Optimal Expansion of the Electricity Grid

Mario Blazquez de Paz*

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Abstract

The current integration process of electricity markets initiated in Europe, e.g. Nordpool, EMCC, and the difficulties to coordinate investment decisions make necessary the development of economic models to evaluate ex-ante the impact that the integration process has on equilibrium outcome. In this paper, I develop a model to analyze the effect that investment decisions in generation and in transmission on different points of the electricity grid has on market equilibrium outcome. In the paper, I show that an increase in transmission capacity between or within a market and an increase in generation capacity within a market reduce equilibrium prices. I also describe with type of policy performs better depending on demand realizations.

KEYWORDS: electricity auctions, transmission constraint, optimal expansion electricity grid.

1 Introduction

An optimal level of investment in the electricity system would involve joint optimization of investments in generation and transmission. From an economic perspective, generation and transmission are complementary goods, if the price of one decreases, the quantity sold of the other increases. The mechanism underlying this phenomenon is simple: consumers are only sensitive to the total price of electricity since they do not consume the generated electricity and the transmission service separately. Consequently, if electricity price falls, they will consume more electricity and demand a greater quantity of the transmission service. Consequently, investments in generation and transmission complement each other. Sometimes, however, investments in generation and transmission are substitutable. For example, in an isolated region with limited interconnection with the grid, a rise in local demand can be satisfied by either reinforcing the line or building a new power plant within the zone. If both investments occur simultaneously, then neither will be profitable because to cover the investment fix cost, the equilibrium price should be high in peaks hours and if both investment occur simultaneously, due to the excess of generation and transmission capacity, the equilibrium price will be low even in peak hours.

*PhD Student at the University of Bologna. Mail: mario.blazquezdepaz2@unibo.it

In order to coordinate investment decisions in generation and transmission, some instruments, like financial transmission rights, are implemented. However, the presence of fixed costs and returns to scale in both types of investments made the optimal investment decisions and the coordination process even more difficult making necessary the introduction of regulatory instruments as planning mechanisms, merchant mechanisms or performance based regulation mechanisms that facilitate optimal investment decisions and improve the coordination between investment decisions in generation and transmission.

The integration of electricity markets like Nordpool or EMCC introduce new coordination investment problems. Under this new scenario, it is not only necessary to coordinate investment decisions in generation and transmission within each single market (country), "Intra-TSO"[1] investment decisions, but also it is necessary to coordinate investment decisions between different markets (countries), "Inter-TSO" investment decisions.

In this paper, I develop a simple model to analyze the effect of investments decisions in generation and transmission capacity within and between markets and I stablish comparisons between equilibrium allocations under different investment policies.

As in the first chapter of my thesis. My analysis proceeds by first extending a simple duopoly model similar to the one in Fabra et al. (2006), which is then varied in several directions. In the basic set up, two suppliers with symmetric capacities and asymmetric (marginal) costs, are allocated in two different regions[2] (North and South) connected by a transmission line. The two firms face a demand in each region that is assumed to be perfectly inelastic and known with certainty when suppliers submit their offer prices. Each supplier must submit a single price offer for its entire capacity. The assumption of price-inelastic demand can be justified by the fact that the vast majority of consumers purchase electricity under regulated tariffs that are independent of the prices negotiated in the wholesale market, at least in the short run.

The assumption that suppliers have perfect information concerning market demand is reasonable when applied to markets in which offers are "short lived", such as in Spain, where there are 24 hourly day-ahead markets each day. In such markets suppliers can be assumed to know the total demand they face in any period with a high degree of certainty. In markets in which offer prices remain fixed for longer periods, e.g., a whole day, like in Australia and in the former markets in England and Wales, on the other hand, it is more accurate to assume that suppliers face some degree of demand uncertainty, or volatility, at the time they submit their offers.

I extend this basic model to analyze the effect that different policies have on equilibrium outcome. In particular, I analyze the impact of an increase in transmission capacity within the markets, "intra-TSO" investment, I anlyze the impact of an increase in generation capacity within a market and finally, I analyze the effect of an increase in transmission

[1] The TSO (transmission system operator) is the regulator organism that guarantees the efficient operation of electricity markets.

[2] In this paper, I assume single price market, i.e. the equilibrium price is unique for both regions even when the transmission line is congested. Therefore, I refer to regions instead of markets. By contrast, in case of nodal pricing markets, when the transmission line is congested, the equilibrium price differs across markets, in such a case, instead of regions, I refer to markets.

capacity from an outside market, "inter-TSO" investment.

Any of the policies proposed induce a reduction on equilibrium prices, but none of them dominates the others, in the sense, that none of them generates lower equilibrium prices. However, many useful policy recommendations can be made. In particular: If the demand is low in both markets, the policy that performs better is the one that increases transmission capacity from an outside market, "inter-TSO" investment; if the demand is concentrated in market North, the policies that performs better are the ones that increase transmission capacity from an outside market, "inter-TSO" investment or increase generation capacity within the market; finally, if the realization of demand is concentrated in market South an increase transmission between markets, "intra-TSO investment, performs better. Moreover, the model can quantify the overall effect of the three policies proposed. The model characterizes the equilibrium for any realization of demand. Therefore, it is easy to compare the difference in equilibrium prices among policies for any realization of demand. In particular, if the equilibrium price does not change both policies perform equal, if the equilibrium price is lower the policy performs better and if the equilibrium price is higher the policy performs worse. Hence, the difference in performance between policies can be easily quantified.

Joskow (2005) identifies the elements necessary to guarantee the optimal expansion the electricity grid and analyzes the effectiveness of these elements in the particular cases of the electricity market in England and PJM. Joskow and Tirole (2005) conclude that under a stringent set of assumptions, the merchant investment model has a remarkable set of attributes that appears to solve the natural monopoly problem and the associated need for regulating electric transmission companies. However, incorporating more realistic attributes of transmission networks and the behavior of transmission owners and system operators leads to the conclusion that several potentially significant inefficiencies may result from reliance on the merchant transmission investment framework.

Bushnell and Stoft (1996) analyze the incentives for electric grid investment that derives from the allocation of property rights to investors. Under certain conditions, this contract network approach can effectively deter detrimental investments in transmission capacity. However, when these conditions are not met, market participants may still find it profitable to undertake network alterations which are detrimental to the network as a whole.

Henze, Noussair and Willems (2012) and Willianson, Jullien, Kiesling and Staropoli (2006) provide interesting experimental analysis of the main regulatory schemes proposed in the theoretical literature to generate an efficient level of transmission and generation capacity.

The article proceeds as follows, in section two, I describe the benchmark model. In section three, I characterize the equilibrium when the transmission line within the markets increases ("Intra-TSO" investment). In section four, I characterize the equilibrium when a new transmission line from an outside market is build ("Inter-TSO" investment). In section five, I characterize the equilibrium when a new generation plant is build within the market. Section six compares the equilibrium outcome generated by these three policies. Finally, section seven concludes. Proofs are in the Appendix.

2 Benchmark model

In this section, I introduce the benchmark model. In the next ones, I will modify it to analyze the effect that an increase in transmission or generation capacity has on equilibrium outcome.

2.1 The model

Set up of the model. There exist two electricity regions, region North and region South, that are connected by a transmission line with capacity T.

There exist two duopolists with capacities k_n and k_s, where subscript n means that the firm is located in region North and subscript s means that the duopolist is located in region South. Suppliers' marginal costs of production are c_n and c_s. In this paper, I analyze the effect that an increase in transmission or generation capacity has on equilibrium. In order to focus in this effect, I will assume that firms are symmetric in capacity $k_n = k_s = k > 0$ and asymmetric in costs, in particular $c_s = c > c_n = 0$. I introduce the asymmetry in cost assumption because I am interested in analyze the effect that an increase in transmission capacity from an outside market, in which the equilibrium price is lower than the marginal cost of the less efficient firm, has on market equilibrium outcome. The level of demand in any period, θ_n in region North and θ_s in region South, is a random variable uniformly distributed that is independent across regions[3] and independent of the market price, i.e., perfectly inelastic. In particular, $\theta_i \in [\underline{\theta}_i, \overline{\theta}_i] \subseteq [0, k+T]$ is distributed according to some known distribution function $G(\theta_i)$, $i = n, s, i \neq j$

The capacity of the transmission line is lower than the installed capacity in each region $T \leq k$, i.e. the transmission line could be congested for some realization of demands (θ_s, θ_n). The transmission line is congested when the firm that is dispatched first in the auction, after satisfy the demand in its own region, can not sell its remain residual generation capacity in the other region because of the transmission constraint.

Timing of the game. Having observed the realization of demands $\theta \equiv (\theta_s, \theta_n)$, each supplier simultaneously and independently submits a bid specifying the minimum price at which it is willing to supply up to its capacity, $b_i \leq P$, $i = n, s$, where P denotes the "market reserve price", possibly determined by regulation.[4] Let $b \equiv (b_s, b_n)$ denote a bid profile. On the basis of this profile, the auctioneer calls suppliers into operation. If suppliers submit different bids, the lower-bidding supplier's capacity is dispatched first. Without loss of generality, assume that $b_n < b_s$. If the capacity of supplier n is not sufficient to satisfy the total demand $(\theta_s + \theta_n)$ in the case of the transmission line not congested, or $(\theta_n + T)$ in the case of the transmission line congested,[5] the higher-bidding

[3] In the majority of electricity markets, demand in one region is higher than in the other region. Moreover, there exists the possibility of some type of correlation between demands across regions. In this paper, I assume uniform distribution and independence of demand. However, the model can be easily modified to introduce different distributions of demand and correlation between demands across regions.

[4] P can be interpreted as the price at which all consumers are indifferent between consuming and not consuming, or a price cap imposed by the regulatory authorities. See von der Fehr and Harbord (1993, 1998).

[5] When the demand in market South is larger than the transmission line capacity $\theta_s > T$, firm n can only satisfy the demand in its own region and T units of demand in market South $(\theta_n + T)$. Below in this section, I will explain with detail the total demand that can be satisfied by each firm and the residual

supplier's capacity, firm s, is then dispatched to serve residual demand, $(\theta_s + \theta_n - k)$ if $(\theta_s > k - \theta_n$ and $\theta_n \in [k-T, k])$, or $(\theta_s - T)$ if $(\theta_s > T$ and $\theta_n \in [0, k-T])$. If the two suppliers submit equal bids, then supplier i is ranked first with probability ρ_i, where $\rho_n + \rho_s = 1$, $\rho_i = 1$ if $\theta_i > \theta_j$, and $\rho_i = \frac{1}{2}$ if $\theta_i = \theta_j$, $i = n, s$, $i \neq j$. The tie breaking rule implemented is such that if the bids of both firms are equal and the demand in region i is greater than the demand in region j, the auctioneer dispatches first the supplier located in region i.[6]

The output allocated to supplier $i, i = n, s$, denoted by $q_i(\theta, b)$, is given by

$$q_i(b;\theta,T) = \begin{cases} min\{\theta_i + \theta_j, \theta_i + T, k_i\} & \text{if } b_i < b_j \\ \rho_i min\{\theta_i + \theta_j, \theta_i + T, k_i\} + & \\ \qquad [1-\rho_i] max\{0, \theta_i - T, \theta_i + \theta_j - k_j\} & \text{if } b_i = b_j \\ max\{0, \theta_i - T, \theta_i + \theta_j - k_j\} & \text{if } b_i > b_j \end{cases} \tag{1}$$

The output function has an important role determining the equilibrium, therefore I will explain it in greater detail. Below, I describe the construction of firm n's output function, the one for firm s is symmetric.

The total demand that can be satisfied by firm n when it submits the lower bid $(b_n < b_s)$ is defined by $min\{\theta_n + \theta_s, \theta_n + T, k\}$. The realization of (θ_s, θ_n) determines three different areas (left panel in figure 1).

$$min\{\theta_n + \theta_s, \theta_n + T, k\} = \begin{cases} \theta_s + \theta_n & \text{if } \theta_n \le k - \theta_s \text{ and } \theta_s < T \\ \theta_n + T & \text{if } \theta_n < k - T \text{ and } \theta_s > T \\ k & \text{if } \theta_n > k - \theta_s;\ \theta_s \in [0, T] \\ & \text{or if } \theta_n > k - T;\ \theta_s \in [T, k+T] \end{cases}$$

When demand in both regions is low, firm n can satisfy the total demand $(\theta_s + \theta_n)$. If the demand in region South is greater than the transmission capacity $\theta_s > T$, firm n cannot satisfy the demand in region South even when it has enough generation capacity to do so, therefore the total demand that firm n can satisfy is $(\theta_n + T)$. Finally, if the demand is large enough the total demand that firm n can satisfy is its own generation capacity.

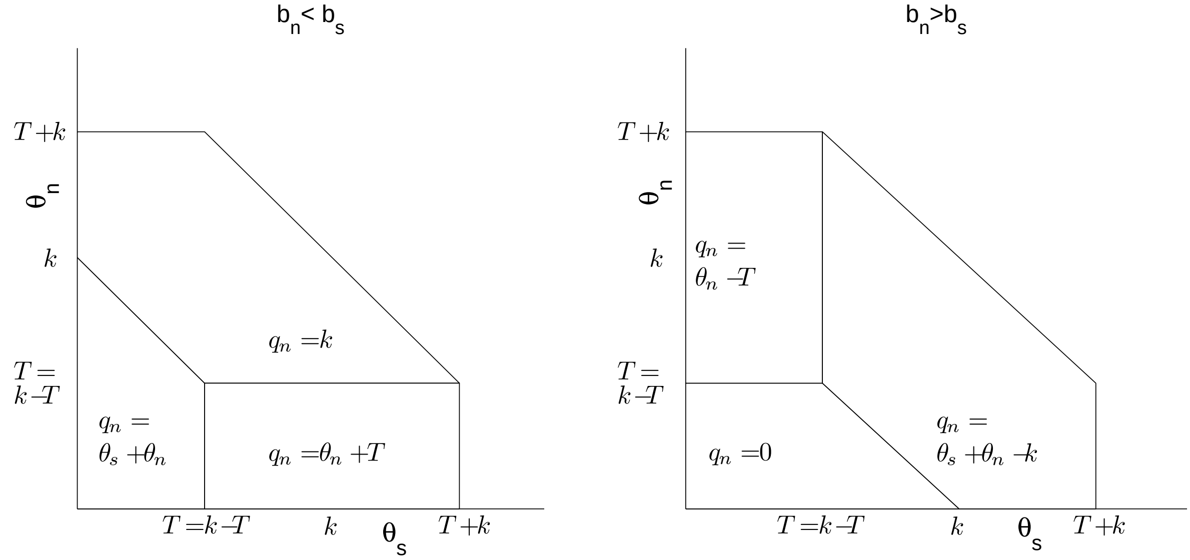
The residual demand that firm n satisfies when it submits the higher bid $(b_n > b_s)$ is defined by $max\{0, \theta_n - T, \theta_s + \theta_n - k\}$. The realization of (θ_s, θ_n) determines three different cases (right panel in figure 1).

$$max\{0, \theta_n - T, \theta_s + \theta_n - k\} = \begin{cases} 0 & \text{if } \theta_n < T;\ \theta_s \in [0, k-T] \\ & \text{or } \theta_n < k - \theta_s;\ \theta_s \in [k-T, k] \\ \theta_n - T & \text{if } \theta_n > T \text{ and } \theta_s \in [0, k-T] \\ \theta_s + \theta_n - k & \text{if } \theta_n > k - \theta_s;\ \theta_s \in [k-T, T+k] \end{cases}$$

demand that can be satisfied by each firm.

[6] This tie breaking rule prioritizes in the dispatch the firm located in the high demand region. However, the equilibrium outcome could change if the tie breaking rule prioritizes in the dispatch the most efficient firm (the one with lower marginal generation cost).

Figure 1: Output function for firm n. ($k_n = k_s = 60, T = 30$)



When demand in both regions is low, firm s satisfies the total demand, therefore the residual demand that remains to firm n is zero. When the total demand is large enough, firm s cannot satisfy the total demand and some residual demand $(\theta_s + \theta_n - k)$ remains to firm n. Due to the transmission constraint, the total demand that firm s can satisfy diminishes. As soon as demand in market North is greater than the transmission capacity $(\theta_n > T)$, firm s cannot satisfy it, therefore some residual demand $(\theta_n - T)$ remains to firm n.

Finally, the payments are worked out by the auctioneer. I will work out the payoff when the auctioneer runs a uniform price auction. When the auctioneer runs a uniform price auction,[7] the price received by a supplier for any positive quantity dispatched by the auctioneer is equal to the higher accepted bid in the auction in each market. Hence, for a given realization of $\theta \equiv (\theta_s, \theta_n)$ and a bid profile $b \equiv (b_s, b_n)$, supplier n's profits, $i = n, s$, can be expressed as

$$\pi_i^u(b; \theta, T) = \begin{cases} (b_j - c_i)k & \text{if } b_i < b_j \text{ and } \theta_i + \theta_j \geq k_i \text{ and } k - \theta_i \leq T \\ (b_i - c_i)q_i(b; \theta, T) & \text{otherwise} \end{cases} \tag{2}$$

As in the case of the production function, the payoff function has an important role determining the equilibrium, therefore I will explain it in greater detail. Below, I describe the construction of firm n's payoff function, the one for firm s is symmetric.

If $b_n < b_s$ and $\theta_n + \theta_s \geq k$ and $k - \theta_s \leq T$. Firm n submits the lower bid, has not enough capacity to satisfy the demand in both regions and after satisfy the demand in its own region, the remaining of its generation capacity is lower than the transmission line capacity (the transmission line is not congested). In such a case, firm s sets the price in the auction and the transmission line is not congested. Hence, the payoff function for firm

[7] The objective of this paper is analyze the effect that investments in different points of the electricity grid has on equilibrium. Therefore, I have decided to work out the equilibrium using uniform price auction because the equilibrium is in pure strategies and this simplify the analysis.

n is equal to $\pi_n^u(b;\theta,T) = (b_s - c_n)k$. In the rest of the cases, the payoff for firm n is its own bid multiply by its dispatch.

2.2 Equilibrium analysis

Lemma 1. In a single electricity price market. When the auction is uniform, the equilibrium price (P^*) is either, the marginal cost (c) of the less efficient firm, or the maximum price allowed by the auctioneer (P).

Proof. An equilibrium in which $P^* \in (0, c)$ is not possible because the most efficient firm can always submits a bid an epsilon lower than c and it is still dispatched first in the auction for a higher equilibrium price. An equilibrium $P^* \in (c, P)$ is not possible because the firm that submits the higher bid in the auction is dispatched last and satisfies the residual demand, in such a case, it prefers to submit the higher bid allowed by the auctioneer □

Based on this ancillary result, I present the main result of this section.

Proposition 1. In a single electricity price market. When the auction is uniform, the characterization of the equilibrium strategies falls into one of the next two categories (figure 2).

i Low demand (area A). The equilibrium pair is in pure strategies and both firms submit a bid equal to the marginal cost (c).

ii High demand (area B). The equilibrium pair is in pure strategies and one of the firms submits the maximum bid allowed by the auctioneer (P).

When the realization of demand is low, both firms have enough generation capacity to satisfy the total demand. Therefore, they compete fiercely to be dispatched first in the auction and the equilibrium price is the typical Bertrand equilibrium with cost asymmetries in which both firms submit a bid equal to the marginal cost of the less efficient firm.

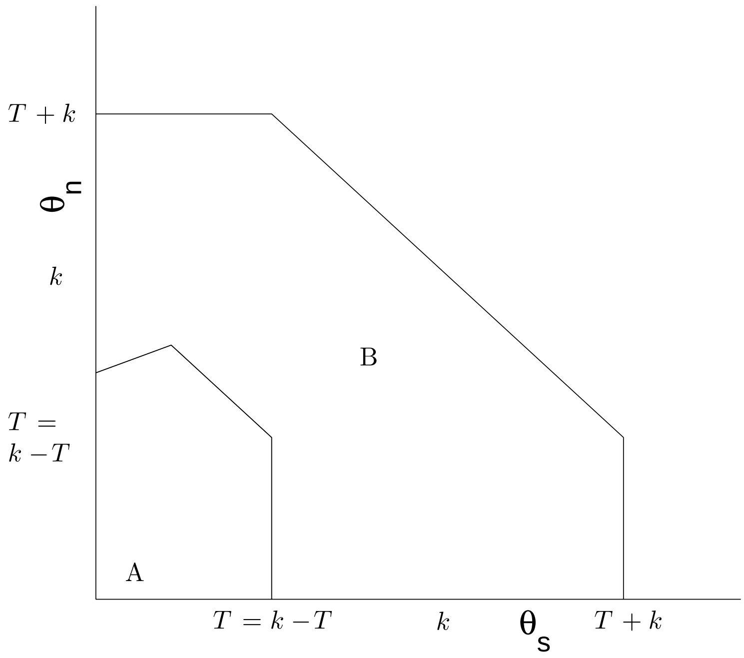
When the realization of demand is high, at least one of the firms (can be both of them) face a positive residual demand. Therefore, it has incentives to submit the maximum bid allowed by the auctioneer and satisfies the residual demand at the price P.

3 "Intra-TSO" investment (increase in transmission capacity within the market)

In this section, I analyze the effect that an increase in transmission capacity within the current market has on equilibrium outcome. To make comparisons among models feasible, from now own, I assume increases in transmission capacity or in generation capacity of the same amount ($\triangle$). In particular, in this section I analyze the size of the transmission capacity within the markets is $T + \triangle$.

As in the benchmark model, lemma 1 and proposition 1 apply. The characterization of the equilibrium is summarized in the central panel of figure 3. As in the benchmark

Figure 2: Single price market. Benchmark model. Equilibrium areas. ($k_n = k_s = 60, T = 30, P = 7, c = 2$)

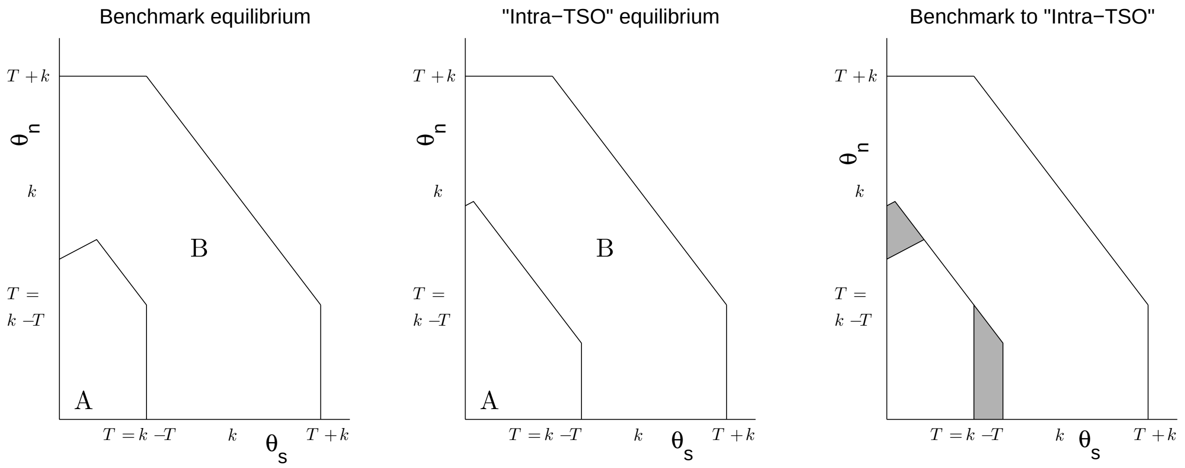


model, the equilibrium characterization falls in two different categories. When the realization of demand is law, the equilibrium is in pure strategies and both firms submit a bid equal to the marginal cost of the least efficient firm. When the realization of demand is high, the equilibrium is in pure strategies and at least one of the firms have incentives to submit the maximum bid allowed by the auctioneer. Therefore, the equilibrium price is equal to the maximum bid allowed by the auctioneer P.

To facilitate the comparison between the equilibrium outcome in the benchmark model and the equilibrium outcome when the transmission line within the markets increases, I have introduced the left panel in figure 3. The left panel in figure 3 characterizes the equilibrium in the benchmark model. Finally, the right panel in figure 3 compares the equilibrium outcome of the benchmark model with the one in which transmission capacity within the markets, "intra-TSO" investment, increases in the amount ($\triangle$). The light grey area shows the combination of demands for which the equilibrium price is lower after an increase in transmission capacity.

When the transmission line increases, the total demand that can be satisfies in the market increases. An increase in transmission capacity increases the flow of electricity between markets and so the peak demand that can be satisfied in each market increases. This increase in market size appears also when a new transmission line is build from an outside market or when new generation capacity is build within a market. However, the effect that an increase in transmission or generation capacity have on market size is not the aim of this paper.

Figure 3: Single price market. Increase transmission capacity within the markets. Equilibrium areas. $(k_n = k_s = 60, T = 40, P = 7, c = 2)$



The left panel characterizes the equilibrium in the benchmark model. The central panel characterizes the equilibrium when the transmission line between markets increases in (△) units. The right panel compares both equilibria.

4 "Inter-TSO" investment (increase in transmission capacity from an outside market)

In this section, I characterize the equilibrium when a new transmission line of capacity $T_n = \triangle$ is build from an outside market.

4.1 The model

Set up of the model. The same set up that in the benchmark model. In addition, a new transmission line is build from an outside market. The new transmission line connects the outside electricity market with the efficient region (North region) in the current market. The new transmission line capacity is $T_n = \triangle$ and the equilibrium price in the outside market is lower than the marginal cost of the inefficient firm $P_n^* < c$.

As I have explained above in the paper, the new transmission line capacity is $T_n = \triangle$. This made possible stablish comparisons between this policy and the one proposed in the previous section. Moreover, in the next section, I characterize the equilibrium when a new firm installs $k_{n1} = \triangle$ units of new generation in the current market. The assumption concerning the size of transmission and generation capacity made possible stablish comparisons between the three policies proposed.

The assumption $P_n^* < c$ is inessential to characterize the equilibrium, but I introduce this assumption to analyze if the construction of a new transmission line from a cheaper market contributes to reduce equilibrium prices in the current market. As in the case of the assumption concerning generation or transmission capacity, to stablish comparisons between policies, in the next section, I assume that the entry firm has a marginal cost equal to $c_{n1} = P_n^*$.

Timing of the game. As in the benchmark model, having observed the realization of demands $\theta \equiv (\theta_s, \theta_n)$, each supplier simultaneously and independently submits a bid specifying the minimum price at which it is willing to supply up to its capacity, $b_i \leq P$, $i = n, s$. It is important to remark that in this section, the firms located in the outside market do not participate directly in the auction in the present market, i.e. the equilibrium price in the outside market is taken as a parameter for the firms located in the current market. In the next section, I analyze the effect that an increase in generation capacity within the current market has on equilibrium outcome. In such a case, all the firms, the new one included, participate in the market submitting bids simultaneously in the auction. Let $b \equiv (b_s, b_n)$ denote a bid profile. On the basis of this profile, the auctioneer calls suppliers into operation. If suppliers submit different bids, the lower-bidding supplier's capacity is dispatched first. Without loss of generality, assume that $b_n = min\{b_n, b_s, P_n^*\}$. If the capacity of supplier n is not sufficient to satisfy the total demand $(\theta_s + \theta_n + \triangle)$ in the case of the transmission line in the current market is not congested, or $(\theta_n + T + \triangle)$ in the case that the transmission line congested, the firms located in the outside market and the higher-bidding supplier's capacity, firm s, is then dispatched to serve residual demand. When, $P_n^* < b_n < b_s$. If the capacity of supplier n is not sufficient to satisfy the total demand $(\theta_s + \theta_n - \triangle)$ in the case of the transmission line not congested, or $(\theta_n + T - \triangle)$ in the case of the transmission line congested, the higher-bidding supplier's capacity, firm s, are then dispatched to serve residual demand. If the two suppliers submit equal bids, then supplier i is ranked first with probability ρ_i, where $\rho_n + \rho_s = 1$, $\rho_i = 1$ if $\theta_i > \theta_j$, and $\rho_i = \frac{1}{2}$ if $\theta_i = \theta_j$, $i = n, s$, $i \neq j$. The tie breaking rule implemented is such that if the bids of both firms are equal and the demand in region i is greater than the demand in region j, the auctioneer dispatches first the supplier located in region i.

The output allocated to supplier $i, i = n, s$, denoted by $q_i(\theta, b)$, is given by

$$q_i(b;\theta,T) = \begin{cases} min\{\theta_i + \theta_j + \triangle, \theta_i + T + \triangle, k_i\} & \text{if } b_i = min\{b_i, b_j, P_n^*\} \\ max\{0, min\{\theta_i + \theta_j - \triangle, \theta_i + T - \triangle, k_i\}\} & \text{if } P_n^* < b_i < b_j \\ \rho_i min\{\theta_i + \theta_j, \theta_i + T, k_i\} + & \\ \quad [1-\rho_i] max\{0, \theta_i - T, \theta_i + \theta_j - k_j\} & \text{if } b_i = b_j \\ max\{0, \theta_i - T, \theta_i + \theta_j - k_j\} & \text{if } b_i = max\{b_i, b_j, P_n^*\} \end{cases} \tag{3}$$

Below, I describe the construction of firm n's output function, the one for firm s is symmetric. The total demand that can be satisfied by firm n when it submits the lower bid $b_n = min\{b_n, b_s, P_n^*\}$ is defined by $min\{\theta_n + \theta_s + \triangle, \theta_n + T + \triangle, k\}$. The realization of (θ_s, θ_n) determines three different areas (left panel in figure 4).

$$min\{\theta_n + \theta_s + \triangle, \theta_n + T + \triangle, k\} = \begin{cases} \theta_s + \theta_n + \triangle & \text{if } \theta_n \leq k - \triangle - \theta_s \text{ and } \theta_s < T \\ \theta_n + T + \triangle & \text{if } \theta_n < k - T - \triangle \text{ and } \theta_s > T \\ k & \text{if } \theta_n > k - \triangle - \theta_s;\ \theta_s \in [0, T] \\ & \text{or if } \theta_n > k - T - \triangle;\ \theta_s \in [T, k+T] \end{cases}$$

When demand in both regions is low, firm n can satisfy the total demand within the market $(\theta_s + \theta_n)$ and the demand in the outside market up to the outside transmission line capacity $(\triangle)$. If the demand in region South is greater than the transmission capacity $\theta_s > T$, firm n cannot satisfy the demand in region South even when it has enough generation capacity to do so, therefore the total demand that firm n can satisfy is $(\theta_n + T)$

within the market and the demand in the outside market up to the outside transmission line capacity ($\triangle$). Finally, if the demand is large enough the total demand that firm n can satisfy is its own generation capacity (k).

The demand that can be satisfied by firm n when it submits an intermediate bid $P_n^* < b_n < b_s$ is defined by $max\{0, min\{\theta_i + \theta_j - \triangle, \theta_i + T - \triangle, k_i\}\}$. The realization of (θ_s, θ_n) determines four different areas (central panel in figure 4).

$$max\{0, min\{\theta_i + \theta_j - \triangle, \theta_i + T - \triangle, k_i\}\} =$$
$$\begin{cases} 0 & \text{if } \theta_n \leq \triangle - \theta_s \\ \theta_s + \theta_n - \triangle & \text{if } \theta_n \leq k + \triangle - \theta_s \text{ and } \theta_s < T \\ \theta_n + T - \triangle & \text{if } \theta_n < k + \triangle - T \text{ and } \theta_s > T \\ k & \text{if } \theta_n > k + \triangle - \theta_s;\ \theta_s \in [0, T] \\ & \text{or if } \theta_n > k + \triangle - T;\ \theta_s \in [T, k+T] \end{cases}$$

When demand in both regions is very low, the electricity flows from the outside market and the residual demand for firm n is zero. When the demand in the current market is higher than ($\triangle$), firm n faces a positive residual demand equal to the total demand within the market $(\theta_s + \theta_n)$ minus the electricity that flows from the outside market ($\triangle$). If the demand in region South is greater than the transmission capacity $\theta_s > T$, firm n cannot satisfy the demand in region South even when it has enough generation capacity to do so, therefore the total demand that firm n can satisfy is $(\theta_n + T)$ minus the electricity that flows from the outside market ($\triangle$). Finally, if the demand is large enough the total demand that firm n can satisfy is its own generation capacity (k).

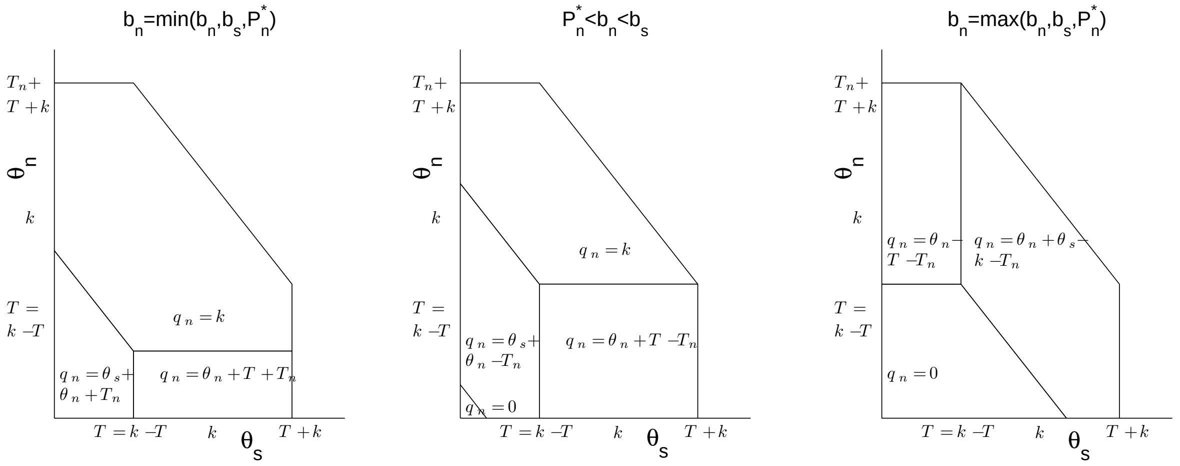
The residual demand that firm n satisfies when it submits the higher bid ($b_n = max\{b_n, b_s, P_n^*\}$ is defined by $max\{0, \theta_n - T - \triangle, \theta_s + \theta_n - k - \triangle\}$. The realization of (θ_s, θ_n) determines three different cases (right panel in figure 4).

$$max\{0, \theta_n - T - \triangle, \theta_s + \theta_n - k - \triangle\} =$$
$$\begin{cases} 0 & \text{if } \theta_n < T + \triangle;\ \theta_s \in [0, k - T] \\ & \text{or } \theta_n < k + \triangle - \theta_s;\ \theta_s \in [k + \triangle - T, k] \\ \theta_n - T - \triangle & \text{if } \theta_n > T + \triangle \text{ and } \theta_s \in [0, k - T] \\ \theta_s + \theta_n - k - \triangle & \text{if } \theta_n > k + \triangle - \theta_s;\ \theta_s \in [k - T, T + k] \end{cases}$$

When demand in both regions is low, the electricity that flows from the outside market plus firm s's generation installed capacity is enough to satisfy total demand, therefore the residual demand that remains to firm n is zero. When the total demand is large enough, the firms located in the outside market and firm s cannot satisfy the total demand and some residual demand $(\theta_s + \theta_n - k - \triangle)$ remains to firm n. Due to the transmission constraint, the total demand that firm s can satisfy diminishes. As soon as demand in market North is greater than the transmission capacity $(\theta_n > T)$, firm s cannot satisfy it. However, the firms located in the outside market can sell its generation capacity up to the transmission line capacity ($\triangle$) in the current market. Therefore, the residual demand for firm n when the transmission line within the current market is congested is $(\theta_n - T - \triangle)$.

Finally, the payments are worked out by the auctioneer.

Figure 4: Output function for firm n. ($k_n = k_s = 60, T = 30, T_n = 10$)



$$\pi_i^u(b;\theta,T) = \begin{cases} (b_j - c_i)k & \text{if } b_i < b_j \text{ and } \theta_i + \theta_j \geq k_i + \triangle \text{ and } k - \theta_i \leq T - \triangle \\ (b_i - c_i)q_i(b;\theta,T) & \text{otherwise} \end{cases} \quad (4)$$

If $b_n = min\{b_n, b_s, P_n^*\}$ and $\theta_n + \theta_s \geq k+ \triangle$ and $k - \theta_s \leq T- \triangle$. Firm n submits the lower bid, has not enough capacity to satisfy the demand in both regions and after satisfy the demand in its own region, the remaining of its generation capacity is lower than the transmission line capacity (the transmission line is not congested). In such a case, firm s sets the price in the auction. Hence, the payoff function for firm n is equal to $\pi_n^u(b;\theta,T) = (b_s - c_n)k$. In the rest of the cases, the payoff for firm n is its own bid multiply by its dispatch.

4.2 Equilibrium analysis

As in the benchmark model, lemma one applies. Using this ancillary result, I present the main result of this section.

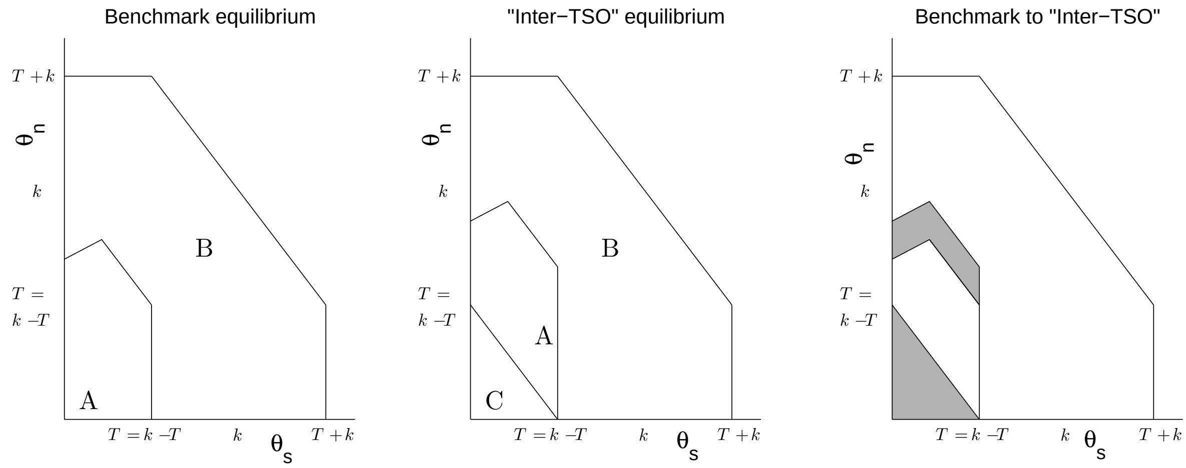
Proposition 2. Consider a single electricity price market. When the auction is uniform the characterization of the equilibrium strategies falls into one of the next three categories (left panel figure 5).

i Very low demand (area C). The equilibrium pair is in pure strategies, the most efficient firm submits a bid equal to the equilibrium price in the outside market (P_n^*) and the less efficient firm submits a bid equal to its marginal cost (c). The equilibrium price in the current market is (P_n^*).

ii Low demand (area A). The equilibrium pair is in pure strategies and both firms submit a bid equal to the marginal cost (c). The equilibrium price in the current market is (c).

iii High demand (area B). The equilibrium pair is in pure strategies and one of the firms submit the maximum bid allowed by the auctioneer (P). The equilibrium price in the current market is (P).

Figure 5: Single price market. Increase transmission capacity from an outside market. Equilibrium areas. ($k_n = k_s = 60, T = 30, T_n = 10, P = 7, P_n^* = 1, c = 2$)



The left panel characterizes the equilibrium in the benchmark model. The central panel characterizes the equilibrium after a new transmission line of $T_n = \triangle$ units of capacity is build from an outside market in which the equilibrium price is P_n^*. The right panel compares both equilibria.

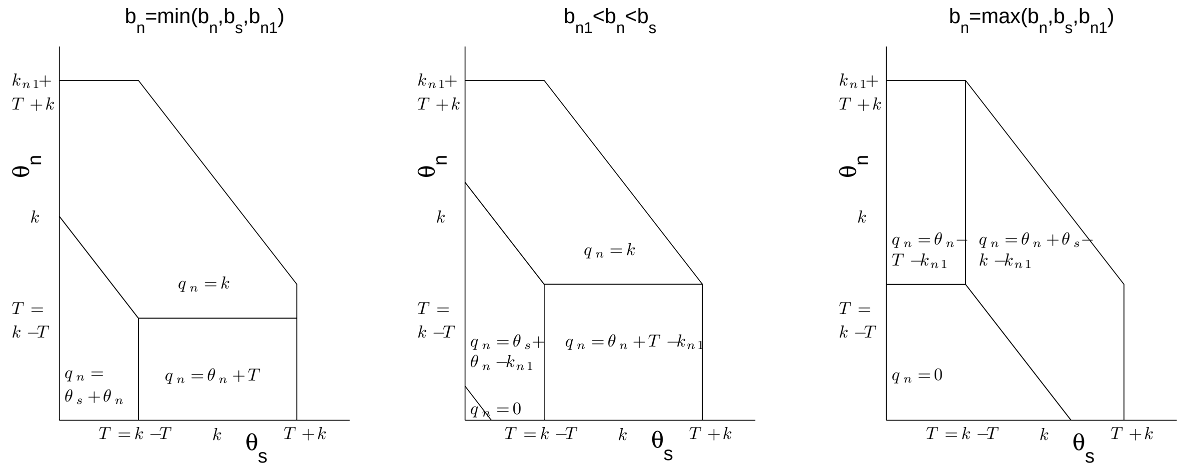
When the realization of demand is very low in the current market, the firms located in the outside market can fully satisfy the demand in the current market. Therefore, firm n has incentives to submit a very low bid (P_n^*) to be dispatched first in the auction and satisfies the demands in the current market and in the outside market. However, when the demand in the current market is high enough, the most efficient firm faces a positive residual demand after the firms in the outside demand market are dispatched. For such combination of demands, firm n submits a bid equal to the marginal cost of the less efficient firm and satisfies the residual demand in the current market after the firms located in the outside market are dispatched. Finally, when the demand is high enough, some of the firms located in the current market submit a bid equal to the maximum price allowed by the auctioneer and satisfy the residual demand.

To facilitate the comparison between the equilibrium outcome in the benchmark model and the equilibrium outcome when a new transmission line from an outside market is build, I have introduced the left panel in figure 5. The left panel in figure 5 characterizes the equilibrium in the benchmark model. Finally, the right panel of figure 5 compares the equilibrium outcome of the benchmark model with the one in which a new transmission line of capacity ($\triangle$) is build from an outside market. The light grey area shows the combination of demands for which the equilibrium price is lower after the new transmission line is build.

5 Increase in generation capacity within the market

In this section, I analyze the effect that an increase in generation capacity has on equilibrium outcome. I characterize the equilibrium when a new firm with marginal generation cost $c_{n1} = P_n^*$ build $k_{n1} = \triangle$ units of new generation capacity in region North. As I have explained before, I introduce the assumptions concerning marginal cost and installed

Figure 6: Output function for firm n. $(k_n = k_s = 60, T = 30, k_{n1} = 10)$



generation capacity to facilitate the comparison with the models described above in the paper.

5.1 The model

The set up and the timing of the game is the same than in the model in which a new transmission line is build from an outside market. The main difference is that in the current model, the firm that enter in region North participates in the auction submitting its own bid (b_{n1}). However, the output and the profit functions are worked out in the same manner than in the previous model. Figure 6 summarizes the output function for firm n.

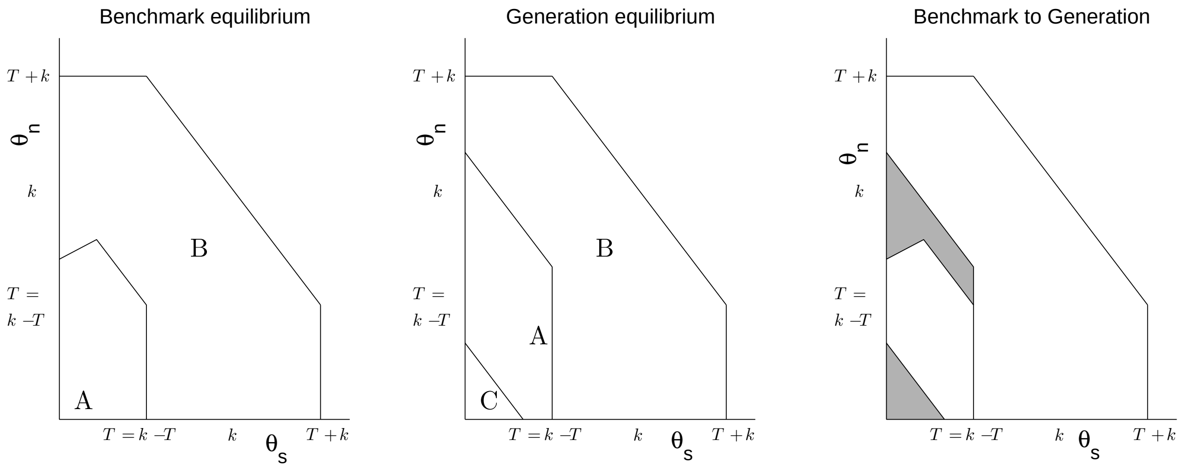
5.2 Equilibrium analysis

As in the model in which a new transmission line is build from an outside market, lemma one and proposition two applies. Therefore, the equilibrium characterization falls in three different categories.

When the realization of demands is very low, the most efficient firm, firm n, submits a bid equal to the marginal cost of the second most efficient firm, firm $n1$. For these combination of demands, the equilibrium price is c_{n1}. When the realization of demand increases, firm n, faces a positive demand in case of be dispatched second in the auction. For this combination of demands, it submits a bid equal to the marginal cost of the least efficient firm, firm s, and the equilibrium price is c. Finally, when the demand is high enough, at least one of the three firms that participate in the auction have incentive to submit the maximum price allowed by the auctioneer.

To facilitate the comparison between the equilibrium outcome in the benchmark model and the equilibrium outcome when a new firm build generation capacity within the market, I have introduced the left panel in figure 7. The left panel in figure 7 characterizes the equilibrium in the benchmark model. Finally, the right panel of figure 7 compares the equilibrium outcome of the benchmark model with the one in which new generation capacity ($\triangle$) is build in the North region. The light grey area shows the combination

Figure 7: Single price market. Increase in generation capacity within the market. Equilibrium areas. ($k_n = k_s = 60, k_{n1} = 10, T = 30, P = 7, c = 2, c_{n1} = 1$)



The left panel characterizes the equilibrium in the benchmark model. The central panel characterizes the equilibrium after a new firm with marginal cost of production $c_{n1} = P_n^*$ installs $k_{n1} = \triangle$ units of new generation capacity in the current market. The right panel compares both equilibria.

of demands for which the equilibrium price is lower after new generation capacity is build.

6 Equilibrium comparison

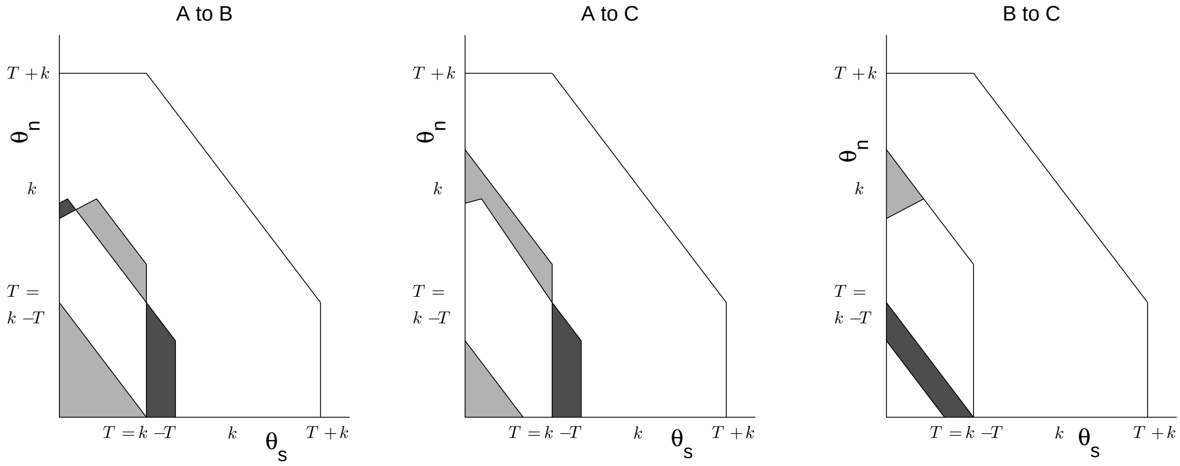
As I have shown in the previous sections, an increase in transmission capacity within the current market, "intra-TSO" investment, an increase in transmission capacity from an outside market, "inter-TSO" investment, and an increase in generation capacity within the current market induce a reduction on equilibrium prices. In this section, I compare the performance between these three different policies.

First, I compare the model that analyzes an increase in transmission capacity within the market with the model that analyzes an increase in transmission from an outside market. As can be observed in the left panel of figure 8, an increase in transmission capacity within the current market generates lower equilibrium prices than an increase in transmission capacity from an outside market in the dark grey areas. However, it generates higher equilibrium prices in the light grey areas.

Second, I compare the model that analyzes an increase in transmission capacity within the market with the model that analyzes an increase in generation capacity in region North. As can be observed in the central panel of figure 8, an increase in transmission capacity within the current market generates lower equilibrium prices than an increase in generation capacity in region North in the dark grey area. However, it generates higher equilibrium prices in the light grey areas.

Third, I compare the model that analyzes an increase in transmission capacity from an outside market with the model that analyzes an increase in generation capacity in region North. As can be observed in the right panel of figure 8, an increase in transmission

Figure 8: Single price market. Equilibrium comparison. $(k_n = k_s = 60, T = 30, P = 7, c = 2)$



The left panel compares the equilibrium outcome generated by an increase in transmission capacity between markets and an increase in transmission capacity from an outside market. The central panel compares the equilibrium outcome generated by an increase in transmission capacity between markets and an increase in generation capacity in the current market. The right panel compares the equilibrium outcome generated by an increase in transmission capacity from an outside market and an increase in generation capacity in the current market.

capacity from an outside market generates lower equilibrium prices than an increase in generation capacity in region North in the dark grey area. However, it generates higher equilibrium prices in the light grey area.

In the majority of electricity markets, the electricity is higher in some markets than in others and some kind of correlation among demands between markets exists. However, in the set up of the model, I assume that the demand is independent and uniformly distributed (this assumptions can be easily modified). Based on these assumptions and from the analysis described in the previous paragraphs many useful policy recommendations can be made. In particular: If the aim of the policy is reduce equilibrium prices when the demand is low in both markets, the policy that performs better is the one that increases transmission capacity from an outside market, "inter-TSO" investment; if the aim of the policy is reduce equilibrium prices when the demand is concentrated in market North, the policies that performs better are the ones that increase transmission capacity from an outside market, "inter-TSO" investment or increase generation capacity within the market; finally, if the objective of the policy is reduce equilibrium prices when the realization of demand is concentrated in market South an increase transmission between markets, "intra-TSO investment, performs better.

Finally, if the government, or the regulator want to evaluate "ex-ante" the economic impact of these policies. The model can quantify the overall effect of each of these policies easily. To quantify the overall effect of each policy is enough to identify the realization of demand, q_i, for which there is a a change in equilibrium price after the policy is implemented and multiply between them both quantities. If I sum this product for any demand in which the equilibrium price changes, I obtain the overall decrease on equilibrium prices for any policy $\sum_i q_i(P^*_{policy} - P^*_{benchmark})$.

7 Conclusion

In this paper, I have analyzed the effect that an increase in transmission or in generation capacity in different points of the electricity gird have on equilibrium outcome. This model is useful to evaluate the effect that the integration of electricity markets e.g., Nordpool or EMCC, has on equilibrium outcome.

An increase in transmission capacity within the current markets, "intra-TSO" investment, an increase in transmission capacity from an outside market, "inter-TSO" investment or an increase in generation capacity within the current market induce a reduction on equilibrium prices.

It is not possible to stablish a rank between these policies because none of them generates lower equilibrium prices for every realization of demand. However, many useful policy recommendations can be established. In particular: If the demand is low in both markets, the policy that performs better is the one that increases transmission capacity from an outside market, "inter-TSO" investment; if the demand is concentrated in market North, the policies that performs better are the ones that increase transmission capacity from an outside market, "inter-TSO" investment or increase generation capacity within the market; finally, if the realization of demand is concentrated in market South an increase transmission between markets, "intra-TSO investment, performs better. Morevoer, the model can quantify the overall effect of the three policies proposed. The model characterizes the equilibrium for any realization of demand. Therefore, it is easy to compare the difference in equilibrium prices among policies for any realization of demand. In particular, if the equilibrium price does not change both policies perform equal, if the equilibrium price is lower the policy performs better and if the equilibrium price is higher the policy performs worse. Hence, the difference in performance between policies can be easily quantified.

In this paper, I have assumed that the electricity market is organized as a single price electricity market. Moreover, I have assumed that the equilibrium price in the outside market is a parameter, i.e. the equilibrium prices are not worked out simultaneously across markets. The relaxation of these assumptions could have important implications for the analysis presented in this paper. In particular, when the equilibrium market is organized as a nodal price market and the equilibrium is worked out simultaneously across markets, the strategies of the firms change substantially and so equilibrium outcome. Moreover, as I have shown in the second chapter of my thesis, when the electricity market is organized as a nodal price market, an increase in transmission capacity changes the payoff functions of the firms and so its investments decisions. In the next future, I would like to explore the possibility of endogenize transmission and generation decisions when the electricity market is organized as a nodal price market.

Annex

Proposition 1. As I have shown in lemma one, there are two pure strategies equilibria in which the equilibrium price is c or P. To find the equilibrium in which the equilibrium price is equal to c, it is necessary to find the combination of demands for which both firms have incentives to submit a bid equal to c. To find the equilibrium in which the equilibrium price is equal to P, it is necessary to find the combination of demands for which at least one firm has incentives to submit a bid equal to P.

First, I work out the realization of demands that made the most efficient firm be indifferent between submit the maximum bid allowed by the auctioneer and submit a bid equal to the marginal generation cost of the least efficient firm. This combination is defined by the next equation: $Pmax\left\{0, \theta_n - T, \theta_n + \theta_s - k\right\} = cmin\left\{\theta_n + \theta_s, \theta_n + T, k\right\}$. This define four possible equations:

$$P(\theta_n - T) = c(\theta_n + \theta_s) \Rightarrow \theta_n = \frac{PT}{P-c} + \frac{c}{P-c}\theta_s \tag{5}$$

$$P(\theta_n - T) = ck \Rightarrow \theta_n = T + \frac{ck}{P}\theta_s \tag{6}$$

$$P(\theta_n + \theta_s - k) = c(\theta_n + T) \Rightarrow \theta_n = \frac{cT}{P-c} + \frac{Pk}{P-c} - \frac{P}{P-c}\theta_s \tag{7}$$

$$P(\theta_n + \theta_s - k) = ck \Rightarrow \theta_n = \frac{ck}{P} + k - \theta_s \tag{8}$$

Second, I work out the realization of demands that made the least efficient firm be indifferent between submit the maximum bid allowed by the auctioneer and submit a bid equal to the marginal generation cost of the least efficient firm. This combination is defined by the next equation: $(P-c)max\left\{0, \theta_s - T, \theta_n + \theta_s - k\right\} = (c-c)min\left\{\theta_n + \theta_s, \theta_s + T, k\right\} = 0$. It is important to remark that in this equation, to work out the profit function of the inefficient firm, it is necessary to subtract its own marginal cost. This define two possible equations:

$$P(\theta_s - T) = 0 \Rightarrow \theta_s = T \tag{9}$$

$$P(\theta_n + \theta_s - k) = 0 \Rightarrow \theta_n = k - \theta_s \tag{10}$$

The area below of the line defined by equations 5, 6, 7, 8, 9 and 10 defines the combination of demands for which both firms have incentives to submit a bid equal to c. Therefore, the equilibrium price is c, area A in figure 2. As soon as the realization of demand is over the line defined by equations 5, 6, 7, 8, 9 and 10, at least one of the firms have incentives to submit a bid equal to P and the equilibrium price is P, area B in figure 2.

Proposition 2. As I have shown in lemma one, there are two pure strategies equilibria in which the equilibrium price is P_n^*, c or P. To find the equilibrium in which the equilibrium price is equal to P_n^*, it is necessary to find the combination of demands for which the efficient firm has incentives to submit a bid equal to P_n^* instead of a bid equal to c. To find the equilibrium in which the equilibrium price is equal to c, it is necessary to find the combination of demands for which both firms have incentives to submit a bid equal to c. To find the equilibrium in which the equilibrium price is equal to P, it is necessary to find the combination of demands for which at least one firm has incentives to submit a bid equal to P.

First, I work out the realization of demands that made the most efficient firm be indifferent between submit a bid equal to the equilibrium price in the outside market and submit a bid equal to the marginal generation cost of the least efficient firm. This combination is defined by the next equation: $cmax\{0, min\{\theta_n + \theta_s - \triangle, \theta_n + T - \triangle, k\}\} = P_n^* min\{\theta_n + \theta_s + \triangle, \theta_n + T + \triangle, k\}$. This define two "relevant"[8] equations:

$$c(\theta_n + \theta_s - \triangle) = P_n^*(\theta_n + \theta_s + \triangle) \Rightarrow \theta_n = \frac{(P_n^* + c)\triangle}{c - P_n^*} - \theta_s \tag{11}$$

$$c(\theta_n + \theta_s - \triangle) = P_n^* k \Rightarrow \theta_n = \frac{P_n^* k}{c} + \triangle - \theta_s \tag{12}$$

The area below of the line defined by equations 11 and 12 defines the combination of demands for which the most efficient firm has incentives to submit a bid equal to P_n^*. Therefore, the equilibrium price is P_n^*, area C in the left panel of figure 5.

Second, I work out the realization of demands that made the most efficient firm be indifferent between submit the maximum bid allowed by the auctioneer and submit a bid equal to the marginal generation cost of the least efficient firm. This combination is defined by the next equation:
$Pmax\{0, \theta_n - T - \triangle, \theta_n + \theta_s - k - \triangle\} = cmax\{0, min\{\theta_n + \theta_s - \triangle, \theta_n + T - \triangle, k\}\}$. This define four possible equations:

$$P(\theta_n - T - \triangle) = c(\theta_n + \theta_s - \triangle) \Rightarrow \theta_n = \frac{P(T + \triangle)}{P - c} - \frac{c\triangle}{P - c} + \frac{c}{P - c}\theta_s \tag{13}$$

$$P(\theta_n - T - \triangle) = ck \Rightarrow \theta_n = T + \triangle + \frac{ck}{P}\theta_s \tag{14}$$

$$P(\theta_n + \theta_s - k - \triangle) = c(\theta_n + T - \triangle) \Rightarrow \theta_n = \frac{cT}{P - c} + \triangle + \frac{Pk}{P - c} - \frac{P}{P - c}\theta_s \tag{15}$$

$$P(\theta_n + \theta_s - k - \triangle) = ck \Rightarrow \theta_n = \frac{ck}{P} + k + \triangle - \theta_s \tag{16}$$

Third, I work out the realization of demands that made the least efficient firm be indifferent between submit the maximum bid allowed by the auctioneer and submit a bid equal to the marginal generation cost of the least efficient firm. This combination is defined by the next equation: $(P - c)max\{0, \theta_s - T, \theta_n + \theta_s - k\} = (c - c)max\{0, min\{\theta_n + \theta_s - \triangle, \theta_n + T - \triangle, k\}\} = 0$. This define two possible equations:

[8] There are equations that are trivial.

$$P(\theta_s - T) = 0 \Rightarrow \theta_s = T \tag{17}$$

$$P(\theta_n + \theta_s - k - \triangle) = 0 \Rightarrow \theta_n = k + \triangle - \theta_s \tag{18}$$

The area below of the line defined by equations 13, 14, 15, 16, 17 and 18 defines the combination of demands for which both firms have incentives to submit a bid equal to c. Therefore, the equilibrium price is c, area A in figure 5. As soon as the realization of demand is over the line defined by equations 13, 14, 15, 16, 17 and 18, at least one of the firms have incentives to submit a bid equal to P and the equilibrium price is P, area B in figure 5.

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