from the literature is that parental socialization efforts are responsive to incentives, both the perceived returns and the costs of cultural transmission. This responsiveness is what distinguish these economic models from the anthropological or sociobiological models of cultural transmission of

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Cavalli Sforza and Feldman (1981) and Boyd and Richerson (1985) where transmission is a mechanical process. However, these models have left unexplored the role of children as adopters of culture.

The interest of economists in the process of cultural transmission, has been motivated, among other things, by the evidence of slow cultural convergence and persistence of cultural diversity, reported in Glazer and Moynihan (1963) and Borjas (1995) et c. This evidence contradicts the predictions of rapid convergence and assimilation of cultures common to the early sociological theories that described American society as a "melting pot". Recent economic models of cultural transmission give one account of why cultures remain diverse. For instance, Bisin and Verdier ((2000a), argue that: "The persistence of cultural traits, we claim, is the consequence of the demand for "cultural pluralism" on the part of ethnic, religious, and racial minorities."

The purpose of this paper is to show that when children are adopters of culture, cultural diversity can persist over time even if parents have homogenous preferences in the sense that parents, regardless of their own cultural heritage, are all trying to promote the same cultural traits and values. More formally, we want to show that the assumption that parents want to turn their children into copies of themselves is neither a necessary, nor a sufficient condition for cultural diversity. Instead, our ambitions is to provide a framework which allows for an understanding of the underlying causes of assimilation and segregation beyond the simple notion that parents have different view of the world that they transmit to their children.

This is, in our view, very important. First, although there are obvious examples of culturally transmitted traits where parents do have an interest in promoting their own variant, e.g. language, religion, this interest is far less obvious when it comes to cultural traits and values associated with low status and poor market outcomes.

Consider, for instance the existence of subcultures of criminality, bad eating habits and low educational ambitions within a community. According to existing theories of cultural transmission such subcultures can only persist if some parents are actively promoting their children to have low moral standards, prefer junk food and skip classes. Regrettably, and arguably, some parents may not care much, or are sometimes not fully aware of the dire consequences for the child’s future well being. But from that to assuming that parents in these disadvantaged environments should actively influence their children to forge values that are conducive to a poor future there is a long way to go. To the extent

\footnote{A somewhat different story is told in Austen-Smith and Fryer (2003), regarding the phenomenon of acting white. There a black-white market skill gap persists because families face trade offs in their nurturing efforts. On the one hand, all families agree that investing in human capital is valuable, but families also care about the returns to some social game. In the Austen-Smith and Fryer model, human capital productive on the labour market and social capital, productive in social interactions in the family or in the local community compete for parental attention in black families, but amount to being the same thing in white families. However, again, the trade off is assumed and the question remains why social capital in some communities is a substitute for human capital, while in other societies, social and human capital are complements.}
that parents can identify that keeping the child off the street, sending the child to school and promoting better food habits are steps in the right direction, it seems reasonable that most parents would at least try to do so. Trying may, however, not be sufficient. First, parents may not try hard enough. Second, if the child is surrounded by enough bad role models, if children, in general, prefer sugar, fat, and TV to fibers, vitamins, and homework; what parents preach will have little impact.

Another point of our paper is that in some instances, cultural variants or traditions may disappear in spite of parental efforts to teach and transmit them to their children. Pinker (2002), takes the example of minority dialects, and argues that parents speaking a minority dialect may try in vain to promote their language. It is not unusual that children never reach proficiency, in particular if most friends only speak the majority dialect. The failure of these parents to preserve their cultural variant is left unaccounted for by the existing theories.

Both these examples point back to the important role of the child in determining the success of parental nurturing efforts.

The contribution of this paper is to bring the child’s adoption of culture into focus and to show that when children are adopters and not merely receivers of cultural transmission from society, diversity need not be driven by a parental demand for cultural pluralism. Neither does such a demand for cultural pluralism on the part of parents, guarantee a culturally diverse population. To this aim, we generalize and extend the insights of earlier economic models of cultural transmission by endogenizing the way children adopt culture from society, the so called oblique transmission.

The of adoption of culture from society has been discussed by Boyd and Richerson (1985) under the term oblique transmission. They define oblique transmission as unbiased if children adopt a cultural variant from society with a probability equal to its proportion in the population, e.g. by picking a model at random and adopting his cultural variant. This is the maintained assumption of previous economic models of cultural transmission. If children adopt variants more or less than proportionally to their presence in society, the oblique transmission process is defined as biased. Various forms of biases are discussed; direct and indirect biases and frequency dependence; and the motivation for their existence is based on the idea that children are adopting cultural variants guided by some form of evaluation and comparison of the merit of alternative variants.

Boyd and Richerson (1985) describe different forms of merit evaluation and the biases they give rise to. Direct and indirect biases can be positive or negative and result from experimentation by the subject (direct) or inferred from how the variant has affected the success of others (indirect). The example they take, is that of how a child adopts a style of holding a ping-pong racket. Direct bias may result if children observe the different styles of holding the racket, then try for themselves, to finally settle for the grip which renders them more successful.

\[ \text{Footnote: Oblique transmission is distinguished from vertical transmission which captures transmission of culture from parents to children. Horizontal transmission, i.e., transmission of culture from individuals of the same generation has also been used. This terminology was first introduced by Cavalli Sforza.} \]
Indirect bias results if children copy the grip of the most successful player(s). Such direct and indirect evaluation of merit can result in positive or negative bias in favour or against a variant (grip) if it is intrinsically - for genetic or other reason - easy or difficult to adopt, or because that variant appears to render those who have adopted it successful or unsuccessful. Yet another way of evaluating merit is discussed, namely that children choose to adopt the grip which is most frequent. This kind of merit evaluation gives rise to frequency dependent bias, or conformism.3

Our model captures the idea that children adopt variants guided by the different forms of evaluating merit and incorporates it into the standard framework of cultural transmission. More specifically, we assume that the transmission of culture is outcome of a socialization process where parental nurturing efforts, in order to maximize child wellbeing, are chosen in response to an oblique transmission process determined by peer group interaction and merit evaluation. We have already argued for why we will not assume that parents are promoting their own cultural variant, instead we allow parents to promote either variant.

Merit guided oblique transmission is shown to give rise to unbiased (linear) oblique transmission only if children assign equal merit to the variants of a trait and if no significance is given to frequency. Any departure from this, results in some form of biased (non-linear) oblique transmission. We then explore the consequences for cultural dynamics of biased (non-linear) oblique transmission under different assumptions about parental preferences, i.e. if parents have a preference for cultural pluralism or if parents agree on which cultural variant is desirable. Biased oblique cultural transmission has very different qualitative implications compared to linear transmission. In particular, biases open up for multiple stable equilibria. Hence, initial conditions regarding the prevalence of cultural variants, may determine if there will be assimilation or diversity in the long run.

Our model of oblique transmission gives rise to explanations of cultural diversity that we would argue are of particular relevance when it comes to the persistence of cultural variants that are penalized by economic markets. We need not interpret this persistence as the result of cultural conservatism on the part of parents. Instead, we show that if young agents are negatively biased against adopting the variant promoted by parents or if children are conformist in their adoption behaviour, variants desired by no one may will persist in the long run.

An implication of this result is that it is not necessarily a violation of parental rights to promote children’s adoption of mainstream traits through "Head Start" or similar programs, as would be the conclusion if diversity was in explicit demand by parents.

We are also able to provide an explanation for why some variants are out-

3 Conformism is not new to economics. It has been show to arise when rational agents use the decisions and behaviors of others as sources of information about the qualities and of a good or the virtues of a trait, Becker (1994), Banerjee (1992), and Bikhchandani, Hirshleifer and Welch (1992). Conformism can also result when agents value status in a group, Bernheim (1994) and Becker and Murphy (2002) or because of mutual externalities from coordination.
competed, which does not rest on the presumption that not even the holders of the variant were willing to promote them. Again, if children are negatively biased against a variant in their adoption of a trait, or if it there is conformism, even variants promoted by parents risk extinction if the fraction of holders is small enough.

The paper is organized as follows. First we present a general framework of cultural transmission where parents can chose to promote the adoption of either of two variants of a cultural trait. Second, we develop a model of merit guided oblique transmission with the aim of capturing the young individuals’ adoption of cultural traits from their older peers. Third, we derive the implications for optimal vertical transmission of different biases in the oblique transmission. Section four characterizes the cultural dynamics. Section five concludes.

2 Cultural transmission

This section considers how young agents come to adopt cultural traits which later in life influence adult behavior and success. We assume that socialization takes place in the family through vertical transmission, and in society through oblique transmission. Vertical and oblique transmission take place in the period children are born so that children reach adulthood the period after. It is from that period on, that cultural values and traits adopted by the child, are important determinants of wellbeing.

Parents are assumed to be altruistic and to be willing to spend resources and nurturing efforts in order to maximize their children’s future wellbeing. In previous models, parents were assumed to socialize their children to their own cultural variant. Here we relax this assumption and allow (non-paternalistic) parents to invest in cultural variants which are different from their own. More precisely, parents invest in the cultural variant that induces behaviors which maximizes their children’s true wellbeing. For instance, overweighted parents may try to induce children not to adopt their own unhealthy eating habits and disorganized parents may try to get their child to keep its room tidy.

Assume a trait with two cultural variants, $a$ and $b$. While children are born naive and malleable, adults remain of the same type through the rest of their life time, once they have been socialized to either of the variants. More formally, parents choose the probability $\tau_i \geq 0$, that the child learns the $i$-variant through vertical transmission. If the parent fails in influencing his child (this happens with probability $(1 - \tau_i)$), the child can still be socialized to the $i$-variant of the trait by society through a process of oblique transmission.

Let $q \geq 0$ be the proportion adults holding the $a$-variant. The remainder are of type $b$. Consider a parent trying to transmit the $a$-variant. If the parent exerts effort $\tau_a \geq 0$, the total probability that the child adopts $a$ is given by:

$$x(\tau_a, q) = \tau_a + (1 - \tau_a)f(q),$$

(1)

where $f(q)$ captures the process of oblique transmission by which the "naive" child is influenced by society (peers).
A parent who instead wants to promote the $b$-variant will choose $\tau_b \geq 0$. If the parent does this, the child can only adopt the $a$-variant as a result of oblique transmission:

$$x(\tau_b, q) = (1 - \tau_b) f(q).$$

(2)

Note that when promoting the $a$-variant, parents can increase the probability that the child adopts the $a$-variant beyond what would be the case if the child was exposed only to peers, while the child of a parent promoting the $b$-variant will necessarily adopt the $a$-variant with smaller probability: $x(\tau_b, q) \leq f(q) \leq x(\tau_a, q)$.

We assume that the oblique transmission function $f(q)$ has the following properties: $f : [0, 1] \to [0, 1]$ is a twice continuously differentiable, increasing function with $f(0) = 0$, $f(1) = 1$ and with at most one $\tilde{q} \in (0, 1)$ such that $f(\tilde{q}) = \tilde{q}$. In the following section we discuss its microfoundations in detail.

Active nurturing is a costly undertaking. We denote by $c_i(\tau)$ the cost born by a parent who chooses nurturing effort $\tau$ to transmit variant $i, i \in \{a, b\}$. We assume that $c_i : [0, 1] \to \mathbb{R}^+$ is a twice continuously differentiable convex function with $c_i(0) = 0$, $c_i'(0) = 0$, $c_i'(\tau)/c_i''(\tau) < 1$ and $\lim_{\tau \to 1} c_i(\tau) = +\infty$. The later assumption guarantees that no parent will completely determine the trait of his child. We assume that the cost of transmitting a particular trait is equal across parental types.\(^4\)

Since the parent’s choice of $\tau_a$ (or $\tau_b$) uniquely determines the probability $x$ that the child adopts the $a$-variant of the trait, we can, using (1) and (2), solve for $\tau_b$ and $\tau_a$ and write the nurturing cost directly as a function of $x$ (instead of $\tau_b$ and $\tau_a$):

$$C(x, q) = \begin{cases} c_a((x - f(q))(1 - f(q))^{-1}) & \text{if } x \geq f(q) \\ c_a((f(q) - x)f(q)^{-1}) & \text{if } x \leq f(q). \end{cases}$$

(3)

The assumptions imposed on $c_i(\tau)$ imply that $C(f(q), q) = 0$, $\lim_{x \to 0} C(x, q) = +\infty$ for all $q \in (0, 1]$ and $\lim_{x \to 1} C(x, q) = +\infty$ for all $q \in [0, 1)$.

Parents care about the child’s wellbeing in adulthood. We denote by $V^{ij}$ the utility a parent of type $i$ attaches to nurturing a child of type $j$. It is not the purpose of this paper to model the determination of $V^{ij}$. However, what is important is that the cultural variant adopted by the child is expected to affect the child’s wellbeing in adult life and that parents care about this. Hence, we will make no assumptions regarding the relative magnitudes of the $V^{ij}$’s. Instead we will consider two possibilities, either parental preferences are homogenous in the sense that the sign of the difference $V^{ia} - V^{ib}$ is the same for parents $i$ and $j$. We will call this case "melting pot"-incentives alluding to the general idea behind the melting pot theories that parents were all striving for a common American Dream. The other possibility is that parents disagree: We will call it demand for pluralism when $V^{ba} - V^{bb} < 0 < V^{aa} - V^{ab}$.

\(^4\)It is straightforward to generalize the analysis and assume that costs differs across type of parents. For instance that costs could be higher when transmitting the "other" variant. The results would not change qualitatively.
In their nurturing efforts, \(i\)-type parents hence choose the probability \(x \in [0,1]\) that the child acquires the \(a\)-variant by maximizing the expected value of offspring wellbeing net of nurturing costs. More formally, parents will solve:

\[
\max_{x \in [0,1]} (xV^{ia} + (1-x)V^{ib}) - C(x,q).
\] (4)

Before we go on to characterize the optimal nurturing efforts of \(a\)– and \(b\)–type parents, we need to explore the determinants of the process of oblique transmission function \(f(q)\). It is clear from (3) that this process is crucial for the effective costs of nurturing.

3 Peers and oblique transmission

We assume that the process of oblique transmission through which the young agent adopts cultural traits from society is the result of interaction with a group of peers. This group of peers, who are already socialized, is randomly selected from the population and we assume that the size of the peer group, \(G\), is exogenously given. Peer group interaction allows us to model the agents adoption of culture as the result of an evaluation of the relative merit of the variants of traits, observed in his peers, by the young agent.

In contrast, the standard assumption of previous economic models of cultural transmission has been that the young agent is randomly matched to one role-model from whom the variant is copied. This process gives rise to an unbiased oblique transmission function, \(f(q) = q\), (see Figure 1.i) and implies that children adopt each of the variants with a probability equal to their proportion in the community. However, as discussed in Boyd and Richerson (1985), oblique transmission need not be unbiased, in particular if young agents adopt cultural traits based on some form of evaluation of their merits.

There are three qualitatively different biases discussed in the sociobiological literature, positive, negative and frequency-dependent.

**Positive bias:** The probability that the naive individual acquires variant \(a\) is always greater than if he had selected a model at random, \(f(q) > q\) \(\forall q \in (0,1)\) (see Figure 1.ii).

**Negative bias:** The probability that the naive individual acquires variant \(a\) is always smaller than if he had selected a model at random, \(f(q) < q\) \(\forall q \in (0,1)\) (see Figure 1.iii).

**Frequency-dependent bias:** When the frequency of variant \(a\) in the community is greater (smaller) than one-half, the probability that a naive individual acquires \(a\) is increased (decreased) relative to the unbiased transmission, \(f(q) \geq q\) for \(q \geq 1/2\) (see Figure 1. iv).
In order to provide micro foundations for biased transmission, we introduce the notion of "merit". More precisely, we assume that after observing the cultural variants of his peers, the naive agent assigns merit to the different variants and follows a probabilistic rule where the probability of adopting a variant is given by its relative merit in the peer group. More specifically, assume that young agents assign merit \( m_i(n, G) \) to trait \( i \), where \( n \) is the number of \( i \)-type peers. As discussed in the introduction, this merit may depend on some form of predisposition, be inferred by a process of experimentation or from the "success" or salience of some peer(s) holding the trait. This inferred merit may also depend on the proportion, \( n/G \), of peers holding the trait. Without loss of generality, let

\[
m_i(n, G) = \tilde{m}_i + k \cdot g\left(\frac{n}{G}\right) \quad i = a, b
\]

where \( \tilde{m}_i \) and \( k \) are non-negative constants, \( g(0) = 0 \), \( g(1) = 1 \) and \( g\left(\frac{G}{G}\right) < g\left(\frac{G+1}{G}\right) \). If \( k = 0 \), merit is constant and independent of frequency.

Denote by \( P(i|n, G) \) the conditional adoption probability, i.e. the probability with which the young agent adopts trait \( i \) when \( n \) agents among the \( G \) peers are of type \( i \).

**Assumption 1:** The conditional probability of adopting the \( a \)-variant of the trait is given by the sum of the merit of the \( a \)-type peers relative to the total merit in the peer group:

\(^5\text{This would be rational if it was the case that evolutionary forces had acted so as to increase the frequency of better variants.}\)
\[ P(a|n, G) = \frac{n \cdot m_a(n, G)}{n \cdot m_a(n, G) + (G - n) \cdot m_b(G - n, G)}. \]  
(6)

It follows from this assumption that the naive agent can only adopt variants that are represented in the peer group and that the probability of adoption of a variant increases with the number of peers who hold the variant.

**Assumption 2.** The oblique transmission function \( f(q) \) is the unconditional probability with which the child adopts the \( a \)-variant,

\[ f(q) = \sum_{n=0}^{G} \binom{G}{n} q^n (1 - q)^{G-n} P(a|n, G). \]  
(7)

Assumption 2 states that the oblique transmission process results from merit based adoption of traits (Ass1) from a randomly selected peer group of size \( G \).

It will prove useful to derive some implications of the properties of the conditional adoption probability for the oblique transmission function \( f(q) \).

**Lemma 1.** \( f(q) = q \) if \( P(a|n, G) = \frac{n}{G} \) for all \( n = 0, 1, ..., G \).

**Proof.** See Appendix.

It is clear from Lemma 1 that if the agent has only one role model, \( G = 1 \), the only possible transmission function is unbiased. If \( G > 0 \), linear or unbiased transmission, \( f(q) = q \), requires very strong conditions on \( P(a|n, G) \). It is also with noting that, any degenerate conditional adoption probability i.e. \( P(a|n, G) \in \{0, 1 \} \forall n \) will give rise to non-linear transmission. Examples of such degenerate adoption rules are to follow the majority or to adopt the trait held by the most successful peers. Such degenerate adoption rules are assumed in the literature on herd behavior and informational cascades, Banerjee (1992) and Bikhchandani, Hirshleifer and Welch (1992), where conformism is indeed the outcome.

Proposition 1 shows, however, that a probabilistic, (non-degenerate) conditional adoption function as proposed in (6), allows us to generate all four transmission functions depicted in Figure 1.

**Proposition 1.** Under assumptions 1 and 2, the oblique transmission function is

(i) unbiased iff \( k = 0 \) and \( \bar{m}_a = \bar{m}_b \).

(ii) positively biased iff \( \bar{m}_a - \bar{m}_b > k(\frac{G-1}{G} - g(\frac{1}{G})) \) and \( \bar{m}_a > \bar{m}_b \).

(iii) negatively biased iff \( \bar{m}_a - \bar{m}_b < k(g(\frac{G-1}{G}) - g(\frac{1}{G})) \) and \( \bar{m}_b > \bar{m}_a \).

(iv) frequency dependent iff \( |\bar{m}_a - \bar{m}_b| < k(\frac{G-1}{G} - g(\frac{1}{G})) \) and \( m_a(G-1, G) < m_b(1, G) \).

**Proof.** See Appendix.

Unless merits are equal and independent of frequency, i.e. \( k = 0 \) and \( \bar{m}_a = \bar{m}_b \), (6) cannot generate unbiased transmission. This is the case because \( f(q) = q \) (Lemma 1, (i)) requires that \( P(a|n, G) = n/G \). Constant, but non equal, merits
always generate positive (negative bias) when \( \tilde{m}_a > \tilde{m}_b \) \( (\tilde{m}_a < \tilde{m}_b) \). Conformist oblique transmission arises only when the magnitude of the difference in the constant part of merits of traits is not too large relative to the importance given to frequency. In other words, conformism only arises if the young agent cares about frequency and perceives the different cultural variants as rather similar.

4 The dynamics of culture

We now return to the parent’s problem, which was to choose \( x \), the probability that the child adopts the \( a \)-variant, in order to maximize expected value of nurturing efforts net of costs:

\[
\max_{x \in [0,1]} (x V^{ia} + (1-x)V^{ib}) - C(x, q). \tag{8}
\]

The first order condition (for an interior solution) is:

\[
V^{ia} - V^{ib} = C_x(x, q) \quad i = a, b. \tag{9}
\]

Let \( x(q, W) \) be the solution to (8) where \( W^i = V^{ia} - V^{ib} \). Lemma 2 characterizes this optimal socialization probability under different assumptions regarding which type of the trait gives the highest wellbeing for the child.

**Lemma 2** Assume that \( C_{qq}(x, q) < 0 \), for all \( x \) then

(i) If \( W^i > 0 \), \( x(q, W^i) \) is increasing in \( q \) and \( W^i \), \( f(q) < x(q, W^i) < 1 \) for all \( q < 1 \), \( x(1, W^i) = f(1) \) and \( x_q(q, W^i) \leq f'(q) \).

(ii) If \( W^i < 0 \), \( x(q, W^i) \) is increasing in \( q \) and decreasing in \( W^i \), \( 0 < x(q, W^i) < f(q) \) for all \( q > 0 \), \( x(0, W^i) = f(0) \) and \( x_q(q, W^i) \leq f'(q) \).

**Proof.** See appendix.

It is worth noting that the optimal effort put forth by parents, \( \tau_i \), is decreasing in the proportion of the population having the trait parents are trying to promote.\(^6\) In the next section we will explore the consequences of non-linear oblique transmission for the dynamics of culture with a particular focus on when diversity is sustainable and when cultural types disappear.

We follow Hauk and Sáez-Martí (2002) and assume a Poisson process for births and deaths, holding the population size constant. Let \( \lambda \) be the probability that an adult survives from one period to the next. Also, let \( (1 - \lambda) \) be the probability that an adult bears a child that reaches adulthood (with certainty) a period later. Hence, each period, a fraction \( (1 - \lambda) \) of the adult population have just reached adulthood after having been born and socialized in the period before. It follows that the fraction of \( a \)-variants in the population evolves as:

\[
q_{t+1} = \lambda q_t + (1 - \lambda)(q_t x(q_t, W^a) + (1 - q_t)x(q_t, W^b)). \tag{10}
\]

\(^6\) Blin and Verdier (2000) refer to this as cultural substitution.
Eliminating time indices from (10) we can write the change in the fraction of a-types as
\[ \Delta q = (1 - \lambda)(q x(q, W^a) + (1 - q) x(q, W^b) - q). \] (11)
The net inflow of a-variants in the population is hence given by a convex combination of a-variant and b-variant parents’ optimal transmission probabilities characterized in Lemma 2, where the weights change with the population share \( q \).

We are interested in finding conditions under which (i) the different variants of the trait coexist in equilibrium even if all parents agree on which type is desirable; and (ii) when one type of the trait may disappear although parents strive to transmit their own traits. We will refer to the idea of agreement on what type of trait is best for children (i) as melting pot incentives \( (W^a = W^b) \) and to the idea that parents wish to promote their own trait as a demand for pluralism (ii).

First, let us show that diversity is sustainable under melting pot incentives. Let \( q(t, q_0) \) denote the path induced by equation (11) when the initial condition is \( q_0 \). Let \( E \) denote the set of steady states. Without loss of generality we will assume that \( W^i \neq 0 \), for \( i = \{a, b\} \).

**Proposition 2 (Melting pot incentives)** Assume \( W^i = W > 0 \ (i=a, b) \) then
(i) \( 1 \in E \)
(ii) if oblique transmission is unbiased or positively biased (in favour of trait a) then \( E = \{1\} \) and \( q(t, q_0) \to 1 \) for all \( q_0 \).
(iii) if oblique transmission is negatively biased (against trait a) \( E = \{q^*, 1\} \) with \( q^* \in (0, 1) \) and \( q(t, q_0) \to q^* \) for all \( q_0 \neq 1 \).
(iv) if oblique transmission is conformist and \( W \) small enough then \( E = \{q_1^*, q_2^*, 1\} \), \( q_1^* < q_2^* \), \( q(t, q_0) \to q_1^* \) for all \( q_0 < q_2^* \) and \( q(t, q_0) \to 1 \) for all \( q_0 > q_2^* \). For large enough \( W \), \( E = \{1\} \) and \( q(t, q_0) \to 1 \) for all \( q_0 \).

**Proof.** See appendix. ■

Proposition 2 shows that diversity is sustainable under melting pot incentives if the desired trait is difficult to acquire through oblique transmission. Because parents are in a sense working uphill, the "unwillingness" of children due to a negative bias or conformist pressure, to acquire the desired variant prevents the undesired variant from disappearing.

When children are conformist, there can be multiple stable steady states, but since all parents are promoting the a-variant, if the proportion of a-variants is high enough to start with, the undesired b-variant is bound to disappear as a result of conformist pressure. This opens up for the possibility of diversity or assimilation depending on initial conditions and has interesting implications for the effects of integration policies. We will discuss this further in the next section.

In Figure 2 we illustrate the cultural dynamics under melting pot incentives when the oblique transmission function is negatively biased and when there is conformism. We have plotted \( \Delta q \) as a function of \( q \in [0, 1] \). Stable rest point are
marked with filled squares mark and unstable rest points with empty squares. We can see that diversity can be sustained even in a situation when all parents promote the same cultural variant. First, when there is a negative bias against adopting the a-variant through peer interaction, culture will always be diverse. Second, when there is conformism, it is clear that the economy converges either to a diversified culture with a "low" fraction of a-types, or if there are enough a-types to start with, the b-variant is out-competed.

![Figure 2. Melting pot incentives](image)

Next we consider the type of parental preferences that have been analyzed in the previous literature, namely the demand for pluralism.

**Proposition 3 (Demand for pluralism)** Assume $W^b < 0 < W^a$ then

(i) $\{0,1\} \subseteq E$.

(ii) if oblique transmission is unbiased $E = \{0,q^*,1\}$ and $q(t,q_0) \rightarrow q^*$ for all $q_0 \in (0,1)$.

(iii) if oblique transmission is negatively biased (against trait a) $q = 1$ is always unstable and $q = 0$ is stable for small enough $W^a$. There may exist interior stable steady states.

(iii) if oblique transmission is positively biased (in favour of trait a) $q = 0$ is always unstable and $q = 1$ is stable for small enough $W^b$. There may exist interior stable steady states.

(iv) if oblique transmission is conformist, 0 is stable for small enough $W^a$, 1 is stable for small enough $W^b$. For small enough $W^a$ and $W^b$ there always exist an interior unstable steady state. For large enough $W^a$ and $W^b$ there always exist at least one interior stable steady state. There may exist multiple interior stable steady states.

**Proof.** See appendix.

Proposition 3 (ii) confirms the standard result in the literature, namely that if parents promote their own trait and oblique transmission is linear, diversity is guaranteed. However, this guarantee fails when oblique transmission is biased, since also desired traits may die out. When the bias is positive or negative, the variant at risk is the one with a bias against it. It is only when parents holding the variant unfavoured by the bias attach enough value to their children adopting it that there is room for coexistence of the two traits in equilibrium. We will
discuss this further in the concluding discussion. Intuitively, conformism can put all minority traits at risk of extinction.

Figure 3 depicts the cultural dynamics when there is demand for pluralism for different oblique transmission mechanisms. Is is evident that variants may be at risk, even if there is demand for pluralism. It is also evident that the cultural dynamics with biased oblique transmission are far more complex than when linearity is assumed.

![Graphs showing cultural dynamics](image)

**Figure 3. Demand for pluralism**

## 5 Conclusion

We have shown that cultural diversity is not guaranteed when parents holding different cultural traits promote their own culture. We have also shown that there can be diversity even if all parents promote the same cultural trait. Furthermore, by showing that biases in the oblique transmission function open up for multiple stable steady states, we have opened up for a richer understanding of the underlying driving forces of cultural assimilation and stratification.

For instance, it need not be correct to interpret the disappearance of a cultural type as resulting from a reversal of parental valuation of the type, which was the only possible interpretation using previous economic models of cultural transmission. Our model opens up for the possibility that a trait can disappear as a result of integration of a community into another one, not because parents change their mind about benefits but because the composition of peer groups change.

This insight is very important for understanding the effects of integration policies. In the previous understanding of cultural dynamics, an ever so small fraction of families with "bad" habits could contaminate a purely "good" neigh-
borhood. When there is conformism, this is no longer the case. Neither is it the case if children are biased in favor for the good trait.

Another important insight from our analysis lies in understanding the reasons for why economically dysfunctional or otherwise unhealthy cultural traits may persists. The previous implicit understanding has been that this is so because these dysfunctional cultural traits are promoted by parents as a result of a parental demand for cultural pluralism. We show that this is not the only possible explanation. In particular, if children are biased in favour of liking unhealthy food or of taking it easy, or even if they just like to eat what others eat or shirk as others do, parents of all types, may try in vain to promote good eating or work habits. When there is conformism or negative biases, it may be the case that, due to different initial conditions, in some communities parents have to put a lot of effort into combatting bad but persistent habits, while in other communities parents who exert scarce little nurturing effort need not worry much.

Our analysis shows that one needs to be careful in drawing conclusions about the reasons for why some "dysfunctional" traits persist and why some cultures are driven out of existence. Parental ambitions and incentives are indeed important, but because young individuals are not passive receivers of oblique transmission, the processes of assimilation and stratification are not only up to parents. A natural extension of the present paper is to start thinking about how parents and (or policy makers) can influence the biases in the process of oblique transmission.

References


6 Appendix

Proof of Lemma 1

\[ \sum_{n=0}^{G} \binom{G}{n} q^n(1-q)^{G-n} \frac{n}{G} = \sum_{n=1}^{G} \binom{G}{n} q^n(1-q)^{G-n} \frac{n}{G} \]

\[ q \sum_{n=1}^{G} \binom{G}{n} q^{n-1}(1-q)^{G-n} \frac{n}{G} = q \sum_{n=0}^{G-1} \binom{G-1}{n} q^n(1-q)^{G-1-n} = q \]

the last equality follows from the binomial theorem:

\[ \sum_{n=0}^{T} \binom{T}{n} q^n p^{T-n} = (q + p)^T. \]

Proof of Proposition 1

From lemma 1 it follows that if \( P(a|n, G) > (\leq)n/G \forall n \) then \( f(q) > (\leq)q \) for \( q \in (0, 1) \).

i) If \( k = 0 \) and \( \tilde{m}_a = \tilde{m}_a \) then \( P(a|n, G) = \frac{n}{G} \forall n \) and Lemma 1, applies.

ii) Assume now that \( k \neq 0 \). \( P(a|n, G) > n/G \) whenever

\[ \tilde{m}_a - \tilde{m}_b > k(g\left(\frac{G-n}{G}\right) - g\left(\frac{n}{G}\right)). \] (12)
The RHS in (12) is decreasing in $n$. $P(a\mid n, G) > n/G \forall n$ whenever (12) holds for $n = 1$.

iii) Assume now that $k \neq 0$. $P(a\mid n, G) < n/G$ whenever

$$m_a - m_b < k\left(g\left(\frac{G-n}{G}\right) - g\left(\frac{n}{G}\right)\right).$$

The RHS in (13) is decreasing in $n$. $P(a\mid n, G) < n/G \forall n$ whenever (13) holds for $n = G - 1$.

iv) Note first that $f'(0) = G(P(a\mid 1, G) - P(a\mid 0, G)) = GP(a\mid 1, G)$ and $f'(1) = G(P(a\mid 1, G) - P(a\mid G - 1, G) = G(1 - P(a\mid G - 1, G)$.

If

$$k\left(\frac{1}{G} - g\left(\frac{G-1}{G}\right)\right) < m_a - m_b < k\left(g\left(\frac{G-1}{G}\right) - g\left(\frac{1}{G}\right)\right)$$

then $P(a\mid 1, G) < 1/G$ and $P(a\mid G - 1, G) > (G - 1)/G$ and, $f'(0) < 1, f'(1) < 1$ and there is a unique $\hat{q} \in (0, 1)$ such that $f(\hat{q}) = \hat{q}$.

When $m_a = m_b$ and $k \neq 0$, $P(a\mid n, G) = 1 - P(a\mid G - n, G)$ and since

$$\binom{G}{n} = \frac{G}{(G-n)},$$

$$f(q) = 1 - f(1 - q)$$

and $f(1/2) = 1/2$.

**Proof of Lemma 2**

Assume that $W > 0$ and $q \neq 1$, then $x(q, W^i) > f(q)$ and $C(x, q) = c_a((x - f(q))(1 - f(q))^{-1})$. If follows from $c''_a > 0$ that $x(q, W)$ is increasing in $W^i$. To see this, from the first order condition we have that

$$W \equiv c'_a\left(\frac{x(q, W) - f(q)}{1 - f(q)}\right)(1 - f(q))^{-1}.$$

Differentiating with respect to $W$ gives:

$$1 = c''_a((x(q, W) - f(q))(1 - f(q))^{-1})(1 - f(q))^{-2}x_w(q, W).$$

Hence, $x_w(q, W) > 0$. To see that $x(q, W)$ is increasing in $q$, differentiate the first order condition with respect to $q$ and rearrange to get:

$$x_q(q, W) = f'(q)\left(\frac{1 - x(q, W)}{1 - f(q)}\right) - c'_a\left(\frac{x_a(q, W)}{c_a(q, W)}\right).$$

The assumption $C_{x_a}(x, q) < 0$ implies that the second term in the RHS is positive. Since $f'(q) > 0$, it follows that $x_q(q, W) > 0$. Moreover $x_q(q, W) \leq f'(q)$.

The proof for Lemma 1(ii) is analogous.

**Proof of Proposition 2**

Since $W^i = W$, it follows from (11) that $\Delta q = 0$ whenever $x(q, W) = q$. i) It follows from the fact that $x(1, W) = 1$.

ii) Since $x(q, W) > f(q)$ when $W > 0$ and $q < 1$ (lemma1) then $\Delta q > 0$ whenever $f(q) \geq q$. Evaluating the derivative of $\Delta q$ with respect to $q$ at $q = 1$ we obtain

$$\frac{d(\Delta q)}{dq}\bigg|_{q=1} = (1 - \lambda)(f'(1) - 1)$$
and \( q = 1 \) is stable whenever \( f'(1) \leq 1 \).

iii) If transmission is frequency dependent, \( f(q) \) has three fixed points, \( q = 0, \bar{q}, 1 \). Since \( x(0, W) > 0 \) and \( \lim_{W \to 0} x(q, W) = f(q) \), for small enough \( W \), \( x(q, W) \) has also three fixed points, \( E = \{ q_1(W), q_2(W), 1 \} \) with \( \lim_{W \to 0} q_1(W) = 0 \) and \( \lim_{W \to 0} q_2(W) = \bar{q} \). For large enough \( W \), \( x(q, W) > q \) for \( q < 1 \) and \( \lim_{W \to \infty} q(t, q_2) = 1 \).

**Proof of Proposition 3**

i) If \( W^b < 0 < W^a \), \( x(0, W^b) = 0, x(q, W^a) = 1 \) and \( \Delta q = 0 \) at \( q = 0, 1 \).

From (11) and the first order conditions,

\[
\Delta q = (1-\lambda)(c^{-1}_a(W^a(1-f(q)))q(1-f(q))-c^{-1}_b((-W^b)f(q))f(q)(1-q)+f-q)
\]

for \( W^b < 0 < W^a \). Taking the derivative with respect to \( q \) and evaluating at 0 and 1 we obtain:

\[
\frac{d(\Delta q)}{dq}_{q=0} = (1-\lambda)(c^{-1}_a(W^a) + f'(0) - 1)
\]

\[
\frac{d(\Delta q)}{dq}_{q=1} = (1-\lambda)(c^{-1}_b(-W^b) + f'(1) - 1)
\]

It is easy to see that:

a) \( f'(0) \geq 1 \) \( \Rightarrow \frac{d(\Delta q)}{dq}_{q=0} > 0 \) and \( f'(0) < 1, \frac{d(\Delta q)}{dq}_{q=0} \leq 0 \) whenever \( c^{-1}_a(W^a) \geq 1 - f'(0) \).

b) \( f'(1) \geq 1 \) \( \Rightarrow \frac{d(\Delta q)}{dq}_{q=1} > 0 \) and \( f'(1) < 1, \frac{d(\Delta q)}{dq}_{q=1} \leq 0 \) whenever \( c^{-1}_b(-W^b) \leq 1 - f'(1) \).

ii) That \( q = 0, 1 \) are unstable follows from a) and b). Since \( c^{-1}_a(W^a(1-q)) \) is decreasing in \( q \) and \( c^{-1}_b((-W^b)q) \) is increasing there is a unique interior rest point \( q^* \). Stability follows from the fact that \( c^{-1}_a(W^a(1-q)) \leq c^{-1}_b((-W^b)q) \) whenever \( q \leq q^* \).

iii) follows from a) and b).

iv) follows from a) and b).