A Non-Technical Presentation of Land Use Theory

Answers to questions and exercises

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Throughout, we have the following:

- All jobs are located in the Central Business District (CBD)
- Only one member of each household commutes to the CBD
- The opportunity cost of commuting time is zero
- The price of housing is the price per square foot of housing per month.

Question 1:

Every dwelling in the city has 2,000 square feet of living space.
The typical household earns a wage of £500 per month to spend on commuting and housing costs.
The monthly costs of commuting are £30 per mile per month.

1a) The budget constraint of a typical household is given by:

$500 = 30\,x + 2{,}000\,R(x)$

1b) The household's willingness to pay for housing is

$$R(x) = \frac{500 - 30x}{2{,}000}$$

This means that the housing price (price per square foot of housing per month) at the CBD is:

$$R(x) = 500/2{,}000 = 0.25$$

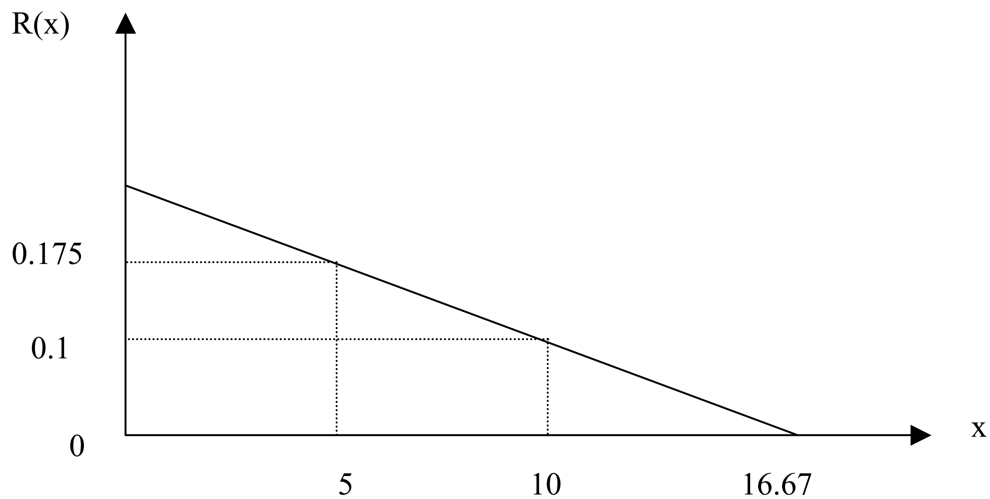
and at 5 miles from the CBD:

$$R(x) = (500 - 30 * 5)/2{,}000 = 0.175$$

The city fringe x_f is a distance from the CBD beyond which nobody wants to live (i.e. the land rent is negative). We have:

$$R(x) = 0 \Rightarrow x_f = \frac{500}{30} = 16.67$$

1c) The slope of this housing-price function is equal to 30/2,000 = 0.015, so we have



1d) The equilibrium-housing price function is negatively sloped because it makes residents indifferent among all locations between differences in commuting costs are offset by differences in housing costs.

Question 2:

Every dwelling in the city has still 2,000 square feet of living space.
There are now two types of households. Type-1 household earns a wage of £1000 per month whereas type-2 household earns £500 per month. Both spend their wages on commuting and housing costs.
The monthly costs of commuting are £30 per mile per month.

2a)

The type-1 household's willingness to pay for housing is

$$R_1(x) = \frac{1000 - 30x}{2000}$$

$$R_2(x) = \frac{500 - 30x}{2000}$$

For type 1-household, the housing price (price per square foot of housing per month) at the CBD is:

$$R(x) = 1000/2000 = 0.5$$

and at 10 miles from the CBD:

$$R(x) = (1000-30 * 10)/2000 = 0.35$$

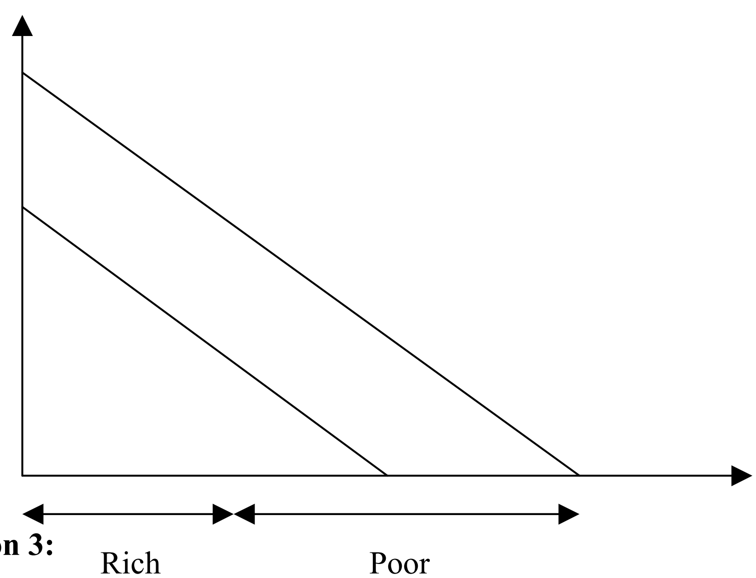
For type 2-household, the housing price (price per square foot of housing per month) at the CBD is:

$$R(x) = 500/2000 = 0.25$$

and at 5 miles from the CBD:

$$R(x) = (500-30 * 10)/2000 = 0.1$$

2b)



Question 3:

Every dwelling in the city has still 2,000 square feet of living space.
There are two types of households. Type-1 household earns a wage of £1000 per month whereas type-2 household earns £500 per month. Both spend their wages on commuting and housing costs.
The monthly costs of commuting are £60 per mile per month for type-1 household and £10 per mile per month for type-2 household.

3a) The type-1 household's willingness to pay for housing is

$$R_1(x) = \frac{1000 - 60x}{2000}$$

$$R_2(x) = \frac{500 - 10x}{2000}$$

For type 1-household, the housing price (price per square foot of housing per month) at the CBD is:

$$R(x) = 1000/2000 = 0.5$$

and at 15 miles from the CBD:

$$R(x) = (1000-60 * 15)/2000 = 0.05$$

For type 2-household, the housing price (price per square foot of housing per month) at the CBD is:

$$R(x) = 500/2000 = 0.25$$

and at 5 miles from the CBD:

$$R(x) = (500-10 * 15)/2000 = 0.175$$

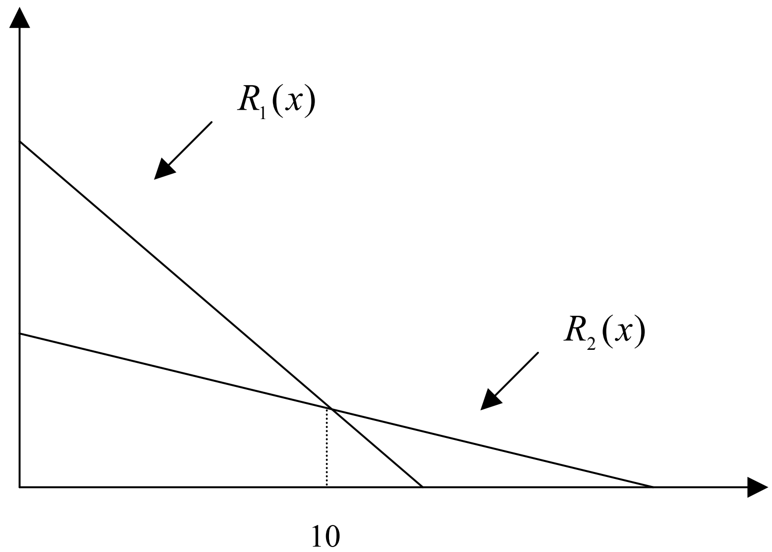
This means that for locations close to the CBD type-1 household outbid type-2 households whereas for locations far away (15 miles from the CBD) we have the contrary.

3b)

Bid rents are equal when

$$R_1(x) = \frac{1000 - 60x}{2000} = R_2(x) = \frac{500 - 10x}{2000}$$

As a result, at $\bar{x}$=10 miles from the CBD bid rents are equal.
Thus we have:



3c)

Since for locations far away poor outbid rich households, we have to solve:

$$R_2(x) = \frac{500 - 10x}{2000} = 0$$

which implies that $x_f = 50$ miles from the CBD.

At the CBD, only type-1 household live there. Therefore, at the CBD, $R(x = 0) = R_1(x = 0) = 0.5$ Pounds per square foot of housing per month

At 10 miles from the CBD, bid rents are equal. Thus $R(x = 10) = R_1(x = 10) = R_2(x = 10) = 0.2$ Pounds per square foot of housing per month.

At 20 miles from the CBD, only type-2 households live there. Thus $R(x = 20) = R_2(x = 20) = 0.15$ Pounds per square foot of housing per month.

Question 4:

There are two types of households. Type-1 household earns a wage of £1000 per month whereas type-2 household earns £500 per month. Both spend their wages on commuting and housing costs.
The monthly costs of commuting are £60 per mile per month for type-1 household and £10 per mile per month for type-2 household.
We now assume that housing space varies both with distance to the CBD and wage. The housing consumption is given by the following function:

$$h_1(wage, x) = \alpha_1 wage + 6.x = \alpha_1 1000 + 6.x \text{ square feet of living space}$$

$$h_2(wage, x) = \alpha_2 wage + x = \alpha_2 500 + x \text{ square feet of living space}$$

with $\alpha_1 > 0$ and $\alpha_2 > 0$.

4a)

The budget constraint for type-1 and type-2 households are respectively given by:

$$1000 = 60x + (\alpha_1 1000 + 6.x) R_1(x)$$

$$500 = 10x + (\alpha_2 500 + x) R_2(x)$$

4b)

Type-1 and 2 households' willingness to pay for housing are

$$R_1(x) = \frac{1000 - 60x}{\alpha_1 1000 + 6x}$$

$$R_2(x) = \frac{500 - 10x}{\alpha_2 500 + x}$$

For type 1-household, the housing price (price per square foot of housing per month) at the CBD is:

$$R_1(x = 0) = \frac{1}{\alpha_1}$$

and at 10 miles from the CBD:

$$R_1(x = 10) = \frac{400}{\alpha_1 1000 + 60}$$

For type 2-household, the housing price (price per square foot of housing per month) at the CBD is:

$$R_2(x=0)=\frac{1}{\alpha_2}$$

and at 5 miles from the CBD:

$$R_1(x=10)=\frac{400}{\alpha_2 500+10}$$

This means that we cannot determine the equilibrium locations without knowing the alphas and the betas.

4c) Assume $\alpha_1=\frac{1}{10}$ and $\alpha_2=\frac{1}{5}$.

We have

$$R_1(x)=\frac{1000-60x}{100+6x}$$

$$R_2(x)=\frac{500-10x}{100+x}$$

and

$$R'_1(x)=-\frac{12{,}000}{(100+6x)^2}<0$$

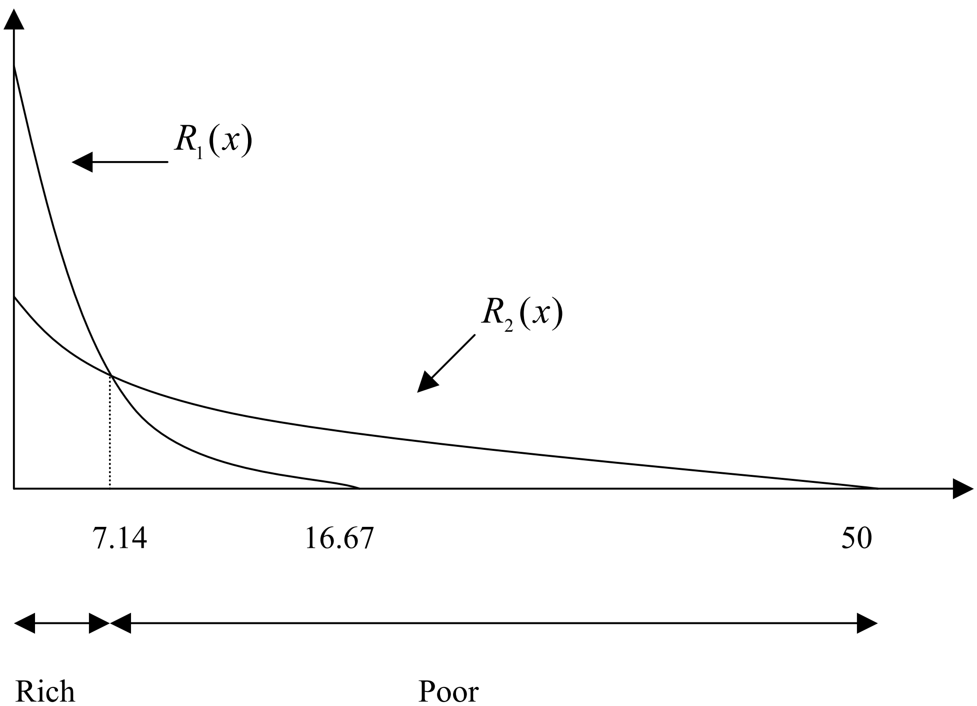
$$R'_2(x)=-\frac{1500}{(100+x)^2}<0$$

It is easy to see that $R_1(x=0)=10>R_2(x=0)=5$ and that $R_1(x=\frac{50}{7}=7.14)=4=R_2(x=\frac{50}{7}=7.14)$ so that bid rents cross each other at $\bar{x}=7.14$ miles from the CBD.
Finally, we have

$$R_1(x) = 0 \Leftrightarrow \frac{1000 - 60x}{100 + 6x} = 0 \Leftrightarrow x = 16.67$$

$$R_2(x) = 0 \Leftrightarrow \frac{500 - 10x}{100 + x} = 0 \Leftrightarrow x_f = 50$$

Even though bid rents are not linear, type-1 households will locate close to the CBD whereas type-2 households reside further away (after 10 miles).



4d) Assume now $\alpha_1 = 1$ and $\alpha_2 = \frac{1}{10}$.

$$R_1(x) = \frac{1000 - 60x}{1000 + 6x}$$

$$R_2(x) = \frac{500 - 10x}{50 + x}$$

and

$$R'_1(x) = -\frac{66,000}{(1000+6x)^2} < 0$$

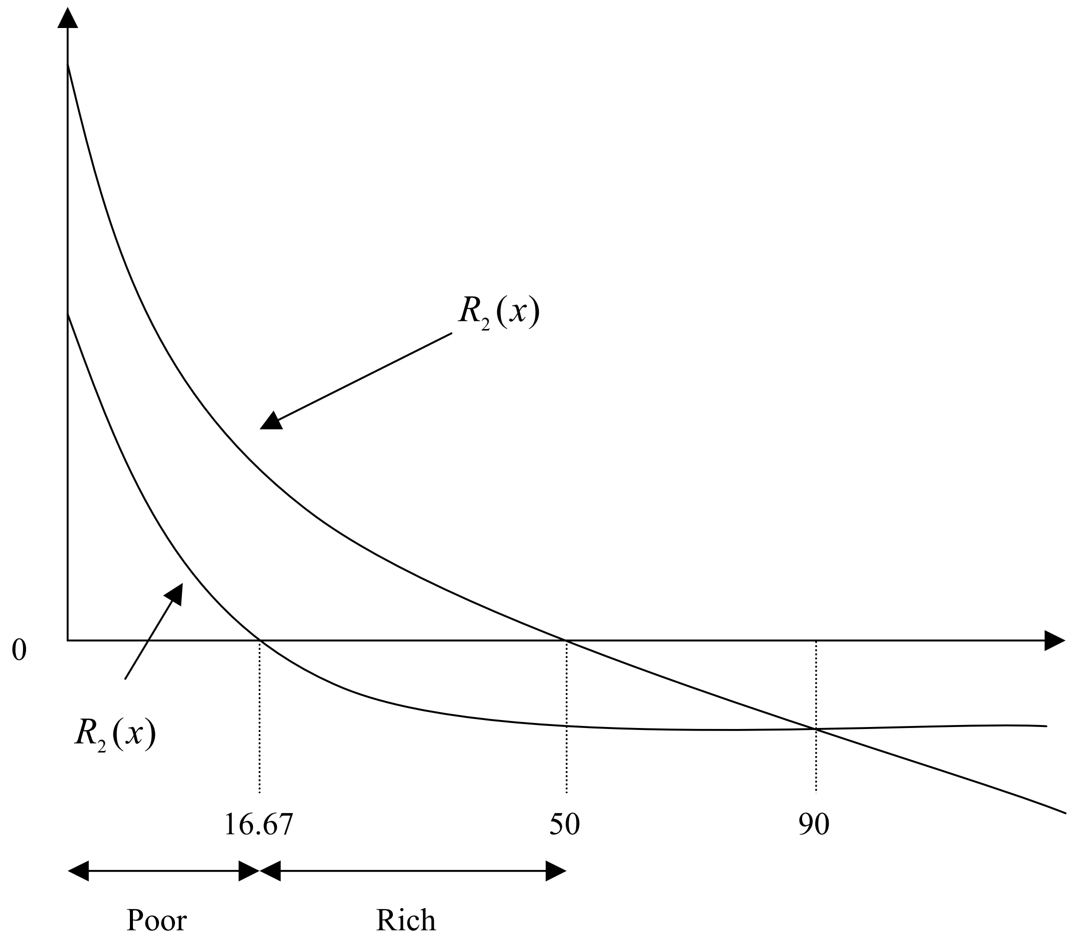
$$R'_2(x) = -\frac{1000}{(50+x)^2} < 0$$

It is easy to see that $R_1(x=0) = 1 < R_2(x=0) = 10$ and that $R_1(x=90) = R_2(x=90) = \text{-}2.86 < 0$ so that bid rents cross each other at $\bar{x} = 90$ miles from the CBD.

Finally, we have

$$R_1(x) = 0 \Leftrightarrow \frac{1000-60x}{1000+6x} = 0 \Leftrightarrow x = 16.67$$

$$R_2(x) = 0 \Leftrightarrow \frac{500-10x}{50+x} = 0 \Leftrightarrow x = 50$$



4e) How can we interpret α_1 and α_2?

We have that when $\alpha_1 < \alpha_2$, then the rich bid away the poor to the outskirts of the city whereas when $\alpha_1 > \alpha_2$, the rich are live far away from the CBD. This is because the alphas represent the degree of preference for housing (more exactly the marginal impact of wages on housing, i.e. at a given distance *x*, if the wage increases by 1%, then housing space increases by $\alpha_1 \%$ for type-1 household and $\alpha_2 \%$ for type-2 household. So when α_1 is small relative to α_2, the rich are very attracted to the CBD to save on commuting costs. When α_1 is large relative to α_2, then the rich are very sensitive to the size of their house and thus prefer to locate further away from the CBD where housing is cheaper.