A Trickle-Down Theory of Incentives with Applications to Privatization and Outsourcing

Fredrik Andersson
A trickle-down theory of incentives with applications to privatization and outsourcing

Fredrik Andersson *
†

December 2008

Abstract

The make-or-buy decision is analyzed in a three-layer principal-management-agent model. There is a cost-saving/quality tradeoff in effort provision. The principal chooses between employing an in-house management and contracting with an independent management; the cost-saving incentives facing the management are, endogenously, weaker in the former case. Cost-saving incentives trickle down to the agent, affecting the cost-saving/quality trade-off. It is shown that weak cost-saving incentives to the management promote quality provision by the agent, and that a more severe quality-control problem between the principal and the management, as well as a higher valuation of quality, make an in-house management more attractive.

Under revision

JEL Classification: D23, L22, L24

Keywords: make-or-buy decision, multitask principal-agent problem, outsourcing

1 Introduction

The make-or-buy decision has intrigued economists for generations. In somewhat different disguises it has been scrutinized by a large number of scholars.1 The different disguises stem from

---

*Correspondence to: Fredrik Andersson, Department of Economics, Lund University, P.O. Box 7082, S-220 07 LUND, SWEDEN. Phone +46-46-222 86 76, fax +46-46-222 46 13, email: Fredrik.Andersson@nek.lu.se.

†I am grateful for comments and suggestions from seminar audiences at University of Southampton and Uppsala University. Financial support from the Bank of Sweden Tercentenary Foundation and from the Jan Wallander and Tom Hedelius Foundation is gratefully acknowledged.

different questions being asked. Some work focuses on the fundamental – but probably somewhat quaint to non-economists – question about the nature of the firm and the forces determining its boundaries; other work is more focused on hands-on trade-offs concerning vertical integration. While this paper falls in the second category by focusing on the choice between in-house production and outsourcing of activities, the most prominent distinguishing feature is the focus on measurement-related determinants of this choice in an (almost) comprehensive-contracting context. In addition, the choice between public and privatized management of “public-sector activities” is an important source of motivation and application of the model presented.

Valuable insights about the make-or-buy decision have been gained within the property-rights approach. The main insight is that in the presence of contractual incompleteness, activities involving two parties for which specific assets are important are more likely to be integrated by one of the parties; the reason is that the owner of the specific asset may be subject to “hold-up” by the other party.2 The hold-up problem, in turn, undermines incentives to invest in specific assets.3

While the property-rights approach is conceptually convincing, it is clearly only part of the story. An additional set of properties that are relevant is the measurement and contractibility characteristics of the activity subject to the make-or-buy decision. There is, moreover, empirical work indicating that measurement aspects are important for explaining the make-or-buy decision: In their work on in-house versus independent sales forces, Anderson and Schmittlein (1984) and Anderson (1985) found measurement-related explanatory variables to stand out most strongly. Holmström and Milgrom (1991 and 1994) and Holmström (1999) have brought these observations to bear in theoretical analyses of the make-or-buy decision.4

In this paper, we employ the measurement approach in trying to answer a number of specific questions relevant when the make-or-buy decision is encountered in practice. More specifically, we consider a three-layer hierarchy with a principal, a management, and an agent. The principal – which may be, for example, the top executives of a firm or an elected body – delegates a task

---

2 A party is subject to hold-up if another party threatens to withdraw from trade – in which case the specific asset would be inefficiently utilized – in order to appropriate all, or a large portion, of the surplus.

3 The seminal contributions are Grossman and Hart (1986) and Hart and Moore (1990); see Hart (1995) and Holmström (1999) for clear and simple accounts of the basic logic.

4 In Holmström and Milgrom (1991) it is argued that strong monetary incentives and rules governing how a task is accomplished are substitutes in structuring appropriate over-all incentives. In Holmström and Milgrom (1994), the complementarities among a set of instruments for affecting performance in a given task, and the implications of such complementarities for empirical work, are analyzed. Holmström (1999) explores how the power to structure incentives – a key trait of the firm – may or may not be determined by asset ownership.
to a management – which may be the middle management of a firm, or a subcontractor; the management, in turn, delegates the actual execution of the task to an agent. The success in the undertaking of the task has a cost-saving and a quality dimension, and the agent can allocate his effort between these dimensions.

The distinction between “make” and “buy” emanates from an agency problem in the relationship between the principal and the management; in the presence of this agency problem it matters whether cost savings accrue to the principal or directly to the management. The accrual of cost savings is the assumed underlying difference between make and buy, and this distinction derives from cost savings being tied to an asset whose disposition is subject to incomplete-contracting limitations. The management is assumed to exert some control over the cost-saving measure, and this distorts cost-saving incentives if cost savings accrue to the principal, but not if cost savings accrue directly to the management. In this framework, the principal provides weaker direct cost-saving incentives when interacting with an in-house management. While we devote parts of Section 2 to justifying the distinction between make and buy along these lines, the intended main contribution is the analysis of the make-or-buy decision given this distinction. The core of the analysis deals with how equilibrium contracts and the equilibrium choice between make and buy depend on the incentive problem faced by the management in rewarding the agent, and by the principal in rewarding the management; in particular, the possibilities for rewarding quality. The main results are that:

- the strength of incentives is subject to trickling down: when the management faces weak incentives, the incentives provided to the agent by the management will be weak as well;
- there is trickling down in effort allocation too: weakening cost-saving incentives for the management will, under plausible circumstances, lead to more care being devoted to quality “on the ground”;
- the more severe the incentive problem between the principal and the management as regards quality measurement, the more likely is the principal to opt for an in-house management;
- the higher the value attributed to quality by the principal, the more likely is the principal to opt for an in-house management.

While several of these results are quite intuitive, they are, arguably, generated in an empirically plausible vertical structure which, importantly, accounts for the “make” and “buy” cases.
Moreover, the results have empirical implications that square well with trade-offs manifest in the context of outsourcing of government activities (see Section 5).

The related work by Acemoglu, Kremer and Mian (2004) builds on the core idea that market incentives sometimes induce too much “signaling effort,” i.e. effort to inflate others’ assessment of performance without promoting performance per se; they mention schooling and delegated asset management as examples where this may be a significant problem. Their analysis is devoted to analyzing why incentives are, in general, weaker in firms and, even more so, in governments, than in markets. They consider a career-concerns model with a “good” and a “bad” component of effort; after showing that market incentives may be excessively strong, they argue that firms can remedy this by creating, by design, a moral-hazard-in-teams problem; they also argue that competition between firms allows remnants of market incentives to trickle down to employees, and that this effect can be avoided by governments. Their paper is thus related to this one in terms of the distinguishing characteristic of firms and governments compared to markets; while they focus on the foundation for this difference, our focus is on the implications for associating activities with modes of organization. Moreover, they work in a “contract-free” environment, and hence do not address questions about the properties of actual incentive contracts.

A related issue that has received some attention is the question how competition affects optimal managerial incentives. Levin and Tadelis (2005) also study the make-or-buy decision by devising a simple theoretical model – driven by contract-administration costs – for generating and testing prediction from contracting by US cities. There are also a number of papers that employ principal-agent models with multiple-level hierarchies, most of them, however, focusing on collusion between the agent and the next level in the hierarchy in concealing information from the principal; Tirole (1986) is a seminal contribution.

A common criticism of the Williamsonian specific-asset story is the lack of a clear account for how and why the hold-up problem is attenuated under vertical integration.

Schmidt (1997) demonstrates the existence of two countervailing effects, providing reasonable conditions for competition to strengthen managerial incentives; Raith (2003) demonstrates that a positive correlation between competition and incentives is likely to arise in cross-section data when there is underlying heterogeneity in terms of product substitutability; Cuiat and Guadalupe (2003) provide empirical evidence of such a positive correlation.

Another instance of related work is Tadelis (2002), who draws on work by Bajari and Tadelis (2001) on fixed-price versus cost-plus contracting in procurement to argue that the complexity of an activity makes “make” a more likely outcome of the make-or-buy decision.

Novaes and Zingales (2004) develop a model with a three-level hierarchy to explore whether the degree of “bureaucratization” – in the sense of the amount of effort devoted to creating input-based performance measures
An important part of the motivation for this paper is the relevance of the analysis for the organization of public-sector activities, public-sector activities referring to activities that are publicly financed or subject to extensive regulation and often provided directly by the public sector. The most important contribution in this literature is Hart, Shleifer and Vishny (1997) who employ an incomplete-contract framework in approaching the question of privatization in general, and the issue of privatizing prisons in particular. In their model, the agent in charge of the operation, the warden, makes two investments, one geared towards cost savings, having adverse consequences for quality, and one geared towards quality-enhancing innovations. While an in-house warden needs the consent of the principal to implement any investment, an independent warden needs consent only for quality-enhancing innovations; in both cases, consent is followed by renegotiation of the incomplete contract. It is shown that an independent warden has excessive incentives for cost savings, and too weak incentives for quality innovations; an in-house warden has too weak incentives for cost savings as well as quality innovations. The Hart-Shleifer-Vishny model is clearly rife with insight concerning public-sector contracting in contexts plagued by contractual incompleteness; it also endogenously obtains two distinct regimes – in-house versus independent operations. The drawback of their approach is that the incentives generated directly by contracts cannot be analyzed since contracts serve mainly as threat points in renegotiation in their framework. Also, the implications for outsourcing within the private sector are not addressed, possibly reflecting the view (shared by us) that the plausibility of the assumptions rely on rigidities in terms of e.g. the duration of relationships that are typical for the public sector. By focusing on the direct implications of contracts, we consider our work complementary to theirs.

The paper proceeds as follows. In the next section, a simple example highlighting the trickling-down effect is analyzed in order to provide some groundwork for the rest of the analysis; in addition, the underpinnings of muted incentives for an in-house management are presented.

---

9 See Domberger and Jensen (1997) for an overview of the issues and a review of some empirical evidence.
10 In particular, it is clear that implicit contracts play a role in within-private-sector outsourcing relationships that is inappropriate in relationships involving the public sector due to e.g. susceptibility to corruption.
11 One distinguishing feature of the public sector is arguably the feature that agencies and agents in one way or another serve multiple principals or multiple goals (Dixit, 2002). Following Wilson (1989) there has been some work on the desirability of creating clear “missions” – essentially undoing multiple-principal problems – for public-sector bodies; Dewatripont, Jewitt and Tirole (1999) provide formal support for this idea in a multi-task career-concerns model. Somewhat relatedly, it may be argued that the intrinsic motivation of agents is more important in the presence of weaker monetary incentives; this idea is explored by Besley and Ghatak (2005).
Next, in Section 3 the two-task agency problem faced by the management is presented; in Section 4 the main results are derived in the full three-layer model, and in Section 5 we discuss applications and elaborations. In Section 6 we conclude the paper. Most of the analysis of the full model is provided in the Appendix.

2 Basic framework

We will consider a three-layer agency model with a principal at one end. The principal has an exogenously given task that she cannot solve by herself; she may be thought of as an elected body or the top management of a firm or corporation. At the other end is an agent, who in the end solves the task; the agent may be thought of as a worker.

The principal cannot delegate the task directly to the agent. There could be several plausible reasons for this. Our assumption is that the intermediate tier in the model, referred to as the management, specializes in extracting good performance measures (he may apply this competence to several independent agents); this implies that any side contract between the principal and the agent would suffer from – and in the extreme be undermined by – manipulation by the agent. The management may be thought of as the middle management within a firm if the task in question is solved in-house, and as the manager-owner of a subcontractor if the task is outsourced.

The preferences of the parties are simple; they will be enriched in the full model below.

- The principal, $P$, is a risk neutral profit maximizer, valuing the successful completion of the task in question at some $B > 0$. Assuming that the task is worthwhile solving, the principal’s key objective is to minimize cost, and, in the following sections, ascertain quality.

- The agent, $A$, cares about income, $y$, and the effort he exerts, $a$. He is risk averse, and his utility from income $y$ and effort $a$ is

$$u_A(y; a) = -\exp\left\{-r_A \left[ y - a^2/2 \right]\right\},$$

with $r_A > 0$ the agents level of absolute risk aversion; the specific utility function is assumed in order for the full model to be reasonably tractable. The agent has reservation payoff $\pi_A$.

- The management, $M$, is risk averse or risk neutral and profit maximizing with preferences
over net remuneration $R$ given by

$$u_M(R) = - \exp\{-r_M R\},$$

with absolute risk aversion $r_M \geq 0$. The management has a reservation payoff $\pi_M$.

To clarify the potential role of the management’s risk aversion we allow it in the introductory example, abstracting from it in the full model for the sake of tractability. The nature of the task will be quite general. In this section we will consider an example where the task is perfectly contractible in all respects but one, which may be thought of as realized cost; in the sequel there will, in addition, be a quality dimension. Contracts are assumed to be linear in the relevant performance measures; this is not important in the example in this section, but necessary to have a workable multitask model below.\(^\text{12}\)

**In-house versus independent management.** The presumption that incentives “originating in” an organization are, in general, weaker than incentives generated in contractual relations between organizations is clearly crucial for the rest of the paper.\(^\text{13}\) We will refer to the two cases as two *regimes*. The distinction between the regimes arises from the combination of the assumption about the accrual of cost savings below, and the agency problem introduced in subsection 2.2; in the absence of any substantive agency problem between the principal and the management, the difference between the two regimes would vanish.

Cost savings accrue through an *asset* that is, at least for practical purposes, indivisible and whose value cannot be subject to a sharing contract. The assumptions about the asset are thus in line with the property-rights approach although it plays the sole role of carrying the benefits of cost savings.\(^\text{14}\) An in-house management is defined by the principal owning the asset; the results of cost-saving efforts thus accrue to the principal, whose payoff is

$$B + x - R^{\text{in-house}}(x),$$

where $x$ is (unbiasedly) measured cost savings and where $R^{\text{in-house}}(x)$ is the remuneration to the management. An independent management, on the other hand, owns the asset and the results of cost-saving efforts accrue directly to the management; the principal’s payoff is then

$$B - R^{\text{indep}}(x)$$

\(^\text{12}\)The most convincing rationale for linear contracts is provided by Holmström and Milgrom (1987).

\(^\text{13}\)As we have noted, this is a widely shared presumption, articulated e.g. by Williamson (1998).

\(^\text{14}\)The property-rights approach is presented by e.g. (Hart, 1995); while Hart dismisses unreflected reliance on “residual income” in modeling, he also stresses the point that residual income in most cases and for good reasons goes together with the *residual control rights* that come with ownership.
where $R_{\text{indep}}(x)$ is the contracted remuneration to the management; the management’s revenues in this case are $R_{\text{indep}}(x) + x$.

2.1 Example

Consider now a case where the agent, in the end, exerts effort, $a$, on a task whose outcome—an inverse measure of realized cost—is, for simplicity, $a$. The contract governing the agent’s reward, however, can be based only on a noisy performance measure

$$x = a + \varepsilon,$$

where $\varepsilon$ is a random variable reflecting the fact that the outcome is affected but not determined by the agent’s effort; $\varepsilon$ is normally distributed with mean zero and variance $v$.

The principal delegates the task to the management, offering a linear contract

$$R = \alpha + \beta x,$$

for constants $\alpha$ and $\beta$. If the management is risk neutral, it would seem natural to impose that $\beta = 1$; as we will discuss shortly, however, this is qualified due to the agency problem below.

The management, in turn, delegates the task to the agent, and the agent’s monetary reward is

$$y = F + mx$$

for constants $F$ and $m$.

Optimal contracts. Given the assumption that contracts are linear, the contract that the management optimally offers to the agent is simple. Since the analysis is a simplified roadmap to the analysis of the multitask model below, which is deferred to the Appendix, we provide the details.

The management solves (where expectations are w.r.t. the distribution of $\varepsilon$)

$$\max_{m,F} -E \exp \{-r_M (\alpha + \beta x - (F + mx))\} = -\exp \left\{-r_M \left(\alpha - F + (\beta - m)a - r_M (\beta - m)^2 v/2\right)\right\}$$

s.t. $-\exp \{-r_A (F + ma - a^2/2 - r_A m^2 v/2)\} \geq \pi_A$,

and $a \in \arg \max \{-\exp \{-r_A (F + ma - a^2/2 - r_A m^2 v/2)\}\}$.

---

15One may note that the results would be the same if the outcome of the task was random too, as long as its expectation was $a$; for example, the outcome could be equal to $x$.

16Using the fact that $E \exp \{-r_A (F + m(a + \varepsilon) - a^2/2)\} = \exp \{-r_A (F + ma - a^2/2 - r_A m^2 v/2)\}$ for $A$, and similarly for $M$. 

---

8
Maximization by the agent yields \( a^* = m \); inserting this and taking logarithms we get

\[
\max \alpha - F + (\beta - m)m - r_M (\beta - m)^2 v/2
\]

s.t. \( F + m^2 - m^2/2 - r_A m^2 v/2 \geq -\ln(-\pi_A)/r_A \).

Solving the constraint – which obviously must bind – for \( F \), we get the simple unconstrained problem of maximizing

\[
\alpha + (\beta - m)m - r_M (\beta - m)^2 v/2 + m^2 - m^2/2 - r_A m^2 v/2 + \ln(-\pi_A)/r_A,
\]

and the first-order condition implies directly that

\[
m = \beta \cdot \frac{1 + r_M v}{1 + (r_A + r_M) v}.
\]

Note that the management’s risk aversion strengthens the incentives for the agent. More important, however, the weaker the incentives faced by the management, whose strength is measured by \( \beta \), the weaker are the incentives provided by the management to the agent; this property is clearly true quite generally in principal-agent models with a risk-averse agent.\(^{17}\)

This extremely simple example highlights a straightforward and natural property that is rarely noted, viz. that incentives trickle down. For example, it provides a simple and, arguably, quite plausible explanation of the frequently made observation that incentives are weaker in non-profit firms than in for-profits, an observation that is sometimes considered puzzling; we will come back to this when we discuss applications.

### 2.2 Origins of muted incentives

We will start by a formal development and then go on to interpretations and intuition.

**Manipulation by the management.** Consider an environment as in the example where the management observes \( x = a + \varepsilon \) as described, but where the management can distort the signal observed by the principal by means of manipulation.\(^{18}\) To keep things straightforward

---

\(^{17}\)One can, for example, easily verify an analogous result in a two-outcome continuous-action model – equilibrium incentives and equilibrium effort are increasing in the payoff difference between the bad and the good outcome for the principal (the argument is available from the author). This means that expanding the principal’s set of instruments would not change the solution qualitatively. This is worth noting since the problem looks superficially similar to the double-marginalization problem in monopoly theory. While it is true that in the example by-passing the management would be beneficial, the main results below deal with cases where the blunting of incentives is desirable, which it is not in the monopoly context.

\(^{18}\)This approach is in line with Baker (1992); it is also employed by Holmström (1999).
and simple, let the principal’s signal be given by

$$z = x + \gamma_\rho d = a + \varepsilon + \gamma_\rho d, \ \rho \in \{\text{in-house, indep}\}$$

(5)

where \(d\) is the management’s manipulation or distortion, and \(\gamma_\rho > 0\) is a constant that may in principle – and, indeed, in practice as we will come back to – depend on the regime; in this section, however, such potential dependence is inconsequential and we drop the subscript. The management suffers disutility \(d^2/2\) (in monetary terms) from manipulation \(d\).

**In-house management.** In the case of an in-house management, cost savings accrue to the principal while the principal must reward the management based on the available measure of cost savings; cf. (1). Since the distortion, \(d\), does not enter the management’s constraints, and since it is separable from the management’s other choice variables (\(m\) and \(F\)), its effect on the management’s problem is simply to add a benefit, \(\beta \gamma d\), and a cost, \(-d^2/2\), to the objective function which adds up to a benefit of \(\gamma^2 \beta^2/2\) since \(d = \beta \gamma\) is optimal for \(M\).

The principal’s problem, however, is affected more substantially. Again, we present some in-text analysis at this point since it illuminates the ensuing more cumbersome analysis in the general case. The principal maximizes her payoff subject to the standard constraints, viz. that the management attains its reservation payoff, and that the management behaves optimally. Formally, the principal solves (using reduced forms in the constraint):

$$\max_{\alpha,\beta} B + E \left( a - \beta z - \alpha \right) = B + (1 - \beta) a - \gamma^2 \beta^2 - \alpha$$

s.t. \(-\exp \left\{ -r_M \left( \alpha + (\beta - m)a + \gamma^2 \beta^2/2 - r_M (\beta - m)^2 v/2 - F \right) \right\} \geq \pi_M\),

and \((m, F)\) chosen optimally by \(M\).

Noting that \(m = \beta (1 + r_M v) / (1 + (r_A + r_M) v)\) and solving the agent’s participation constraint for \(F\) as in (3) we have

$$\max_{\alpha,\beta} B + (1 - \beta) m - \gamma^2 \beta^2 - \alpha$$

s.t. \(\alpha + (\beta - m)m + \gamma^2 \beta^2/2 - r_M (\beta - m)^2 v/2 - \left[ - \ln \left( -\pi_A \right) / r_A + r_A m^2 v/2 - m^2/2 \right] \geq - \ln(-\pi_M)\);

the objective function is, substituting the constraint and simplifying,

$$B + m - m^2/2 - \gamma^2 \beta^2/2 - r_M (\beta - m)^2 v/2 - r_A m^2 v/2 + \ln \left( -\pi_A \right) / r_A + \ln(-\pi_M).$$

The first-order condition w.r.t. \(\beta\) implies, using \(dm/d\beta = (1 + r_M v) / (1 + (r_A + r_M) v)\), and expressing \(m\) in terms of \(\beta\),

$$\beta = \frac{(1 + r_M v) (1 + (r_A + r_M) v)}{\gamma^2 (1 + (r_A + r_M) v)^2 + (1 + r_A v) (1 + r_M v)^2 + r_M v (r_A v)^2};$$

(6)
note, in particular, that when the management is risk neutral, \( r_M = 0 \),

\[
\beta = \frac{1}{\gamma^2 (1 + r_A v) + 1}.
\]

showing that \( \beta = 1 \) if \( \gamma = 0 \), while \( \beta < 1 \) if \( \gamma > 0 \); the expression in (6) generalizes this dependence on \( \gamma \), showing that whenever \( \gamma > 0 \), incentives provided to an in-house management are limited by manipulation.

**Independent management.** In the case of an independent management, cost savings accrue to the management, and the reward to the management is by construction the cost savings plus what is contracted in addition; cf. (2). This means that, with \( \beta \) denoting the incentive provided by the principal in addition to the cost savings themselves, the objective function of the management is then (recall that \( a^* = m \) in equilibrium)

\[
- \exp \left\{ -r_M \left( \alpha + (1 + \beta - m) a + \gamma^2 \beta^2 / 2 - r_M \left( 1 + \beta - m \right)^2 v / 2 - F \right) \right\}.
\]

Formally, the principal solves \(((m,F) \text{ chosen optimally by } M)\):

\[
\max_{a,\beta} \quad B + E \left( -\beta z - \alpha \right) = B - \beta a - \gamma^2 \beta^2 - \alpha
\]

s.t. \( - \exp \left\{ -r_M \left( \alpha + (1 + \beta - m) a + \gamma^2 \beta^2 / 2 - r_M \left( 1 + \beta - m \right)^2 v / 2 - F \right) \right\} \geq \pi_M. \)

In order to facilitate interpretation, it is useful to note that the effective incentive intensity is \( \beta = \tilde{\beta} + 1 \) in the case of an independent management; indeed, it is readily shown that

\[
m = (\beta + 1) \cdot \frac{1 + r_M v}{1 + r_A v + r_M v}
\]

in this case. Working through this in a way similar to the in-house case above, one obtains the solution

\[
\tilde{\beta} = \frac{-r_A v r_M v (1 + (r_A + r_M) v)}{\gamma^2 (1 + (r_A + r_M) v)^2 + (1 + r_A v) (1 + r_M v)^2 + r_M v (r_A v)^2} \leq 0;
\]

note that \( \tilde{\beta} = 0 \) (and thus \( \beta = 1 \)) for \( r_M = 0 \), and note also that when \( r_M > 0 \) the principal provides insurance to the management to an extent that decreases with manipulability, \( \gamma \).

**Interpretations.** In the framework presented, the principal provides incentives to the management to control costs; \( a \) is realized cost savings measured by \( x \), and \( z \) is a distorted measure of cost savings. This is a natural specification if, for example, \( a \) measures long-term cost savings, while \( x \) and \( z \) are accounting measure of cost savings, \( z \) to some extent being controlled by the management.
The interpretation in the case of an in-house management, with cost savings accruing to the principal, is simple and clear: with strong incentives for measured cost savings come incentives for manipulation, and the greater the management’s manipulation possibilities, the weaker are optimal incentives.

If revenues accrue to the management, on the other hand, manipulation has a bite only in so far as the principal offers additional incentives (positive or negative) on top of the cost savings themselves, corresponding to $\beta = 1$; quite obviously the principal finds it optimal not to offer any such additional incentives to a risk-neutral management. The key is of course that the objective of the party controlling the performance measure is the true cost savings. For interpretations, consider a subcontractor to a firm with a contract sharing cost savings; clearly, the subcontractor has incentives to inflate cost estimates whenever there is cost sharing in the sense that $\beta < 1$; $\gamma > 0$ amounts to this being possible.

Let us finally relate the implications of the current set-up to those of the property-rights approach reviewed briefly in the introduction. The “residual rights of control” coming with ownership of assets play a role somewhat related to that of the accrual of cost savings here. In both cases, this is in some sense a manifestation of control. Here this is disadvantageous as far as we see now, although it will be advantageous for second-best reasons under certain circumstances below; in the property-rights approach the costs and benefits of control depend on whether the incentives gained by one party are more or less valuable than those lost by the other party.

3 A two-task management-agent model

In this section we will develop and briefly outline a two-task “management-agent model” that will serve as our basic framework for the remainder of the paper; for tractability, the management will be assumed risk neutral (we will come back this in Section 5).\textsuperscript{19} The thrust of the model – as well as of other multitask principal-agent models – is that the incentive problem has an effort allocation dimension in addition to the effort extraction dimension that is the defining element of most principal-agent models. In our application below, we will distinguish between a cost-saving dimension and a quality dimension.

In formal terms, the model produces two output measures, $x_1$ and $x_2$, that depend stochastically on two effort (input) dimensions, $a_1$ (cost savings) and $a_2$ (quality), controlled by the

\textsuperscript{19}Obviously, it would normally be called a principal-agent model, but for the sake of consistency with the development of the full model below we call the two layers management and agent.
agent. The management cares about \( x_1 \) and \( x_2 \) but, again, this could be modified to include randomness with no consequences for the results. We assume that

\[
x_i = a_i + \varepsilon_i, \quad i = 1, 2,
\]

where \( \varepsilon_i \) is noise, assumed to be normally distributed with mean zero and variance \( v_i \), and assumed independent across \( i \). Note that this formulation – combined with the cost-saving and quality interpretations of the two dimensions that we adopt – gives us a cost-saving dimension and a quality dimension of effort as well as measured output.

The rest of the setting follows the two lower tiers of the example in the previous section closely. The management, being a risk neutral profit maximizer, offers the agent a contract that specifies monetary compensation that is constrained to be linear in the performance measures:

\[
y = F + m_1 x_1 + m_2 x_2.
\]

The agent has preferences over monetary compensation and effort, \((a_1, a_2)\), according to the von Neumann-Morgenstern utility function (letting \( r_A = r \) for brevity)

\[
u(y; a) = -\exp\{-r [y - c(a)]\}, \quad \text{where } c(a) = a_1^2 + 2\kappa a_1 a_2 + a_2^2;
\]

the parameter \( \kappa \) measures the degree of substitutability between \( a_1 \) and \( a_2 \) in the agent’s disutility-of-effort function. The agent has reservation payoff \( \pi_A \).

The management values the two dimensions of realized output at \( \beta_1 \) and \( \beta_2 \) per unit, and the problem faced by the management is thus

\[
\begin{align*}
\max & \quad E [\beta_1 a_1 + \beta_2 a_2 - (F + m_1 x_1 + m_2 x_2)] \\
\text{s.t.} & \quad -\exp\{-r [F + m_1 (a_1 + \varepsilon_1) + m_2 (a_2 + \varepsilon_2) - (a_1^2 + 2\kappa a_1 a_2 + a_2^2)]\} \geq \pi_A, \\
& \quad \text{and } a \in \arg \max -E \exp\{-r [F + m_1 (a_1 + \varepsilon_1) + m_2 (a_2 + \varepsilon_2) - (a_1^2 + 2\kappa a_1 a_2 + a_2^2)]\}.
\end{align*}
\]

The solution, which is derived in the Appendix, is

\[
m_1 = \frac{2rv_2(\beta_1 - \beta_2\kappa) + \beta_1}{4r^2(1 - \kappa^2)v_1v_2 + 2rv_1 + 2rv_2 + 1},
\]

and

\[
m_2 = \frac{2rv_1(\beta_2 - \beta_1\kappa) + \beta_2}{4r^2(1 - \kappa^2)v_1v_2 + 2rv_1 + 2rv_2 + 1}.
\]

\( F \) is determined residually. The key insight added by the effort-allocation dimension is – unsurprisingly but importantly – that there is, in general, an interdependence between the two

\footnote{The seminal contribution to the development of this framework is Holmström and Milgrom (1991).}
output dimensions in the sense that incentives provided for one component of the result affect inputs and results in both dimensions. This interdependence is a bit unwieldy even as we rule out stochastic dependence between the noise terms and assume that each output measure depends only on one input. Nevertheless, some general — and for our purposes important — properties can be demonstrated by considering some special cases. We will take the case when \( a_1 \) and \( a_2 \) are substitutes in the agent’s utility function — i.e. when \( \kappa > 0 \) — as the main case and only occasionally note results for the other case; the complements case \( (\kappa < 0) \) gives the effort-extraction problem a “free-lunch flavor” that seems unnatural in most applications.

- First, consider the case where \( a_2 \) has no intrinsic value to the management so that \( \beta_2 = 0 \) (note that this case departs somewhat from our assumptions below). This gives

\[
m_1 = \frac{\beta_1 (2rv_2 + 1)}{4r^2(1 - \kappa^2)v_1v_2 + 2rv_1 + 2rv_2 + 1}; \quad m_2 = \frac{-2rv_1\kappa \beta_1}{4r^2(1 - \kappa^2)v_1v_2 + 2rv_1 + 2rv_2 + 1},
\]

and we see that as long as the two inputs, \((a_1, a_2)\), are substitutes, the agent is punished for a high \( x_2 \).

- Secondly, consider the case where the informativeness about effort in one dimension of output, say 2, grows small, i.e. when \( v_2 \to \infty \). In this case

\[
m_1 = \frac{2r(\beta_1 - \beta_2 \kappa)}{4r^2(1 - \kappa^2)v_1 + 2r}; \quad m_2 = 0,
\]

and we see that the incentives provided for \( x_1 \) must be used to control both dimensions of effort; from the expression one sees e.g. that if the uninformative dimension is important enough — more precisely if \( \beta_1 < \beta_2 \kappa \) — output in the remaining dimension is punished.

The last case is important because the main case below will be relatively closely related to it. It also highlights the general point that there are important circumstances under which weak incentives are desirable for “second-best reasons.”

### 4 Incentives in the three-layer two-task model

We will now consider the general principal-management-agent model where the technology of the project delegated to the agent is that specified by the two-task model. The preferences of the principal and the management are the same as in the example — \( P \) and \( M \) are both risk neutral profit maximizers, \( M \) having reservation payoff \( \pi_M \) — and \( A \)’s preferences and action possibilities were specified in the previous section. As in subsection 2.2, \( M \) can manipulate the cost-saving measure in such a way that the principal observes

\[
z_1 = x_1 + \gamma d_1 = a_1 + \gamma d_1 + \varepsilon_1.
\]

In addition, we
assume that the principal has a similar informational disadvantage concerning the observation of the quality-related performance measure, $x_2$. The management observes $x_2 = a_2 + \varepsilon_2$, while the principal observes

$$z_2 = a_2 + qd_2 + \varepsilon_2,$$

where $q$ is a non-negative constant which we assume to be the same across regimes, and $d_2$ is a distortion of the signal controlled by the management, carrying a cost $d_2^2/2$.

As in the example, cost savings accrue to the principal when an in-house management is employed but directly to an independent management. We do not assume any such distinction concerning quality; since the management does not care about quality directly (as opposed to money) there is no room for such a mechanism. When an in-house management is employed, the principal’s objective is

$$V = B + a_1 + pa_2 - R\text{in-house}(z),$$

where the quality-related performance may weigh more or less heavily in the principal’s payoff according to the parameter $p \geq 0$, and she offers a linear contract to $M$:

$$R\text{in-house}(z) = \beta_0 + \beta_1 z_1 + \beta_2 z_2.$$

With an independent management, $V = B + pa_2 - R\text{indep}(z)$, with $R\text{indep}(z) = \beta_0 + a_1 + \tilde{\beta}_1 z_1 + \beta_2 z_2$.

Consider now the principal’s problem, which is stated and analyzed in the Appendix. For the case of an in-house management, it is

$$\max_{\beta} \quad B + (1 - \beta_1)a_1 + (p - \beta_2)a_2 - \beta_1^2 \gamma^2 - \beta_2^2 q^2 - \beta_0$$

s.t. $\beta_0 + (\beta_1 - m_1(\beta)) a_1 + (\beta_2 - m_2(\beta)) a_2 + \beta_1^2 \gamma^2 / 2 + \beta_2^2 q^2 / 2 - F(m_1(\beta), m_2(\beta)) \geq \pi_M,$

where are determined by the $M$’s optimization problem. The case with an independent management is similar except for $\beta_1$ being replaced by $\tilde{\beta}_1 = \beta_1 - 1$.

There are two important things to note about this way of formulating the principal’s problem: First, $\beta_1$ and $\beta_2$ vary freely as a function of $\gamma$ and $q$ (and $p$ below); as we will elaborate in the next subsection, this is a natural parameterization for comparative statics of the relative desirability of an independent versus an in-house management. Secondly, when the quality measure is manipulable ($q > 0$) it is no longer true in general that $\beta_1 = 1$ for an independent and risk neutral management, but it is still true that $\beta_1$ is unambiguously larger ceteris paribus for an independent management; this is seen by noting that the terms in (10) involving $\beta_1 a_1$ cancel if one substitutes the constraint (and that the objective function is concave in $\beta$ as we note below).
4.1 Comparative statics

Absent manipulability – i.e. for $\gamma = q = 0$ – the problem faced by the principal is simple: The management delegates the project to the agent with equilibrium incentives according to (7) and (8). The principal just forwards her incentives to the management, $\beta_1 = 1$ and $\beta_2 = p$.

In the presence of manipulability, on the other hand, incentives will depend on $\gamma$ and $q$. The comparative statics of $\beta_1$ and $\beta_2$ as manipulability of cost savings gains strength ($\gamma$ grows) are straightforward and unsurprising: $\partial \beta_1 / \partial \gamma < 0$ and $\partial \beta_2 / \partial \gamma$ has the opposite sign to $\kappa$, that is, when $\kappa$ is positive, $\partial \beta_2 / \partial \gamma < 0$. Consider next equilibrium effort, $(a_1, a_2)$, and its dependence on the attenuation of cost-saving incentives. Equations (A.4) and (A.5) in the Appendix show that while the dependence of equilibrium effort on $\gamma$ is in general ambiguous, we have an unambiguous result when $v_2$ is large – i.e. when the observation of the quality dimension is a poor indicator of quality. In that case, $a_1$ is unambiguously decreasing in $\gamma$, while $a_2$ is increasing in $\gamma$ when $\kappa > 0$ (and decreasing if $\kappa < 0$). Formally:

**Proposition 1** Cost-saving incentives, $\beta_1$, are attenuated as manipulability of the cost-saving measure ($\gamma$) grows. Incentives for quality provision, $\beta_2$, are attenuated (strengthened) if $\kappa > 0$ ($\kappa < 0$). When quality measurement is sufficiently imprecise (i.e. $v_2$ is sufficiently large), equilibrium cost-saving effort ($a_1$) is decreasing in $\gamma$, and equilibrium effort exerted on quality ($a_2$) is increasing (decreasing) in $\gamma$ for $\kappa > 0$ ($\kappa < 0$).

As an immediate corollary of the proposition, together with the fact that an independent management provides stronger cost-saving incentives, it follows that opting for an independent management rather than an in-house one makes the agent “on the ground” focus more on cost savings at the expense of quality when there is “competition” between the two components of effort, i.e. when $\kappa > 0$. By tilting incentives towards cost savings at the management level, the activities on the ground are tilted in the same direction due to the trickling-down effect. From a positive perspective, the result thus seems to corroborate commonsensical notions of, for examples, the consequences of outsourcing and privatization.

Considering comparative statics with respect to $q$, we find that $q$ affects $\beta_2$ negatively, while the effect on $\beta_1$ has the opposite sign of $\kappa$, i.e., a harder quality-control problem as measured by $q$ leads to an attenuation of cost-saving incentives too when $\kappa > 0$.

In the presence of manipulability, equilibrium incentives to the management depend non-trivially on other variables, such as the valuation of quality, $p$, and the incentive problem faced by the management (as measured by $v_1$ and $v_2$). The dependence on $p$ is clear cut: The incentive intensity for quality, $\beta_2$, depends positively on $p$; i.e., a higher valuation of quality by
the principal increases the optimal reward for quality to the management. More interestingly, for \( q > 0 \) – i.e. in the presence of manipulability of the quality measure – \( \partial \beta_1 / \partial p \) has the opposite sign of \( \kappa \); i.e., when efforts are substitutes a higher valuation of quality makes the principal want to blunt cost-saving incentives. This is quite important since it confirms that *there is a trickling-down effect in the effort-allocation dimension as well*: In the presence of an agency problem between \( P \) and \( M \), blunting cost-saving incentives for the management helps shifting the agent’s effort towards the quality dimension; moreover, this blunting of cost-saving incentives is an optimal response to a higher valuation of the quality of output.

### 4.2 The trade-offs

The comparative statics of the optimal contract offered by the principal to the management can straightforwardly be translated into statements about how the optimal mode of governance – i.e. make or buy – are affected by manipulability, \( q \), and the importance of quality, \( p \).

The choice between make (employing an in-house management) and buy (contracting with an independent management) is a choice between facing a cost of *providing* cost-saving incentives due to manipulation by an in-house management, and a cost of *muting* cost-saving incentives (relative to \( \beta_1 = 1 \)) due to manipulation by an independent management. Since the objective function is well-behaved in being concave (and, in fact, quadratic) in \((\beta_1, \beta_2)\) – as is shown in the Appendix – an exogenous shift leading to a reduction in \( \beta_1 \) can be identified with a shift that makes choosing an in-house management more attractive. The following proposition follows directly from the analysis in the previous subsection.

**Proposition 2** Suppose that cost-saving effort and quality effort are substitutes in the agent’s utility function (\( \kappa > 0 \)). Then an in-house management is more attractive relative to contracting with an independent management: (i) the more severe is the quality-control problem faced by the principal in dealing with the management (i.e. the larger is \( q \)), and, for \( q > 0 \), (ii) the more valuable is quality for the principal (the higher is \( p \)).

While the proposition has clear qualitative implications for the choice of regime it may allow an independent management with substantively muted cost-saving incentives for significant ranges of other parameters if \( \gamma \) is not too large. If, however, there is more room for manipulation of measures of realized costs by an independent management (\( \gamma_{\text{indep}} > \gamma_{\text{in-house}} \)) – as is arguably quite reasonable – there is correspondingly less room for cost-sharing arrangements between the principal and an independent management, and an in-house management will be optimal for a larger range of parameters.
In the analysis we have assumed that $q$ is equal across the two regimes. While this may or may not be reasonable, the results only depend on the much weaker property that manipulability across activities varies similarly in the two regimes: As long as this is the case the notion of an activity with a high degree of manipulability is well defined, and the prediction that the higher this degree, the more likely is the activity to be performed in-house, stands.

An additional question to ask would be how the optimal contract between the principal and the management depends on the measurement problem, i.e. how $(\beta_1, \beta_2)$ depends on $v_2$. This question turns out not to have a clear answer however.

5 Applications and elaborations

In this section we will elaborate on the results and discuss robustness and extensions.

**Robustness.** In a way, the assumed information structure is simple. In particular, the principal and the management observe performance measures of the agent’s activity that are closely related to one another. Apart from being in the interest or tractability, however, it seems hard to argue that there is likely to exist substantial asymmetries of a different nature that would be directly relevant to our inquiry. As to risk aversion, we have abstracted from it in the analysis of the full model; looking at the examples in Section 2 it seems clear that it would not alter our results substantially.

**Empirical implications.** There are two sets of empirical implications tied to Propositions 1 and 2 respectively. The main implication of the first proposition – that in-house production is a way of securing quality when quality measurement is imprecise – is, in our view, that there is some truth to the often-heard argument that privatization or outsourcing may be a threat to quality. This is interesting and important but does not lead much further since it does not tell anything about optimal contracts.

The implications of Proposition 2 – that in-house production is more likely the more severe the quality-control problem between the principal and the management, and the more important is quality – on the other hand, produce empirically testable hypotheses about governance of activities. To give a simple example, it would seem to predict that management-type activities and research-and-development activities be organized in-house rather than independently. It also provides a potential rationale for the pervasiveness of publicly provided elementary education,
although this view is sometimes challenged as we will note below.\textsuperscript{21}

**Privatization and outsourcing.** Although we believe that the framework developed can prove useful in systematic empirical investigations of determinants of outsourcing and privatization, this is not the place for explorations in such directions. Instead, we will confine ourselves to discussing a salient example. It is generally held that garbage collection is a prime example of an activity, often performed in-house by local governments, that can be contracted out in a way that leads to substantial cost savings without jeopardizing quality.\textsuperscript{22} Snow removal is an activity that may superficially look similar to garbage collection. There are, however, scattered evidence from Sweden – in particular from a major overhaul of snow removal in the city of Stockholm – indicating that contracting out is likely to work much less well in this case. There are, we believe, two distinguishing features that may explain the difference: uncertainty and measurement problems. While uncertainty clearly plays a role, the measurement issue is fundamental: In order to establish a contractually viable relationship between the effort exerted in snow removal and “snowfreeness,” an elaborate measurement apparatus is necessary. However ambitious – and costly – such an apparatus is construed, it is still bound to rest heavily on vague criteria. The upshot is that in-house provision – with the accompanying weak incentives – is likely to be a substitute for some of the measurement effort and, in the end, possibly preferable.\textsuperscript{23} This example illustrates, in our view, two important points: first, measurement is key; secondly, whether or not an activity is suitable to outsourcing/contracting out has little to do with its production technology and all to do with contracting possibilities.\textsuperscript{24}

**Firms, governments and non-profits.** In this paper, our focus is on the make-or-buy decision concerning a particular activity, and how the decision is affected by the nature of the activity. The driving mechanism is the muted cost-saving incentives induced by in-house production. A related question is whether the constraint that cost-saving incentives

\textsuperscript{21}This example is pushed by Acemoglu, Kremer and Mian (2004) too.

\textsuperscript{22}See e.g. Savas (1977).

\textsuperscript{23}Note that the endogeneity of the imprecision of measurement alluded to takes us a little bit beyond the model, but not in a consequential way.

\textsuperscript{24}The last point is re-infored by noting that steps towards privatization in schooling in the sense of student/parent choice combined with some extent of free entry seems to work well in many circumstances; see e.g. Hoxby (2002). The reason seems to be that quality control can be decentralized to students/parents under voucher-type competition. The model in this paper is not directly applicable to such an environment, but the example illustrates the point that the production technology and the “softness” / “hardness” of the activity are not the key determinants of its suitability to choice and private provision.
in-house not be too strong may be different for different types of organizations. There is, for example, reasons to believe that incentives in non-profit organizations are weaker than in for-profit organizations.\textsuperscript{25} The difference between firms and governments in this regard is, moreover, corroborated by Acemoglu, Kremer and Mian (2004). In terms of our model, $\gamma$ might differ across types of organizations, and this would have relatively straightforward implications for organizational choice. This may, for example, throw light on the prevalence of non-profits in some sectors, like hospitals and schools, and it may also rationalize calls for prohibiting for-profit actors in certain types of activities.\textsuperscript{26} In pursuing this line of thought, the trickle-down property of incentives seems particularly pertinent.

6 Concluding remarks

In this paper we have tried to approach the make-or-buy decision in a comprehensive-contracting framework emphasizing the measurement aspects of cost savings and quality. We have shown that incentives trickle down from the principal-management contract to the management-agent contract, and that this produces the result that outsourcing, roughly, is less attractive, the harder is the quality-control problem. Finally, we have discussed implications for outsourcing and privatization.

There are a number of substantive questions raised but unanswered by this paper. First, the current theory seems – as we have noted – readily extended to analyzing the choice among, for example, for-profit, non-profit and government operation; this extension seems worthwhile, and it may offer tractable empirical implications as a side benefit. Secondly, while our approach to modeling the underlying reason for the distinction between make and buy is, in our own view, convincing, this deserves further investigation. More generally, one would like to see further integration between measurement-based and asset-ownership-based theories of make-or-buy.\textsuperscript{27} This model, or variants of it, seem potentially useful in such an undertaking.

\textsuperscript{25}See Roomkin and Weisbrod (1999) for an empirical explorations of hospitals, and Glaeser and Shleifer (2001) for a simple theoretical model.

\textsuperscript{26}This alludes to a rather fierce debate in Sweden about whether or not for-profit hospitals should be prohibited.

\textsuperscript{27}Holmström (1999) takes some preliminary steps.
Appendix

Solving the management-agent model  The problem can be written

\[
\begin{align*}
\max & \quad [(\beta_1 - m_1)a_1 + (\beta_2 - m_2)a_2 - F] \\
\text{s.t.} & \quad \exp(-r(F + m_1a_1 + m_2a_2 - \frac{r}{2}m_1^2v_1 - \frac{r}{2}m_2^2v_2 - [a_1^2 + 2\kappa a_1 a_2 + a_2^2])) \geq u_0
\end{align*}
\]

and optimality for the agent, the first-order conditions for which are

\[
m_1 - 2(a_1 + \kappa a_2) = 0; \quad m_2 - 2(\kappa a_1 + a_2) = 0.
\]

Maximization yields

\[
a^*_1 = \frac{m_1 - \kappa m_2}{2(1 - \kappa^2)}; \quad a^*_2 = \frac{m_2 - \kappa m_1}{2(1 - \kappa^2)}
\]

and the objective function is (with \(a^*_1\) and \(a^*_2\) inserted, \(\tilde{u} = -\ln(-u_0)/r\) and after substituting the constraint)

\[
\begin{align*}
(\beta_1 - m_1)\frac{m_1 - \kappa m_2}{2(1 - \kappa^2)} + (\beta_2 - m_2)\frac{m_2 - \kappa m_1}{2(1 - \kappa^2)} + m_1\frac{m_1 - \kappa m_2}{2(1 - \kappa^2)} + m_2\frac{m_2 - \kappa m_1}{2(1 - \kappa^2)} \\
-\frac{r}{2}m_1^2v_1 - \frac{r}{2}m_2^2v_2 - \frac{1}{4(1 - \kappa^2)^2}[(m_1 - \kappa m_2)^2 + 2\kappa(m_1 - \kappa m_2)(m_2 - \kappa m_1) + (m_2 - \kappa m_1)^2] - \tilde{u}.
\end{align*}
\]

Simplifying by multiplying by \(2(1 - \kappa^2)\), we have

\[
\begin{align*}
\beta_1(m_1 - \kappa m_2) + \beta_2(m_2 - \kappa m_1) - r(1 - \kappa^2)m_1^2v_1 - r(1 - \kappa^2)m_2^2v_2 \\
-\frac{1}{1 - \kappa^2}\left[\frac{1}{2}(m_1 - \kappa m_2)^2 + \kappa(m_1 - \kappa m_2)(m_2 - \kappa m_1) + \frac{1}{2}(m_2 - \kappa m_1)^2\right] - \tilde{u}.
\end{align*}
\]

The first-order conditions w.r.t. \((m_1, m_2)\) are:

\[
\begin{align*}
\beta_1 - \beta_2 - 2r(1 - \kappa^2)v_1 m_1 - \frac{1}{1 - \kappa^2}[(m_1 - \kappa m_2) + \kappa[(m_2 - \kappa m_1) - \kappa(m_1 - \kappa m_2) - \kappa(m_2 - \kappa m_1)]] = 0, \\
\beta_2 - \beta_1 - 2r(1 - \kappa^2)v_2 m_2 - \frac{1}{1 - \kappa^2}[-\kappa(m_1 - \kappa m_2) + \kappa[(m_1 - \kappa m_2) - \kappa(m_2 - \kappa m_1) + (m_2 - \kappa m_1)]] = 0;
\end{align*}
\]

or, simplifying,

\[
\begin{align*}
\beta_1 - \beta_2 - 2r(1 - \kappa^2)v_1 m_1 + \frac{1 - \kappa^2}{1 - \kappa^2} m_1 + \frac{k^3 - \kappa}{1 - \kappa^2} m_2, \\
\beta_2 - \beta_1 - 2r(1 - \kappa^2)v_2 m_2 + \frac{1 - \kappa^2}{1 - \kappa^2} m_1 + \frac{k^3 - \kappa}{1 - \kappa^2} m_2.
\end{align*}
\]

This can be written

\[
\begin{pmatrix}
2r(1 - \kappa^2)v_1 + 1 & -\kappa \\
-\kappa & 2r(1 - \kappa^2)v_2 + 1
\end{pmatrix}
\begin{pmatrix}
m_1 \\
m_2
\end{pmatrix}
= \begin{pmatrix}
\beta_1 - \beta_2 \kappa \\
\beta_2 - \beta_1 \kappa
\end{pmatrix},
\]

21
and the solution is
\[
\begin{pmatrix}
m_1 \\
m_2
\end{pmatrix} = \frac{1}{D} \begin{pmatrix}
2r(1 - \kappa^2)v_2 + 1 & \kappa \\
\kappa & 2r(1 - \kappa^2)v_1 + 1
\end{pmatrix} \begin{pmatrix}
\beta_1 - \beta_2 \kappa \\
\beta_2 - \beta_1 \kappa
\end{pmatrix}.
\]

The determinant is
\[
D = 4r^2(1 - \kappa^2)^2v_1v_2 + 2r(1 - \kappa^2)v_1 + 2r(1 - \kappa^2)v_2 + 1 - \kappa^2 = \\
(1 - \kappa^2) [4r^2(1 - \kappa^2)v_1v_2 + 2rv_1 + 2rv_2 + 1];
\]
note that it is positive. Solving, we obtain
\[
m_1 = \frac{2rv_2(\beta_1 - \beta_2 \kappa) + \beta_1}{4r^2(1 - \kappa^2)v_1v_2 + 2rv_1 + 2rv_2 + 1},
\]
and
\[
m_2 = \frac{2rv_1 (\beta_2 - \beta_1 \kappa) + \beta_2}{4r^2(1 - \kappa^2)v_1v_2 + 2rv_1 + 2rv_2 + 1}.
\]

**Proof of Propositions 1 and 2.** *The principal’s problem in the three-layer model.*

The argument is made for an in-house management; the conclusions are the same – with almost exactly the same argument – for the other case. The principal’s problem is, suppressing B in the principal’s objective since it does not affect solutions, and using the fact that manipulation satisfies \(d_1^* = \beta_1 \gamma\) and \(d_2^* = \beta_2 q_2\),

\[
\max_{\beta} E[a_1 + pa_2 - (\beta_0 + \beta_1 (x_1 + \gamma d_1) + \beta_2 (x_2 + q d_2))] = (1 - \beta_1)a_1 - \beta_1^2 \gamma^2 + (p - \beta_2)a_2 - \beta_2^2 q^2 - \beta_0
\]

s.t. \(E(\beta_0 + (\beta_1 - m_1)x_1 + (\beta_2 - m_2)x_2 + \beta_1^2 \gamma^2/2 + \beta_2^2 q^2/2 - F) \geq \pi_M\)

and \(m\) maximizes \(E(\beta_0 + (\beta_1 - m_1)x_1 + (\beta_2 - m_2)x_2 + \beta_1^2 \gamma^2/2 + \beta_2^2 q^2/2 - F)\) s.t. \(M\)'s constraints

where \(\pi_M\) is \(M\)'s reservation utility. Taking expectations, this can be written

\[
\max_{\beta} (1 - \beta_1)a_1 + (p - \beta_2)a_2 - \beta_1^2 \gamma^2 - \beta_2^2 q^2 - \beta_0
\]

s.t. \(\beta_0 + (\beta_1 - m_1(\beta))a_1 + (\beta_2 - m_2(\beta))a_2 + \beta_1^2 \gamma^2/2 + \beta_2^2 q^2/2 - F(m_1(\beta), m_2(\beta)) \geq \pi_M\),

with \(m_1\) and \(m_2\) chosen optimally, and \(F\) determined by the participation constraint.

Substituting the constraint, the objective function (denoting it \(\phi\)) is

\[
\phi(\beta_1, \beta_2) = (1 - m_1(\beta))a_1 + (p - m_2(\beta))a_2 - \beta_1^2 \gamma^2/2 - \beta_2^2 q^2/2 - F(m) - \pi_M
\]

where, with \(\hat{u}\) the agent’s (re-normalized) reservation utility,

\[
F(m) = \hat{u} + \left(\frac{m_1 - km_2}{2(1 - \kappa^2)}\right)^2 + 2kr\frac{m_1 - km_2 m_2 - km_1}{2(1 - \kappa^2)} + \frac{m_2 - km_1}{2(1 - \kappa^2)} ^2 + r\frac{m_1^2 v_1 + r m_2^2 v_2 - m_1 \frac{m_1 - km_2}{2(1 - \kappa^2)} - m_2 \frac{m_2 - km_1}{2(1 - \kappa^2)} }.
\]
Substituting for the $a$’s and simplifying a bit:

$$\phi(\beta_1, \beta_2) = (1 - m_1(\beta)) \frac{m_1 - \kappa m_2}{2(1 - \kappa^2)} + (p - m_2(\beta)) \frac{m_2 - \kappa m_1}{2(1 - \kappa^2)} - \beta_1^2 \gamma^2 / 2 - \beta_2^2 q^2 / 2 - F(m) - \pi_M.$$  

Importantly, the objective is jointly concave in $\beta$. This follows from the fact that $\phi$ is concave in $m$, while $m$ is linear—and thus weakly concave with a zero second derivative—in $\beta$ from (7) and (8).

Differentiating w.r.t. $(m_1, m_2)$ gives (following the optimality conditions for the agent’s incentives):

$$\frac{\partial \phi}{\partial m_1} = 1 - p \kappa - (2r(1 - \kappa^2)v_1 + 1) m_1 + \kappa m_2,$$

$$\frac{\partial \phi}{\partial m_2} = p - \kappa + \kappa m_1 - (2r(1 - \kappa^2)v_2 + 1) m_2.$$

This should be inserted into

$$\frac{\partial \phi}{\partial \beta_1} = \frac{\partial \phi}{\partial m_1} \frac{\partial m_1}{\partial \beta_1} + \frac{\partial \phi}{\partial m_2} \frac{\partial m_2}{\partial \beta_1} - \beta_1 \gamma^2 = 0,$$

(A.1)

$$\frac{\partial \phi}{\partial \beta_2} = \frac{\partial \phi}{\partial m_1} \frac{\partial m_1}{\partial \beta_2} + \frac{\partial \phi}{\partial m_2} \frac{\partial m_2}{\partial \beta_2} - \beta_2 q^2 = 0,$$

(A.2)

and at this stage we may note that for $\gamma = q = 0$ any solution to

$$\frac{\partial \phi}{\partial m_1} = \frac{\partial \phi}{\partial m_2} = 0$$

is clearly a solution to the principal’s problem too, and with $\beta_1 = 1$ and $\beta_2 = p$ the $m$’s solving the system will coincide with equilibrium $m$’s. This confirms the already-noted fact that setting $\beta_1 = 1$ and $\beta_2 = p$ is optimal for the principal in the absence of manipulation.

To say something about cases where there are distortions or constraints making a first-best contract (in this context, i.e. in the absence of further distortions) between $P$ and $M$ infeasible, we need to develop (A.1) and (A.2) explicitly, however. To do this, we note

$$m_1 = \frac{2rv_2(\beta_1 - \beta_2 \kappa) + \beta_1}{4r^2(1 - \kappa^2)v_1v_2 + 2rv_1 + 2rv_2 + 1}; m_2 = \frac{2rv_1(\beta_2 - \beta_1 \kappa) + \beta_2}{4r^2(1 - \kappa^2)v_1v_2 + 2rv_1 + 2rv_2 + 1};$$

for notational convenience, denote the denominator of these expressions:

$$N = 4r^2(1 - \kappa^2)v_1v_2 + 2rv_1 + 2rv_2 + 1.$$

We then have

$$\frac{\partial m_1}{\partial \beta_1} = \frac{2rv_2 + 1}{N}; \frac{\partial m_1}{\partial \beta_2} = \frac{-\kappa 2rv_2}{N};$$

$$\frac{\partial m_2}{\partial \beta_1} = \frac{-\kappa 2rv_1}{N}; \frac{\partial m_2}{\partial \beta_2} = \frac{2rv_1 + 1}{N}.$$
The second derivatives of the objective function are:

\[
\begin{align*}
\frac{\partial^2 \phi}{\partial \beta_1^2} &= \frac{1}{N} \left\{ 
\begin{array}{c}
[1 - p \kappa - (2r(1 - \kappa^2)v_1 + 1) m_1 + \kappa m_2] (2rv_2 + 1) \\
+ [p - \kappa + \kappa m_1 - (2r(1 - \kappa^2)v_2 + 1) m_2] (-\kappa 2rv_1)
\end{array}
\right\} - \beta_1 \gamma^2 = 0, \\
\frac{\partial^2 \phi}{\partial \beta_2^2} &= \frac{1}{N} \left\{ 
\begin{array}{c}
[1 - p \kappa - (2r(1 - \kappa^2)v_1 + 1) m_1 + \kappa m_2] (-\kappa 2rv_2) \\
+ [p - \kappa + \kappa m_1 - (2r(1 - \kappa^2)v_2 + 1) m_2] (2rv_1 + 1)
\end{array}
\right\} - \beta_2 \gamma^2 = 0.
\end{align*}
\]

The second derivatives of the objective function are:

\[
\begin{align*}
\frac{\partial^2 \phi}{\partial \beta_1^2} &= \frac{1}{N} \left\{ 
\begin{array}{c}
-(2r(1 - \kappa^2)v_1 + 1) (2rv_2 + 1) + \kappa^2 2rv_1 \\
- \kappa (2rv_2 + 1) + (2r(1 - \kappa^2)v_2 + 1) \kappa 2rv_1
\end{array}
\right\} - \gamma^2 < 0,
\end{align*}
\]

or, simplifying,

\[
\frac{\partial^2 \phi}{\partial \beta_1^2} = -\frac{\kappa^2 2rv_1 + \kappa 2rv_2 + 1}{N} - \gamma^2 < 0,
\]

or, simplifying,

\[
\frac{\partial^2 \phi}{\partial \beta_2^2} = -\frac{\kappa 2rv_2 + (2r(1 - \kappa^2)v_1 + 1) (2rv_1 + 1)}{N} - q^2 < 0,
\]

or, simplifying,

\[
\frac{\partial^2 \phi}{\partial \beta_2^2} = -\frac{\kappa 2rv_2 + \kappa 2rv_1 + 1}{N} - q^2 < 0,
\]

and

\[
\frac{\partial^2 \phi}{\partial \beta_1 \partial \beta_2} = \frac{1}{N} \left\{ 
\begin{array}{c}
[(2r(1 - \kappa^2)v_1 + 1) \kappa 2rv_2 + \kappa (2rv_1 + 1)] (2rv_2 + 1) \\
+ [\kappa 2rv_2 + (2r(1 - \kappa^2)v_2 + 1) (2rv_1 + 1)] (\kappa 2rv_1)
\end{array}
\right\},
\]

or, simplifying,

\[
\frac{\partial^2 \phi}{\partial \beta_1 \partial \beta_2} = \frac{\kappa (2rv_1 + 2rv_2 + 1)}{N} > 0.
\]

Using this, we can write down the Hessian,

\[
H = \frac{1}{N} \left( \begin{array}{cc}
-(\kappa^2 2rv_1 + \kappa 2rv_2 + 1) - \gamma^2 \tilde{N} & \kappa (2rv_1 + 2rv_2 + 1) \\
\kappa (2rv_1 + 2rv_2 + 1) & -(2rv_1 + \kappa^2 2rv_2 + 1) - q^2 \tilde{N}
\end{array} \right)
\]

and its inverse,

\[
H^{-1} = \frac{\tilde{N}}{\tilde{D}} \left( \begin{array}{cc}
-(2rv_1 + \kappa^2 2rv_2 + 1) - q^2 \tilde{N} & -\kappa (2rv_1 + 2rv_2 + 1) \\
-\kappa (2rv_1 + 2rv_2 + 1) & -(\kappa^2 2rv_1 + 2rv_2 + 1) - \gamma^2 \tilde{N}
\end{array} \right),
\]

with

\[
\tilde{D} = \tilde{D} + q^2 \tilde{N} (\kappa^2 2rv_1 + 2rv_2 + 1) + \gamma^2 \tilde{N} (2rv_1 + \kappa^2 2rv_2 + 1) + \gamma^2 q^2 \tilde{N}^2 > 0,
\]

where \( \tilde{D} \) is the determinant of the Hessian when \( \tilde{N} \) is factored out and the terms \(-\gamma^2 \tilde{N}\) and \(-q^2 \tilde{N}\) are ignored. Thanks to the concavity of \( \phi \), we know that \( \tilde{D} \) is positive (and it is relatively easily found to be \( \tilde{D} = (1 - \kappa^2) N \)).
Comparative statics with respect to $\gamma$.

The comparative-statics equation with respect to $\gamma^2$ is

$$H \left( \begin{array}{c} \frac{\partial \beta_1}{\partial \gamma^2} \\ \frac{\partial \beta_2}{\partial \gamma^2} \end{array} \right) = \left( \begin{array}{c} \beta_1 \\ 0 \end{array} \right),$$

and we have

$$\left( \begin{array}{c} \frac{\partial \beta_1}{\partial \gamma^2} \\ \frac{\partial \beta_2}{\partial \gamma^2} \end{array} \right) = \frac{\tilde{N}}{\tilde{D}} \left( \begin{array}{c} -\beta_1 \{ (2rv_1 + \kappa^2 2rv_2 + 1) + q^2 \tilde{N} \} \\ -\beta_1 \{ (2rv_1 + 2rv_2 + 1) \} \end{array} \right).$$

clearly, $\partial \beta_1 / \partial \gamma^2 < 0$ (and hence, obviously, $\partial \beta_1 / \partial \gamma < 0$) while the sign of $\partial \beta_2 / \partial \gamma^2$ is opposite that of $\kappa$.

Comparative statics of effort levels. Now, note that

$$\frac{\partial \beta_2}{\partial \beta_1} = \frac{\kappa (2rv_1 + 2rv_2 + 1)}{2rv_1 + \kappa^2 2rv_2 + 1 + q^2 \tilde{N}}, \quad (A.3)$$

and consider $a$’s dependence on $\gamma$ through $\beta$: We have

$$a^*_1 = \frac{m_1 - \kappa m_2}{2(1 - \kappa^2)}; \quad a^*_2 = \frac{m_2 - \kappa m_1}{2(1 - \kappa^2)},$$

and

$$m_1 = \frac{(2rv_2 + 1) \beta_1 - \kappa \beta_2}{N}; \quad m_2 = \frac{(2rv_1 + 1) \beta_2 - \kappa \beta_1}{N}.$$

Thus:

$$\frac{\partial a^*_1}{\partial \beta_1} = \frac{1}{N} \left( (2rv_2 + 1) + \kappa^2 \right), \quad \frac{\partial a^*_1}{\partial \beta_2} = \frac{1}{N} \left( -\kappa [1 + (2rv_1 + 1)] \right),$$

$$\frac{\partial a^*_2}{\partial \beta_1} = \frac{1}{N} \left( -\kappa [(2rv_2 + 1) + 1] \right), \quad \frac{\partial a^*_2}{\partial \beta_2} = \frac{1}{N} \left[ \kappa^2 + (2rv_1 + 1) \right].$$

Looking at the effect of varying $\gamma$ on cost-saving incentives,

$$\frac{da^*_2}{d\gamma^2} = \left( \frac{\partial a^*_2}{\partial \beta_1} + \frac{\partial a^*_2}{\partial \beta_2} \frac{\partial \beta_2}{\partial \beta_1} \right) \frac{\partial \beta_1}{\partial \gamma^2},$$

where, $\partial \beta_2 / \partial \beta_1$ comes from (A.3) above; factoring out the positive constant, we have

$$\frac{da^*_2}{d\gamma^2} = \frac{1}{N} \left( -\kappa [(2rv_2 + 1) + 1] + [\kappa^2 + (2rv_1 + 1)] \right) \frac{\partial \beta_2}{\partial \beta_1} \frac{\partial \beta_1}{\partial \gamma^2}. \quad (A.4)$$

The main conclusion coming from this is that the first term is leading for large enough $v_2$, i.e. when the quality dimension is indeed hard to measure. We also have analogously,

$$\frac{da^*_1}{d\gamma^2} = \frac{1}{N} \left( [v_2 + 1 + \kappa^2] + [\kappa [1 + (2rv_1 + 1)]] \right) \frac{\partial \beta_2}{\partial \beta_1} \frac{\partial \beta_1}{\partial \gamma^2}, \quad (A.5)$$

and we find that here, too, the first term is leading for large enough $v_2$, implying – expectedly – that cost-saving effort is reduced.
Comparative statics with respect to $q$. The comparative-statics expressions are

$$\begin{pmatrix}
\frac{\partial \beta_1}{\partial q} \\
\frac{\partial \beta_2}{\partial q}
\end{pmatrix} = \frac{\bar{N}}{\bar{D}} \begin{pmatrix}
-\beta_2 \{\kappa (2rv_1 + 2rv_2 + 1) \\
-\beta_2 \{\kappa^2 2rv_1 + 2rv_2 + 1 \} - \gamma^2 \bar{N}
\end{pmatrix}.$$ 

The key observations to make are that the sign of $\frac{\partial \beta_1}{\partial q}$ is opposite to the sign of $\kappa$, and that $\frac{\partial \beta_2}{\partial q}$ is unambiguously negative.

Comparative statics with respect to $p$. The analysis follows similar lines as those underlying the comparative statics with respect to $q$: the derivatives are

$$\begin{pmatrix}
\frac{\partial \beta_1}{\partial p} \\
\frac{\partial \beta_2}{\partial p}
\end{pmatrix} = \frac{\bar{N}}{\bar{D}} \begin{pmatrix}
-\kappa (2rv_1 + 2rv_2 + 1) - q^2 \bar{N} \\
-\kappa (2rv_1 + 2rv_2 + 1) - (\kappa^2 2rv_1 + 2rv_2 + 1) - \gamma^2 \bar{N}
\end{pmatrix} \begin{pmatrix}
(2rv_1 + 2rv_2 + 1) / \bar{N} \\
(2rv_1 + \kappa^2 2rv_2 + 1) / \bar{N}
\end{pmatrix},$$

$$\begin{pmatrix}
\frac{\partial \beta_1}{\partial p} \\
\frac{\partial \beta_2}{\partial p}
\end{pmatrix} = \frac{\bar{N}}{\bar{D}} \begin{pmatrix}
-q^2 \kappa (2rv_1 + 2rv_2 + 1) \\
-q^2 (2rv_1 + \kappa^2 2rv_2 + 1)
\end{pmatrix}. $$

Expectedly, $\frac{\partial \beta_2}{\partial p} > 0$; it is also worth noting that for $q = 0$ there is no effect on $\beta_1$.

References


