Human Capital, Rent Seeking, and a Transition from Stagnation to Growth

by Nils-Petter Lagerlöf and Thomas Tangerås
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Abstract

We present a growth model where agents divide time between rent seeking in the form of resource competition; and working in a human capital sector, interpreted as trade or manufacturing. Rent seeking exerts negative externalities on the productivity of human capital, generating multiple steady states. Adding shocks to the model — in the form of violence in the rent seeking process, and changes in the size of the contested resource base — the model can replicate a long phase with stagnant incomes and high levels of rent seeking, interrupted by small failed growth spurs; this is eventually followed by a permanent transition to a sustained growth path where rent seeking vanishes in the limit. We illustrate the workings of the model with simulations and argue that the results, and what drives them, fit with some broad historical facts about growth, rent seeking, and the so-called natural resource curse.

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1 Introduction

A couple of hundred years ago Europe embarked on what is known as the Industrial Revolution: it started to exhibit sustained growth in per-capita incomes. This came together with many other social transformations. Levels of education started to rise, public school reforms were introduced, democracy started to spread, voting rights were extended, the labor force moved from agriculture to industry and from rural to urban areas, and the so-called Demographic Transition set in: first mortality rates fell, and thereafter fertility rates.

In a sense, the Industrial Revolution seems to have also brought with it a reallocation of agents’ time and efforts, away from rent seeking and into productive activities. In pre-industrial Europe, and (to even larger extent) in early human civilizations, those classes whose incomes were mainly based on the land they held, or the conquest of new land – that is, the landed aristocracy and the military – seem to have had higher status, power, wealth, and incomes, than the classes who made their living from skills, rather than land. Industrialization resulted in the rise of a skilled urban class, and the demise of the landed aristocracy. This was reflected in the transition from agriculture to industry, and the process of urbanization. It also fits with changes in career choices of university graduates in England around the time of the Industrial Revolution, away from, for instance, the land-owning profession and into more urban, high-skilled professions, such as banking (Doepke and Zilibotti 2005, Table 2).

This paper links a growing literature on what caused the transition from stagnation to growth to an even larger literature on rent seeking. We set up a growth model where agents divide time between rent seeking and productive activities. Rent seeking could be, for example, military competition for land and other resources: that is, a pure zero-sum game. The productive activity is one where agents use their human capital, which could be interpreted as trade or manufacturing. The model generates an endogenous transition from a situation where per-capita incomes are stagnant and agents spend all their
time in rent seeking, to a path where per-capita incomes grow at a sustained rate and rent seeking vanishes in the limit. Our results fit with many stylized facts about long-run development.

The first important component of our model is an externality from rent seeking on the productivity of human capital. One way to think of this is that rent seeking can be associated with violence. When rent seeking is the most violent — amounting to outright warfare — it adversely affects, for example, trade (see discussion later).

Another element of our model is stochastic. We consider two settings. In the first, we let the parameter which links rent seeking to human capital productivity be stochastic. We interpret this as the economy being hit by violence shocks. These determine how violent forms rent seeking takes: the more violent, the worse are the detrimental effects from rent seeking on human capital productivity.

The central mechanism driving the results is that the economy alters its response to the shocks over the course of development. The distribution from which the shocks are drawn is exogenous and time-invariant, but as rent seeking declines the economy becomes insulated from the shocks. When human capital levels are low agents spend all their time in rent seeking so violence shocks have devastating effects; when human capital levels are high agents spend almost no time in rent seeking so the shocks have negligible effects.

This gives rise to quite rich dynamics. The economy can be stuck for very long in a stagnant trap with low levels of human capital and per-capita incomes, and agents engaging only in rent seeking. Once a mild enough violence shock arrives agents re-allocate some time away from rent seeking into the human capital sector. Since rent seeking is a zero-sum activity, this raises incomes of all agents. This leads to more human capital investment, thus further mitigating rent seeking and violence also in the next period. If the economy is spared adverse shocks for sufficiently long it converges to a path with sustained growth in human capital and per-capita incomes, and
declining rent seeking. In the limit rent seeking vanishes altogether, making the economy immune to violence shocks.

Early on in this take-off, however, the economy is vulnerable, and a bad violence shock can push the economy back into poverty. Our simulations illustrate how an economy can go through temporary spurts – phases of drops in rent seeking and rising human capital levels – followed a few periods later by a reversion into full rent seeking and human capital falling back to its earlier constant levels. This fits with the observed rise-and-fall patterns of urban and prosperous regions in European history (De Long and Shleifer 1993), as well as growth miracles and “melt-downs” in modern times.

We also consider a setting where the economy is hit by resource shocks. These change the size of the pie that agents compete for when engaging in rent seeking. Many of the qualitative results are the same as in the setting with violence shocks: a long phase of stagnation and “false starts” is followed eventually by a growth take-off and the vanishing of rent seeking; once the economy has embarked on a balanced growth path it becomes insulated from further resource shocks, and does not contract back into poverty.

Also in this setting the most interesting result refers to how the economy’s response to the shocks changes over the course of development. At early stages of development, when agents spend all their time in rent seeking, an increase in resources raises incomes. This is consistent with the pattern of early human civilizations, whose prosperity depended largely on their resource endowments. Resource depletion (man-made or otherwise) often led to their downfall: consider, for example, the Maya and Easter Island civilizations (Brander and Taylor 1998, Diamond 2005).

When the economy has just begun to experience a growth take-off the resource shocks have the reverse effect: a positive resource shock reduces incomes. This is because they increase rent seeking and thus lower productivity of human capital. Such detrimental effects plague many countries in the world today, which is known as the natural resource curse. For example, oil discoveries have ruined countries like Nigeria (Sala-i-Martin and Subra-
Finally, on the balanced growth path resource shocks cease to have any
effect at all, since incomes earned from resource competition come to be
dwarfed by those earned from human capital. This result is in line with
the fact that resource endowments do not explain much of the variation in
per-capita incomes between different rich countries in the world today.

The rest of this paper is organized as follows. Next Section 2 relates this
paper to some existing work. Section 3 cites some empirical facts that are
consistent with our results and/or assumptions. Section 4 sets up the model
where the stochastic element comes from so-called violence shocks. Section
5 analyzes a setting where the shocks instead change the size of the resource
base. A concluding discussion is given in Section 6.

2 Existing literature

This paper relates to several strands of literature, much of which will be
discussed in more detail as we present the model and document some sup-
porting facts. However, to give an overview we may think of our story in
light of two broad fields of literature: long-run growth models; and models
of rent seeking and conflict.

The literature on long-run growth tries to explain the take-off from stag-
nation to growth that started in Western Europe a couple of centuries ago,
as well as many related changes occurring in the same process and around
the same time (Goodfriend and McDermott 1995; Tamura 1996, 2002, 2005;
Galor and Weil 2000; Jones 2001; Kögel and Prskawetz 2001; Galor and
Moav 2002; Hansen and Prescott 2002; Lagerlöf 2003a,b, 2005a; Doepke and
Zilibotti 2005; see Galor 2005 for an overview). Some of these (for instance,
Galor and Weil 2000) emphasize an element of endogenous fertility, which
we abstract from here.\footnote{However, as discussed later, our exogenous resource shocks could be interpreted as
changes in population pressure.} Others focus on the transition from agricultural to
industrial production (for example, Kögel and Prskawetz 2001, Hansen and Prescott 2002, and Tamura 2002), or the rise of an urban class (Doepke and Zilibotti 2005); this relates more closely to the theme of this paper.

The major contribution we make to this literature is twofold. First, we think of the transition from stagnation to growth, and from rural/agricultural production to urban/industrial, in the context of a parallel transition away from rent seeking and conflict into production based on human capital. This brings many new insights. In particular, we argue that several results referring to rent seeking, war, and the natural resource curse are consistent with some interesting long-run growth facts (as discussed later in Section 3).

Second, our model contains an element of stochastic shocks, the effects of which are altered endogenously in the course of development. This enables our model to explain both an eventual permanent take-off from stagnation to sustained growth, and several “false starts” preceding it.2

There is also a large literature on rent seeking and conflicts. Some early seminal contributions include Hirshleifer (1988, 1989), Grossman (1991), Murphy et al. (1991, 1993), Skaperdas (1991, 1992), Grossman and Kim (1995); for a recent overview see Skaperdas (2003). We share with this literature a general idea that some resources are allocated not through trade on markets, but rather through (potentially violent) conflict. One difference is that these papers often (but not always) use static models; to explain changes over time in rent seeking and other variables we need a dynamic model.3

2In this respect our paper shares some mechanical features with Lagerlöf (2003a,b), who allow for stochastic shocks to population (epidemics) in a model with endogenous fertility, to explain a transition from stagnation to growth. Likewise, Lagerlöf (2005b) allows for war shocks in a model of resource competition and endogenous mortality. Different from these papers, here we endogenize the choice between rent seeking and working in a human capital sector.

3Of course, some papers do analyze rent seeking and conflicts using dynamic settings: see, for example, Murphy et al. (1991), Tornell and Lane (1999), Gonzales (2005), and Tangerás and Lagerlöf (2005). However, these models are not designed to account for take-offs from stagnation to growth. In that respect, our model relates more closely to the long-run growth literature cited above.
Many of these papers model externalities from rent seeking to productive activities, which can give rise to multiple equilibria with different levels of rent seeking. For example, in environments with many rent seekers producers may face higher probability of having their output expropriated, thus making productive activities less profitable, which can sustain an equilibrium where many agents choose to engage in rent seeking. Alternatively, entrepreneurial activity can destroy the scope for rent seeking through various channels, generating similar self-reinforcing mechanisms. (See, for example, Acemoglu 1995, Baland and Francois 2000, Torvik 2002, and further references therein.) We allow for similar externalities (although in a somewhat more black-boxed manner) and our model could in principle also generate such multiplicity (see Footnote 7); in our dynamic setting this would correspond to indeterminate growth paths. However, for the purpose of this paper we restrict attention to the case where, in each period, there is only one equilibrium. Instead, we focus on the dynamic aspects of the model and the associated multiplicity of steady states, and the results when introducing shocks to this environment.

3 Empirical support

3.1 The transition from stagnation to growth

To simplify, one can say that in the world today Western Europe and some of its colonial offshoots have high and growing per-capita incomes; most other countries have low and non-growing per-capita incomes. However, today’s rich countries had stagnant per-capita incomes up until about the late 18th, or early 19th, centuries. Over the period 1500-1820 the average per-capita GDP growth rate for 12 Western European countries (among others, the UK, Germany and France) is estimated to have been a modest 0.14% per year; over the period 1820-70 the growth rate rose to 1.04%, and over the 1870-1913 period to 1.33% (Maddison 2003, Table 8b). This take-off from stagnation to growth is commonly known as the Industrial Revolution. (See,
for example, Galor 2005 for an overview.)

Up until not too long ago most growth models could not account for such
take-offs. Rather, they would typically predict a monotonically declining
growth rate as an economy approaches its balanced growth path (conver-
gence). We thus add to a new generation of growth models where such
transitions can be explained (as discussed in Section 2 above).

### 3.2 Human capital is less subject to rent seeking

A central mechanism in our model is that an increase in human capital in-
duces agents to reallocate time away from rent seeking to working in the
human capital sector. For this to make sense we must assume that (income
from) human capital cannot be expropriated, or taxed. That is, rent seek-
ing is here defined as competition over resources other than human capital,
typically land or natural resources.

This should not be interpreted too literally. Throughout human history
human capital has at least occasionally been heavily taxed. However, in
European history human capital seems to have been more difficult for rulers
to tax than income from land, since it is more mobile. For example, in
medieval times when despotic monarchs tried to extract tax revenues from
prosperous European city states this eventually led to the cities’ decline,
and the rise of freer cities elsewhere (De Long and Shleifer 1993, De Long
2000). This mechanism also relates to migrations following religious and
minority persecution in European history. Following the pogroms during
the first crusade and the Spanish Inquisition, literate and educated Jews
migrated to Lithuania and the Ottoman Empire, respectively, contributing
to the economic upswing of those areas (Ben-Sasson 1976). In fact, abundant
human capital may in and by itself facilitate mobility: as argued by Botticini
and Eckstein (2005, Section 7), high Jewish skill levels dating millennia back
can explain historical Jewish migrations, and Jewish Diaspora to this day. It
seems plausible that high skill levels facilitated migration among non-Jewish
people too.
One form of rent seeking could of course be slavery, which amounts to taxing or expropriating someone’s labor. However, this need not be the same as taxation or expropriation of human capital. Fenoaltea (1984) argues that slavery has mostly been used to extract low-skilled labor; high-skilled tasks tend to be performed by free workers. In that sense, the transition away from rent seeking in our model could at least broadly be thought of in the context of the transition from slavery to free labor.

3.3 The rise and fall of the natural resource curse

If rent seeking is detrimental to growth, resource abundance can, by inducing more rent seeking, delay a take-off from stagnation to growth and make countries poorer. This is, in essence, the natural resource curse, and it seems to describe the experience of many poor countries in the world today (see, for example, Sachs and Warner 1995; Gylfason et al. 1999; Atay 2001; Sala-i-Martin and Subramanian 2003; Isham et al. 2005).

The existing cross-country empirical literature, however, focuses on modern times. In a historical perspective natural resources may have played the opposite role. Humans have obviously relied on different types of resources at different stages of development, but a general pattern may still be discerned. The earliest civilizations arose in what was then the Fertile Crescent, and forests and fertile soils have been necessary for the evolvement and survival of many other historic civilizations. The downfall of, for instance, the Maya and Easter Island civilizations followed the depletion of their resource bases (Brander and Taylor 1998, Diamond 2005). That is, natural resources was at that stage of development largely a blessing (and their depletion was the curse).

Resource abundance started to be more of a curse concurrent with a number of economic changes beginning in the centuries leading up to the Industrial Revolution. A case in point could be Spain. Initially, the discovery and extraction of silver and gold in the American colonies had a direct positive effect on Spain’s incomes, in the sense that it enabled military expansion.
However, in the longer run the resulting fiscal expansion created adverse institutional baggage, such as the practice of selling tax exemptions, which in turn delayed Spain’s industrialization. This process has been described and modelled by Drelichman (2005); see also Acemoglu et al. (2005). In a sense, Spain’s windfall gains thus had opposite effects in the long and short run.

Finally, among rich and growing countries in the world today natural resources seem to be neither curse, or a blessing. In a sense, this relates to Sala-i-Martin and Subramanian (2003) who find that the resource curse does not apply to countries with good institutions, which typically means rich and growing countries. Some anecdotal and casual empiricism would suggest the same: for example, the income gap between the United States and Europe has little to do with abundance of natural resources on either side of the Atlantic. Resource scarcity has not prevented Luxembourg, Switzerland, or Japan from growing rich; neither has resource abundance (in the form of, for example, oil) been a curse for Norway or Alaska.

To sum up, our readings of the facts suggest that an abundance of resources was once a blessing (in early human civilizations); later a curse (in economies on the verge of a take-off); and eventually had little effect at all, positive or negative (in countries exhibiting sustained growth).

### 3.4 Negative externalities from rent seeking

As described already, we assume that rent seeking negatively affects the productivity of human capital. There are many ways to motivate this. One can think of productivity of human capital as dependent on something we could call “institutions.” Sala-i-Martin and Subramanian (2003) and Isham et al. (2005) study the links between natural resource abundance, per-capita incomes, and institutions across countries today, where institutions are measured in similar ways as in the recent related literature (for example, Acemoglu et al. 2001, 2002), as indices over property rights protection, etc. They find that the link from natural resources to economic outcomes indeed works through institutions.
Another motivation could be that rent seeking – in the sense of competition for land and resources – sometimes becomes violent, thus potentially impeding activities which are based on human capital, such as trade. Looking at data over trade and war across a large set of countries since 1870, Glick and Taylor (2005) find that wars do reduce trade (not only for belligerent nations but also neutrals). Moreover, the negative trade effects of war do not go away when peace breaks out, but linger on long after. For example, Glick and Taylor (2005) find that the negative effects on trade from the first world war continued almost into the 1930’s, and the effects of the second world war until the mid 1950’s.

4 The Model

Agents live in overlapping generations for two periods, as children and adults. Adults consume and invest in their children’s human capital; children consume nothing and are passive (other than receiving human capital investment from their parents). Each agent has one offspring so population is constant, and population size is denoted $P$.

Adults earn income from two sources. They can engage in rent seeking, here meaning zero-sum competition for a finite resource (for instance land) which yields a fixed total amount of output, $A$; and they can work in something which may be thought of as a trading or manufacturing, where their pay depends on their human capital. The human capital of an agent being adult in period $t$ is denoted $H_t$.

Each agent has one unit of time to divide between working in the human capital sector and engaging in rent seeking. The income earned from rent seeking is given by a Tullock-type of contest function: an agent who spends $r_t \in [0, 1]$ units of time in rent seeking receives a fraction of the total pie ($A$) proportional to her fraction of the total amount of time spent in rent seeking; that fraction is given by:

$$\frac{r_t}{r_t + (P - 1)R_t},$$

(1)
where $R_t$ is the average time spent in rent seeking among other agents, and (recall) $P$ is total population.\textsuperscript{4}

The next component of our model is a negative externality from rent seeking on the productivity of human capital. In this section we shall interpret this externality as an effect of violence in society. That is, rent seeking can take different forms, amounting to peaceful disputes, outright warfare, or anything in between. The more violent forms that rent seeking takes, the more adverse are the effects on the productivity of human capital.

Moreover, in this section we let the degree of violence in rent seeking be random. More precisely, the total amount of violence is given by $R_t \rho_t$, where (recall) $R_t$ is the average time each agent spends in rent seeking, and $\rho_t$ is a random variable drawn independently at each $t$ from a uniform distribution on the interval $[\underline{\rho}, \overline{\rho}]$, where $0 \leq \underline{\rho} < \overline{\rho} < 1$. The higher is $\rho_t$ the more violent forms rent seeking takes, if any rent seeking takes place at all (that is, if $R_t > 0$). If there is no rent seeking ($R_t = 0$), there is no violence.

The income earned per unit of time spent in the human capital sector, by an agent endowed with $H_t$ units of human capital, is given by $\Omega(R_t \rho_t) H_t$, where $\Omega'(R_t \rho_t) < 0$. We assume that $\Omega(0) \equiv B > 0$. This formulation implies that if rent seeking is eliminated ($R_t = 0$) the economy becomes insulated from violence shocks.

For the rest of this paper we shall use the following functional form, which generates nice closed-form solutions:

$$\Omega(R_t \rho_t) = B \left(1 - R_t \rho_t\right). \quad (2)$$

Note that $\overline{\rho} < 1$ and $R_t \leq 1$ ensures that $\Omega(R_t \rho_t) > 0$ for all feasible $R_t$ and $\rho_t$.

\textsuperscript{4}Formally, we may define this fraction as equal to $1/P$ if $r_t + R_t = 0$, and as in (1) if $r_t + R_t > 0$. As can be understood when setting up the maximization problem later, $R_t = 0$ cannot be an equilibrium: an atomistic agent taking $R_t = 0$ as given and setting $r_t = 0$ gets a share $1/P < 1$, but would grab the whole pie by instead setting $r_t$ arbitrarily small but strictly positive.
The total income of an atomistic agent who spends $r_t$ units of time in resource competition is thus given by

$$Y_t = \left[\frac{r_t}{r_t + (P - 1)R_t}\right] A + (1 - r_t)B (1 - R_t\rho_t) H_t.$$  \hfill (3)

The utility function of the same agent is given by

$$U_t = (1 - \beta) \ln c_t + \beta \ln H_{t+1},$$  \hfill (4)

where $c_t$ is the adult’s own consumption and $H_{t+1}$ is the human capital invested in the (single) child.\footnote{Letting the child’s human capital directly enter the parent’s utility function is the easiest way to create a link from income today to human capital tomorrow. One could alternatively assume that parents care about their children’s total income, or welfare. However, that would greatly complicate the analysis without generating much further insight.} Purely for simplicity we assume that one unit of human capital is produced using one unit of the consumption good:

$$c_t = Y_t - H_{t+1}.$$  \hfill (5)

The agent chooses $H_{t+1}$ and $r_t$ to maximize (4), subject to (3) and (5), and the constraint that $r_t \in [0, 1]$.\footnote{Note that agents choose $r_t$ knowing $\rho_t$, that is, either before, or simultaneously with, the realization of $\rho_t$. This is necessary for the realization of $\rho_t$ to have an impact on $r_t$ (and thus $R_t$), which is a central part of our story.} This gives:

$$H_{t+1} = \beta Y_t,$$  \hfill (6)

and

$$\frac{(P - 1)R_t A}{[r_t + (P - 1)R_t]^2} \geq B (1 - R_t\rho_t) H_t,$$  \hfill (7)

where the weak inequality is strict if $r_t = 1$, and holds with equality if $r_t < 1$.

### 4.1 Equilibrium rent seeking

Since all agents are identical, in a symmetric equilibrium they choose the same $r_t$, that is: $r_t = R_t$. As shown in Section A.1 of the appendix, (7) gives...
the equilibrium amount of time in rent seeking as follows:

\[
R_t = \begin{cases} 
1 & \text{if } H_t \leq \hat{H}_t \\
\frac{1}{\rho_t} \left( \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{\rho_t \gamma}{BH_t}} \right) & \text{if } H_t \geq \hat{H}_t 
\end{cases}
\] (8)

where

\[
\hat{H}_t = \frac{\gamma}{B(1 - \rho_t)},
\] (9)

and

\[
\gamma = \frac{A(P - 1)}{P^2}.
\] (10)

In words, \(\hat{H}_t\) is the threshold level that \(H_t\) must exceed for agents to allocate any time at all to working in the human capital sector. This threshold is increasing in \(\rho_t\). That is, the more violent forms that rent seeking takes the more likely are agents to allocate all their time to rent seeking, as follows from the productivity of human capital being decreasing in the degree of violence.

Note that \(R_t\) is decreasing in \(H_t\) for all \(H_t \geq \hat{H}_t\), and that \(R_t\) goes to zero as \(H_t\) goes to infinity. That is, accumulation of human capital reduces rent seeking by inducing agents to spend more time in the human capital sector instead.

As described in Section A.1 of the appendix, the quadratic equation which defines the set of equilibria [see (28)] also has the root

\[
R_t = \frac{1}{\rho_t} \left( \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\rho_t \gamma}{BH_t}} \right).
\]

However, this equilibrium (if it exists) has many peculiar features: it implies that total violence, \(R_t\rho_t\), is decreasing in the violence shock, \(\rho_t\); and that \(R_t\) is increasing in \(H_t\), so that sustained growth in \(H_t\) makes \(R_t\) go to one rather than zero. Moreover, for \(\rho_t < 1/2\) the positive root cannot be an equilibrium since that would imply \(R_t > 1\); thus, if \(\overline{\rho}\) (the upper bound for \(\rho_t\)) is less than 1/2 this root can never be an equilibrium.
4.2 Income and dynamics

We can set $R_t = r_t$ in (3) to derive an expression for equilibrium income:

$$Y_t = \frac{A}{P} + (1 - R_t)B(1 - R_t\rho_t)H_t.$$  

Using equilibrium rent seeking in (8), and the definition of $\gamma$ in (10), it is shown in Section A.2 of the appendix that we can write this as:

$$Y_t = \begin{cases} \frac{A}{P} & \text{if } H_t \leq \hat{H}_t \\ \frac{A}{P} + BH_t \left( \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\gamma\rho_t}{BH_t}} \right) & \text{if } H_t \geq \hat{H}_t \end{cases},$$  \hspace{1cm} (11)

where (recall) $\hat{H}_t$ is defined in (9), and $\gamma$ in (10).

To see the intuition behind (11) it is useful to first close down the rent seeking externality, that is, setting $\rho_t = 0$. For $H_t$ above $\hat{H}_t$ income then becomes $\frac{A}{P^2} + BH_t$. A marginal increase in $H_t$ thus raises income by $B$, independently of how much time is spent in rent seeking. Intuitively, there is a direct effect on income while holding fixed the level of rent seeking, as given by $(1 - R_t)B$. There is also a time reallocation effect, as given by $-(\partial R_t/\partial H_t)BH_t$. The latter effect occurs because rent seeking is a zero-sum activity; therefore, when all agents spend less time in rent seeking everyone’s income rises. With the functional forms used here the combined marginal effect of a rise in human capital is the same as if there was no rent seeking at all (that is, $B$).\(^8\) If $\rho_t > 0$ an increase in human capital has a third effect, by reducing rent seeking and thus raising productivity in the human capital sector.

Using the expressions for human capital investment in (6) and income in

\(^8\)Using (27) in the appendix [or applying l’Hôpital’s rule to (8)] one can see that $\rho_t = 0$ gives $R_t = \gamma/(BH_t)$. Using (3), setting $r_t = R_t$ and $\rho_t = 0$, income in equilibrium is seen to equal $Y_t = A/P + (1 - R_t)BH_t$. It is then straightforward to verify that

$$\frac{\partial Y_t}{\partial H_t} = (1 - R_t)B - \left( \frac{\partial R_t}{\partial H_t} \right) BH_t = B.$$
(11) we get a dynamic equation for human capital:

\[ H_{t+1} = \begin{cases} 
\frac{\beta A}{T} & \text{if } H_t \leq \hat{H}_t \\
\beta \left[ \frac{A}{2} + B H_t \left( \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{2\rho t}{BH_t}} \right) \right] & \text{if } H_t \geq \hat{H}_t
\end{cases} \tag{12} \]

where (recall again) \( \hat{H}_t \) is defined in (9), and \( \gamma \) in (10).

It can also be seen from (12) that sustained growth in \( H_t \) requires that \( \beta B > 1 \), which we hereafter assume holds.\(^9\)

It is informative to analyze how the model behaves if \( \rho_t \) is held fixed, at some level \( \rho \). Figure 1 illustrates the dynamics for one numerical example (used in Section 4.4 later), in a 45-degree diagram where \( \rho \) takes either one of two values. For small enough \( \rho \) (such as \( \rho = 0.3 \) in this numerical example) the economy exhibits sustained growth regardless of initial conditions. For higher \( \rho \) (here \( \rho = 0.5 \)) the economy is characterized by multiple steady states: if the economy starts off with a human capital level above a critical point it exhibits sustained growth; if it starts off slightly below it jumps (almost) immediately to a steady state with constant human capital. This multiplicity hinges on the link from rent seeking to human capital productivity (as captured by \( \rho \)) being strong enough. Low levels of human capital today means a lot of rent seeking today; the larger is \( \rho \) the more adverse are the effects from rent seeking on the productivity of human capital, and thus on incomes today and human capital tomorrow.

### 4.3 Transitions

A setting where \( \rho \) is constant can thus generate underdevelopment traps through a standard multiple steady-state story, but it fails to explain how a prolonged period of stagnant incomes can be followed by a transition to sustained growth. As we shall see next, letting \( \rho_t \) fluctuate randomly can generate such a pattern.

\(^9\)That is, \( H_{t+1}/H_t \) goes to \( \beta B \) as \( H_t \) goes to infinity, so if \( \beta B > 1 \) it holds that \( H_t \) can grow indefinitely.
Given the assumption that $\rho_t$ is uniformly distributed on $[\rho_h, \rho_l]$ it can be seen that $\hat{H}_t$ is uniformly distributed on $[H, \overline{H}]$, where

$$H = \frac{\gamma}{B(1 - \rho)}$$

and

$$\overline{H} = \frac{\gamma}{B(1 - \overline{\rho})}.$$ 

(13)

(14)

Since the distribution of $\rho_t$ is time independent, so is the distribution of $\hat{H}_t$. That is, the threshold fluctuates over time but is stationary and does not follow any trend.

We assume that

$$H < \frac{\beta A}{P} < \overline{H},$$

(15)

where $H$ and $\overline{H}$ are given by (13) and (14). This means that a hypothetical economy which starts off with $H_0 = \beta A/P$, and is constantly hit by the maximum violence shock ($\rho$) is stuck in a stagnant trap with constant human capital and per-capita incomes. If the same economy is instead constantly hit by the minimum violence shock ($\overline{\rho}$) it breaks out of poverty and onto a sustained growth path.

Human capital stays constant at $\beta A/P$ as long this falls below the fluctuating threshold, $\hat{H}_t$. Sooner or later comes a period when $\rho_t$ and thus $\hat{H}_t$ are sufficiently low, so that the actual human capital level, $\beta A/P$, comes to exceed the threshold. As a result, agents start allocating less time to rent seeking. This leads to less violence, a higher return to human capital, higher incomes, and thus also more human capital invested in the next generation. This in turn makes the actual human capital level in the next period more likely to exceed the threshold realized in that period, in turn making it more likely that human capital rises further. This creates a virtuous cycle where, period by period, the economy reaches higher human capital levels, insulating it from further shocks.

Early on in this take-off the economy stays vulnerable; a severe enough violence shock can wipe out all gains and push the economy back into poverty.
But once human capital exceeds the maximum threshold level, $\mathcal{T}$ in (14), the economy can no longer fall back into the poverty trap. From there on the economy embarks on a sustained growth path where human capital grows indefinitely, and in the limit agents spend all their time in the human capital sector.

### 4.4 Simulation results

We next simulate the model. First we choose values for the exogenous parameters $\beta$, $A$, $P$, and $B$, and calculate the associated value for $\gamma$ from (10). We then generate a sequence of violence shocks, $\rho_t$. Setting an initial value for human capital, $H_0$, the algorithm is then straightforward: given $\rho_0$ and $H_0$, we use (12) to compute $H_1$; using $\rho_1$ and $H_1$ we compute $H_2$ the same way; and so on. This generates a sequence of human capital stocks, which then gives the amounts of time allocated to rent seeking in each period from (8), and levels of income from (11).

Figure 2 shows the result of one such simulation. The parameter values are chosen as follows: $A = 100$; $P = 1000$; $\bar{p} = 0.9$; $\underline{\rho} = 0.3$; $B = 5$; and $\beta = 0.346$. This is one set of values consistent with (15). The choices of $B$ and $\beta$ imply that the economy grows by 2.2% per year on the balanced growth path, if each period corresponds to 25 years. $^{10}$ Initial human capital is set equal to its stagnant level, $\beta A/P$.

Since $\rho_t$ is random no two runs are identical, but they all have the same qualitative features. As seen in Figure 2 the economy is stuck in a stagnant state for a number of generations (about 50 in this case), where human capital is roughly constant. Throughout this stagnant phase of development the economy makes several “false starts,” that is, short spurts of growth associated with temporary reductions in rent seeking. These are triggered by mild violence shocks (low $\rho_t$’s) making the threshold human capital level

$^{10}$The annual per-capita income growth rate is calculated as $(Y_{t+1}/Y_t)^{(1/25)} - 1$. This converges to $(\beta B)^{(1/25)} - 1$ on the balanced growth path, which equals about 2.2%, given how we set $\beta$ and $B$. 
fall below actual human capital. In the same manner, they are punctured by a subsequent severe shock (a high $\rho_t$), which pushes the threshold human-capital level above actual human capital, thus making human capital revert back to its stagnant level, $\beta A/P$, in the next period. Only by experiencing a series of sufficiently mild shocks can the economy break out permanently from stagnation, as the actual human capital levels become high enough to insulate the economy from even the worst shocks.

The productivity of human capital is given by $B(1 - R_t \rho_t)$; see (2). Its path is shown in the lower-right panel of Figure 2. Note that the path is volatile as long as $R_t$ is relatively close to one. Once human capital starts to grow and $R_t$ goes to zero, human capital productivity stabilizes at the level $B$ (here set to 5). Likewise, the annualized income growth rate in the lower-left panel fluctuates around zero until it rises and stabilizes at 2.2% per year.

5 Resource shocks

In the setting presented thus far changes in the total amount of resources – that is, the size of the pie for which agents compete, $A$ – has two opposing effects on incomes. On the one hand, a larger pie has a direct positive effect on incomes as all competing agents get more to share; on the other, it induces more time allocated to rent seeking, thus reducing productivity in the human capital sector, leading to lower incomes from human capital.

Next we examine a setting where the size of the pie fluctuates randomly over time. To this end, we make two simplifications. First we let the violence parameter (linking rent seeking to human capital productivity) be constant, so that we can skip the time index on $\rho$.

Second, we let both $A$ and $P$ go to infinity, while keeping the ratio $A/P$ finite and strictly positive; this ratio can then be seen to equal the parameter $\gamma$ as defined in (10). We let this parameter evolve over time according to this
simple process:

\[ \gamma_{t+1} = \gamma_t \varepsilon_t, \]  

(16)

where \( \varepsilon_t \) is uniformly distributed on \([\underline{\varepsilon}, \overline{\varepsilon}]\), and \( 0 < \underline{\varepsilon} < \overline{\varepsilon} \).

Since \( \gamma_t \) measures resources per agent the process in (16) has many interesting interpretations. A rise in \( \gamma_t \) (a high draw of \( \varepsilon_t \)) could amount to the discovery or conquest of new land, or a new resource, or the invention of a new technology used to harvest the land/resource, or a fall in the population competing for the land/resource. A fall in \( \gamma_t \) (a low draw of \( \varepsilon_t \)) could be due to the depletion of the land/resource, or a rise in the population competing for it.

### 5.1 Equilibrium rent seeking

The preferences and budget constraints are the same as in (3) to (5), so the model can be solved exactly as in Section 4. The only difference is that \( \gamma_t \) now carries a time index, and \( \rho \) does not. Analogous to (8) the expression for equilibrium rent seeking thus becomes:

\[
R_t = \begin{cases} 
1 & \text{if } H_t \leq \widehat{H}_t \\
\frac{1}{\rho} \left( \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{\rho \gamma_t}{B H_t}} \right) & \text{if } H_t \geq \widehat{H}_t 
\end{cases},
\]  

(17)

where, analogous to (9),

\[
\widehat{H}_t = \frac{\gamma_t}{B (1 - \rho)}.
\]

### 5.2 Income and dynamics

Income is now given by an expression which differs slightly from (11): the first row now becomes \( \gamma_t \); and the first term in the second row becomes \( \gamma_t / P \), which equals zero (since we let \( P \) go to infinity keeping \( \gamma_t \) finite). That is, for \( H_t \leq \widehat{H}_t \), \( Y_t = \gamma_t \); and for \( H_t \geq \widehat{H}_t \), \( Y_t = B H_t \left( 1/2 - \sqrt{1/4 - \rho \gamma_t / (B H_t)} \right) \).
Recalling (6), this gives the following difference equation for \( H_t \), analogous to (12):

\[
H_{t+1} = \begin{cases} 
\beta \gamma_t & \text{if } H_t \leq \hat{H}_t \\
\beta B H_t \left( \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{c_t}{BH_t}} \right) & \text{if } H_t > \hat{H}_t 
\end{cases}
\]

where (recall again) \( \hat{H}_t = \gamma_t/[B(1 - \rho)] \).

As before, we assume that \( \beta B > 1 \), ensuring the existence of a balanced growth path.

### 5.2.1 The effects from changes in \( \gamma_t \)

It is interesting to note that, at any given level of \( H_t \), the effects from a change in \( \gamma_t \) on \( R_t \) and \( H_{t+1} \) (and on \( Y_t \)) depend on whether, or not, the economy has entered a stage of development where agents spend some time outside of rent seeking. More precisely, using (17) and (18) we can see that:

\[
\frac{\partial R_t}{\partial \gamma_t} = \begin{cases} 
0 & \text{if } H_t < \hat{H}_t \\
> 0 & \text{if } H_t > \hat{H}_t 
\end{cases}
\]

and

\[
\frac{\partial H_{t+1}}{\partial \gamma_t} = \begin{cases} 
> 0 & \text{if } H_t < \hat{H}_t \\
< 0 & \text{if } H_t > \hat{H}_t 
\end{cases}
\]

where (recall once again) \( \hat{H}_t = \gamma_t/[B(1 - \rho)] \).

In poor economies, where \( H_t \) falls below \( \hat{H}_t \), agents spend all their time in rent seeking. Therefore, an increase in \( \gamma_t \) generates no reallocation of time between the different sectors; it only has a positive effect on incomes and human capital investment. When \( H_t \) starts to exceed \( \hat{H}_t \) and agents spend a positive amount of time in the human capital sector, a rise in \( \gamma_t \) induces agents to reallocate more time into rent seeking. The result is lower productivity of human capital, working through the rent seeking externality \( (\rho) \), which lowers incomes and human capital investment.
It is also interesting to note that
\[ \lim_{H_t \to \infty} \frac{\partial H_{t+1}}{\partial \gamma_t} = \lim_{H_t \to \infty} \frac{\partial R_t}{\partial \gamma_t} = 0. \] (21)

That is, resource shocks are neutral on the balanced growth path, where incomes earned from human capital dwarf those earned from rent seeking.

Our results are therefore consistent with the facts discussed in Section 3.3. In pre-industrial societies an abundance of fertile land, forests, and other resources constituted the basis for the very existence of civilizations, and the depletion of those resources led to many civilizations’ collapse (Brander and Taylor 1998, Diamond 2005).

At intermediate stages of development, as the growth take-off has just set in, resource shocks start being detrimental to income. This captures the presence of a natural resource curse among poor countries today (Sala-i-Martin and Subramanian 2003, Isham et al. 2005), and in, for instance, Spain several centuries ago (Drelichman 2005).

Finally, as the economy reaches the most advanced stage of development, and human capital levels go to infinity, the resource shocks have no effect at all.

## 5.3 Transitions

Just like in the setting with violence shocks, the setting with resource shocks can generate a long phase of stagnant levels of human capital, with occasional false starts, eventually followed by a transition to sustained growth. To see how the mechanics work, consider an economy where human capital falls below the threshold in some period \( t \), that is, \( H_t < \tilde{H}_t = \gamma_t/[B(1-\rho)] \). Since income is then given by the amount of resources per agent (that is, \( Y_t \) equals \( \gamma_t \)) in the next period human capital equals \( H_{t+1} = \beta \gamma_t \) [recall (6)]. Human capital thus falls below the threshold in period \( t+1 \) if \( H_{t+1} = \beta \gamma_t < \tilde{H}_{t+1} = \gamma_{t+1}/[B(1-\rho)] \); and, vice versa, human capital exceeds the threshold if this

22
inequality is reversed. Using (16) we can thus write:

\[
H_{t+1} \begin{cases} > \Rightarrow H_{t+1} \iff \frac{\gamma_{t+1}}{\gamma_t} = \varepsilon_t \begin{cases} < \Rightarrow \beta B(1 - \rho). \end{cases} \end{cases}
\]

That is, for agents to start reallocating time away from rent seeking the change in resources per agent (that is, \(\varepsilon_t\)) must be small enough. Put another way, the level of resources per agent in the present period must be small enough, thus ensuring that competition for those resources is not too tempting; but resources per agent in the preceding period must also be sufficiently high to make human capital investment in that period (and thus human capital levels in the present) high enough to make rent seeking not too tempting today.

Analogous to (15) we next assume that

\[
\varepsilon < \beta B(1 - \rho) < \overline{\varepsilon}. \tag{23}
\]

This ensures that a hypothetical economy which is constantly hit by the most resource increasing shock (\(\overline{\varepsilon}\)) is stuck in a stagnant trap with agents spending all their time in rent seeking, and human capital and per-capita incomes being stagnant. The same economy being constantly hit by the most resource reducing shock (\(\varepsilon\)) instead breaks out of poverty and converges onto a sustained growth path.

### 5.4 Simulation results

We next simulate the model letting \(\gamma_t\) follow the process in (16). We set the upper and lower bounds for \(\varepsilon_t\) to \(\varepsilon = 0.68\) and \(\overline{\varepsilon} = 1.32\). This implies that \(\varepsilon_t\) has a mean equal to one; thus, in any period \(\gamma_t\) is equally likely to grow as it is to contract.

We let initial resources per agent, \(\gamma_0\), correspond to \(A/P\) in the simulation in Section 4.4, that is \(\gamma_0 = 100/1000 = 0.1\). We set \(\rho = 0.6\), which is the average of the upper and lower bounds for \(\rho\) used in the simulation in Section 23.
4.4. The remaining parameter values are also chosen as in Section 4.4. In particular, we set $B = 5$ and $\beta = 0.346$ so that, if each period corresponds to 25 years, per-capita incomes grow by 2.2% per year on the balanced growth path. Initial human capital, $H_0$, is set to $\beta \gamma_0$ [cf (18)]. These values imply that (23) holds, so that a transition to sustained growth occurs sooner or later.

Figure 3 shows human capital being stagnant for many generations and then taking off onto sustained growth at some stage. More precisely, human capital first drifts slowly; in the run shown in Figure 3 the drift is slightly downwards. However, this drift is notably dwarfed by the trend after the economy has started to exhibit sustained growth.

The stagnant phase continues for about 75 periods, and throughout this phase the economy displays short spurts of growth associated with temporary reductions in rent seeking, just as in the simulations in Section 4.4 where $\rho$ fluctuated. An initial reduction in rent seeking due to a low draw of $\varepsilon_t$ can be interrupted by a subsequent high draw, generating an expansion in the pie which agents compete for, and thus inducing a rise in rent seeking. Only when human capital levels are high enough is the economy insulated from such shocks.

One difference compared to the results in Section 4.4 is that both the actual and threshold human capital levels now fluctuate, since both depend on the path of $\gamma_t$. This is also why the annualized growth rate is here so much more volatile during the stagnant phase: resource shocks affect incomes earned from rent seeking; the violence shocks only affected incomes earned from human capital.

The most interesting observation shows up in the lower-right panel of Figure 3, showing the paths of three variables: the gross growth rate of

\[ \text{\footnotesize \[11\]This would not be the case if we had specified a process for $\gamma_t$ which exhibits mean reversion. Such mean reversion typically arises in Malthusian models with endogenous fertility, where population tends to approach a constant level proportional to the amount of land and/or the level of technology. The formulation chosen here has the advantage of generating the simple conditions in (22) and (23).} \]
income, as given by $Y_{t+1}/Y_t$ (not annualized); the resource shock, $\varepsilon_t$; and the correlation between these two variables over 10 periods (centered on the latter of the two mid periods).

This correlation coefficient measures the impact of the resource shocks, and its time path roughly illustrates the time pattern of the natural resource curse, as described in Sections 3.3 and 5.2.1. As long as $R_t = 1$ it holds that $Y_{t+1}/Y_t = \gamma_{t+1}/\gamma_t = \varepsilon_t$, so the gross growth rate and the shock are perfectly correlated. As rent seeking declines ($R_t < 1$) the correlation becomes negative. The correlation goes to zero on the balanced growth path, since the gross growth rate goes to $\beta B$, which is constant and thus uncorrelated with $\varepsilon_t$.\footnote{See (18) and note that $Y_{t+1}/Y_t$ approaches $H_{t+1}/H_t$ on the balanced growth path.} This is harder to see in Figure 3, but in simulations over longer time horizons the correlation coefficient can indeed be seen to converge to zero.

6 Conclusions

We have presented a growth model where agents divide time between rent seeking and using their human capital. Rent seeking is modelled as competition for a non-growing resource, like land; the human capital sector could be interpreted as trade or manufacturing.

A central element of this model is that rent seeking exerts negative externalities on the productivity of human capital. This is modelled in quite a black-boxed manner, by simply assuming that the productivity of human capital decreases with equilibrium rent seeking; we discuss and motivate this further below.

We also introduce a stochastic element: first in the form of violence shocks, capturing variation in how violent forms rent seeking takes; and then as so-called resource shocks, capturing changes in the size of the pie that agents compete for.

The first result we want to emphasize is the dynamic patterns that our model generate. First of all, the model produces an endogenous take-off
from stagnation to growth. That is, the economy can be stuck in a state of stagnation with low levels of human capital and per-capita incomes for a long while (several tens of generations in our simulations). Eventually, when a favorable enough shock arrives human capital rises, simultaneously with a decline in rent seeking. However, if a reverse shock arrives too soon after the take-off the economy can revert back to its initial poverty trap. Only by experiencing sufficiently favorable shocks over sufficiently many periods can the economy transit permanently onto a balanced growth path. Once human capital has reached high enough levels, and rent seeking fallen low enough, the economy becomes insulated from even the worst shocks.

The transition from stagnation to growth that the model generates is consistent with the growth take-off that started in Western Europe with the Industrial Revolution (see, for example, Galor and Weil 2000 and Galor 2005). The several “false starts” that precede the take-off are consistent with the rise and fall of, for example, several European city states in pre-industrial times (De Long and Shleifer 1993, De Long 2000). Moreover, once growth has taken off the economy becomes immune to even the worst shocks; this fits with the observation that no OECD country has (yet) crashed back to pre-industrial income levels.

These patterns arise in a setting with either violence shocks or resource shocks. The setting with resource shocks generates one additional interesting result, which fits some long-run facts about the so-called natural resource curse. At early stages of development, when agents spend time only in rent seeking, changes in the size of the resource base has only positive effects on incomes: as the size of the pie that people compete for rises, so does the piece each agent receives in equilibrium; this is consistent with the observation that resource endowments, such as forests and fertile soils, constituted the fundamental basis for the development of early human civilizations. When a growth take-off has just set in an increase in resources leads to lower human capital and per-capita income levels, because it induces a reallocation of time into rent seeking, and thus lower productivity of human capital; this
captures the negative growth effects that natural resources have in countries which have just started to grow, known as the natural resource curse. As the economy has safely embarked on a balanced growth path, however, the detrimental effects of resource shocks vanish: as incomes earned from human capital come to dwarf those earned from resource competition, shocks to the resource base have vanishingly small impact on incomes; this is also consistent with the observation that per-capita income gaps across OECD countries seems to show little consistent variation (negative or positive) with natural resource abundance.

To sum up, the model replicates a three-stage process in which the economy evolves from being subject, first to a natural resource blessing, then to a natural resource curse, then to a situation where natural resources are irrelevant for living standards. This three-stage process seems largely consistent with the facts, but has not yet been replicated in previous theoretical work.13

Several of the assumptions driving our results could probably be derived endogenously in richer frameworks. Most obviously, the negative link from rent seeking to productive activities could be endogenized. For example, if productive activities in general were themselves subject to rent seeking, in the form of taxation or theft, then more rent seeking would lower the reward to such activities; this would be close to the mechanisms at work in some of the earlier rent seeking literature, as discussed in Section 2. However, in our model productive activities – that is, production based on human capital – are by assumption free from rent seeking. In fact, this is a crucial assumption of the model, since it drives the mechanism through which human capital growth generates a substitution away from rent seeking.

13 Baland and Francois (2000) set up a model where a resource boom (interpreted as, for example, a rise in commodity prices) exerts less detrimental (or even positive) effects in countries with less rent seeking. (See also Torvik 2002.) Baland and Francois also cite empirical evidence from the 1970’s oil boom supporting this pattern. However, different from us they do not discuss the very first stage of development – that is, early human civilizations – at which an increase in natural resource endowments seems to have exerted a positive effect in income.
But there are other ways to interpret the negative externality from rent seeking to human capital productivity. The endogeneity of institutions is one. For example, in societies with abundant resources (say land) the most talented agents may choose to grab as much land as possible and force other agents to work on it, as slaves or serfs, and build military skills to fight over land. They would have little interest in building institutions designed to protect land-less agents living in urban centers. By contrast, in environments where land is relatively scarce, and competition is intense and lethal over the little land there is, some talented agents may instead specialize in more skill-intensive activities, like trade. They may then have an interest in using their talents to build institutions which help enforce contracts and the rule of law, which would make such activities more profitable.

There may be other and better ways to model this. The point is that, regardless of what the precise mechanisms are, the negative effects from rent seeking on human capital productivity could arise also when rent seeking does not amount to direct expropriation of income from human capital, but rather competition for some other resource, such as land.

Another element of our model which could be endogenized is the workings of the shocks. The violence shocks could be thought of as arising from choices made by agents as to how violent they act when competing for resources. For example, there may be more and less violent technologies with which agents compete for existing resources. Using a violent technology may be more costly but also result in a greater share of the pie. One starting point could be the approach taken in our own earlier work, where we endogenize the choice that agents make about whether, or not, to participate in a conflict (Tangerås and Lagerlöf 2005).

A final element which could be endogenized is the resource shocks. Since these amount to changes in the ratio of total resources over population \((A/P)\), they could be modeled in a setting where the resource base \((A)\) and population \((P)\) evolve endogenously over time, and are subject to stochastic shocks. The long-run growth literature cited in Section 2 contains many examples of
related models.

A Appendix

A.1 Equilibrium rent seeking

Imposing symmetry ($r_t = R_t$) on the first-order condition for $r_t$ in (7) it is seen that

$$\frac{(P-1)R_tA}{[R_t + (P-1)R_t]^2} \geq B (1 - R_t \rho_t) H_t.$$ (24)

Taking into account that $R_t = 1$ if the inequality is strict, this can be elaborated on to give

$$R_t = \min\left\{1, \frac{\gamma}{B(1 - R_t \rho_t) H_t}\right\},$$ (25)

where (recall) $\gamma = (P - 1)A/P^2$ [see (10)]. From (25) it can be seen that an equilibrium with $R_t = 1$ exists if $\gamma/[BH_t(1 - \rho_t)] \geq 1$, or

$$H_t \leq \frac{\gamma}{B(1 - \rho_t)} = \hat{H}_t.$$ (26)

This gives the first case in (8). From (25) it is also seen that an equilibrium with $R_t < 1$ must satisfy

$$R_t = \frac{\gamma}{B(1 - R_t \rho_t) H_t}$$ (27)
or, after some algebra,

$$R_t^2 - \left(\frac{1}{\rho_t}\right)R_t + \frac{\gamma}{\rho_t BH_t} = 0$$ (28)

which has solutions:

$$R_t = \frac{1}{2\rho_t} \pm \sqrt{\left(\frac{1}{2\rho_t}\right)^2 - \frac{\gamma}{\rho_t BH_t}}$$

$$= \frac{1}{\rho_t} \left(\frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{\rho_t \gamma}{BH_t}}\right).$$ (29)

Disregarding the larger root (as discussed in Footnote 7) gives the second case in (8).
A.2 Income

Using (8) it easily seen that,

$$B(1 - R_t \rho_t) H_t = BH_t \left( \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\rho_t \gamma}{BH_t}} \right) \quad (30)$$

for $R_t < 1$. Then (27) says that

$$B (1 - R_t \rho_t) H_t R_t = \gamma = \frac{(P - 1)A}{P^2} \quad (31)$$

from using the definition of $\gamma$. Using (30) and (31), together with the expression for income in (3), gives

$$Y_t = \frac{A}{P} + [1 - R_t] B(1 - R_t \rho_t) H_t$$
$$= \frac{A}{P} + B(1 - R_t \rho_t) H_t - R_t B(1 - R_t \rho_t) H_t$$
$$= \frac{A}{P} + BH_t \left( \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\rho_t \gamma}{BH_t}} \right) - \frac{(P - 1)A}{P^2} \quad (32)$$

which is one of the cases in (11).
References


[10] De Long, J.B., 2000, Overstrong against thyself: war, the state, and growth in Europe on the eve of the industrial revolution, in Olson,


[35] ——, 2005b, War, resource competition, technology, and population, mimeo, York University.


Figure 1: 45-degree diagram for different fixed levels of $\rho$. 
Figure 2: Simulations with violence shocks.
Figure 3: Simulations with resource shocks.