Inflow Uncertainty in Hydropower Markets*

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Abstract

We analyse effects of uncertainty on market performance in hydropower systems under alternative assumptions about market structure. Uncertainty creates distinct possibilities for exercising market power that cannot be captured in deterministic models. The qualitiative and quantitative features of these possibilities depend on the characteristics of residual demand facing individual firms, which again depends on consumer preferences and generation and transmission technologies.

1 Introduction

Stochastic water inflow may create opportunities for manipulating outcomes in hydro-dominated electricity markets. For example, in the winter of 2002-2003, when the Nordic electricity market experienced unusually low water inflow and extraordinarily high prices, a number of observers - including market participants, politicians and consumer representatives - claimed that electricity producers took the opportunity to generate “too much” power in the summer, creating excessive scarcity and unnecessarily high prices in the winter. To determine whether market power was actually exercised has proven very difficult. The aim of this paper is to enhance our understanding of whether, and under what conditions, market power is affected by the stochastic of water inflow.

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There is by now a substantial literature on strategic behaviour in electricity market, but most of this literature has concentrated on thermal systems.[1] Hydropower systems differ from thermal systems in a number of respects, including the possibility of storing energy (water), large variations in production possibilities (i.e. inflow) and small variable costs and high flexibility in the short run. These features have important implications for the functioning of markets in general and for the potential for strategic behaviour in particular. On the one hand, small variable costs and high flexibility leads to relatively low price volatility and limited opportunities for exercising market power in the short-run; on the other hand, variations in production possibilities and limited storage capacity leads to high price volatility and greater potential for exercising market power in the longer run, over seasons and years.

To the extent that hydro power has been considered in the literature on strategic interaction, the concern has tended to be how its presence affects behaviour in an otherwise thermal-dominated market. For example, Crampes and Moreaux (2001) use a deterministic, two-period Cournot model of a mixed hydro and thermal market to demonstrate that the presence of hydro producers - who have the opportunity to shift output between periods - implies that also thermal power production is determined by intertemporal considerations. Bushnell (2003) bases his numerical analysis of the degree of competition on the Western US market on a similar Cournot framework, in which producers control both hydro and thermal generation facilities.[2]

While the above literature takes into account some of the features of hydro technologies, others are neglected; in particular, these models are deterministic and do not include any stochastic elements. In the literature that does include such elements, either attention has been directed at particular issues or results are driven by specific assumptions about the nature of uncertainty. For example, Johnsen (2001) uses a two-period model with stochastic inflow to study trade between areas with different production technologies, but the uncertainty element is relevant only because of transmission constraints. Mathiesen *et al.* (2003) analyse the possibility of supply shortages in dry years, but their main concern is not uncertainty, but reservoir constraints. Skaar and Sørgard (2006) discuss implications of mergers in hydropower systems with temporary bottlenecks. Førsund (2007) provides a more general discussion

[1] See for example, Borenstein and Bushnell (1999), Cardell et al. (1997), von der Fehr and Harbord (1993) and Green and Newbery (1992).

[2] von der Fehr and Sandsbråten (1997) consider trade between areas dominated by, respectively, hydro and thermal power, but their model is based on the assumption of perfect competition.

of hydropower systems for the two extremes of perfect competition and monopoly, but the relevance of uncertainty for the exercise of market power is not considered in any great detail.[3] In the two-period model of Garcia *et al.* (2001), results depend critically on the assumption of a rectangular demand function (implying a positive probability of a price equal to zero in the second period). Finally, Centeno *et al.* (2007) provides a discussion of the effect of uncertain inflow on prices and output in a numerical model based on the Spanish electricity market.

The aim of this paper is to pin point in more detail the impact of uncertainty. It is well known that variations in demand over time may create opportunities for exercising market power by intertemporal price discrimination, even in models with no uncertainty (the same is true for variations in supply conditions, in combination with limitations on storage or transmission capacities); see for instance Førsund (2007). The present analysis therefore takes as its starting point a set up in which, in the absence of uncertainty, market power does not affect market performance. Specifically, we first consider a two-period model in which demand is identical across periods, there are no (effective) constraints on storage and producers find it optimal to utilise all available water. Uncertainty is introduced in the form of stochastic second-period inflow. It is demonstrated that, unless demand is linear in price, uncertainty matters; in particular, firms with market power produce a different amount in the first period than they would have done if there was no uncertainty (or they did not have market power); whether they produce more or less than the deterministic benchmark depends on the shape of the demand function.

We subsequently extend the model to analyse how different assumptions about supply, demand and market structure affect the way producers choose to exercise market power. The impact of demand is determined by the higher order derivatives of the inverse demand function (i.e, the third and fourth derivatives of the utility function), which may be interpreted in relation to the concept of "prudence" and increase in prudence, cf. Menegatti (2001). Moreover, what matters is the shape of the net inverse demand function facing any given firm, which depends on both the shape of demand and the reaction function of competitors, see Borenstein *et al.* (1999); specifically, we discuss how the cost structure of a competitive fringe influences behaviour of firms with market power.

Our analysis suggests that one way to approach the issue of stochastic

[3] Bjerkholt and Olsen (1984) discuss how uncertainty affects social-planner investment decisions in a hydro-dominated system, and point out that assumptions about the nature of uncertainty and demand are crucial. However, they do not discuss market behaviour.

is by framing it in the context of the short-term marginal (opportunity) cost of hydro output, which is the value of water in future periods. The water value depends on the distribution of both demand and water inflow, as well as the shape of (net) demand facing the producer. When we concentrate attention on inflow uncertainty, it is not because demand uncertainty is unimportant, but that, compared to variations in inflow, variations in demand tend to be limited. For example, over the last ten years annual demand (consumption) of Norwegian electricity consumers has varied between 117 and 123 TWh, while water inflow has varied between 85 and 140 TWh. This is the main reason why numerical models of the Nordic power market, like the Balmorel model and Samkjøringsmodellen[4], do take account variations in inflow but not of variations in demand.

The variations in future water availability create an interesting parallel between hydropower generation and utilisation of non-renewable resources of unknown size, see for instance Kemp (1976). The present model consequently shares similarities with models of optimal exploitation of an uncertain reserve of exhaustible natural resources; see for instance Pindyck (1980) and Loury (1978), and, for a survey of the literature, Cairns (1990). However, while the time horizon of the hydropower problem is two seasons, or at most a few years, exploitation of an exhaustible natural resource typically has a time horizon of decades. Moreover, the time horizon in hydropower generation is essentially exogenous, determined by the cycle of water inflow (and, possibly, storage capacity), while in production of non-renewable resources the time horizon is endogenous, determined by the intensity of utilisation. The nature of uncertainty is also different, because hydropower producers typically have much more precise information about both production possibilities and demand conditions than do producers of non-renewable resources.

The paper is organized as follows. In Section 2, a two-period model with uncertainty is presented. Section 3 is devoted to a description of the first-best solution or outcome under perfect competition, while Section 4 contains the analysis of a symmetric oligopoly of hydropower producers. Section 5 is devoted to a discussion of the importance of the nature of uncertainty. In Section 6, the shape of (residual) demand facing a given firm is discussed. Section 7 contains concluding remarks.

[4] See www.balmorel.com and http://www.sintef.no/content/page1____4941.aspx for documentations on these models.

2 A Two-Period Model with Uncertainty

We consider a two-period model with N identical hydropower producers or firms. Short-term variable costs equal zero. Water inflow to the reservoirs of firm i in period t is denoted q_{it} (these, as well as all other quantities, are measured in energy equivalents). Water inflow in the first period is deterministic and equal to $\overline{q}_{i1}$. In the second period, inflow is stochastic; in particular, $q_{i2} = \overline{q}_{i2} + \tilde{\varepsilon}_i$, where $\overline{q}_{i2}$ is expected inflow and $\tilde{\varepsilon}_i = \varphi_i \varepsilon_i$, with φ_i a constant and ε_i an independently and randomly distributed variable with $E\{\varepsilon_i\} = 0$ and $cov(\varepsilon_i, \varepsilon_j) = 0$, $i \neq j$. For simplicity in order to avoid the extreme values that occur in for example a normal distribution, we assume that ε_i only takes on the values σ_i and $-\sigma_i$ with equal probability, i.e., $P(\varepsilon_i = \sigma_i) = P(\varepsilon_i = -\sigma_i) = 0.5$. The variance of $\tilde{\varepsilon}_i$ thus becomes $var(\tilde{\varepsilon}_i) = \varphi_i^2 \sigma_i^2$. We let x_{it} denote output of firm i in period t.

The reservoir level of firm i at the beginning of Period 1 is r_{i0}, while the required level of reservoirs of firm i at the end of Period 2 is r_i. The resource constraints for producer i in Period 1 may thus be expressed as

$$0 \leq x_{i1} \leq r_{i0} + q_{i1}.$$

In other words, production in Period 1 must be non-negative and less than or equal to the sum of inflow and water initially in the reservoir.

Similarly, in Period 2 a water balance restriction adds up total production and inflow over the two periods:

$$0 \leq x_{i2} \leq r_{i0} + q_{i1} - x_{i1} + q_{i2} - r_i.$$

We assume $x_{i2} = r_{i0} + q_{i1} - x_{i1} + q_{i2} - r_i$ is binding (i.e., no spilling of water), and define $\bar{Q}_i = r_{i0} + q_{i1} + \overline{q}_{i2} - r_i$ as the deterministic part of available water over the two periods. The total amount of water available for production over the two periods is then $Q_i = \bar{Q}_i + \varphi_i \varepsilon_i$. It follows that $x_{i2} = \bar{Q}_i + \varphi_i \varepsilon_i - x_{i1}$. We may think of the two-period model as consisting of a summer and a winter period, where there is uncertainty about inflow in the winter period, and the problem for the producer is to utilise the water so that the reservoir is at the required level before spring flooding.[5] In the spring, overflow will occur unless reservoirs are sufficiently emptied, implying that saving too much water in the second period brings the value of water to zero. Implicitly, we assume that marginal revenue is always positive so that producers have an incentive to bring reservoirs down to the level required to avoid flooding. Alternatively, the no-spilling assumption may be justified with

[5] Førsund (2007) applies two-period models.

reference to regulation that restricts spilling (in Norway, spilling is only allowed in exceptional circumstances and actual spilling is monitored by the regulator, Norwegian Water Resources and Energy Directorate NVE).

We further assume that the inverse demand function is identical across periods, with the inverse demand function in period t given by $p(X_t)$, with $p'(X_t) < 0$, and $X_t = \sum_{i=1}^{N} x_{it}$ is total electricity consumption in that period. This symmetry assumption allows us to concentrate on the effect of uncertainty in inflow, because, as we shall see below, under certainty (i.e. with $\varphi_i \equiv 0$, all i) and the no-withholding constraint (i.e. no-spilling or transferring of water to later periods), market power does not affect market outcomes when inverse demand functions are identical over time. Hence the assumption of identical inverse demand functions isolates the effect of uncertainty. We elaborate on this issue below. For simplicity, we ignore discounting.

3 Perfect Competition

We first identify the Pareto Optimum or first-best solution, defined as the allocation of output that maximizes the expected sum of consumer and producer surpluses, defined in the standard fashion. This outcome of course corresponds to the outcome with perfect competition, i.e. price-taking behaviour of firms. Let $\bar{Q} = \sum_{i=1}^{N} \bar{Q}_i$ and $\varphi\varepsilon = \sum_{i=1}^{N} \varphi_i \varepsilon_i$. If we normalise the variance of ε to 1, i.e. set $\sigma = 1$ and $\sigma^2 = 1$, the standard deviation of inflow becomes φ. We want to solve the following maximization problem:

$$\max_{X_1, X_2} E\left\{\int_0^{X_1} p(y)\,dy + \int_0^{X_2} p(y)\,dy\right\} \quad \text{st. } X_2 = \bar{Q} + \varphi\varepsilon - X_1. \quad (1)$$

The first-order condition for this problem may be written

$$p\left(X_1^f\right) = E\left\{p(\bar{Q} + \varphi\varepsilon - X_1^f)\right\}, \quad (2)$$

where X_t^f denotes the first-best solution in period t, $t = 1, 2$. In other words, the price in the first period should equal the expected price in the second period. It is immediately clear that in the special case of no uncertainty, i.e., with $\varphi \equiv 0$, identical inverse demand functions imply that production in the first period should equal production in the second period, i.e., $X_1^f = X_2^f = \frac{\bar{Q}}{2}$.

From Jensen's Inequality (cf. Berck and Sydsæter, 1991), it follows that

$$p\left(X_1^f\right) = E\left\{p\left(X_2^f\right)\right\} \geqslant p\left(E\left\{X_2^f\right\}\right)$$

when p is convex and so $X_1^f \leq \frac{\overline{Q}}{2} \leq E\left\{X_2^f\right\}$ (with strict inequalities if p is strictly convex). With convex inverse demand, an increase in supply leads to a smaller reduction in marginal willingness to pay than the increase in willingness to pay following a corresponding reduction in supply. Therefore, in expected terms willingness to pay is higher when supply is uncertain than when it is not. Consequently, compared to the case of no uncertainty, welfare is increased if more output is shifted from Period 1 to Period 2.

Figure 1 shows the optimal resource allocation for a social planner under certainty and uncertainty, respectively, when the inverse demand function is convex. For simplicity of exposition we define $p_t = p(X_t)$ for $t = 1, 2$. Under certainty, the planner equalizes both price and output over the two periods. Under uncertainty, it is optimal to reduce production in the first period relative to the level that would be optimal under certainty, by equalizing first-period price with the second-period expected price.

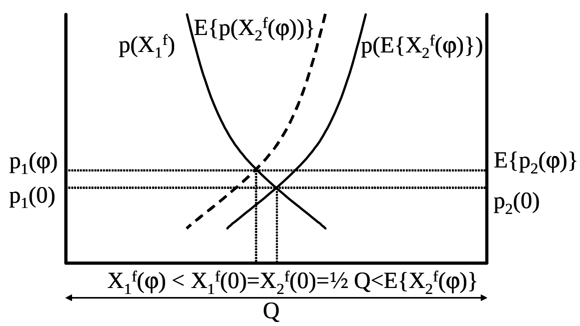

Figure 1. The optimal resource allocation for the social planner

We may consider the impact of uncertainty in more detail by examining the effect of an increase in the variation in inflow, φ. Differentiating (2) with respect to φ and solving we get

$$\frac{dX_1^f}{d\varphi} = \frac{E\left\{\varepsilon \cdot p'\left(\bar{Q} + \varphi\varepsilon - X_1^f\right)\right\}}{p'\left(X_1^f\right) + E\left\{p'\left(\bar{Q} + \varphi\varepsilon - X_1^f\right)\right\}}, \tag{3}$$

where the denominator is negative, because $p' < 0$. By rewriting the numerator of (3), we find

$$\begin{aligned} E\{\varepsilon \cdot p'\} &= E\{\varepsilon\} \cdot E\{p'\} + cov(\varepsilon, p') \\ &= cov(\varepsilon, p'), \end{aligned}$$

where the last equality follows from the assumption $E\{\varepsilon\} = 0$.

Consider again the case when the inverse demand function is convex, i.e., $p'' > 0$. Then

$$cov\left(\varepsilon, p'\left(\bar{Q} + \varphi\varepsilon - X_1^f\right)\right) > 0,$$

and therefore

$$\frac{dX_1^f}{d\varphi} < 0.$$

In other words, when the inverse demand function is convex, an increase in the variation of inflow (keeping expected inflow constant) implies a smaller optimal output in the first period. Smaller output implies that price will be higher in the first period, and, given that from (2) price in the first period equals the expected price in the second period, the increase in uncertainty also increases the expected price in the second period.

It is immediate that in the converse case of a concave inverse demand function, i.e., $p'' < 0$, we have the exact opposite result; output in the first period is increasing in demand uncertainty.

The above analysis is summarized in the following proposition.

Proposition 1 *At the first-best allocation,*

i) $X_1^f = \frac{\overline{Q}}{2}$ *when* $\varphi = 0$.

ii) $\frac{dX_1^f}{d\varphi} \gtreqless 0 \Leftrightarrow p'' \lesseqgtr 0$

ii) $\frac{dp_1^f}{d\varphi} = \frac{dEp_2^f}{d\varphi} \lesseqgtr 0 \Leftrightarrow p'' \lesseqgtr 0$

4 Imperfect Competition

We now turn to the case of imperfect competition, where firms are assumed to choose quantities in Cournot fashion. Specifically, we assume firms first choose first-period outputs simultaneously and subsequently, having observed the realization of second-period inflows, choose second-period outputs.

The no-withholding assumption implies that in the second period firms supply whatever output can be produced from available water. We can therefore concentrate attention on first-period behaviour, assuming that firms take into account equilibrium play in the second period. Consequently, in the first period firm i solves the problem of maximizing expected profit over both periods, or

$$\max_{x_{i1}} E\left\{p(X_1)x_{i1} + p(X_2)x_{i2}\right\}, \tag{4}$$

subject to the condition

$$x_{i2} = \bar{Q}_i + \varphi_i\varepsilon_i - x_{i1}.$$

The first-order condition for this problem may be written as

$$p_1' x_{i1} + p_1 = E\left\{p_2'\left[\bar{Q}_i + \varphi_i\varepsilon_i - x_{i1}\right] + p_2\right\}, \tag{5}$$

where for simplicity we have dropped the arguments of p_1 and p_2. In other words, at maximum profit marginal revenue in the first period equals expected marginal revenue in the second period.

In the special case of certainty, i.e., $\varphi_i = 0$, the symmetry of demand over time implies that firms produce the same amount in both periods, i.e. $x_{i1} = x_{i2} = \frac{\bar{Q}_i}{2}$ for all i. This implies that the price will also be the same in both periods, i.e., $p_1 = p_2$. In this case, therefore, oligopolistic producers are unable to benefit from exercising their market power.[6]

Returning to the case of uncertainty, consider first a setting with symmetric producers. Then, by adding (5) over all N producers, we may express the difference between (expected) prices in the two periods as

$$p_1 - E\left\{p_2\right\} = \frac{1}{N} E\left\{p_2' X_2 - p_1' X_1\right\}. \tag{6}$$

Holding aggregate expected output $\bar{Q} = \bar{Q}_i N$ fixed, market prices would move to the social optimum as N approaches infinity.

We next consider the case of a monopoly, or perfect cartel. Let $\varphi\varepsilon = \sum_{i=1}^{N} \varphi_i\varepsilon_i$ and again normalise the variance of ε to 1, i.e. let $\sigma = 1$ and $\sigma^2 = 1$. The first-order condition for the monopoly producer can then be expressed as:

$$M\left(X_1^m\right) = E\left\{M\left(\bar{Q} + \varphi\varepsilon - X_1^m\right)\right\}, \tag{7}$$

where X_1^m denotes optimal monopoly production and

$$M\left(X\right) = p\left(X\right) + p'\left(X\right)X. \tag{8}$$

We may apply the same reasoning to the marginal-revenue function as we did in the previous section to the inverse demand function; in particular, first-period output is smaller (and hence price is higher) when

[6] Note that this result depends crucially on the no-withholding assumption. If firms could increase profits by spilling water, production and prices would remain constant across periods, but aggregate output would be smaller, and prices would be higher, compared to first best.

marginal revenue is convex, and *vice versa*. Moreover, when the marginal revenue function is convex (concave), an increase in the variation in inflow in the second period results in a lower (higher) optimal production in the first period. The intuition is also similar: for example, when marginal revenue is convex, a large supply realisation leads to a smaller reduction in marginal revenue than the increase following a correspondingly small realisation; therefore, the more variation there is in supply the higher is expected marginal revenue in the second period and so the more output it pays to move from the first period to the second.

The above analysis is summarised in the following proposition.

Proposition 2 *At maximum profit for the monopoly producer, we have*
i) $X_1^m = \frac{\overline{Q}}{2}$ *when* $\varphi = 0$.
ii) $\frac{dX_1^m}{d\varphi} \gtreqless 0 \Leftrightarrow M'' \lesseqgtr 0$
ii) $\frac{dM_1^m}{d\varphi} = \frac{dEM_2^m}{d\varphi} \lesseqgtr 0 \Leftrightarrow M'' \lesseqgtr 0$

Straightforward differentiation gives

$$M''(X) = 3p''(X) + p'''(X)X.$$

It follows that a sufficient condition for the marginal revenue function M to be convex is that the inverse-demand function is convex and that its third derivative is positive (conversely, marginal revenue is concave if the second and third derivatives of the inverse-demand function are negative). In the case of a quadratic inverse-demand function, marginal revenue is convex if the inverse-demand function is convex, and *vice versa*. With linear inverse demand, marginal revenue is linear also.

In order to compare the monopoly outcome to that of perfect competition, it is convenient to consider marginal revenue as a function of price rather than output. We can rewrite the marginal revenue (8) as

$$\widetilde{M}(p) = p + \frac{x(p)}{x'(p)},$$

where $x(p)$ is the demand function and $x'(p)$ is the derivative of the demand function.

Hence, we rewrite the condition (7) as a function of price:

$$\widetilde{M}(p_1) = E\widetilde{M}(p_2),$$

where p_2 depends on ε through $p_2 = p(Q + \varphi\varepsilon - X_1)$.

Applying Jensen's Inequality, we have

$$\widetilde{M}(p_1) = E\left\{\widetilde{M}(p_2)\right\} \gtreqless \widetilde{M}(E\{p_2\}) \Longleftrightarrow \widetilde{M}'' \gtreqless 0.$$

It follows that the first-period price exceeds the expected second-period price if marginal revenue (as a function of price) is convex, and *vice versa*:

$$p_1 \gtreqless E\{p_2\} \Longleftrightarrow \widetilde{M}'' \gtreqless 0.$$

Therefore, since from (2) $p_1 = E\{p_2\}$ when competition is perfect, we have that output in the first period is less under monopoly than under perfect competition if marginal revenue as a function of price is convex, and *vice versa*:

Proposition 3 *Comparing outcomes under monopoly and perfect competition, we have:*

$$X_1^m \lesseqgtr X_1^f \Longleftrightarrow \widetilde{M}'' \gtreqless 0.$$

Above we noted that marginal revenue is linear if demand is linear, so in this case outcomes under monopoly and perfect competition coincide. Furthermore, we may rewrite marginal revenue as a function of market price as follows:

$$\widetilde{M}(p) = p[1 - \gamma(p)],$$

where $\gamma = \frac{X}{p}p'$ is the absolute value of the inverse demand elasticity. It follows that if the demand elasticity is constant, marginal revenue has the same shape as the inverse-demand function and, moreover, since marginal revenue is linear in price, in expected terms prices are equalised across periods and the monopoly solution coincides with that under perfect competition.

We further have

$$\widetilde{M}''(p) = -2\gamma'(p) - p\gamma''(p).$$

In other words, a sufficient condition for marginal revenue (as a function of market price) to be convex is that the (absolute value of) the inverse demand elasticity is decreasing and concave (conversely, marginal revenue is concave if the inverse demand elasticity is increasing and convex). If the inverse demand elasticity is linear in price, marginal revenue is convex if the elasticity is decreasing, and *vice versa*. A decreasing inverse demand price elasticity implies that the price effect of increasing supply is larger for high prices (or small output) than for low prices (or large output); in other words, high realisations of output necessitates only small reductions in price while small realisations of output allows for large increases in price; therefore the monopolist has greater incentive to move output from the first to the second period than a welfare maximising planner and so under monopoly output becomes smaller, and price higher, in Period 1 compared to first best or perfect competition.

5 Differentiated Duopoly

So far we have studied monopoly and oligopoly with identical producers. In practice, producers differ in various ways, particularly with respect to the nature of the uncertainty they face. Such differences may result from idiosyncratic features, such as technology, or variations in extraneous factors, such as the weather. For example, windpower production has different stochastic properties to hydropower production, while wind and hydrological conditions vary between geographical areas. Moreover, combinations of production possibilities may affect producers exposure to risk; for example, if a producer has both windpower and hydropower, negative correlation between wind and water inflow reduces the total variance of aggregate production possibilities.

In this section we analyse how the distribution of uncertainty among producers affects the market outcome, by considering a duopoly market in which the two producers face the same expected inflow but variances of inflow may differ. Specifically, we assume that inflows are perfectly correlated, but that the producers' shares of total uncertainty differ. Let $\varepsilon_1 = [1-\lambda]\,\varphi\varepsilon$ and $\varepsilon_2 = \lambda\varphi\varepsilon$, where λ is a parameter that satisfies $\lambda \in (0, \frac{1}{2})$; in other words, we order firms such that Firm 2 is the producer with the smaller risk exposure. The variance of inflow in the second period for the two producers are $var\,(\varepsilon_1) = var\,([1-\lambda]\,\varphi\varepsilon)$ and $var\,(\varepsilon_2) = var\,(\lambda\varphi\varepsilon)$, respectively. Note that aggregate uncertainty in the system is $\varphi\varepsilon$, with variance $\varphi^2 var\,(\varepsilon)$, so that the level of λ does not affect the total variance in the system.

We further assume that in all other respects producers are identical. Specifically, the deterministic part of water inflow to the reservoirs are the same, and so are the reservoir levels at the beginning of Period 1 and the required level of reservoirs the end of Period 2. It follows that the deterministic parts of output available over the two periods are identical, i.e. $\overline{Q}_1 = \overline{Q}_2 = \overline{Q}$. The relevant intertemporal resource constraints may then be written

$$x_{12} = Q + \varphi\,[1-\lambda]\,\varepsilon - x_{11}. \tag{9}$$

$$x_{22} = Q + \varphi[\lambda]\varepsilon - x_{21}. \tag{10}$$

From the first-order conditions for the two producers (5), we find

$$x_{11} - x_{21} = \frac{E\,\{p_2'\varphi\,[1-2\lambda]\,\varepsilon\}}{p_1' + E\,\{p_2'\}} = [1-2\lambda]\,\frac{\varphi cov\,(p_2', \varepsilon)}{p_1' + E\,\{p_2'\}}. \tag{11}$$

It follows that, since $1 - 2\lambda \geq 0$ and $p_1' + E\,\{p_2'\} < 0$, the sign of $x_{11} - x_{21}$ is the opposite of that of $cov\,(p_2', \varepsilon)$. We have earlier shown that $cov\,(p_2', \varepsilon)$ is positive for $p_2'' > 0$ and negative for $p_2'' < 0$. Consider the case in which the inverse demand function is convex, i.e. $p'' > 0$, so that $cov\left(p_2', \varepsilon\right) > 0$. Then Producer 1, who faces more uncertainty, produces less than Producer 2 in the first period. The intuition is that, since the demand function is convex, large realisations of output reduces price by relatively less than the increase in price following a correspondingly small realisation in output. Consequently, the higher is the variation in output, the larger is the average price received in the second period and hence the stronger is the incentive to shift output from Period 1 to Period 2. In the converse case, in which inverse demand is concave, i.e. $p'' < 0$, so $cov\left(p_2', \varepsilon\right) < 0$, we have the opposite result: the producer with the more uncertain inflow produces more in the first period than the producer with less variance in inflow.

We analyse the effects of a small increase in the total standard deviation of inflow, φ, when λ is kept constant. By totally differentiating and solving the system (5) (see the Appendix for details), we get:

$$\frac{dx_{11}}{d\varphi} = \frac{dX_1}{d\varphi}\frac{p_1' + p_1''x_{11} + E\left\{p_2' + p_2''x_{12}\right\}}{p_1' + E\left\{p_2'\right\}} + \frac{E\left\{[1+\lambda]\, p_2'\varepsilon + p_2''\varepsilon x_{12}\right\}}{p_1' + E\left\{p_2'\right\}}. \tag{12}$$

$$\frac{dx_{21}}{d\varphi} = \frac{dX_1}{d\varphi}\frac{p_1' + p_1''x_{21} + E\left\{p_2' + p_2''x_{22}\right\}}{p_1' + E\left\{p_2'\right\}} + \frac{E\left\{[2-\lambda]\, p_2'\varepsilon + p_2''\varepsilon x_{22}\right\}}{p_1' + E\left\{p_2'\right\}}. \tag{13}$$

The effects on the two producers' output in the first period differ depending on the parameter λ as well as their initial output profiles. When the producers are identical, i.e. $\lambda = 0.5$, effects are identical also. Moreover, starting from the point of no uncertainty, i.e. $\varphi = 0$, where $x_{11} = x_{21} = x_{12} = x_{22}$, find

$$\frac{dx_{11}}{d\varphi} - \frac{dx_{21}}{d\varphi} = [1 - 2\lambda]\frac{cov\left(p_2', \varepsilon\right)}{p_1' + E\left\{p_2'\right\}}.$$

In other words, $\frac{dx_{11}}{d\varphi} > \frac{dx_{21}}{d\varphi}$ if $cov\left(p_2', \varepsilon\right) < 0$, and *vice versa*. This corresponds to the result we found above: here, an introduction of a small measure of uncertainty leads to a stronger reaction by the firm who faces a larger share of this uncertainty.

For overall efficiency, total output, but not individual producers' share of production, is important. We obtain the aggregate effect on output in the first period of an increase in the standard deviation of inflow by adding (13) and (12) and solving:

$$\frac{dX_1}{d\varphi} = \frac{E\left\{3p_2'\varepsilon + p_2''\varepsilon X_2\right\}}{3p_1' + p_1''X_1 + E\left\{3p_2' + p_2''X_2\right\}}. \tag{14}$$

The distribution of variance among the two producers does not affect the impact on aggregate output. Hence, from a competition policy point of view, distribution of uncertainty across producers does not matter, at least in the present context. Note that this result is in contrast to the standard result in oligopoly theory that competition between equal firms is more intense than competition between asymmetric firms; see Borenstein *et al.* (1999) for an application to electricity markets.

The above analysis is summarized in the following proposition.

Proposition 4 *Consider a duopoly, in which producers are identical in all respects except that* $\varepsilon_1 = [1-\lambda]\,\varphi\varepsilon$ *and* $\varepsilon_2 = \lambda\varphi\varepsilon$*, where* $\lambda \in \left(0, \frac{1}{2}\right)$*. Then*

i) $x_{11} \gtreqless x_{21} \Leftrightarrow p'' \lesseqgtr 0$.

ii) $X_1 = x_{11} + x_{12}$ *does not depend on* λ.

6 The Shape of Residual Demand

We have found that the impact of uncertainty on hydro-producer behaviour depends crucially on the shape of demand facing individual producers.

From standard consumption theory, we know that the second derivative of the inverse demand function may be expressed as the third derivative of the utility function. A positive second derivative of the inverse demand function therefore corresponds to a positive third derivative of the utility function, which is the condition for precautionary saving (characterizing the consumer as "prudent"; see eg. Menegatti, 2001). The precautionary-saving result is discussed in Bjerkholt and Olsen (1994).[7] In our context, it implies that when consumers are prudent uncertainty provides an independent motive for storing water in reservoirs or enhancing supply security. The case of a concave inverse demand function corresponds to consumer dis-saving, in which case storage and supply security should be reduced. In other words, under perfect competition firms produce less (more) than the deterministic benchmark when consumers are prudent (not prudent).

When competition is imperfect, behaviour depends on higher order derivates of the inverse demand function also. The third derivative of the inverse demand function is the fourth derivative of the utility function and there is only a weak economic interpretation of this entity. Menegatti (2001) points out that a positive fourth derivative of the utility function implies an increase in prudence. The interpretation of the special case where $p'' > 0$ and $p''' > 0$ is that the consumer is a precautionary saver with prudence that increases with consumption.

[7] See also Førsund (2007, chapter 2).

Now, the demand facing a firm with market power does not depend solely on consumer demand, but also on the supply of its competitors. Supply decisions depend again on production technology and other constraints, such as bottlenecks in the transmission network. In the Nordic market for instance, hydropower producers face competition from thermal production. In such mixed markets, the shape of residual demand, and hence the impact of uncertainty on the behaviour of hydro producers, will depend not only on consumer preferences but also on technology in both generation and transmission.[8]

To see this, suppose that in addition to hydropower there is thermal capacity in the system operated by a competitive fringe. The fringe equates price to marginal cost, or $c'(q) = p$, where q is fringe output and c is fringe costs. Then residual demand facing a monopolist hydropower producer is given by $x = d - q$, where d is consumer demand. The first- and second-order derivates of the residual demand function is given by $x' = d' - q'$ and $x'' = d'' - q''$, respectively.

One might be tempted to argue that, since the demand response of consumers is generally low in electricity markets, characteristics of residual demand is determined first and foremost by the character of competitor supply. However, this is not necessarily so. First, consumer response to prices can be considerable, as was demonstrated during the winter of 2002-2003, when the Nordic electricity market experienced unusually low water inflow. Second, what matters here is not the slope of the demand function, but its curvature; even a step demand function may change slope rather quickly.

Neverthless, the character of the supply side of the market may well be important in determining the shape of residual demand facing hydropower producers. Figure 2 contains a plot of marginal cost at the aggregate level in the Nordic power market.[9] Marginal cost is quite flat in the area realised in wet and normal years, but relatively steep in the area realised in dry years. Interpreted in the simple model above, the convexity of the marginal-cost curve translates into a concave supply function for the fringe (i.e. $q'' < 0$), which tends to make the inverse residual demand function convex.

Bottlenecks in the transmission system - a prevalent feature of most electricity markets, including the Nordic market - affect residual demand also. Take the case of Norwegian hydro producers as an example. In dry years, there will be import to Norway, while in wet years there will

[8]See for instance Borenstein *et al.* (1999) and Førsund (2007, chapter 5) for discussions of mixed markets.

[9]Costs for each technology are given as average variable costs on a yearly basis, which are mostly determined by productive efficiency and the cost of fuel.

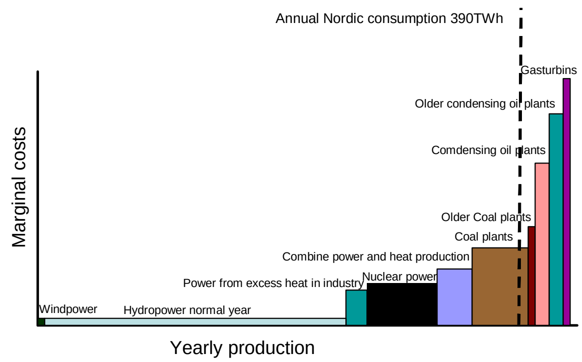

Figure 1: Marginal costs in the Nordic power market

be export. Moreover, in very dry years import capacity will be binding. Consequently, bottlenecks in the transmission system means that supply function of (foreign) competitors in effect becomes vertical for sufficiently high prices, thereby exagerating any tendency to convexity of the residual demand function facing Norwegian hydro producers.

Moreover, in the absence of bottlenecks residual demand will be influenced both by demand and supply from outside regions, whereas when bottlenecks arise residual demand will be determined by local demand only.[10] Continuing the above example of Norwegian hydro, this implies that the inverse demand elasticity tends to be both increasing and convex (due to the "kink" at the point where the transmission constraint binds). If so, our theory suggests that producers with market power will produce more, and store less, when faced with inflow uncertainty than producers without such power, thereby accentuating scarcity when water availability turns out to be low and so undermining security of supply.

7 Concluding Remarks

In this paper we have analysed effects of uncertainty in inflow on hydropower markets under alternative assumptions about market structure.

It turns out that effects depend on the shape of the demand for electricity. When the inverse demand function is convex, competitive firms optimally respond to uncertainty by saving more output (i.e. water) for the future, thereby accommodating to the fact that low realisations of inflow are relatively more costly to consumers than the gains from corresponding high realisations. Convex demand may be interpreted

[10] Johnsen (2001) has an example of this for a small region in Norway

as consumer prudence, when uncertainty should lead to precautionary saving. When demand is concave, optimality is achieved by saving less.

Firms with market power are concerned not with price *per se* but marginal revenue, which depends on higher order differentials of the (inverse) demand function. When demand is linear, or elasticity of demand is constant, outcomes under imperfect competition coincide with those under perfect competition. More generally, imperfect competition may lead to either more or less saving of output than under perfect competition, depending on the relation between marginal revenue and market price. In particular, whether firms with market power save more or less than firms without such power depends not only on consumer prudence but also on the increase in prudence following a change in output.

The shape of demand faced by individual firms depends not only on consumer preferences but also on the behaviour of competitors. Specifically, residual demand depends on underlying technology, both in generation and transmission. The impact of uncertainty on behaviour therefore requires detailed knowledge not only about consumer demand but also technological characteristics of the power system.

Variations in exposure to uncertainty affect the behaviour of individual producers, but not necessarily aggregate output. Aims by individual producers to reduce exposure to risk by changing technology, or by shifting risk to competitors, may therefore be counterproductive from the point of view of the system as a whole.

The analysis has been conducted in a simplified set up aiming at isolating the effect of uncertainty. In particular, we have considered identical demand across time and assumed that there is no possibility (or incentive) to spill water. Relaxing these assumptions would increase the scope for exercising market power.

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8.1 Asymmetric Uncertainty in a Duopoly Market

We have two producers with the first-order condition from (5) with the production constraints given by (9) and (10). Aggregate production in the second period is given by:

$$X_2 = Q_1 + Q_2 + \varphi\varepsilon - X_1. \tag{15}$$

Total differentiating the equation (5) i=1,2 with constraints and (15) yields the system of equations:

$$p_1' dX_1 + p_1'' dX_1 x_{i1} + p_1' dx_{i1} - p_2' dX_2 - p_2'' dX_2 x_{i2} - p_2' dx_{i2} = 0,$$

where i=1,2 and

$$dX_2 = d\varphi\varepsilon - dX_1,$$
$$dx_{12} = d\varphi \left[1-\lambda\right] \varepsilon - dx_{11},$$
$$dx_{22} = d\varphi \left[\lambda\right] \varepsilon - dx_{21}.$$

By solving the above system for producer 1 we get:

$$\begin{aligned} & dX_1 \left(p_1' + p_1' x_{11} + E\left\{p_2' + p_2'' x_{12}\right\}\right) + dx_{11}\left(p_1' + E\left\{p_2'\right\}\right) \\ = & \, d\varphi \left(E\left\{p_2'\varepsilon + p_2'' x_{12} + p_2' \left[1-\lambda\right]\varepsilon\right\}\right). \end{aligned} \tag{16}$$

Solving (16) with respect to $\frac{dx_{11}}{d\varphi}$ gives (13).

By solving the above system for producer 2 we get:

$$\begin{aligned} & dX_1 \left(p_1' + p_1' x_{21} + E\left\{p_2' + p_2'' x_{22}\right\}\right) + dx_{21}\left(p_1' + E\left\{p_2'\right\}\right) \\ = & \, d\varphi \left(E\left\{p_2'\varepsilon + p_2'' x_{22} + p_2' \lambda\varepsilon\right\}\right). \end{aligned} \tag{17}$$

Solving (17) with respect to $\frac{dx_{21}}{d\varphi}$ gives (12).

We know that $dX_1 = dx_{11} + dx_{21}$, and solving for $\frac{dX_1}{d\varphi}$ we get (14).