Delinquent Networks

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Introduction

Important literature in criminology and sociology on the social aspects of crime

(Sutherland, 1947, Sarnecki, 2001 and War, 2002)

Main Results

The positive correlation between self-reported delinquency and the number of delinquent friends reported by adolescents has proven to be among the strongest and one of the most consistently reported findings in the delinquency literature. In other words, the most important explanatory variable that explains the crime decision of a particular individual is the number of delinquent friends that this individual has.

Very little has been done in economics (at least theory)

Empirical evidence: Peer effects are very strong in criminal decisions.

Case and Katz (1991), data from the 1989 NBER survey of youths living in low-income Boston neighborhoods.

Moving a youth with given family and personal characteristics to a neighborhood where 10 percent more of the youths are involved in crime than in his or her initial neighborhood is to raise the probabibility the youth will become involved in crime by 2.3 percent.

Glaeser, Sacerdote and Scheinkman (QJE, 1996).

Accross crimes, crime committed by younger people have higher degrees of social interaction

Accross cities, the degree of social interactions is larger in those communities where families are less intact.

Ludwig, Duncan and Hirschfield (QJE, 2001), Moving to Opportunity (MTO) experiment that relocates families from high- to low-poverty neighborhoods.

They find that this policy reduces juvenile arrests for violent offences by 30 to 50 percent of the arrest rate for control groups.

Chen and Shapiro (2003) find strong peer effects in prison by showing that worsening prison conditions significantly increases post-release crime.

Damm and Dustmann (2008): Does growing up in a neighborhood in which a relatively high share of youth has committed crime increase the individual's probability of committing crime later on?

Danish natural experiment that randomly allocates parents of young children to neighborhoods with different shares of youth criminals.

Area fixed effects: One standard deviation increase in the share of youth criminals in the municipality of initial assignment increases the probability of being charge with an offense at the age 18-21 by 8 percentages point (or 23 percent) for men.

This neighborhood crime effect is mainly driven by property crime.

Bayer et al. (2009) consider the influence that juvenile offenders serving time in the same correctional facility have on each other's subsequent criminal behavior.

Strong evidence of learning effects in criminal activities: Exposure to peers with a history of committing a particular crime increases the probability that an individual who has already committed the same type of crime recidivates with that crime.

AIM: To wedge a bridge between the underlying social setting where individuals are embedded, and the individual crime decisions.

Two prominent (THEORY) exceptions are Sah (1991) and Glaeser, Sacerdote and Scheinkman (1996).

Sah (JPE, 1991) examines the influence of the social surrounding on individual expectations.

Individuals' perceptions of punishment probabilities evolve endogenously with the information on the costs of crime these individuals gather locally, this information being in turn related to the level of local involvement in crime activities.

The paper examines the coupled dynamics of the individual sense of impunity (or lack of), the associated individual crime behavior, and the local patterns of crime.

Glaeser, Sacerdote and Scheinkman (QJE, 1996) provide a model of crime decisions with positive interrelationships among such decisions.

The positive covariance across agents' decisions about crime generates an overall variance in crime rates higher than predicted by the simple cost-benefit trade off at the individual level.

The positive covariance is obtained with a population of ex ante heterogeneous agents

rational agents, who decide to become criminals after arbitrating the costs and benefits associated to this activity

and imitators, who simply blindly imitate the behavior of rational agents.

Given a topology for social interactions (in the paper, agents are arranged on a circle), individual decisions taken by rational agents spill over nearby imitators and generate clusters of agents undertaking identical activities (either criminals or workers).

This model yields to an index of the degree of social interactions which is later estimated for a variety of different crimes in the U.S.

Our model.

Compared to the standard crime model (Becker, 1968) three main innovations.

First, the criminal decision is not binary since individuals not only decide to become a criminal or not but also determine how much effort to put in criminal activities.

Second, (on the benefit part), the booty is not exogenous but depends positively on the effort level of the criminal in question and negatively of the efforts of the other criminals.

Third, on the cost part, criminals may benefit from being friends with other criminals or more exactly from belonging to the same network of relationships.

Motivation

- Delinquent decisions (Petty crime)
  - Each player makes an effort regarding delinquent behavior.
  - Not organized, noncooperative.
  - Efforts display some degree of stategic complementarities.

Motivation

- Delinquent decisions (Petty crime)
  - Are more central delinquents more active delinquents?
  - Is there any simple recipe to determine who are the delinquents to keep track of, based on their pattern of connections?
  - What agents should be watched/neutralized in order to maximally reduce delinquency?

Motivation

- Delinquent decisions (Petty crime)
  - A policy maker may worry about welfare *inside* the group or *outside* the group.
  - Is there any simple recipe to determine who are the delinquents to keep track of, based on their pattern of connections?
  - What agents should be watched/neutralized in order to maximally reduce delinquency?

Literature

- Games on networks
  - Jackson (2007)
  - Ballester, Calvó-Armengol & Zenou (2006)
  - Calvó Armengol and Zenou (2004).
  - Bramoullé & Kranton (2007)
  - Goyal & Moraga (2001)
  - Galeotti et al. (2006)
- Social Interactions
  - Becker (1968)
  - Glaeser , Sacerdote and Scheinkman (1996).
  - Benabou (1994).
- Social Network Analysis
  - Bonacich (AJS 1987)
  - Borgatti (2003)
  - Freeman (1979)
  - Granovetter (AJS 1973)

What we do

- Use a model of delinquent behavior and evaluate the different policies:
  - Impact on aggregate delinquent activity.
  - Implementation: algorithimc considerations/complexity.

The general model

- A set of players N
- Strategies $x_i \in \mathbb{R}_+$
- Utility

$$u_i(\mathbf{x}) = \alpha x_i - x_i^2 + \sum_{j \neq i} \sigma_{ij} x_i x_j$$

- Second derivatives

$$\mathbf{\Sigma} = (\sigma_{ij})_{i,j \in N}$$

- Pure-strategy Nash equilibria.

The general model

- A decomposition trick

$$u_1(\mathbf{x}) = 1 - x_1^2 - 0.5x_1x_2$$
$$u_2(\mathbf{x}) = 1 - x_2^2 + 0.2x_1x_2$$

$$\Sigma = \begin{pmatrix} -1 & -0.5 \\ 0.2 & -1 \end{pmatrix}$$

$$\begin{pmatrix} -1 & -0.5 \\ 0.2 & -1 \end{pmatrix} = -0.5\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + \begin{pmatrix} -0.5 & 0 \\ 0.7 & -0.5 \end{pmatrix}$$

$$= -\underbrace{0.5}_{\gamma}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} - \underbrace{0.5}_{\beta}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \underbrace{0.7}_{\lambda}\underbrace{\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}}_{\substack{\mathbf{G} \\ \text{network}}}$$

The general model

- The utility function *before* the decomposition

$$u_i(\mathbf{x}) = \alpha x_i - x_i^2 + \sum_{j \neq i} \sigma_{ij} x_i x_j$$

- The utility function *after* the decomposition

$$u_i(\mathbf{x}) = \alpha x_i - \frac{1}{2}(\beta - \gamma) x_i^2 - \gamma \sum_j x_i x_j + \lambda \sum_j g_{ij} x_i x_j$$

The general model

- **Theorem.** Let $\lambda^* = \lambda/\beta$

  If

  $$\beta > \lambda\rho(\mathbf{G})$$

  the unique Nash equilibrium of the game is:

  $$x_i^*(\mathbf{\Sigma}) = K_\Sigma \bullet b_i(\mathbf{G}, \lambda^*)$$

- Structure of peer effects:

$$x_i^*(\mathbf{\Sigma}) = \frac{b_i(\mathbf{G}, \lambda^*)}{b(\mathbf{G}, \lambda^*)} x^*(\mathbf{\Sigma})$$

The general model

- Katz-Bonacich centrality index
  - It is a measure of influence/power.
  - Assigns to each node in a network, the number of walks starting from it

$$b_i(\mathbf{G}, a) = \sum_{k=0}^{\infty} a^k b_i^{[k]}(\mathbf{G})$$

  - a is the *decay factor* of incluence: how distant friends matter.
  - Total centrality of the network:

$$b(\mathbf{G}, a) = \sum_{i=0}^{n} b_i(\mathbf{G}, a)$$

The general model

- Aggregate decision increases with the density of the network of complementarities.

The delinquency model

- Utility function

$$u_i(x,g) = \underbrace{y_i(x)}_{proceeds} + \underbrace{p_i(x,g)}_{apprehension} \quad \underbrace{f}_{fine}$$

- Assume

$$y_i(x) = x_i \max\{1 - \delta \sum_{j=1}^{n} x_j, 0\}$$

$$p_i(x,g) = p_0 x_i \max\{1 - \phi \sum_{j=1}^{n} g_{ij} x_j, 0\}$$

The delinquency model

$$u_i(x,g) = \underbrace{(1-\pi)x_i - \delta x_i^2}_{Own\ concavity} - \underbrace{\delta \sum_{j \neq i}^{n} x_i x_j}_{Global\ Substituability} + \underbrace{\pi\phi \sum_{j=1}^{n} g_{ij} x_i x_j}_{apprehension}$$

where $\pi = p_0 f$

Nash equilibrium

For all $\mathbf{y} \in \mathbb{R}^n$, $y = y_1 + ... + y_n$ is the sum of its coordinates.

Define $b(g, \theta) = \sum_{i=1}^{i=n} b_i(g, \theta)$, denote $\theta = \pi\phi/\delta$ and let $\rho\left(g\right)$ be the *spectral radius* of the adjacency matrix $\mathbf{G}$.

Proposition 0.1 *If $\theta\rho\left(g\right) < 1$, then there exists a unique Nash equilibrium $\mathbf{x}^*$, which is interior, and given by:*

$$\mathbf{x}^* = \frac{1-\pi}{\delta\left[1 + b(g,\theta)\right]}\mathbf{b}(g,\theta) \quad (1)$$

The key player problem

- Remove an agent so as to minimize the aggregate delinquent activity.

$$i^* \in \arg\min_i x^*(\mathbf{\Sigma}_{-i})$$

- Geometrically

$$i^* \in \arg\min_i b(\mathbf{G}_{-i}, \lambda^*)$$

The key player problem

- **The intercentrality measure.**
  - Accounts for the number of walks *traversing* each agent in the network.
  - Consistent with existing literature on centrality measures: betweeness (Freeman)
  - Measures contribution of a player to other players' centralities.

The key player problem

- The intercentrality measure.

$$c_i(\mathbf{G}, a) = \frac{b_i(\mathbf{G}, a)}{m_{ii}(\mathbf{G}, a)}$$

where $m_{ii}(\mathbf{G}, a)$ counts the number of walks starting and ending at player *i*.

The key player problem

- **Theorem.** The delinquent to eliminate from the network is the one with the highest intercentrality level.
- **Corollary**. The cost of finding the key player is reduced by the intercentrality formula

$$c_i(\mathbf{G}, a) = \frac{b_i(\mathbf{G}, a)}{m_{ii}(\mathbf{G}, a)}$$

Example Network g in Figure 1 with eleven delinquents.

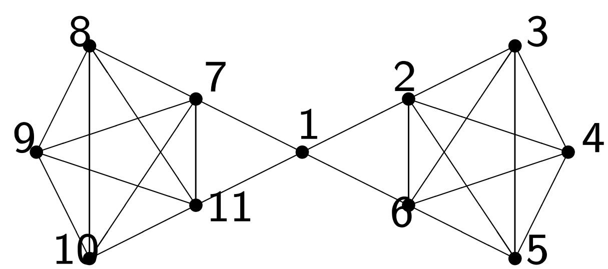


Figure 1

$\theta = \pi\phi/\delta$

$\delta = \phi = 1$

θ	0.1			0.2		
Player Type	x_i	b_i	d_i	x_i	b_i	d_i
1	0.077	1.75	2.92	0.072	8.33	41.67*
2	0.082*	1.88*	3.28*	0.079*	9.17*	40.33
3	0.075	1.72	2.79	0.067	7.78	32.67

Comparing policies

For each delinquent i, define:

$$\eta_i(g) = n \frac{x^*(g) - x^*(g_{-i})}{\sum_{j=1}^{n} \left[x^*(g) - x^*(g_{-j}) \right]}.$$

This is the ratio of returns (in delinquency reduction) when i is the selected target versus a random selection with uniform probability for all delinquents in the network.

$d^{av}(g, \theta)$ average of the intercentrality measures in network g

$\sigma_{\mathbf{d}}(g, \theta)$ standard deviation of the distribution of this intercentrality measures.

Proposition 0.2 *Let i^* be the key player in g for a given θ. Then,*

$$\eta_{i^*}(g) \geq 1 + \frac{\sigma_{\mathbf{d}}(g, \theta)}{d^{av}(g, \theta)}.$$

This establishes a lower bound on the ratio of returns in delinquency reduction when the key player is removed.

Standard punishment policy (Becker)

$x^* = \mathbf{1}^T.\mathbf{x}^*$, equilibrium aggregate delinquency activity.

$$\frac{\partial x^*}{\partial \pi} = \underbrace{-\frac{x^*}{1-\pi}}_{\text{direct negative effect}} + \underbrace{\frac{\phi}{\delta}\frac{\partial x^*}{\partial \theta}}_{\text{indirect positive effect}}$$

where

$$\frac{\partial x^*}{\partial \theta} = \frac{(1-\pi)}{\delta}\frac{1}{[1+b(g,\theta)]^2}\frac{\partial b(g,\theta)}{\partial \theta} > 0$$

Numerical simulations for $\delta = 0.1$. Figures 2a ($\phi = 0.8$)

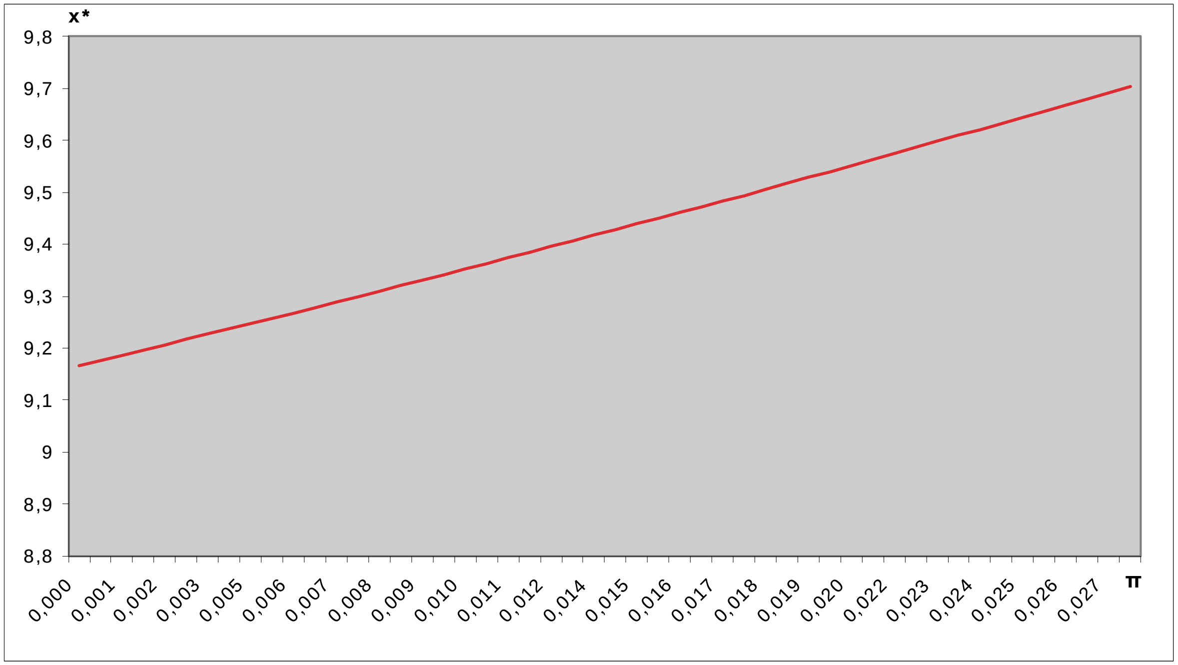


Figure 2a. Impact of deterrence π on total level of delinquent activity for $\delta = 0.1$ and $\phi = 0.8$.

When $\phi = 0.8$, the network is more connected and there are a lot of synergies between delinquents.

Bonacich centralities have high values, meaning that both direct and direct links matter very much.

In that case, increasing punishment π *increases* total delinquent activity because the indirect positive effect dominates the direct negative effect.

Figure 2b ($\phi = 0.1$).

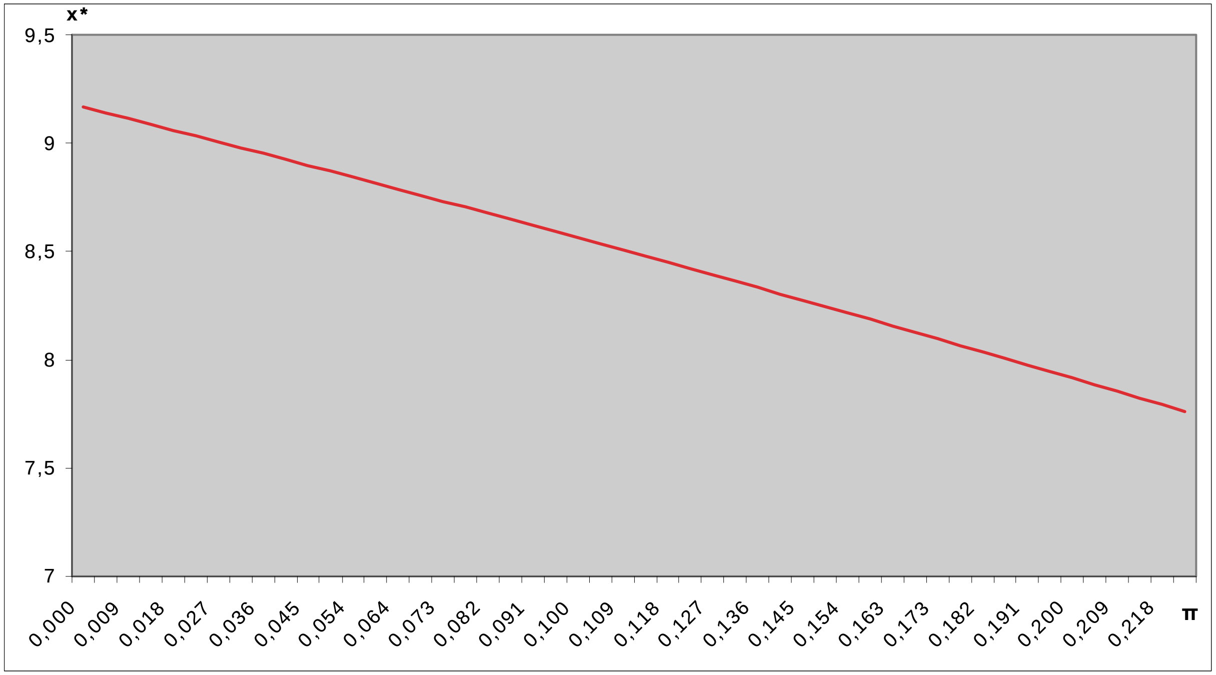


Figure 2b. Impact of deterrence π on total level of delinquent activity for $\delta = 0.1$ and $\phi = 0.1$.

Interactions are not important between criminals since $\phi = 0.1$.

Friends of friends have not that much influence on delinquents.

Network effect becomes unimportant compared to the deterrence effect and an increase in punishment π reduces total delinquency x^*.

Finding the key link

Policy that targets *links* rather than *individuals*.

How to remove optimally a link (or a set of links) between two individuals in order to minimize the total delinquency level.

A social planner would like to optimally reduce the (communication) externalities among delinquents subject to a restriction in the number r of bilateral influences that can be targeted.

Another example is to put a delinquent teenager in another school where there are less delinquent (MTO). By doing so, this delinquent will stop his activities and communication with other delinquents in the older school.

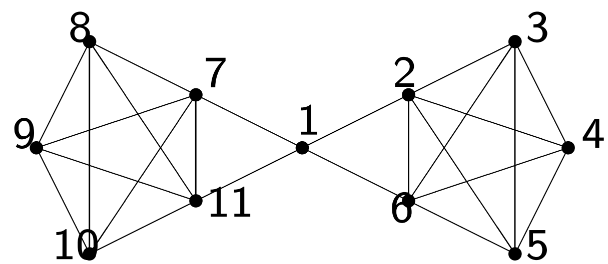


Figure 1

As in the case of the key player, even when $r = 1$, the optimal choice can depend on the strength of complementarities:

Removed link $\{i, j\}$	Reduction in $b(g, 0.1)$	Reduction in $b(g, 0.22)$
$\{1, 2\}$	0.59	185.99*
$\{2, 6\}$	0.63*	180.84
$\{2, 3\}$	0.58	164.37
$\{3, 4\}$	0.53	148.95

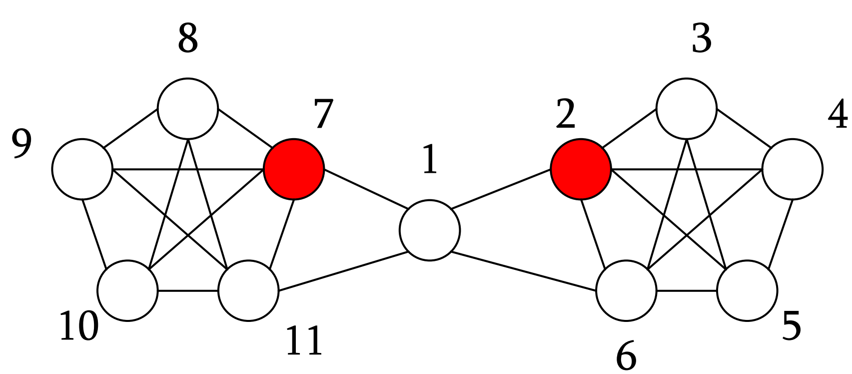
The key group

- The key group: Choose *s* delinquents so as to reduce total activity.
  - The key group is the set of delinquents with highest group intercentrality in the network.

The key group

- Relationship to the key player. Let $S = \{i_1, \ldots, i_s\}$

$$c_S(\mathbf{G}, a) = c_{i_1}(\mathbf{G}, a) + c_{i_2}(\mathbf{G}_{-\{i_1\}}, a) + \ldots + c_{i_s}(\mathbf{G}_{-\{i_1, i_2, \ldots, i_{s-1}\}}, a)$$



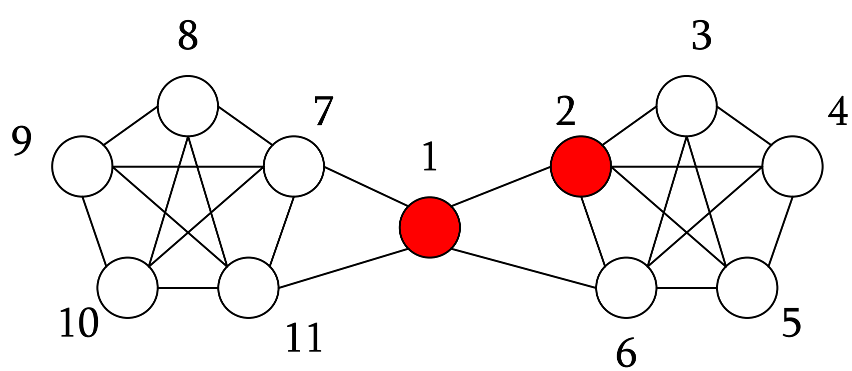
Removed Group S	$d_S(g, a)$
$\{2,7\}^*$	67.22
$\{2,8\}$	64.01
$\{3,8\}$	59.39
$\{1,2\}$	56.66
$\{2,6\}$	50.41
$\{2,3\}$	46.96
$\{3,4\}$	42.15

The key group

- Computational complexity
  - There cannot exist any fast procedure to compute the key group.
    - Choosing the *s* most intercentral. NO.
    - Greedy: choosing the most intercentral at each stage. NO.
    - The problem is *NP-hard*. Approximation algorithms on submodular functions.

The key group

- A greedy algorithm may fail *(a=0.2)*



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$\{3,4\}$	42.15

- Approximation error

$$\frac{67.22-56.66}{67.22} \approx 16\%$$

The key group

- **Proposition.** When using a greedy algorithm to reduce the pool of delinquents, the approximation error is bounded above by

$$\frac{1}{e} \approx 36.79\%$$

  – Group intercentrality is a submodular function

Joining delinquency networks

Allow individuals to choose whether they want to participate in the crime market or not in the first stage.

Two-stage game.

Definition 0.1 *The extended game is a two stage game where:*

- *In stage 1, each player $i \in N$ decides whether to participate ($c_i = 1$) or not ($c_i = 0$) to the crime market.*

- *In stage 2, let S be the set of players who decided to participate. Then, these players play the game in g_S.*

- *The final utilities are:*

$$U_i(S, \mathbf{x}_S, g) = \begin{cases} u_i(\mathbf{x}_S, g_S) & \text{if} \quad i \in S \\ \omega & \text{otherwise} \end{cases}$$

Study the *subgame perfect equilibrium in pure strategies* of this extended game.

Definition 0.2 *The set S is supported in equilibrium if there exists a ω and a subgame perfect equilibrium where the set of players who decide to participate is S, given the outside option ω. S is also called an (equilibrium) participation pool of the game at the wage level ω.*

Let $\mathcal{E}(\omega)$ be the family of sets supported by ω at equilibrium in the extended game.

Proposition 0.3 *Let $S \subseteq N$ and $\theta\rho(g) < 1$ for all $j \in N \backslash S$. Then, the set S is supported at equilibrium by the outside option ω if and only if:*

$$\max_{j \in N \backslash S} \frac{b_j(g_{S \cup \{j\}}, \theta)}{1 + b(g_{S \cup \{j\}}, \theta)} \leq \frac{1}{1-\pi}\omega\delta \leq \min_{i \in S} \frac{b_i(g_S, \theta)}{1 + b(g_S, \theta)}$$

Remark 0.1 *Whenever*

$$\omega > \frac{(1-\pi)^2}{4\delta},$$

all agents outside the delinquency pool is an equilibrium, that is, $\varnothing$ *is supported as an equilibrium by* ω*.*

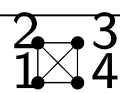
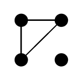
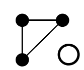
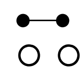
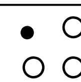
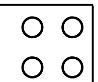
Whenever an equilibrium exists, multiplicity of equilibria is a natural outcome of the extensive form game.

Pairwise-stable networks

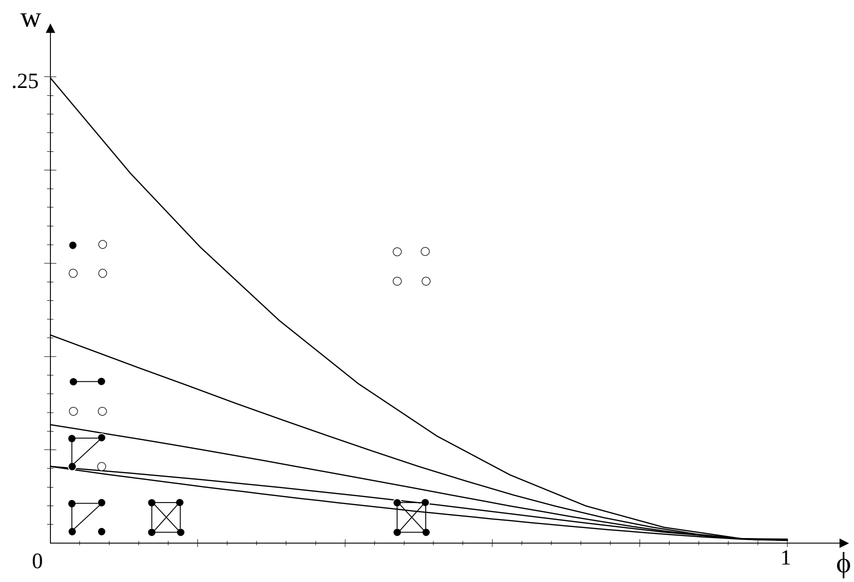
3 stages.

Only for $n = 4$

Last-stage equilibrium effort levels $\left(x_1^*, \ldots, x_4^*\right)$

						
x_1^*	$\frac{1-\phi}{5-3\phi}$	$\frac{1-\phi}{5-4\phi}$	$\frac{1-\phi}{4-2\phi}$	0	0	0
x_2^*	$\frac{1-\phi}{5-3\phi}$	$\frac{1-\phi}{5-4\phi}$	$\frac{1-\phi}{4-2\phi}$	$\frac{1-\phi}{3-\phi}$	$\frac{1-\phi}{2}$	0
x_3^*	$\frac{1-\phi}{5-3\phi}$	$\frac{1-\phi}{5-4\phi}$	$\frac{1-\phi}{4-2\phi}$	$\frac{1-\phi}{3-\phi}$	0	0
x_4^*	$\frac{1-\phi}{5-3\phi}$	$\frac{(1-\phi)(2-\phi)}{10-8\phi}$	0	0	0	0

Multiple equilibria: Equilibrium configurations as a function of w and ϕ.



The equilibrium number of criminals decreases with w (holding ϕ constant) and with ϕ (holding w constant)

Finding the key player with criminal participation decision

No network formation

$x^*(g, \omega)$ maximum aggregate equilibrium delinquency level when the delinquency network is g and the labor market wage is ω.

This delinquency level is equal to the total amount of delinquency in the worst case scenario of maximum delinquency.

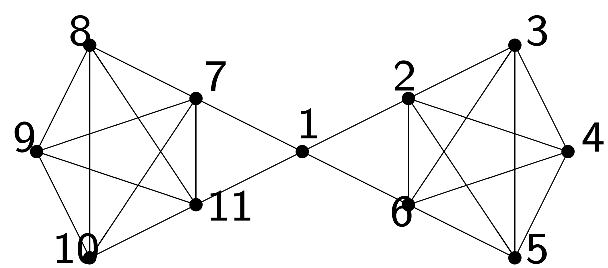
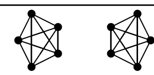
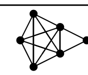


Figure 1

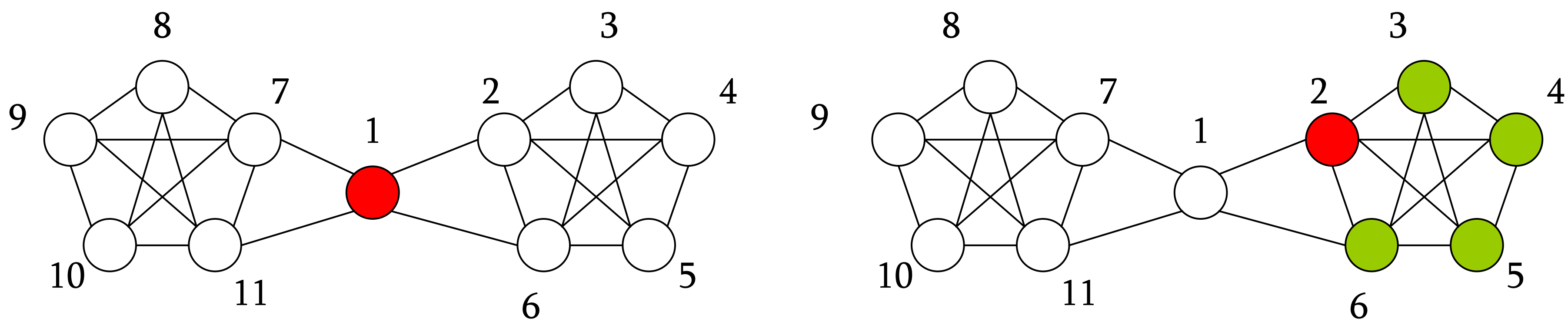
When $\theta = 0.2$ and $\omega = 0$, the key player was the player acting as a bridge, i.e. delinquent 1.

What happens if we now consider the endogenous delinquency network formation in the two-stage game?

	$\omega = 0.001$	$\omega = 0.003$
$x^*(g_{-1}, \omega)$	0.7843	0.7843
$x^*(g_{-2}, \omega)$	0.7847	0.7785
Key Player	1	2
Final delinquency pool		

Extensions

- Endogenous participation $a = 0.2$
  - When $w=0$, the key delinquent is 1.
  - When $w = 5$, the key delinquent is 2.



  - Snowball effect.

This example implicitly explains how one policy (providing a higher ω) increases the effectiveness of another policy (choosing the key player) in order to reduce delinquency.

These policies are *complementary* from the point of view of their effects on total delinquency, although we are aware that they may be substitute if we had considered a budget-restricted planner who had to implement costly policies.

The key-group problem with criminal participation decision

There are now three effects that should be taken into account when choosing the set of players to be removed:

(i) A *direct effect* due to the reduction of their initial delinquent activity. The choice is here biased towards the most Katz-Bonacich central players.

(ii) An *indirect effect* due to the (lower) incentives of the remaining players. In this dimension, group-intercentrality is the relevant variable to consider.

(iii) A possible *snow-ball effect* because the removal of a player may induce a process where the remaining players (sequentially) find it profitable to leave the pool of delinquents and to participate to the labor market. This effect depends on the magnitude of the outside option ω.

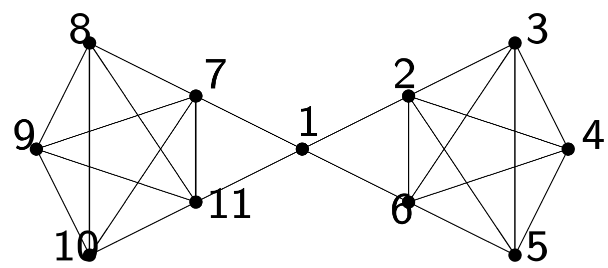


Figure 1

	g_{-1}			g_{-2}		
Pool S	ω_L	ω_H	$b(g_S, \theta)$	ω_L	ω_H	$b(g_S, \theta)$
$\{2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$	0.00	12.50	50.00	—	—	—
$\{2, 3, 4, 5, 6\}$	0.50	12.50	25.00	—	—	—
$\{1, 6, 7, 8, 9, 10, 11\}$	—	—	—	1.02	1.70	39.47
$\{1, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$	—	—	—	0.00	4.17	51.33
$\{1, 7, 8, 9, 10, 11\}$	—	—	—	1.70	7.03	36.25
$\{7, 8, 9, 10, 11\}$	—	—	—	7.03	12.50	25.00
$\{1, 3, 4, 5, 6\}$	—	—	—	0.95	1.25	12.37
$\{3, 4, 5, 6\}$	—	—	—	1.25	3.12	10.00

Juvenile Delinquency and Conformism

Theoretical model

The network $N = \{1, \ldots, n\}$ finite set of agents.

$n-$square adjacency matrix $\mathbf{G}$ of a network $\mathbf{g}$

i and j are directly connected in $\mathbf{g}$ if and only if $g_{ij} = 1$, and $g_{ij} = 0$, otherwise.

$g_{ij} = g_{ji}$ and $g_{ii} = 0$.

Set of individual i's best friends (direct connections) is:

$N_i(\mathbf{g}) = \left\{ j \neq i \mid g_{ij} = 1 \right\}$, which is of size g_i (i.e. $g_i = \sum_{j=1}^{n} g_{ij}$).

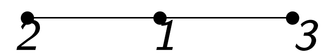
The *reference group* of each individual i is $N_i(\mathbf{g})$, i.e. the set of his/her best friends, which does not include him/herself.

Let $\gamma_{ij} = g_{ij}/g_i$, for $i \neq j$, and set $\gamma_{ii} = 0$. By construction, $0 \leq \gamma_{ij} \leq 1$.

Note that $\boldsymbol{\gamma}$ is a row-normalization of the initial friendship network $\mathbf{g}$,

$\mathbf{G}$ and $\boldsymbol{\Gamma}$ are the adjacency matrices of, respectively, $\mathbf{g}$ and $\boldsymbol{\gamma}$.

Example 0.1 *Consider the following friendship network* $\mathbf{g}$*:*



Then,

$$\mathbf{G} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \qquad \textit{and} \qquad \mathbf{\Gamma} = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Preferences

$e_i(\mathbf{g})$ crime effort level of criminal i in network $\mathbf{g}$.

$e_i^{av}(\mathbf{g})$ average crime effort of the g_i best friends of i

$$e_i^{av}(\mathbf{g}) = \frac{1}{g_i} \sum_{j=1}^{j=n} g_{ij} e_j$$

$$u_i(e_i, e_i^{av}) = \underbrace{a + b_i\, e_i}_{\text{Benefits}} - \underbrace{p\, e_i\, f}_{\text{Costs}} - c\, e_i^2 - \underbrace{d\, (e_i - e_i^{av})^2}_{\text{Peer effects (Conformism)}}$$

with $a, c, d > 0$, and $b_i > 0$ for all i.

b_i ex ante heterogeneity:

$$b_i(\mathbf{x}) = \sum_{m=1}^{M} \beta_m x_i^m + \frac{1}{g_i} \sum_{m=1}^{M} \sum_{j=1}^{n} \theta_m g_{ij} \, x_j^m$$

Individual loses utility $d(e_i - e_i^{av})^2$ from failing to conform to others.

Utility function: standard costs/benefits structure (a la Becker) with an added element, of conformism:

Best friends define a social norm and each individual wants to minimize the social distance between his/her crime level and that of his/her reference group

Individual outcomes results from both idiosyncratic characteristics and peer effects

Payoffs are interdependent and agents choose their levels of activity simultaneously

Bilateral influences:

$$\frac{\partial^2 u_i(e_i, e_i^{av})}{\partial e_i \partial e_j} = \begin{cases} -2\,(c+d) < 0\text{, when } i = j \\ 0\text{, when } i \neq j \text{ and } g_{ij} = 0 \\ 2d > 0\text{, when } i \neq j \text{ and } g_{ij} = 1 \end{cases}.$$

A simple symmetric case

Ex ante, all individuals/criminals are identical, i.e. same ex ante idiosyncratic heterogeneity, so that $b_i = b$.

Proposition 0.1 *Assume that $b_i = b$ and $b > p\,f$. Then, the conformity game has a unique Nash equilibrium in pure strategies, which is given by:*

$$e_i^* = e_i^{av*} = \frac{b - p\,f}{2c}$$

In particular, the higher the deterrence, the lower the crime level.

Ex ante heterogeneity criminals

Proposition 0.2 *Consider the general case when all individuals have ex ante idiosyncratic and peer heterogeneities, and different tastes for conformity. Assume that* $b_i > p\,f$ *for all* i*. Then, there exists a unique Nash equilibrium where each individual* i *provides the following crime effort:*

$$\begin{aligned} e_i^* &= \frac{d}{c+d}\,e_i^{av} + \frac{b_i}{2\,(c+d)} - \frac{p\,f}{2\,(c+d)}, \\ &= \left(\frac{d}{c+d}\right)\frac{1}{g_i}\sum_{j=1}^{j=n} g_{ij}\,e_j + \frac{b_i}{2\,(c+d)} - \frac{p\,f}{2\,(c+d)}, \end{aligned}$$

which is increasing with the average crime effort of the reference group e_i^{av} *and decreasing with deterrence* pf*, i.e.*

$$\frac{\partial e_i^*}{\partial e_i^{av}} > 0 \text{ and } \frac{\partial e_i^*}{\partial pf} < 0$$

Empirical strategy

Theoretical model: behavioral foundation for the so-called *spatial lag model* (see, e.g. Anselin, 1988)

$$e_{i,\kappa} = \phi \frac{1}{g_{i,\kappa}} \sum_{j=1}^{n_\kappa} g_{ij,\kappa} e_{j,\kappa} + \sum_{m=1}^{M} \beta_m x_{i,\kappa}^m + \frac{1}{g_{i,\kappa}} \sum_{m=1}^{M} \sum_{j=1}^{n_\kappa} \gamma_m g_{ij,\kappa} x_{j,\kappa}^m + \nu_{i,\kappa}$$

$\mathbf{G} = \left\{ g_{ij} \right\}$: $n \times n$ spatial weight matrix that formalizes the network structure of the agents (with $g_{ij} = 1$ if i and j are conected, and $g_{ij} = 0$, otherwise),

Maximum Likelihood: $\widehat{\phi}$, $\widehat{\boldsymbol{\beta}}$, $\widehat{\boldsymbol{\gamma}}$

In matrix notation:

$$\mathbf{e} = \phi \mathbf{G} \mathbf{e} + \mathbf{X} \boldsymbol{\beta} + \mathbf{G} \mathbf{X} \boldsymbol{\gamma} + \boldsymbol{\eta} + \boldsymbol{\nu}$$

The reduced-form equation:

$$\mathbf{e} = [\mathbf{I} - \phi \mathbf{G}]^{-1} \mathbf{X} \boldsymbol{\beta} + [\mathbf{I} - \phi \mathbf{G}]^{-1} \mathbf{G} \mathbf{X} \boldsymbol{\gamma} + [\mathbf{I} - \phi \mathbf{G}]^{-1} (\boldsymbol{\eta} + \boldsymbol{\nu}) \,.$$

Data

Dataset of friendship networks in the United States from the National Longitudinal Survey of Adolescent Health (AddHealth)

Richness of the information provided by the AddHealth data

Pupils were asked to identify their best friends from a school roster

Information on the characteristics of nominated friends

9,322 pupils distributed over 166 networks

Friendship networks

Friendship information is based upon actual friends nominations.

Pupils were asked to identify their best friends from a school roster (up to five males and five females)

The limit in the number of nominations is not binding

Less than 1% of the students in our sample show a list of ten best friends

Less than 3% a list of five males and roughly 4% name five females

On average, they declare to have 6.04 friends with a small dispersion around this mean value (the standard deviation is equal to 1.32).

A link exists between two friends if at least one of the two individuals has identified the other as his/her best friend (undirected networks)

We also consider directed networks: 12% of relationships are not reciprocal

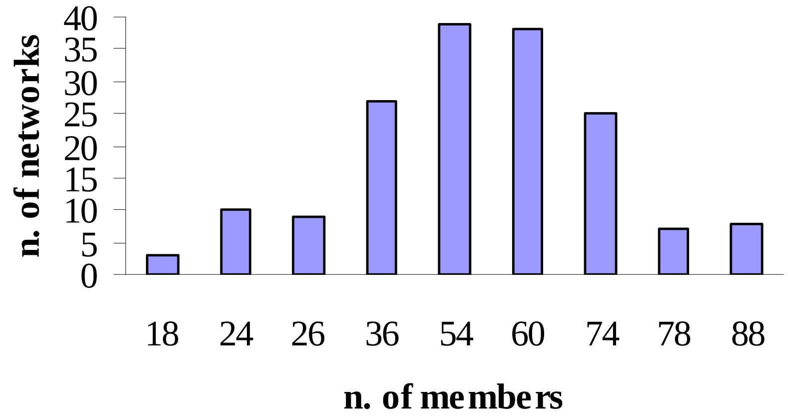


Figure 1. The empirical distribution of adolescent networks

The average and the standard deviation of network size are 49.51 and 16.80, respectively

Criminal activity

Addhealth contains an extensive set of questions on juvenile delinquency, ranging from light offenses that only signal the propensity towards a delinquent behavior to serious property and violent crime

Delinquency index

15 delinquency items:

1) paint graffiti or signs on someone else's property or in a public place

2) deliberately damage property that didn't belong to you

3)lie to your parents or guardians about where you had been or whom you were with

4)take something from a store without paying for it

5)get into a serious physical fight

6)hurt someone badly enough to need bandages or care from a doctor or nurse

7) run away from home

8) drive a car without its owner's permission

9) steal something worth more than $50

10) go into a house or building to steal something;

11) use or threaten to use a weapon to get something from someone

12) sell marijuana or other drugs

13) steal something worth less than $50

14) take part in a fight where a group of your friends was against another group

15) act loud, rowdy, or unruly in a public place.

Each response is coded using an ordinal scale ranging from 0 (i.e. never participate) to 1 (i.e. participate 1 or 2 times), 2 (participate 3 or 4 times) up to 3 (i.e. participate 5 or more times)

The delinquency index is a composite score

The mean is 1.03, with considerable variation around this value (the standard deviation is equal to 1.22).

Estimation issues

Are we really capturing peer effects?

or

Are we only capturing the effects of

- exogenous peer characteristics
- correlation in tastes of people that sort in the same group
- the presence of unobservable factors correlated with our measure of peer effects?

1) The reflection problem (Manski, 1993)

Is it possible to disentangle the endogenous effects, i.e. the influence of peer outcomes, from the (contextual) exogenous effects, i.e. the influence of exogenous peer characteristics?

It arises because in the standard approach individuals interact in groups, that is individuals are affected by all others in their group and by none outside the group

In social networks groups overlaps

Bramoullé et al. (2006): under mild conditions, whenever one considers explicitly a network, then the reflection problem ceases to exist since one can identify separately the two different effects

2) **Correlated effects/Selection issues**

Is it possible to disentangle "endogenous effects" from "correlated effects", i.e. those due to the fact that individuals in the same group tend to behave similarly because they face a common environment?

Correlated effects might originate from the possible sorting of agents into "groups"

If the variables that drive this process of selection are not fully observable, potential correlations between (unobserved) group-specific factors and the target regressors are major sources of bias.

In our case: two types of selection issues

"group" can be the network or the peers

Selection on observables

Our particularly large information on individual, parental, school, neighborhood variables should reasonably explain the process of selection into groups

Selection on unobservables

Assume agents self-select into different networks in a first step, and that link formation takes place within groups in a second step.

Bramoullé *et al.* (2006): if link formation is uncorrelated with the observable variables, this two-step model of link formation generates network fixed effects.

Assuming additively separable network heterogeneity, a within group specification is able to control for selection issues

Bramoullé et al. (2006): by subtracting from the individual-level variables the network average, social effects are again identified and one can disentangle endogenous effects from correlated effects

Our empirical model has a network-specific component of the error term

Traditional (pseudo) panel data fixed effects estimator

Network-specific unobserved effects are controlled for

Evidence

1) network-fixed effects account for unobservable factors driving the allocations of agents into networks

2) Once observables and network-fixed effects are controlled for linking decisions are un uncorrelated with peer-level observables

We consider individual variables that are commonly believed to induce self-selection into teenagers' friendship groups (Parental education, Parental care, Mathematics score, Motivation in education, School attachment, Social exclusion)

3) **Individual and school unobserved heterogeneity**

Existence of unobservable individual and school characteristics that may be correlated with our variable of interest

Proxies for typically unobserved individual characteristics

To control for differences in individual ability and leadership propensity: mathematics score, indicator of self esteem and the level of physical development compared to the peers

School dummies to account for school-specific unobservable effects

3) **Deterrence measures**

Our aim is to identify the existence and extent of peer effects and it is not a precise assessment of the magnitude of the deterrence effect

Identification of deterrence measures (arrest rate, public spending) on crime

-local levels of crime may affect local arrest rates or spending

-local unobservable characteristics which determine both crime rates and deterrence policies

Our network-based empirical strategy cannot help

To control for deterrence policy effects we include county-level dummies

They account for observable and unobservable area-of-residence variables (policing practicing, ethnic concentration, low informal social control, lack of educational or economic opportunities,etc....)

Different model specifications where different sets of controls have been added

(i) Standard individuals' characteristics and behavioral factors (e.g., socio-demografic factors, family background, motivation in education)

(ii) Gradually introduce protective factors (e.g., relationship with teachers, social exclusion, school attachment, parental care)

(iii) Then include proxies aiming at capturing the quality of social interactions (derived characteristics of direct friends)

(iv) Finally, controls for unobservable individual and school characteristics

Results

The estimated effect of ϕ*,which measures the taste for conformity, is positive and highly statistically significant*

The impact is not negligible in magnitude

A one-standard deviation increase in individual i taste for conformity or in the (average) criminal activity of his/her peers raises individual is level of activity by about 5% of a standard deviation

Table 3: Maximum likelihood estimation results

Dependent variable: delinquency index

Variable	All crimes
Conformism / peer effects (ϕ)	0.0612** (0.0305)
Individual socio-demographic variables	yes
Family background variables	yes
Protective factors	yes
Residential neighborhood variables	yes
Contextual effects	yes
School dummies	yes
County dummies	yes
Network fixed effects	yes
pseudo-R^2	0.4766

Notes:

- Estimated coefficients and standard errors (in parentheses) are reported
- Estimation using SpaceStat v1.93 (Anselin, 1995).
- Control variables are those listed in Table A.1
- ** indicates statistical significance at the 5 percent level

Type of crime-specific peer effects

The literature on local interactions has uncovered some interesting differences between different types of crime

For instance, Ludwig et al. (2000) find that neighborhood effects are large and negative for violent crime but have a mild positive effect on property crime

In contrast, Glaeser et al. (1996) find instead that social interactions seem to have a large effect on petty crime, a moderate effect on more serious crime and a negligible effect on very violent crime

The basic idea of our theoretical model is that agents' criminal behavior is driven by their desire to reduce the discrepancy between their own crime effort and that of their reference group (i.e. their best friends). We find that such a model is validated by our data for juvenile crime as a whole

The richness of the information provided by the AddHealth data on juvenile crime enables us also to test our conformism model for different types of crime

Specifically, we analyze whether the magnitude of the peer effects depends on the type of crime committed

We split the offences reported in our data in three groups (with increasing costs of committing crime).

Type-1 crimes contains (i) to paint graffiti or sign on someone else's property or in a public place; (ii) to lie to the parents or guardians about where or with whom having been; (iii) to run away from home; (iv) to act loud, rowdy, or unruly in a public place.

Type-2 crimes consists of (i) to get into a serious physical fight; (ii) to hurt someone badly enough to need bandages or care from a doctor or nurse; (iii) to drive a car without its owner's permission; (iv) to steal something worth less than \$50.

Type-3 crimes encompasses (i) to take something from a store without paying for it; (ii) to steal something worth more than \$50; (iii) to go into a house or building to steal something; (iv) to use or threat to use a weapon to get something from someone; (v) to sell marijuana or other drugs.

Less than 20 percent of the teenagers in our sample confess to have committed the more serious offences.

We estimate the following modified version of our empirical model

$$
\begin{aligned}
e_{i,\kappa,l} &= \phi_l \frac{1}{g_{i,\kappa}} \sum_{j=1}^{n_{i,\kappa}} g_{ij,\kappa}\, e_{j,\kappa} + \sum_{m=1}^{M} \beta_m x_{i,\kappa}^m \\
&\quad + \frac{1}{g_{i,\kappa}} \sum_{m=1}^{M} \sum_{j=1}^{n_\kappa} \gamma_m g_{ij,\kappa} x_{j,\kappa}^m + \varepsilon_{i,k,l}
\end{aligned}
$$

where $e_{i,k,l}$ is now the index of crime of type l committed by individual i in network κ,

ϕ_l : taste for conformity for *type-l crime*

Results

The estimated effect of ϕ_l, which measures the taste for conformity for type-l crime, remains always significant and positive regardless of the seriousness of the crime considered

It decreases in magnitude when moving from light to more serious crimes

Table 3: Maximum likelihood estimation results

Dependent variable: delinquency index

Variable	Type 1	Type 2	Type 3
Conformism / peer effects (ϕ)	0.0688** (0.0320)	0.0499** (0.0241)	0.0079** (0.0035)
Individual socio-demographic variables	yes	yes	yes
Family background variables	yes	yes	yes
Protective factors	yes	yes	yes
Residential neighborhood variables	yes	yes	yes
Contextual effects	yes	yes	yes
School dummies	yes	yes	yes
County dummies	yes	yes	yes
Network fixed effects	yes	yes	yes
pseudo-R^2	0.4915	0.4111	0.4599

Notes:

- Estimated coefficients and standard errors (in parentheses) are reported
- Estimation using SpaceStat v1.93 (Anselin, 1995).
- Control variables are those listed in Table A.1
- ** indicates statistical significance at the 5 percent level

A one-standard deviation increase in individual i's taste for conformity for type-1 crimes translates roughly into a

9.8 *percent increase in standard deviations of individual i's criminal activity when petty crimes (type-1 crimes) are considered,*

whereas this effect amounts to 6.3 *and only to* 1.4 *for intermediary (type-2 crimes) and serious crimes (type-3 crimes), respectively*

This evidence is in line with the findings of Glaeser et al. (1996) who show that social interactions are more important for petty crimes.

Weak and Strong Ties in Crime

Granovetter (1973, 1974, 1983) argued that weak ties are superior to strong ties for providing support in getting a job.

Weak ties are acquaintances and strong ties are close friends.

Granovetter found that neighborhood based close networks were limited in getting information about possible jobs.

In a close network, everyone knows each other, information is shared and so potential sources of information are quickly shaken down, the network quickly becomes redundant in terms of access to new information.

In contrast, Granovetter stresses the strength of weak ties involving a secondary ring of acquaintances who have contacts with networks outside ego's network and therefore offer new sources of information on job opportunities.

Theoretical model

Consider a population of individuals of size one.

Individuals are either noncriminal or involved in criminal activities.

Dyads Individuals belong to mutually exclusive two-person groups, referred to as *dyads*.

Two individuals belonging to the same dyad hold a *strong tie* with each other.

Dyad members do not change over time. A strong tie is created once and for ever, and can never be broken.

Individuals can be in either of two different states: criminal or not criminal.

Dyads, which consist of paired individuals, can thus be in three different states, which are the following:

(i) both members are criminals $-$we denote by d_2 the number of such dyads;

(ii) one member is criminal and the other is not (d_1);

(iii) both members are not criminal (d_0).

Aggregate state

$c(t)$ and $u(t)$ criminal rate and the noncriminal rate at time t,

$c(t), u(t) \in [0, 1]$,

$$\begin{cases} c(t) = 2d_2(t) + d_1(t) \\ u(t) = 2d_0(t) + d_1(t) \end{cases} \tag{1}$$

Total population normalized to 1:

$$c(t) + u(t) = 1 \tag{2}$$

or, alternatively,

$$d_2(t) + d_1(t) + d_0(t) = \frac{1}{2} \tag{3}$$

Social interactions Time is continuous and individuals live for ever.

Individuals randomly meet by pairs repeatedly through time.

At each period, any given individual is matched with his/her dyad partner with probability $1 - \omega$, while he/she is matched randomly to any other individual in the population with complementary probability ω.

We refer to matchings inside the dyad partnership as *strong ties* $(1-\omega)$ and to matchings outside the dyad partnership as *weak ties* or random encounters (ω).

By definition, weak ties are transitory and only last for one period.

Within each matched pair, information is exchanged, as explained follows.

Information transmission Individuals are aware of some criminal activity at exogenous rate λ.

Delinquents pass this information on to their current matched partner, be it a strong tie or a weak tie.

Information about crime is thus essentially obtained through friends and relative (i.e. strong and weak ties).

This information transmission protocol defines a Markov process. The state variable is the relative size of each type of dyad.

p the rate at which criminal are caught.

When a criminal is caught, he/she spends some time in prison and then gets out.

Petty crimes: time spent in prison is short enough so that the strong tie's status does not change during this period of time.

Flows of dyads between states Net flow of dyads from each state between t and $t + dt$ is given by:

$$\begin{cases} \overset{\bullet}{d}_2(t) = h(c(t))d_1(t) - 2pd_2(t) \\ \overset{\bullet}{d}_1(t) = 2g(c(t))d_0(t) - [p + h(c(t))]\, d_1(t) + 2pd_2(t) \\ \overset{\bullet}{d}_0(t) = pd_1(t) - 2g(c(t))d_0(t) \end{cases}$$

where $h(c(t)) = (1 - \omega + \omega c(t))\lambda$ is the probability to hear from a crime opportunity either by a weak or a strong tie (λ is the rate at which 'potential' criminals hear from a crime opportunity)

and $g(c(t)) \equiv \omega c(t)\lambda$ is the probability to hear from a crime opportunity only by weak ties.

These dynamic equations reflect the flows across dyads. Graphically,

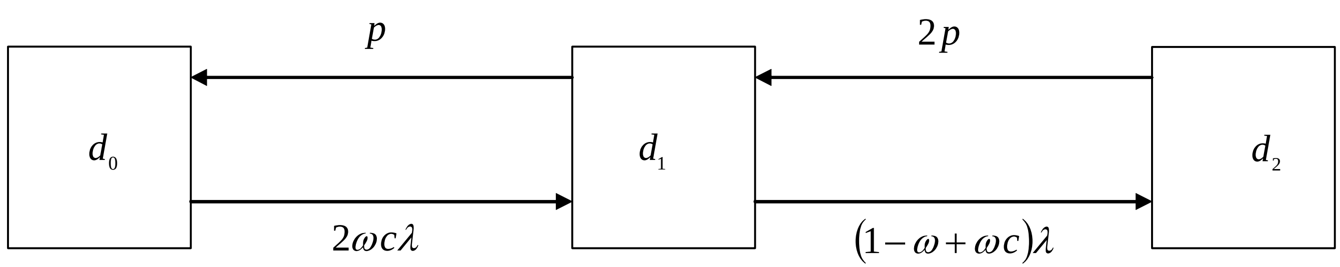


Figure 1. Flows in the crime market

The system reduces to a two-dimensional dynamic system in $d_2(t)$ and $d_1(t)$ given by:

$$\begin{cases} \dot{d}_2(t) = h(c(t))d_1(t) - 2pd_2(t) \\ \dot{d}_1(t) = 2g(c(t))\left(1/2 - d_2(t) - d_1(t)\right) - \left[p + h(c(t))\right] d_1(t) + 2pd_2(t) \end{cases}$$

where:

$$c(t) = 2d_2(t) + d_1(t)$$

Steady-state equilibrium analysis

Steady-state (d_2^*, d_1^*, d_0^*). We get:

$$d_2^* = \frac{(1 - \omega + \omega c^*)\lambda}{2p} d_1^* \tag{4}$$

$$d_1^* = \frac{2\omega c^* \lambda}{p} d_0^* \tag{5}$$

where

$$d_0^* = \frac{1}{2} - d_2^* - d_1^* \tag{6}$$

$$c^* = 2d_2^* + d_1^* \tag{7}$$

Definition 0.1 *A steady-state labor market equilibrium is a four-tuple $(d_2^*, d_1^*, d_0^*, c^*)$ such that equations (4), (5), (6) and (7) are satisfied.*

Define $Z = (1 - \omega) / \omega$, $B = p / (\lambda\omega)$. We have the following result.

Proposition 0.1

(i) *There always exists a steady-state equilibrium $\mathcal{U}$ where all individuals are noncriminals and only d_0-dyads exist, that is $d_2^* = d_1^* = c^* = 0$, $d_0^* = 1/2$ and $c^* = 0$.*

(ii) *If*

$$p < \lambda[\omega + \omega(4 - 3\omega)]/2 \tag{8}$$

there exists a steady-state equilibrium $\mathcal{C}$ where $0 < c^* < 1$ is defined by

$$c^* = \frac{B^2}{2d_0^*} - B - Z > 0, \tag{9}$$

and $0 < d_0^* < 1/2$ is the unique solution of the following equation:

$$-\frac{Z}{B}d_0^{*2} - \frac{(1+Z)}{2}d_0^* + \left(\frac{B}{2}\right)^2 = 0 \tag{10}$$

Also, the other dyads are given by:

$$d_1^* = \frac{2c^*}{B} d_0^* \tag{11}$$

$$d_2^* = \frac{(Z + c^*)\, c^*}{B^2} d_0^* \tag{12}$$

Comparative statics analysis

Focus on the interior equilibrium $\mathcal{C}$, where $0 < c^* < 1$.

Proposition 0.2 *Consider the steady-state equilibrium* $\mathcal{C}$ *and assume that* $2p/(\omega\lambda) < d_0^*$. *Then, increasing the percentage of weak ties* ω *decreases the number of* d_0-*dyads but increases the crime rate* c^* *in the economy, i.e.*

$$\frac{\partial d_0^*}{\partial \omega} < 0 \qquad , \qquad \frac{\partial c^*}{\partial \omega} > 0$$

The effects of ω *on* d_1^* *and on* d_2^* *are however ambiguous.*

ω measures the proportion of social interaction that occurs outside the dyad, the inter-dyad interactions.

When ω is high, the social cohesion among criminals is low, and delinquents and non-delinquents are in close contact with each other.

Increasing ω induces more transitions from non-crime to crime and thus c^*, the crime rate in the economy, increases. Even though c^* increases, the effect of ω on d_2^* and d_1^* is ambiguous.

To summarize, the effect of weak ties ω on crime c^* is positive because when ω increases, the transition from non-crime to crime increases since $\frac{\partial d_0^*}{\partial \omega} < 0$, even though we do not know if criminals are more likely to be part of a d_1 or d_2-dyad.

To summarize

So far, crucial role of information about crime opportunities

Explain the positive relationship between crime rate and the fraction of weak tie dyads.

Having more weak ties increases the range of interactions and information is spread more efficiently and evenly throughout the economy.

Alternative mechanisms that could give rise to both a positive and negative relationship between crime rate and weak ties.

Model with *social sanctions*.

If a person is surrounded by individuals who are (or appear to be) morally opposed to crime, then he/she is likely to share their aversion.

These social sanctioning effects may work to *increase* crime if delinquency is seen as a badge of honor in a population.

When juveniles see others committing crimes, they infer that their peers value law-breaking; they are then more likely to break the law themselves, which leads other juveniles to draw the same inference and engage the same behavior. In this respect, violence and crime can become status-enhancing.

As a result, depending whether there are strategic complementarities or substituabilities in crime, the relationship between crime and weak ties can be positive or negative. Thus there can be social effects (through weak ties) that could increase or decrease the crime rate.

Empirical analysis

Data and definition of variables

National Longitudinal Study of Adolescent Health (Add Health)

How to measure weak ties ω?

For each individual, the percentage of all friends of best friends over all the individuals in the network will be the counterpart of ω.

Percentage of strong and weak ties each individual i has in network $\mathbf{g}_K$:

$$1 - \omega_{i,\kappa}(\mathbf{g}_\kappa) = \frac{\sum_{j=1}^{N} g_{ij,\kappa}}{n_\kappa}$$

$$\omega_{i,\kappa}(\mathbf{g}_\kappa) = \frac{n_\kappa - \sum_{j=1}^{N} g_{ij,\kappa}}{n_\kappa}$$

Equation to be tested:

$$c_{i,\kappa} = \beta\,\omega_{i,\kappa} + \sum_{l=1}^{L} \gamma^l x_{i,\kappa}^l + \eta_\kappa + \varepsilon_{i\kappa}, \qquad \text{for } i = 1, ..., N_\kappa;\ \kappa = 1, ..., K$$

Results

Positive and significant impact of weak ties on crime.

Magnitude of the effects: the positive impact of weak ties on crime is quite small.

If we consider a change in the percentage of weak ties from two third to three quarter, we obtain an increase of the crime index by around 0.012 and an increase in the average probability of committing crime of roughly 0.018 percent.

Complicated (potentially opposite) effects of weak ties on crime, large results should not be expected.