Price Competition with Convex Costs

[WORK IN PROGRESS]

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- The "Bertrand paradox"

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  3. Study incentives make cost reducing investments.

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  - Each firm is assumed to divide its production evenly over the production plants.

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  - Firm i faces the following demand:

$$D_i(p_i, p_j) = \begin{cases} D(p_i) & \text{if } p_i < p_j \ \forall j \\ m_i D(p_i)/(m) & \text{if } p_i = p_j \forall j \\ 0 & \text{if } p_i > p_j \end{cases}$$

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$$\hat{\pi}_i(p, m_i) = \frac{m_i p D(p)}{m_i} - m_i C\left(\frac{D(p)}{m_i}\right) \text{ if } p = p_i < p_j \ \forall i \neq j$$

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  - Duopolistic profit:

$$\pi_i(p, m_i, m_j) = \frac{m_i p D(p)}{m} - m_i C\left(\frac{D(p)}{m}\right) \text{ if } p = p_i = p_j \ \forall i$$

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Proposition (1)

There exists a NE. In this equilibrium both firms set the unique price $\underline{p}$.

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- Equilibrium selection?

Welfare Properties

- Assume that D is a differentiable function with $D'(p) < 0$ for all $p < p^{Max}$. We also make the corresponding assumption on C, i.e. C is a differentiable function such that $C'(q) \geq 0$.

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- The social welfare function is given by

$$W(p) = S(p) + \Pi(p)$$

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- A necessary condition for a common (interior) price p to be socially optimal is $(W'(p) = 0)$.

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- $p^s \in [\underline{p}, \bar{p}]$, i.e. implementable as a NE.

Investments

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- Demand:

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- Cost:

$$C(D(p)) = c/(2) * (a - b * p)^2$$

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- For example, if $c = 2$ $b \leq 0.46$.

A sub-game perfect equilibria with no incentives to invest

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- It is easy to the that the proposed strategy pair constitutes a sub-game perfect equilibrium.

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Thank you!