Designing Electricity Auctions in the Presence of Transmission Constraints

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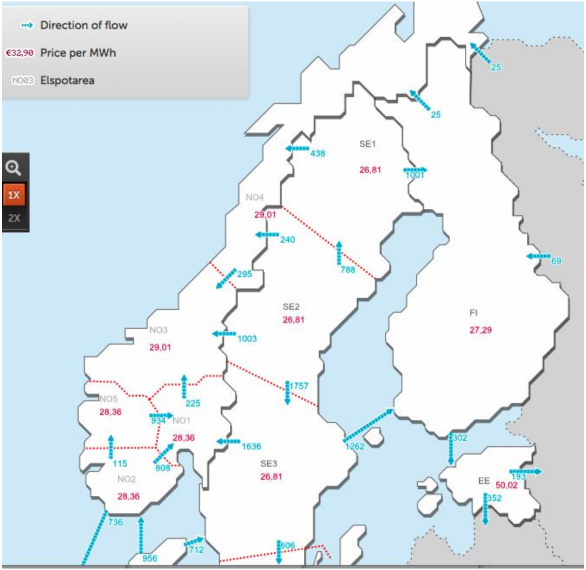
Motivation



The flow of electricity is **congested** when the line's capacity is smaller than the generation capacity

Transmission constraints affect the **equilibrium price** in electricity auctions (Offer, 1995; Bohn et al., 1999)

Figure: Nordpool (February 2014)

Research question

Study the effect of **transmission constraints** on electricity markets' equilibrium allocations

Impact of Transmission Constraints

- In the **absence of transmission constraints**, markets are as if integrated. Equilibrium allocation determined by total demand
- As the **transmission line gets congested**, markets' integration shrinks. Equilibrium allocation determined by
  - Local demand that local firm can satisfy
  - Local demand that firms in other markets can satisfy via the transmission line

Main Results 1

Assume **firms are symmetric in costs and capacity**

1. When the realization of **demand is low** a pure strategy equilibrium exists
2. When the realization of **demand is high** and capacity constraint bind, a **symmetric** mixed strategy equilibrium exists
3. When the realization of **demand is intermediate** and the transmission constraint binds, the mixed strategy equilibrium is **asymmetric**

Main Results 2

In the **intermediate demand region**, if the **capacity of the transmission line increases**:

1. Prices decrease across markets
2. Profits of firm in market with high demand decrease
3. Profits of firm in market with low demand increase when transmission capacity is low

Intuition 1

1. When the realization of **demand is low** and firms are unconstrained, then **competition** between firms drives prices to marginal cost
2. When the realization of **demand is high** and firms' capacity constraints are binding, then the model reduces to a **standard symmetric Bertrand game with capacity constraints**
   - Symmetric mixed strategy equilibrium
3. When the realization of **demand is intermediate** and the transmission constraint is binding, then the firm with higher residual capacity cannot transmit it to the other market
   - Asymmetric mixed strategy equilibrium

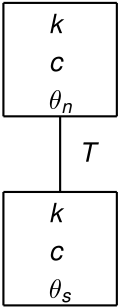
Intuition 2

- When transmission capacity increases, **competition across markets fiercer**
- But why then profits of firm in market with low demand increase when transmission capacity increases?
  - The **equilibrium price** in the low demand market decreases
  - But the **size of the market** that the firm has access to increases
- When transmission capacity is low, a marginal increase in transmission capacity implies that the **second effect dominates**

Contribution

1. **Literature on electricity auctions**: Bohn, Caramanis and Schweppe (RAND, 1984); Borenstein, Bushnell and Stoft (RAND, 2000); Joskow and Tirole (RAND, 2000); Fabra, von der Fehr and Harbord (RAND, 2006)
   - Markets' equilibrium allocation in the presence of congested transmission line
2. **Literature on models with competition under capacity constraints**: Kreps and Scheinkman (RAND, 1983); Osborne and Pitchik (ET, 1986)
   - Introduce transmission constraint: a firm can satisfy its own market's demand, but only the demand up to the capacity of the transmission line in the other firm's market

Model Set-up



- Two markets connected by a **transmission line** with capacity T
- One firm i in each market, with $i = n, s$, each with **capacity** $k \geq T$ and marginal cost of production c
- **Demand** is denoted by θ_i, where $\theta_i \in \left[0, \overline{\theta}_i\right], \forall i = n, s$
- **Discriminatory price auction**: Each firm receives its own offer price. P is the **reservation price** set by the auctioneer
- **Parametric assumptions**:
  - $(k + T) \geq \overline{\theta}_i, \forall i = n, s$
  - $2k \geq \overline{\theta}_n + \overline{\theta}_s$

Timing of the game

1. Level of **demand in each market** (θ_n, θ_s) observed by the firms
2. Each supplier submits a **bid** $b_i \leq P$
3. Based on value of submitted bids, the auctioneer calls **suppliers into operation**

$$q_i(b;\theta,T) = \begin{cases} min\{\theta_i+\theta_j, \theta_i+T, k\} & \text{if } b_i < b_j \\ max\{0, \theta_i - T, \theta_i+\theta_j-k\} & \text{if } b_i > b_j \\ \rho_i min\{\theta_i+\theta_j, \theta_i+T, k\} + [1-\rho_i]max\{0, \theta_i-T, \theta_i+\theta_j-k\} & \text{if } b_i = b_j \end{cases}$$

4. **Profits** of firm *i* equal to:

$$\pi_i^d(b;\theta,T) = \left\{(b_i - c)q_i(b;\theta,T)\right.$$

5. **Congestion rents** paid back to the transmission owner by the auctioneer

$$CR(b;\theta,T) = (b_j - b_i)min\{T, k-\theta_i\}$$

Equilibrium analysis

Lemma 1. If the **demand of electricity is low**, then a unique pure strategy equilibrium exists. Instead, if the **demand of electricity is high**, it does not exist an equilibrium in pure strategies.

Sketch of the proof:

1. $b_i = b_j = c$: i has incentive to raise its bid and serve its residual demand
2. $b_i = b_j > c$: i has incentive to undercut j and be dispatched first
3. $b_j > b_i \geq c$: i has incentive to shade the bid of j

Equilibrium analysis

Proposition 1. If the **demand of electricity is low**, then the equilibrium is such that $b_n = b_s = c$ and no electricity flows through the transmission line.

If the **demand of electricity is intermediate or high** the equilibrium is in mixed strategies. Specifically, firm i chooses a random bidding strategy according to a **cumulative distribution function** $F_i(b)$ equal to

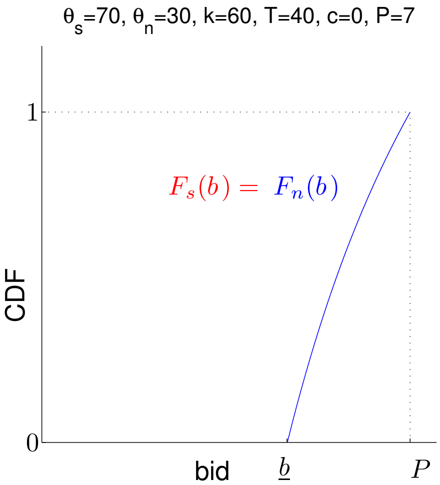
$$F_i(b) = \frac{L_j(b) - L_j(\underline{b})}{L_j(b) - H_j(b)} \forall i, j$$

where: $L_i(b) = min\{\theta_i + \theta_j, \theta_i + T, k\}$ and $H_i(b) = max\{0, \theta_i - T, \theta_i + \theta_j - k\}$ and whose **support** S is

$$S = \left[max\left\{\underline{b}_i, \underline{b}_j\right\}, P\right]$$

▸ Existence ▸ CDF ▸ Support

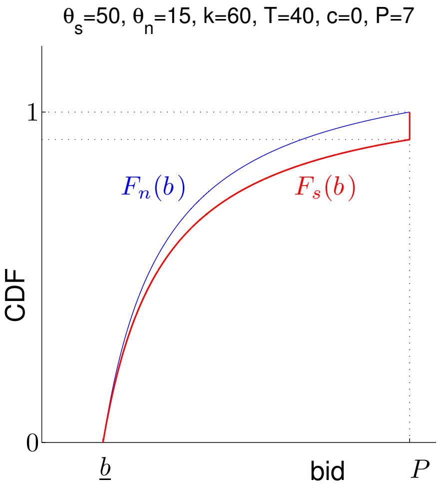
Parametric Example — Symmetric Mixed Strategy Equilibrium



Scarcity in generation capacity, $k < min\{\theta_s + \theta_n, \theta_i + T\}$, implies expected price above marginal cost

Expected equilibrium price and profits are **equal** even though realization of demand differs across markets

Parametric Example — Asymmetric Mixed Strategy Equilibrium



Transmission constraint binds for firm in market *N*, $k - \theta_n > T$

$F_s(b)$ (first-order) stochastically dominates $F_n(b)$

Due to scarcity in transmission capacity, **asymmetric expected prices** across markets

Comparative Statics

Proposition 2. If the demand of electricity is intermediate, an increase in the transmission capacity T:

i. Reduces the lower bound of the strategies' support ($\underline{b}$)

ii. Reduces the expected value of bids in both markets ($E(b_n)$ and $E(b_s)$)

iii. Decreases the profits of the firm located in the market with high demand

iv. Increases the profits of the firm located in the low demand market when the transmission capacity is low, $T \leq \underline{T}$, increases the same firm's profits otherwise, $T > \underline{T}$

Parametric Example — Comparative Statics

Table : $\theta_s = 55, \theta_n = 5, k = 60, c = 0, P = 7$

	$\underline{b}$	$F_s(P)$	$E(b_s)$	$E(b_n)$	$\overline{\pi}_s$	$\overline{\pi}_n$
$T = 60$	0	1	0	0	0	0
$T = 50$	0.58	0.92	2.03	1.58	35	32.08
$T = 40$	1.75	0.75	4.17	3.23	105	78.75
$T = 30$	2.92	0.58	5.47	4.37	175	102.08
$T = 20$	4.08	0.42	6.28	5.28	245	102.08
$T = 10$	5.25	0.25	6.76	6.04	315	78.75
$T = 0$	7	0	7	7	385	35

Discriminatory v. Uniform Electricity Auctions

- Main model assumption: discriminatory electricity auction
  - **Discriminatory auction**: price received by a firm for its output is equal to its own offer price
  - **Uniform auction**: price received by a firm is equal to the higher accepted bid in the auction
- Fabra, von der Fehr and Harbord (RAND, 2006): uniform and discriminatory auctions **do not perform equally** under single node assumption
- How does the discriminatory auction design perform, with respect to the uniform one, under the **two-node assumption**?

Discriminatory v. Uniform Electricity Auctions

Irrelevance result: in a two-node market in which **transmission rights are assigned to the grid owner**, discriminatory and uniform price auctions perform equally

Let $b_i < b_j$, then:

$$\pi_i^u(b;\theta,T) = (b_i - c)\theta_i + (b_j - c)min\{T, k-\theta_i\} - (b_j - b_i)min\{T, k-\theta_i\} =$$
$$= (b_i - c)min\{\theta_i + \theta_j, \theta_i + T, k\} = \pi_i^d(b;\theta,T)$$

Intuition: in the uniform auction, the **owner of transmission rights captures the rents** that arise when the prices in the two markets differ

This **closes the gap** between the profit functions in the two auction designs

Conclusions

1. I develop a model to characterize the equilibrium allocation in discriminatory spot electricity auctions run in **markets connected by a transmission capacity**
2. Even though firms are fully symmetric, when the transmission constraint binds **an asymmetric mixed strategy equilibrium** arises
3. An **increase of the transmission capacity** makes competition fiercer, decreases profits in high demand market and increases profits in low demand market when transmission capacity is low
4. These results robust to **uniform auction**

Thank you!

Tack så mycket!

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Annex 1. Existence of Mixed Strategy Equilibrium

Existence of mixed strategy equilibrium. Dasgupta and Maskin (1986).

1. The discontinuity of $\pi_i(b_i, b_j), \forall i, j$ is **restricted to the strategies** such that $b_i = b_j$.
2. By lowering its price from a position where $b_i = b_j$, a firm discontinuously increases its profit. Therefore, $\pi_i(b_i, b_j)$ is left **lower semi-continuous** in b_i.
3. $\pi_i(b_i, b_j)$ is **bounded**.
4. $\pi_i(b_i, b_j) + \pi_j(b_i, b_j)$ is **continuous**.

Work out the equilibrium. Varian (1980), Deneckere and Kovenock (1986)...

Proposition 1

Annex 2. Cumulative Distribution Function

First step, the **payoff function** for any firm is:

$$\begin{aligned}\pi_i^l(b) &= b\left[F_j^l(b)max\left\{0,\theta_i - T,\theta_i+\theta_j-k\right\} + (1-F_j^l(b))min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}\right] = \\ &= -bF_j^l(b)\left[min\left\{\theta_i+\theta_j,\theta_i+T,k\right\} - max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}\right] + \\ &\quad bmin\left\{\theta_i+\theta_j,\theta_i+T,k\right\}\end{aligned} \tag{1}$$

Second step, $\pi_i^l(b) = \overline{\pi}_i^l \forall b \in S_i, i = n, s$, where S_i is the support of the mixed strategies. Then,

$$\begin{aligned}\overline{\pi}_i^l &= -bF_j^l(b)\left[min\left\{\theta_i+\theta_j,\theta_i+T,k\right\} - max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}\right] + \\ &\quad bmin\left\{\theta_i+\theta_j,\theta_i+T,k\right\} \Rightarrow \\ F_j^l(b) &= \frac{bmin\left\{\theta_i+\theta_j,\theta_i+T,k\right\} - \overline{\pi}_i^l}{b\left[min\left\{\theta_i+\theta_j,\theta_i+T,k\right\} - max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}\right]}\end{aligned} \tag{2}$$

Third step, at $\underline{b}^l$, $F_i^l(\underline{b}^l) = 0 \forall i = n, s$. Then,

$$\overline{\pi}_i^l = \underline{b}^l min \{\theta_i + \theta_j, \theta_i + T, k\} \tag{3}$$

Fourth step, plug in 3 into 2, I obtain the mixed strategies for both firms.

$$\begin{aligned} F_j^l(b) &= \frac{bmin \{\theta_i + \theta_j, \theta_i + T, k\} - \underline{b}^l min \{\theta_i + \theta_j, \theta_i + T, k\}}{b [min \{\theta_i + \theta_j, \theta_i + T, k\} - max \{0, \theta_i - T, \theta_i + \theta_j - k\}]} = \\ &= \frac{L_i^l(b) - L_i^l(\underline{b})}{L_i^l(b) - H_i^l(b)} \forall i = n, s \end{aligned} \tag{4}$$

Proposition 1

It is easy to verify that equation $F_j^I(b)\forall i, j$ is indeed a **cumulative distribution function**.

First, in the third step, I have established that $F_j^I(\underline{b}) = 0$.

Second, $F_j^I(b)$ is an **increasing function** in b. At $\underline{b}$, $L_i^I(\underline{b}) = H_i^I(b)$, for any $b > \underline{b}$, $L_i^I(\underline{b}) < H_i^I(b)$; moreover, $\frac{\partial L_i^I(b)}{\partial b} > 0$, $\frac{\partial L_i^I(\underline{b})}{\partial b} = 0$ and $\frac{\partial H_i^I(b)}{\partial b} > 0$, therefore, $\frac{\partial \left(L_i^I(b) - L_i^I(\underline{b})\right)}{\partial b} > \frac{\partial \left(L_i^I(b) - H_i^I(b)\right)}{\partial b}$.

Third, $F_j^I(b) \leq 1 \forall b \in S_i$.

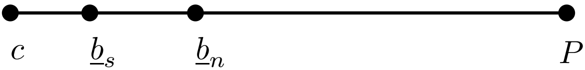
Finally, $F_j^I(b)$ is **continuous** in the support because $L_i^I(b) - L_i^I(\underline{b})$ and $L_i^I(b) - H_i^I(b)$ are continuous functions in the support.

Proposition 1

Annex 3. Support Mixed Strategies Equilibrium

Lemma 2. In a mixed strategy equilibrium the **minimum bid** ($\underline{b}_i$) in the support of the strategies for each firm will satisfy
$\underline{b}_i min \left\{ \theta_i + \theta_j, \theta_i + T, k \right\} = Pmax \left\{ 0, \theta_i - T, \theta_i + \theta_j - k \right\}$.

Moreover, **the support of the strategies for both firms** in the mixed strategy equilibrium will be $\left[max \left\{ \underline{b}_i, \underline{b}_j \right\}, P \right]$.



Proposition 1