Restructuring Electricity Markets when Demand is Uncertain: Effects on Capacity Investments, Prices and Welfare

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Abstract
We examine the effects of reorganizing electricity markets on capacity investments, retail prices and welfare when demand is uncertain. We study the following market configurations: (i) integrated monopoly, (ii) integrated duopoly with wholesale trade, and (iii) separated duopoly with wholesale trade. Assuming that wholesale prices can react to changes in retail prices (but not vice versa), we find that generators install sufficient capacity to serve retail demand in each market configuration, thus avoiding black-outs. Furthermore, aggregate capacity levels and retail prices are such that the separated (integrated) duopoly with wholesale trade performs best (worst) in terms of welfare.

Keywords: Electricity, Investments, Generating Capacities, Vertical Integration, Monopoly and Competition.

JEL-Classification: D42, D43, D44, L11, L12, L13

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1 Introduction

Electricity markets around the world have recently been reformed to improve their economic performance. In many countries, legislators have allowed competition into statutory, vertically integrated monopolies and implemented regulations such as vertical unbundling or even full vertical separation to prevent harmful strategic behavior.\(^1\) Yet, the reform process is far from complete, and there is no consensus that a single model is best (Pittman, 2003). In particular, there is a concern that deregulation—i.e., introducing imperfect competition into statutory monopolies—may undermine infrastructure investments (see, e.g., Buehler et al. (2004)) and even cause adverse welfare effects. Although there are some studies which investigate into the incentives to install capacities in (more) competitive electricity markets (see, e.g., von der Fehr and Harbord (1997), Castro-Rodriguez et al. (2001), Boom (2002) and (2003) as well as Borenstein and Holland (2005)), there is, at least to our knowledge, no formal analysis of the interplay of the vertical structure and the introduction of competition on capacity investment in generation and on electricity prices, and how they affect welfare.

The present paper provides an analytical framework for studying the effects of restructuring electricity. More specifically, we study different configurations of an electricity market model that vary with respect to (a) the vertical structure and (b) the extent to which firms compete. In this model, retail demand is linear and stochastic, and retail prices are set before wholesale prices are determined in a unit price auction according to von der Fehr and Harbord (1997) and (1993). This implies, in particular, that wholesale prices can react to changes in retail prices, whereas retail prices cannot reflect changes in wholesale prices. We also require that electricity generators decide on their capacity levels before retail prices are known.

To analyze the effects of restructuring, we compare the equilibrium outcomes under (i) integrated monopoly, (ii) integrated duopoly with wholesale trade, and (iii) separated duopoly with wholesale trade. The vertical structure as well as the market structure in generation and retail is exogenous for the

\(^1\)In the UK, for example, the industry was vertically separated in three generating firms, in the National Grid company and in 12 regional distribution companies by the Electricity Act in 1989. Some regional distribution companies, however, vertically integrated into generation later on (Newbery, 1999, 2005). The Californian restructuring bill from 1996 also forced the regulated utilities to sell lots of their generation facilities (Borenstein, 2002). The European Union ruled in its Directive 2003/54/EC concerning common rules for the internal market in electricity adopted on 26 June 2003 that electricity generating firms which are also integrated into the transmission and distribution of electricity have to be functionally disintegrated.
different scenarios.

Our main results are the following. First, capacity investments are highest under integrated duopoly and lowest under integrated monopoly. Second, retail prices are lowest under separated duopoly and highest under integrated duopoly. Third, the combined investment and price effects of restructuring are such that the separated duopoly yields the highest social welfare, whereas the integrated duopoly yields the lowest social welfare. Together, our findings suggest that restructuring electricity is likely to increase both capacity investments and welfare, if it implements (imperfect) competition and vertical separation.

The intuition for this result is as follows. Introducing competition requires firms to make strategic investment and pricing decisions. Under integrated duopoly, strategic investment and pricing decisions yield high investments and retail prices, as firms face the risk of being unable to serve own retail demand and, consequently, being exploited by their competitor during the wholesale auction. Vertical separation eliminates both the risk of excessive retail demand and the direct influence of generators on retail prices. As a result, both capacities and retail prices are lower than under integrated duopoly. Yet, as total capacity is always sufficient to serve retail demand at the relevant retail price, irrespective of market configuration, the reduced investment does not pose a problem from a social welfare point of view.

The remainder of the paper is structured as follows. In Section 2, we introduce our analytical framework. In Section 3, we derive the capacity investment and retail price that maximize the expected profit of an integrated monopolist. In Sections 4 and 5 we derive the equilibrium under separated and integrated duopoly, respectively. Section 6 compares the equilibrium outcomes and provides a detailed discussion of the intuition for our results. Section 7 concludes.

2 Analytical Framework

In this section, we outline the analytical framework for the various market configurations considered below. We first consider the demand side. Suppose that the retail customers’ surplus function is given by

\[ V(x; \varepsilon, r) = U(x, \varepsilon) - rx = x - \varepsilon - \frac{(x - \varepsilon)^2}{2} - rx, \]

where \( x \) is the consumed electricity, \( r \) is the retail market price paid per unit of electricity, and \( \varepsilon \) is a demand shock, uniformly distributed on the
Maximizing $V(x; \varepsilon, r)$ with respect to $x$ yields the following retail demand for electricity

$$x(r, \varepsilon) = \max\{1 + \varepsilon - r, 0\}. \quad (2)$$

If there is more than one retailer, retail customers subscribe to the retailer offering the lowest retail price, as electricity is a homogeneous good. If the prices offered by the retailers are identical, consumers choose each retailer with equal probability.

As to the supply side, we consider three different market configurations that differ in terms of the number of active firms and the industry’s vertical structure (see Table 1): (i) integrated monopoly, (ii) integrated duopoly with wholesale trade (2 integrated firms), and (iii) separated duopoly with wholesale trade ($2 \times 2$ firms).

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<th>Integration</th>
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<tr>
<td>Competition</td>
<td>integrated duopoly separated duopoly</td>
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For simplicity, we assume that the marginal cost of generating electricity is constant and normalized to zero. The total costs of electricity generator $i = A, B$ are thus given by

$$C(k_i) = zk_i, \quad (3)$$

where $z$ is the constant unit cost of capacity and $k_i$ the generating capacity installed by firm $i$.

In the two duopoly models we impose the following timing of the game motivated by the real world:

- In stage 1, electricity generators $i = A, B$ decide on their capacity $k_i$ before the level of retail demand is known. In the integrated duopoly we assume that capacity decisions are taken simultaneously. In the separated duopoly we additionally analyze sequential investment decisions and consider the case where generator $A$ gets to decide before generator $B$.

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2We abstract from the so-called chain of monopolies for reasons that will become clear below.

3Firm indices may be ignored when there is only one generator.
• In stage 2 retailers \( \ell = C, D \) simultaneously offer linear retail prices \( r_\ell \) in the separated duopoly whereas the two integrated generators \( i = A, B \) simultaneously offer linear retail prices \( r_i \) in the integrated duopoly. Consumers buy from the firm with the lower retail price, or, if prices are identical, from each firm with probability one half.

• In stage 3, the demand shock \( \varepsilon \in [0, 1] \) is realized.

• In stage 4, generators \( A \) and \( B \) submit price bids \( p_A \) and \( p_B \) for their full capacity \( k_i, i = A, B \) to an auctioneer. The auctioneer then determines the market clearing wholesale price \( p \) (whenever possible) and the amount of electricity each generator may supply to the grid.\(^4\)

• Finally, in stage 5, if supply and demand cannot be balanced, a blackout occurs, and agents do not exchange deliveries and payments. If demand and supply can be equated, retail customers are served and both retailers and generators receive their respective payments.

We model the timing of the benchmark case of an integrated monopoly as close as possible to these duopoly scenarios. Therefore the monopoly firm chooses its capacity and its retail price before the uncertainty concerning the demand level has been resolved.

It is crucial to note that the retail price in the two duopoly settings is determined before the wholesale price. This is in marked contrast to the standard literature on vertically related industries, where the timing is typically reversed, i.e. the wholesale price is determined before the retail price (see Section 6.2 for further details). The difference is motivated by the special characteristics of the electric power industry, where retailers typically specify the terms of delivering electricity before demand is known and then buy electricity on behalf of their customers on the wholesale market. That is, the \textit{retail market clears in the long run}, whereas the \textit{wholesale market clears in the short run}. This implies, in particular, that the wholesale price is a function of the retail price, whereas the retail price cannot react to changes in the wholesale price. Therefore, the chain of monopolies is not a sensible structure: The upstream monopolist would always be able to fully extract the downstream monopolist’s profit by setting the wholesale price equal to the retail price determined in the previous stage.\(^5\) The retail monopolist

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\(^4\)In the integrated monopoly, the wholesale price is irrelevant for the outcome, as it solely allocates profits to upstream and downstream facilities.

\(^5\)This particular form of a price squeeze is possible as retailers must commit to a retail price before the wholesale price is determined.
would thus be indifferent between all admissible retail prices, leaving the equilibrium outcome indeterminate.

Despite of our special timing, the determination of the wholesale price remains crucial for the outcome in the various market configurations, as it affects the returns on investment for an electricity generator. For the sake of concreteness, we assume that the wholesale price is determined according to a unit price auction introduced by von der Fehr and Harbord (1997) and (1993). Unit price auctions were used for the Electricity Pool in England and Wales before the reform in 2001, and are still in use elsewhere, e.g. for the Nord Pool in Scandinavia, or the Spanish wholesale market.

This unit price auction requires each firm \( i \) to bid a price \( p_i \) at which it is willing to supply its total capacity. The auctioneer will then attempt to balance supply and demand on the grid. To do so, he arranges the bids in ascending order and determines the marginal bid that is just necessary to equate supply and demand. The price of the marginal bid is the spot market price that is payed to all generators for each unit that is dispatched on the grid (irrespective of the bids made by these generators). The capacity of suppliers bidding below the spot market price is dispatched completely, whereas the marginal supplier is allowed to deliver just the amount of electricity necessary to balance supply and demand. If the supplied capacity at a certain bid price is insufficient to satisfy demand, but would be more than sufficient to satisfy demand at the next higher bid price, the auctioneer sets the spot market price in between the two bid prices so as to balance supply and demand.

Note that in our framework, the auctioneer may be unable to find a wholesale price that equates supply and demand, since neither retail demand nor generation capacity can respond to changes in the wholesale price. Therefore,

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6 An alternative approach, based on Klemperer and Meyer (1989), has been suggested by Green and Newbery (1992). They assume that firms bid differentiable supply functions, whereas von der Fehr and Harbord (1997) and (1993) assume that they bid step functions.

7 See Bergman et al. (1999), Crampes and Fabra (2005) and Newbery (2005).

8 That is, we abstract from the problem of strategic capacity withholding (see Crampes and Creti (2001), and Le Coq (2002).

9 For simplicity, we ignore transmission constraints, although they might interact with constraints in the generating capacity. See Wilson (2002) for insights into this problem and for the analysis of isolated transmission constraints Borenstein et al. (2000), Joskow and Tirole (2000) and Léautier (2001)

10 Note the difference to Kreps and Scheinkman (1983), where the undercutting firm receives its own price per unit sold even if its capacity is too low to serve the market, so that some customers have to pay the price of the competitor with the next higher price.

11 In line with Wilson (2002) we consider an integrated system because participation in the auction is compulsory if a generating firm wants to sell electricity.
a black-out may occur, where demand cannot be served, and firms receive no payments. Since in our framework demand does not respond to changes in the wholesale price and since the total amount of installed capacities cannot be influenced by the wholesale price, the auctioneer may also fail to find a price that balances supply and demand in the market. Then a black-out occurs. No firm can sell and deliver electricity, and all the firms realise zero profits. Thus, we abstract from any sort of rationing by the auctioneer or the retailers of electricity.\footnote{See Joskow and Tirole (2004) for an analysis of a market where retailers propose not only prices to consumers but also rationing rules which they want to apply. Although in reality domestic consumers are sometimes rationed, the rationing rules are usually not spelled out in any sort of contract with their retailers.} This assumption maximizes the punishment for the generators if their aggregate capacity is too small, thus also maximizing the incentive to install capacity. For the sake of simplicity we also abstract from the fact that in reality firms compete repeatedly on the wholesale and on the retail market.\footnote{Therefore collusion which has been analysed by Fabra (2003) and by Dechenaux and Kovenock (2005) for the wholesale market is out of the scope of this analysis.} As retailers face no other costs than those from buying electricity on behalf of their customers in the separated duopoly case, they are active if and only if the wholesale price is not larger than the relevant retail price ($p \leq r$). For $p > r$, the retailers must declare bankruptcy and exit the market, so that generators cannot sell electricity either.

We now proceed to the analysis of the equilibrium outcomes in the various market configurations. We begin with the benchmark case of Integrated Monopoly.

### 3 Integrated Monopoly

Consider the pricing and investment decisions taken by a vertically integrated monopolist. Recall that, in this case, there is no wholesale market for electricity. The monopolist thus simply chooses the retail price $r^m$ and the capacity level $k^m$ that maximize expected profits

$$
\pi(r, k) = \begin{cases} 
\int_{\max\{r-1, 0\}}^{1} r(1 + \varepsilon - r)d\varepsilon - zk, & \text{if } r \geq 2 - k, \\
\int_{\max\{r-1, 0\}}^{k-1+r} r(1 + \varepsilon - r)d\varepsilon - zk, & \text{if } \max\{0, 1-k\} \leq r < 2 - k, \\
-zk, & \text{if } r < \max\{0, 1-k\}.
\end{cases}
$$

The first element of $\pi(r, k)$ is relevant when the retail price $r$ is so large that demand is always smaller than capacity, even for the highest possible demand
shock $\varepsilon = 1$. The second element is relevant when $r$ is in an intermediate range, such that demand is smaller than capacity for the lowest possible demand shock $\varepsilon = 0$ and larger than capacity for the highest possible demand shock $\varepsilon = 1$. Finally, the third element is relevant if the retail price is low enough that demand is always larger than capacity (even for the smallest possible demand shock $\varepsilon = 0$).

Our first result gives the retail price and capacity $(r^m, k^m)$ that maximize the expected profit of an integrated monopolist.

**Proposition 1 (integrated monopoly)** For capacity costs $z \leq 1/2$, the profit maximizing choices $(r^m, k^m)$ are given by

\[ r^m = \frac{3}{4} + \frac{z}{2}; \quad k^m = \frac{5}{4} - \frac{z}{2}. \] (4)

For capacity costs $z > 1/2$, the integrated monopoly is not sustainable, as $\pi(r^m, k^m) < 0$.

**Proof:** Boom (2003).

Intuitively, Proposition 1 states that if capacity costs are not too high, the monopolist will earn non-negative profits and choose both the retail price and its capacity so as to avoid a black-out (in which case the monopolist would realize a negative profit due to sunk capacity costs). In this case, the retail price increases in the cost of capacity, whereas the installed capacity decreases in the cost of capacity. If capacity costs are too high, even optimal choices of $(r^m, k^m)$ do not yield non-negative profits, and the integrated monopoly is thus not a sustainable market configuration.

Next, we consider the separated duopoly with competition both at the upstream and downstream level of the industry.

### 4 Separated Duopoly (2×2 Firms)

Consider a market configuration with two independent firms competing in electricity generation and two independent firms which compete on the retail market for electricity. As usual, we use backwards induction to derive the

\[ \text{In this case, the condition } k \geq (1 + \varepsilon - r) \text{ becomes equal to } r \geq 2 - k. \text{ Furthermore, the lower bound of the integral assures a positive demand.} \]

\[ \text{The former requires } r \geq \max\{0, 1 - k\}, \text{ and the latter } r < 2 - k. \text{ The upper bound of the integral assures that a black-out does not occur, i.e. capacity is sufficient to satisfy demand.} \]
subgame perfect Nash equilibrium of this game. Given the timing described in Section 2, we therefore start with the wholesale market. Next, we study the retail market. Finally, we analyze the investment decisions of electricity generators.

4.1 Wholesale Market

If the generators’ combined capacities are sufficient to satisfy retail demand at the equilibrium retail price in the separated duopoly, \( r^* \), i.e. \( k_A + k_B \geq x(r^*, \varepsilon) \), the wholesale price determined in the unit price auction is given by

\[
p(p_A, p_B) = p_i \begin{cases} 
   \text{if } p_i < p_j \text{ and } k_i \geq x(r^*, \varepsilon) \text{ or } \\
   \text{if } p_i \geq p_j \text{ and } k_j < x(r^*, \varepsilon) \leq k_A + k_B,
\end{cases}
\]

with \( i, j = A, B \) and \( i \neq j \). If the combined capacity is insufficient to satisfy demand, \( k_A + k_B < x(r^*, \varepsilon) \), the auctioneer cannot find a wholesale price that equates supply and demand, and a black-out occurs.

The volume of electricity \( y_i \) that generator \( i = A, B \) can sell is also a function of the price bids \( p_A \) and \( p_B \). It is given by

\[
y_i(p_A, p_B) = \begin{cases} 
   \min\{k_i, x(r^*, \varepsilon)\} & \text{if } p_i < p_j, \\
   \frac{\min\{k_i, x(r^*, \varepsilon)\} + \max\{0, x(r^*, \varepsilon) - k_i\}}{2} & \text{if } p_i = p_j, \\
   \max\{0, x(r^*, \varepsilon) - k_j\} & \text{if } p_i > p_j,
\end{cases}
\]

with \( i, j = A, B \) and \( i \neq j \). Thus, using (5) and (6), the profit of generator \( i = A, B \) at this stage of the game is

\[
\pi_i(p_A, p_B) = p(p_A, p_B)y_i(p_A, p_B).
\]

Since the firms’ bidding behavior in our setting is equivalent to that derived by Crampes and Creti (2001) and Le Coq (2002) for given capacity levels, we omit the details here. Intuitively, best response price bidding requires each firm to either undercut the rival or bid the maximum price \( p_i = r^* \) at which the retailers are just indifferent between participating in the market or exiting it. In the latter case there would be no demand in the wholesale market and the generators could not sell their electricity. Our next proposition characterizes the resulting Nash equilibria in price bids.

**Proposition 2 (wholesale prices)** Depending on the capacity levels \((k_A, k_B)\) and the retail price \( r^* \), there are the following types of Nash equilibria in price bids:
(i) If $k_A + k_B < x(r^*, \varepsilon)$, any pair $(p_A, p_B)$ forms a Nash equilibrium in price bids. No wholesale price can equate supply and demand, and a black-out occurs.

(ii) If $k_i \geq x(r^*, \varepsilon) > k_j$, with $i, j = A, B$ and $i \neq j$, the Nash equilibrium in pure strategies is characterised by $p_i = r^*$ and $p_j < r^*(x(r^*, \varepsilon) - k_j)/k_i$. The resulting equilibrium wholesale price is $p^* = r^*$, and firms sell the quantities $y_i = x(r^*, \varepsilon) - k_j$ and $y_j = k_j$.

(iii) If $k_A + k_B \geq x(r^*, \varepsilon) > \max\{k_A, k_B\}$, there are two types of Nash equilibria in pure strategies: one with $p_A = r^*$ and $p_B < r^*(x(r^*, \varepsilon) - k_B)/k_A$, and another with $p_B = r^*$ and $p_A < r^*(x(r^*, \varepsilon) - k_A)/k_B$. The wholesale price is the same ($p^* = r^*$) for both types of equilibria, but the quantities sold in equilibrium differ: in the former $y_A = x(r^*, \varepsilon) - k_B$ and $y_B = k_B$, whereas in the latter $y_A = k_A$ and $y_B = x(r^*, \varepsilon) - k_A$.

(iv) If $\min\{k_A, k_B\} \geq x(r^*, \varepsilon)$ the Nash equilibrium $p_A = p_B = 0$ is unique. The resulting equilibrium wholesale price is $p^* = 0$, and firms sell the quantities $y_A = y_B = x(r^*, \varepsilon)/2$.

**Proof:** See appendix A of Le Coq (2002) or the proofs of proposition 1-3 in Crampes and Creti (2001), using that the constant marginal cost of generating electricity is identical and equal to zero by assumption, whereas the maximum price with positive demand is $p = r^*$. ■

Proposition 1 is illustrated in Figure 1. Area $A$ coincides with case (i), where demand exceeds aggregate installed capacity and a black-out must therefore occur. Areas $B$ and $D$ are associated with case (ii): In area $B$, firm $A$ is the large firm, whereas firm $B$ is the small firm; in $D$, these roles are reversed. In both cases, the large firm bids the maximum price $r^*$, whereas the small firm bids just low enough to avoid undercutting by the large firm. In area $C$, which corresponds to case (iii), the difference in installed capacities is smaller than in either $B$ or $D$, and two types of equilibria are possible: Either the large or the small firm bids the maximum price, and the other firm bids low enough to avoid undercutting. In both cases the equilibrium wholesale price is $p^* = r^*$. Finally, area $E$ corresponds to case (iv). Here, each firm has sufficient capacity to satisfy demand alone. Therefore, the price bidding yields a unique Bertrand type equilibrium where $p^* = 0$.

Clearly, multiple Nash equilibria in pure strategies exist for the cases (i)–(iii), as any lower bid that avoids both undercutting and negative profits is admissible. Yet, for cases (i) and (ii), all Nash equilibria in pure strategies are pay-off equivalent. For case (iii), however, the different types of equilibria
are not pay-off equivalent, as the volume of dispatched electricity \( y_i(p_A, p_B) \) depends on the type of equilibrium played. For these capacities we assume the following:\(^\text{16}\)

**Assumption 1** If the generating capacities satisfy \( k_A + k_B \geq x(r^*, \varepsilon) > \max\{k_A, k_B\} \), generators are assumed to co-ordinate on a Nash equilibrium where the firm with the larger capacity bids the maximum price and the firm with the smaller capacity bids low enough to avoid undercutting by the large firm. If generators have equal capacities, they play each type of equilibria with equal probability.

### 4.2 Retail Market

Retailers compete à la Bertrand. Therefore a retailer cannot realize a positive profit if its price is higher than its competitor’s price. In addition the demand shock must be, on the one hand, large enough to generate a positive demand

\(^{16}\)There are indications that with asymmetric capacities this assumption is equivalent to choosing the risk dominant Nash equilibrium
and, on the other hand, small enough in order to ensure that the minimum capacity on the market is sufficient to satisfy demand. The latter ensures that case (iv) of Proposition 1 is realized which coincide with area $E$ in figure 1. This is the only area in 1 where the retailers can earn positive profits. Thus, the demand shock must satisfy:

$$
\varepsilon > r - 1 \equiv \varepsilon \quad \text{and} \quad \varepsilon \leq \min\{k_A, k_B\} + r - 1 \equiv \bar{\varepsilon}.
$$

Taking this into account, the expected profit of retailer $\ell = C, D$ is given by

$$
\pi_\ell(r_\ell, r_t) = \begin{cases} 
0 & \text{if } r_\ell > r_t, \\
\frac{1}{2} \int_{\max\{0,\varepsilon,1\}}^{\max\{0,\min\{\varepsilon,1\}\}} r_\ell(1 + \varepsilon - r_\ell) d\varepsilon & \text{if } r_\ell = r_t, \\
\int_{\max\{0,\varepsilon,1\}}^{\max\{0,\min\{\varepsilon,1\}\}} r_\ell(1 + \varepsilon - r_\ell) d\varepsilon & \text{if } r_\ell < r_t,
\end{cases}
\quad (7)
$$

with $\ell, t = C, D$, and $\ell \neq t$. The profit function (7) indicates that retailers undercut each other until they reach zero profits. Therefore, the following proposition holds:

**Proposition 3 (retail prices)** Depending on the capacity levels $(k_A, k_B)$, there are the following subgame perfect Nash equilibria in retail prices.

(i) If $\min\{k_A, k_B\} \geq 1$ there is a unique Nash equilibrium in pure strategies with $r_C = r_D = 0$.

(ii) If $\min\{k_A, k_B\} < 1$ all Nash equilibria in pure strategies are characterised by $r_C \leq 1 - \min\{k_A, k_B\}$ and $r_D \leq 1 - \min\{k_A, k_B\}$.

**Proof:** Suppose that $r_\ell > r_t$ with $\ell, t = C, D$ and $\ell \neq t$. This can only be an equilibrium, if $r_t \leq 1 - \min\{k_A, k_B\}$ and $r_\ell \leq 1 - \min\{k_A, k_B\}$, because otherwise firm $\ell$ can increase its profits by undercutting and firm $t$ by increasing its price. Suppose, alternatively, that $r_\ell = r_t$, then either $r_\ell = r_t = 0$ must hold, if $\min\{k_A, k_B\} \geq 1$, or $r_\ell = r_t < 1 - \min\{k_A, k_B\}$, if $\min\{k_A, k_B\} < 1$, because otherwise each retailer can double its profit by slightly undercutting its rival.

The retailers cannot realize any positive profit because of Bertrand competition, no matter whether the equilibrium is unique if capacities satisfy $\min\{k_A, k_B\} > 1$, or whether there are multiple equilibria as in the case of $\min\{k_A, k_B\} < 1$. In order to solve for the multiplicity problem in the latter case, we need another assumption on equilibrium selection for these capacity levels.
Assumption 2: If \( \min\{k_A, k_B\} < 1 \) holds, then the retailers choose the Nash equilibrium with \( r_C = r_D = 1 - \min\{k_A, k_B\} \).

Assumption 2 means that the retailers select the equilibrium where both of them choose the highest price which generates zero profits.

4.3 Capacity Investments

Generator \( i = A, B \) decides on its level of generating capacity, anticipating how this decision affects competition both on the retail and the wholesale market. Note that the Bertrand competition on the retail market shifts any possible rent to the generators of electricity. The wholesale price coincides with \( p^* = r^* = \max\{0, 1 - \min\{k_A, k_B\}\} \), if the demand does not exceed the aggregate capacity of both generators. Given a positive price, the market demand is then characterized by \( x(r^*, \varepsilon) = 1 + \varepsilon - 1 + k_A + k_B \), if firm \( i \)'s capacity exceeds the one of its rival, firm \( i \) bids high and may serve the residual demand \( \max\{x(r^*, \varepsilon) - k_j, 0\} = \max\{1 + \varepsilon - 1 + k_j - k_j, 0\} = \varepsilon \). If firm \( i \)'s capacity is lower than the one of its rival, firm \( i \) bids low and may deliver its total capacity up to the level of demand, meaning \( \min\{k_i, 1 + \varepsilon - 1 + k_i\} = k_i \). If the capacities of the two firms are identical then firm \( i \) bids high and low with probability one half each. Since the generators cannot sell electricity if a blackout occurs, \( x(r^*, \varepsilon) \leq k_A + k_B \) has to be satisfied which is equivalent to \( \varepsilon + \min\{k_A, k_B\} \leq k_A + k_B \) or \( \varepsilon \leq \max\{k_A, k_B\} \), if \( \min\{k_A, k_B\} \geq 1 \) holds, which explains the upper integration limit in (8).

It turns out that generator \( i \)'s best response is to choose a higher capacity as its competitor with

\[ k_i = 1 > k_j, \]
if its competitor’s capacity is relatively low or to choose a lower capacity with

\[ k_i = \max\{0, \min\{(k_j - z)/(2k_j), (1 - z)/2\}\} \]

its rival’s capacity is relatively high. This is rather natural because, if the rival has a small capacity, the residual demand served by the firm with the large capacity is relatively large, as well as the price generated on the wholesale market. Therefore it pays to install a large capacity. If the rival’s capacity is relatively large it pays to install a small capacity which is then completely sold and which ensures a higher price on the wholesale market.

From the analysis of the overall best responses the following proposition results for the first pattern of timing where the two firms choose their capacities simultaneously.

**Proposition 4 (simultaneous capacity choices)** The level of capacity costs determines whether a subgame perfect Nash equilibrium in pure strategies exists with simultaneous capacity choices.

(i) If \( 0 \leq z < 1/3 \), there are two asymmetric subgame perfect Nash equilibria in pure strategies, with capacities \( k_i^* = 1 \) and \( k_j^* = (1 - z)/2 \), \( i, j = A, B \) and \( i \neq j \).

(ii) If \( 1/3 \leq z < 1/2 \), there is no subgame perfect Nash equilibria in pure strategies.

(iii) If \( 1/2 \leq z \), there is a unique subgame perfect Nash equilibrium where generators install no capacity.

**Proof:** Firm \( i \)’s best response functions are derived in Appendix A. Solving for the intersections of firm \( i \)’s and firm \( j \)’s best responses in capacities yields the Proposition. ■

Figure 2 illustrates that, for intermediate levels of capacity costs, there is no Nash equilibrium in pure strategies when capacities are chosen simultaneously. This non-existence problem disappears, however, if capacities are chosen sequentially.

**Proposition 5 (sequential capacity choices)** With sequential capacity choices, the game always has a unique equilibrium.

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17See (20) or (21) in Appendix A for the detailed description of firm \( i \)’s best response function.
Figure 2: Best Responses in Capacities

(i) If $0 \leq z < 1/3$, there is a unique subgame perfect Nash equilibrium in pure strategies where firm A chooses $k_A^* = (1 - z)/2$ and firm B chooses $k_B^* = 1$.

(ii) If $1/3 \leq z < 1/2$, there is a unique subgame perfect Nash equilibrium in pure strategies where firm A chooses $k_A^* = 1 - 2z$ and firm B chooses $k_B^* = 1$.

(iii) If $1/2 \leq z$ holds, there is a unique subgame perfect Nash equilibrium in pure strategies where generators install no capacity.

Proof: Substituting firm B’s best response function $k_B(k_A)$ which is either equivalent to (20) or (21) into $\Pi_A(k_A, k_B)$ from (18) or (19) in Appendix A and maximizing with respect to $k_A$ results in the Proposition.

Note that the first mover $A$ always wants to be in the position of the small provider with the smaller capacity than its rival $B$, because being the smaller provider, which always bids low, but receives the high bid of the larger provider in the wholesale market as the wholesale price and sells his to-
tal capacity, is always more profitable than bidding high and serving only the residual demand.

5 Integrated Duopoly

In this section, we briefly review the results for the integrated duopoly with wholesale trade analyzed in Boom (2003), which contains further details on this market configuration (including the proofs omitted here).

5.1 Wholesale Market

In this market configuration, generators may trade with each other (rather than with retailers) on the wholesale market. When the unit price auction is held, total market demand is fixed and given by $x(r^d, \varepsilon)$ with $r^d = \min\{r_A, r_B\}$ being the price consumers must pay on the retail market. The retail demand of firm $i$, in turn, is

$$d_i(r_A, r_B, \varepsilon) = \begin{cases} x(r_i, \varepsilon) & \text{if } r_i < r_j, \\ \frac{1}{2} x(r_i, \varepsilon) & \text{if } r_i = r_j, \\ 0 & \text{if } r_i > r_j, \end{cases}$$

(9)

If total capacity is sufficient to satisfy retail demand, i.e. $k_A + k_B \geq x(r^d, \varepsilon)$, the unit price auction yields the same wholesale price given in equation (5) as with separated retailers, if we substitute $r^d$ for $r^*$. The volume of electricity that integrated generator $i$ can sell coincides with (6), if, again, $r^d$ is substituted for $r^*$. Thus, firm $i$’s revenues are

$$\pi_i(r_i, r_j) = r_i d_i(r_i, r_j, \varepsilon) + p(p_i, p_j) \left[ y_i(p_i, p_j, \varepsilon) - d_i(r_i, r_j, \varepsilon) \right].$$

(10)

Equation (10) states that an integrated generator earns the retail price for each unit of electricity demanded by its subscribers plus the wholesale price times the difference between the units dispatched to the grid and own retail demand. This implies, in particular, that an integrated generator is forced to become a net payer in the wholesale market if its retail demand exceeds own capacity.

Our next proposition characterizes the resulting Nash equilibria in bid prices.

Proposition 6 (wholesale prices) Depending on the capacity levels $(k_A, k_B)$ and the retail demand $x(\min\{r_A, r_B\}, \varepsilon)$, there are the following Nash equilibria in price bids:

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(i) If \( k_A + k_B < x(\min\{r_A, r_B\}, \varepsilon) \), any pair \((p_A, p_B)\) forms a Nash equilibrium in price bids. No wholesale price can equate supply and demand, and a black-out occurs.

(ii) If \( k_A + k_B \geq x(\min\{r_A, r_B\}, \varepsilon) \) but one firm, say \( A \), cannot satisfy own retail demand, \( k_A < d_A(r_A, r_B, \varepsilon) \), then bids satisfy \( p_B = \bar{p}(r_A, r_B, \varepsilon) \) which becomes the wholesale price \( p(r_A, r_B, \varepsilon) \) and \( p_A \leq \hat{p}(r_A, r_B, \varepsilon) < \bar{p}(r_A, r_B, \varepsilon) \), where

\[
\bar{p}(r_A, r_B, \varepsilon) = \frac{r_A d_A(r_A, r_B, \varepsilon)}{d_A(r_A, r_B, \varepsilon) - k_j} \quad \text{and} \quad \hat{p}(r_A, r_B, \varepsilon) = \frac{r_A d_A(r_A, r_B, \varepsilon)}{\min\{k_B, x(\min\{r_A, r_B\}, \varepsilon) - d_B(r_A, r_B, \varepsilon)\}}. (11)
\]

(iii) If \( k_i \geq d_i(r_A, r_B, \varepsilon) \) for \( i = A, B \), the Nash equilibrium \( p_A = p_B = 0 \) is unique. The resulting equilibrium wholesale price is \( p^* = 0 \), and firms earn revenues of \( \pi_i(r_A, r_B) = r_i d_i(r_A, r_B, \varepsilon) \).

**Proof:** See Boom (2003), Appendix B.

Proposition 6(i) describes the case where aggregate capacity is insufficient to serve retail demand, so that a black-out is inevitable. In case (iii), both generators have sufficient capacity to serve their own retail demand, so that they undercut each other until their bids equal zero and revenues accrue only on the retail market. Finally, in case (ii), generator \( A \) must buy electricity on the wholesale market to serve own retail demand. Therefore, it cannot avoid becoming a net payer in the wholesale auction. Generator \( A \) can reduce its net demand position, however, by undercutting its competitor, i.e., bidding a price \( p_A \) that is low enough that \( B \) does not have an incentive to deviate from the maximum price \( \bar{p} \). Yet, even after undercutting, \( B \) appropriates all rents and the revenues of \( A \) are zero.

### 5.2 Retail Market

Deriving the equilibrium retail prices is fairly tedious under integrated duopoly (see Boom (2003)). We therefore confine ourselves to a brief discussion of the pricing strategies available and then state the result without giving proofs.

In the retail market, each integrated generator has three strategies at its disposal: First, it can undercut its rival and corner the market. This strategy yields positive revenues only if the demand shock is such that retail demand is positive and the undercutting generator’s capacity is sufficient to serve it.
Second, it can match the price of its competitor, splitting total retail demand in half. Then, expected revenues depend on the relative capacities of the two firms: For the smaller firm, revenues are as in the undercutting case, except that it attracts only half the demand. For the firm with the larger capacity, however, revenues are different, as it can appropriate the rival’s rent if its capacity is sufficiently large to make up for a lack of capacity of the smaller firm. Finally, it can set a higher price than its competitor, in which case it will not get any subscribers. However, it will earn positive revenues if the competitor cannot serve total retail demand and own capacity is sufficient to make up for the difference.

**Proposition 7 (retail prices)** Depending on the level of capacities \((k_A, k_B)\), there are the following Nash equilibria in retail prices.

(i) If \(k_A = k_B = k < \sqrt{5/2}\), the pareto-dominant Nash equilibrium results in retail prices

\[
r^d = r_A = r_B = \begin{cases} 
2 - \sqrt{2k} & \text{if } 0 \leq k < 1/\sqrt{2}, \\
\frac{1}{2}(3 - \sqrt{4k^2 - 1}) & \text{if } \frac{1}{\sqrt{2}} \leq k < 1/\sqrt{5/2}.
\end{cases}
\]  

(13)

(ii) With not too asymmetric capacities, the unique Nash equilibrium results in \(r^d = r_A = r_B = 0\).

(iii) With asymmetric capacities and \(k_B < \max\{(k_A - 1)/2, \frac{1}{8}\}\), the pareto-dominant Nash equilibrium results in \(r^d = r_A = \max\{\frac{3}{4}, 2 - k_A\} < r_B\). (An analogous equilibrium exists where the roles of \(A\) and \(B\) are reversed.)

(iv) If \(\frac{1}{8} \leq k_B < (k_A - 1)/2\), the pareto-dominant Nash equilibrium results in \(r^d = r_A = r_B = 1 - 2k_B\). (An analogous equilibrium exists where the roles of \(A\) and \(B\) are reversed.)

(v) If \(k_A + k_B < 1\), the equilibria cannot be pareto ranked, but they are payoff-equivalent as both firms realize zero revenues.

**Proof:** See Boom (2003), Appendix C. ■
5.3 Capacity Investments

Again, we briefly discuss the strategies available to a generator and then state the result without proofs (see Boom (2003) for details). If the competitor has a very low capacity, a firm can either choose a very large capacity and corner the market, or it can match the competitor’s capacity to generate positive revenues (for a smaller own capacity, revenues are zero). If the competitor’s capacity is larger (but still relatively small), cornering the market is no longer an option; positive revenues are still possible, however, for a capacity much larger than that of the competitor. For a still larger capacity of the competitor, positive revenues from installing a higher capacity are impossible. Finally, for a very large competitor’s capacity, own revenues are independent of own capacity. For later reference, we summarize the possible outcomes in our next proposition.

**Proposition 8 (capacity investments)** Depending on the level of capacity costs, there are the following pareto-dominant Nash equilibria.

(i) If $0 \leq z < 0.2118$, there is a unique equilibrium where firms choose capacity levels $k_A = k_B = \hat{k}$, with

$$\hat{k} = \arg\max_k \left\{ \frac{1 - 4k^2 + 3\sqrt{4k^2 - 1}}{8} - zk \right\}.$$ 

(ii) If $0.2118 \leq z \leq 1/(2\sqrt{2})$, the equilibria that are not pareto-dominated are characterized either by both firms choosing $\hat{k}$ or by firm A choosing the monopoly capacity $k_m$, defined in Proposition 1, and firm B choosing $k_B = 0$ or vice versa.

(iii) If $1/(2\sqrt{2}) < z < \frac{1}{2}$, there are two equilibria with firm A choosing $k_m$ and firm B choosing $k_B = 0$ or vice versa.

(iv) For $z \geq \frac{1}{2}$, there is a unique equilibrium where firms choose the capacity levels $k_A = k_B = 0$.

**Proof:** See Boom (2003), Appendix C. 

Proposition 8 indicates that uniqueness cannot be achieved, even when considering only pareto-dominant subgame-perfect Nash equilibria.
6 Comparing Market Configurations

In this section, we first construct rankings of the various market configurations in terms of capacities, retail prices and welfare and discuss the intuition. Second, we highlight the differences to the standard literature on vertically related industries.

6.1 Rankings for Capacities, Retail Prices and Welfare

We first consider the capacity levels installed by the generators in the various market configurations. We denote the aggregate competitive capacity level in the case of integrated generators by $k_d = 2\hat{k}$, the aggregate capacity level in the case of separated retailers by $k^* = k^*_A + k^*_B$ and the monopoly capacity level by $k_m$.

**Proposition 9 (ranking of capacities)** Suppose that (i) capacity decisions are either taken sequentially by the separated generators or that $0 \leq z \leq 1/3$, and (ii) that integrated generators co-ordinate on the pareto-dominant competitive equilibrium. Then the ranking of aggregate capacity levels is

$$k_d \geq k^* \geq k_m.$$  

(14)

**Proof:** Follows from comparing Propositions 1, 5 and 8. ■

Proposition 9 states that generating capacity is the lowest in the integrated monopoly and highest in the integrated duopoly. Vertically separated duopoly generators provide an intermediate level of aggregate capacity.

To understand the intuition for this result, first consider the investment incentive of an integrated monopoly generator. Introducing another integrated generator implies that there is both upstream and downstream competition. If the former monopoly generator is now unable to serve its own retail demand, he faces the risk of having to buy electricity from the competitor and to give up all rents from selling electricity. The same is true for the competitor of the former monopoly generator. To avoid such an outcome, each generator will invest more than it would be willing to invest as a monopolist ($k_d > k_m$). Second, consider the impact of vertical separation on the investment incentive of a duopolistic generator. After vertical separation, generators trade with retailers rather than themselves on the wholesale market. Therefore, separated duopoly generators no longer face the risk of having to give up all rents from selling electricity, and thus install smaller capacities.
than integrated duopoly generators \((k^d > k^*)\). In addition, Proposition 9 shows that the positive effect of introducing competition on capacity investments dominates the adverse effect of vertical separation, so that separated duopoly generators install a higher aggregate capacity than the integrated monopoly \((k^* > k^m)\).

Our next result gives a ranking of retail prices in these equilibria when the two generators are integrated \((r^d)\), in the monopoly case \((r^m)\) and when the two generators are separated from the two retailers \((r^*)\).

**Proposition 10 (ranking of retail prices)** Suppose that (i) capacity decisions are either taken sequentially by the separated generators or that \(0 \leq z \leq 1/3\), and (ii) that integrated generators co-ordinate on the pareto-dominant competitive equilibrium. Then the ranking of retail prices is given by

\[
r^d \geq r^m \geq r^*
\]  

**Proof:** Follows from comparing Propositions 1, 3 and 7

Proposition 10 indicates that the integrated duopoly yields not only the highest aggregate capacity, but also the highest retail price of all market configurations under study. The separated duopoly, in turn, yields the lowest retail price. The intuition for the high retail price under integrated duopoly parallels that for the high installed aggregate capacity: Integrated duopoly generators face the risk of being unable to serve their own demand, which may be reduced by setting a high retail price (i.e., keeping demand low). This incentive is absent under both integrated monopoly and separated duopoly. Also note that retail prices are lowest in the separated duopoly, as retail competition compresses the downstream mark-up.

Finally, we consider the welfare levels attained in the integrated competitive duopoly \((W^d)\), in the separated duopoly \((W^*)\) and in the integrated monopoly \((W^m)\).

**Proposition 11 (ranking of welfare levels)** Suppose that (i) capacity decisions are either taken sequentially by the separated generators or that \(0 \leq z \leq 1/3\), and (ii) that integrated generators co-ordinate on the pareto-dominant competitive equilibrium. Then the ranking of welfare levels is given by

\[
W^* \geq W^m \geq W^d
\]  

**Proof:** Since black-outs do not occur irrespective of market configuration, social welfare is given by

\[
W(k) = \int_0^1 U(x(r, \varepsilon), \varepsilon) d\varepsilon - zk
\]
where $k$ denotes total capacity. Substituting $U(x(r, \varepsilon), \varepsilon)$ from (1), $x(r, \varepsilon)$ from (2) and plugging in equilibrium values for $r$ and $k$ for each market configuration yields the associated welfare levels. Comparing these welfare levels completes the proof.

Proposition 11 indicates that the separated duopoly yields the best results in terms of welfare. Also, the integrated monopoly performs better than the integrated duopoly. To grasp the intuition for this result, it is important to note that irrespective of the market configuration, total installed capacity is always large enough to satisfy retail demand at the relevant retail price, so that black-outs never occur in our setting. This immediately implies that increasing capacity, holding retail prices constant, increases capacity costs rather than supply security. These increases in capacity costs must be weighed against the effects of changes in retail prices for the construction of the welfare ranking. Since both total capacity and retail prices are higher in the integrated duopoly than in the successive duopoly, welfare must be lower in the integrated duopoly. The welfare effect of changing from integrated monopoly to separated duopoly is less obvious: Total capacity is higher, but retail prices are lower in the separated duopoly. Proposition 11 shows that the positive effect of lower retail prices dominates the negative effect higher capacity costs, so that the separated duopoly performs better than the integrated monopoly.

6.2 Discussion

It is useful to discuss our findings in light of the literature on vertically related industries. A well-known finding of this literature is that vertical separation, combined with imperfect competition, gives rise to a vertical externality problem. That is, when making strategic pricing or investment decisions, upstream firms do not take into account the effect of these decisions on the profits of downstream firms (and vice versa). Due to this vertical externality, firms (i) typically set inefficiently high (linear) prices, and (ii) tend to make inefficiently low investments. Vertical integration eliminates this vertical externality and therefore increases welfare. The literature also suggests that more intense competition on either the upstream or the downstream market helps compress mark-ups and increase investment.

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18 The classic reference is Spengler (1950). Tirole (1989) and Perry (1989) provide surveys of outcomes in vertically related industries. See e.g. Abiru et al. (1998) for a more recent contribution.

19 See, e.g., Buehler et al. (2004).
thereby raising welfare.\footnote{For instance, in the extreme case of perfect competition in either the upstream or downstream market, the vertical externality disappears.} Given these results, we should expect the integrated duopoly to perform best in terms of welfare, as it combines vertical integration with competition: Capacity investment should be highest, and retail prices should be lowest. Yet, according to Propositions 9–11, this is not the case: Capacity investments are highest under integrated duopoly, but retail prices are also highest (rather than lowest), and welfare is even lower than under integrated monopoly.

To understand why the standard predictions turn out to be inadequate in our setting, it is important to note the following crucial differences to the literature:

**Reversed Timing.** The timing of upstream and downstream decisions is reversed. In our setting, it is the retail market that clears in the long run, whereas it is the wholesale market in the standard literature. This implies, in particular, that wholesale prices can react to changes in retail prices in our setting, whereas retail prices cannot react to changes in wholesale prices. Therefore, inflating the upstream price merely shifts rents from the downstream to the upstream market, without affecting the retail price. That is, holding capacity levels constant, increasing the upstream price still has a negative externality on downstream profits, but leaves total welfare unaffected.

**Investment Effects.** In our setting, higher capacity investments tend to decrease (rather than increase) welfare. Recall that, in our setting, capacity levels and retail prices are chosen such that black-outs do not occur. Therefore, changes of market configuration that give rise to higher levels of capacity do increase generation costs, but leave supply security unaffected. That is, the only way increases in capacity can positively affect welfare is over a higher demand that may be served without a black-out occurring. This does, however, only occur if retail prices decrease as in the separated duopoly scenario, but not if they increase as in the integrated duopoly.

**Unit Price Auction.** Upstream prices are determined in a unit price auction rather than a standard oligopoly model. Together with the reversed timing described above, the unit price auction implies that integrated duopoly generators face the risk of foregoing all rents if they cannot serve their own retail demand. To avoid giving up rents, integrated duopoly generators are willing to make large capacity investments. This cannot happen in an oligopoly model with standard timing, because they can always increase their retail prices according to the capacity installed in the upstream production.
7 Concluding Remarks

This paper has established three main results on the effects of restructuring electricity: First, capacity investments are highest under integrated duopoly and lowest under integrated monopoly. Second, retail prices are lowest under separated duopoly and highest under integrated duopoly. Third, the combined investment and price effects of restructuring are such that the separated duopoly yields the highest social welfare. Together, our findings suggest that restructuring electricity is likely to increase both capacity investments and welfare, if it implements (imperfect) competition and vertical separation.

Note that our findings are in contrast to what one might expect, having standard results of the vertical integration literature in mind: There, introducing imperfect competition tends to undermine investment incentives and increase retail prices due to the vertical externality associated with a double mark-up. Our analysis shows that these standard predictions crucially rely on assumptions on (i) the timing of the game, the (ii) the welfare effects of investment, and (iii) the mechanism determining wholesale prices.

Future research should deal with a number of generalizations. First, it would be interesting to allow for endogenous (and possibly asymmetric) vertical integration along the lines suggested by Buehler and Schmutzler (2005a) and (2005b). Doing so would enrich our understanding of the firms’ strategic investment decisions. Second, the discrimination of non-integrated competitors has rarely been considered in the context of electricity. Third assuming another distribution of demand shocks might create the potential for black-outs in equilibrium. The latter might challenge our welfare ranking because the higher retail prices and higher generating capacities with vertical integration might become valuable from a social welfare point of view. Finally, it would be useful to have a model with more than two upstream competitors, so that the strong extra investment incentive generated by the risk of not being able to serve own downstream demand would be mitigated.
Appendix

A The Derivation of Firm $i$’s Best Response in Capacity.

For $k_j \geq 1$ firm $i$’s profit function (8) translates into

$$\Pi_i(k_i, k_j) = \begin{cases} -zk_i & \text{if } k_i \geq 1, \\ (1 - k_i)k_i - zk_i & \text{if } 0 \leq k_i \leq 1. \end{cases}$$ (18)

If $0 \leq k_j < 1$ holds, firm $i$’s profit function becomes

$$\Pi_i(k_i, k_j) = \begin{cases} \frac{1-k_i}{2} - zk_i & \text{if } k_i \geq 1, \\ \frac{(1-k_i)k_i^2}{2} - zk_i & \text{if } k_j < k_i \leq 1, \\ \frac{1}{2} \left[ \frac{(1-k_j)k_i^2}{2} + (1 - k_i)k_i k_j \right] - zk_i & \text{if } k_i = k_j, \\ (1 - k_i)k_i k_j - zk_i & \text{if } 0 \leq k_i < k_j. \end{cases}$$ (19)

The best response of firm $i$ which is derived from maximizing (18) or (19), respectively, with respect to $k_i$ yields

$$k_i(k_j) = \begin{cases} \frac{1-z}{2} & \text{if } k_j \geq 1, \\ \frac{k_j-z}{2k_j} & \text{if } \frac{1-z-\sqrt{1-2z-2z^2}}{3} \leq k_j \leq 1, \\ 1 & \text{if } 0 \leq k_j \leq \frac{1-z-\sqrt{1-2z-2z^2}}{3}, \end{cases}$$ (20)

for $0 \leq z \leq 1/3$. If $1/3 < z \leq 1/2$ holds, the maximization of (18) and (19) with respect to $k_i$ results in

$$k_i(k_j) = \begin{cases} \frac{1-z}{2} & \text{if } k_j \geq 1, \\ \frac{k_j-z}{2k_j} & \text{if } z \leq k_j \leq 1, \\ 0 & \text{if } 1 - 2z \leq k_j \leq z, \\ 1 & \text{if } 0 \leq k_j \leq 1 - 2z. \end{cases}$$ (21)
References


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