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Revealed Preference Tests of Utility Maximization and Weak Separability of Consumption, Leisure and Money with Incomplete Adjustment

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ABSTRACT

Swofford and Whitney (1987) investigated the validity of two types of assumptions that underlie the representative agent models of modern macroeconomics and monetary economics. These assumptions are utility maximization and weak or functional separability that is required for an economic aggregate to exist.

To reinvestigate the structure of the representative consumer's preferences we develop a mixed integer programming revealed preference test with incomplete adjustment. We find that both a narrow official US monetary aggregate, M1, and a broad collection of assets are weakly separable. We further find that a modern analog of money as suggested by Friedman and Schwartz (1963) is also weakly separable. We also find that consumption goods and leisure are separable from all monetary goods. We find no evidence that official US M2 or MZERO are consistent with utility maximization and weak separability. That is, the assets in these measures do not meet the requirement for forming an aggregate over goods that is consistent with economic theory. Finally, we find that three broad categories of consumption goods, durables, nondurables and services, do not meet the weak separability conditions required for forming a consumption aggregate. However, a consumption aggregate of nondurables and services is weakly separable.

1. Introduction

Two important assumptions of modern macroeconomic modeling are utility maximization and at least weak separability of the representative consumer's utility function. The first assumption provides the starting point for modeling dynamic general equilibrium models leading to steady state relationships that are then approximated for empirical analysis. The weak separability assumption has important implications for demand based studies and is often implicity made in many areas of economic research.[2]

Weak separability for the consumption goods and leisure from "money" is recognized for restricting the effect of money on real economic activity. This is explicitly embedded when specifying the functional form of the one period utility function. However, the question of the appropriate components of the consumption or monetary aggregate is rarely addressed. Examples include the questions of whether the consumption aggregate includes durable goods and whether the monetary aggregate includes money market mutual fund.

Since macroeconomic and monetary economic researchers are interested in the behavior of aggregate economic variables, it is important to have a good idea of what level of aggregation would best approximate the variables from theory. Weak separability between "money" and other goods and assets is required for the existence of a monetary aggregate. Weak separability of major categories of consumption expenditure from leisure and monetary goods must hold if

[2] To make the model solutions tractable many researchers make an assumption of intertemporal additive separability. That assumption is not tested in this paper.

we expect consumption to have a stable relationship with income. Weak separability tests can also be used to evaluate the important question of whether money is separable from consumption and leisure.

Using revealed preference analysis, Swofford and Whitney (1987) investigated the issue of weak separability of groups of money goods that might form a monetary aggregate consistent with economic theory of aggregation over goods. We revisit this analysis now for a couple of reasons. One reason is that in the decade before the Swofford and Whitney paper innovations produce various new goods like money market mutual funds. Thus, the structure of preferences reported in Swofford and Whitney (1987) might have changed since the 1970 to 1985 period covered by their data. A second reason is that the 2008 financial meltdown and ensuing great recession has made what is the appropriate measure of money and liquidity in the economy a pressing issue. Barnett and Chauvet (2011) placed much of the blame for the recent financial crisis on faulty measures of monetary aggregates relied upon and disseminated by the Federal Reserve. A third reason is that recent advances in revealed preference testing using mixed integer programming have made testing necessary and sufficient conditions for weak separability more tractable.[3] Finally, these more tractable tests can be made to allow for incomplete adjustment.

The results presented in this paper are stronger than those in Swofford and Whitney (1987) because advances in revealed preference testing allows us to check necessary and sufficient conditions more easily, rather than just being able to present results of necessary tests as for some of the results presented in Swofford and Whitney (1987). While there are some differences, the results from 1987 hold up remarkably well.

[3] Using revealed preference methods to test whether a utility function is weakly separable was originally suggested by Varian (1983). These results are in turn an extension of the theory developed by Afriat (1967) and later refined by Varian (1982), who summarized Afriat's original results in what he named Afriat's theorem.

We find that from 2000 to 2012 a narrow official US monetary aggregate, M1, a modern analog to money as defined by Friedman and Schwartz (1963) and a broad collection of assets meet the necessary and sufficient conditions for weak separability. We also find that consumption goods and leisure are separable from all the monetary goods in our data set. Further we find no evidence that official US M2 or MZERO are consistent with utility maximization and weak separability. That is, the assets in these potential aggregates do not meet the requirements for forming an aggregate over goods that is consistent with economic theory. Finally, we find that the three categories of consumption goods do not meet the weak separability conditions required for forming a consumption aggregate.

We start this paper by reviewing why weak separability is an important maintained hypothesis that needs testing. We also review how economic monetary aggregates are conceptually superior to other aggregates in the next section of the paper.

2. Economic Monetary Aggregates

Macroeconomic models typically start with a representative consumer faced with maximizing utility over a lifetime.[4] In an intertemporal framework with time additive utility, Barnett (1980) shows, weak separability of some monetary good is required to construct monetary aggregates broader than currency that is consistent with economic theory of aggregation over goods.

Barnett (1980) also showed, the FED's reported aggregates are neither composed nor constructed in a manner consistent with economic theory. The Fed currently reports two simple sum monetary aggregates, M1, currency plus demand deposits plus travelers checks plus other checkable deposits, and M2, M1 plus savings deposits, including retail money market funds, plus

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[4] As Hicks (1956) says concerning representative agents, "To assume that the representative consumer acts like an ideal consumer is a hypothesis worth testing".

time deposits plus small CDs. In the past the FED has reported broader measures, such as M3 and L that include such assets as large time deposits, overnight repurchase agreements and commercial paper.[5] Additionally, components of the various measures have changed. For example today's M2 includes deposits at thrift institutions but M2 initially only included deposits at commercial banks. New financial products, such as money market deposits accounts and sweeps, have led to adjustments in the definitions of official FED aggregates. While some of these changes were made with a priori criteria such as liquidity and capital certainty, the changes were largely driven by increasing instability between the aggregates and economic activity.

All FED aggregates current and past have been reported as the sum of their components. Barnett pointed out that using the simple sum is akin to measuring transportation services in a city by summing up roller skates, cabs, trains, etc. You would not expect the sum to provide a meaningful measure of transportation services; yet, the Fed treats \$100 in checking as providing the same monetary services as \$100 locked away in a certificate of deposit.

Barnett (1980) made this argument forcefully in which he presents a sound theoretic case for a Divisia quantity index based on much the same theories used to construct other economic aggregates like price indices and GDP. Similar indexes have been constructed for various FED defined aggregates by the St. Louis Federal Reserve Bank. Although the theoretical basis for choosing the components are well understood in the literature on economic aggregates, less effort has been devoted to using this theory to find the appropriate components.

If we think of a utility function containing K individual goods and services, then there are well defined conditions for forming an equivalent function composed of J, less than K, aggregates. In particular, a necessary condition for separating services provided by monetary assets into an aggregate separate from consumption and leisure is that the groups must be at least

[5] FED last reported M3 and L in March 2006.

weakly separable from other goods. The existence of weakly separable preferences over money and consumption services is a question that can only be investigated empirically. Given the broad spectrum of assets providing some degree of monetary services, it is important to determine which ones belong in the separable monetary aggregate. Borrowing from Barnett not only would we not want to add roller skates and subway trains to measure transportation services in the economy, we probably do not want to consider roller skates as contributing to transportation services at all. [6]

We use currency and demand deposits as our "anchor" since they clearly provide the monetary services that we wish to measure. Hence, all the aggregates considered below include currency and demand deposits. Beyond these, we allow the data to reveal the broadest monetary group that is separable from consumption and leisure.

In the next section we briefly discuss the revealed preference tests used to check for utility maximization and weak separability. We show how binary mixed integer programming can be used to check if inequalities required by weakly separable utility maximization hold.

3. An Integer Programming Approach to Weak Separability with Incomplete Adjustment

Suppose the K goods and services in the market are observed at T time periods. Let $\boldsymbol{x}_t = (x_{1t}, \dots, x_{Kt})$ denote the observed quantity-vector at time $t \in T$, with the corresponding price-vector $\boldsymbol{p}_t = (p_{1t}, \dots, p_{Kt})$. Suppose the data set $\{\boldsymbol{p}_t; \boldsymbol{x}_t\}_{t \in T}$ is split into two sub-groups with prices $\boldsymbol{z}_t = (p_{1t}, \dots, p_{Ht})$ and $\boldsymbol{r}_t = (p_{H+1t}, \dots, p_{Kt})$ and corresponding quantities $\boldsymbol{w}_t = (x_{1t}, \dots, x_{Ht})$ and $\boldsymbol{y}_t = (x_{H+1t}, \dots, x_{Kt})$, for $H < K$. Consider the list $\mathbb{T} = \{\boldsymbol{z}_t, \boldsymbol{r}_{t,}; \boldsymbol{w}_t, \boldsymbol{y}_t\}_{t \in T}$, which we subsequently will refer to simply as 'the data'. We say that the data $\mathbb{T}$ can be *rationalized* by a weakly separable utility function if there exists a well-behaved, i.e. continuous,

[6] See Barnett, Offenbacher and Spindt (1984, p.1051).

concave and strictly increasing, utility function U and a well-behaved sub utility function V, such that $\{\boldsymbol{w}_t; \boldsymbol{y}_t\}_{t \in T}$ solves the utility maximizing problem (in every time period):

$$\max_{\{\boldsymbol{w},\boldsymbol{y}\}} U\big(\boldsymbol{w}, V(\boldsymbol{y})\big) \ s.t.\ \boldsymbol{z}_t\boldsymbol{w} + \boldsymbol{r}_t\boldsymbol{y} \leq \boldsymbol{z}_t\boldsymbol{w}_t + \boldsymbol{r}_t\boldsymbol{y}_t. \tag{1}$$

There exist a number of different revealed preference procedures to test whether the data can be rationalized by a weakly separable utility structure; See for examples Varian (1983), Fleissig and Whitney (2003) and Cherchye, Demunyck, De Rock and Hjertstrand (2012). All these procedures are static in the sense that they lack capacity to allow for dynamic elements or various kinds of habit persistence such as adjustment costs and the formation of expectations in the data.

Swofford and Whitney (1994) generalized the static weak separability approach by taking habit formation and other incomplete adjustment effects into consideration.[7] Specifically, their test (from now on referred to as the SW test) allows for incomplete adjustment for expenditures in the sub-utility function, which formally amounts to adding the extra budget restriction:

$$\boldsymbol{r}_t\boldsymbol{y} \leq \boldsymbol{r}_t\boldsymbol{y}_t,$$

in the utility maximization problem (1), where $\boldsymbol{r}_t\boldsymbol{y}_t$ is the expenditure on the $\boldsymbol{y}$ goods in time $t \in T$. The basic idea of incomplete adjustment is that the expenditure $\boldsymbol{ry}$ may not be the optimal allocation that maximizes the utility. To see this, let ψ be the shadow price associated with the expenditure $\boldsymbol{ry}$.[8] If $\psi = 0$ then $\boldsymbol{ry}$ is optimally adjusted, while if $\psi < 0$ (> 0) then $\boldsymbol{ry}$ is less (more) than desired. Given this, ψ can be interpreted as a measure of the degree to which $\boldsymbol{ry}$ is adjusted to optimal levels.

[7] Incomplete adjustment concerns the agents' inability to fully adjust their optimal consumption of goods within the observed period. For example, consumers facing a price level shock may take more than one period to adjust their money balances. In this case, it is usually total money balances, and not the way it is held, that is presumed to require time to adjust.

[8] In other words, ψ is the Lagrange multiplier corresponding to the extra budget restriction $\boldsymbol{r}_t\boldsymbol{y} \leq \boldsymbol{r}_t\boldsymbol{y}_t$ in the utility maximization problem (1).

Although theoretically appealing, the SW test is difficult to implement because of its computational complexity (we discuss this in more detail below). For this reason, the remainder of this section puts forward a new computationally tractable revealed preference test that like the SW test allows for incomplete adjustment.

The SW test as well as the test procedure developed in this paper are based on the following theorem, which provides necessary and sufficient non-parametric revealed preference conditions for when a data set can be rationalized by a well-behaved weakly separable utility function that allows for incomplete adjustment in the block of separable goods $\boldsymbol{y}$.

Theorem 1. *(Swofford and Whitney, 1994). Consider the data set* $\mathbb{T} = \{\boldsymbol{z}_t, \boldsymbol{r}_{t,}; \boldsymbol{w}_t, \boldsymbol{y}_t\}_{t\in T}$.

Conditions (SW1)-(SW3) are equivalent:

(SW1) There exists a weakly separable, concave, monotonic, continuous and non-satiated utility function that allows for incomplete adjustment in the weakly separable goods $\boldsymbol{y}$*, and that rationalizes the data* $\mathbb{T}$.

(SW2) There exist numbers $V_t, U_t, \mu_t > 0, \lambda_t > 0$ *and* ψ_t*, such that the following inequalities hold:*

$$V_t - V_s - \mu_t \boldsymbol{r}_t(\boldsymbol{y}_t - \boldsymbol{y}_s) \geq 0 \qquad \text{(sw2\_i)}$$

$$U_t - U_s - \lambda_t \boldsymbol{z}_t(\boldsymbol{w}_t - \boldsymbol{w}_s) - \frac{(\lambda_t + \psi_t)}{\mu_t}(V_t - V_s) \geq 0 \qquad \text{(sw2\_ii)}$$

$$\lambda_t + \psi_t > 0. \qquad \text{(sw2\_iii)}$$

(SW3) There exist numbers $V_t, u_t, \mu_t > 0, \lambda_t > 0$ *and* ψ_t *such that the following inequalities hold:*

$$V_t - V_s - \mu_t \boldsymbol{r}_t(\boldsymbol{y}_t - \boldsymbol{y}_s) \geq 0 \qquad \text{(sw3\_i)}$$

$$if\ \frac{\lambda_t \mu_t}{(\lambda_t + \psi_t)} \boldsymbol{z}_t(\boldsymbol{w}_t - \boldsymbol{w}_s) + (V_t - V_s) \geq 0 \quad then\ u_t - u_s \geq 0 \qquad \text{(sw3\_ii)}$$

$$if\ \frac{\lambda_t \mu_t}{(\lambda_t + \psi_t)} \boldsymbol{z}_t(\boldsymbol{w}_t - \boldsymbol{w}_s) + (V_t - V_s) > 0 \quad then\ u_t - u_s > 0 \qquad \text{(sw3\_iii)}$$

$$\lambda_t + \psi_t > 0. \qquad \text{(sw3\_iv)}$$

Swofford and Whitney (1994) proved that conditions (SW1) and (SW2) are equivalent while the equivalence between (SW2) and (SW3) is established in a slightly different form by Cherchye, Demunyck, De Rock and Hjertstrand (2012). Conditions (SW2) and (SW3) provide means to construct two different methods to verify whether the data set $\mathbb{T} = \{\boldsymbol{z}_t, \boldsymbol{r}_{t,}; \boldsymbol{w}_t, \boldsymbol{y}_t\}_{t \in T}$ is rationalizable by a weakly separable utility function with incomplete adjustment in the separable $\boldsymbol{y}$-goods. The SW test constitutes the first method and consists of using non-linear programming techniques to check whether there exists a solution to the inequalities *(sw2_i)-(sw2_iii)* in condition (SW2). If such a solution exists, this optimization problem yields a solution to the unknowns $V_t, U_t, \mu_t, \lambda_t$ and ψ_t. Given these numbers, Swofford and Whitney (1994) interpreted,

$$IA_t = \frac{|\psi_t|}{\lambda_t}, \tag{2}$$

as a measure of incomplete adjustment. Note here that (2) is the increment of utility from spending an additional monetary unit on the separable goods relative to the marginal utility of total expenditure. As such, it is a ratio of marginal utilities and therefore invariant to monotonic transformations of the utility index. A problem with the SW test, as with any other test procedure based on condition (SW2), is that it is very computationally challenging since it requires minimizing a non-linear objective function subject to at least $T(T-1)$ non-linear and $T(T-1)$ linear constraints.[9]

Consider instead the inequalities *(sw3_i)-(sw3_iv)* in condition (SW3) in Theorem 1. As they are currently presented, these inequalities provide no computational advantage over the original inequalities, *(sw2_i)-(sw2_iii)*. This follows because *(sw3_ii)* and *(sw3_iii)* are still non-linear in the term $\lambda_t\mu_t/(\lambda_t + \psi_t)$, and consequently difficult to implement. But the inequalities can be transformed into a set of linear inequalities, which makes it possible to solve them using

[9] In their empirical application, Swofford and Whitney (1994) had to divide up the sample of 62 observations into two overlapping samples of 40 observations before being able to solve the minimization problem.

simple mixed integer linear programming techniques. This results in a computationally simpler test than would be the case for any other test based on *(sw2_i)-(sw2_iii)* including the SW test.

The basic idea behind our procedure is that the non-linear term $\lambda_t \mu_t/(\lambda_t + \psi_t)$ can without loss of generality be written as:

$$\frac{\lambda_t \mu_t}{(\lambda_t + \psi_t)} = \mu_t + \varepsilon_t^p - \varepsilon_t^n, \tag{3}$$

where $\varepsilon_t^p \geq 0$ and $\varepsilon_t^n \geq 0$ should be interpreted as 'slack' terms. Recall that ψ_t denotes the shadow price corresponding to the expenditure $\boldsymbol{r}_t \boldsymbol{y}_t$. Clearly, we have $\psi_t = 0$ if and only if $\varepsilon_t^p = \varepsilon_t^n$, which means that the expenditure on the separable $\boldsymbol{y}$-goods at time $t \in T$ is optimally adjusted if and only if $\varepsilon_t^p = \varepsilon_t^n$. Now, substituting (3) into *(sw3_i)-(sw3_iv)* gives us the following set of inequalities:

$$V_t - V_s - \mu_t \boldsymbol{r}_t (\boldsymbol{y}_t - \boldsymbol{y}_s) \geq 0 \qquad \textit{(sw3\_i*)}$$

$$if\ \theta_t \boldsymbol{z}_t (\boldsymbol{w}_t - \boldsymbol{w}_s) + (V_t - V_s) \geq 0 \quad then\ u_t - u_s \geq 0 \qquad \textit{(sw3\_ii*)}$$

$$if\ \theta_t \boldsymbol{z}_t (\boldsymbol{w}_t - \boldsymbol{w}_s) + (V_t - V_s) > 0 \quad then\ u_t - u_s > 0 \qquad \textit{(sw3\_iii*)}$$

$$\theta_t = \mu_t + \varepsilon_t^p - \varepsilon_t^n, \qquad \textit{(sw3\_iv*)}$$

where $\theta_t > 0$ and $\mu_t > 0$.

Although *(sw3_i*)-(sw3_iv*)* have the substantial benefit that they are linear which makes them much more suitable for empirical analysis, they are not quite operational in its current form. The reason for this is that we need to link the left-hand sides of *(sw3_ii*)* and *(sw3_iii*)* to the corresponding right hand sides of these inequalities. Here, we use binary (0-1) variables to capture the logical relation between the inequalities. The following theorem provides a formal link between *(sw3_i*)-(sw3_iv*)* and a set of operational inequalities by introducing $T(T-1)$ binary variables (denoted by $\{X_{t,s}\}_{s,t\in T}$).[10]

Theorem 2. *The data set* $\mathbb{T} = \{\boldsymbol{z}_t, \boldsymbol{r}_{t,}; \boldsymbol{w}_t, \boldsymbol{y}_t\}_{t\in T}$ *satisfies (sw3_i*)-(sw3_iv*) if and only if there exist numbers* $V_t, u_t \in [0,1[\,, \theta_t \in\,]0,1[, \mu_t > 0$ *and binary variables* $X_{t,s} \in \{0,1\}$ *such that:*

$$V_t - V_s - \mu_t \boldsymbol{r}_t(\boldsymbol{y}_t - \boldsymbol{y}_s) \geq 0 \qquad \textit{(mip\_i)}$$

$$u_t - u_s < X_{t,s} \qquad \textit{(mip\_ii)}$$

$$(X_{t,s} - 1) \leq u_t - u_s \qquad \textit{(mip\_iii)}$$

$$\theta_t \boldsymbol{z}_t(\boldsymbol{w}_t - \boldsymbol{w}_s) + (V_t - V_s) < X_{t,s} A_t \qquad \textit{(mip\_iv)}$$

$$(X_{t,s} - 1)A_t \leq \theta_t \boldsymbol{z}_t(\boldsymbol{w}_t - \boldsymbol{w}_s) + (V_t - V_s) \qquad \textit{(mip\_v)}$$

$$\theta_t = \mu_t + \varepsilon_t^p - \varepsilon_t^n, \qquad \textit{(mip\_vi)}$$

where A_t *is a fixed number larger than* $\boldsymbol{z}_t \boldsymbol{w}_t + 1$.

We propose to check if the inequalities *(mip_i)-(mip_vi)* have a solution by solving the following mixed integer programming (MIP) problem (with respect to $V_t, u_t, \theta_t, \mu_t, X_{t,s}, \varepsilon_t^p$ and ε_t^n)[11]:

$$\min F = \sum_{t=1}^{T} \left(\varepsilon_t^p + \varepsilon_t^n\right) \quad s.t.$$

$$(mip\_i) - (mip\_vi)$$

$$0 \leq \{V_t, u_t\} < 1$$

$$0 < \theta_t < 1$$

$$\mu_t > 0$$

[10] The proof of Theorem 2 follows by making use of already existing results: *(mip_i)* and *(mip_vi)* are equivalent to *(sw3_i*)* and *(sw3_iv*)* by definition. The equivalence between *(mip_ii)-(mip_v)* and *(sw3_ii*)-(sw3_iii*)* is proven (in a slightly different form) in Cherchye, Demuynck, De Rock and Hjertstrand (2012, Theorem 4).
[11] To handle the strict inequalities in this MIP problem, we use weak inequalities and subtract or add, depending on the restriction, a very small but fixed number to the inequalities.

$$X_{t,s} \in \{0,1\}$$

$$\{\varepsilon_t^p, \varepsilon_t^n\} \geq 0.$$

If there exists a feasible solution to this problem, then preferences are weakly separable in the $\boldsymbol{y}$-goods. Specifically, if a solution exists and $F = 0$, then preferences are weakly separable and the expenditures on the separable $\boldsymbol{y}$-goods are optimally adjusted at all $t \in T$.[12] On the other hand, if there exists a feasible solution with $F > 0$, then the expenditures on the separable goods are not optimally adjusted for at least one $t \in T$. In this case, we may be interested in calculating the required "adjustment". Recalling Swofford and Whitney's (1994) measure of the degree of incomplete adjustment defined by (2), and solving for $|\psi_t|/\lambda_t$ in (3) gives the following expression for IA_t (in terms of the numbers μ_t, ε_t^p and ε_t^n):

$$IA_t = \frac{|\psi_t|}{\lambda_t} = \frac{|\varepsilon_t^p - \varepsilon_t^n|}{(\mu_t + \varepsilon_t^p - \varepsilon_t^n)}, \quad (4)$$

for all $t \in T$. Summarizing our test procedure, the first step consists of solving the MIP problem. And if there exists a feasible solution with $F > 0$ then, in a second step, we suggest to calculate the required adjustment from (4) using the solutions μ_t, ε_t^p and ε_t^n from the MIP problem. With the mixed integer programming problem now specified, we turn to the matter of data. We next set forth the data categories and sources.

4. Data

We examine four categories of consumption goods plus leisure:

1. SER: expenditures on services
2. NDUR: expenditures on nondurables
3. DUR: expenditures on durables

[12] More precisely, recall that there is no incomplete adjustment (i.e. $\psi_t = 0$) if and only if $\varepsilon_t^p = \varepsilon_t^n$. But since the objective function minimizes the 'slacks', we have $\varepsilon_t^p = \varepsilon_t^n$ if and only if $\varepsilon_t^p = \varepsilon_t^n = 0$. This means that there is no incomplete adjustment if and only if $\varepsilon_t^p = \varepsilon_t^n = 0$.

4. LEIS: leisure.

The consumption categories are all quarterly data that is seasonally adjusted and divided by civilian labor force age sixteen and over. The prices of services and nondurables are the respective implicit price deflators. The price of durables is a user cost. A ten percent annual depreciation rate was applied each quarter to annualize expenditures on durables to make it compatible with annualized expenditures on services and nondurables. Expenditures are from the Bureau of Economic Analysis and civilian labor force over age 16 data is from the Current Population Survey. Leisure is calculated as 98 hours minus average hours worked per week during the quarter. Hours and hourly wage rates were obtained from the Economic Report of the President.

The monetary assets and associated user costs are obtained from Historical Financial Statistics, and consist of:

5. CUR+DD: currency plus demand deposits
6. TC: traveler's checks
7. OCDCB and OCDTH: other checkable deposits at commercial banks and thrifts.
8. SD-CB and SD-TH: savings deposits at commercial banks and thrifts.
9. STDCB and STDTH: small time deposits at commercial banks and thrifts.
10. MMMFR and MMFI: retail and institutional money market funds.
11. TB: treasury bills
12. CP: commerical paper.
13. LTD: large time deposits

The monetary good are deflated by the implicit price deflator and the civilian labor force sixteen and over to yield real per capita balances. Their user costs are multiplied by the implicit price deflator to yield nominal prices of a dollar of real balances.

Thus we hypothesize a representative consumer with the above 17 arguments in the utility function. The results from testing this hypothesis and various weak separability hypotheses are presented in the following section.

5. Results

First, the data as outlined above were found to be consistent with utility maximization. This finding also means the data are consistent modeling of an optimizing representative agent.

In Table 1 we present some structures we found to be consistent with weakly separable utility maximization using the mixed integer programming revealed preference test with incomplete adjustment discussed above. This is an interesting set of positive results. Structure 1 shows that all the assets in M1 are weakly separable from consumption goods, leisure and other monetary goods. Thus people may be using M1 as money. Structure 2 shows that the monetary assets that are a modern analog to what Friedman and Schwartz (1963) found to be money over a broad swath of US history are weakly separable from consumption goods, leisure and other monetary goods. Thus the public may also be using money assets similar to those being used over the period Friedman and Schwartz studied. Structure 3 shows that a very board collection of monetary goods including currency are weakly separable from consumption goods and leisure. Thus the public may be using a very broad monetary aggregate like that proposed by Barnett (1980).

Neither of the tests of these three utility structures found any evidence of incomplete adjustment. That is, the objective function of each mixed integer programming problem for these structures were zero, implying no incomplete adjustment.

To investigate how powerful these tests are, we complement our analysis by conducting a series of diagnostic tests. First, we calculate the power in terms of the probability of detecting random behavior following the suggestion by Bronars (1987). In particular, we calculate the probability of rejecting the revealed preference test given that the model does not hold.[13] It is important that the model in question have good power since one that passes the revealed preference restrictions have little value if it has little discriminatory power (i.e. the restrictions are difficult to reject for the given data). We found an optimal power of 100% for all of the above tests; thus, the tests are able (with very high probability) to reject irrational uniform behavior when this is the case. As a second diagnostic test, we also report the recently proposed predictive success measure by Beatty and Crawford (2011).[14] This measure was introduced as a tool of comparing and choosing between models. Although our analysis do not explicitly require us to choose between the different models, it is nevertheless interesting to investigate whether there may be models that are able to better explain the data. The predictive success for all three tests was found to be 1, which is also the highest attainable number. Thus, by definition, there cannot exist a competing model that better explain the data in terms of predictive success.

Picking the best monetary aggregate for a direct or indirect policy target from among the various monetary aggregates that meet the weak separability criteria for the existence of a monetary aggregate is beyond the ability of revealed preference tests. These aggregates will

[13] We simulated 1000 random series for the 17 arguments in the utility function from a uniform distribution on the given budget hyperplane for the corresponding prices and total expenditure. The power is calculated as one minus the proportion of the randomly generated bundles that are consistent with weak separability.

[14] This measure is calculated as the difference between the pass rate (either 1 or 0) and 1 minus the power.

need to be sorted out via other (parametric) empirical methods, based on for example time series procedures.

We can, however, make on comment on sorting the aggregates identified in Table 1. As Greenspan (1995) pointed out, sweeping of demand deposits into interest bearing account may be making the FED official M1 definition of money less useful. The aggregates broader than M1 in Table 1 internalize sweeps that Jones, Dutkowsky and Elger (2005) found to be important in identifying appropriate monetary aggregates.[15]

There are two additional results in Table 1. Structure 4 shows that durables, nondurables, services and leisure are weakly separable from all monetary goods. While this rules out that money could affect the marginal rate of substitution between leisure and any of the three categories of consumption, it also rules out the existence a consumption good composed of durables, nondurables and services. However, as structure 5 shows, the necessary condition for a consumption aggregate of nondurables and services is met.

In Table 2 we present several structures we found not to be consistent with weakly separable utility maximization. Negative results are important as they rule can be used to rule out certain groups of assets as monetary aggregates.

Structure 6 in Table 2 hypothesizes a sub-utility function and thus a monetary aggregate containing the assets in M2. This result means that the public behavior is not consistent with using an M2 monetary aggregate and policy that focused directly or indirectly on such an aggregate would likely be unsuccessful. Structure 7 in Table 2 hypothesizes a sub-utility function and thus a monetary aggregate containing the assets in MZM, which is a monetary aggregate composed of all assets with a zero maturity. This result suggests that MZM is also not an

[15] We do not use data on sweeps as the FED has discontinued their data on sweeps, but since both the broadest admissible aggregate and the modern analog to Friedman and Schwartz money included MMDA, the effects of sweeps would be internalized in those aggregates.

appropriate aggregate, and policy directly or indirectly targeting it would likely be unsuccessful. Finally, we find that three categories of consumption goods are not weakly separable from leisure and monetary goods indicating that one could not expect a consumption aggregate of those categories to have a stable relationship with current income.

Overall, we have found as we did in 1987 that the public may be using a relatively narrow monetary aggregate like M1 or Friedman and Schwartz (1963) money. Unlike 1987 we found that the public may be using a very broad monetary aggregate. We still do not find support for intermediate monetary aggregates M2 or MZM.

Further, we again as in 1987 find no evidence that the public is using a relatively narrow monetary aggregate that included money market mutual funds. This is despite the checkable feature associated with some of these assets.

6. Summary and Conclusions

To test for the existence of monetary aggregates in US quarterly data on consumption goods, leisure and monetary goods from 2000 to 2011, we develop a mixed integer programming revealed preference test that allows for incomplete adjustment in the weakly separable goods. We found interesting positive and negative results:

- We find the data are consistent with utility maximization. That means the data are also consistent with real business cycle modeling.
- We find evidence for a relatively narrow monetary aggregate like M1 or Friedman and Schwartz (1963) money. Unlike 1987 we found that the public may be using a very broad monetary aggregate.

- We do not find support for intermediate monetary aggregates M2 or MZM. Further again as in 1987, we find no support in the data for an intermediate aggregate that includes money market mutual funds.
- We find that three categories of consumption goods are not weakly separable from leisure and monetary goods. This means a consumption aggregate of durables, nondurables and services are not consistent with our data.
- Finally, we find that nondurables and services are separable from other goods. This means a consumption aggregate of nondurables and services meet the requirements for aggregation over goods.

References

Afriat, Sydney N. (1967). "The Construction of Utility Functions from Expenditure Data", *International Economic Review* 8, 66-77.

Barnett, William, A. (1980). "Economic Monetary Aggregate: An Applications of Index Number and Aggregation Theory", *Journal of Econometrics* 14, 11-48.

Barnett, William, A. and Marcelle Chauvet (2011). "How Better Monetary Statistics Could Have Signaled the Financial Crisis", *Journal of Econometrics* 161, 6-23.

Barnett, William, A., Edward K. Offenbacher and Paul A. Spindt (1984). "The New Divisia Monetary Aggregates", *Journal of Political Economy* 92, 1049-1085.

Beatty, Tim and Ian Crawford (2011). "How Demanding is the Revealed Preference Approach to Demand?", *American Economic Review* 101, 2782-2795.

Bronars, Stephen G. (1987). "The Power of Nonparametric Tests of Preference Maximization," *Econometrica* 55, 693-698.

Cherchye, Laurens, Thomas Demuynck, Bram De Rock and Per Hjertstrand (2012), "Revealed Preference Tests for Weak Separability: an integer programming approach", working paper.

Deaton, Angus and John Muellbauer (1980). *Economics of Consumer Behavior*, Cambridge: Cambridge University Press.

Fleissig, Adrian R. and Gerald A Whitney (2003). "A New PC-Based Test for Varian's Weak Separability Conditions", *Journal of Business and Economic Statistics*, 21, 133-144.

Friedman, Milton and Anna J. Schwartz (1963). *A Monetary History of the United States, 1867-1960*, Princeton: Princeton University Press (for the National Bureau of Economic Research).

Greenspan, Alan (1995) “Monetary Policy Report to the Congress”, *Federal Reserve Bulletin*, August, 778-783.

Hicks, John R. (1956). *Value and Capital* Oxford: Oxford University Press.

Jones, Barry E., Donald H. Dutkowsky and Thomas Elger (2005) “Sweep programs and Optimal Monetary Aggregation*”, Journal of Banking & Finance* 29, 483-508.

Swofford, James L. and Gerald A. Whitney (1986). “Nonparametic Tests of Utility Maximization and Weak Separability for Consumption, Leisure and Money”, *Review of Economic and Statistics*, 69, 458-464.

_____ (1986). “Flexible Functional Forms and the Utility Approach to the Demand for Money: A Nonparametric Analysis”, *Journal of Money, Credit, and Banking*, 18, 383-389.

_____ (1987). “Nonparametic Tests of Utility Maximization and Weak Separability for Consumption, Leisure and Money”, *Review of Economic and Statistics* 69, 458-464.

_____ (1994). “A Revealed Preference Test for Weakly Separable Utility Maximization with Incomplete Adjustment”, *Journal of Econometrics* 60, 235-249.

Varian, Hal R. (1982). “The Nonparametic Approach to Demand Analysis”, *Econometrica*, 50, 945-973

____ (1983). “Nonparametric Tests of Consumer Behavior”, *Review of Economic Studies*, 50, 99-110.

Table 1

Some Structures For Which Weak Separability Does Obtain

1. U(DUR, NDUR, SER, LEIS, **V(CUR+DD, TC, OCD-CB, OCD-TH)**, SD-CB, SD-TH, MMMF-R, STD-CB, STD-TH, MMMF-I, T-BILLS, CP, LTD)

2. U(DUR, NDUR, SER, LEIS, **V(CUR+DD, TC, OCD-CB, OCD-TH, SD-CB, SD-TH)**, MMMF-R, STD-CB, STD-TH, MMMF-I, T-BILLS, CP, LTD))

3. U(DUR, NDUR, SER, LEIS, **V(CUR+DD, TC, OCD-CB, OCD-TH, SD-CB, SD-TH, MMMF-R, STD-CB, STD-TH, MMMF-I, T-BILLS, CP, LTD)**)

4. U(**V(DUR, NDUR, SER, LEIS)**, CUR+DD, TC, OCD-CB, OCD-TH, SD-CB, SD-TH, MMMF-R, STD-CB, STD-TH, MMMF-I, T-BILLS, CP, LTD)

5. U(**V(NDUR, SER)**, DUR, LEIS, CUR+DD, TC, OCD-CB, OCD-TH, SD-CB, SD-TH, MMMF-R, STD-CB, STD-TH, MMMF-I, T-BILLS, CP, LTD)

Table 2

Some Structures For Which Weak Separability Does NOT Obtain

6. U(DUR, NDUR, SER, LEIS, **V(CUR+DD, TC, OCD-CB, OCD-TH, SD-CB, SD-TH, MMMF-R, STD-CB, STD-TH)**, MMMF-I, T-BILLS, CP, LTD))

7. U(DUR, NDUR, SER, LEIS, **V(CUR+DD, TC, OCD-CB, OCD-TH, SD-CB, SD-TH, MMMF-R, MMMF-I)**, STD-CB, STD-TH, T-BILLS, CP, LTD)

8. U(**V(DUR, NDUR, SER)**, LEIS, CUR+DD, TC, OCD-CB, OCD-TH, SD-CB, SD-TH, MMMF-R, STD-CB, STD-TH, MMMF-I, T-BILLS, CP, LTD)