Competition vs. Regulation in Mobile Telecommunications

Johan Stennek and Thomas P. Tangerås
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Johan Stennek                Thomas P. Tangerås
RIIE and CEPR                RIIE
(Johan.Stennek@riie.se)     (Thomas.Tangeras@riie.se)

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Abstract

This paper questions whether competition can replace sector-specific
regulation of mobile telecommunications. We show that the monopol-
istic outcome prevails independently of market concentration when
access prices are determined in bilateral negotiations.

A light-handed regulatory policy can induce effective competition.
Call prices are close to the marginal cost if the networks are sufficiently
close substitutes. Neither demand nor cost information is required.

A unique and symmetric call price equilibrium exists under sym-
metric access prices, provided that call demand is sufficiently inelastic.
Existence encompasses the case of many networks and high network
substitutability.

Key Words: network competition, two-way access, access price com-
petition, entry, regulation, network substitutability.
JEL: L510, L960

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financial support. Address: Research Institute of Industrial Economics (RIIE), P.O. Box
55665, SE-102 15 Stockholm, Sweden.
1 Introduction

The liberalization of mobile telecommunications has dramatically reduced market concentration in the OECD area. Table 1 summarizes the development in 30 countries between 1989 and 2004.

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Source: OECD Communications Outlook 2005.

Within ten years of the reforms, all incumbents had lost their monopoly positions and today, most duopolies have also been replaced by even less concentrated market structures.

Most countries promote competition also by reducing network differentiation. Consumers can easily compare the networks’ prices on special Internet webpages, and those who want to switch networks may retain their phone number. Everybody can reach everybody else independently of to which network they belong, since the networks are required to interconnect. Universal service obligations reduce the vertical differentiation across mobile networks.

The OECD countries have also adopted sector-specific regulation, applied by sector-specific authorities, to ensure viable competition. Sector-specific rules are typically viewed as intermediary solutions.1 In the words of the European Commission (2005):

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"Regulation is seen as essentially a temporary phenomenon, required to make the transition from the formerly monopolistic telecommunications industry to a fully functioning market system. ... As the sector evolves, operators will increasingly build their own infrastructures and compete more effectively. ... [R]egulation can be rolled back, and competition law ... will replace sector-specific intervention."

We take issue with this view, and argue that regulation and competition play complementary roles also in the long run. Therefore, we show that the required interventions under certain conditions are minimal and do not require any cost or demand information.

**The limits to call price competition** Telecom markets differ from most other markets, since the networks have both a horizontal and a vertical relation. While they compete for customers in the retail market, they cooperate in the wholesale market by providing call termination. The access prices paid by the networks for terminating calls in the rivals’ networks means that they transfer some of the revenues they collect from their customers to their competitors. The networks therefore have weak incentives to reduce their call prices to capture market shares. The seminal papers by Armstrong (1998) and Laffont, Rey and Tirole (1998a and b), henceforth A-LRT, show that competition is sometimes so weak that the monopoly outcome prevails if there are only two networks.

In essence, the competitive conditions are determined in the termination market. Access prices are typically set in bilateral negotiations, i.e. every pair of networks meets separately to decide the access price between the two of them. By agreeing to charge high access prices, the networks commit to
artificially high marginal costs and therefore, high call prices. Since interconnection agreements are legally enforceable, there is no need for complicated punishment mechanisms to sustain the collusive price. With two networks and bilateral access price negotiations, the market is effectively a monopoly.

**The limits to access price competition** Entry beyond duopoly opens up for access price competition due to the competitive externalities between the different bilateral negotiations. Reducing their common access price commits two networks to more aggressive call pricing and to capture market shares from third parties. Such bilateral access price competition has not been analyzed in the economic literature until now.

Our results demonstrate that any pair of networks indeed has the incentive to reduce its access price below the monopoly level to poach customers from third-party rivals. The networks’ incentives to reduce access prices increase with every new network entering the market, since subscription demand is more elastic the more competitors there are. However, the access charges payable to third-party rivals soften access price competition considerably. The larger the number of networks, the larger is the fraction of calls terminated by third parties, and the larger is the flow of income drained from the two partners. The two effects of entry, the increase in access price competition and the increased share of calls subject to double margins, are both proportional to the size of the two networks’ customer base and cancel at the monopoly call price.

Our first main result is therefore that entry has no effect on call prices, if few restrictions are put on the networks’ price setting strategies. The collusive outcome prevails independently of the market structure.

In an extension of our analysis, we demonstrate that access price collusion
is a problem also with two-part tariffs when networks are close substitutes, provided that arbitrage prevents the networks from setting negative subscription fees. In fact, the outcome is unaffected by two-part tariffs.

**The limits to regulation** The insufficiency of competition may explain the continued use of or the reversion to classical price regulation. The European regulatory framework for electronic communications is one example. Operators with significant market power may be enjoined to set cost-oriented access prices.

Unfortunately, however, when it comes to the burdens of regulation, telecommunications are not different from other markets. Detailed regulation requires accurate information about the networks’ costs: appropriate cost concepts need to be defined and the necessary data collected. Perhaps more importantly, cost-based regulation may distort the incentives for cost-containment and investments. The burdens of regulation are revealed by the frequent appeals of regulatory decisions and the numerous court cases pending in several European countries.

**How to reduce prices with little information** The failures of the unconstrained market and the burdens of detailed regulation call for a third alternative. Is it possible to devise *structural rules* for the networks’ market behavior – rules that are simple and informationally undemanding, yet effective in preventing monopolization?

We propose an *STR-regulation* combining four well-known structural rules: *(i)* interconnection is mandatory; *(ii)* networks are not allowed to charge different prices for off-net and on-net calls; *(iii)* access prices must be reciprocal and *(iv)* below a cost-independent ceiling.

Our second main result is that *STR*-regulation forces the equilibrium
call prices to fall towards the marginal cost as networks become increasingly closer substitutes. When networks are near-perfect substitutes, any access price ceiling is sufficient to push call prices down to the marginal cost – detailed information about costs is therefore not required. The access price ceiling may not only be set very high, but ceilings below the marginal cost of call termination also work well. A special case of STR is the Bill-and-Keep regime, i.e. an access price equal to zero.

The regulation works as follows. Mandatory interconnection, reciprocity of the access price and the ban on call price discrimination minimize the networks’ ability to differentiate themselves. With increasing network substitutability, subscribers are more mobile, which intensifies competition. Due to the access price ceiling, the networks are unable to offset competitive pressure by jacking up the access price.

The analysis brings out the complementarity between regulation and competition. Without regulation, access price collusion leads to monopolization, independent of the number of networks. Without competition, the STR-rules are all meaningless. While most people seem to agree that a sector-specific regulation is necessary during the transition from monopoly to competition,\(^2\) our work shows that regulation may be required also in the long run, when there are several competing networks in the market, all have built up sizable customer bases, and when access price collusion may be a more acute problem than price squeezes.\(^3\) This stands in contrast to the commonly held view that competition and regulation are substitutes, two recent examples being

\(^2\)There is also some anecdotal evidence that sector-specific regulation may contribute to lower prices. In 2001, mobile call charges were much higher (more than twice as high) in New Zealand which had until then almost exclusively relied on antitrust rules, than in the UK and in the US which relied on sector-specific rules. Australia and Chile with models somewhere between the two extremes also had an intermediate price level (Kerf et al., 2005).

\(^3\)For more on price squeezes, see Bouckaert and Verboven (2004) and Valletti (2003).

Methodological contributions  From a methodological point of view, our first main contribution is the derivation of a sufficient condition for the existence of a pure strategy call price equilibrium for any degree of network substitutability and any number of networks. Our analysis shows that an equilibrium exists, provided that call demand is sufficiently inelastic. Existence is far from trivial in two-way access situations since the networks’ profit functions are not necessarily concave. A network losing from somewhat undercutting the rival may still benefit from a large price cut to (nearly) corner the market and avoid paying the access price. A-LRT’s solutions, an access price close to the termination cost or low network substitutability are not useful for our purposes. We do not want to tie access prices to cost or demand data, and we want to assess the performance of the regulation also when networks are close substitutes.

Our second methodological contribution is to extend the analysis of bilateral access pricing from duopoly to general $n$-network oligopoly. Provided that demand is sufficiently inelastic, we prove the existence of a semi-symmetric perfect Bayesian equilibrium, sustained by passive beliefs. This result is non-trivial since, with bilateral access price negotiations, there will be competition also in the access pricing stage.

Related Literature  The literature on two-way access with more than two networks is small. Calzada and Valletti (2005) focus on how access prices can be used to deter entry. Our focus is on post-entry access pricing. Their model is one of multilateral negotiations, whereas our emphasis is on competitive access prices, i.e. bilateral negotiations. Finally, our results hold for any degree of network substitutability, whereas Calzada and Valletti restrict
the attention to the case of low network substitutability. Jeon (2005) considers direct access price regulation. He shows that an access price markup proportional to the call price markup of the competitors induces the Ramsey outcome. We analyse regulatory policies that do not require cost information. Gilo and Spiegel (2004) analyze the implications of transit when a third party seeks access to two interconnected networks, but they abstract from competition.

Doganoglu and Tauman (2002) contain results on network substitutability related to ours. They prove the existence of a unique symmetric call price equilibrium in a model with two networks independently of network substitutability under two alternative sets of assumptions: (i) that the access price is above the termination cost but below the demand intercept, or (ii) that a network’s access price is a linear function of the competitor’s call price. We extend their analysis by not imposing any restrictions on the access price, by ruling out asymmetric equilibria, and by allowing for general $n$-network oligopoly. Our analysis also brings out the crucial role played by the price elasticity of call demand.

Related work emphasizing the role of inelastic call demand includes Armstrong (2003). He studies competition between two networks for heterogeneous subscribers under the assumption of perfectly inelastic call demand.


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4The basic duopoly framework of A-LRT has also been extended in other directions, such as the gradual evolution of market shares following entry from monopoly to duopoly (de Bijl and Peitz, 2002), asymmetric networks (Carter and Wright, 2003), non-linear pricing (Dessein, 2003) and investments (Valletti and Cambini, 2005).
2 Duopoly

Consider a market for mobile telephony with two competing networks. We first consider the duopoly case to highlight the role of network substitutability and the access price ceiling and then discuss the role of further entry and access price competition in the next section.

The interaction is described as a game with four stages. The networks negotiate access prices and then unilaterally set call prices. The customers choose a subscription and finally decide how many calls to make.

Telecom networks typically charge their customers both a call price and a subscription fee. Our analysis of subscription fees is relegated to section 4.1. The call price is sometimes subdivided into an opening fee and a per-minute charge, but we assume all calls to be of equal length and each network to specify an overall call price, $p_i$.

There also exists an alternative, but inferior means of communication – think of finding a pay-phone – which is always available and does not require any subscription. The price of alternative calls is exogenously set at $v$, including the disutility of the additional effort required. The outside option guarantees that call prices are bounded when demand is very inelastic.

Call Demand In stage four, every consumer subscribing to network $i$ makes $q_i$ mobile calls at (the non-discriminatory) price $p_i$ and $q_0$ alternative calls at price $v$ to every other subscriber (balanced call pattern), so as to maximize quasi-linear utility

$$U(q_i, q_0) = \left( q_i + q_0 - \frac{1}{2} (q_i + q_0)^2 \right) \frac{1}{v} - p_i q_i - v q_0.$$  

Consequently, the demand for mobile calls is linear and equal to $D(p_i) = 1 - \varepsilon p_i$ for $p_i \leq v$ and zero for $p_i > v$. Since the price-elasticity of demand is
\[ \eta(p_i) = \varepsilon p_i / (1 - \varepsilon p_i), \] we will refer to a low \( \varepsilon \) as a low elasticity of demand.

**Subscription Demand** In stage three, consumers choose networks. Consumers base their choice of network on the net benefits of the networks over the outside option. The indirect utility of the outside option is \( U(0, D(v)) = (1 - \varepsilon v)^2 / 2\varepsilon \), and the net benefit of network \( i \) is

\[
V(p_i) = U(D(p_i), 0) - U(0, D(v)) = (v - p_i) \left( 1 - \varepsilon \frac{v + p_i}{2} \right),
\]

for \( p_i \leq v \) and zero for all prices above \( v \). The price elasticity of the net benefit function is \( \sigma_{vp}(p_i) = - (\partial V(p_i) / \partial p_i) (p_i / V(p_i)) = D(p_i) p_i / V(p_i) \).

We employ a random utility model, and network \( i \)'s market share is equal to

\[
S_i(p_i, p_j) = \frac{V(p_i)^{\frac{\gamma}{\gamma+1}}}{V(p_1)^{\frac{\gamma}{\gamma+1}} + V(p_2)^{\frac{\gamma}{\gamma+1}}},
\]

when at least one network charges a price strictly below \( v \). To derive equation (2), assume that a subscriber selects network \( i \) over \( j \) only if \( V(p_i) \exp \{ \delta_i \} \geq V(p_j) \exp \{ \delta_j \} \), where \( \delta_i \) and \( \delta_j \) are two double exponentially distributed utility terms, independent across subscribers and networks.\(^5\)

The price-elasticity of the demand for subscriptions is

\[
\sigma_i(p_i, p_j) = - \frac{\partial S_i}{\partial p_i} \frac{p_i}{S_i} = \frac{1}{\gamma} (1 - S_i) \sigma_{vp}(p_i).
\]

If \( \gamma \) is close to zero, subscription demand is very elastic and the network with the lowest price captures most of the subscribers. If \( \gamma \) is very large, subscription demand is very inelastic and the networks divide the market approximately equally, independently of prices. The network substitutability parameter \( \gamma \) captures many different factors such as customer heterogeneity in combination with product differentiation, switching costs, and bounded rationality.

The advantage of the random utility model over the commonly used Hotelling model is that the market is fully covered at all prices with positive demand, independently of the degree of network substitutability. The advantage of using the net benefit rather than the indirect utility function is that a network failing to provide subscribers with any net benefit over the outside option \( p_i \geq v \) ends up with a zero market share, if the competitor offers a positive net benefit \( p_j < v \). This has the implication that no network will ever be tempted to raise its price above \( v \), so as to make its profit entirely from incoming calls.

**Call prices** In stage two, the networks set (non-negative) call prices taking the access price as given. Under \( STR \), the profit of network \( i \in \{1, 2\} \) is given by

\[
\pi_i = S_i \left[ S_i (p_i - c) D (p_i) + S_j (p_i - a - c_o) D (p_i) + S_j (a - c_t) D (p_j) \right], \tag{4}
\]

where \( c_t \) is the marginal cost of call termination, \( c_o \) is the marginal cost of call origination, \( c = c_t + c_o \), and \( a \) is the reciprocal access price. It is assumed that the marginal cost is lower than the willingness to pay for the first unit, i.e. \( c < v < \varepsilon^{-1} \). The square bracket represents the profit per subscriber. The first term represents on-net calls and the second outgoing off-net calls. The third term represents incoming off-net calls.

Assuming that a pure strategy equilibrium exists, let \( p_i^d \) be the duopoly equilibrium call price of network \( i \), and \( S_i^d \) its equilibrium market share. Clearly, the equilibrium call prices will typically depend on the access price.

**Access price** In the first stage, the networks negotiate a reciprocal access price, so as to maximize the joint profit

\[
S_1^d (p_1^d - c) D (p_1^d) + S_2^d (p_2^d - c) D (p_2^d).
\]

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Since the access price only indirectly affects profits via its effect on equilibrium call prices, it constitutes an instrument of collusion. If possible, the access price will be set to implement the monopoly price, that is \( p_1^d = p_2^d = p^m \). The monopoly price is characterized by the Lerner formula \( (p - c) / p = 1/\eta (p) \) if \( p \leq v \) and it is otherwise equal to \( v \).

To guarantee that a network will not have an incentive to make phony calls to the other network, the marginal cost of off-net calls must be non-negative, i.e. \( a \geq -c_o \). Therefore, it is required that the regulatory cap on the access price satisfies \( \pi \geq -c_o \).

### 2.1 Equilibrium

**Call Prices** Network \( i \)'s marginal profit is

\[
\frac{\partial \pi_i}{\partial p_i} = S_i D (p_i)
\]

\[
+ \frac{\partial S_i}{\partial p_i} \left[ S_i (p_i - c) D (p_i) + S_j (p_i - a - c_o) D (p_i) + S_j (a - c_i) D (p_j) \right] 
\]

\[
+ S_i (p_i - c - S_j (a - c_i)) D' (p_i) 
\]

\[
+ S_i \frac{\partial S_i}{\partial p_i} (a - c_i) [D (p_i) - D (p_j)].
\]

(5)

The first three lines represent the standard trade-off between price and sales; an increased price leads to a higher mark-up on every call, but reduces the customer base and reduces call demand. The fourth line represents two "composition effects," resulting from the subscribers switching to the competing network as a result of a price increase: access costs are increased, but so are access revenues. The composition effect may be positive or negative.

Our objective is to study competition when networks are close substitutes and with little restrictions on access prices. Unfortunately, a pure strategy equilibrium may fail to exist under those circumstances. When networks
are close substitutes, they will have to set a high access price so as to deter marginal price cuts. However, a large price cut may still be profitable as it allows the deviating network to seize nearly all consumers, thereby avoiding the access costs. A price equal to the marginal cost cannot be an equilibrium either, because a network can change its price a little and earn a positive profit. Equation (5) shows that a small increase in the price above the marginal cost would be profitable if call demand were inelastic (since \( a \geq -c_o \)).

The root of the existence problem is the traffic flowing between networks, represented by the composition effect, the final term in (5). The two solutions devised by A-LRT are not useful in our context. Their first solution is to assume that the networks are poor substitutes, i.e. \( \partial S_i / \partial p_i \approx 0 \). Then, the market shares are insensitive to price changes and the composition effect vanishes. The networks are essentially local monopolists, and set prices with an effective marginal cost equal to \( c + S_j (a - c_t) \). Their second solution is to assume that the access price is close to the termination cost, i.e. \( a \approx c_t \). Then, the traffic between the networks is of minor importance to profits and the composition effect vanishes.

Our solution to the existence problem is instead to require call demand to be inelastic.

**Lemma 1** Consider a market with two networks under STR. There exists a unique and symmetric pure strategy equilibrium in call prices if demand is sufficiently inelastic (that is, if \( \epsilon \) sufficiently low). The equilibrium price \( p^d (\gamma, a) \in [c, v) \) is implicitly given by

\[
\frac{p^d - c}{p^d} = \frac{1}{\eta (p^d) + \sigma (p^d)} \left[ 1 + \frac{1}{2} \left( \frac{a - c_t}{p^d} \right) \eta (p^d) \right].
\]

The equilibrium price is increasing in the access price and the degree of net-
work differentiation.

This result is a corollary to Lemma 5 in Appendix A.1.

A low elasticity of demand has a similar effect on the profit function as an access price close to the termination cost. The idea is that the traffic between the networks is of minor importance to profits when the difference in demand is small. The difference in equilibrium demand is indeed small when demand is inelastic since the equilibrium prices will be contained in \([c, v]\). Since \(|D(p_1) - D(p_2)| \leq \varepsilon (v - c)\), the difference drops towards zero and the composition effect vanishes as demand becomes more inelastic.

**Access Prices** Since the networks charge the same call price in equilibrium, each network earns half the monopoly profit at that price, i.e. 
\[\pi^d = \frac{1}{2} (p^d - c) D(p^d).\] They have a common interest in setting an access price inducing a call price as close as possible to the monopoly price, \(p^m\). That is, when feasible, the networks set
\[\alpha^m = c_s + \frac{p^m \sigma_{vp}(p^m)}{\gamma n^2(p^m)}\]  
(7)
and enjoy the full monopoly profit. To see that \(\alpha^m\) induces the monopoly outcome, simply substitute the Lerner rule into the equilibrium condition (6) and solve for \(\alpha^m\). If the monopoly access price exceeds the access price ceiling, \(\alpha^m > \pi\), the networks set the access price at the ceiling, since \(p^d\) is increasing in \(a\), and since the network’s profits are increasing in price, whenever the price is below the monopoly level. In sum:

**Lemma 2** Consider a market with two networks under STR. There is a unique equilibrium access price, given by 
\[a^d = \min \{\alpha^m, \pi\},\] provided that call price demand is sufficiently inelastic.
2.2 Policy Implications

The closer substitutes are the networks, the higher is the monopoly access price \( \alpha^m \). The necessary access price even increases without any bound (i.e. \( \alpha^m \to \infty \)), as the networks become closer to being perfect substitutes (\( \gamma \to 0 \)). When networks are close substitutes, any cap on access prices must consequently be binding. In fact, by limiting the networks’ ability to offset the competitive pressure by charging high access prices, the networks are forced to marginal cost pricing when networks are close substitutes, i.e. \( \lim_{\gamma \to 0} p^d (\gamma, \overline{\pi}) \to c \). Thus, (the proof is in Appendix A.2):

**Proposition 1** STR induces duopoly networks to charge call prices as close to the marginal cost as is desired, independent of the access price ceiling, provided that the networks are sufficiently close substitutes, and assuming that demand is sufficiently inelastic.

Our first conclusion is that it is the combination of STR and competition (i.e. having a second network and high network substitutability) that drives down call prices. Network substitutability is crucial for efficiency. If the networks are poor substitutes (\( \gamma \to \infty \)) and the access price ceiling generous (\( \overline{\pi} > c_1 \)), then STR induces the networks to set the access price close to the marginal cost of termination (\( a^d \to c_1 \)) and charge the monopoly price. The crucial role of having a second network is evident from inspecting the construction of the STR-rules; that is, STR cannot replace competition. Reversely, increased network substitutability and having a second network will not have any effect on call prices, unless the access price ceiling \( \overline{\pi} \) is binding. That is, competition cannot replace regulation.

Our second conclusion is that the access price ceiling can be set arbitrarily high, and that the informational requirement is therefore minimal. The only
restriction is that $\pi \geq -c_o$. The access price ceiling may not only be set very high, but ceilings below the marginal cost of call termination also work well. In particular, STR combined with Bill-and-Keep ($\pi = 0$) would do the trick.

**Structural versus Cost-Based Regulation** A possible cost-based regulation (henceforth "CBR") is to peg access prices down to the marginal cost of call termination, i.e. $\pi^{CBR} = c_t$. When the networks have the same costs, CBR leads to reciprocal access prices. The networks will also face the same marginal costs for on-net and off-net calls, and Ramsey pricing prescribes the same prices for off- and on-net calls. There is no price discrimination and no tariff-mediated network externalities. In equilibrium, the networks would set the same call price, characterized by

$$\frac{p^{CBR} - c}{p^{CBR}} = \frac{1}{\eta (p^{CBR}) + \sigma (p^{CBR})}.$$  

Since $p^d$ is increasing in the access price, it is clear that STR induces a higher price than CBR, whenever $\pi > \pi^{CBR} = c_t$.

One way of viewing STR is as a slight weakening of CBR, preserving the reciprocity of access prices and the absence of call price discrimination, but disconnecting the access price ceiling from the production cost. The advantage of STR is that it does not require any cost information, and the advantage of CBR appears to be a lower call price. Note, however, that STR provides stronger incentives for cost containment than CBR by making the networks residual claimants on any efforts to reduce the termination costs. Taking these incentives into account, it may well be the case that $p^{CBR} > p^{STR}$.

The relative efficiency of CBR is small in situations of high and low net-

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6A more detailed comparison of STR and CBR in terms of dynamic efficiency is left to future research.
work substitutability. If networks are very poor substitutes, both policies perform equally poorly, thereby inducing the monopoly price \( \lim_{\gamma \to \infty} p^{C_{BR}} = \lim_{\gamma \to \infty} p^d = p^m \). If networks are very close substitutes, both policies perform equally well, inducing marginal cost pricing.

We conclude that \( STR \) is a substitute for cost-based regulation, and that \( STR \) is likely to perform better whenever information is sparse or investment incentives important.

3 Oligopoly

The game unfolds as in the case with two networks. The networks first set reciprocal access prices in pair-wise negotiations and then set call prices simultaneously and independently. Based on the observed call prices, consumers subscribe to a network and then make the desired amount of calls. Consistent with actual practice, we assume networks to only observe their own access prices when setting the call price. Since the game has imperfect information, we use (a refinement of) perfect Bayesian equilibrium as a solution concept.

Access price negotiations For simplicity, we follow the tradition in the literature and do not model the bilateral access price negotiations in stage 1 as extensive form games. Instead, we take the short-cut and assume that each pair of networks has delegated its choice of access price to a separate agent. Thus, an agent called \( A_{ij} \) sets the reciprocal access price \( a_{ij} = a_{ji} \in [-c_o, \overline{a}] \) for traffic between networks \( i \) and \( j \). Agent \( A_{ij} \)'s objective is to maximize the sum of \( i \) and \( j \)'s expected profit, to be defined below. There are \( n(n-1)/2 \) agents and they all determine the access prices simultaneously.
and independently of each other.\textsuperscript{7}

Access price $a_{ij}$ is subsequently reported to the owners of networks $i$ and $j$, but to nobody else. In stage 2, network $i$ therefore knows $a_i = (a_{i1}, ..., a_{i-1}, a_{i+1}, ..., a_{in})$ and must form its beliefs about all other access prices. Let $a$ be the entire vector of access prices, and $a_{-i} = a \setminus a_i$ the access prices not observable to network $i$. The beliefs are represented by the probability density function $\mu_i(a_{-i}|a_i)$.

**Equilibrium concept**  A perfect Bayesian equilibrium consists of $n(n - 1)/2$ access prices $a_{ij}^*$, $n$ call price mappings $p_i^*(a_i)$, and $n$ belief mappings $\mu_i^*(a_{-i}|a_i)$. An equilibrium has three defining characteristics:

First, network $i$ sets the call price $p_i^*(a_i)$, which maximizes the expected profit

$$\pi_i^*(p_i, a_i) = \int \pi_i(p_i, p_{-i}^*(a), a_i) \mu_i^*(a_{-i}|a_i) da_{-i}$$

where $p_{-i}^*(a) = (p_i^*(a_1), ..., p_{i-1}^*(a_i-1), p_{i+1}^*(a_{i+1}), ..., p_n^*(a_n))$ are the equilibrium call price mappings of $i$’s competitors, and where

$$\pi_i^*(p_i, p_{-i}^*(a), a_i) = S_i^i(p_i, p_{-i}^*(a)) \left((p_i - c) D(p_i) + \sum_{k \neq i} S_k^i(p_i, p_{-i}^*(a)) (a_{ik} - c_i) (D(p_k^*(a_k)) - D(p_i))\right),$$

is the ex post profit of network $i$.

Second, agent $A_{ij}$ sets the reciprocal access price $a_{ij}^* = a_{ji}^*$, which maximizes

$$\pi_i^c(p_i^*(a_{ij}, a_{i-j}^*), (a_{ij}, a_{i-j}^*)) + \pi_j^c(p_i^*(a_{ji}, a_{j-i}^*), (a_{ji}, a_{j-i}^*))$$

taking all other access prices $a_{i-j}^* = a_i^* \setminus a_{ij}^*$ and $a_{j-i}^* = a_j^* \setminus a_{ji}^*$ as given.

\textsuperscript{7This is essentially the Nash-equilibrium-in-Nash-bargaining-solutions approach, introduced by Davidson (1988) and Horn and Wolinsky (1988). See Björnerstedt and Stenkek (2006) for a non-cooperative foundation for this approach.}
Third, Bayesian updating prescribes networks observing all their access prices to be at their equilibrium levels to assume that also all other access prices are at their equilibrium levels, that is \( \mu^*_i (a^*_{-i} | a^*_i) = \infty \). Beliefs are not defined off the equilibrium path. We invoke passive beliefs – the assumption that a network continues to believe that all other access prices are at their equilibrium levels, also following a deviation; that is, \( \mu^*_i (a^*_{-i} | a_i) = \infty \) also if \( a_i \neq a^*_i \).

We also restrict the attention to semi-symmetric passive-belief perfect Bayesian equilibria, i.e. equilibria prescribing all access prices to be the same and equal to \( a^* \) (which is now a scalar).

### 3.1 Call Price Equilibrium

Call price demand is the same as in the case with two networks, namely \( D(p) = 1 - ep \) for all \( p \leq v \), otherwise \( D(p) = 0 \). The random utility model of subscription demand can easily be extended to \( n \) networks. Network \( i \)'s market share is then given by \( S_i = V(p_i)^{\frac{1}{\gamma}} / \sum_{j=1}^{n} V(p_j)^{\frac{1}{\gamma}} \), where the net benefit function \( V(p_i) \) is defined in (1). The price-elasticity of subscription demand is still given by (3) and equal to \( \sigma(p, n) = (1 - n^{-1}) \sigma_{vp}(p) / \gamma \) if all networks charge the same price \( p \).

Our characterization of the equilibrium call price mappings only focuses on the information sets where either no deviation has occurred or only one pair of networks has deviated in the access pricing stage (the proof is in Appendix A.3):

**Lemma 3** Consider a market with \( n \) networks under STR. Assume that the equilibrium prescribes all network-pairs to agree on the same access price \( a \in [-c_o, A] \).
1. There exists a unique and symmetric equilibrium in the continuation game following universal agreement on $a$, provided that call demand is sufficiently inelastic. The equilibrium call price $p^* (\gamma, a, n) \in [c, v)$ is implicitly given by

$$
\frac{p^* - c}{p^*} = \frac{1}{\eta(p^*) + \sigma(p^*, n)} \left[ 1 + \frac{n - 1}{n} \frac{a - c}{p^*} \eta(p^*) \right]. \quad (8)
$$

The price $p^*$ is increasing in the access price and the degree of network differentiation. Entry reduces call prices if, and only if, the access price is sufficiently low relative to network substitutability to ensure an equilibrium price below the monopoly level prior to entry.

2. There exists a unique equilibrium in the continuation game following a single deviation to $\tilde{a} \in [-c_0, a]$ by $A_{ij}$, provided that call demand is sufficiently inelastic. Networks $i$ and $j$ set the same call price $p^* (\gamma, (\tilde{a}, a), n)$ implicitly defined in equation (19) in the Appendix. The price $p^* (\gamma, (\tilde{a}, a), n)$ is increasing in the access charge $\tilde{a}$. All networks except $i$ and $j$ set the call price $p^* (\gamma, a, n)$.

The interesting thing to note here is that entry has an ambiguous effect on call prices. Upon inspection of (8), we see that entry affects the call price through two channels. The elasticity of subscriber demand $\sigma(p^*, n)$ increases with entry, which tends to push down the call price. This is the standard competition effect of entry, affecting prices in most industries. In telecom, the networks’ effective marginal cost also increases with entry. The effective marginal cost is defined as $C(a, n) = c + \frac{n-1}{n} (a - c)$, taking into account that a share of calls are terminated off-net and are therefore subject to an access price mark-up. This double-margins effect pulls the call price in the opposite direction. Either effect may dominate, but they cancel out exactly at the monopoly price.
Even if the number of firms grows without any bound, the price will not necessarily be pushed down to the marginal cost. This is true also when networks are close substitutes and the access price is low. One reason is that every single network has a fraction of loyal customers since every network offers its own variety to the market. This is a well-known effect of the random utility model (cf. Anderson, de Palma and Thisse, 1992). Another reason, specific to network industries, is that the access price markup will increase in importance, the more fragmented the market becomes.

### 3.2 Access price Equilibrium

When networks $i$ and $j$ (formally, agent $A_{ij}$) set their access price, they assume all other network-pairs to stick to the recommended access price $a$. A marginal deviation in the access price to $\tilde{a} \neq a$, with the purpose of inducing the call price to deviate from $p^* = p^*(\gamma, a, n)$ to $p^* (\gamma, (\tilde{a}, a), n)$, has the following effect on the expected joint profit

$$\frac{\partial(\pi_i + \pi_j)}{\partial p_i} = S_iD(p^*) + S_i \left( p^* - \tilde{C} (a, n) \right) D'(p^*) + \left( \frac{\partial S_i}{\partial p_i} + \frac{\partial S_j}{\partial p_i} \right) (p^* - c) D(p^*),$$

(9)

where $\tilde{C} (a, n) = c + (1 - S_i - S_j) (a - c_i)$ is the joint marginal cost of the two networks. Note that the maximization problem facing the agent is similar to that facing an individual network. The optimal price is a trade-off between a higher mark-up on the one hand and lower call demand and smaller customer bases on the other hand. The difference is that the network pair internalizes both the competition effect ($\partial S_j/\partial p_i$) and the double margins effect ($C(a, n) - \tilde{C}(a, n) = S_j(a - c_i)$) on the rival-cum-partner’s profit. Internalization of the competition effect is the standard cartel motive and pulls in the direction of a high access price $\tilde{a}$, to discourage poaching. Internal-
ization of the double margins is the standard motive for vertical integration and pulls in favor of a lower access price \( \tilde{a} \).

To determine the equilibrium access and call price, use full market coverage \( \partial S_j / \partial p_i = - (n - 1)^{-1} \partial S_i / \partial p_i \) and the equilibrium relation (8) to get

\[
\frac{\partial (\pi_i^x + \pi_j^x)}{\partial p_i} = - \left( \frac{p^* - c}{p^*} - \frac{1}{\eta(p^*)} \right) \frac{\eta(p^*) D(p^*)}{n(n - 1)}. \tag{10}
\]

It immediately follows that the two networks will reduce their access price if the prescribed access charge induces a call price above the monopoly price, i.e. \( p^*(\gamma, a, n) > p^m \). In this case, the double margins effect dominates the cartel effect. The two networks would like to increase their access price in the opposite case, \( p^*(\gamma, a, n) < p^m \), because the cartel effect then dominates double margins. This final case can be an equilibrium only if the prescribed access price is binding, \( a = \pi \), and a price hike is impossible. In case the access price induces the monopoly call price

\[
a^m(n) = c_t + \frac{n}{n - 1} \frac{p^m \sigma(p^m, n)}{\eta(p^m)^2} = c_t + \frac{1}{\gamma} \frac{p^m \sigma_{vp}(p^m)}{\eta(p^m)^2} = a^m, \tag{11}
\]

no pair has any local incentive to change the access price. To see that \( a^m(n) \) implements the monopoly call price, simply substitute the Lerner rule into the equilibrium relation (8), and use the expression for \( \sigma(p^m, n) \).

**Lemma 4** Consider a market with \( n \) networks under STR. There is a unique semi-symmetric equilibrium access price, given by \( a^* = \min \{ a^m(n), \pi \} \), provided that call price demand is sufficiently inelastic (\( \varepsilon \) is low).

**Proof.** See Appendix A.4.

The equilibrium under bilateral negotiations is the one maximizing industry profit. The access price is exactly the same as if set jointly by all networks. Hence, the outcome of bilateral negotiations is the collusive outcome, independently of market structure. This result is robust and arises
in any symmetric and fully covered market with a balanced call pattern. Recall that entry has two countervailing effects. On the one hand, entry beyond duopoly strengthens access price competition, since a smaller share of the (horizontal) competitive externalities between networks can be internalized in any bilateral negotiation. To see this, note that the cross-price subscription elasticity \( \left( \frac{\partial S_i}{\partial p_j} \right) \left( \frac{p_j}{S_i} \right) = (n \gamma)^{-1} \sigma_{\text{up}} (p^*) \) is decreasing in the number of competing networks in symmetric equilibrium. However, the share of the (vertical) double-margins externality that can be internalized in any bilateral negotiation is smaller. In symmetric equilibrium, the flow of traffic between two specific networks only constitutes a fraction \( n^{-1} \) of the total traffic of each network. Since the degrees to which the two externalities are internalized are both proportional to the two networks’ market shares, the two effects cancel at the monopoly price.

Note also that in this model, the access price is actually independent of the market structure, i.e. \( a^* = a^d \). Unlike when competition is increased through increased network substitutability, the collusive outcome can be sustained without altering the access price whenever competition is increased through entry. This is a corollary to the result that entry does not affect the equilibrium call price, if the call price is at the monopoly level prior to entry.

### 3.3 Policy Implications

The ineffectiveness of access price competition implies that viable competition is unlikely to arise, if the networks are allowed to set any access price, i.e. in the absence of regulation. Entry has an effect on call prices if, and only if, the access price ceiling is binding, \( \alpha^m > \overline{\alpha} \). There are two ways of ensuring a binding access price ceiling. The first is to set the ceiling sufficiently low, the second is to reduce network differentiation to increase \( \alpha^m \). In sum:
Proposition 2  Entry beyond duopoly leads to (weakly) lower call prices. If the access price ceiling is too generous and the networks too differentiated, the monopoly price prevails independently of the number of networks. If the access price ceiling is sufficiently low or networks are sufficiently close substitutes, entry reduces call prices.

This result demonstrates that entry beyond duopoly is a complement to regulation and not a substitute, since entry has an effect only when the access price ceiling is binding.

It is possible that our model underestimates the effect of entry on call price competition. A new network is like a new variety in this model, and product space is never overcrowded. The presence of loyal subscribers tends to limit the intensity of call price competition as new networks enter the market. The effect of a crowded product space can be incorporated into the model by considering a more general network differentiation parameter $\gamma (n, \theta)$, where $\theta$ now signifies switching costs etc., and where $\partial \gamma / \partial n < 0$. In this case, competition dominates double margins even at the monopoly price. Hence, unilateral deviations from the monopoly price become increasingly profitable as entry occurs, which tends to drive the equilibrium access price $a^m (n)$ up to the ceiling. If $\lim_{n \to \infty} \gamma (n) = 0$, additional entry would eventually drive call prices down to the marginal cost.

However, the substantial costs of building new networks, the technical limitations to unbounded entry and the anti-competitive effects of access pricing, lead us to question whether reduced network differentiation is not a more fruitful approach than entry in achieving a competitive environment in telecommunications.
4 Extensions

4.1 Two-Part Tariffs

Telecom networks typically use two-part tariffs, with a call price $p_i$ and a subscription fee $F_i$. It has been argued that access price collusion may then not be a problem. This conclusion is based on Laffont, Rey and Tirole’s (1998a-b) result that networks using two-part tariffs do not have any incentive to raise their access prices above the cost of termination. Assuming the networks to be poor substitutes, the networks set the call price equal to the effective marginal cost and use the subscription fee to extract the resulting consumer surplus. They set the access price equal to the marginal cost, so as to avoid distortions in the call price, since the maximization of the industry profits is the same as the maximization of the social surplus.

We show that the effect of two-part tariffs to a large extent depends on network differentiation. (The formal analysis is relegated to Appendix A.5.) If the networks are nearly perfect substitutes, the subscription fee is competed down to zero and the networks barely break even. If arbitrage possibilities prevent the networks from setting negative subscription fees, they can profit from setting an access price above the termination cost, since they would then have a positive margin on calls.

In fact, the possibility of two-part tariffs does not affect the equilibrium, provided that the subscription fees must be non-negative and the networks are sufficiently close substitutes. In any symmetric equilibrium, the access price is equal to $\max \{ \overline{\pi}, \, a^m \}$, the call price is equal to $p^d$, and the subscription fee is set to zero.

In reality, the true arbitrage condition may be somewhat below zero in case the networks can frame a negative fee as a partial subsidy of handsets,
but this is of no consequence for our results as long as the subscription subsidy cannot be too large.

4.2 Anti-competitive Arbitrage

At the turn of the millennium, the main Swedish mobile carrier Telia launched a campaign offering late night calls at SEK .75 per minute. As termination charges were well above that level, an arbitrage opportunity on off-net calls arose. For example, the access price charged by the main competitor Comviq at the time was SEK 1.60, which opened a per minute arbitrage window of SEK 0.85=1.60-.75 less the marginal cost of termination. A small company called Faxback identified the arbitrage opportunity and struck a deal with Comviq. Comviq agreed to pay Faxback SEK 1.20 per minute for all calls made by Faxback’s Telia subscriptions to a certain phone number in Comviq’s network. Soon thereafter, Faxback connected a large set of Telia mobile phones to its computers and started making eight-hour nightly nonsense calls. After a while, Telia’s computerized intelligence system discovered the plot. The campaign was eventually withdrawn and Faxback was sued for fraudulent behavior. In a recent verdict, Faxback was freed by the Stockholm City court which deemed the arbitrage legal.

The interesting point is that arbitrage of the Faxback type is anti-competitive. Arbitrage effectively eliminates the incentive to undercut the competitor by establishing a call price floor. In the notation of the present paper, arbitrage arises whenever \( a - p > c_t \). The no-arbitrage condition is \( p \geq a - c_t \). Assume that the access price ceiling is generous and call demand not too inelastic, i.e. \( p^{m} \leq \bar{\pi} - c_t \), but subscription demand is very elastic so that \( \alpha^{m} > \bar{\pi} \). If

\footnote{The only sound heard during the calls was the whistling rooster from the Robin Hood movies.}
arbitrage were infeasible, the equilibrium access price would be \( a^* = \bar{\pi} \) and the call price \( p^* (\gamma, \bar{\pi}, n) < p^m \). However, with arbitrage, the access price \( a = p^m + c_t \leq \bar{\pi} \) is sufficient to sustain \( p^m \) as the equilibrium.

These results suggest that policy makers should take steps to prevent arbitrage. Arbitrage would be eliminated under a Bill-and-Keep regime, since arbitrage would imply a negative call price, \( p < -c_t \). Second, Faxback-arbitrage is only feasible if agreements of the Comviq-type are legal. Third, direct arbitrage is feasible only if networks are allowed to operate affiliates with a significant amount of subscriptions in a competitor’s network.

### 4.3 More on Call Demand Elasticity

The practical relevance of our proposed policy hinges on the sensitivity of our results to the price elasticity of demand. If it were the case that equilibrium is only guaranteed for unrealistically low demand elasticities, \( STR \) would not produce call prices close to the marginal cost, but most likely a situation with fluctuating prices. Our proof of existence suggests that the upper bound on \( \varepsilon \) is reduced as \( \gamma \) is pushed towards zero.

To gauge the significance of the price elasticity of demand, we use a numerical simulation. Fortunately, the simulation indicates that the elasticities can be set quite generously. To calibrate the model, we look at the situation in the Swedish market prior to the imposition of access price caps in the late 1990’s. At the time, the call price was approximately \( p = 5 \) SEK per minute (divide by 10 to translate into Euro). Estimates of the short-term marginal cost per minute were not too far from \( c = 0.1 \), and it may be assumed that call termination and call origination were equally expensive, i.e. \( c_o = c_t = 0.05 \). Absent regulation, the observed price was probably close to its monopoly level, and the negotiated access price, which was around \( a = 3 \)
per minute at the time, was sufficient to sustain collusion.

Using the Lerner rule, $(5 - 0.1) / 5 = \eta(5)^{-1}$, we get an equilibrium elasticity of call demand around $\eta^m = 1$. This elasticity was elevated by the lack of competition, and the deep elasticity parameter can be calibrated to be around $\varepsilon = 0.1$. Substituting the prices, estimated costs and $\eta^m = 1$ into the pricing equation (6), i.e.

$$\frac{5 - 0.1}{5} = \frac{1}{1 + \sigma^m} \left( 1 + \frac{13 - 0.05}{2} \right),$$

we may infer the approximate subscriber elasticity to be $\sigma^m = 0.3$. Assuming the price of a pay-phone call (including the disutility of using such a device) to be $v = 9$, the deep network substitutability parameter can be calibrated to be around $\gamma = 3.5$, recalling $\sigma^m = D(p^m) / 2\gamma V(p^m)$.

The question is now whether this situation can be construed as an equilibrium of the model. The answer is yes: substituting the observed and inferred numerical values into the profit function, the profit function is nicely concave whenever the competitor charges the monopoly price.

The next issue is to study the effect of a policy shift in line with STR. Consider first a reduction in $\gamma$, but without any access price ceiling. For instance, increasing network substitutability to $\sigma' = 1$ (approximately corresponding to $\gamma' = 1$) would imply that the networks have to set an access price of more than $\sigma' = 10$ to induce the monopoly price.

Consider a further increase of the network substitutability, say to $\gamma'' = 0.5$ (which would correspond to $\sigma = 2$ at a symmetric monopoly price), but now assume that the regulator imposes a price ceiling of $\sigma = 15$. At the maximal access price, the monopoly price can no longer be sustained as an equilibrium and the call price falls to $p'' = 4.4$. Successive increases in network substitutability, first to $\gamma''' = 0.1$ and then to $\gamma'''' = 0.05$, lead to successive falls in the equilibrium price, first to $p''' = 1.5$ and then to
\[ p''' = 0.9.\] All cases are summarized in the following table.

<table>
<thead>
<tr>
<th>Case</th>
<th>( \pi )</th>
<th>( a )</th>
<th>( \gamma )</th>
<th>( p )</th>
<th>( \sigma (p) )</th>
<th>( \eta (p) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original situation</td>
<td>-</td>
<td>3</td>
<td>3.5</td>
<td>5</td>
<td>0.3</td>
<td>1</td>
</tr>
<tr>
<td>Experiment 1</td>
<td>-</td>
<td>10</td>
<td>1</td>
<td>5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Experiment 2</td>
<td>15</td>
<td>15</td>
<td>0.5</td>
<td>4.4</td>
<td>1.6</td>
<td>0.8</td>
</tr>
<tr>
<td>Experiment 3</td>
<td>15</td>
<td>15</td>
<td>0.1</td>
<td>1.5</td>
<td>1.8</td>
<td>0.2</td>
</tr>
<tr>
<td>Experiment 4</td>
<td>15</td>
<td>15</td>
<td>0.05</td>
<td>0.9</td>
<td>2</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Note that the equilibrium prevails under the successive reductions of \( \gamma \), despite keeping \( \varepsilon = 0.1 \) fixed. The reduction of the price elasticity of call demand, \( \eta \), is due to the reduced price. These demand elasticities are quite low, but broadly consistent with econometric evidence (approximately 0.5 on US data, see e.g. Hausman, 2002).

As is evident from Figure 1, the profit function becomes increasingly peaked around the equilibrium price as \( \gamma \) decreases. We anticipate further reductions in \( \gamma \) to have no effect on the existence of equilibrium. Hence, for plausible initial values of \( \gamma \) and \( \varepsilon \), and with a very generous access price ceiling \( \pi = 15 \), changes in the degree of network substitutability have no effect on the existence of equilibrium.

The source of the existence problem is that a network may find it profitable to significantly cut its price to corner the market. However, at high degrees of network substitutability, the price charged by the competitor will be close to the marginal cost and thus, the possibility of undercutting the rival diminishes as networks become highly substitutable. Our simulations indicate that at realistic values, the strategic effect working through the competitor’s price is sufficient to render price cuts unprofitable, even with highly substitutable networks.
5 Concluding Remarks

Several empirical studies suggest that telecommunications has a surprisingly large impact on economic growth. One study indicates that a third of the growth in the OECD area over a 20-year period can be attributed to the direct and indirect impact of telecommunications (Röller and Waverman, 2001). Later research suggests that mobiles have a large impact on growth, and especially in developing countries, where fixed lines may not be viable (Waverman, Meschi and Fuss, 2005). The creation of effective competition in the mobile telecom industry may thus have far more important consequences that previously understood. This paper investigates the role that regulation should play to create effective competition.

Our three main points Our results show that competition and regulation should be viewed as complements in reducing the prices of mobile phone calls.
Our analysis is not so much concerned with the process of entry as with different levels of concentration in mature markets. Regulation is therefore also a complement to competition in the long run.

We also show that the required interventions may be limited to defining structural rules (STR-regulation) for the networks’ pricing rather than to setting access price ceilings close to some measure of cost. The necessary information may therefore be minimal and the problems of cost-containment avoided.

Finally we show that the call price competition and access price competition induced by entry are offset by an increase in the networks’ effective cost, as a larger share of calls are terminated in the rivals’ networks and therefore subject to the access price markup. Also considering the substantial costs of building new networks, our results suggest that other methods for reducing call prices may be preferred. We have demonstrated that efforts to reduce network differentiation may be one such alternative. In fact, the STR ban on price discrimination arguably also helps simplifying the consumers’ price comparisons and may contribute to competition also for this reason.

Our analysis primarily focuses on the sufficiency of STR-regulation in forcing prices down towards the marginal cost in a setting with a high degree of network substitutability. The necessity of the access price cap is immediate from our analysis. What about the other elements of the policy? Absent mandatory interconnection, each network has an incentive to make the networks incompatible, thereby creating strong network externalities, with the aim of driving the rivals out of the market. As shown in a companion paper which assumes perfect substitutability of networks (Stennek and Tangeräs, 2006), non-reciprocal access prices and price discrimination between off- and on-net calls can be used to sustain strong network externalities even with
mandatory interconnection.

**Future research**  Note also that with increasing returns to scale in the industry, the marginal cost is below the average cost, and Ramsey pricing is a more appropriate welfare benchmark than marginal cost pricing. It is not socially optimal to strive for perfectly substitutable networks. We leave the question of optimal network substitutability to future research.

In our model all consumers have the same call demand and, therefore, the networks offer only one contract each. In reality most networks offer a menu of contracts, presumably to price discriminate between consumers with different call patterns. Dessein (2003) analyzes non-linear pricing in duopoly. We expect entry to both increase competition and to raise the perceived cost of calls even heterogenous consumers. Access price competition with heterogenous consumers is left for future research.

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**A Appendix**

**A.1 Proof of Lemma 1**

Lemma 1 is an immediate corollary to Lemma 5.

**Lemma 5** Consider a market with $n$ networks under STR and assume that all access prices are common knowledge, but not necessarily the same. There exists a pure strategy equilibrium in call prices if demand is sufficiently inelastic (that is, if $\varepsilon$ sufficiently low). If, in addition, all access prices are identical and equal to $a \in [-c_v, \pi]$, the equilibrium $z^* (\gamma, a, n) \in [c, v)$ is unique and symmetric and implicitly given by

$$\frac{z^* - c}{z^*} = \frac{1}{\eta (z^*) + \sigma (z^*, n)} \left[ 1 + \frac{n - 1}{n} \frac{a - c_t}{z^*} \eta (z^*) \right]. \quad (12)$$

The equilibrium price $z^*$ is increasing in the access price and the degree of network differentiation. It is decreasing in the number of networks for all prices below the monopoly price and increasing in the number of networks for all prices above the monopoly price.

**Proof.** We will show that there exists a unique and symmetric pure strategy equilibrium for all $\varepsilon < \varepsilon (\gamma, \pi)$ and some $\varepsilon (\gamma, \pi) > 0$. The equilibrium price is above the marginal cost, $c$, but below a certain highest price $P (\gamma, \varepsilon, \pi) \in (c, v)$. Let $p_{-i} = (p_1, \ldots, p_{i-1}, p_{i+1}, p_n)$, so that $p = (p_i, p_{-i})$. With $n$ networks, the profit of network $i \in N = \{1, 2, \ldots, n\}$ is

$$\pi_i (p) = S_i \left( (p_i - c) D (p_i) + \sum_{k \neq i} S_k \left( a_{ik} - c_k \right) (D (p_k) - D (p_i)) \right),$$

36
where $a_{ik}$ is the reciprocal access price between networks $i$ and $k$. The marginal profit is equal to

$$\frac{\partial \pi_i}{\partial p_i} = \frac{\partial S_i}{\partial p_i} (p_i - c) D (p_i) + S_i (D (p_i) + (p_i - c) D' (p_i))$$

$$+ \sum_{k \neq i} (a_{ik} - c_i) S_k \left( \frac{\partial S_k}{\partial p_i} + \frac{s_k}{s_k} \frac{\partial S_k}{\partial s_k} \right) D (p_k) - D (p_i)) - S_i D' (p_i).$$

(13)

Substitute $\frac{\partial S_i}{\partial p_i} = -\gamma^{-1} S_i (1 - S_i) \frac{D(p_i)}{V(p_i)}$ and $\frac{\partial S_k}{\partial p_i} = \gamma^{-1} S_i S_k \frac{D(p_i)}{V(p_i)}$ into the marginal profit and rewrite to obtain

$$R_i(p) = \frac{\partial \pi_i}{\partial s_i} \frac{\gamma V(p_i)}{S_i D(p_i)} = - (1 - S_i) (p_i - c) D (p_i) + \gamma \left( 1 + (p_i - c) \frac{D(p_i)}{D(p_i)} \right) V (p_i)$$

$$- \sum_{k \neq i} (a_{ik} - c_i) S_k \left( (1 - 2S_i) (D (p_k) - D (p_i)) + \gamma \frac{D(p_k)}{D(p_i)} V (p_i) \right)$$

(14)

Since $\gamma V (p_i) / S_i D (p_i) > 0$ for all $p_i < v$, $\text{sgn}\{\partial \pi_i / \partial p_i\} = \text{sgn}\{R_i(p)\}$ for all $p_i < v$.

**Existence** The existence proof proceeds in four claims. The first two claims establish that a network will never set a price above $P (\gamma, \varepsilon, \overline{z}) \in (c, v)$ nor below $c$, given that everybody else charges a call price at or above $c$, at least one competitor sets a price in $[c, v)$ and $\varepsilon$ is sufficiently small.

**Claim 1** There exists an $\varepsilon_1 (\gamma, \overline{z}) > 0$ such that for all $\varepsilon < \varepsilon_1 (\gamma, \overline{z})$, $p_i < c$ is a strictly dominated strategy.

**Proof.** Note that $R_i (p)$ can be rewritten as

$$R_i(p) = - (1 - S_i) (p_i - c) D (p_i) - \gamma \left( S_i c + \sum_{k \neq i} (a_{ik} + c_o) S_k \right) \frac{D(p_i)}{D(p_i)} V (p_i)$$

$$+ \gamma \left( 1 - \frac{\varepsilon p}{1 - \varepsilon p} \right) V (p_i) - \varepsilon \sum_{k \neq i} S_k (1 - 2S_i) (a_{ik} - c_i) (p_i - p_k),$$

where we have used $D (p) = 1 - \varepsilon p$ in the last line. The sum of the terms on the first line is strictly positive for $p_i \leq c$. The expression on the second line is strictly positive for $\varepsilon$ sufficiently small. Hence, $\pi_i (p) < \pi_i (c, p_{-i})$ for all $p_i < c$ and for all $p_{-i}$, provided that $\varepsilon$ is sufficiently small. ■

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Let $M_{-i}$ be the set of networks not including $i$ that charge a call price strictly below $v$. Let $\overline{p}(p_{-i})$ be the maximal of these prices for the case that $M_{-i} \neq \emptyset$.

**Claim 2** Assume that $\varepsilon < (v+c)^{-1}$. If $p_k \geq c$ for all $k \neq i$ and $p_j < v$ for some $j \neq i$, then there exists a $\mathcal{P}(\gamma, \varepsilon, \overline{\alpha}) \in (c, v)$ such that $\partial \pi_i / \partial p_i < 0$ for all $p_i \in [\max\{\mathcal{P}; \overline{p}(p_{-i})\}, v)$. $\mathcal{P}$ is increasing in $\gamma, \varepsilon$ and $\overline{\alpha}$.

**Proof.** By manipulating terms and using linearity of demand, we get

$$R_i(p) = -\left(\frac{1}{2} (1 - \varepsilon (v+c)) (p_i - c) - \gamma V (p_i) \varepsilon \gamma \max\{0; \overline{\alpha} V(p_i) \frac{V(p_i)}{D(p_i)}\} \right)$$

$$- \varepsilon \gamma (p_i - c + \max\{0; \overline{\alpha}\} - \sum_{k \neq i} S_k a_{ik} + (1 - S_i) c_i) \frac{V(p_i)}{D(p_i)}$$

$$- \left((\frac{1}{2} - S_i) (1 - \varepsilon (v+c)) + \varepsilon (1 - S_i) (v - p_i + 2cS_i) (p_i - c)\right)$$

$$- \varepsilon \sum_{k \neq i} S_k (1 - 2S_i) (c (p_k - c) + (a_{ik} + c_o) (p_i - p_k)).$$

Define the term in parenthesis in the first line as $H(p_i) = H(p_i, \gamma, \varepsilon, \overline{\alpha})$. Note that $H$ is strictly increasing in $p_i$ since $\varepsilon < (v + c)^{-1}$, $-V'(p_i) = D(p_i)$ and $-\frac{d}{dp_i} \left[ V(p_i) \frac{V(p_i)}{D(p_i)} \right] = \frac{(1-\varepsilon p_i)^2 + (1-\varepsilon v)^2}{2(1-\varepsilon v)^2} > 0$. Moreover, $H(c) < 0$ and $H(v) > 0$ since $V(v) = 0$. Hence, there exists a unique $\mathcal{P}(\gamma, \varepsilon, \overline{\alpha}) \in (c, v)$ implicitly defined by $H(\mathcal{P}) = 0$, with the property that $H(p_i) > 0$ for all $p_i > \mathcal{P}$. Note also that the second line is strictly negative for all $p_i \in [\mathcal{P}, v)$, whereas the two final lines are non-positive for all $v > p_i \geq \overline{p}(M_{-i})$ and $p_k \geq c$ for all $k \neq i$. Hence, $R_i(p) < 0$ for all $p_i \in [\max\{\mathcal{P}(\gamma, \varepsilon, \overline{\alpha}); \overline{p}(M_{-i})\}, v)$. The properties of $\mathcal{P}(\gamma, \varepsilon, \overline{\alpha})$ follow from implicit differentiation of $H(\mathcal{P}, \gamma, \varepsilon, \overline{\alpha}) = 0$.

**Claim 3** There exists a pure strategy Nash equilibrium, $z^* \in [c, \mathcal{P}(\gamma, \varepsilon, \overline{\alpha})]^n$, for every $\varepsilon < \varepsilon_2(\gamma, \overline{\alpha})$, and for some $\overline{\varepsilon}_2(\gamma, \overline{\alpha}) > 0$.

**Proof.** First, note that claims 1 and 2 guarantee that every network will set a price in $[c, \mathcal{P}]$ given that all other networks set a price in $[c, \mathcal{P}]$, provided that
\( \varepsilon \) is sufficiently small. Second, note that \( \pi_i \) is continuous in \( p \) on the domain \( p \in [c, P]^n \). The existence proof amounts to verifying quasi-concavity of \( \pi_i \) in \( p_i \) on \( [c, P] \) for all \( \varepsilon < \varepsilon_2 (\gamma, \pi) \) and some \( \varepsilon_2 (\gamma, \pi) > 0 \). Claim 1 implies \( R_i(c, p_{-i}) > 0 \), and Claim 2 implies \( R_i(P, p_{-i}) < 0 \) for all \( p_{-i} \in [c, P]^{n-1} \).

Hence, there exists a \( \hat{p}_i (p_{-i}) \in (c, P) \) which satisfies \( R_i(\hat{p}_i, p_{-i}) = 0 \). If \( \hat{p}_i \) is uniquely defined, \( \pi_i \) is single-peaked and therefore strictly quasi-concave in \( p_i \) on \([c, P]\). We now demonstrate that \( R_i(p) \) is strictly decreasing in \( p_i \) in the interval \([c, P]\), provided that \( \varepsilon \) is sufficiently small. Note that

\[
\frac{\partial R_i(p)}{\partial p_i} = - \left( \gamma + (1 - S_i) \left( 1 + \frac{1}{\gamma} S_i \frac{D(p_i)}{V(p_i)} (p_i - c) \right) \right) D(p_i) \\
- \varepsilon \left( \gamma \frac{V(p_i)}{D(p_i)} - \sum_{k \neq i} S_i S_k (a_{ik} - c_t) \right) \\
- \varepsilon \gamma \left( p_i - c - \sum_{k \neq i} S_k (a_{ik} - c_t) \right) \frac{d}{dp_i} \left[ \frac{V(p_i)}{D(p_i)} \right] \\
- \varepsilon \sum_{k \neq i} \left( a_{ik} - c_t \right) S_i S_k \left( 3 - 4 S_i \right) \frac{D(p_i)}{V(p_i)} (p_i - p_k) \\
- \varepsilon \left( \sum_{k \neq i} (a_{ik} - c_t) S_k (1 - 2 S_i) - (1 - S_i) (p_i - c) \right),
\]

where we have used Roy’s identity \( V'(p_i) = -D(p_i) \), the explicit expressions for \( \partial S_i / \partial p_i \) and \( \partial S_j / \partial p_i \) as well as linear demand \( D(p) = 1 - \varepsilon p \) and \( D'(p) = -\varepsilon \). Since \( \lim_{\varepsilon \to 0} V(p) / D(p) = v - p \),

\[
\lim_{\varepsilon \to 0} \frac{\partial R_i(p)}{\partial p_i} = - \left( \gamma + (1 - S_i) \left( 1 + \frac{1}{\gamma} S_i \frac{p_i - c}{v - p_i} \right) \right) < 0 \forall p_i \leq \mathcal{P} (\gamma, \varepsilon, \overline{\pi}) < v.
\]

Hence, \( R_i(p) \) is strictly decreasing in \( p_i \) in the interval \([c, \mathcal{P}]\) provided that \( \varepsilon \) is sufficiently small. \( \blacksquare \)

Existence is ensured for all \( \varepsilon \) smaller than the minimum of \( \varepsilon_1 (\gamma, \pi) \) used in Claim 1, \( (v + c)^{-1} \) used in claim 2 and \( \varepsilon_2 (\gamma, \pi) \), used in Claim 3.
Uniqueness  Claim 1 establishes that all equilibrium prices must be at or above the marginal cost provided that $\varepsilon$ is sufficiently small. The uniqueness proof proceeds in four claims. Claim 4 establishes that all equilibrium prices are contained in $[c, P]$ provided that $\varepsilon$ is sufficiently small. Claim 5 establishes that any equilibrium in which two networks charge symmetric access prices forces them to charge identical call prices, provided that $\varepsilon$ is sufficiently small. This holds the implication that all networks charge the same call price if all access prices are the same. Finally, Claim 6 verifies that there can be one symmetric equilibrium at most. Let $z^*$ denote an equilibrium and $z^*_i$ the equilibrium price of network $i$.

Claim 4 Assume that $\varepsilon < (v + c)^{-1}$ and $\varepsilon < \varepsilon_1 (\gamma, \bar{\pi})$ defined in Claim 1 are both satisfied. In any equilibrium $z^*$, $z^*_i \in [c, \mathcal{P})$ for all $i \in N$ with $\mathcal{P} (\gamma, \varepsilon, \bar{\pi})$ defined in Claim 2.

Proof. We first demonstrate that at least one network charges a call price strictly below $v$ in equilibrium. Suppose, on the contrary, that $p_i \geq v \forall i \in N$. The industry profit is $\sum_{k \in M} S_k (v - c) D (v) \leq (v - c) D (v)$ in this case, where $M$ is the (possibly empty) set of networks which charge a call price exactly equal to $v$. It follows that at least one network, say network $j$, earns a profit strictly below $(v - c) D (v)$. Any deviation by $j$ to $v - \delta$, $\delta > 0$, would render $j$ the monopoly status and profit $(v - \delta - c) D (v - \delta)$. By setting $\delta$ arbitrarily close to but below $v$, $j$ could strictly increase its profit. Having established that at least one network charges a call price below $v$ in equilibrium, we now show that all networks set a price strictly below $v$ in equilibrium. Suppose wlog that $p_i \geq v$ in equilibrium. Since, in this case, $M_{-i} \neq \emptyset$ and $p_k \geq c$ for all $k \neq i$, we know from Claim 2 that $\pi_i (\max\{\mathcal{P}; \bar{\pi} (p_{-i})\}, p_{-i}) > \pi_i (p) = 0$ for all $p_i \geq v$, and so $p_i \geq v$ cannot
be an equilibrium. Having established that all networks charge a call price strictly below \( v \) in equilibrium, we now show that all networks set a price strictly below \( P \) in equilibrium. Assume wlog that network \( i \) charges the maximal price, i.e. \( p_i \geq \overline{p}(\underline{p}_{-i}) \). For any \( p_i \in (P, v) \), \( i \) will strictly profit by lowering its call price, see Claim 2; hence, the maximal equilibrium price must necessarily be strictly below \( P \).

**Claim 5** Assume that networks \( i \) and \( j \) set symmetric access prices, \( a_{ik} = a_{jk} \forall k \neq i, j \). There exists an \( \varepsilon_3 (\gamma, \overline{p}) > 0 \) such that for all \( \varepsilon < \varepsilon_3 (\gamma, \overline{p}) \) any call price equilibrium \( z^* \in [c, P] \), satisfies \( z_i^* = z_j^* \).

**Proof.** Any interior equilibrium \( p_k \in (c, v) \forall k \in N \) must satisfy the two first-order conditions \( R_i(p) = 0 \) and \( R_j(p) = 0 \), where \( R_i(p) \) was defined in equation (14) and \( R_j(p) \) can be equivalently defined. It follows that every interior equilibrium must satisfy \( R_i(p) - R_j(p) = 0 \). \( R_i(p) \) is strictly decreasing in \( p_i \) in the interval \([c, P]\) provided that \( \varepsilon \) is sufficiently small; see (15). Note that

\[
\frac{\partial R_j(p)}{\partial p_i} = \frac{1}{\gamma} S_i S_j \frac{D(p_i)}{V(p_i)} (p_j - c) D(p_j) + \varepsilon (a_{ij} - c_i) S_i (1 - 2S_j) + \varepsilon \sum_{k \neq j} S_i S_k (a_{jk} - c_j) \left( 1 - \frac{1}{\gamma} \frac{D(p_i)}{V(p_i)} (1 - 4S_j) (p_j - p_k) \right) - \varepsilon (a_{ij} - c_i) S_i (1 - S_i) \left( 1 - \frac{1}{\gamma} (1 - 2S_j) \frac{D(p_i)}{V(p_i)} (p_j - p_i) \right),
\]

where we have used the explicit expressions for \( \partial S_i / \partial p_i \) and \( \partial S_j / \partial p_i \) and \( \partial S_k / \partial p_i \) as well as linear demand \( D(p) = 1 - \varepsilon p \) and \( D'(p) = -\varepsilon \). Since

\[
\lim_{\varepsilon \to 0} V(p) / D(p) = v - p,
\]

\[
\lim_{\varepsilon \to 0} \frac{\partial R_j(p)}{\partial p_i} = \frac{1}{\gamma} S_i S_j \frac{p_j - c}{v - p_i} \geq 0 \forall p_i < v, p_j \geq c.
\]

(16)

To summarize, \( R_i(p) - R_j(p) \) is strictly decreasing in \( p_i \) in the interval \([c, P]\), provided that \( \varepsilon \) is sufficiently small. For every \( p_j \in [c, P] \), therefore, there can
be at most one solution \( \hat{p}_i \in [c, P] \) to \( R_i(\hat{p}_i, p_{-i}) - R_j(\hat{p}_i, p_{-i}) = 0 \), provided that \( \varepsilon \) is sufficiently small. Since \( i \) and \( j \) charge symmetric access prices, \( R_i(p_j, p_{-i}) = R_j(p_j, p_{-i}) \). Thus, \( R_i(p) \neq R_j(p) \) for all \( p_i \neq p_j \), which excludes the possibility of an asymmetric equilibrium. ■

**Claim 6** There exists at most one symmetric equilibrium \( z^*_i = z^* \in [c, v) \) \( \forall i \in N \). The equilibrium price is the implicit solution to equation \((8)\).

**Proof.** By substituting \( z^*_i = z^* \) \( \forall i \) and \( a_{ik} = a \) \( \forall i, k \) into the first-order condition \( R_i(z^*) = 0 \) and using \( D(z^*) = 1 - \varepsilon z^* \), we find that any symmetric equilibrium must be a solution to \( g(z^*, \gamma, a, n) = 0 \), where

\[
g(p, \gamma, n, a) = 1 - \varepsilon \left( 2p - c - 1 - \frac{n - 1}{n} (a - c_i) \right) - \frac{1}{\gamma} \frac{n - 1}{n} (p - c) \frac{D^2(p)}{V(p)}
\]

is a third-degree polynomial. The derivative

\[
\frac{\partial g(p, \gamma, n, a)}{\partial p} = -2 \varepsilon - \frac{1}{\gamma} \frac{n - 1}{n} \left( \frac{D^2(p)}{V(p)} + (p - c) \frac{d}{dp} \left[ \frac{D^2(p)}{V(p)} \right] \right),
\]

is negative for all \( p \in [c, v) \) since

\[
\frac{d}{dp} \left[ \frac{D^2(p)}{V(p)} \right] = \frac{D^2(p)}{V^2(p)} \left( 2D'(p) V(p) + D^2(p) \right) = \frac{D(p)}{V^2(p)} (1 - \varepsilon v)^2 > 0.
\]

Since \( g(p, \gamma, a, n) \) is strictly decreasing in \( p \) \( \forall p \in [c, v) \), there exists at most one solution to \( g(z^*, \gamma, a, n) = 0 \) in \( z^* \in [c, v) \). It is easy to rewrite \( g(z^*, \gamma, a, n) = 0 \) as \((12)\). ■

Defining \( \varepsilon(z, \gamma) \) as the minimum of \( \varepsilon_1(\gamma) \), \( (v + c)^{-1} \), \( \varepsilon_2(\gamma, \bar{p}) \) and \( \varepsilon_3(\gamma, \bar{p}) \) completes the existence and uniqueness proof.

**Comparative statics** The symmetric equilibrium \( z^* \) is the implicit solution to \( g(z^*, \gamma, a, n) = 0 \), where \( g(p, \gamma, a, n) \) is defined in equation \((17)\). Thus, \( dz^*/d\gamma = - (\partial g/\partial \gamma) / (\partial g/\partial p) \), \( dz^*/da = - (\partial g/\partial a) / (\partial g/\partial p) \) and...
\[ \frac{dz^*}{dn} = -\left( \frac{\partial g}{\partial n} \right) / \left( \frac{\partial g}{\partial p} \right). \] Since \( \frac{\partial g}{\partial p} < 0 \), \( \frac{\partial g}{\partial a} = (n-1) \varepsilon/n > 0 \) and \( \frac{\partial g}{\partial \gamma} = (n-1) (z^* - c) D^2 (z^*) / (V (z^*) n \gamma^2) \geq 0 \), the first two results follow. Using eq. (12), it can easily be verified that
\[ \frac{\partial g}{\partial n} = \left( \frac{z^* - c}{z^*} - \frac{1}{\eta (z^*)} \right) \eta (z^*) D (z^*) n (n-1), \]
which completes the comparative statics exercise. \( \blacksquare \)

### A.2 Proof of Proposition 1

When the price elasticity of call demand is sufficiently low, the unique equilibrium price \( p^d \in [c, v] \) is given by (6). Using \( D (p^d) = 1 - \varepsilon p^d \), \( \eta (p^d) = \varepsilon p^d / (1 - \varepsilon p^d) \), and \( \sigma (p^d) = p^d / (1 - \varepsilon p^d) / 2 \gamma (v - p^d) (1 - \varepsilon (v + p^d) / 2) \), we get the following bounds on \( p^d (\gamma, \pi) \):
\[ 0 \leq p^d - c = \frac{2 + (\pi - c_1) \frac{\varepsilon}{1 - \varepsilon p^d}}{1 - \varepsilon p^d} \gamma + \frac{\varepsilon c}{2 \varepsilon} \frac{1}{1 - \varepsilon (v + p^d) / 2} \gamma \leq \frac{(v + c) (c - c_1)}{2 \varepsilon} (c + \pi + c_0) \gamma, \quad (18) \]
where the second inequality follows from \( \varepsilon > 0 \), \( a \leq \pi \), \( p^d \in [c, v] \) and \( \varepsilon < (v + c)^{-1} \) (for the last restriction, see claim 4). Clearly, \( \lim_{\gamma \to 0} p^d (\gamma, \pi) = c. \) \( \blacksquare \)

### A.3 Proof of Lemma 3

Assume throughout the proof of this Lemma that \( \varepsilon < \bar{\varepsilon} (\gamma, \pi) \) where \( \bar{\varepsilon} (\gamma, \pi) \) was defined in the proof of Lemma 5. Moreover, let the equilibrium access price be \( a \) (where \( a \) is now a scalar), and assume that all access prices except possibly \( a_{ij} = a_{ji} = \bar{a} \) are equal to \( a \). Applying passive beliefs, network \( i \)'s expected profit in the continuation game is
\[
\pi^e_i (p_i, (\bar{a}, a)) = S_i \left( p_i, p^*_j (\bar{a}, a), p^*_{-ij} (a) \right) \left[ (p_i - c) D (p_i) \right.
+ S_j \left( p_i, p^*_j (\bar{a}, a), p^*_{-ij} (a) \right) (\bar{a} - c_i) \left( D (p^*_j (\bar{a}, a)) - D (p_i) \right)
+ \sum_{k \neq i,j} S_k \left( p_i, p^*_j (\bar{a}, a), p^*_{-ij} (a) \right) (a - c_i) \left( D (p^*_k (a)) - D (p_i) \right]\right],
\]
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where $p^*_{-ij}(a)$ is the vector of equilibrium prices not including $p_i^*(\tilde{a}, a)$ and $p_j^*(\tilde{a}, a)$, and a symmetric expression can be found for $\pi_j^*(p_j, (\tilde{a}, a))$. Network $k \neq i, j$’s expected profit is 
\[
\pi_k(p_k, p^*_{-k}(a), a) = S_k(p_k, p^*_{-k}(a)) ((p_k - c) D(p_k) + \sum_{l \neq k} S_l(p_k, p^*_{-k}(a)) (a - c_l) (D(p_l^*(a_l)) - D(p_k)))
\].

**Part 1** Note that the maximisation problem facing network $k$ is the same as in Lemma 5, with the exception that here, all access prices are equal to $a$. All networks face the same maximisation problem and by virtue of Lemma 5, the equilibrium is unique and symmetric and given by $p^* = p^*(\gamma, a, n) = z^* (\gamma, a, n)$.

**Part 2** Networks $k \neq i, j$ do not observe the deviation by $i$ and $j$, hence all $k \neq i, j$ anticipate that everybody else will charge the call price $z^*(\gamma, n, a)$. Hence, no network $k \neq i, j$ has an incentive to deviate from $z^*(\gamma, n, a) \in [c, v)$. The maximisation problem facing network $i$ and $j$ is the same as in Lemma 5, with the exception that here all access prices except $a_{ij}$ are equal to $a$ and $a_{ij} = \tilde{a}$. Claims 1 and 2 lead us to the conclusion that $p_i \in [c, P)$ and $p_j \in [c, P)$. As is evident from the proof of Claim 3, $\pi_i^*(p_i, (\tilde{a}, a))$ is strictly quasi-concave in $p_i$ and $\pi_j^*(p_j, (\tilde{a}, a))$ is strictly quasi-concave in $p_j$. Hence, there exists a set of prices $p_i^*(\gamma, (\tilde{a}, a), n)$ and $p_j^*(\gamma, (\tilde{a}, a), n)$ that are mutual best responses to one another, which proves equilibrium in the continuation game.

Next, we demonstrate uniqueness. Any pair of equilibrium prices $(\tilde{p}_i, \tilde{p}_j)$ is interior and must satisfy $\partial \pi_i^*(\tilde{p}_i, (\tilde{a}, a)) / \partial p_i = 0$ and $\partial \pi_j^*(\tilde{p}_j, (\tilde{a}, a)) / \partial p_j = 0$, or equivalently, $R_i(\tilde{p}_i, \tilde{p}_j) = 0$ and $R_j(\tilde{p}_i, \tilde{p}_j) = 0$, where $R_i$ was defined in (14). For $\varepsilon$ sufficiently low, $\partial R_i / \partial p_i < 0$, and $\partial R_j / \partial p_j \geq 0$ see (15).
Hence, for every \( p_i \in [c, P] \), there exists a unique \( \hat{p}_j(p_i) \in (c, P) \) implicitly defined by \( R_j(p_i, \hat{p}_j) = 0 \). Every equilibrium must therefore satisfy \( r_i(\hat{p}_i) = R_i(\hat{p}_i, \hat{p}_j(\hat{p}_i)) = 0 \) and \( \hat{p}_j = \hat{p}_j(\hat{p}_i) \). The equilibrium is unique if \( r_i(p_i) \) is monotonic in the domain \((c, P)\). Note that

\[
\frac{\partial r_i}{\partial p_i} = \frac{\partial R_i}{\partial p_i} + \frac{\partial R_i}{\partial p_j} \frac{d\hat{p}_j}{dp_i} = \left( \frac{\partial R_i}{\partial p_i} \frac{\partial R_j}{\partial p_j} - \frac{\partial R_j}{\partial p_i} \frac{\partial R_i}{\partial p_j} \right) \frac{\partial R_j}{\partial p_j}.
\]

Using (15) and (16),

\[
\lim_{\varepsilon \to 0} \left( \frac{\partial R_i}{\partial p_i} \frac{\partial R_j}{\partial p_j} - \frac{\partial R_i}{\partial p_i} \frac{\partial R_j}{\partial p_j} \right) = \left( \gamma + 1 - S_j \right) \left( \gamma + (1 - S_i) \left( 1 + \frac{1}{\gamma} S_j \frac{p_j - c}{v - p_j} \right) \right)
+ S_j \left( 1 - S_j \right) \left( 1 + \frac{1}{\gamma} \left( 1 - S_i \right) \right) \frac{p_j - c}{v - p_j}
+ \frac{1}{\gamma} S_i S_j \left( 1 - S_j - S_i \right) \frac{p_j - c}{v - p_j} \frac{p_j - c}{v - p_j},
\]

which is strictly positive for all \( p_i \in [c, P] \) and \( p_j \in [c, P] \). Hence, \( r_i(p_i) \) is strictly decreasing in \( p_i \) if \( \varepsilon \) is sufficiently low.

Since \( i \) and \( j \) are symmetric in the access prices, their call prices must also be symmetric, i.e. \( \bar{p}_i = \bar{p}_j = \bar{p}_i \) if \( \varepsilon \) is sufficiently low, see Claim 5.

Plugging the symmetric prices into \( \partial \pi^e_i (p_i, (\bar{a}, a)) / \partial p_i = 0 \) yields \( \bar{p}(\bar{a}, a) = p^* (\gamma, (\bar{a}, a), n) \) implicitly defined by

\[
\frac{\bar{p} - c}{p} = \frac{1}{\eta(\bar{p}) + \sigma_i(\bar{p}, p^*)} \left[ 1 + \left( S_i (\bar{p}, p^*) \left( \frac{\bar{a} - \alpha_i}{p} \right) + (1 - 2S_i (\bar{p}, p^*)) \left( \frac{\alpha - \alpha_i}{p} \right) \right) \eta(\bar{p}) \right]
- \left[ \sigma_j (\bar{p}, p^*) \left( \frac{(\bar{a} - \alpha_i)}{p} \right) S_i (\bar{p}, p^*) + (1 - 3S_i (\bar{p}, p^*)) \sigma_i (\bar{p}, p^*) \right] \left( \frac{(\bar{a} - \alpha_i)}{p} \right) \frac{D(p^*) - D(\bar{p})}{D(\bar{p})}.
\]

(19)

where \( \sigma_{ji} = (\partial S_j / \partial p_i) (p_j / S_j) \) is the cross-price subscriber elasticity and, recall, \( p^* = p^* (\gamma, a, n) \). As is easily verified, \( \bar{p}(a, a) = p^* (\gamma, a, n) \).

Comparative statics: Differentiation of the equilibrium condition \( r_i(\hat{p}_i) = 0 \) yields \( d\bar{p} / d\bar{a} = -(\partial r_i(\hat{p}_i) / \partial \bar{a}) / (\partial r_i(\hat{p}_i) / \partial p_i) \). Since \( \partial r_i(\hat{p}_i) / \partial p_i < 0 \) and \( \partial r_i(\hat{p}_i) / \partial \bar{a} = -\gamma S_j D'(\bar{p}) V(\bar{p}) / D(\bar{p}) \) in symmetric equilibrium, \( d\bar{p} / d\bar{a} > 0 \) follows. □
A.4 Proof of Lemma 4

Existence In the continuation game, $i$ and $j$ set the same call price $\tilde{p}_i = \tilde{p}_j = p^*(\gamma, (\tilde{a}, a), n)$ and all other networks set $p^*(\gamma, a, n)$. By monotonicity of the call price in $\tilde{a}$, the range of call-prices implementable by agent $A_{ij}$ is $\Omega(a) = [p^*(\gamma, (-c_a, a), n), p^*(\gamma, (\pi, a), n)]$. The joint profit $\pi_{ij}(\tilde{a}, a)$ of the two networks can be separated into $\pi_{ij}(\tilde{a}, a) = x(p^*(\gamma, (\tilde{a}, a), n), a) + \varepsilon y(p^*(\gamma, (\tilde{a}, a), n), a)$, where

$$x(p, a) = S(p) \left( (p - c) (1 - \varepsilon p) + \varepsilon^{n-2} (a - c_i) (p - p^*(\gamma, a, n)) \right)$$

and

$$y(p, a) = S(p) \left( \frac{2}{n} - S(p) \right) (a - c_i) (p - p^*(\gamma, a, n)),$$

and $S(p) = 2V(p)^{\frac{1}{n}} / (2V(p)^{\frac{1}{n}} + (n - 2) V(p^*(\gamma, a, n))^{\frac{1}{n}})$ is the total market share of networks $i$ and $j$. Since $p^*(\gamma, (a, a), n) = p^*(\gamma, a, n)$, $S(p^*(\gamma, (a, a), n)) = 2n^{-1} x(p^*(\gamma, (a, a), n), a) = 2n^{-1} (p^*(\gamma, a, n) - c) D(p^*(\gamma, a, n))$, $y(p^*(\gamma, (a, a), n), a) = 0$ and $\partial y/\partial p|_{p=p^*(\gamma,(a,a),n)} = 0$. In the search for the optimal access price, the following result due to Armstrong (1998) is convenient:

**Lemma 6** Let $f$ and $z$ be twice continuously differentiable functions of a scalar variable defined on $[\underline{p}, \bar{p}]$. Let $f$ have a unique maximand $p^*$, and assume that $f''(p^*) < 0$ and $z(p^*) = z'(p^*) = 0$. Then for all sufficiently small (but positive) $\varepsilon$, $p^*$ is the unique maximand of the function $f(p) + \varepsilon z(p)$.

The usefulness of this Lemma becomes obvious once we realize that

**Claim 7** $x(p, a)$ has a unique maximand $\tilde{p}(a)$ in $\Omega(a)$ for every $a \in [-c_a, \pi]$.

The maximand has the following properties: $\tilde{p}(a^m(n)) = p^*(\gamma, a^m(n), n) = p^m$ and $\partial^2 x / \partial p^2 |_{p=p^m} < 0$ provided that $a^m(n) \leq \pi$, $\tilde{p}(\pi) = p^*(\gamma, \pi, n)$ and $\partial^2 x / \partial p^2 |_{p=\tilde{p}(\pi)} < 0$, provided that $a^m(n) > \pi$. 

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Proof. We first show that \( x(p,a) \) has a unique maximand in \( \Omega(a) \). The set \( \Omega_+(a) \) of \( p \)'s in \( \Omega(a) \) for which \( x(p,a) \geq 0 \) is non-empty due to the fact that \( p^*(\gamma,a,n) \in \Omega(a) \) and \( x(p^*(\gamma,a,n),a) \geq 0 \) since \( p^*(\gamma,a,n) \in [c,v) \). Obviously, every maximand of \( x(p,a) \) must be contained in \( \Omega_+(a) \). We now demonstrate that \( x(p,a) \) has a unique maximand \( \hat{p}(a) \) in \( \Omega_+(a) \). By continuity of \( x \), the set of \( p \)'s for which \( x < 0 \) is open. Hence, \( \Omega_+(a) \) is closed. Since \( \Omega_+(a) \) is even bounded, it is compact, hence has a maximum. \( x(p,a)/S(p) \) strictly concave in \( p \) renders \( \Omega_+(a) \) convex since \( S > 0 \). Differentiate:

\[
\frac{\partial x}{\partial p} = \frac{s'(p)}{s(p)} x(p) + S(p) \left( 1 + \varepsilon c + \varepsilon \frac{(n-2)}{n} (a - c) - 2\varepsilon p \right)
\]

and

\[
\frac{\partial^2 x}{\partial p^2} = 2 \frac{s'(p)}{s(p)} \frac{\partial x}{\partial p} + \left[ \frac{d}{dp} \left( \frac{s'(p)}{s(p)} \right) \right] x - 2\varepsilon S(p).
\]

Note that

\[
\frac{d}{dp} \left( \frac{s'(p)}{s(p)} \right) = \frac{1}{\gamma} S'(p) \frac{D(p)}{V(p)} - \frac{1}{\gamma} \left( 1 - S(p) \right) \frac{d}{dp} \left( \frac{D(p)}{V(p)} \right) < 0,
\]

since \( S'(p) < 0 \) and \( \frac{d}{dp} \left( \frac{D(p)}{V(p)} \right) > 0 \), see the proof of Claim 2. Hence, \( \partial^2 x/\partial p^2 < 0 \) for all \( p \in \Omega_+(a) \) satisfying \( \partial x/\partial p \geq 0 \). Convexity of \( \Omega_+(a) \) means that there can be at most one solution \( x'(\hat{p}) = 0 \) in \( \Omega_+(a) \). Hence, \( x \) is single-peaked in \( p \in \Omega_+(a) \) and therefore has a unique maximand \( \hat{p} \in \Omega_+(a) \).

Next, let us characterise the optimum. Use \( p^* = p^*(\gamma,a,n) \) in (8) to get

\[
\frac{\partial x(p,a)}{\partial p} \bigg|_{p=p^*} = -\frac{2}{n(n-1)} \left[ \frac{p^* - c}{p^*} - \frac{1}{\eta(p^*)} \right] \eta(p^*) D(p^*).
\]

Assume first that \( a = a^m(n) \leq \bar{a} \). In this case, \( p^* = p^m \) and therefore \( \partial x(p,a^m(n))/\partial p \bigg|_{p=p^m} = 0 \). It follows that \( \hat{p}(a^m(n)) = p^m \) is the unique maximand of \( x \) in \( \Omega(a^m(n)) \). Assume next that \( a = \bar{a} < a^m(n) \). In this case \( p^* < p^m \) and therefore \( \partial x(p,\bar{a})/\partial p \bigg|_{p=p^*} > 0 \). \( \partial^2 x/\partial p^2 < 0 \) for all \( p \in \Omega_+(a) \) satisfying \( \partial x/\partial p \geq 0 \) implies that \( x \) is strictly increasing in \( p \) for all \( p \in \Omega_+(a) \).
in this case. Hence, \( \hat{p}(\pi) = p^*(\gamma, \pi, n) \) is the unique maximand of \( x \) in \( \Omega(\pi) \).

By virtue of Claim 7, the properties of \( y(p) \) and Lemma 6, \( a^m(n) \) is the unique maximand of \( \pi_{ij}(\bar{a}, a^m(n)) \) for \( a^m(n) \leq \pi \), and \( \pi \) is the unique maximand of \( \pi_{ij}(\bar{a}, \pi) \) for \( \pi < a^m(n) \) provided \( \varepsilon \) is sufficiently small but positive. This completes the existence proof. ■

**Uniqueness** It cannot be the case that \( a < a^m(n) < \pi \) in symmetric equilibrium. For in this case, \( p^*(\gamma, a, n) < p^m \), \( \partial x(p, a)/\partial p|_{p=p^*} > 0 \) and \( A_{ij} \) would benefit from setting \( \bar{a} > a \) to induce a call price \( \bar{p} > p^* \). It cannot be the case that \( a \in (a^m(n), \pi] \) in symmetric equilibrium, either. For in this case, \( p^*(\gamma, a, n) > p^m \), \( \partial x(p, a)/\partial p|_{p=p^*} < 0 \) and \( A_{ij} \) would benefit from setting \( \bar{a} < a \) to induce a call price \( \bar{p} < p^* \). Finally, it cannot be the case that \( a < \pi \leq a^m(n) \) in symmetric equilibrium. For in this case, \( p^*(\gamma, a, n) < p^m \) and \( A_{ij} \) would benefit from setting \( \bar{a} > a \) to induce a call price \( \bar{p} > p^* \). ■

**A.5 Two-part Tariffs**

Consider the case where two networks each charge a subscription fee \( F_i \geq 0 \) in addition to the non-discriminatory call price \( p_i \geq 0 \). The subscriber’s indirect utility is \( V(p_i) - F_i \) and \( i \)’s customer base \( S_i = \left( (V(p_i) - F_i)^{\frac{1}{2}} \right) \left( (V(p_1) - F_1)^{\frac{1}{2}} + (V(p_2) - F_2)^{\frac{1}{2}} \right)^{-1} \). Each network maximises the Lagrangean \( L_i = \pi_i + \lambda_i F_i \), where

\[
\pi_i = S_i \left[ (p_i - c) D(p_i) + S_j (a - c_i) (D(p_j) - D(p_i)) + F_i \right]
\]

is the network’s profit. Any symmetric equilibrium \( p_1 = p_2 = p, F_1 = F_2 = F \), is given by the solutions to

\[
\frac{\partial L_i}{\partial p_i} = \frac{\partial S_i}{\partial p_i} \left[ (p - c) D(p) + F \right] + \frac{1}{2} \left( D(p) + (p - c - \frac{1}{2} (a - c_i)) D'(p) \right) = 0,
\]

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\[
\frac{\partial L_i}{\partial F} = \frac{\partial S_i}{\partial F} \left[ (p - c) D(p) + F \right] + \frac{1}{2} + \lambda = 0
\]

and \(\lambda F = 0, \lambda \geq 0\). Subtract \(\partial L_i / \partial F\) from \(\partial L_i / \partial p_i\) and use \(D(p_i) \frac{\partial S_i}{\partial p_i} = \frac{\partial S_i}{\partial p_i}\) to get

\[
F \lambda = -\frac{1}{2} \left( p - c - \frac{1}{2} (a - c_i) \right) F \frac{\eta(p)}{p} = 0. \quad (20)
\]

There are two types of equilibria, the standard solution \(p(a) = c + \frac{1}{2} (a - c_i)\),

\[
F(\gamma, a) = \frac{1}{(1 + 2\gamma)} \left( 2\gamma V (c + \frac{1}{2} (a - c_i)) - \frac{1}{2} (a - c_i) D \left(c + \frac{1}{2} (a - c_i)\right) \right) > 0
\]

(21)

where \(F\) was obtained by substituting \(p = c + \frac{1}{2} (a - c_i)\) into \(\partial L_i / \partial p_i\) and using the symmetric relation \(\frac{\partial S_i}{\partial p_i} = -\frac{1}{2\gamma} \frac{D(p)}{V(p) - F}\), and the corner solution \(F = 0\) and \(p\) given by \(\partial \pi_i / \partial p_i = 0\), ie \(p = p^d(\gamma, a)\). By using (21), we see that \(F(\gamma, a) \geq 0\) if and only if

\[
\gamma > \overline{\gamma}(a) = \frac{1}{4} (a - c_i) \frac{D \left(c + \frac{1}{2} (a - c_i)\right)}{V \left(c + \frac{1}{2} (a - c_i)\right)}. 
\]

The inequality is violated for all \(\gamma\) sufficiently low (but positive) provided \(a > c_i\). Note also that \(\overline{\gamma}'(a) > 0\) for all \(a > c_i\) by the fact that \(\frac{d}{dp} \left[ \frac{D(p)}{V(p) - F} \right] > 0\), see the proof of Claim 2. Hence, there exists a \(\hat{a}(\gamma) = \overline{\gamma}^{-1}(\gamma)\), such that the standard solution applies if and only if \(a \leq \hat{a}(\gamma)\), and the corner solution applies if and only if \(a \geq \hat{a}(\gamma)\).

Consider next the profit maximising choice of \(a\). Since we are interested in

the case with high network substitutability and a generous access price ceiling \(\bar{\pi} > c_i\), assume \(\gamma\) to be sufficiently low to ensure \(\alpha^m > \hat{a}(\gamma)\) and \(\bar{\pi} > \hat{a}(\gamma)\)

(recall that \(\alpha^m\) is decreasing in \(\gamma\), whereas \(\hat{a}'(\gamma) > 0\) and \(\hat{a}(0) = c_i\)). For all \(a \in [-c_o, \hat{a}]\), the symmetric equilibrium profit is

\[
\pi(a) = \frac{\gamma}{1 + 2\gamma} \left[ \frac{1}{2} (a - c_i) D \left(c + \frac{1}{2} (a - c_i)\right) + V \left(c + \frac{1}{2} (a - c_i)\right) \right]
\]

and the marginal profit

\[
\pi'(a) = \frac{1}{4} \frac{\gamma}{1 + 2\gamma} D' \left(c + \frac{1}{2} (a - c_i)\right) (a - c_i)
\]
which implies that $c_t$ is the optimal choice of $a$ in $[-c_o, \widehat{a}]$. For all $a \in [\widehat{a}, \overline{a}]$, the symmetric equilibrium profit is

$$
\pi (a) = \frac{1}{2} \left( p^d (\gamma, a) - c \right) D \left( p^d (\gamma, a) \right),
$$

which we know reaches its maximum at $\min\{\alpha^m, \overline{a}\}$. The profit function is non-monotonic in $a$ with two local maximands $c_t$ and $\min\{\alpha^m, \overline{a}\}$. Which of these is the global maximand depends on $\gamma$. Note that $\pi (\alpha^m) = \frac{1}{2} (p^m - c) D (p^m)$ is independent of $\gamma$ and $\pi (c_t) = \frac{\gamma}{1+2\gamma} V (c)$ vanishes in the limit as $\gamma \to 0$. Hence, $\pi (\alpha^m) > \pi (c_t)$ for $\gamma$ sufficiently low. Comparing $\pi (c_t)$ and $\pi (\overline{a})$ is more difficult since also $\lim_{\gamma \to 0} \pi (\overline{a}) = 0$. Note, however, that

$$
\frac{\pi (\overline{a})}{\pi (c_t)} = \frac{1 + 2\gamma \frac{D (p^d (\gamma, \overline{a}))}{V (c)} (p^d (\gamma, \overline{a}) - c)}{2 \frac{D (p^d (\gamma, \overline{a}))}{V (c)} (p^d (\gamma, \overline{a}) - c)}
$$

$$
= \frac{1 + 2\gamma \frac{D (p^d (\gamma, \overline{a}))}{V (c)} (\overline{a} - c_t) \varepsilon}{2 \frac{D (p^d (\gamma, \overline{a}))}{V (p^d (\gamma, \overline{a}))} D (p^d (\gamma, \overline{a}))}
$$

where the second equality follows from (18).

$$
\lim_{\gamma \to 0} \frac{\pi (\overline{a})}{\pi (c_t)} = 1 + \frac{(\overline{a} - c_t) \varepsilon}{2D (c)} > 0
$$

implies that even $\pi (\overline{a}) > \pi (c_t)$ for $\gamma$ sufficiently low. It follows that the equilibrium access charge is $\min\{\alpha^m, \overline{a}\}$, the subscription fee is 0 and the call price $p^d (\gamma, a^d)$ in symmetric equilibrium, provided that $\overline{a} > c_t$ and networks are sufficiently close substitutes.