

# ENDOGENOUS MERGERS IN CONCENTRATED MARKETS\*

Henrik Horn  
World Trade Organization  
Institute for International Economic Studies, Stockholm University  
CEPR

Lars Persson  
The Research Institute of Industrial Economics, Stockholm

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## Abstract

This paper proposes an approach for predicting the pattern of mergers when different mergers are feasible. It generalizes the traditional IO approach, employing ideas on coalition-formation from cooperative game theory. The model suggests that in concentrated markets, mergers are conducive to market structures with large *industry* profits, and thus points to a conflict between private and social incentives. It is shown how mergers may be undertaken in order to preempt other possible, and socially more desirable, mergers. The model also throws light on the formation of research joint ventures and tariff-jumping foreign direct investment.

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## 1. Introduction

The forces behind mergers are poorly understood, despite their evident empirical importance. In particular, the literature has very little to say about why certain firms merge and not others, in the industries of most anti-trust concern — those with relatively few firms. The traditional analysis of merger incentives was originated by Stigler (1964, 1971) and Salant *et al* (1983), and developed by e.g. Deneckere and Davidson (1985), Perry and Porter (1985), and McAfee and Williams (1992). It considers a merger between an arbitrary number of firms in the context of some particular oligopoly model. Firms are said to have incentives to merge if the profit of the merged entity is higher than their combined profits in the initial, no-merger equilibrium — we will refer to this as the “traditional criterion” for incentives for merger. It has been suggested, however, that even if the traditional criterion is fulfilled, firms may refrain from merging, since it might increase profits of an individual firm more to stand outside a merger than to participate, when the merger yields positive externalities on outside firms. This “hold-up” problem hence presupposes that there is a third alternative, one where the firm does not participate in the merger, but where the other firms merge.

The purpose of this paper is to generalize the conventional approach to merger formation. The traditional criterion for merger incentives is natural when the choice is between two market structures only, where one is a strict concentration of the other caused by a single merger. However, there are typically several different merger constellations that are feasible, and we will therefore allow the market participants to choose between *all* the feasible constellations, in order to determine the market structure. Following the literature, we will refer to such an analysis where more than one merger is possible, as one of “endogenous” merger determination.<sup>1</sup> More generally, the paper can be seen as suggesting a theory of

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<sup>1</sup>The importance of endogenizing merger formation is recognized by Salant *et al.*, who also sketch a model that could incorporate such a feature. Deneckere and Davidson (1983) present a two-stage non-cooperative model of endogenous merger formation, where it is assumed that the market structure is determined before the negotiations of the terms of the merger. However, this representation seems more suitable for the case of formation of cartels, which is also one

endogenous market structure for industries with a limited number of firms, and in which the means of production are in fixed supply.

One can distinguish at least three approaches to the modeling of endogenous merger formation. Kamien and Zang (1990, 1991) treat it as a normal form game model in which firms post bids for other firms and asking prices for their own, and where the equilibrium market structure arises as a result of this bidding. A distinguishing feature of this approach is the simultaneity of the posting of the bids and asking prices, which makes it seem descriptive of situations where there are no negotiations between firms, and where changes in ownership of firms instead take place through some form of trading mechanism, such as a stock exchange. We will return this approach in Section 3.

In contrast, the second and third approaches focus on the ability of firms to freely communicate about mergers, and to sign binding contracts. Merger formation is here treated as some form of bargaining. These approaches thus seem descriptive of situations where a limited number of firms and owners are involved. The second approach would portray the merger formation as a non-cooperative extensive form bargaining game, adopting a model such as the one analyzed by e.g. Chatterjee *et al.* (1993), or Ray and Vohra (1998). While in many ways being the most natural approach, it has some undesirable features. In particular, the predictions about the mergers that will be formed would be sensitive to the order in which firm make offers and counter-offers. However, this feature seems to be an artifact of the modeling procedure, rather than an aspect of first-hand importance to merger formation.<sup>2</sup>

This paper follows a third approach, and treats the merger formation as a cooperative game, for a combination of two reasons. First, and as is often stressed, by sacrificing some details in the description of an extremely complicated strategic interaction, a cooperative approach may provide some insights into situations where players are free to communicate and write binding contracts — as firms

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interpretation given by the authors. See e.g. Bloch (1992), Yi (1997, 1998), and Yi and Shin (1995) for recent contribution to this approach.

<sup>2</sup>See Ray and Vohra (1995) for a discussion of the problem of modeling coalition formation.

typically are in the process of merger formation — insights that are hard to achieve in non-cooperative models.<sup>3</sup> Secondly, by using a cooperative approach we also generalize the traditional merger analysis, which implicitly relies on cooperative reasoning. The traditional criterion for merger incentives is based on the idea that if a merger represents a Pareto improvement for a group of firms, they should somehow be able to agree on a division of this gain. But, it is not specified why and how the parties agree on this division. Here we generalize this approach by taking its logic one step further, including comparisons not only between two arbitrarily chosen market structures, but between *all* possible market structures.

The paper uses a cooperative approach to coalition formation, “partition function form games”, that dates back to Thrall and Lucas (1963). The partition function form includes the payoffs of all players in any possible coalition structure. Following Shenoy (1979), the partition function form game is expressed as an “abstract system”, which is a very general description of a game.<sup>4</sup> It consists of two components: a set of outcomes, and a binary relation *dom* defined on these outcomes, with the interpretation that  $M^j$  “dominates”  $M^i$  whenever  $M^j \text{ dom } M^i$ , where  $M^i, M^j \in \mathcal{M}$ . Here, the set of outcomes will be ownership structures, and the *dom* relation will be a generalization of the traditional criterion for merger incentives that can be applied to any pair of structures. Hence, it can rank structures where neither is strictly more concentrated than the other, or where more than one merger is involved. By applying a solution concept (stability criterion) to this game, in our case the core, one can predict an equilibrium ownership (coalition) structure. Thus, we will rank every feasible structure against every other structure, and view structures that are undominated as equilibrium

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<sup>3</sup>See e.g. the overviews of cooperative theory in Greenberg (1994), and Osborne and Rubenstein (1994). There seems to be a much more eclectic view of cooperative game theory among game theorists than among non-specialists who apply game theoretic concepts.

<sup>4</sup>This concept originates from von Neumann and Morgenstern (1947), and its relevance to a large number of cooperative and non-cooperative approaches is discussed by Greenburg (1994). A recent cooperative application is Espinoza and Inarra (1996), who employ the notion of von Neumann-Morgenstern stable sets of payoffs to derive “merger stable” equilibria for markets where firms are identical.

ownership structures. As argued by Hart and Kurz (1983), such an approach can be seen as a first step toward a full-fledged dynamic analysis, in that it characterizes its final outcome.

The model of endogenous merger formation is first applied to the case where complete monopolization is permitted and yields maximal industry profits among all possible market structures. It is shown in Section 3 that the hold-up problem is not as pervasive here as in other approaches, under fairly general circumstances regarding technology and demand. The reason for this is that in our framework a refusal by some party to sign a contract to which it is signatory implies that the whole contract is void. Then, if one player is leaving the monopoly, the others can always lure him back with a better offer, as long as the monopoly enjoys the advantage of yielding the highest industry profit. We find this feature intuitively appealing, since we believe that while the hold-up problem might be descriptive of markets with a large number of owners, it is much less plausible that a limited number of owners in a concentrated industry which can communicate and sign binding agreements would wait indefinitely to reap the gains from merger, as a result of constantly trying to become profitable outsiders. If the parties can communicate and sign binding contracts then the outcome should also be efficient.

Section 4 derives conditions under which the most concentrated structure will arise, also when a monopoly is not permitted to form. It shows also that if cost savings from mergers only stem from savings of fixed costs, then the equilibrium market structure maximizes industry profits among all permitted structures.

Section 5 briefly draws some conclusions concerning circumstances under which private incentives are correct from a socially point of view, and when they diverge. It also shows that equilibrium mergers need neither eliminate the most inefficient firms, nor create the most efficient ones. The section ends with a few remarks on merger policy when mergers are endogenous. When evaluating a proposed merger the alternative to the merger is not necessarily the outset, but may be another proposed merger. The design of an optimal policy is considerably more complicated when these features have to be taken into account.

A distinguishing feature of the present model, when compared to other models

of endogenous mergers, is that this model can handle asymmetries between firms, and thus predict the mergers that are more likely to emerge in situations where firms are asymmetric. For instance, it is shown how mergers may be undertaken in order to pre-empt other possible, and socially more desirable, mergers. The reasons for this is partly the fact that merger formation in these cases are partly driven by preemptive motives: a firm's alternative to merging need not be to remain in the initial position, but to stand outside some other merger. A merger can thus partly be driven by a desire to avoid negative repercussions that the merging parties would experience if other mergers were not prevented.

This mechanism turn out to be central for the results derived in Section 6, where the model is applied to three issues related to conventional merger analysis. For example, it is shown how research joint ventures may be formed in order to pre-empt other possible, and socially more desirable, joint ventures. It is also illustrated how high trade barriers may lead to mergers among domestic firms, where these domestic mergers are partly driven by the incentive for domestic firms to pre-empt international mergers. This result contrasts to the “tariff-jumping” argument which holds that high barriers provoke international mergers.

## **2. The model**

The standard approach in cooperative game theory relies on the “coalition function” or “characteristic function” form, which largely ignores externalities.<sup>5</sup> In these games it is implicitly assumed that the payoff levels members of a coalition can attain are independent of the actions chosen by the players outside this coalition. This description sharply contrasts with the situation we envisage where the outcome of the merger formation should be expected to be influenced both by the profits firms make when merging, as well as by their profits when other firms merge. In order to capture this type of situation, we will describe merger formation as a partition function form game.

Consider an industry in which production requires the usage of an asset that is

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<sup>5</sup>See e.g. Shubik (1983, p. 354) and Greenberg (1994, p. 1308).

in fixed supply. Each unit of this asset is indivisible and is under the control and ownership of a separate owner. The interaction in the industry takes place in two stages. In the first the owners form firms. A firm needs one unit of the asset to be operational, so each separate owner can run a firm. But owners can also form coalitions — merge — in order to create firms that control more than one unit of the asset. An *ownership structure*  $M^j$  is a partition of the set  $\mathcal{N} = \{1, \dots, n\}$  of owners into coalitions. Let  $\mathcal{M}$  be the *set of all possible ownership structures* and let  $\mathcal{K}^j$  be the *set of all firms (coalitions) in  $M^j$* .

The firms that are formed in the first stage compete non-cooperatively in standard fashion in an oligopolistic market in the second stage. Let the profits of firm  $k$  in market structure  $M^j$  be denoted  $\pi_k^j$ , etc.. When agreeing on a merger, owners can decide on any division of the firm’s profit, as long as the sum of the payments equals the profit of the firm. However, owners cannot make any payments between coalitions in the first stage, nor can there be any transfers between firms in the second stage.<sup>6</sup>

In line with much of cooperative game theory, and in a trivial sense with the traditional IO approach to the analysis of merger incentives, we will derive the equilibrium coalition structure(s) by means of binary comparisons of such structures. A central concept will be the “dominance relation” *dom*. The idea behind this relation is that if a ownership structure  $M^i$  is “dominated” by another structure  $M^j$ , the former will not be the outcome of the merger game, since it is in the interest of owners who have the power of enforcing  $M^j$  over  $M^i$  and vice versa — who are “decisive” with respect to  $M^j$  and  $M^i$  — to enforce  $M^j$  whenever the alternative threatens to be  $M^i$ . In particular, for  $M^j$  to dominate  $M^i$ , all owners who are involved in the formation and breaking up of mergers between the two structures must in some sense prefer the dominating structure to the other structure. As mentioned before, we are mainly interested in cases

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<sup>6</sup>In the paper we abstract from potential conflicts between managers and owners, conflicts that are at the forefront in a large literature in finance. Related, all assets owned by the same owner are employed in the same firm. We therefore depart from the large literature in finance which focuses on conflicts between owners and managers, and between different groups of owners.

where there is a limited number of firms in the market. Since firm owners can typically freely communicate about mergers, it is reasonable to view the owners as participating in some form of bargaining over the two structures.

Three issues need to be resolved in order to utilize this approach: First, who can influence the ranking of a given pair of structures? Secondly, what criterion does this group employ for the ranking? Thirdly, how should all the rankings of the various possible structures be weighted together? These three questions, which are central to any cooperative game, are addressed in three subsections below. However, before turning to these details we present an example that demonstrates the considerations the model are meant to capture.

### 2.1. An example

Assume there are three oligopolistic firms, 1, 2, and 3, at the outset. They are allowed to merge, but not to form a monopoly. The possible market structures are  $M^O = \{1, 2, 3\}$ ,  $M^A = \{12, 3\}$ ,  $M^B = \{13, 2\}$ , and  $M^C = \{23, 1\}$ , where in structure  $A$  firms 1 and 2 are merged, and firm 3 is on its own, etc. The profits of the firms in the possible market structures are

$$\begin{aligned} \pi_1^O = \pi_2^O = \pi_3^O = 0 & & \pi_{12}^A = 70, \pi_3^A = 50 \\ \pi_{13}^B = 100, \pi_2^B = 0 & & \pi_{23}^C = 90, \pi_1^C = 5 \end{aligned}$$

The task of predicting the merger that is likely to be formed is complicated by the fact that for any payoff vector that is proposed, there are other payoff vectors that are better for both participants of another merger in some other structure — that is, there exists no “coalition structure core.” This tendency to move away from any proposed market structure implies that there is no simple candidate for an equilibrium market structure.<sup>7</sup>

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<sup>7</sup>The traditional approach to merger analysis would only permit a comparison of structure  $M^O$  with either structure  $M^A$ ,  $M^B$  or  $M^C$ , since these are the only comparisons involving a strict concentration, stemming from a single merger. For example, in the terminology above, the traditional criterion would amount to the criterion  $M^A \text{ dom } M^O$  if  $\pi_{12}^A > \pi_1^O + \pi_2^O$ . This solution implicitly, and in a trivial sense, relies on the core:  $M^A$  is the proposed solution if  $M^A$  is



Clearly, the profits that each merged entity achieves in the different constellations — the values 70, 100, and 90 — should influence the identity of the merger that is formed. From this perspective, the merger between firms 1 and 3 seems most likely. However, the firms are not only differently profitable in the different merger constellations, but are also differently profitable outside mergers, and should therefore be differently concerned to merge. Suppose that firms 1 and 3 contemplate merging, in order to divide the profit of 100. Firm 2 could then propose structure  $M^A$ , in which firms 1 and 2 merge. The resulting profit of 70 is smaller than the 100 achieved through the merger between firms 1 and 3. But, in order to support market structure  $M^B$  instead of  $M^A$ , firm 3 may not accept anything less in the merger with firm 1 than the 50 it would get in the proposed structure  $M^A$ , as an outsider. Hence, firm 1 would be left with a maximum of 50 in the merger with firm 3. Firm 2 would then be willing to give firm 1 a payoff of 60, say, in order to induce it to merge with firm 2. This would leave firm 2 with 10, which is better than the 0 obtained if 1 and 3 merge. Hence, we may expect structure  $M^A$  to dominate structure  $M^B$  as a candidate for a solution to the merger game. A similar reasoning would conclude that structure  $M^A$  in this sense dominates structures  $M^C$  and  $M^O$ . Being the only undominated market structure, structure  $M^A$  would be our unique suggestion for a solution to the merger formation game — the only market structure that is stable with respect to the possibility of firms to merge is  $M^A$ .

## 2.2. Decisive owners

Let  $\mathcal{H}^i$  be a subset of the set of firms (coalitions) in  $M^i$ ,  $\mathcal{H}^i \subseteq \mathcal{K}^i$ , and let  $\mathcal{O}(\mathcal{H}^i)$  be the set of owners participating in the firms in  $\mathcal{H}^i$ .

**Definition 1** A decisive group of owners with respect to ownership structures  $M^i$  and  $M^j$ , denoted  $\mathcal{D}_g^{ij}$ , and the sets of firms  $\mathcal{H}_g^i$  and  $\mathcal{H}_g^j$  that comprise these owners in the respective structure, have the following properties:

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undominated, which it is whenever  $M^A$  and  $M^O$  are the only possible structures, and  $M^A \text{ dom } M^O$ .

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  1.  $\mathcal{D}_g^{ij} = \mathcal{O}(\mathcal{H}_g^i) = \mathcal{O}(\mathcal{H}_g^j) \neq \emptyset$ .
  2.  $\mathcal{H}_g^i \cap \mathcal{H}_g^j = \emptyset$ .
  3.  $\nexists \mathcal{D} \subset \mathcal{D}_g^{ij} \mid \mathcal{D}$  fulfills 1 and 2.

Hence, this set of owners is obtained by forming a union of firms in  $M^i$  and a union of firms in  $M^j$ , such that the two unions thus formed contain the same set of owners, the two unions do not have any firms in common, and there is no strict subset of the set thus formed that can be constructed in the same way. Subscript  $g$  indicates that in general there may exist more than one decisive group of owners with respect to two ownership structures.

There is a very simple idea behind this perhaps somewhat opaque definition. It can be seen in the following example, in which there are 8 owners, and two ownership structures  $M^A = \{1, 2, 3, 4, 5, 6, 7, 8\}$  and  $M^B = \{1, 2, 3, 4, 5, 6, 7, 8\}$ . Owners 6, 7 and 8 belong to identical firms in both structures, and since payments between firms are not allowed, these owners cannot influence the ranking of  $M^A$  and  $M^B$  — they are not “decisive” with respect to these two structures. Now turn to owners 1, 2 and 3. If  $M^A$  is formed, 3 will not participate in a merger. In order to prevent this, which would be in the interest of 3 if the owner’s profit in this structure is small, 3 may offer to forego surplus in structure  $M^B$  in the merger with owner 2. On the other hand, owner 1 may counter with an offer to forego surplus in structure  $M^A$ , to induce a merger with owner 2. Or, 2 might be willing to sacrifice a large part of the profit made by firm 23 in  $M^B$ , in order to induce owner 3 to participate in a merger. Owner 1 may then need to offer a large part of the profit in  $M^A$  to persuade 2 to merge. In either case, by being linked to owner 2 in structure  $M^B$ , 3 is able to “bargain” with 1 over 2’s participation in a merger. In a similar vein, if the profit of owner 5 in structure  $M^A$  is low, this owner may offer a large share of the cake to owner 4, in order to induce the latter to participate in the merger in structure  $M^B$ . Thus, owners 1, ..., 5 have the possibility, and usually also the incentive, to affect the ranking of  $M^A$  and  $M^B$ , and are therefore “decisive” with respect to these two structures. But, it is not possible for owners 1, 2 and 3, to transfer resources to owners 4 and 5, and vice

versa; hence the need to distinguish between separate groups of decisive owners.

### 2.3. The dominance relation

Then, when should the set of decisive owners be said to prefer one structure to another? Let  $\mathcal{D}_g^{ij}$  be an arbitrary decisive group with respect to  $M^i$  and  $M^j$ . We will say that  $M^j$  *dominates*  $M^i$  via  $\mathcal{D}_g^{ij}$  if and only if the combined profit of the decisive group is larger in  $M^j$  than in  $M^i$ ,

$$\sum_{k \in \mathcal{H}_g^j} \pi_k^j > \sum_{k \in \mathcal{H}_g^i} \pi_k^i \quad (2.1)$$

where  $(\mathcal{H}_g^i, \mathcal{H}_g^j, \mathcal{D}_g^{ij})$  fulfill Definition 1. With more than one decisive group, these groups may dominate in opposite directions. For  $M^j$  to *dominate*  $M^i$ , written  $M^j \text{ dom } M^i$ , we require that domination holds for *each* decisive group with respect to  $M^i$  and  $M^j$ :

**Definition 2**  $M^j \text{ dom } M^i$  iff (2.1) holds for each decisive group w.r.t.  $M^i$  and  $M^j$ .

Note that this implies that we cannot have both that  $M^j \text{ dom } M^i$  and  $M^i \text{ dom } M^j$ ; that is, the *dom* relation is asymmetric.

Note that *our dominance relation includes the criterion for merger incentives employed in the traditional merger literature as a special case* which arises when  $M^i$  is a structure without mergers, and  $M^j$  one where a single group of owners have merged. Our criterion then exactly coincides with the standard criterion: the sets of decisive owners are identical in both cases, as are the dominance relations. But, the dominance relation employed here is more general in two respects: it is applicable to situations where there is more than one concentrative merger, and where neither ownership structures is a strict concentration of the other.

### 2.4. Equilibrium ownership and market structures

The definition of decisive groups and of the *dom* relation specifies how to rank any pair of ownership structures. The remaining issue is to specify how these rankings

are to be used in order to predict the outcome of the merger formation. To this end we define as *Equilibrium Ownership Structures* (EOS) those structures that are in the *core*:

**Definition 3** The set of EOS is  $\mathcal{M} \setminus \{M' \in \mathcal{M} \mid \exists M \in \mathcal{M} \text{ such that } M \text{ dom } M'\}$ .

This stability concept is largely chosen for its analytical convenience. But, it is also in a trivial sense in keeping with the traditional merger literature, as noted above. Of course, there are some well-known problems with the core. For instance, it may contain more than one element. Conversely, it may be empty, in which case we are not able to predict the equilibrium ownership structure. This possibility arises from the intransitivity of the *dom* relation: it is possible that  $M^i \text{ dom } M^j$ , and  $M^j \text{ dom } M^h$  while  $M^h \text{ dom } M^i$ , since different decisive groups are involved in the three dominance rankings. In the applications below, it will be necessary to impose enough structure so as to avoid these kinds of problems.<sup>8</sup>

## 2.5. The dominance relation and individual payoffs

It has been assumed that if the combined profits of the decisive group is larger in one structure than in the other, the merging owners in the structure with larger profits will somehow agree on a division that makes all members of the decisive group prefer this to the alternative. We will here briefly point to one possible way in which this dominance relation could instead be couched in terms of individual payoffs.

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<sup>8</sup>Note that since we for the sake of analytical simplicity often will employ oligopoly models with certain symmetries, different EOS may lead to identical *Equilibrium Market Structures* (EMS), i.e., to markets with the same number of firms, and the same distribution of production technologies across firms. For instance, in the case where the underlying oligopoly market is a homogenous product Cournot market where firms have identical technologies, a merger between any pair of firms gives rise to the same market structure, and as a consequence to the same outcome in terms of the distribution across firms of outputs and of profits, as a merger between any other pair of firms. Under such circumstances we will not be able to predict exactly which of the ownership structures with identical outcomes that will result, but may still predict the equilibrium market structure. However, with sufficient asymmetry between the firms each EOS will give rise to a separate EMS.

When forming a merger in the first stage, participating owners can freely choose how to distribute among themselves the profits the firm will make in the second stage. A vector of *payoffs*  $r^j = (r_1^j, \dots, r_n^j)$  specifies a distribution of these profit among the owners in ownership structure  $M^j$ . The only constraint that is imposed on payoffs is that each firm distributes exactly the profit it makes in the product market competition stage to its owner(s).<sup>9</sup> We thus say that a payoff vector  $r^j$  is  $M^j$ -feasible if for each firm  $k$  in market structure  $M^j$  it holds that

$$\sum_{h \in k} r_h^j = \pi_k^j \quad (2.2)$$

where  $h$  is an arbitrary owner. The set of all payoff vectors that are feasible in  $M^j$  is denoted  $\mathcal{F}^j$ . We will say that  $M^j$  dominates  $M^i$  via  $\mathcal{D}_g^{ij}$ , written  $M^j \text{ dom}_{\mathcal{D}_g^{ij}} M^i$ , if and only if there does *not* exist a pair of feasible payoffs  $(r^i, r^j)$  such that all owners in  $\mathcal{D}_g^{ij}$  are better off in  $M^i$  than in  $M^j$ , i.e.,  $M^j \text{ dom}_{\mathcal{D}_g^{ij}} M^i$  if and only if

$$\nexists (r^i, r^j) \mid r^i \in \mathcal{F}^i, r^j \in \mathcal{F}^j, r_h^i \geq r_h^j, \forall h \in \mathcal{D}_g^{ij} \quad (2.3)$$

An alternative definition of dominance would then be the following:

**Definition 2'**  $M^j \text{ dom } M^i$  iff (2.3) holds for each decisive group w.r.t.  $M^i$  and  $M^j$ .

The following Lemma, which is proved in Horn and Persson (1996), shows that Definition 1, which is a generalization of the traditional criterion for merger incentives, can be seen as a reduced form of Definition 2:

**Lemma 1.** *Definitions 2 and 2' give the same ranking of ownership structures.*

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<sup>9</sup>We thus permit both positive and negative payoffs. One possible situation in which an owner would agree to a negative payoff, would arise if there were costs of exiting the market. But, the basic thrust of the results in the ensuing sections would continue to hold if we restrict payoffs to be non-negative. However, the analysis requires some additional assumptions, and becomes more complicated. Moreover, for a sufficiently symmetric setup, the results below would be unaffected by such a non-negativity constraint.

### 3. Is there a “hold-up problem”?

A commonly discussed issue in the merger literature is the hold-up problem. A manifestation of such a problem would be that a monopoly is not formed, even though its formation is both permitted and profitable according to the traditional criterion for merger incentives. As will be seen, the present approach does not share this feature under weak assumptions about the underlying technologies and demand.

We will throughout the analysis assume that a weaker version of the traditional criterion for merger incentives holds. Let the maximally permitted degrees of concentration be captured by the minimum permitted number of firms  $\bar{k}$ .<sup>10</sup> The following assumption is maintained throughout:

- Any single, purely concentrative, merger that reduces the number of firms from  $k$  to  $\bar{k}$ , where  $\bar{k} < k \leq n$ , increases the combined profit of the merging firms. (A1)

We thus assume that the traditional criterion holds for any purely concentrative merger that yields the most concentrated structure permitted. The assumption is expressed in a non-formal way in order to be more transparent and to save on formalism. Two points should be emphasized. First, (A1) does *not* assume away the hold-up problem, since it is fully compatible with a situation in which outside firms’ increase in profits are larger than the increase in profits from participating in the merger, in both cases compared to the outset. Secondly, we allow for the possibility that a purely concentrative merger that leads to a structure with more than  $\bar{k}$  firms decreases the combined profit of the merging firms. Thus, it is not necessary that *all* concentrative mergers fulfill the traditional criterion. Condition (A1) is fulfilled in virtually every oligopoly model employed in the merger literature, for the case where the merger results in a monopoly.<sup>11</sup>

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<sup>10</sup>From a policy point of view it might have been more natural to measure the degree of concentration by, say, a Herfindal index. However, this would be analytically much more involved. Also, in the fixed cost savings models considered below this choice would obviously not matter.

<sup>11</sup>For instance, if the initial number of firms is three and a monopoly is permitted, (A1) does

Our first result establishes that the hold-up problem is not a feature of the present framework in the case where a monopoly is allowed to form:

**Proposition 1.** *Assume that (A1) holds, and that a monopoly is permitted to form. Then the monopoly structure is the unique EOS.*

**Proof.** Since all players are included in the same decisive group when ranking the monopoly structure  $M^M$  to an arbitrary ownership structure  $M^i$ ,  $M^M \text{ dom } M^i$  iff the monopoly profit is larger than the combined profits of the oligopolists in  $M^i$ , and this is ensured by (A1). The asymmetry of the dominance relation implies that  $M^M$  is then also undominated. ■

The fact that the hold-up problem does not appear here but in other studies, is due to differences in equilibrium concepts, since (A1) holds in most oligopoly models as long as merger to monopoly is allowed, as noted above. To better understand these differences we will relate our approach to the other two approaches to endogenous merger determination that were mentioned in the Introduction. Kamien and Zang (1990, 1991) provided a formal foundation for the hold-up problem in which owners first simultaneously post bids for other firms, and asking prices for their own firms, and the pattern of mergers is then determined on the basis of these bids. In a later stage there is product market competition. Somewhat over-simplifying, the basic mechanism can be illustrated as follows: consider a market with  $n$  symmetric firms and a candidate equilibrium where all firms make identical bids that permit a monopoly to be formed, which give each firm  $1/n$  of the monopoly profit. This need not be a Nash equilibrium since any firm could deviate by making an unacceptable asking price, in the expectation of making a larger profit by becoming a duopolist that competes with the remaining  $n - 1$  merged firms. Hence, monopolization breaks down.<sup>12</sup>

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not require that a merger to a duopoly is profitable for the merging partners, but only that a complete monopolization is. (A1) holds in the basic Salant *et al* (1983) model without fixed costs, since in their model  $\pi(1) > n\pi(n)$ , for  $n > 1$ .

<sup>12</sup>The authors discuss the possibility to write contingent contract, but claim that this appears to be a direct violation of U.S. antitrust laws. However, it is shown by Yi (1995) that if owners can in the first stage make offers that are conditioned on the market structure, the equilibrium

A crucial aspect of the approach of Kamien and Zang is that it requires that the equilibrium in the merger formation game is a simultaneous move Nash equilibrium in unconditional asking prices and bids. This description seems most suitable for a situation where there are many firms in the market, and where consequently a firm's choice of whether or not to participate in a merger has limited impact on the incentives of the other firms. Our framework is more descriptive of situations where firms cannot deviate from a proposed merger without expecting other parties to react, if they so wish. In actuality, a refusal by some party to sign a contract to which it is a signatory, typically implies that the whole contract is void, even if the other parties have signed it. A deviating owner must therefore take into account the market structures that might arise as a result of the deviation. Therefore, while the hold-up problem might be descriptive of markets with a large number of owners, it seems less plausible that a limited number of owners in a concentrated industry would wait indefinitely to reap the gains from merger, as a result of constantly trying to become profitable outsiders.

Turn next to the relationship between the current approach to endogenous merger formation and one which would rely on non-cooperative sequential bargaining models. Ray and Vohra (1998) point to two main features of their analysis: first, just as here it employs a partition function approach in order to capture inter-coalitional interactions. Second, the players are assumed to be "far-sighted" in that they fully take into account the dynamic implications of a particular decision with regard to a merger. Here the two approaches differ. To see how, consider the following three firm example from their paper:<sup>13</sup>

$$\begin{aligned}\pi_1^O &= \pi_2^O = \pi_3^O = 0 \\ \pi_{12}^A &= \pi_{13}^B = \pi_{23}^C = \frac{1}{2} & \pi_3^A &= \pi_2^B = \pi_1^C = 2 \\ \pi_{123}^M &= 3\end{aligned}$$

According to the reasoning in Ray and Vohra, these firms are likely to achieve market structure becomes more concentrated.

<sup>13</sup>Strictly speaking, the example in Ray and Vohra refers to provision of a public good, and not mergers in an oligopolistic market.



an outcome which is inefficient for the industry, that is, they will not monopolize the market. When the firms consider forming the monopoly, one firm has an incentive to defect knowing that the other two firms would have an incentive to remain merged once faced with the fact of the defection. While this seems to capture some of the considerations of the firms in such a situation, it does not appear to be the full story. As pointed out by Ray and Vohra, “*This statement is fraught with numerous complexities that we have found best to avoid, in the interest of making some progress on the question of coalition formation. If any binding agreement can, in principle be renegotiated, then the outcome should be efficient. After all, if as in the example above, the other two can try to lure him back with the promise of a better offer...*” (pp. 3-4). As pointed out above, a main idea behind our model is to capture these possibilities.

Our analysis lacks the farsightedness emphasized by Ray and Vohra, however, in its reliance on the core solution concept. One coalition structure may be dominated by another structure, that in turn is dominated by a third. It might be argued that the first domination in this chain is not credible, since it comes through domination by a structure that itself is dominated, and that thus cannot be part of the solution. The essential consequence of permitting such domination, as is done here, is to restrict the set of solutions unnecessarily. This is a price we pay in order to avoid an extremely complicated forward-looking analysis.<sup>14</sup>

Another weakness of the current approach is the lack of determination of payoffs within merged entities. One possibility of addressing this, would be to add a payoff division rule to the equilibrium mergers, for instance that, payoffs are equally divided, or according to Shapley values. One would then assume that this division rule only applied to equilibrium merger(s). Of course, the proposed payoff vector would typically not be immune against deviations, since there typically does not exist a core payoff vector.

To conclude, the current approach is only one of several possible ways of endogenizing mergers. As always, the value of any particular approach lies in

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<sup>14</sup>See Aumann and Myerson (1988), and Chwe (1995), for other approaches building on farsightedness.

the insights it yields. In our view, the current model captures in a relatively simple fashion mechanisms that are not obvious *a priori*, but once illuminated appear intuitively compelling. It therefore improves our understanding of certain aspects of merger formation. We thus see our approach as complementary to those of Kamien and Zang, and of Ray and Vohra.

#### 4. Equilibrium ownership and market structures

If a monopoly is permitted, it will typically be formed, and if so, no further analysis is needed, since both the equilibrium ownership and market structure are determined. However, it may perhaps intuitively be argued that the hold-up problem did not arise above exactly because of this possibility for firms to form a monopoly, but that this is often not possible in practice. We will therefore in the rest of this paper assume that monopolies are not permitted to form, e.g. due to interventions by a competition authority, or since it is not associated with maximal industry profits (see the discussion in the Concluding remarks section). The following fairly general result limits the set of ownership structures that are candidates for EOS:

**Proposition 2.** *Assume that (A1) holds. Then, in any EOS owners are partitioned into the minimum permitted number of firms.*

**Proof.** Any structure  $M^i$  with more than  $\bar{k}$  firms is by (A1) dominated by some structure with  $\bar{k}$  firms which is a pure concentration of  $M^i$ . Thus, only structures with  $\bar{k}$  firms are candidates for being EOS. ■

This Proposition provides another piece of evidence in favor of the view expressed above, that the owners should be able to avoid what is possibly the worst of outcomes, the least concentrated industry structure.<sup>15</sup> But, it falls short of establishing that structures with  $\bar{k}$  firms are undominated — it may generally be that *all* structures are dominated because of the above-mentioned intransitivity of the dominance relation. It is difficult to find general conditions that ensure

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<sup>15</sup>Note that for the initial, no-merger structure to be the single EOS, it is neither sufficient nor necessary that the inequality in (A1) is reversed for all market structures.

that the set of EOS is non-empty in the case when monopoly is not permitted. There is one central exception, however, which is when duopolies are permitted to form. This case is of interest in its own right since we are interested in mergers in concentrated markets. But, it also points to certain features of merger formation that should be present also in cases where only less concentrated structures are permitted. Note, however, that while in traditional oligopoly analysis there is usually no qualitative difference between duopolies and less concentrated structures, there is here a fundamental difference from the point of view of merger formation.

#### 4.1. Duopolies

It will be useful to specify some properties of the merger technology in order to proceed. Many merger models would share the following feature:

- A purely concentrative merger that leads to a structure with  $\bar{k}$  firms increases the profits of outside firms. (A2)

The most immediate case where (A2) need not be fulfilled arises when the merger creates strong variable cost synergies, in the terminology of Farrell and Shapiro (1990a). (A2) is clearly compatible with the existence of a hold-up problem, since if the positive externalities are strong enough it will make outsiders profit more than insiders from a merger.

We are now set to show that a concentrated equilibrium market structure is not specific to the case where monopoly is permitted (for the proof, see the Appendix):

**Proposition 3.** *Assume that (A1) and (A2) hold, and that all ownership structures are permitted except a monopoly. Then, the set of EOS is non-empty, and consists of those duopoly structures that give rise to the maximal industry profit among all structures.*

Proposition 3 not only shows that the tendency toward concentration that was established for the case when a monopoly is permitted also exists when a duopoly

is the maximally permitted degree of concentration. It also demonstrates that, under (A1) and (A2) the equilibrium duopoly structure gives rise to the maximal *industry* profits among *all* structures, except for a monopoly.

It is difficult to derive results which are equally strong to those in Propositions 1 and 3 for less concentrated structures, because of the intransitivity of the dominance relation. This intransitivity obviously does not arise in the case of monopoly, since all owners then belong to the same decisive group when ranking this structure with any other structure. All that matters then is that the monopoly profit is greater than the aggregate profit in any other structure. Nor does it appear when the maximally permitted degree of concentration is a duopoly, since all owners belong to the same decisive group also when two arbitrary duopoly structures are ranked. Intuitively, when only one decisive group is involved, as when monopoly or duopolies are allowed, owners have unlimited possibilities of transferring payments among themselves. When there is more than one decisive group, however, it is not possible to transfer payments between owners in different decisive groups. These different groups of owners may then disagree as to which ownership structure to prefer.

#### **4.2. Less concentrated structures**

We now turn to situations in which neither monopolies nor duopolies are permitted. In order to determine the equilibrium ownership and market structures, we need to impose more structure on the profits of the different firms, both in and outside merged entities. In the following two subsections we will to this end utilize two versions of the standard Cournot model that are commonly employed in the merger literature — one where firms differ in their fixed costs and mergers save on fixed costs, and one where firms have different constant marginal costs, and a merger enable firms to produce with the most efficient technology in control by the merging firms.<sup>16</sup>

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<sup>16</sup>Of course, there are several other oligopoly models employed in the literature, such as those in Deneckre and Davidson (1985), and Perry and Porter (1985). But, for the sake of brevity we limit the analysis to the two mentioned in the main text.

Before turning to these models, it should perhaps be stressed that we have by no means “assumed” that the EMS are the market structures that maximize industry profits. This can immediately be seen from the following numerical example with four owners. Let all firms make zero profits in all structures, except for in  $M^A = \{12, 3, 4\}$  where profits are  $\pi_{12}^A = 6$ , and  $\pi_3^A = \pi_4^A = 1$ , and in  $M^B = \{13, 2, 4\}$  where profits are  $\pi_{13}^B = 4$ ,  $\pi_2^B = 2.5$ , and  $\pi_4^B = 2.5$ . In this case  $M^A$  is the only EOS. To see this, note that both these structures dominate all other structures, and by asymmetry are not dominated by these structures. The only decisive group w.r.t.  $M^A$  and  $M^B$  is  $\{1, 2, 3\}$ , and since the combined profits of owners in this group is higher in  $M^A$  than in  $M^B$ , the former dominates the latter. However, *industry* profits is higher in the latter than in the former.

#### 4.2.1. Fixed cost savings

The most obvious reason for firms to merge is to save on fixed costs. This motive has been a main object of study in the traditional literature, and is also often claimed in actual merger cases. It is captured in its most basic form in the following model, which will be referred to as the Fixed Cost Model: an industry produces a homogenous product, there is Cournot competition, firms have identical variable cost technologies, but may differ in their fixed costs. By merging firms may save on fixed costs, but the variable cost technologies are unaffected by the merger. Owners are labeled such that if they were to run independent firms, the fixed costs in these firms would be  $f_1 < f_2 \dots < f_n$ .

To characterize equilibrium market structures in this model, we need to complement the model description with an assumption concerning the merger technology. The standard approach in the IO literature is to assume that a merged firm only needs to use one of the operations that give rise to fixed costs, and that it can choose the most efficient one:

- The fixed cost in a merged firm is the minimum of those of the merging firms. (A3)

This assumption has the convenient implication of reducing the number of dif-

ferent market structures that need to be taken into consideration. For instance, with three owners, a merger between owners 1 and 3 gives rise to the same market structure as a merger between owners 2 and 3. Of course, the flip side of the coin is that we cannot distinguish between these two ownership structures.

The following result is proved in the Appendix:

**Proposition 4.** *In the Fixed Cost Model, assuming (A1) holds, (i) the set of EOS is non-empty, and contains all structures that have the minimal industry costs among structures with  $\bar{k}$  firms; and (ii) if also (A3) holds, the set of EOS is non-empty, and in all EOS the assets of owners  $1, \dots, \bar{k}$  are employed in separate firms, and all ownership structures with this distribution of assets are EOS.*

The first part of the Proposition thus shows that regardless of the manner in which the fixed costs savings come about, the market structure with the maximally concentrated ownership structure belongs to the set of EOS. The second part establishes that if in addition the merger technology is of the common type specified in (A3), then the unique EMS is the structure in which industry fixed costs are minimized given  $\bar{k}$ , and where industry profits hence are maximized. The private efficiency of the “market merger mechanism” that was demonstrated above for the cases where monopolies and duopolies were permitted — the fact that the EMS maximized industry profits among all structures with a degree of concentration not exceeding  $\bar{k}$  — is hence a feature also of this standard Cournot model, regardless of the minimum permitted degree of concentration!

#### 4.2.2. Variable cost rationalizations

In the model to be considered next, firms differ in their constant variable (marginal) costs. The question of whom to merge with then gets an additional, strategic, dimension since prices and outputs may be differentially affected by different mergers. Both these models are of course very simplified descriptions of how mergers allow firms to combine assets. But we believe that the two models capture the two basic positive effects of a merger on firms’ cost functions. For models where

the merger of assets are explicitly modeled, see for instance Perry and Porter (1985), and Farrell and Shapiro (1990b).

The following model is denoted the Variable Cost Model: an industry produces a homogenous product, there is Cournot competition, there are no fixed costs, and firms have constant variable costs, which may differ across firms. Label owners such that if they were to run independent firms, their variable costs would be  $c_1 < c_2 < \dots < c_n$ .

Two further assumptions will be used. First, as in the previous model, the merger technology is specified. The simplest possibility is that firms can use the most efficient of the technologies to which they have access:

- The constant variable cost in a merged firm is the minimum of the constant variable costs of the merging firms. (A4)

The second assumption serves to dispatch of situations in which industry profits are higher when certain firms have very high marginal costs:

- The industry profit is maximal among all structures with  $\bar{k}$  firms when owners  $1, \dots, \bar{k}$  (co-)own different firms. (A5)

As will be shown below, this assumption can be seen as a restriction on the degree to which firms' costs may differ. Clearly, this property may well be fulfilled in specific cases, but it does not have any claim to general validity.

The following result is established in the Appendix:

**Proposition 5.** *In the Variable Cost Model, assuming that (A1), (A4) and (A5) hold, the set of EOS is non-empty, and contains all ownership structures with  $\bar{k}$  firms in which the assets of owners  $1, \dots, \bar{k}$  are employed in separate firms.*

The Proposition establishes together with Proposition 2 that the merger market mechanism will lead to the maximally permitted degree of concentration. Thus, the Variable Cost Model is in this respect identical to the Fixed Cost Model. However, it does not fully characterize the set of EOS, since it leaves open the possibility that other structures with  $\bar{k}$  firms than those that give rise to a market structure with costs  $c_1, \dots, c_{\bar{k}}$  belong to the set of EOS.

## 5. Private vs. social incentives for mergers

The typical question concerning welfare addressed in the IO literature is whether some exogenously chosen, purely concentrative, merger increases welfare. But, equipped with a theory of endogenous mergers it is natural to investigate the broader issue of the social desirability of the “merger market mechanism”. Not unexpectedly, it is hard to derive general conclusions. The difficulty stems from the fact that mergers in oligopoly models typically both enhance productive efficiency, and contribute to monopolization.

In order to disentangle these effects, consider first the following Conglomerate Fixed Cost Model: there are  $n$  owners who sell in  $n$  separate markets. The firms may have different variable costs, but these are unaffected by mergers. Firms differ in their fixed costs, and owners are labeled such that if the none of the owners were to merge, the fixed costs would be such that  $f_1 < f_2 \dots < f_n$ . We then have the following clear-cut result:

**Corollary 1.** *In the Conglomerate Fixed Cost Model, assuming (A1) and (A3) hold, the EOS are the structures that (i) minimize industry costs, and (ii) are socially preferred among all structures.*

**Proof.** Part (i) follows directly from Proposition 4 upon noting that variable costs are not affected by mergers, and that the present model in all other respects is a special case of the Fixed Cost Model, for any maximal permitted degree of concentration  $\bar{k}$ . To establish Part (ii), note that all structures have the same prices and variable costs by the assumption that the markets are separated. Thus, since the set of EOS minimizes industry cost by (i), it yields the socially preferred market structure. ■

The distinguishing feature of this situation is that mergers do not have any externalities on parties outside the industry. They are still associated with intra-industry externalities, since each merger excludes the possibility of some other mergers from taking place. However, the merger market mechanism internalizes these effects, and thus leads to an outcome that is privately efficient, and since there are no other externalities the merger market is also socially efficient. Now, it



may be argued that it is already well understood that such conglomerate mergers have positive welfare effects. But, note that what is shown is not that a particular conglomerate merger increases welfare, but that among *all possible* such mergers, those take place that yield the highest welfare.

Contrary to what intuition might suggest, the merger market mechanism need not lead to the creation of the most efficient firm, nor to the elimination of the most inefficient firm. This is easily seen in the case of three owners,  $i, j$  and  $k$ . Instead of specifying a particular merger technology, let a merger between owners  $i$  and  $j$  result in a firm with a fixed cost  $f_{ij}$ , etc..

**Corollary 2.** *In the Conglomerate Fixed Cost Model, with three owners, and assuming (A1) holds, the EOS involves a merger between owners  $i$  and  $j$  iff  $f_{ij} + f_k < \min(f_{ik} + f_j, f_{jk} + f_i)$ , with  $i \neq j \neq k \neq i$ .*

**Proof.** By Proposition 3 we know that structures in the EOS are the duopolies that give rise to maximal industry profits among all duopoly structures except monopoly. Since there are no differences between mergers in their effects on revenues and variable cost, the duopoly structure that maximizes industry profits is the structure that minimizes industry costs. ■

Hence, the equilibrium merger is *not* necessarily associated with the lowest fixed cost for the merging owners, i.e., it is *not* necessarily the most efficient firm that is created through the mergers. It may even be the one that is associated with the highest fixed cost among the three possible merged entities. Furthermore, there may be a merger between firms  $i$  and  $j$ , say, even if  $f_k$  is larger than  $f_i$  and  $f_j$ . Hence, the most inefficient firm is *not* necessarily eliminated. Nevertheless, the equilibrium merger is the one which is associated with the highest welfare.

Intuitively, the merger market mechanism takes into consideration the “opportunity cost” of mergers. For instance, in the case with three owners, the most efficient single-asset firm might be needed in order to create the most efficient two-asset firm. But, the “opportunity cost” of creating this firm may be higher than for creating the other firms, since this firm may forego more profits when merging than the others do. Since the merger process takes these aspects into

consideration, the most efficient firm is not necessarily created.

Private and social incentives are hence perfectly aligned in the case of conglomerate mergers. There is a degree of correspondence also in the case of the Fixed Cost Model considered above. Proposition 4 suggests that in this model the EOS are still privately efficient since they give rise to the market structure with the maximal aggregate profit. Moreover, the EOS are socially efficient in the sense that they give rise to the market structure with the highest welfare among all *equally concentrated* market structures, since all structures with a given degree of concentration yield the same consumer surplus. Of course, from a social point of view a less concentrated structure may be preferred. But, among the structures with the maximally permitted degree of concentration, the forces driving mergers actually lead to the socially preferred market structure. Thus, the policy maker would in this special case only need to care about the degree of concentration, and could leave to market forces to determine the market structure.

The picture is much more complicated when there are asymmetries between firms' variable costs, since now the differences in impacts of mergers on firms' revenues will matter for the outcome. The proof of the following result is brought in the Appendix:

**Corollary 3.** *In the Variable Cost Model, assuming that (A1) and (A4) hold, that outputs are strategic substitutes, and that all structures except a monopoly are permitted, the set of EOS is non-empty and consists of the duopoly structures where one firm faces the variable cost  $c_1$ , and the other firm has the variable cost  $c_2$  if  $c_n < c'$  and the cost  $c_n$  if  $c_n > c''$ , where  $c_1 < c' \leq c'' < c_n$ .*

The Corollary shows that the incentive to merge in order to rationalize production dominates the incentive to merge to limit competition for low values of the variable cost in the high cost firm, but that for sufficiently high variable cost in the high cost firm the *anti-competitive* incentive instead dominates.<sup>17</sup> Intuitively, when firms merge they may either end up in a structure with two efficient firms both producing with low variable cost but with relatively intense competition, or

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<sup>17</sup>It was this latter possibility that (A5) served to avoid in Proposition 5.

they may “buy” themselves less competition at the price of costly production in the high cost firm. When the higher cost becomes sufficiently high, the efficient firm approaches becoming a monopolist in the market, and thus the high cost firm spends very little on inefficient production.<sup>18</sup>

Let us end by a few remarks about policy. The general picture that emerges from the above analysis is that one should not in general expect the “merger market mechanism” to lead to socially efficient market structures. There thus seems to be a role for welfare-enhancing merger policies, since the EOS will in general be determined both by the cost saving motives highlighted in the Conglomerate Fixed Cost Model, as well as by the anti-competitive motives. But, the endogenous nature of mergers implies that the optimal design of merger policy is here substantially more involved than in the traditional case of exogenous mergers. For instance, when evaluating the optimal response to a suggested merger, it is important to take into account those market structures that might arise both as a result of accepting as well as of rejecting the proposed merger. In principle, the competition authority must forecast what further mergers that may be provoked as a result of accepting the proposed merger, and what mergers that are prevented from taking place. Similarly, it must also assess those other mergers that might be proposed if the current proposal is rejected.

The endogeneity of the merger formation also points to another complicating factor for the design of merger policy: the set of EOS may partly depend on dominance relations between structures *not* in the set of EOS. For instance, it may in general be that neither of structures  $M^j$  and  $M^i$  dominate each other, but that structure  $M^j$  is the only equilibrium structure absent policy, since  $M^i$  is dominated by a third structure  $M^k$ . A merger policy that does not permit  $M^k$  to potentially be formed, might then make  $M^i$  a possible equilibrium outcome.

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<sup>18</sup>Strictly speaking, in order to prevent a monopoly to be formed, the cost in the high cost firm must be sufficiently low in order for the firm to maintain a positive market share.

## 6. Applications

In this section we briefly demonstrate how the model developed above can throw light on other issues of policy concern. The purpose is not to derive general results, but rather to hint at aspects of these issues that have not been highlighted so far.

### 6.1. Research joint ventures

The social desirability of research joint ventures (RJV) has recently attracted interest in the literature. As in the case of mergers, the typical approach in the literature has been to study the impact of collaboration between exogenously chosen groups of firms.<sup>19</sup> But, the consequences of these agreements will obviously depend on the characteristics of the firms that collaborate, as well as of those not participating. It is therefore of interest to determine which of all possible RJVs that market forces will select. Note that an agreement to form a RJV is basically an agreement to form a coalition. Firms are free to communicate about RJVs, and they can commit to such arrangements through legally binding contracts, even though the firms retain considerable discretion over the efforts put into the collaboration. Furthermore, the surplus created through RJVs can be quite freely distributed, since firms have in practise significant freedom in deciding on both the reported total costs of RJV projects, as well as on how to share these costs. Assuming that firms can be taken to be unconstrained in the transfer of resources within RJVs, we can thus directly apply the approach developed above.

Formally, we employ a variant of the Variable Cost Model with three firms 1,2,3, who face a linear industry demand  $p = 1 - \sum x_i$ . Any constellation of these three firms, including all three of them, are allowed to form a costless RJV that with certainty results in a new process technology with associated variable cost  $\bar{c}$ , but the firms can also abstain from undertaking a RJV. The project cannot be undertaken by any of the firms alone (perhaps because neither firm has sufficient know-how to carry out the research alone), but each firm can only participate in one RJV. The new technology obtained through the RJV is more efficient than

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<sup>19</sup>One of the few exceptions are Bloch (1995), and Yi and Shin (1995).

the technologies employed by firms 2 and 3, but less so than the technology of firm 1. For simplicity, let the initial variable costs of firms 2 and 3 be  $c_2 = 1/6$ , and  $c_3 = 1/3$ , and let the new technology give rise to the variable cost  $\bar{c} = 1/9$ ; we thus assume that  $c_1 < 1/9$ . There are four possible RJVs — the three constellations where two firms participate, and the one where all firms take part. But, regardless of which RJV that is formed, firm 1 will continue to use its own technology since it is more efficient. Thus, if 1 and 2 undertake the RJV the costs of firms 1, 2 and 3 are, respectively,  $c_1$ ,  $1/9$  and  $1/3$ . The corresponding costs when firms 1 and 3 form a RJV are  $c_1$ ,  $1/6$  and  $1/9$ , and when firms 2 and 3 form a RJV they are  $c_1$ ,  $1/9$  and  $1/9$ , which are also the costs if all firms participate.

Will there be a joint venture, and if so, between which firms? On the one hand, firms 2 and 3 have the greatest incentive, in the sense that their variable costs would fall by the most with the new technology. But, as has been demonstrated above, this is only half of the story. Indeed, straightforward calculations yield the following (see the Appendix):

**Proposition 6.** *The equilibrium RJV is between firms 1 and 2 for all  $c_1 < \bar{c}$ .*

Hence, despite the fact that firm 1 has no interest in the new technology *per se*, and despite the fact that firm 3 would stand the most to gain from participating, the predicted RJV involves firms 1 and 2. Why? Since leaving firm 3 outside the agreement ensures that one of the firms is relatively uncompetitive, and this is conducive to industry profits.

We have for simplicity assumed that the RJV is costless. As a result, since the RJV lowers production costs, and thus intensifies competition, it must be socially desirable. However, three features should be pointed out. First, note that among the four possible firm constellations in the RJV, the market selects the RJV which results in the highest industry costs, and which is the *worst* RJV from a social point of view. Secondly, if there were fixed costs associated with the RJV, it would be possible that the RJV chosen by the market actually reduces welfare, while simultaneously some other RJV would increase welfare. Thirdly, the model predicts that there will *not* be a RJV involving all three firms — this

constellation has the disadvantage of destroying industry profits by spreading the low cost technology too widely, without bringing the usual advantage of a monopoly in terms of market power.

This simplistic depiction of RJV obviously abstracts from many of the aspects of RJVs that are considered central in the literature, such as the inherent uncertainty of R and D, the difficulties in appropriating its outcome, and the importance for welfare of the overall level of R and D that is undertaken in the industry. We have also abstracted from the possible interaction between incentives to form RJVs and incentives for mergers; this issue is to be dealt with in a companion paper. For instance, it is clear that RJVs, by changing the technology, might affect the profitability of consequent mergers. However, simple as the example is, it highlights a feature of RJV that should be of central importance — that among all possible RJVs, market forces might select those that are less desirable from a social point of view.

## 6.2. Market structure with capacity constrained firms

Throughout the analysis, firms have been assumed to be unconstrained in their capacity to produce any volume they like. However, there are many situations when this is not the case. For instance, firms may be constrained by production quotas imposed for environmental reasons, or they may face import or export quotas. Air lines have a limited number of landing slots, mining and oil firms have concessions on extraction of certain quantities, and broadcasters need licenses. Such output restrictions provide special incentives to merge, as can easily be seen.<sup>20</sup>

Let there again be three firms 1,2 and 3, facing a linear demand  $p = 1 - \sum x_i$ . Assume that the owners are not allowed to form a monopoly, but that duopolies are allowed (for example, there could be savings of fixed costs in the background). There are four possible market structures, a triopoly and the three different duopoly constellations. Each firm has a quota  $q_i$ , and the firms can

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<sup>20</sup> Depending on the particular application, firms may or may not be able to retain their quotas if they merge with other quota holders.

produce costlessly up to the quota, but it is not possible to produce more at any cost. Let  $q_3 < q_2 < q_1 < 1/4$ , this is a sufficient condition for the quotas to be small enough to bind in the triopoly equilibrium. Assume also that they are large enough so that a merged entity is not capacity constrained,  $q_i + q_j > 3/8$ . What is the equilibrium market structure? The following result is proven in the Appendix:

**Proposition 7.** *The equilibrium market structure involves a merger between the two firms with the largest quotas.*

The intuition behind the Proposition is simple: in the Cournot equilibrium, the merging firms reduce their combined output. The outside firm will thereby get an incentive to increase its volume, but can't do this because of the capacity constraint. The merging firms, on the other hand, will underutilize their quotas. The market is therefore most monopolized when the smallest quota is being used, and hence the owner of that quota will not merge.

A trivial, but possibly important, aspect of quantity constraints is that they tend to increase the profitability of mergers. In the example above this is illustrated by the fact that there are gains from mergers, even though there are no cost savings. This is a much more general phenomenon than what is suggested by this example, which follows immediately whenever the firms outside the merger cannot increase their outputs in response to the contraction of output that the merging firms undertake. Hence, one should expect incentives for mergers to be more pronounced in industries where capacity constraints are important, as long as these are not sufficiently restrictive to make the aggregate output volume less than the monopolist's optimal volume. Related, one should expect import quotas to increase incentives for domestic mergers.

### 6.3. Tariffs and foreign direct investment

It is often argued that high trade costs induce foreign firms to undertake direct investment in order to avoid the trade barriers, so called tariff-jumping FDI. This reasoning implicitly presumes that the assets bought by the foreign firms are in

infinitely elastic supply — if they were not, the FDI would affect the prospects of investments for competitors, and these effects would have to be taken into account in the investment decision. This elastic supply of assets may be descriptive of certain markets. However, in practise the majority of inward FDI in e.g. the U.S. does not take the form of purchases of infinitely elastically supplied assets, but of mergers and acquisitions of firms in the same industry. For instance, mergers and acquisitions accounted for over 91% of overall foreign direct investment in the U.S. in 1994, and averaged over 81% during 1989-1994.<sup>21</sup> As we shall see, the mode of investing in the foreign country might make a significant difference to the plausibility of the argument that high trade-barriers induce FDI.

We will employ a version of the Variable Cost Model, where there are three firms serving a market with linear demand  $p = 1 - \sum x_i$ . Firms 1 and 2 are located in the market, and the foreign firm 3 is located abroad. There are no costs of production, but firm 3 faces a per unit trade cost  $t$  as a result of its location and/or its nationality. But by merging with one of the local firms, firm 3 can avoid the trade cost. The most direct interpretation of this cost is that of a tariff. But, in this linear Cournot model the trade cost can also be interpreted as a transport cost, since after a merger with a local firm all production could take place through the local firm.

Again we allow for duopolies to be formed, but rule out monopoly. There are thus two type of mergers that could be formed. One is a domestic merger between firms 1 and 2. The other is an international merger, which could be interpreted as a foreign direct investment. There are two identical mergers of this type, one between firms 1 and 3, and the other involving firms 2 and 3. Note that while the model incorporates the standard type of reason for forming an international firm — to lower costs of serving the market (albeit here for just one of the firms) — there is no conventional reason for forming the domestic firm. The following result is established in the Appendix:

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<sup>21</sup> Figures compiled by JETRO (1996). In certain sectors, such as autos, new investments have nevertheless been important.



**Proposition 8.** *The EOS gives rise to the triopoly for  $0 < t < 1/15$ , an international merger for  $1/15 < t < 1/3$ , either the triopoly or an international merger for  $1/3 < t < 2/5$ , the triopoly for  $2/5 < t < \sqrt{2} - 1$ , and the domestic merger for  $\sqrt{2} - 1 < t \leq 1/2$ .*

That is, for very low levels of the trade cost there will not be any merger for standard reasons. For somewhat higher levels (but lower than  $1/3$ ) the traditional gain from “tariff-jumping” dominates other effects, and the trade barrier thus induces an international merger, as the conventional argument suggests. In the range  $1/3 < t < 1/2$ , the foreign firm is choked off from the market in the triopoly, but maintains a positive market share in the case where the domestic firms merge. The domestic merger brings smaller aggregate profits compared to the international merger (which induces savings of trade costs) for  $t < 2/5$ , and this merger is furthermore worse than the triopoly for firms 1 and 2 for  $t < \sqrt{2} - 1$  (i.e., condition (A1) is not fulfilled). However, *when the trade barrier is sufficiently large, there will be a domestic merger*, contrary to the standard reasoning! This standard argument neglects the fact that the level of the trade barrier affects not only the incentives to form an international firm, but also those to merge domestically. When the barrier is sufficiently high, the gains from a domestic merger is even larger than those from an international merger, due to the high degree of monopolization the former brings. This result, which is far more general than what the present example seems to suggest, thus points to the possibility that the assumption of an infinitely elastic supply of assets in the foreign market, an assumption which is implicitly maintained in the whole foreign direct investment literature, may actually be quite restrictive.<sup>22</sup>

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<sup>22</sup>In a companion paper we employ the approach developed here, to examine determinants and welfare effects of mergers in an international market in the context of more general oligopoly models.

## 7. Concluding remarks

Virtually the whole industrial organization literature on mergers considers the incentives for, and the consequences of, mergers between an exogenously chosen group of firms. We have in this paper tried to take the analysis one step further by endogenously determining the identity of the firms that will merge. The purpose has not been to provide a theory describing the dynamic process of mergers. The creation of such a theory is indeed desirable, as long as it does not rely on arbitrary timing assumption, etc., and as long as it yields economic insights into the merger process. Instead, the paper has proposed a criterion for the stability of a market structure to mergers. The approach extends the basic idea in the merger literature — what we refer to as the traditional criterion for mergers — and it adapts concepts from cooperative game theory in order to capture the distinguishing feature of oligopolistic markets — the existence of externalities between firms. The general picture that emerges when applying the model to standard concentrated Cournot markets, is that market forces tend to be conducive to market structures with large industry profits, as opposed to large profits of specific groups of firms, as long as the standard criterion for concentrative mergers is fulfilled for mergers to very concentrated structures.

Taken literally, the analysis suggests that the only reason why industries are not completely monopolized is that it is either unprofitable, or not allowed by anti-trust authorities. It is obviously very hard to determine how far these explanations might go. But, even if they cannot fully explain the fact that industries are not monopolized, we think that they are more important than seems to be commonly believed. With regard to the profitability of monopolization, it is clear that monopolization is likely to bring gains from coordination of decisions in output markets in the short to medium run. However, monopolization is also likely to have less advantageous consequences in the longer run for production costs, and for e.g. innovativeness. Hence, it can not be taken for granted that industry profits are larger with monopolization than with less concentrated structures.

Concerning the role of anti-trust, it is interesting to note Stigler's (1950) view

that when mergers become legal, and efficient capital markets emerged at the end of the 19th century, mergers to create close to monopolies occurred in many industries in the U.S. This view is also supported by Scherer and Ross (1990), who write about the Great Merger Wave of 1887-1904, that “[i]ts outstanding characteristic was the simultaneous consolidation of numerous producers into enterprises dominating the market they supplied.” Stigler argues that the main explanation of why mergers for monopoly became much more uncommon in the U.S. in the beginning of the 20th century was due to antitrust law: “The era of merger for monopoly ended in this country roughly in 1904, when the *Northern Securities* decision made it clear that this avenue to monopoly was also closed by the antitrust laws.” Stigler continues “The Sherman Law seems to have been the fundamental cause for the shift from merger for monopoly to merger for oligopoly.” In conclusion, as long as merger for monopoly or a dominant positions were allowed in concentrated structures in the U.S., they took place frequently and at fast pace.

A distinguishing feature of the present model, when compared to other models of endogenous mergers, is that this model can handle asymmetries between firms, and thus predict the mergers that are more likely to emerge in situations where firms are asymmetric. The model was employed to issues concerning research joint ventures, mergers when firms are capacity constrained, and tariff-jumping foreign direct investment. In all three cases the analysis pointed to conflicts between consumer interests and the functioning of the merger market mechanism — coalitions were formed between the wrong firms from the point of view of consumers. The reasons for this were the fact that merger formation in these cases were largely driven by preemptive motivations: a firm’s alternative to merging need not be to remain in the initial position, but to stand outside some other merger. A merger can thus partly be driven by a desire to avoid negative repercussions that the merging parties would experience if other mergers were not prevented.

Finally, the model should also be applicable to other instances of coalition formation than mergers, such as the formation of customs unions, or monetary unions.

## Appendix

### Proof of Proposition 3

Any structure  $M$  with more than two firms is by Proposition 2 dominated by some duopoly structure which is a pure concentration of  $M$ . Thus, only duopolies are candidates for EOS.

Let  $\mathcal{M}^2$  be the set of ownership structures that yield maximum profit among duopoly structures. When making a dominance ranking between two structures that are both in  $\mathcal{M}^2$ , the decisive group contains all owners. Since aggregate profits are the same, neither structure dominate the other. When making a dominance ranking between  $M^j \in \mathcal{M}^2$  and some duopoly structure  $M^i \notin \mathcal{M}^2$ , the decisive group will again contain all owners, and thus  $M^j \text{ dom } M^i$  since  $\pi^j > \pi^i$  by definition. Hence, any structure not in  $\mathcal{M}^2$  is dominated by structures in  $\mathcal{M}^2$ , and by the asymmetry of the dominance relation, no structure in  $\mathcal{M}^2$  is dominated by any duopoly structure not in  $\mathcal{M}^2$ . Structures in  $\mathcal{M}^2$  are therefore undominated with respect to all duopoly structures.

It remains to show that  $M^j$  is undominated w.r.t. structures comprising more than two firms. Note that when a dominance ranking involves a duopoly, there will be either one or two decisive groups. If there is one decisive group, it may or may not involve all owners, and if there are two decisive groups, they must together comprise all owners. Let  $M^i$  be some arbitrary structure with more than two firms, and let  $M^j$  be an arbitrary structure in  $\mathcal{M}^2$ .

(a) With one decisive group comprising all players, (A1) and (A2) ensure that a strict concentration of  $M^i$  to some duopoly  $M^{i'}$  increases industry profits. But, since structures in  $\mathcal{M}^2$  have the highest industry profits among the duopolies,  $\pi^j \geq \pi^{i'} > \pi^i$ . Hence,  $M^j \text{ dom } M^i$ . By the asymmetry of the *dom* relation,  $M^j$  is undominated w.r.t.  $M^i$ .

(b) With one decisive group comprising less than all players, this group must in  $M^j$  be a pure concentration of the firms in the group in  $M^i$ . Hence, since the profit of the decisive group by assumption (A1) is higher in  $M^j$  than in  $M^i$ ,  $M^j \text{ dom } M^i$ , and by (A2),  $\pi^j > \pi^i$ . It follows from the asymmetry of the *dom* relation

that  $M^j$  is undominated w.r.t.  $M^i$ .

(c) With two separate decisive groups, (A1) and (A2) ensure that a strict concentration of  $M^i$  to some duopoly  $M^{i'}$  increases industry profits. But, since structures in  $\mathcal{M}^2$  have the highest industry profits among the duopolies,  $\pi^j \geq \pi^{i'} > \pi^i$ . Hence, profits are higher for at least one of the decisive groups, and consequently  $M^j$  is not dominated by  $M^i$ .

Hence, the set of EOS is identical to the set  $\mathcal{M}^2$ , and these structures are associated with the maximal industry profits among all structures except monopoly. ■

#### Proof of Proposition 4

*Part (i).* Consider first rankings where both structures have  $\bar{k}$  firms. Let  $\mathcal{M}^{\bar{k}}$  be the set of ownership structures that yield maximal industry profits among structures with  $\bar{k}$  firms. We first show that structures in  $\mathcal{M}^{\bar{k}}$  are undominated w.r.t. all other structures with  $\bar{k}$  firms. When making a dominance ranking between some  $M^j \in \mathcal{M}^{\bar{k}}$ , and some  $M^i \notin \mathcal{M}^{\bar{k}}$  with  $\bar{k}$  firms, let  $\pi_g^j$  and  $\pi_g^i$  be, in the respective structure, the combined profits of the firms in a typical decisive group  $g$  w.r.t. these structures. Then, since  $\pi^j > \pi^i$  by definition,

$$\sum_g (\pi_g^j - \pi_g^i) + \sum_h (\pi_h^j - \pi_h^i) > 0 \quad (7.1)$$

where the first summation is over all decisive groups w.r.t.  $M^j$  and  $M^i$ , and the second summation is over all firms  $h$  whose owners are in  $\mathcal{U}^{ij} \equiv \mathcal{N} \setminus \cup_g \mathcal{D}_g^{ij}$ , which is the set of owners who are *not* decisive with respect to  $M^i$  and  $M^j$ . But, since there is an equal number of firms in the two structures, and the firms whose owners are not decisive make the same profits in the two structures, the latter sum is zero. There must then be at least some decisive group for which the profit is larger in structure  $M^j$  than in  $M^i$ , and hence the former is undominated w.r.t. the latter. This reasoning obviously holds independently of the number of decisive groups, and also in the case where  $\mathcal{U}^{ij} = \emptyset$ .

Next turn to rankings between some  $M^j \in \mathcal{M}^{\bar{k}}$  and some structure  $M^i$  with  $k^i > \bar{k}$  firms. Then, for any pure concentration of  $M^i$  to a structure  $M^{i'}$  with  $\bar{k}$  firms,  $\pi^{i'} > \pi^i$ , by (A1) and by the positive externalities from mergers that is

an inherent feature of this type of Cournot model. Therefore, since  $\pi^j \geq \pi^i$  by definition, we have that  $\pi^j > \pi^i$ , and expression (7.1) then applies again. Since  $M^{i'}$  is a pure concentration of  $M^i$ ,  $\mathcal{U}^{ij} = \mathcal{U}^{ii'}$ , which may but need not be empty, the last term is zero. There must consequently be some decisive group with a larger profit in  $M^j$  than in  $M^i$ , and  $M^j$  is thus undominated w.r.t.  $M^i$ .

Hence, since structures in  $\mathcal{M}^{\bar{k}}$  are neither dominated by other equally concentrated structures, or by less concentrated structures, they are elements of the set of EOS. Furthermore, since revenues net variable costs are the same in all structures with  $\bar{k}$  firms, structures in  $\mathcal{M}^{\bar{k}}$  must have lower industry fixed costs than other structures with  $\bar{k}$  firms.

*Part (ii).* By (A3)  $\mathcal{M}^{\bar{k}}$  is now the set of ownership structures with  $\bar{k}$  firms in which the assets of owners  $1, \dots, \bar{k}$  are employed in different firms. Note, first, that by Proposition 2, the only candidates for EOS are structures with  $\bar{k}$  number of firms.

Secondly, consider an arbitrary structure  $M^i$  with  $\bar{k}$  firms that is not in  $\mathcal{M}^{\bar{k}}$ . This structure is always dominated by *some* other structure with  $\bar{k}$  firms. To show this, some further notation is needed. With a slight abuse of notation, let  $\{1, \dots, \bar{k}\}$  denote the set of owners  $1, \dots, \bar{k}$ , and let  $\mathcal{S}^i \subset \{1, \dots, \bar{k}\}$  denote those among these owners who (co-)own firms in structure  $M^i$  that do *not* have any other owner from  $\{1, \dots, \bar{k}\}$ . Then  $\{1, \dots, \bar{k}\} \setminus \mathcal{S}^i$  are those owners who co-own firms with owner from  $\{1, \dots, \bar{k}\}$ . Let the owner with the lowest number in  $\{1, \dots, \bar{k}\} \setminus \mathcal{S}^i$  be denoted  $k'_1$ , and denote the firm which this owner co-owns as  $K'$ . Let  $k'_2$  be the owner in  $\{1, \dots, \bar{k}\}$  with the second lowest ranking in  $K'$ . Finally, let  $K'' \in M^i$  be a firm whose owner(s) do not belong to  $\{1, \dots, \bar{k}\}$  — such a firm must exist — and let  $k''_1$  be the owner with the lowest number in this possibly single-owner firm. Now consider a structure  $M^j$  which is derived from  $M^i$  by letting owner  $k'_1$  and owners of  $K''$  merge. The decisive group will then be owners in the set  $\{\mathcal{O}(K') \cup \mathcal{O}(K'')\}$ . The total fixed costs of this group is by (A3)  $f_{k'_1} + f_{k'_2}$  in  $M^j$ , and  $f_{k'_1} + f_{k''_1}$  in  $M^i$ . The latter cost exceeds the former, and  $M^i$  is hence dominated by  $M^j$ . The argument above can be applied to any structure  $M^i \notin \mathcal{M}^{\bar{k}}$ , and hence all these

structures are dominated.<sup>23</sup>

Hence, all structures that are not in  $\mathcal{M}^{\bar{k}}$  are dominated. Since it was shown in Part (i) for an arbitrary merger technology fulfilling (A1), that structures not in  $\mathcal{M}^{\bar{k}}$  could not dominate structures in  $\mathcal{M}^{\bar{k}}$ , this must hold also with the merger technology specified in (A3), as long as it fulfills (A1). Hence, the set of EOS is equal to the set  $\mathcal{M}^{\bar{k}}$ . ■

### Proof of Proposition 5

A parallel argument to that in Part (i) of the proof of Proposition 4 can be made, with one modification. First, in a ranking of two structures with an equal number of firms, the firms who's owners are not decisive face competitors with the same, and in some cases lower marginal costs. They must therefore make lower profit in  $M^i$ . Consequently, the second sum is (7.1) negative. It follows *a fortiori* that there must be at least some decisive group for which the profit is larger in structure  $M^j$  than in  $M^i$ . ■

### Proof of Corollary 3

Cournot competition and (A4) imply that there are positive externalities from a concentrative merger. Then by (A1) Proposition 3 is applicable. We know that the set of EOS consists of those duopoly structures that give rise to the maximal industry profit.

The profit of firm  $i = l, h$  is  $\pi_i = P(q_l + q_h)q_i - c_i \cdot q_i$ , where  $c_h \geq c_l$ . The FOCs imply  $q_l > q_h$  for  $c_h > c_l$ . Totally differentiating the FOCs,

$$\frac{\partial q_i}{\partial c_i} = \frac{1}{D}(2P' + q_j P'') \quad \frac{\partial q_i}{\partial c_j} = -\frac{1}{D}(P' + q_i P''); \quad i \neq j = h, l$$

where

$$D \equiv (P')^2 + (P' + q_l P'')P' + (P' + q_h P'')P' > 0$$

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<sup>23</sup>The following example may clarify the reasoning: Let  $M^i = \{1, 23, 4, 5\}$  and let  $\bar{k} = 4$ . Here  $S^i = \{1, 4, 5\}$ ,  $K' = \{23\}$ ,  $k'_1 = 2$ ,  $k'_2 = 3$ ,  $K'' = \{5\}$  and  $k''_1 = 5$ . Structure  $M^i$  is then dominated by  $M^j = \{1, 25, 3, 4\}$ , since for the decisive group w.r.t.  $M^i$  and  $M^j$  is  $\{2, 3, 5\}$ , we have that  $f_{k'_1} + f_{k'_2} = f_2 + f_3 < f_2 + f_5 = f_{k''_1} + f_{k''_1}$ .

since outputs are strategic substitutes. The aggregate equilibrium profit is

$$\pi(c_l, c_h) = P(q_l + q_h)(q_l + q_h) - c_l q_l - c_h q_h; \quad q_i = q_i(c_l, c_h)$$

(i)  $c_l = c_1$ , since aggregate profits is higher the lower is  $c_l$  :

$$\frac{\partial \pi(c_l, c_h)}{\partial c_l} = \frac{P'}{D} [(q_h - q_l)P' + (q_h - 2q_l)(P' + q_h P'') - q_l(P' + q_l P'')] < 0$$

(ii) At  $c_h = c_l$ ,  $q_l = q_h \equiv q$ :

$$\frac{\partial \pi(c_l, c_l)}{\partial c_h} = -\frac{2qP'}{D} [P' + qP''] < 0$$

There will thus be a range  $(c_l, c')$  in the neighborhood of  $c_h = c_l$ , in which  $\pi(c_l, c_h) < \pi(c_l, c_l)$ .

(iii) Let  $\bar{c}$  be such that  $q_h = 0$  for  $c_h \geq \bar{c}$ . Since, the monopolist's profit with cost  $c_l$  exceeds that of a duopoly with the same cost  $c_l$  in both firms, there must be a range of values of  $(c', \bar{c}(c_l))$  such that  $\pi(c_l, c_h) > \pi(c_l, c_l)$ . ■

### Proof of Proposition 6

Let  $x_i$  be the output of firm  $i$ , and let  $\pi_i$  be its profit. With linear demands and costs the equilibrium output and profit of firm  $i$  are:

$$x_i = \frac{1}{4}(1 - 3c_i + c_j + c_k) \quad \pi_i = \frac{1}{16}(1 - 3c_i + c_j + c_k)^2; \quad i \neq j \neq k$$

There are five structures to consider:  $M^O = \{1, 2, 3\}$ ,  $M^A = \{12, 3\}$ ,  $M^B = \{13, 2\}$ ,  $M^C = \{1, 23\}$ , and  $M^M = \{123\}$ .

(i) In structure  $M^O$  the profits of firms 1 and 2 are, respectively,

$$\pi_1^O = \frac{9}{64}(1 - 2c_1)^2 \quad \pi_2^O = \frac{1}{576}(5 + 6c_1)^2$$

(ii) The equilibrium outputs are in  $M^A$  :

$$x_1^A = \frac{1}{36}(13 - 27c_1), \quad x_2^A = \frac{1}{4}(1 + c_1), \quad x_3^A = \frac{1}{36}(1 + 9c_1)$$

The resulting profits of firms 1 and 2, and the industry profit, are

$$\pi_1^A = \frac{1}{1296}(13 - 27c_1)^2 \quad \pi_2^A = \frac{1}{16}(1 + c_1)^2 \quad \pi^A = \frac{1}{1296}(251 - 522c_1 + 891c_1^2)$$



(iii) The equilibrium outputs and the industry profit are in  $M^B$ :

$$x_1^B = \frac{1}{72}(23 - 54c_1), \quad x_2^B = \frac{1}{72}(11 + 18c_1), \quad x_3^B = \frac{1}{24}(5 + 6c_1)$$

$$\pi^B = \frac{1}{5184}(875 - 1548c_1 + 3564c_1^2)$$

(iv) The equilibrium outputs and the industry profit are in  $M^C$ :

$$x_1^C = \frac{1}{36}(11 - 27c_1), \quad x_2^C = \frac{1}{36}(7 + 9c_1), \quad x_3^C = \frac{1}{36}(7 + 9c_1)$$

$$\pi^C = \frac{1}{432}(73 - 114c_1 + 297c_1^2)$$

(v) When all three firms participate, the aggregate profit is the same as in structure  $M^C$ .

Note that for outputs to be positive we must have that  $c_1 < 11/27$ . But, since  $c_1 < 1/9$  by assumption, this inequality is always fulfilled. It can now straightforwardly be seen that  $M^A$  dominates all the other structures, since

$$\pi^A - \pi^B = \frac{1}{1728}(43 - 180c_1) > 0$$

$$\pi^A - \pi^C = \frac{1}{324}(8 - 45c_1) > 0$$

$$\pi_1^A + \pi_2^A - \pi_1^O - \pi_2^O = \frac{1}{2592}(23 + 108c_1) > 0$$

$$\pi^A - \pi^M = \pi^A - \pi^C > 0$$

Hence, the equilibrium RJV involves firms 1 and 2. ■

### Proof of Proposition 7

Let  $M^O = \{1, 2, 3\}$ ,  $M^A = \{1, 2, 3\}$ ,  $M^B = \{1, 3, 2\}$ ,  $M^C = \{1, 2, 3\}$ . The best-reply function of firm  $i$  in triopoly is  $x_i = \frac{1}{2}(1 - x_j - x_k)$ . Since the maximum quota is  $1/4$ , the minimum best reply is  $1/4$  for each firm in the triopoly. Hence, all quotas bind, and the equilibrium profits of firm  $i$  are  $\pi_i^O = (1 - q_i - q_j - q_k)q_i$ .

Consider next duopolies. The best reply of a duopolist  $i$  against another firm  $j$  is  $x_i = \frac{1}{2}(1 - x_j)$ . In the duopoly structure  $M^A$ , if firm 3 uses its quota fully, it still produces less than  $1/4$ , and hence the best reply for firm 1,2 is less than  $3/8$ , which in turn is less than  $q_1 + q_2$  by assumption. Firm 3's best reply against an output

of firm 12 of less than  $3/8$  is larger than  $5/16$ , which is larger than its quota. Hence, by the linearity of the model, the unique Nash equilibrium is  $x_{12}^A = 3/8$ , and  $x_3^A = 1/4$ . The resulting industry profit is  $\frac{1}{4}(1 - q_3^2)$ . A symmetric reasoning to the above shows that in the two other duopoly structures, the quota of the outside firm will also bind. It follows that industry profits in these structures can be obtained by replacing  $q_3$  with  $q_1$  and  $q_2$  in the above expression. The duopoly structure with maximal profit thus arises when the outside firm has the smallest quota.

It remains to verify that  $M^A$  dominates  $M^O$ . This follows immediately from the facts that the combined profits of firms 1 and 2 in the triopoly is  $(1 - (q_1 + q_2) - q_3)(q_1 + q_2)$ , while the profit of firm 12 in structure  $M^A$  is  $(1 - x_{12} - q_3)x_{12}$ . These expressions are identical except for that  $x_{12}$  replaces  $q_1 + q_2$  in the second expression. Hence, since  $x_{12}^A$  maximizes this expression, and since  $q_1 + q_2 > x_{12}^A$ , the profit is larger in the duopoly structure for the two firms, and hence  $M^A$  dominates  $M^O$ . ■

### Proof of Proposition 8

Let  $M^O = \{1, 2, 3\}$ ,  $M^A = \{12, 3\}$ ,  $M^B = \{13, 2\}$ , and  $M^C = \{1, 23\}$ . The triopoly quantities and profits are, for  $t < 1/3$ ,

$$x_1^O = x_2^O = \frac{1}{4}(1 + t) \quad x_3^O = \frac{1}{4}(1 - 3t)$$

$$\pi_1^O = \pi_2^O = \frac{1}{16}(1 + t)^2 \quad \pi_3^O = \frac{1}{16}(1 - 3t)^2$$

and  $x_1^O = x_2^O = 1/3$ ,  $x_3^O = 0$ ,  $\pi_1^O = \pi_2^O = 1/9$ , and  $\pi_3^O = 0$ , for  $1/3 < t < 1/2$ .

In the case of a merger between the domestic firms 1 and 2,

$$x_{12}^A = \frac{1}{9}(1 + t) \quad x_3^A = \frac{1}{9}(1 - 2t)$$

$$\pi_{12}^A = \frac{1}{9}(1 + t)^2 \quad \pi_3^A = \frac{1}{9}(1 - 2t)^2$$

and with an international merger  $x_{13}^B = x_2^B = x_1^C = x_{23}^C = 1/3$ , and  $\pi_{13}^B = \pi_2^B = \pi_1^C = \pi_{23}^C = 1/9$ .

There are three types of dominance rankings that are of interest, since structures  $M^B$  and  $M^C$  are completely symmetric:

(i) The difference in aggregate profits in the two type of duopolies is  $\pi^A - \pi^B = t(5t - 2)/9$ . Hence, for  $0 < t < 2/5$ ,  $M^B \text{ dom } M^A$ , and vice versa for  $2/5 < t < 1/2$ .

(ii) In the range  $t < 1/3$ ,  $M^O \text{ dom } M^B$  iff  $1 - 18t + 45t^2 < 0$ , i.e., for  $0 < t < 1/3$ , while  $M^B \text{ dom } M^A$  for  $1/15 < t < 1/3$ . For  $1/3 < t < 1/2$  both structures give the same profit for the decisive group, and hence neither dominates the other.

(iii) In the range  $t < 1/3$ ,  $M^A \text{ dom } M^O$  iff  $t < 2/7$ , while the opposite holds for  $2/7 < t < 1/3$ . In the range  $1/3 < t < 1/2$ ,  $M^O \text{ dom } M^A$  iff  $1 - 2t - t^2 > 0$ , i.e., for  $1/3 < t < \sqrt{2} - 1$ , while  $M^A \text{ dom } M^O$  for  $\sqrt{2} - 1 < t < 1/2$ .

The above implies that  $M^O$  is undominated for  $0 < t < 1/15$  and for  $1/3 < t < \sqrt{2} - 1$ ,  $M^B$  is undominated for  $1/15 < t < 2/5$ , while  $M^A$  is undominated for  $\sqrt{2} - 1 < t < 1/2$ . ■

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