# GAMES AND MARKETS

## ESSAYS ON COMMUNICATION, COORDINATION AND MULTI-MARKET COMPETITION

**Mattias Ganslandt**

Established 1939

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**Foreword**

The Research Institute of Industrial Economics (IUI) has a long tradition in studying international competition. This ambition is currently pursued within the project "Industrial Organization and International Specialization". In the five essays contained in this volume Mattias Ganslandt applies game theoretic methods to such diverse problems as the role of capacity in multi-market entry deterrence, international market integration, arbitrage in international trade, the effects of strategic uncertainty on equilibrium selection in coordination games and, finally, the role of communication in teams.

This book has been submitted as a doctoral thesis at the Department of Economics, University of Lund. It is the 56th dissertation completed at IUI since its foundation in 1939.

IUI would like to thank the thesis advisor, Hans Carlsson at the University of Lund, for his encouragement and support.

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Stockholm in April 1999

Ulf Jakobsson
Director of IUI

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Stockholm in March 1999
Mattias Ganslandt

# Contents

# 1

# Introduction

This thesis consists of two parts, dealing with equilibrium selection in noncooperative games and strategic interaction in international markets, respectively. The main issue raised in each part is different from the other, but a common method based on noncooperative game theory is used throughout the thesis.

In the first part of the thesis, presented in Chapters 2 and 3, the general question is what mechanism can induce players to expect the same equilibrium in coordination games. In Chapter 2, tacit coordination is analyzed in noisy games. The model is employed to generate predictions about the outcome in a symmetric coordination game. Chapter 3 introduces a model with structured and costly pre-play communication. The model is used to generate hypotheses on how the outcome of the game is related to the cost of communication and capacity for transmitting information.

The second part of the thesis, presented in Chapters 4, 5 and 6, studies different issues related to strategic interaction in international markets. Chapter 4 deals with credibility in multi-market competition. The analysis focuses on the incumbent's possibilities to exploit first-mover advantages as it competes with potential entrants in several markets. Chapter 5 presents a model of arbitrage in a multi-market oligopoly. More specifically, the model is used to generate hypotheses on how the market structure is related to market-specific barriers and scale economies in the transportation technology. The last essay, presented in Chapter 6, analyzes price discrimination in a simple two-country model. The aim of the analysis is to evaluate the effects of costly arbitrage and free-riding in an international context.

The rest of this introduction provides a background for the studies and summarizes the main findings. Section 1.1 presents the background to the essays on equilibrium selection and section 1.2 presents a brief background to the studies on strategic interaction in international markets.

## 1.1 Coordination in Games

When decisions are decentralized in a group of individuals and the payoff to an individual member depends on the efforts chosen by other members of the team, then there may exist multiple equilibria. As long as the members of the group coordinate successfully, each member is satisfied with his individual choice. In many situations, however, players can coordinate in many different ways and the group may collectively prefer some of the outcomes.

This multiplicity of equilibria is an important problem to solve. A theory that can only predict that the outcome of a noncooperative game is an equilibrium, without specifying which equilibrium, is a weak and uninformative theory. A more desirable theory should select one equilibrium as the solution of the game (cf. Harsanyi and Selten, 1988), and thus a theory of equilibrium selection is required.

In games with multiple equilibria, the prediction that a Nash equilibrium is played relies on the assumption that some mechanism leads all players to expect the same equilibrium. In noncooperative game theory, two types of mechanisms have been considered: equilibrium selection based on strategic information and equilibrium selection based on non-strategic information. The general problem is to identify the conditions determining the equilibrium and make precise and plausible predictions; which is the topic of the first part of this thesis.

## *Uncertainty and Coordination*

Chapter 2 presents a theoretical analysis of a simple coordination game originally due to Bryant (1983), where several players simultaneously choose efforts from a compact interval. The lowest effort determines the output of a public good. Each player's payoff is determined by the minimum effort in the group minus the cost of his own effort. In this game players face a hard coordination problem as the game has a continuum of Pareto-ranked equilibria.[1]

A similar problem occurs in the Stag hunt game illustrated in Figure 1.1. In this game, player 1 selects a row and player 2 a column, where H represents a high effort and L a low effort. The payoffs are given as the intersection of a row and a column, where player 1's payoff is specified first. This game is interesting because it has two strict equilibria, (H,H) and (L,L), where the former is Pareto efficient. Players receive 3 if they successfully coordinate in (H,H). However, the H strategy involves a risk, since coordination failure yields 0 to a player choosing H, while the L strategy guarantees the player a payoff of 2. The question is: Should we expect players to choose (H,H) or (L,L)?

[1] Bryant (1994) discusses the macroeconomic relevance of this game.

| | H | L |
|---|---|---|
| H | 3,3 | 0,2 |
| L | 2,0 | 2,2 |

Figure 1.1. The Stag Hunt

The problem of multiple equilibria partly stems from the assumption about excessively rational and well-informed agents in traditional game theory. One way of tackling this problem is to compare the original model with a perturbed variant with slightly modified assumptions about information and knowledge.[2] In this manner, the concept of trembling-hand perfect equilibrium (Selten, 1975) requires that any solution of the game should survive small mistakes from the players. Unfortunately, the coordination problem in Bryant's model remains unsolved, since any strict Nash equilibrium not only survives trembling-hand perfection but also Kohlberg and Mertens' (1986) considerably stronger requirement of strategic stability.

Recently, researchers have considered a more general approach to equilibrium selection. One example is Harsanyi and Selten's (1988) general theory of equilibrium selection. In choosing between two strict equilibria as solution candidates, Harsanyi and Selten's theory uses two different criteria: payoff dominance and risk dominance.[3] In a situation where the players are uncertain whether one equilibrium or the other will be the outcome of the game each player will choose subjective probabilities over the other players' strategies considering the relative risk of different strategies. A solution is obtained through a procedure yielding a unique equilibrium.

In some respects, however, it would be more satisfactory to derive a unique solution from a noncooperative model rather than a model based on a more or less ad hoc mechanism for the formation of beliefs.[4]

[2] Crawford (1991) provides an evolutionary explanation of the outcome in experiments based on Bryant's game. See also Crawford (1995), for a dynamic explanation.

[3] The former is based on collective rationality; if one equilibrium gives all players higher payoffs, then (according to Harsanyi and Selten) each player can be quite certain that the other player will opt for this equilibrium. The latter criterion, on the other hand, is based on individual rationality.

[4] cf. Nash's (1951) program of providing noncooperative foundations for axiomatic solution concepts. For instance, Carlsson and van Damme (1993a), (1993b) introduce an incomplete information model based on a perturbation of the players' payoff information. The game to be played is determined by a random draw from a class of games, including the Stag hunt game in Figure 1.1. The uncertainty results in a unique solution of the actual game to be played.

This is the topic of the first essay in this thesis. More specifically, the aim is to answer the question: What strategies can we expect players to choose in a coordination game, if there is uncertainty about how strategies are translated into efforts?

For this purpose, we introduce noise in Bryant's (1983) model. An error term is added to each player's choice before his effort is determined. In order to derive precise predictions, limit equilibria of games with vanishing noise are studied, which gives us the *noise-proof equilibria.*

If the utility function is sufficiently concave, the noise-proof equilibrium is an interior point or the lowest effort in the continuum of equilibria in the original game. Hence, the most efficient point cannot be sustained as an equilibrium in the noisy version of the game. The inefficiency in the unique noise-proof equilibrium increases in the number of players. Interestingly, this result agrees with the findings in experiments based on Bryant's game (cf. Van Huyck et al, 1990).

### *Communication and Coordination*

Chapter 3 introduces a model where players can use communication to coordinate in a specific equilibrium.

It has been argued that if an equilibrium arises as the result of costless negotiations between the players, they will never settle down in an equilibrium Pareto dominated by another equilibrium outcome.[5] Dominated equilibria are never *renegotiation proof* (cf. Fudenberg and Tirole, 1991, p. 175). Thus, according to this argument, players will reach an efficient outcome in any coordination game, even if the environment and the most efficient behavior are very complex. Why then, are most rules of behavior simple?

A factor which may influence the outcome of a coordination game, and motivate players to choose a simple rather than a revenue-maximizing strategy, is that pre-play communication is usually neither costless nor unrestricted. In chapter 3, I study how the primitive conditions for transmission of information, e.g. the capacity of the information channel and the cost of transmission, affect the solution to a game with multiple Nash equilibria.[6] The general question is: Could an outside observer who knows the struc-

[5] This is not an uncontroversial hypothesis. As Aumann (1990) observes, in some coordination games each player has an incentive to signal a specific intention regardless of his own intended play. Thus, it is not clear that messages in symmetric coordination games are informative.

[6] In cheap-talk models, on the other hand, it is assumed that talk is costless and messages do not vary in complexity (cf. Crawford and Sobel, 1982).

tural conditions for communication, i.e. the cost of communication and the qualities of the channel for transmission, make a prediction of the outcome in a specific situation?

We consider a set-up with asymmetric information. Both players know that any pure strategy is a best reply to itself, but only one player knows the revenue-maximizing equilibrium.[7] The informed player can send a costly message to his uninformed counterpart through a channel admitting binary code only. The same code is used in a number of situations. Does this mean that pre-play communication favors revenue-maximizing Nash equilibria?

The general problem to be solved by the players is to find the best labeling and code, given the ex ante payoffs and the cost of communication.[8]

In Chapter 3, an optimal labeling and code for a class of coordination games are derived. Short code strings are associated with strategies in simple coordination problems (with few equilibria). Long code strings are associated with strategies in games with many equilibria.

When the cost of communication is high and the efficiency gains are small, the informed player has a motive to choose a short code string as a message. The corresponding strategy is highly regular and the equilibrium is most likely inefficient in the underlying game.

However, when the cost of communication is low, or the potential efficiency gains are large, it is no longer true that players will always choose the shortest code string as a message. Nevertheless, players choose sequences of actions with relatively short descriptions, if they are approximately as good as the revenue-maximizing strategy. In this case, equilibrium selection is a trade-off between efficiency in the underlying game and how easily the equilibrium strategy is described.

## 1.2 Competition in International Oligopolies

The new trade theory recognizes that international markets are often imperfectly competitive, thus building on insights in the industrial organization literature. One insight from this literature is that market integration may have a pro-competitive effect.

In the presence of increasing returns to scale, trade can reduce domestic monopoly power and increase social welfare. In particular, welfare is higher under free trade, if firms produce a larger total quantity (e.g. Markusen,

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[7] In two related studies, Farrell (1993) and Rabin (1990) analyze models in which an informed player can send a message to his uninformed counterpart.

[8] This problem is closely related to the issues raised in the theory of teams (see, for instance, Marschak and Radner, 1972).

1981, Helpman and Krugman, 1985) or more varieties of the goods (Krugman, 1979).[9]

For small countries, the pro-competitive effect of market integration is particularly simple. If borders are open and barriers to entry for importing firms are low, the fact that a domestic industry has a high concentration of sales by a few firms need not be a problem. In this case, imports can discipline market concentration in the domestic market.

While this relationship between market integration and concentration is straightforward in theory, reality is much more complex. Despite several decades of liberalization of international trade, retail prices do not seem to have been reduced to the anticipated levels. Persistent price differentials appear to be widespread, both across and within countries.[10]

This suggests that the present theory should be extended in several ways. For instance, it should be recognized that barriers to entry and barriers to trade interact in important ways, which is the focus of the second part of this thesis.

### *Strategic Investments and Multi-Market Competition*

One way raising profits for an incumbent firm is to acquire or maintain monopoly power, which requires exclusion of entrants as well as absorption, intimidation or cartelization of competitors. Chapter 4 deals with the issue of how an incumbent firm can maintain monopoly power in a multi-market game.[11]

The first avenue of strategic entry deterrence in multi-market games is predation. Predation models aim at explaining why and when firms are willing to incur losses when battling with an entrant. This strand of the literature started with Selten's (1978) chain-store game. A single multi-market incumbent faces potential entry by a series of local firms, each of which plays only once but observes all previous actions. Each period, a potential entrant decides to enter or stay out of a particular market. The incumbent firm earns a higher profit if monopoly prevails, but has a short-run incentive to accommodate if entry occurs. Milgrom and Roberts (1982), Kreps and Wilson (1982) and Easley, Masson and Reynolds (1985)

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[9] Markusen (1981) concludes that the pro-competitive effect of trade generally increases world real income, but not necessarily the real income of each trading country.

[10] Empirical evidence strongly suggests that deviations from the law of one price in traded goods are important (e.g. Isard, 1977, Giovannini, 1988, Engel, 1993, Engel and Rogers, 1996, Goldberg and Knetter, 1997).

[11] For a general survey on multi-market competition, see Witteloostuijn and Wegberg (1992).

introduce private information and show that an incumbent firm has an incentive to fight entrants to build a reputation. The predation literature concludes that entry deterrence is more likely if the incumbent is present in many markets, since building a reputation is more profitable in the long run.

The second avenue of strategic entry deterrence in multi-market games is preemption through strategic investment. A general feature of models with strategic investment is that the incumbent installs capacity before the potential entrant can make any strategic moves. Unlike output levels, capacity is an irreversible decision and has a commitment value in the post-entry game. In a multi-market game, however, capital is not completely market-specific and import competition can undermine the commitment value of capacity.[12]

Chapter 4 analyzes whether a multi-market firm needs more capacity to deter entry if capacity can be redistributed between markets without cost. We derive sufficient conditions for entry deterrence in the multi-market game. It is shown that to deter entry, the first mover installs a production capacity which is strictly larger than the capacity needed to deter entry when parts of the capacity can be assigned to specific markets. Thus, the main implication of the multi-market model is that entry deterrence is less likely, due to the possibility of output shifting.[13] Contrary to the conclusion from the predation literature, Chapter 4 concludes that multi-market competition can be pro-competitive in a game with capacity commitments.

### *International Competition and Market Access*

Chapter 5 studies the microeconomic foundations for the commonly used integrated and segmented markets assumptions in international trade theory.[14] This distinction has, in particular, been used for policy analysis. For instance, Helpman and Krugman (1985) present an oligopoly model of international trade where welfare in autarchy is compared to an integrated equilibrium. Smith and Venables (1988) compare a segmented and an inte-

[12] Calem (1988), Anderson and Fischer (1989), Venables (1990) and Wegberg (1995) analyze multi-market competition when the post-entry game is Cournot.

[13] It should be noted that strategic investment in capacity is not the only method for a multi-market firm to influence potential entrants (see, for instance, Smith, 1987, and Horstmann and Markusen, 1987).

[14] The integrated market assumption implies that producers set a single quantity or price at the world level and let arbitrageurs determine the distribution of sales to national markets (e.g. Markusen, 1981; Helpman and Krugman, 1985). At the opposite extreme, with segmented market behavior, firms choose strategic variables in each market separately (e.g. Brander, 1981; Brander and Krugman, 1983).

grated equilibrium to evaluate the welfare effects of realizing the internal market in the European Community.[15] The integrated equilibrium removes the monopoly power that firms have in a particular market and replace it by an average degree of monopoly power. This is a strongly pro-competitive policy and the gains for some industries are substantial.

In a more general way, it has been argued that international trade increases competition. The argument is that domestic industries are forced to behave more competitively when they are faced with intensified international competition.[16]

There are several factors, however, which can have a restrictive impact on the strength of the pro-competitive effect of market integration. For instance, barriers to entry for importing firms and restrictions on arbitrage can moderate the effect substantially.

Chapter 5 introduces a simple oligopoly model of international competition. The model builds on two assumptions: economies of scale in the transportation technology and market-specific access costs. It is shown that imports can fail to discipline domestic market concentration, when the market-specific barriers and fixed costs in transportation are high.

The pro-competitive effect of trade liberalization can be reinforced, if market barriers are dismantled at the same time as trade costs are reduced. Furthermore, the results indicate that while there are strong microfoundations for macroeconomic price convergence in the form of purchasing power parity under constant returns to scale in transportation technology and low barriers to entry, price convergence need not occur if these conditions are violated.

## *Price Discrimination and Arbitrage*

Chapter 6 focuses on price discrimination in international markets. In particular, the analysis focuses on the effects of free-riding and costly arbitrage.

Empirical evidence suggest that third-degree price discrimination is the most likely form in international markets, i.e. consumers in different countries are charged different prices, but each consumer faces a constant price for all units of output purchased (cf. Phlips, 1983). For instance, in a sit-

[15] See also Baldwin and Venables (1995) for a survey on several similar studies.

[16] This hypothesis has been tested in several studies, see Caves (1985) for a survey. In particular, Levinsohn (1993) finds some support for the idea, using firm-level data. Jacquemin and Sapir (1991) argue that "extra-EC imports exercise a strong disciplinary effect on European industry, and that further external liberalisation would be helpful in reinforcing such an effect, especially given the oligopolistic nature of most sectors presently subject to the highest EC internal barriers." See also Norman (1991).

uation where a monopolist must incur a variable trade cost to export a product to a foreign market, the optimal monopoly price in the foreign market is lower than the domestic price and third-degree price discrimination occurs (for a survey, see e.g. Varian, 1989, or Tirole, 1988).[17]

Much of the discussion about third-degree price discrimination is due to its welfare effects. The negative effect of the monopoly distortion (i.e. the different marginal valuations in different markets) must be weighed against the positive effect of an increase in the aggregate output when price discrimination is permitted. Specifically, it has been shown that welfare is higher under uniform pricing if the total output does not increase under discriminatory prices (Robinson, 1933).[18]

This result, however, depends crucially on a static comparison between uniform prices and discriminatory prices. In particular, a uniform price is obtained with regulation or costless arbitrage and the monopolist has a single decision variable in each market (output or price). If either of these assumptions is relaxed, the welfare effects might be quite different. More precisely, costly arbitrage and free-riding on the provision of market specific services can moderate or reverse the positive welfare effect of price equalization.

First, it has been shown that cross-hauling in international trade can moderate and even reverse the pro-competitive effect of trade. Brander and Krugman (1983) show that the gains from trade in an international oligopoly are substantially moderated if trade is costly.

Second, the literature on intrabrand competition suggests that the welfare consequences of arbitrage can be negative in the presence of spillovers. Externalities and spillovers exist if consumers can use the local services provided by one firm, while buying the goods from another. For instance, Telser (1960) and Perry and Porter (1990) consider the provision of pre-sale information in multiple markets. It is shown that arbitrage may prevent the provision of such information, since the services cannot be appropriated by the retailer providing them. In this case vertical restraints, e.g. exclusive territories, can be welfare-enhancing. [19]

The effects of spillovers and trade costs have not been analyzed in a

[17] A similar result is obtained in oligopoly models of intra industry trade where a firm's domestic price is higher than its f.o.b. price for foreign consumers (see Brander, 1981, Brander and Krugman, 1983).

[18] Several contributions consider the output effect of price discrimination, see e.g. Battallio and Ekelund (1972), Edwards (1950), Finn (1974), Schmalensee (1981), Varian (1985), Schwartz (1990). For a good survey, see Tirole (1988) and Varian (1989).

[19] Posner (1981), Mathewson and Winter (1984), (1986) discuss how vertical territorial arrangements can enhance local investments in the presence of externalities, in some detail. See also Marvel and McCafferty (1984).

formal model with third-degree price discrimination, however. It is still an open question whether the benefits of a reduced price differential outweigh the cost of arbitrage and the reduced level of local services. This is the topic of the last essay in this thesis.

Chapter 6 considers a situation where a single manufacturing firm produces a homogenous good for two markets with different willingness to pay. The manufacturing firm also invests in market-specific services which are complementary to the product. Arbitrage is introduced and it is shown that the positive welfare effects of price-equalization can be reversed, when the barriers to entry in the arbitrage sector are determined by costly investments by the manufacturing firm. Moreover, the incentives for market-specific investments can be too weak if arbitrage is permitted. In particular, uniform prices can reduce welfare in the long-run, if arbitrageurs free-ride on market-specific investments by manufacturing firms in high-valuation markets.

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# 2

# Noisy Equilibrium Selection in Coordination Games

This chapter is co-authored with Hans Carlsson

## 2.1 Introduction

Consider a noncooperative game situation where a number of agents are faced with several strict Pareto-ranked Nash equilibria. Many people would argue that Pareto-optimality provides a natural coordination principle, since the players have a common interest in achieving efficiency and the Pareto-optimal equilibrium is strict and, thus, self-enforcing. The experimental evidence in Van Huyck, Battalio and Beil (1990) (henceforth VHBB) casts serious doubts on this argument suggesting that, in the presence of strategic uncertainty, efficiency considerations are much less important for the individual agent than the relative risk of available strategies. The present paper presents a simple perturbation, in the spirit of trembling-hand perfection, which may provide a theoretical foundation for VHBB's results. Our basic model is a variant of Bryant's (1983) minimum effort game, which is easily adapted to the VHBB framework: A number of players simultaneously choose costly efforts and the lowest effort determines the output of a public good. The game has a continuum of Pareto-ranked strict Nash equilibria.

We perturb the game by assuming that each player makes a slight error when choosing his effort, technically by adding noise to the strategies before they are translated into efforts. The perturbation accounts for the fact that, in most realistic settings, agents' information about the payoff structure and their expectations about each other are neither as precise nor as accurate as traditional game-theoretic analyses presume. The noise lends itself to several interpretations. In some contexts, in particular when action spaces are continuous, it seems natural to assume that players cannot choose actions with perfect accuracy. The noise may also result from slightly imperfect information about the productivity of the different agents' efforts or it may be seen as an expression of the individual player's strategic uncertainty.

In order to derive precise predictions, we study the limit equilibria of sequences of games with vanishing noise. This gives us the noise-proof equilibria of the original game, which turns out to be a surprisingly strong refinement. Subject to a genericity condition, it always determines unique solutions for the class of games being studied. These solutions show a good correspondence with the tendencies observed in the VHBB experiments. Moreover noise proofness has the following noteworthy properties.

First, noise proofness is a robust refinement, in the sense that it does not depend on the details of the noise structure. Second, the noisy games are supermodular which means that, in view of a well-known result by Milgrom and Roberts (1990), the noise-proof solution can be derived by an iterated dominance argument. Due to this property, the noise-proof solution can be considered as the approximate outcome of a broad class of learning processes. Finally, the noise-proof equilibrium has an interesting link with the notion of potential: Our basic model is a potential game and the noise-proof solution maximizes the potential.

As our basic model only encompasses a rather narrow class of coordination games, the question whether noise proofness can also be successfully applied to other classes of games is of great importance. Thus, to extend our analysis, we briefly examine another class of coordination games, that is, median games, which has been studied in experiments by Van Huyck, Battalio and Beil (1991) and Van Huyck, Battalio and Rankin (1996). Our analysis highlights the importance of the fineness of the action grid used in experiments. With a sufficiently fine action grid, the subjects tend to converge to the Pareto-optimal equilibrium, which is the only noise-proof one.

Sections 2 and 3 present the basic model, the perturbation and the results on noise proofness. Section 4 compares these results with the experimental evidence in VHBB. Section 5 discusses risk-dominance and Section 6 deals with median games. Section 7 reviews the related literature and concludes. The Appendices contain proofs and a more general set-up which, in particular, is more in line with the strategic uncertainty interpretation of the noise.

## 2.2 A Model with Noisy Coordination

Our basic model is the following game, denoted $\Gamma$.[1] $n$ players simultaneously choose strategies $e_1, ..., e_n$, which may be interpreted as efforts, from an

[1] The tacit coordination game $\Gamma$, is a variant of Bryant's (1983) minimum-effort game.

interval $[0, M]$. The lowest effort, $\min\{e_1, ..., e_n\}$, determines the output of a public good (or equivalently, a private good equally divided among the players). For a vector of real numbers $r = (r_1, ..., r_m)$, we let $\underline{r}$ denote $\min\{r_1, ..., r_n\}$. Using this notation, the payoff to player $i$ under strategy profile $e = (e_1, ..., e_n)$ may be written

$$u_i(e) = g(\underline{e}) - e_i, \tag{2.1}$$

where $g : \mathbb{R} \to \mathbb{R}$ is a concave and continuously differentiable function expressing the value of the public good. We assume that $g'(0) > 1$ and that there is, at most, one point $e^1$, $0 < e^1 \leq M$, such that $g'(e^1) = 1$. It is clear that all pure strategy equilibria are symmetric. Conversely, for each $t$ in $[0, e^1]$, there exists a Nash equilibrium with $e_i = t$ for all $i$. Hence there is a continuum of Nash equilibria, ranked by the common effort.

## 2.3 Noise-Proof Equilibrium

In the perturbed variant of the above model, denoted $\Gamma^\varepsilon$, players choose strategies from $[0, M]$, which are translated into efforts by the addition of noise terms. Hence, the actual efforts are random variables which we will denote $e_i$. We begin by studying a very simple version with additive noise. Thus,

$$e_i = s_i + \varepsilon \cdot X_i, \tag{2.2}$$

where $\varepsilon > 0$ is a scale parameter and $X_i$ is a random variable independent of $s_i$. We assume that $X_i$ are i.i.d., take values on $[-1, 1]$, have zero mean and continuously differentiable distributions. Some of these assumptions are quite strong and not entirely satisfactory. Later, it will be shown that our results survive a much more attractive set of assumptions.

The present assumptions have the advantage of allowing the following, very simple, analysis. Letting $f(\cdot\,|s)$ denote the conditional density of the lowest effort under strategy profile $s = (s_1, ..., s_n)$, the payoff to player $i$ can be written

$$u_i(s) = \int g(e) f(e\,|s)\, de - s_i, \tag{2.3}$$

where the first term is the expected utility of the public good and the second term equals the expected effort. Now, suppose that we have a symmetric interior equilibrium in a noisy game. The necessary first-order conditions imply the following equation:[2]

[2]For $s = (s_1, ..., s_n)$ and $\Delta \in R$, we have $f(e + \Delta\,|s_1 + \Delta, ..., s_n + \Delta) = f(e\,|s)$, by

$$\frac{1}{n}\int g'(e)\,f(e\,|s)\,de = 1 \tag{2.4}$$

The intuition is straightforward: $1/n$ is the probability that the realized effort of a given player will be the (uniquely) lowest effort under a symmetric strategy profile and the integral on the left-hand side is the expected marginal utility of the public good under the given strategy profile. Hence, the left-hand side equals the expected marginal benefit for a player from increasing his own strategy, while the right-hand side clearly is the corresponding marginal cost.

We will now characterize the *noise-proof equilibria* of the original game $\Gamma$, i.e. the possible limit equilibria of sequences $\{\Gamma^{\varepsilon}\}_{\varepsilon\downarrow 0}$ of noisy games with vanishing noise. We assume that there is, at most, one point $e^n$, $0 < e^n \leq M$, such that $g'(e^n) = n$. If no such point exists, we set $e^n = 0$ when $g'(0) < n$ and $e^n = M$ when $g'(M) > n$. Clearly, the assumption that $e^n$ is uniquely defined by these conditions, as well as the corresponding assumption on $e^1$, is a genericity condition, i.e. it should hold for almost all specifications.

**Proposition 1** *$\Gamma$ has a unique noise-proof equilibrium $\widehat{s}$. $\widehat{s}$ is in pure strategies and $\widehat{s}_i = e^n$ for all $i$.*

A proof will be provided later. To see why a sequence of pure-strategy equilibria corresponding to $\{\Gamma^{\varepsilon}\}_{\varepsilon\downarrow 0}$ must converge to $(e^n, ..., e^n)$, note that as $\varepsilon \longrightarrow 0$, the support of $f(\cdot\,|s)$ will shrink to $\underline{s}$ for any $s$. Hence, the limit of condition (2.4) for an interior solution becomes

$$g'(\underline{\widehat{s}}) = n \tag{2.5}$$

and, thus, $\widehat{s} = (e^n, ..., e^n)$ as all pure-strategy equilibria are symmetric. The boundary solutions $\widehat{s} = (0, ..., 0)$ and $\widehat{s} = (M, .., M)$ result when $g'(0) \leq n$ and $g'(M) \geq n$, respectively.

For a different intuition for the result, let us compare a player's situation in a game with noise and one without noise. Specifically, consider a symmetric strategy profile $e = (\underline{e}, ..., \underline{e})$ in a game without noise and player $i$'s payoff if he deviates to $e_i'$. Player $i$'s marginal utility of increasing his effort equals $g'(e_i') - 1$ as long as $e_i' < \underline{e}$, but drops to $-1$ at $e_i' = \underline{e}$. The large

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additivity. Hence,

$$(*)\quad \partial f(e\,|s)\,/\partial e + \sum \partial f(e\,|s)\,/\partial s_i = 0.$$

The first-order condition corresponding to player $i$'s maximization problem is $\int (\partial f(e\,|s)\,/\partial s_i)\,g(e)\,de = 1$. Summing over $i$ and exploiting (*), we obtain $-\int (\partial f(e\,|s)\,/\partial e)\,g(e)\,de = n$ and the condition 2.4 results, using integration by parts.

number of equilibria in games without noise is closely linked to this discontinuity. For $e$ to be an equilibrium, it suffices that $g'(\underline{e}) \geq 1$. When noise is added and the discontinuity disappears, we get the much more stringent equilibrium condition (2.4): At the interior equilibrium $s = (\underline{s}, ..., \underline{s})$ of a noisy game, the expected marginal benefit from an increased effort must be exactly matched by the marginal cost.[3]

Note that whenever $e^n < M$, then $e^n < e^1$ and, thus, the noise-proof equilibrium is inefficient. Moreover, the inefficiency increases in the number of players. This phenomenon is easy to explain in the light of Equation (2.4). The individually optimal trade-off implies an effort level, which is too low from a social point of view, since it disregards the positive externalities of an increased effort. This kind of inefficiency is a characteristic of games with positive spillovers (see, e.g., Cooper and John, 1988).

A highly noteworthy property of the noise-proof equilibrium is that it can be derived using iterated strict dominance in the noisy games. This result follows from the fact that these games are supermodular, i.e. characterized by (weak) strategic complementarity: A player's incentive to increase his strategy grows, if another player increases his strategy. To spell out the result formally, let, for a given noisy game $\Gamma^\varepsilon$, $D_i^\varepsilon$ denote the pure strategies for player $i$, which are serially undominated, i.e. they survive iterated elimination of strictly dominated strategies in $\Gamma^\varepsilon$. Let

$$\begin{aligned} a_i^\varepsilon &\equiv \inf D_i^\varepsilon \qquad a^\varepsilon \equiv (a_1^\varepsilon, ..., a_n^\varepsilon) \\ b_i^\varepsilon &\equiv \sup D_i^\varepsilon \qquad b^\varepsilon \equiv (b_1^\varepsilon, ..., b_n^\varepsilon)\,. \end{aligned}$$

From the paper by Milgrom and Robert (1990) on supermodular games, we know that $a^\varepsilon$ is the lowest and $b^\varepsilon$ the highest Nash equilibrium of $\Gamma^\varepsilon$. Thus, exploiting Proposition 1:

**Proposition 2** *For all $i$* $\lim_{\varepsilon \to 0} a_i^\varepsilon = \lim_{\varepsilon \to 0} b_i^\varepsilon = e^n$.

This weakening of the solution concept enlarges the range of mechanisms by which a noise-proof equilibrium may be established. In particular, it will be the approximate outcome of a broad range of learning processes in slightly noisy games.[4] This feature does not lack importance. As will be

[3] A similar phenomenon occurs when a Nash bargaining game is perturbed. The basic, unperturbed model has a continuum of strict equilibria, due to the discontinuity of payoffs. When noise is added, payoffs become continuous and only one equilibrium, the Nash bargaining solution, survives the perturbation (see, e.g., Carlsson, 1991).

[4] Milgrom and Roberts (1990) show that, under weak conditions on the learning process, strategy choices will eventually be confined to the serially undominated set in supermodular games.

shown below, the experimental evidence suggests that learning is empirically more relevant than the rationalistic equilibration mechanisms traditionally considered in game theory.

Finally, we note an interesting connection between noise proofness and the notion of potential. A game with strategy profiles $S$ and payoffs $u_i$ is a potential game if there exists a function $P : S \longrightarrow R$, such that for all $i$ and $s_{-i}$ we have

$$u_i(s_i, s_{-i}) - u_i(\widetilde{s}_i, s_{-i}) = P(s_i, s_{-i}) - P(\widetilde{s}_i, s_{-i}) \tag{2.6}$$

for all $s_i, \widetilde{s}_i \in [0, M]$. Monderer and Shapley (1996) have pointed out that the games in VHBB are potential games and this is also valid for the basic model in this paper (to see this, set $P(e) = g(\underline{e}) - \sum e_i$). The deeper significance of the potential is as yet unclear, but it has been suggested, partly on the basis of the experimental findings in VHBB, that the potential-maximizing equilibrium can be used for predicting the outcome in a potential game. The interesting point is that, in the class of games under consideration, the equilibrium that maximizes the potential always coincides with the noise-proof equilibrium. This connection suggests that noise proofness can help us gain a better understanding of the potential and vice versa.

## 2.4 Experimental Evidence

VHBB study coordination games with discrete strategy sets $\{1, ..., 7\}$ and payoff functions

$$u_i(e) = a\underline{e} - be_i, \tag{2.7}$$

where $a > b \geq 0$. Groups of 2 to 16 subjects played series of one-stage simultaneous move games. No communication was allowed before or during play.

The experiments essentially comprise three different games, called A, B and C. Games A and B used groups of 14-16 subjects. Game C used small groups of 2 subjects, randomly selected from the entire set of subjects. In games A and C, the payoff parameters were set at $a = 0.20$ and $b = 0.10$. In game B parameters were set at $a = 0.20$ and $b = 0$. After each period game, the minimum action was publicly announced. The treatments in VHBB (1990) are summarized in table 2.1.

It is easily verified that any symmetric profile is a Nash equilibrium and, if $b > 0$, a strict one. Moreover, these equilibria are Pareto-ranked by the common effort, $(7, ..., 7)$ being the only efficient equilibrium. Using a

| Treatment | Payoff function | No. of subjects |
|---|---|---|
| A | $0.20 \cdot \underline{e} - 0.10 \cdot e_i$ | 14-16 |
| B | $0.20 \cdot \underline{e}$ | 14-16 |
| C | $0.20 \cdot \underline{e} - 0.10 \cdot e_i$ | 2 |

TABLE 2.1. VHBB (1990) experimental treatments

simple renormalization and applying Proposition 1 to continuous versions with strategy sets $[1,7]$ of these games, it can be shown that $(1, ..., 1)$ is the unique noise-proof equilibrium if and only if $a < bn$, while $(7, ..., 7)$ is the unique noise-proof equilibrium if $a > bn$. If $a = bn$, all symmetric pure strategy equilibria are noise-proof. Hence, $(1, ..., 1)$ is the unique noise-proof equilibrium in (the continuous version of) game A, $(7, ..., 7)$ is the unique noise-proof equilibrium in game B, while any symmetric pure strategy profile is a noise-proof equilibrium in game C.[5]

Disregarding some unimportant features, the experimental procedure can be described as follows. Games A and B were played repeatedly five or ten times by fixed groups of subjects. Game C was repeated three or five times, each player being randomly matched with a new opponent in each period. After each period game, the minimum action chosen in that game was publicly announced to the participants.[6]

The experimental results vary significantly between the games. In game A, the initial efforts of the subjects were widely dispersed and then approached the lowest effort $e_i = 1$. By period ten, 72 percent adopted the minimum effort. In game B, there was immediately a strong tendency to choose the highest effort. This tendency was subsequently reinforced and by period five, 96 percent of the subjects choose $e_i = 7$. In game C, finally, the subjects' efforts varied substantially in all periods without showing any clear trend.

These results are encouraging for the notion of noise proofness. In the two games, A and B, with unique noise-proof equilibria, a vast majority of the subjects conform to this solution, at least after some periods of learning. In game C, on the other hand, where noise proofness is indeterminate, the subjects' choices fail to converge and the chances of coordinating with

[5] Actually game B is not subsumed under the above model. A simple adjustment, however, shows that (7,...,7) is the unique noise-proof equilibrium of game B. (Note that (7,...,7) is weakly dominant in this game and that a higher effort always weakly dominates a lower one.) Similarly, it is straightforward to extend our analysis to the non-generic case corresponding to game C.

[6] We will not consider the experiments in which game C was played repeatedly by fixed pairs of players. This treatment typically resulted in repeated game effects which the present framework is not suited to account for.

your opponent remain small through all the periods. It should be noted that a number of competing hypotheses and solution concepts turn out to be considerably less successful, when confronted with the VHBB evidence. The hypothesis that the players will coordinate on the Pareto-optimal equilibrium, as reflected in Harsanyi and Selten's (1988) notion of payoff dominance, for example, is contradicted by the behavior in games A and C. Game C gives no support to the idea that players want to maximize their security level. Harsanyi and Selten's (1988) notion of risk-dominance fares well in games A and B where its prediction coincides with the noise-proof equilibrium. In game C, however, risk-dominance selects the equilibrium with $e_i = 4$, which is not supported by the experimental evidence (see Crawford, 1991).

A noteworthy feature in the experiments is that coordination, when it takes place, results from an adaptive process rather than the players' deductive reasoning. Hence, the evidence does not favor the rationalistic view on equilibrium selection but, instead, highlights the role of strategic uncertainty and learning. The predictive success of noise proofness may be viewed as a result of the combination of its nice learning properties and the noise mimicking the players' strategic uncertainty.

## 2.5 Noise-Proofness and Risk-Dominance

In this section, we will discuss the relationship between noise-proofness and risk-dominance in more detail. Harsanyi and Selten's (1988) general theory of equilibrium selection discriminates between strict equilibria. In Harsanyi and Selten's theory, payoff-dominance should have absolute precedence and players should have no trouble coordinating their expectations at the commonly preferred equilibrium point. This is a unique point in the minimum game, with the highest effort by all players, i.e. $e_i = e^1$ for all $i$. As noted in the previous section, this prediction is far from the play observed in the experiments conducted by VHBB (1990). A variant of the theory that eliminates the precedence to payoff-dominance seems more promising.

The risk-dominance concept selects a unique equilibrium in the minimum game.[7] The definition is based on a comparison between equilibrium points, two by two. In a game with more than two equilibrium points, the risk-dominant equilibrium is a point not risk-dominated by any other point. A

[7] Harsanyi and Selten's theory is based on finite choice sets. However, in the minimum game we can take the limit as the distance between two compared equilibrium points go to zero to obtain an approximation with continuous strategy spaces.

hypothetical process starts from a situation where it is common knowledge that either of two points will be the solution.

According to Harsanyi and Selten, players reason in the following way. Player $i$ attaches subjective probability $z_i$ to the event that all opponents choose the first equilibrium and $(1 - z_i)$ to the event that they choose the second. Beliefs are independent and uniformly distributed. Player $i$ chooses a best reply to his beliefs. Adaption is achieved by using the tracing procedure, which is simple in this particular game. Iteration comes to an end at the first iteration since the situation is symmetric and all players have a unique best reply to his prior. We assume that there is, at most, one point $m \in [0, M]$ with $g'(e^m) = m$, for $m \in \mathbb{R}_{++}$. In appendix B, the risk-dominant equilibrium is computed. We obtain the following characterization of the risk-dominant equilibrium in the minimum game

$$(g'(e) - 1)^{n-1} - g'(e)^{n-2} = 0, \tag{2.8}$$

where $e^m$ solves the equation. The risk-dominant equilibrium is $e_i = e^m$ for all $i$. Risk-dominance and the approach proposed in section 3 yield the same result in a game with two players. With more than two players risk-dominance selects a different equilibrium point than our approach. More precisely, the efforts in the risk-dominant equilibrium are higher than the efforts in the noise-proof equilibrium, i.e. $e^m > e^n$ for all $n > 2$.

The different outcomes are due to the approaches differing in their assumptions on players correlation in beliefs on the strategies of their opponents. Risk-dominance rely on more or less ad hoc thought processes to model the players' reasoning about the game, while the approach presented in section 3 is based on a fully specified noncooperative game. The relative advantages of the different theories cannot be settled a priori.

However, the risk-dominant equilibrium and the limit equilibrium selected in the noisy game are both well defined. Thus, further experiments can show which approach gives the better predictions in Bryant's coordination game. Using a simple renormalization ($b = 1$) in a linear version of the game with efforts in $[0, M]$, the unique equilibrium is determined by payoff parameter $a$ and the number of players $n$. The strategy profile $(M, ...M)$ is the unique noise proof equilibrium if $a > n$, and $(0, .., 0)$ if the opposite holds. Correspondingly, $(M, ...M)$ is the risk-dominant equilibrium if $\ln(a^{-1}) / \ln(1 - a^{-1}) + 1 > n$, and $(0, .., 0)$ if the opposite holds.

The different concepts are illustrated in Figure 2.1. The unique noise proof equilibrium shifts from $(M, ..., M)$, below the lower line in the figure, to $(0, .., 0)$, above this line. The unique risk-dominant equilibrium shifts from $(M, ..., M)$, below the upper line in the figure, to $(0, .., 0)$, above this line. Points A and C illustrate games A and C in VHBB (1990). At any point like D between the upper and lower lines, however, risk-dominance

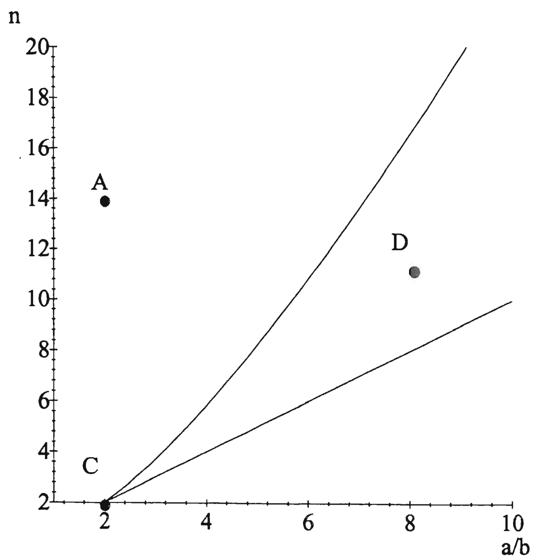


FIGURE 2.1. Risk-Dominance and Noise-Proofness

and noise proofness select different equilibria. A modified experiment can, thus, possibly test the different theories.

## 2.6 Median Games

In another study, Van Huyck, Battalio and Beil (1991) present results from experiments on median (or average opinion) games. These games are characterized by discrete strategy sets $\{1, ..., X\}$ and payoff functions

$$u_i(x) = a\widetilde{x} - b(\widetilde{x} - x_i)^2 \qquad i = 1, ..., n, \tag{2.9}$$

where $a > 0$, $b > 0$ and $\widetilde{x}$ denotes the median of the strategy profile $x = (x_1, ..., x_n)$. Thus, a player's payoff is increasing in the median and decreasing in the distance between his own strategy choice and the median. Clearly, each symmetric pure strategy profile is a strict Nash equilibrium. Moreover, the equilibria of the game are Pareto-ranked, the highest equilibrium $(X, ..., X)$ being the only efficient one.

$(X, ..., X)$ is also the unique noise-proof equilibrium. For this purpose, note that in a (pure-strategy) equilibrium of a slightly noisy median game, each player's strategy must be very close to the median. Hence, on average, the expected benefit from increasing one's strategy by $\Delta > 0$ is approximately equal to $a\Delta/I - b\Delta^2$, which is positive for sufficiently small $\Delta$.

Van Huyck, Battalio and Beil (1991) report an experiment on a median game with $a = 0.1$, $b = 0.05$, $X = 7$, and $n = 9$. The game was played repeatedly during ten periods by fixed groups of subjects and the median group action was announced to the group after each period. The extreme history dependence is the most striking feature of the experimental results: In all groups, the median selected in the initial period remained the median group action in each subsequent period. The median selected was either 4 or 5.

The fact that the experimental behavior did not correspond to the noise-proof solution may seem discouraging for the present approach. We will argue, however, that the discrepancy is a natural result of the divergence between the incentives which players face in discrete vs continuous median games. Our argument builds on a stylized discrete version of a noisy median game, which also allows us to understand the observed history dependence and to highlight the importance of the fineness of the action grid applied in experiments. We consider a situation where, at some stage of a repeated median game, the players' expectations are influenced by some historic precedent. We assume that the group median in period $t$ was some action $\widetilde{x}_t \in (2, ..., X-1)$ and that each player assigns a high probability to that action being chosen by most of his co-players in period $t+1$. However, he also takes the possibility that one or several other players may deviate from $\widetilde{x}_t$ into account. Specifically, he assumes that each co-player will choose $\widetilde{x}_t + 1$ with probability $\delta$, $\widetilde{x}_t - 1$ with probability $\delta$ and $\widetilde{x}_t$ with probability $1-2\delta$, where $\delta \in (0, 0.5]$ and the actions of different players' are statistically independent.

Given this set-up, we intend to investigate whether there exist values of $\delta$, for which the individual player would prefer to deviate from $\widetilde{x}_t$ to $\widetilde{x}_t+1$ or $\widetilde{x}_t-1$ in period $t+1$. In the case of median games, it is easily shown that only upward deviations can be rational. Since it is also obvious that deviations cannot be rational for $\delta$ sufficiently close to zero, it makes sense to look for the smallest value of $\delta$, to be denoted $\underline{\delta}$, at which upward deviations start to become profitable. For the Van Huyck, Battalio and Beil's (1991) median game, no such $\underline{\delta}$ exists: For any $\delta \in (0, 0.5]$ the player will prefer to play the status quo action $\widetilde{x}_t$. (Proofs of this and other results stated in this section are given in Appendix C.) This suggests that the various pure strategy equilibria of this game are highly robust, even to large amounts of strategic uncertainty, and, thus, explains why a status quo, once established, might be very unlikely to change.

The robustness of the equilibria in Van Huyck, Battalio and Beil's (1991) game is partly linked to the coarseness of the action grid of the game. If the interval between adjacent actions, to be denoted $h$, is reduced from $h = 1$ to, say, $h = 0.1$, a status quo equilibrium at $\widetilde{x}_t$ will only be robust for $\delta < 0.203$, i.e. we have $\underline{\delta} = 0.203$. For $h = 0.01$, we get $\underline{\delta} = 0.102$.[8] Hence, the chances of observing a convergence towards the Pareto-optimal equilibrium should be higher in experiments using a finer action grid. This prediction is verified in an experiment on a median game reported in Van Huyck, Battalio and Rankin (1996). This game had seven players and $h = 0.06$ which yields the critical value $\underline{\delta} = 0.173$. Most groups of subjects either play optimally throughout or tend to increase their actions until the maximum value is reached. To complete the analysis, it is also interesting to note that game A - as defined in Section 3 - has $\underline{\delta} = 0.048$ (for $n = 15$), i.e., for $\delta$ above this value, the player wants to decrease his action from a current status quo. (Due to the linearity of this model, this value is independent of the action grid size.) It is tempting to associate the rather rapid convergence to the lowest action observed in the experiments with the low $\underline{\delta}$-value of this game.

To conclude, we hope to have shown that far from being in contradiction with noise proofness, the various experiments on median games actually lend additional support for the empirical relevance of this notion. The above analysis also highlights the importance of the fineness of the action grid being used in experiments. In the presence of strategic uncertainty, the players' incentives to deviate from a given status quo is much smaller in a game with a coarse action grid than in variants with finer grids.

## 2.7 Related Literature and Concluding Comments

In a closely related paper, Anderson, Goeree and Holt (1996) study a different perturbation of linear minimum-effort games.[9] In their model, each player's decision error has a specific functional form, thus implying that the likelihood of choosing a particular action is positively related to the expected payoff associated with that action. As the likelihood of nonoptimal decisions goes to zero, the equilibrium of this model converges to a unique solution which coincides with the noise-proof equilibrium in generic games.

Although Anderson et al. arrive at essentially the same conclusion as we

[8] The explanation of this phenomenon lies in the fact that the punishment for a player who deviates from the group median is quadratic and, thus, decreases at a faster rate than the linear potential reward. This is also the key to understanding why the Pareto-optimal equilibrium is the only noise-proof one.

[9] We are indebted to a referee for signaling the existence of this study which we were not aware of when writing the first version of the present paper.

do, their approach differs considerably from ours with regard both to the scope and to the nature of the perturbation. While we consider a broad class of perturbations delimited by a number of intuitively defendable assumptions (see Appendix A), Anderson et al.'s perturbation is restricted to one particular functional form where the players' degree of rationality is the only variable parameter. The difference in nature between the approaches can be described by a comparison with existing refinements for finite games. While noise proofness might be considered as an adaptation of trembling-hand perfection (THP) to games with continuous strategy spaces, Anderson et al.'s refinement is more akin to proper equilibrium. [10]

Carlsson and van Damme (1993) is another related study, which analyses a class of coordination games where each player has two actions. They perturb the games by means of global payoff uncertainty, i.e. by embedding them into a larger incomplete information game where payoffs are determined randomly and privately observed with noise by each player. Using iterated strict dominance and letting the noise vanish they manage to select unique solutions for almost all games in the given class. Interestingly, if we apply their approach to minimum games with strategy sets $\{1, 2\}$ and payoffs as in (2.7), it selects $(1, ..., 1)$ if $a < bn$ and $(2, ..., 2)$ if $a > bn$. Hence, we get exactly the same criterion as when applying noise proofness to the continuous versions of these games, with strategy sets $[1, 2]$. This concordance between approaches based on very different setups and perturbations is quite remarkable and promising for the prospects of a more unified theory of equilibrium selection built on a strictly noncooperative foundation.

A common view on coordination games maintains that, since every strict Nash equilibrium survives the traditional refinements, any such equilibrium should be considered a potential solution. We agree with Crawford (1995) that this view is not of much help for explaining the systematic discrimination between strict equilibria that was observed in the VHBB experiments. Crawford also argues that you need exogenous belief parameters in order to explain the coordination process. Although this is probably true when it comes to the dynamics of this process, our results indicate that, by using

[10] On these refinements, see e.g. Fudenberg and Tirole (1991). The comparison between noise proofness and THP is complicated by a number of issues. First, relying on the assumption that small errors are more likely than large ones, noise proofness exploits the topology of the strategy space in a way which would not always make sense in a finite model. Second, noise proofness derives its strength partly from a perturbation where errors are continuously distributed. As discussed in Carlsson (1991), if THP is adapted to continuous-action games without this assumption, the results may be very different.

a more powerful refinement, its limiting outcome can be predicted without invoking exogenous parameters. Our results also stress the importance of not identifying refinements with deductive equilibration mechanisms. Combined with Milgrom and Roberts' results, noise proofness yields powerful predictions about the limiting outcomes of adaptive processes.

Our aim has been to show that noise proofness and the perturbation from which it is derived are interesting and, potentially, very powerful tools for the analysis of games with multiple equilibria. The present paper, admittedly, deals only with a rather narrow class of games. An important task for future research will, thus, be to extend the analysis to broader classes of games.[11] Another task will be to confront noise proofness with sharper experimental tests. In particular, it would be interesting to conduct experiments on games with non-linear payoffs and unique interior noise-proof equilibria. Finally, the role of the fineness of the action grid should be further explored.

[11] For a related application to signaling games, see Carlsson and Dasgupta (1997).

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## Appendix A: A More General Set-Up

The proof of Proposition 1 as sketched in section 3 exploits a number of strong and somewhat unsatisfactory assumptions. For instance, the zero mean condition on the $X_i$ implies, somewhat awkwardly, that one must allow for the possibility of negative efforts. Presently, we will see that this and other deficiencies in our model can be repaired, by using a more general class of perturbations, without invalidating our main result. An attractive feature of the new set-up is that it encompasses the case where one of the players does not make any error in the noisy game. This case fits well with a strategic uncertainty interpretation where players do not actually make any errors; the noise is just an expression of each individual player's uncertainty about the exact choices of the others.

We still let the random efforts $E_i$ take values on a bounded interval $[s_i - \varepsilon, s_i + \varepsilon]$. We assume that the distribution of player $i$'s effort, to be denoted $F_i(\cdot\,|s_i)$, depends on $s_i$ only and that the different $E_i$ are statistically independent, but we allow the error terms $E_i - s_i$ to depend on $s_i$ and we do not need any symmetry assumptions on the noise variables. It is natural to assume the sets $\{E_i(s_i)\}_{s_i \in [0,M]}$ to be ordered by first-order stochastic dominance, i.e. if $s_i \geq s_i'$, then $F_i(e_i\,|s_i) \leq F_i(e_i\,|s_i')$ for all $e_i$. We consider sequences $\{\Gamma^k\}$ of noisy games with vanishing noise in the sense that the support shrinks to a single point, i.e. $\varepsilon^k$ converges to 0.[12] Using the notation $E(s)$ for the random variable corresponding to the lowest effort given strategy profile $s$ and letting $\overline{r}$ denote the expectation of a random variable $r$, we also make the following assumptions:

I. $\overline{E}_i(s_i)$ *is continuously differentiable and* $d\overline{E}_i^k(s_i)/ds_i$ *converges uniformly to* 1 *for all* $i$.

II. $\overline{E}(s)$ *is continuously differentiable and* $\sum_i \partial\overline{E}^k(s)/\partial s_i$ *converges uniformly to* 1.

Assumption I seems rather innocuous considering that $\overline{E}_i^k$ converges to $s_i$. Assumption II can be given a similar motivation. Since $\overline{E}^k(s)$ converges

[12] The assumption of vanishing supports could be replaced by convergence in probability and appropriate restrictions on the tails of the probability distributions. Also note that, since we do not impose a zero mean condition on the error terms $E_i - s_i$, we can avoid the possibility of negative efforts.

to $\underline{s}$, $\sum_i \partial \overline{E}^k(s)/\partial s_i$ must converge to 1 on average on any arbitrarily small interval.[13]

We claim that Propositions 1 and 2 remain valid under the above assumptions. To show this, we start by noting that, integrating by parts, we get, for $A \leq \underline{s} - \varepsilon$ and $B \geq \underline{s} + \varepsilon$,

$$\text{(A.1)} \qquad \int f(e\,|s)\,g(e)\,de = g(B) - \int_A^B F(e\,|s)\,g'(e)\,de$$

$$\text{(A.2)} \qquad \overline{E}(s) = \int f(e\,|s)\,e\,de = B - \int_A^B F(e\,|s)\,de.$$

Next let us establish the key property that the noisy games are supermodular. It suffices to show that $\partial^2 u_i(s)/(\partial s_i \partial s_j) \geq 0$ for all $s$, $i$ and $j$, $i \neq j$. Using (A.1) we have

$$\text{(A.3)} \qquad \partial u_i(s)/\partial s_i = -\int \partial F(e\,|s)/\partial s_i \cdot g'(e)\,de - d\overline{E}_i(s_i)/ds_i$$

As $g'(e)$ is always nonnegative it suffices to show that $\partial^2 F(e\,|s)/(\partial s_i \partial s_j)$ is always nonpositive. Noting that $F(e\,|s) = 1 - \prod_j [1 - F_j(e\,|s_j)]$ by definition and $\partial F_i(e\,|s)/\partial s_i \leq 0$ by first-order stochastic dominance, this result follows straightforwardly.

By supermodularity, any noisy game $\Gamma^k$ has a lowest and a highest Nash equilibrium in pure strategies. We let $a^k$ denote the lowest equilibrium in $\Gamma^k$. Obviously, then, $\Gamma$ has a lowest and a highest *noise-proof* equilibrium $\mathbf{a} = (a, ..., a)$ and $\mathbf{b} = (b, ..., b)$ for some $a, b \in [0, M]$, $a \leq b$. If $a \in (0, M)$, there exists $\{a^k\}$ converging to $\mathbf{a}$ such that, for all $k$, the first-order condition

$$\text{(A.4)} \qquad -\int \partial F^k(e\,|a^k)/\partial s_i \cdot g'(e)\,de - \partial \overline{E}_i^k(a^k)/\partial s_i = 0$$

is satisfied. As $\partial F^k(e\,|a^k)/\partial s_i$ is always nonpositive and strictly negative only close to $\underline{a}^k$ for large $k$, it is clear that the integral in (A.4) can be approximated by $g'(\underline{a}^k) \int [\partial F^k(e\,|a^k)/\partial s_i]\,de$ for large $k$. Using (A.2) and assumption I, we get

$$g'(\underline{a}^k)\,\partial \overline{E}^k(a^k)/\partial s_i \approx 1$$

for large $k$. Summing over $i$ and exploiting assumption II yields the limit condition

$$g'(a) = n$$

In a similar way, we get $g'(b) = n$ and, thus, $a = b = e^n$ if $b \in (0, M)$. The cases with extreme solutions are easily handled by substituting the

[13] We let the reader verify that the above assumptions are indeed compatible with the case where one of the players does not make any error.

appropriate inequalities for equation (A.4). This completes the proof of Proposition 1.

The proof of Proposition 2 as sketched in section 3 relies only on the supermodularity property and, thus, remains valid.

# Appendix B: The Risk-Dominant Equilibrium

In this appendix we derive the unique risk-dominant equilibrium in $\Gamma$. Compare two equilibrium point $\mathbf{e} = (e, ..., e)$ and $\mathbf{e}' = (e + \Delta e, ..., e + \Delta e)$, where $\Delta e > 0$. The prior of player $i$ is $p_i(e) = \Delta e \cdot (g(e + \Delta e) - g(e))^{-1}$ and $p_i(e') = 1 - p_i(e)$. Equilibrium $\mathbf{e}$ riskdominates $\mathbf{e}'$ if

$$\left(1 - \Delta e \cdot (g(e + \Delta e) - g(e))^{-1}\right)^{n-1} (g(e + \Delta e) - g(e)) - \Delta e \leq 0 \quad (2.10)$$

Next, we proceed with a comparison between two equilibrium points $\mathbf{e} = (e, ..., e)$ and $\mathbf{e}'' = (e - \Delta e, ..., e - \Delta e)$. The prior of player $i$ is $p_i(e) = 1 - p_i(e'')$ and $p_i(e'') = \Delta e \cdot (g(e) - g(e - \Delta e))^{-1}$. Equilibrium $\mathbf{e}$ riskdominates $\mathbf{e}''$ if

$$\left(1 - \Delta e \cdot (g(e) - g(e - \Delta e))^{-1}\right)^{n-1} (g(e) - g(e - \Delta e)) - \Delta e \geq 0 \quad (2.11)$$

A unique risk-dominant equilibrium must satisfy both conditions. We can find a solution at the limit. If both conditions are satisfied at the limit then they are also satisfied for every $\Delta e > 0$. Hence, we can use the definition of the derivative ($\Delta e \downarrow 0$) to obtain a characterization of the risk-dominant equilibrium in the minimum game:

$$(g'(e) - 1)^{n-1} - g'(e)^{n-2} = 0 \quad (2.12)$$

# Appendix C: Computing $\underline{\delta}$

We consider an $n$-person median game with action grid size $h$ and payoffs

$$u_i(x) = a\widetilde{x} - b(\widetilde{x} - x_i)^2 \qquad , i = 1, ..., n \tag{2.13}$$

such that $n = 2z + 1$ for some $z \in Z_+$. Given a status quo $\widetilde{x}$ and a player $i$, let $k_+$ (resp. $k_-$) denote the number of players other than $i$ who choose $\widetilde{x} + h$ (resp. $\widetilde{x} - h$) in the next period. Under the assumptions in the main text and for $j \in \{0, ..., n-1\}$, player $i$ considers the events $k_+ = j$ and $k_- = j$ to have the same probability:

$$\Pr\{k_+ = j\} = \Pr\{k_- = j\} = \binom{n-1}{j} \delta^j (1-\delta)^{n-1-j} \tag{2.14}$$

The expected net gain for player $i$ from choosing $\widetilde{x} + h$ rather than $\widetilde{x}$ equals

$$\begin{aligned} \nu_+(\delta) \;=\;& \Pr\{k_+ > z\} bh^2 + \Pr\{k_+ = z\} ah \\ & - \Pr\{k_- > z\} 4bh^2 \\ & - (1 - \Pr\{k_+ > z\} - \Pr\{k_+ = z\} - \Pr\{k_- > z\}) bh^2 \end{aligned} \tag{2.15}$$

while his expected net gain from choosing $\widetilde{x} - h$ rather than $\widetilde{x}$ equals

$$\begin{aligned} \nu_-(\delta) \;=\;& \Pr\{k_- > z\} bh^2 - \Pr\{k_- = z\} ah \\ & - \Pr\{k_+ > z\} 4bh^2 \\ & - (1 - \Pr\{k_- > z\} - \Pr\{k_- = z\} - \Pr\{k_+ > z\}) bh^2 \end{aligned} \tag{2.16}$$

A comparison between (2.15) and (2.16), taking (2.14) into account, shows that $\nu_+(\delta) > \nu_-(\delta)$. To find the critical $\delta$ for a particular set of parameters, one should look for

$$\underline{\delta} = \inf\{\delta \in (0, 0.5]; \nu_+(\delta) > 0\} \tag{2.17}$$

To find the critical $\delta$ for the minimum effort game with payoff functions

$$u_i(x) = a\underline{e} - be_i \qquad , i = 1, ..., n \tag{2.18}$$

and unit grid size, one should look for

$$\underline{\delta} = \inf\{\delta \in (0, 0.5]; \nu_-(\delta) > 0\} \tag{2.19}$$

where

$$\nu_-(\delta) = b - \Pr\{k_- = 0\} a \tag{2.20}$$

(The proof that $\nu_-(\delta) > \nu_+(\delta)$ for $a = 0.2$ and $b = 0.1$ in these games is left to the reader.)

# 3

# Simplicity and Communication in Coordination Games

## 3.1 Introduction

This paper investigates how collective behavior in coordination games is determined by the transmission of information. Pre-play communication should help players avoid coordination failures. Furthermore, transmission of information should help players optimize their collective behavior. Does this mean that pre-play communication is a guarantee of successful coordination, and does pre-play communication favor Pareto-optimal Nash equilibria in the underlying game?

Many interesting games studied in the game theoretic literature exhibit multiple *strict* Nash equilibria.[1] While intuition might suggest that players should be able to coordinate in a Pareto-optimal equilibrium, the usual refinements in game theory fail to select an efficient, or even unique, outcome. This conflict between intuition and formal analysis has constituted the basis for several efforts among game theorists.

The first approach allows agents to send costless pre-play signals before choosing their actions. This costless pre-play communication is called *cheap talk*. Unfortunately, cheap talk does not help players to coordinate in the efficient outcome. There exist equilibria where players have decisions rules that are constant and therefore unaffected by the message received from the other players (cf. Crawford and Sobel, 1982, Green and Stokey, 1982). Hence, both problems of coordination, i.e. the problem of equilibrium selection and the problem of social inefficiency, remain unsolved.

The second approach suggests that if an equilibrium arises as the result of costless negotiations between the players, then team members should be able to coordinate in a Pareto-optimal outcome. It is argued that it must not be profitable for any player to propose that a strategy combi-

[1] Much recent discussion in game theory has focused on experiments based on simple coordination problems. These coordination problems have been used by game theorists to test various hypotheses on learning, equilibrium selection and strategic uncertainty. For examples and references, see Van Huyck, Battalio and Beil (1990), (1991), (1993); Van Huyck et al (1995); Van Huyck, Battalio and Rankin (1996) and Van Huyck, Cook and Battalio (1997).

nation be abandoned for another equilibrium, where everybody is better off. Only Pareto-optimal equilibria are *renegotiation-proof* (Fudenberg and Tirole, 1991, p. 174f). Renegotiation proofness predicts complex behavior in situations where complex behavior is collectively optimal.[2]

In reality, however, players do neither as poorly in terms of equilibrium selection as some cheap talk models suggest, nor as well in terms of efficiency as renegotiation proofness predicts. Therefore some underlying assumptions must be changed.

The model in this paper is used to analyze how the structure of a common language influences the equilibrium selection problem in coordination games, where players are allowed to transmit messages to coordinate their behavior. In order to simplify the analysis, it is assumed that the coordination game neither involves conflicts of interest, such as the Battle of Sexes, nor problems of trustworthiness, as in the Stag Hunt Game.[3] Instead, we focus on a variant of Binmore's (1994) Dodo game. All players have identical interests and there are no incentives to send insincere messages. The game has many strict Nash equilibria, and the players have asymmetric information. The informed player knows the relative Pareto ranking of all Nash equilibria before players choose their actions, while the uninformed player expects all symmetric pure strategy combinations to be strict Nash equilibria with the same payoff *ex ante*.

The pre-play communication is modelled in two steps. Before one player is informed about the ranking of equilibria, both players communicate without cost and players can create a common language, i.e. a labeling and a code, which is optimal for a class of coordination games. Once the informed player has learned the Pareto-ranking of equilibria, every message is costly. At this stage a message has to be transmitted through a costly channel admitting binary code only.

Players can use communication both to coordinate their expectations in a specific equilibrium and to optimize their collective behavior. We can illustrate our basic results in a simple version of the game:

[2] Harsanyi and Selten's (1988) "general theory of equilibrium" selection discriminates between strict equilibria. In HS's theory, payoff-dominance should have absolute precedence and players should have no problems in coordinating their expectations at the commonly preferred equilibrium point. Thus, HS's theory predicts the same outcome as renegotiation proofness in games of mutual interests.

[3] In some games, each player is better off if he can convince the other player to choose a high effort, regardless of his own intended play. Aumann (1990) argues that it is not clear that players should expect their opponents to believe their announcements.

| | H | L |
|---|---|---|
| H | 2,2 | 0,0 |
| L | 0,0 | 1,1 |

$G_1$

| | H | L |
|---|---|---|
| H | 1,1 | 0,0 |
| L | 0,0 | 2,2 |

$G_2$

In this Dual-Dodo game two players choose H or L. Player 1 selects a row and player 2 selects a column and Nature determines the state of the world ($G_1$ or $G_2$). The payoffs are given as the intersection of a row and a column, where player 1's payoff is specified first. Before either player is informed about the choice of Nature, they can meet and decide how to communicate after Nature has informed player 1 about its choice. Assume that they decide to play $(H, H)$, if no information is transmitted. In other words, they choose the H strategy as a "convention". Next, they can decide that if player 1 transmits a signal to player 2, they should both change strategies to L, i.e. $(L, L)$.

If Nature selects $G_1$, they are both satisfied with the tacit convention and no information is transmitted, but what if the other state of the world occurs? It immediately follows that the players are ready to give up one unit of utility each to transmit a message, which would trigger $L$-play. If the cost is higher, they will remain in the $(H, H)$ equilibrium.

Thus, if the cost of communication is sufficiently high, players will choose the strategy described by the empty string. This equilibrium is the most simple one in two ways. First, the empty string is the shortest description available in the language chosen by the players. In that sense, the equilibrium is the most easily described. Second, the equilibrium is simple because the behavior is not conditional on the state of the world: players would choose the same action not withstanding the choice of Nature. The purpose of the rest of this paper is to generalize this result.

The paper is organized as follows. In section 2, the main features of the general model are described. The labeling and coding procedures are described in section 3 and 4, respectively. Section 5 presents the results and section 6 concludes.

## 3.2 The Binary Choice Game

Consider a simple coordination problem where two players are required to choose between two actions, called $a_1$ and $a_2$. Before players choose their actions, Nature has decided which of the two equilibrium profiles is dominant, i.e. which strategy profile is associated with a "superior" and an "inferior" outcome respectively. When Nature selects $A_1$, strategy profile $(a_1, a_1)$ dominates $(a_2, a_2)$ and when choosing $A_2$, the dominance relation is reversed. In a superior equilibrium, each player gets $x$ and in an inferior equilibrium they both get 1. If players fail to coordinate, they both get 0. The two payoff matrices $A_1$ and $A_2$ are defined as follows:

$$\begin{array}{c|c|c|} & a_1 & a_2 \\ \hline a_1 & x,x & 0,0 \\ \hline a_2 & 0,0 & 1,1 \\ \hline \end{array} \qquad \begin{array}{c|c|c|} & a_1 & a_2 \\ \hline a_1 & 1,1 & 0,0 \\ \hline a_2 & 0,0 & x,x \\ \hline \end{array} \tag{3.1}$$

$$A_1 \qquad\qquad\qquad A_2$$

where $x > 1$. In this game, both symmetric strategy profiles are strict Nash equilibria. The binary choice game is a meta-game, where the players face the coordination problem described above $T$ times.

As in Gauthier (1975), each player will choose an option under a description. We consider the coordination problem to be defined by the agents' descriptions of the game. It is assumed that all players make a mutual distinction between the one-period actions $a_1$ and $a_2$ before the game starts. Following Sugden (1995), we shall use the term "label" for the description by which players recognizes pure strategies. Labeling is a function $L_i$, assigning a label $L_i(s_i)$ to each strategy $s_i \in S_i$ of each player $i$, such that each pure strategy of each player has a distinct label. The rules of the meta-game are then defined as follows.

First, the players construct a *common language*, i.e. a labeling and a code, before the number of periods in the game is determined and a specific payoff structure is chosen. We assume that player 1 can transmit a message to player 2 through a channel admitting transmissions in binary code only.[4] For this purpose, fix an alphabet $\mathcal{A} = \{0, 1\}$. Let $\mathcal{A}^*(c)$ denote the set of all strings $z = z_1 z_2 z_3 ... z_c$ of length $c$ with elements $z_k \in \mathcal{A}$. Define the union of all strings $\mathcal{A}^* = \bigcup_{c \geq 1} \mathcal{A}^*(c)$. The message which is transmitted from player 1 to player 2 is *a suggestion* what players should do in the coordination game. A suggestion is a list specifying a strategy for each player. The suggestion is *consistent,* if the strategy profile is a mutual best response (see Farrell, 1988). We assume that player 1 will only make consistent suggestions. As any strict equilibrium is a symmetric pure strategy profile, this assumption implies that a consistent suggestion can be reduced to a description of a single pure strategy.

Second, the number of periods in the game is drawn, i.e. $T \in \Omega$, where

[4] Consequently, only few strategies can be described with short code strings, so that the descriptions of equilibrium strategies vary in complexity, cf. Chaitin (1975).

$\Omega = (1, 2, ..., \overline{T})$. It is assumed that there is a probability function $\pi : \Omega \rightarrow (0, 1)$, such that

$$\sum_{T \in \Omega} \pi(T) = 1 \tag{3.2}$$

and $\pi(T) > 0$ for all $T \in \Omega$. Next, Nature selects a sequence of matrices which determines payoffs in every period. Let $A^t \in \{A_1, A_2\}$ be the payoff matrix in period $t$ and $A = \left(A^1, ..., A^T\right)$ be the sequence of matrices defining the payoff structure. There is a probability function $p_T : \times_{t \in T} \{A_1, A_2\} \rightarrow (0, 1)$, such that $p_T(A) = 2^{-T}$ for all $A \in \times_{t \in T} \{A_1, A_2\}$. At the end of the second stage player 1 is informed about $A$ and $T$ without noise.

Third, the informed agent, i.e. player 1, can send a message $m$ coded in alphabet $\mathcal{A}$, which is received by player 2. The complexity of a message $m$ coded in $\mathcal{A}$, is defined as the length of the string. The cost of transmission is $w$ per bit.

Fourth, players choose strategies. The strategy of player $i$ is an ordered string of actions, written $s_i \in S_i$, where $S_i = \times_{t \in \overline{T}} \{a_1, a_2\}$. We assume that in a $T$ period game, the sequence of actions is truncated after the $T$:th element. Finally, the game is played over $T$ periods and players receive payoffs. The payoff is the average period revenue minus the cost of communication. There is no observation of actions or payoffs until the game is over.

## 3.3 Labels

The players will choose an appropriate language for the entire class of payoff structures. For this purpose the players can proceed in the following manner. They attach one label to each action in the games in one period, call them $y_1$ and $y_2$. Moreover, the players will associate the two labels with two pure strategies in every game in more than one period. Next, the players will choose labels, $y_3$ and $y_4$, for the sequences of actions in the two-period games which remain to be named. These labels are also associated with two pure strategies in every game of more than two periods. Next, players will choose four labels, $y_5, ..., y_8$, in the games of three periods, for sequences of actions not yet labeled. Continue in this way to name $2^{k-1}$ sequences of actions in the $k$-period games and let these labels be associated with strategies in every game in $\overline{T}$ periods. Denote the set of all labels with $Y = \{y_1, y_2, ...\}$.

The procedure described above leaves many questions unresolved. The procedure only implies that when $k$ is small a label $y_k$ is used in a wider range of games. For instance $y_1$ and $y_2$ must be attached to the actions in the one-period game but it is arbitrary for which strategies these labels are used in games of more than one period. However, we can apply one more assumption to give the code more structure.

For both players, a $T$-period game can be decomposed into $\lceil T/k \rceil$ games in $k$ periods, where the last game is possibly truncated.[5] Consider a strategy called $\widetilde{y}$ in a $k$-period game. This strategy is a sequence of actions. Repeat this sequence of actions $\lceil T/k \rceil$ times. The resulting strategy is labelled $\widetilde{y}$ in the $T$-period game. We refer to this assumption as *invariance with respect to decomposition*.[6]

This assumption implies that our labeling procedure is highly structured. Indeed, there is a unique labeling, up to symmetric transformations, which satisfies this condition. For instance, the one-period game labels $y_1$ and $y_2$ would describe uniform sequences of actions in any $T$-period game. Any $T$-period game can be decomposed into $T$ one-period games, with a uniform action labelled $y_1$ or $y_2$. Denote repetition with $*$. Labels $y_1$ or $y_2$ refer to $(a_1)*$ and $(a_2)*$. Correspondingly, labels $y_3$ or $y_4$ refer to $(a_1, a_2)*$ and $(a_2, a_1)*$. We can proceed to construct this labeling in the same manner for $y_5$, $y_6$ etc.

## 3.4 Optimal Coding

We can now proceed to the problem of coding. A *code* is a function $\varphi : Y \to \mathcal{A}^*$ and the elements of $\varphi(Y)$ are called code-strings.

The players' goal is to find a code maximizing the expected payoff. Introduce the function $bin : \mathbb{N} \to \mathcal{A}^*$, where $bin$ is a binary expansion of $n \geq 0$, such that $(n)_2 = 1bin(n)$. By definition, $bin(1) = \lambda$. To simplify the notation, let $\log k \equiv \lfloor \log_2(k) \rfloor$, where $\lfloor \cdot \rfloor$ denotes the "floor" of the real (rounding downwards). Define $\log 0 \equiv 0$.

We consider two situations. In the first case, the empty string can be used as a message, in the second case it cannot. The following condition is defined:

(C) $\lambda$ is a code-string,

where $\lambda$ is defined as the empty string. Condition C is satisfied in the first case and violated in the second. In the first case, the following result is obtained:

[5] $\lceil \alpha \rceil$ denotes the "ceiling" of the real $\alpha$, (i.e. rounding upwards).

[6] In Herbert Simon's (1959) words, "man is not only a concept forming, but also a patternfinding, animal."

**Proposition 1** *If condition C applies, then* $\varphi(y_k) = bin(k)$ *for* $k = 1, 2, ...$ *is an optimal code.*

The probability distribution over sequences of payoff matrices is uniform. Therefore, all pure equilibrium strategies are identical with respect to the expected revenue. We will make use of the following simplifying lemma.

**Lemma 2** *The expected revenue in any equilibrium in pure strategies is* $\frac{1}{2}(x+1)$*, before Nature has selected* $A$*.*

**Proof.** The expected revenue in any equilibrium before Nature has selected $A$ is

$$E[u] = \frac{1}{T} 2^{-T} \sum_{k=0}^{T} \binom{T}{k} ((T-k)x + k) = \frac{1}{2}(x+1), \tag{3.3}$$

which concludes the proof. ■

We can now provide the proof of the main result.

**Proof.** *Step 1.* There are $2^n$ unique code-strings of length $n$ in $\varphi$ and in $\mathcal{A}^* \bigcup \{\lambda\}$, for all $n \geq 0$. Thus, $\varphi$ uses all strings in $\mathcal{A}^* \bigcup \{\lambda\}$. *Step 2.* The length of a code-string $\varphi(y_k)$ is $|\varphi(y_k)| = |bin(k)| = \log k$, which is increasing in $k$. *Step 3.* Using the lemma, the expected value of the sequence of equilibria generated by the labels $(y_k, y_k)$ for $T \geq 1 + \log(k-1)$ is

$$\sum_{t=1+\log k}^{\overline{T}} \left[ \pi(t) \frac{1}{2}(x+1) - \pi(t) \cdot w \cdot |\varphi(y_k)| \right]. \tag{3.4}$$

The expected revenue (the first part in the squared brackets) is independent of the code. Thus, we can reduce the problem of finding an optimal code to a minimization problem of the expected cost. The optimal code must solve:

$$\min_{\varphi} w \sum_{k=1}^{2^{\overline{T}}} \left( \sum_{t=1+\log k}^{\overline{T}} [\pi(t) \cdot |\varphi(y_k)|] \right). \tag{3.5}$$

From *step 1,* it follows that all code-strings in $\mathcal{A}^* \bigcup \{\lambda\}$ are used. Therefore, the assumption that $\pi(t) > 0$ for all $t$ implies that short code-strings must be used for small $k$, i.e. $|\varphi(y_k)|$ must increase monotonically in $k$ (follows from step 2). ■

When condition C applies, the players decide to associate the empty string with a strategy in the one-period game and, therefore, to a specific strategy in any game in more than one period. If the players wish to play this strategy, they do not need to transmit any information through the channel.

In practice, the strategy labeled with the empty string is a convention in the game. Under the assumption of invariance with respect to decomposition, this means that the players had decided to define a uniform sequence of actions, $(a_1)*$ or $(a_2)*$, as is the convention.

For the second case, when condition C does not apply, we obtain a similar result:

**Proposition 3** *If condition C does not apply, then* $\varphi'(y_k) = bin(k+1)$ *for* $k = 1, 2, ...$ *is an optimal code.*

**Proof.** *Step 1.* There are $2^n$ unique code-strings of length $n$ in $\varphi$ and in $\mathcal{A}^*$. Thus, $\varphi$ uses all strings in $\mathcal{A}^*$. *Step 2.* The length of a code-string $\varphi'(y_k)$ is $|\varphi'(y_k)| = |bin(k+1)| = \log(k+1)$, which is increasing in $k$. *Step 3.* Using the lemma we can see that the expected value of the sequence of equilibria generated by $(y_k, y_k)$ for $T \geq 1 + \log(k-1)$ is

$$\sum_{t=1+\log k}^{\mathcal{T}} \left[\pi(t)\frac{1}{2}(x+1) - \pi(t) \cdot w \cdot |\varphi'(y_k)|\right]. \tag{3.6}$$

The expected revenue (the first part in the squared brackets) is independent of the code. Therefore, the problem of finding an optimal code can be reduced to a minimization problem of the expected cost. The optimal code must solve:

$$\min_{\varphi} w \sum_{k=1}^{2^{\mathcal{T}}} \left( \sum_{t=1+\log k}^{\mathcal{T}} [\pi(t) \cdot |\varphi'(y_k)|] \right). \tag{3.7}$$

From step 1, it follows that all code-strings in $\mathcal{A}^*$ are used. Therefore, the assumption that $\pi(t) > 0$ for all $t$ implies that short code-strings must be used for small $k$, i.e. $|\varphi'(y_k)|$ must increase monotonically in $k$. ■

In the second case, we obtain a symmetric code. Both strategies in the one-period game are associated with one-bit code strings. This situation is reasonable if player 2 is genuinely uninformed. For instance, consider a situation where both players know the rules of the game, but the uninformed player does not know at which point in time the game will occur. In that case, the first bit of the message has a very high coordination value.

The results in propositions 1 and 2 are not surprising. Players will use all strings of length zero before they using code-strings of length one, and strings of length one before using code-strings of length two, and all strings of length two before using strings of length three etc. In other words; they will attach a label to each node in a binary tree. Thus, the problem of finding a code is reduced to the problem of associating code-strings of a given length with some particular labels. To minimize the expected average length, it suffices to attach the most likely labels with the shortest strings. The codes $\varphi$ and $\varphi'$ are two such examples.

## 3.5 Simplicity and Efficiency

Player 1 can send a message $m$, coded in an alphabet $\mathcal{A}$, to player 2. After communication, player 1 and 2 each chooses a strategy $s_1$ and $s_2$, respectively. Players choose strategies simultaneously and for all periods. No observations are done until the game ends. Finally, players receive payoffs determined by $A$ and strategies $s_1$ and $s_2$. The cost of transmission is $w$ per bit.

If the efficiency gains are small and communication is costly, it is always profitable to coordinate these in a Nash equilibrium with the shortest description. More precisely,

**Proposition 4** *Assume that condition C applies. If $x-1<w$, then player 1 would choose to transmit $\lambda$ as a message to the uninformed player. None of the players would incur any cost of communication.*

**Proof.** (i) The minimum payoff of the least complex message is $\underline{u}=1$. The maximum payoff transmitting further steps of a more complex message is $\overline{u}=x-w$. Now, $x-w<1$ if $x-1<w$ ■

Second, we proceed to the case where the empty string cannot be used as a message. A similar result holds if condition C does not apply:

**Proposition 5** *Assume that condition C does not apply. If $x-1<2w$, then player 1 would choose to transmit a 1 bit code-string to the uninformed player.*

**Proof.** (i) The minimum payoff, of the least complex message, is $\underline{u}=\frac{1}{2}(x+1)-w$. The maximum payoff transmitting further steps of a more complex message is $\overline{u}=x-2w$. Now, $\frac{1}{2}(x+1)-w>x-2w$ if $x-1<2w$. ■

It is worth noting that the value of communication is high in both cases. If players chose an equilibrium strategy at random, the expected payoff would be $\frac{1}{4}(x+1)$, which is clearly lower than the expected payoff in the first case and lower than the expected payoff in the second case, if $w<\frac{1}{4}(x+1)$. Second, if our attention is restricted to a labeling satisfying invariance with respect to decomposition, the Nash equilibrium with the shortest description is a strategy profile with a sequence of actions with the most regular pattern. In this case, we can expect players to choose the same action in every period, if the cost of transmitting information is high.

**Corollary 6** *As the labeling satisfy invariance with respect to decomposition the expected equilibrium strategy is a uniform sequence of actions, i.e.* $(a_1)*$ *or* $(a_2)*$*, if (i)* $x-1 < w$ *and condition C applies, or, (ii)* $x-1 < 2w$ *and condition C does not apply.*

The results in propositions 3 and 4 suggest that high costs of communication and small differences between the revenues in different equilibria, give both players incentives to keep the transmission of information at a minimum level. Both results are rather extreme in the following sense: players would not transmit more than the minimum number of bits, even if that resulted in a successful coordination in the equilibrium with the highest revenue. If communication costs are high, the players prefer to transmit the shortest string available, even if they only succeed in coordinating in the least efficient equilibrium, since the efficiency gains are outweighed by the additional cost of transmitting extra bits. In the game studied in this paper, this is equivalent to choosing the most regular pattern of behavior, if the labeling is invariant with respect to decomposition.

However, short descriptions and simple strategies do not merely exist at high communication costs. At lower levels of communication costs the problem of choosing an optimal equilibrium is a trade-off between how efficient an equilibrium strategy is in the underlying game and how easy it is to describe. This can be illustrated with two simple examples.

EXAMPLE 1. Consider a labeling which is invariant with respect to decomposition. Let $y_3$ denote $(a_1, a_2)*$. As condition C applies, the code-string for this label is one bit. For this strategy, there exists exactly one state of the world for which the sequence of actions is optimal with respect to revenues. However, for every $T$, there exist $T$ sequences of payoff matrices where $(a_1, a_2)*$ is almost optimal in terms of revenue, i.e. it is optimal in every period except one. For each of these sequences, approximately half the matrices are $A_1$ and $A_2$, respectively. Naturally, that means that a uniform sequence of actions, $(a_1)*$ or $(a_2)*$, is far from optimal with respect to revenues.

If the state is one of the sequences close to $(a_1, a_2)*$ and $T > 7$, the first bit transmitted from the informed to the uninformed player will increase each agent's payoff with at least $\frac{1}{4}(x-1) - w$. The second bit transmitted would only increase the payoff with $\frac{1}{8}(x-1) - w$. More precisely,

$$\frac{w}{x-1} \in (0.125, 0.250) \tag{3.8}$$

is a sufficient condition for ensuring that it is optimal for the informed player to choose the equilibrium strategy with the 1-bit code-string, $\varphi(y_3)$, rather than trying to coordinate in the equilibrium with the highest revenue or in the equilibrium with the shortest description (the empty string $\lambda$). Indeed, the optimal choice of both players is a trade-off between ease of describability and efficiency. Players approximate the perfect fit with some sequence of actions that are easily described in order to save communication costs.

EXAMPLE 2. The previous example was devoted to showing how the players can approximate a specific strategy with a sequence of actions, with a short description. This example will show how players choose messages at some given number of periods, as the cost of communication varies. We are interested in the expected average length of the message.

For this purpose, define the average length of the code-string with respect to $p_T$ as:

$$L_{\varphi}(w) = \sum_{A \in \times\{A_1, A_2\}} \left[ p_T(A) \left| \varphi(y(A,w)) \right| \right], \tag{3.9}$$

where $\varphi(y(A,w))$ is the optimal code-string in state $A$ at cost $w$.

Again, consider a labeling which is invariant with respect to decomposition. Assume that condition C applies and let $T = 8$ and $x = 1.5$. The two extreme cases can easily be solved for. As the communication cost is zero, $w = 0$, the players would naturally choose to coordinate in the revenue-maximizing outcome in any state of the world. The average length of the message transmitted would be $L_{\varphi}(0) \approx 6.01$. At the other extreme, when the cost of communication is high, $w > 0.5$, the informed player would choose the empty string as a message in every state of the world. In this case, the average length of the message transmitted would be $L_{\varphi}(0.5) = 0$.

To see what the average code length would be at intermediate levels of the communication cost, we have conducted some numerical simulations. Let $\widetilde{n}$ denote the number of states where players choose a different strategy than the most efficient one in terms of revenues and, correspondingly, let $\widetilde{n}_1$ denote the number of states where the players choose $\lambda$ rather than the most efficient in the underlying game. The number of deviations from the revenue-maximizing strategy, i.e. $\widetilde{n}$ and $\widetilde{n}_1$, should be related to the total number of states, which is 256.

The results of these simulations are reported in Table 3.1. The average length of the code-string transmitted decreases monotonically as the cost of communication increases. It is worth noting that players do not change to the shortest description at some threshold, but rather change their behavior gradually. At relatively low levels of $w$, players would start to play strategies which are more easily described. For instance, when $w = 0.04$, the probability that players play a more easily described strategy is 0.86. Hence, players are very likely to change from the revenue-maximizing strategy to some more easily described sequence of actions. However, the probability that they play the strategy with the shortest description is only 0.23. In 159 of 256 states, the informed player would transmit some, but not all, information about the payoff structure to the uninformed player.

| $w$ | $L_\varphi$ | $\tilde{n}$ | $\tilde{n}_1$ |
|---|---|---|---|
| 0.00 | 6.012 | 0 | 0 |
| 0.01 | 5.902 | 4 | 4 |
| 0.02 | 4.484 | 77 | 16 |
| 0.03 | 2.492 | 185 | 31 |
| 0.04 | 1.629 | 219 | 60 |
| 0.05 | 1.105 | 235 | 86 |
| 0.07 | 0.637 | 247 | 111 |
| 0.10 | 0.559 | 250 | 120 |
| 0.20 | 0.184 | 252 | 208 |
| 0.30 | 0.035 | 254 | 246 |
| 0.40 | 0.004 | 254 | 254 |
| 0.50 | 0.000 | 255 | 255 |

TABLE 3.1. Numerical simulation.

In terms of communication, players would start from a situation with zero communication costs, where the uninformed player perfectly learns the Pareto-optimal behavior and then changes gradually to a situation where the uninformed player remains without any knowledge about the state of the world at very high costs of transmission of information. In terms of communication costs, players would not incur any cost of transmission at zero and very high $w$. At intermediate levels, however, they would use the channel for transmission of information and the expected cost of communication would then be strictly positive.

## 3.6 Conclusions

It is shown that choosing a message (and an equilibrium) is a trade-off between how efficient the strategy is in the underlying game and how easily described it is.

If communication is costly, players will coordinate in a Nash equilibrium where the sequence of actions has a short description. The equilibrium seems simple to the players, since it is obtained with a description occurring in a wide range of games, including the least complex coordination problems (with few strict Nash equilibria). In this way, the observed equilibrium behavior is: (i) easy to describe since the code-string attached to the strategy is short, and (ii) simple because it replicates the behavior in a much less complex decision problem. Thus, we expect team-behavior to be highly regular if players communicate in a structured and costly way.

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# 4

# Strategic Investment in Multi-Market Games

## 4.1 Introduction

Established firms can restrict or prevent competition, due to first-mover advantages.[1] Despite the fact that most industrialized countries have regulations against monopolization, recent empirical evidence suggests that entry deterrence is common business practice.[2]

Following Spence (1977) and Dixit (1980), this paper considers a model where established firms can invest in capacity to the extent that entry by other firms is deterred. While previous studies have characterized entry deterrence in a single market, this paper analyzes entry deterrence in a multi-market game.

The crucial condition for strategic entry deterrence is that the incumbent can make early decisions, in order to restrict its future freedom of action. While this might be possible in the single-market game, the conditions may change when firms compete in many markets. Even if the cost of capacity is sunk, the multi-market incumbent can redistribute some of its capacity from markets with competition, to markets without. Thus, the firm maintains some degrees of freedom when acting in more than one market.

This paper is therefore based on two sets of questions: What is the scope for an incumbent to exploit its first-mover advantage in a multi-market game? Does an incumbent firm have an incentive to make a commitment

[1] Strategic variables considered in the literature on entry deterrence include price (Bain , 1996, Sylos-Labini, 1962, Gaskins, 1971, Kamien and Schwartz, 1971, Matthews and Mirman, 1983), cost (Smiley and Ravid, 1983, Spence, 1981), patent policy (Gilbert and Newberry, 1982), product variety (Schmalensee, 1978), advertising (Comanor and Wilson, 1967) and capacity (Spence, 1977, Dixit, 1980, Gelman and Salop, 1983, Allen, 1993).

[2] For a summary of different features of national competition laws in industrial countries, see OECD (1996), and for empirical evidence on strategic entry deterrence, see Smiley (1988), Bunch and Smiley (1992), and Allen et al (1995). It should be noted, however, that American case law has placed a heavy burden on plaintiffs to prove that a capacity expansion is clearly meant to hurt competitors and harm competition, which would be the case if such conduct were to be considered illegal (see Dobson et al, 1994).

to a specific market, in order to prevent competition in that market, even if such commitment is costly?

If multi-market competition facilitates entry-deterrence, it should be expected that integrated markets are more concentrated than segmented markets. On the other hand, if the opposite holds and multi-market competition obstructs the incumbent's possibilities to restrict competition, integrated markets should be expected to be less concentrated. Hence, the issue of market-linkages is important for any theory of market integration.

This paper considers a market situation described as a multi-stage game, where the incumbent first selects a global capacity, then competes with a number of entrants determined at the local level. In this respect, this model differs from most previous studies of multi-market interaction, where it is often assumed that firms are allowed to make decisions at the multi-market level exclusively, referred to as the integrated market hypothesis, or at the local level, referred to as the segmented market hypothesis.[3]

In the model presented in this paper, each firm is assumed to exhibit a symmetric Leontief technology with a fixed unit-cost of production. Furthermore, demand is considered to be independent between markets and firms compete in strategic substitutes in the last stage of the game. The incumbent firm is free to redistribute its global capacity between different markets. Hence, there is a strategic link between different markets.[4]

The possibility to redistribute global capacity between markets makes entry-deterrence more difficult and more costly than in a single-market game. To deter entry, the multi-market firm must install capacity beyond the level required in a single-market game.[5] Interestingly, the per-market

[3] Venables (1990) and Ben-Zvi and Helpman (1992) are two exceptions. In their models, capacity decisions are made on an integrated basis and other decisions, e.g. price and sales decisions, on a national basis. The model in this paper closely resembles Venables' as well as Ben-Zvi and Helpman's models in its attempt to analyze the importance of investment when capacity can be used on a multi-market level, while sales decisions are taken on a local basis.

[4] See Witteloostuijn and Wegberg (1992), for an extensive summary on multi-market competition models where existing firms are potential entrants. In particular, Bulow, Geanakopolos and Klemperer (1985) present a multi-market model relating to our analysis. They study a multi-market game where two firms compete in one market, but where one of the firms is a monopolist in a second market. If the two markets exhibit joint economies, then a positive shock in one market has positive effects on entry deterrence in the other market, provided that the products are strategic substitutes or strategic complements. In our model, however, the unit-cost is fixed and Bulow, Geanakopolos and Klemperer's analysis does not apply.

[5] This paper is not concerned with the relative profitability of entry deterrence and accomodation. In Ganslandt (1997), it has been shown that entry deterrence is profitable, if sufficient conditions are satisfied.

capacity installed to deter entry can be strictly larger than the largest subgame-perfect investment in the single-market game. However, no capacity will be left idle in equilibrium.[6]

In an extension of the model, it is demonstrated that the results also hold for strategic complements, if sufficient conditions apply. It is concluded that in many reasonable cases, the incumbent is obliged to install extra capacity in order to deter entry in the multi-market game.

If the capacity that would deter entry is beyond the monopoly output, the multi-market incumbent has an incentive to induce market segmentation. In particular, the incumbent may induce market segmentation through bundling of products and services. Firms can bundle their tradable products with locally produced and consumed nontradables. If the product cannot be used without local services, the capacity is assigned to the local market, provided that the marginal cost of expanding the local capacity of services in other markets is sufficiently high. In this respect, these results relate to Horn and Shy (1996), where market segmentation is endogenously determined through bundling of tradables with nontradables.

The paper is organized as follows. Section 2 introduces four versions of the multi-market game. Section 3 is devoted to the first version of the game, which is similar to Selten's (1978) chain store game. In this version, a multi-market firm competes sequentially with several potential entrants in distinct markets. Section 4 studies the second version of the multi-market game, where the incumbent competes with $n$ firms simultaneously, after the capacity choice has been made. Section 5 deals with the third version, where the multi-market firm competes with a second large player, which is a potential entrant in all $n$ markets. Section 6 introduces market commitments and analyzes under what circumstances the incumbent will serve markets from a single multi-market plant as opposed to many local plants. Section 7 shows that our main result holds if firms compete in strategic complements, if sufficient conditions apply. Section 8 illustrates three applications and section 9 concludes.

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[6] It should be noted that these results do generally not hold. In a similar two-firm, two-stage game with iso-elastic demand, the incumbent will hold excess capacity which is idle and will be utilized only in the event of entry. This result is easily shown in a simple model, originally set up by Bulow, Geanakopolos and Klemperer (1985). Similar results with multiple incumbent firms are shown by Barham and Ware (1993).

## 4.2 Multi-Market Entry Deterrence

Four versions of a multi-market game are considered. In the first three versions, production capacity is assumed to be used at the multi-market level. The incumbent is not allowed to assign parts of its total capacity to local markets. Instead, capacity can be redistributed between different markets, without additional costs. The first three cases differ with respect to potential competition and timing.

In the first version of the multi-market game, analyzed in section 3, an incumbent meets sequential competition from local entrants. The sequential structure is plausible when firms independently try to specify a certain product. They consider entry as soon as the product specification is correct and they have raised enough money for local production. This first happens to firm one, then to firm two etc. In this first version of the game, it is assumed that potential entrants only consider local entry. One rational for this assumption is that the firm has to succeed in its domestic market before it can raise money for multi–market expansion.

In the second version of the multi-market game, analyzed in section 4, the incumbent faces simultaneous competition from local entrants. The simultaneous structure arises when the incumbent owns a global patent expiring at the same time in all local markets. In this case, local competitors already have a correct specification of the product. As soon as the patent expires, they immediately consider entry in the local market. In the second version of the game, the assumption that potential competitors only consider local entry is maintained.

In the third version of the multi-market game, analyzed in section 5, the incumbent faces simultaneous competition from a single multi-market competitor in all markets. This market structure is plausible if the first competitor to finish the process of product specification immediately considers a multi-market strategy, or if a global patent expires in all markets simultaneously and the potential entrant can raise enough money for multi-market entry.

After the analysis of the first three versions of the multi-market game, the assumptions about the incumbent's possibilities to restrict competition are changed. In the fourth version of the game, analyzed in section 6, the incumbent is allowed to assign parts of its capacity to local markets. The choice of a certain production organization is a trade-off between the cost of entry-deterrence with the multi-market capacity and the cost of market assignments.

## 4.3 Sequential Competition from Local Entrants

A multi-market firm, type $m$, has advertised its product and now meets demand for its product in $n$ markets, numbered 1 to $n$. In each market, there is a potential entrant, type $e$, who might raise enough funding from creditors to establish a firm in market $t$, selling the same product as the multi-market enterprise.

Entry in a local market is associated with a fixed cost $A$, which can be considered an advertising cost, that makes consumers in the local market aware of the entrant. Advertising makes all consumers in the market aware of the firm and its products, but does not affect aggregate demand for the homogenous goods. There is no personal arbitrage, since consumers are only aware of firms advertising in their home market. Accordingly, prices need not be internationally equalized.

In the first version of the multi-market game, we focus on a situation where each potential competitor considers advertising in a single market only and, consequently, intends to remain local. At the beginning of the game none of the potential entrants has a sufficiently correct specification for starting production. But as time passes, one after another, they finish the process of specification and raise enough credit to enter the local market. This will first happen to entrant 1, then to entrant 2, etc. As soon as a player has specified the product correctly, he must decide to enter or stay out of the market. If he decides to stay out, he is no longer a potential competitor.[7] If a local firm enters a market, the incumbent and the entrant choose outputs simultaneously and the market clears as a duopoly. If the potential entrant stays out, monopoly will prevail.

After this description of the market situation in the first version of the multi-market game, we turn to a formal specification of the model. The game, $\Gamma_n^1$, has $n+1$ players, player $m$ and player $1, ..., n$ $(n \geq 1)$. There are $n$ separate markets, labelled $1, .., n$. The game is played over a sequence of periods $0, ..., n$. In period 0, the incumbent, player $m$, must choose a pre-entry capacity $k$, which is immediately announced to all players. At the beginning of period $t = 1, ..., n$, player $t$ decides to enter or stay out of market $t$. Player $t$'s decision is announced to all players. If player $t$ decides to enter, player $m$ and player $t$ will choose $x_t^m$ and $x_t^e$ simultaneously, where subscripts refer to markets and superscripts to firm-type. If player $t$ decides to stay out of market $t$ monopoly will prevail in that market. The output decision is immediately announced to all players. At the end of period $t$, the

[7] This assumption is made to simplify the analysis. It is not restrictive. Indeed, it can be shown that a potential entrant will not benefit from delaying its entry decision.

market clears and payoffs are distributed to player $m$ and player $t$. Next, for $t = 1, ..., n-1$ period $t+1$ begins and is played according to the same rules. The game ends after period $n$.

Player $m$'s payoff is the sum of $n$ partial payoffs for $t = 1, ..., n$. Player $m$'s revenue in market $t$ is $v\left(x_t^m, x_t^e\right)$. The cost of capital is additive and the marginal cost is $c > 0$. The objective of player $m$ is to maximize its total payoff:

$$\pi^m\left(k, x_1^m, .., x_n^m, x_1^e, .., x_n^e\right) = \sum_{t=1,..,n} v\left(x_t^m, x_t^e\right) - ck \tag{4.1}$$

and it is required that $x_1^m + .. + x_n^m \leq k$. Setting up a firm, i.e. entering market $t$, is associated with a fixed cost $A > 0$ for player $t$. Player $t$'s revenue is $v\left(x_t^e, x_t^m\right)$. Marginal capital cost is $c > 0$ and additive. The objective of player $t = 1, ..., n$ is to maximize its payoff:

$$\pi^e\left(x_t^e, x_t^m\right) = \begin{cases} v\left(x_t^e, x_t^m\right) - cx_t^e - A & \text{if it enters} \\ 0 & \text{if it stays out} \end{cases} \tag{4.2}$$

Next, we introduce some notation before proceeding with the analysis. I will define strategic substitutes, introduce a necessary and sufficient condition on entry-deterrence and define the deterrence level.

We shall call $x_t^i$ a *strategic substitute* for $x_t^j$, if the partial cross-derivative of the profit function with respect to the strategic variables is strictly negative. Strategic substitutes imply that when a firm has a more aggressive strategy, the optimal response of the other firm is to play *less* aggressively. The condition that $x_t^i$ is a strategic substitute for $x_t^j$ is referred to as S:

$$\text{(S)} \qquad \frac{\partial^2 \pi\left(x_t^i, x_t^j\right)}{\partial x_t^i \partial x_t^j} < 0$$

Second, a best-reply function with a non-binding capacity restriction on player $m$ in the one-period game $\Gamma_1^1$, denoted $\beta^m\left(x_1^e\right)$, is introduced. Correspondingly, the entrant's best reply function is denoted $\beta^e\left(x_1^m\right)$. The best reply functions $\beta^m\left(x_1^e\right)$ and $\beta^e\left(x_1^m\right)$ are implicitly defined by

$$\frac{\partial v\left(\beta^m\left(x_1^e\right), x_1^e\right)}{\partial x_1^m} = 0, \quad \frac{\partial v\left(x_1^m, \beta^e\left(x_1^m\right)\right)}{\partial x_1^e} - c = 0 \tag{4.3}$$

If a potential competitor decides to enter in period 1, this gives the following Nash equilibrium, when the capacity constraint is non-binding for the incumbent: $\{\overline{x}_1^m, \overline{x}_1^e\}$, where $\overline{x}_1^m = \beta^m\left(\overline{x}_1^e\right)$, $\overline{x}_1^e = \beta^e\left(\overline{x}_1^m\right)$. If $k \leq \overline{x}_1^m$, the incumbent will use the entire capacity, but with $k > \overline{x}_1^m$ some capacity will be left idle. The unique Nash equilibrium in the subgame with entry is $\{\widehat{x}_1^m(k), \widehat{x}_1^e\}$ where $\widehat{x}_1^m(k) = \min\{k, \overline{x}_1^m\}$ and $\widehat{x}_1^e = \beta^e(\widehat{x}_1^m(k))$.

In the second subgame in the second stage, with no entry, we obtain the following Nash equilibrium, when the capacity constraint is non-binding for the incumbent: $\{\overline{\overline{x}}_1^m, 0\}$, where $\overline{\overline{x}}_1^m = \beta^m(0)$. Thus, $\overline{\overline{x}}_1^m$ is the monopoly level the incumbent would choose, if the cost of capacity was sunk and capacity did not restrict output.

When the firms compete in strategic substitutes, the potential entrant's profit is decreasing in the incumbent's output. However, the incumbent does not choose an output above the limit $\overline{x}_1^m$, if the potential competitor enters the local market. Thus, under condition (S), it is a necessary condition for entry deterrence that the profit of the potential entrant is non-positive in a Nash equilibrium with a non-binding capacity restriction for the incumbent. This condition will be denoted D:

$$\text{(D)} \qquad v(\overline{x}_t^e, \overline{x}_t^m) - c\overline{x}_t^e - A \leq 0$$

If the necessary deterrence condition D is satisfied, condition S is a sufficient condition for entry deterrence. However, it can easily be shown that S is not a necessary condition for the result. In particular, the result can hold, even if the strategic variables are strategic complements.

If D is satisfied and player $t$ would earn a positive profit as a monopoly it follows from the Theorem of Intermediate Values that the profit of the entrant must be equal to zero at some positive level of output by the incumbent. This *deterrence level* will be denoted $\widetilde{x}$ and defined:

$$\pi(\beta^e(\widetilde{x}), \widetilde{x}) - c\beta^e(\widetilde{x}) - A = 0 \tag{4.4}$$

Thus, if the established firm successfully commits to an output $\widetilde{x}$, it deters entry. It is also assumed that $\widetilde{x}$ is above the output level of a natural monopoly. In other words, the entry-deterring incumbent in our model is operating beyond the scale of operation it would choose, if it did not face potential entry.

Next, three results from the first version of the multi-market game, $\Gamma_n^1$, can be shown. First, D is a sufficient condition on entry deterrence in the multi-market game. Second, if firms compete in strategic substitutes, then D is not only a sufficient, but a necessary, condition for entry deterrence. Third, if both conditions S and D are satisfied, the incumbent installs strictly more than $n \cdot \widetilde{x}$ to deter entry in $\Gamma_n^1$.

If D is satisfied the local entrant does not earn a positive profit in $\{\overline{x}_t^m, \overline{x}_t^e\}$, and would thus stay out of the local market. To see that D is a sufficient condition for entry deterrence, assume that the incumbent has installed more capacity in period 0 than he will ever use. Thus, every market can be treated independently and the unique Nash equilibrium in every market $t$ is $\{\overline{x}_t^m, \overline{x}_t^e\}$, and entry deterrence is thus possible.

**Proposition 1** *If D is satisfied, then entry deterrence is possible in* $\Gamma_n^1$.

**Proof.** ($D \Rightarrow$ entry deterrence is possible). Let the pre-commitment capacity be very large. The capacity constraint is not binding in any subgame. The objective of the incumbent is to maximize its profit with respect to $x_t^m$, for all $t$,

$$\frac{\partial v\left(x_t^m, x_t^e\right)}{\partial x_t^m} = 0 \qquad \forall t. \tag{4.5}$$

This problem is additively independent and each market can be considered as a separate one-market game $\Gamma_1^1$. If the capacity constraint is not binding, the unique Nash equilibrium with entry is $\left\{\overline{x}_t^m, \overline{x}^t\right\}$, where $\overline{x}_t^m = \beta^0\left(\overline{x}^t\right)$, $\overline{x}^t = \beta^t\left(\overline{x}_t^m\right)$. Since $v\left(\overline{x}^t, \overline{x}_t^m\right) - c\overline{x}^t - A \leq 0$, player $t$ will choose to stay out and monopoly prevails. ■

Next, we will show that, the deterrence condition (D) is not only a sufficient, but also a necessary condition on entry deterrence, if firms compete in strategic substitutes. Strategic substitutes (S) imply that the profit of a potential entrant is monotonically decreasing in the incumbent output. If $k$ does not restrict output, then $\left\{\overline{x}_t^m, \overline{x}^t\right\}$ is the unique Nash equilibrium with entry in market $t$. Furthermore, $\overline{x}_t^m$ is the highest output the incumbent will select with any capacity $k$. Hence, if the potential competitor earns a positive profit in $\left\{\overline{x}_t^m, \overline{x}^t\right\}$, the same will hold in any Nash equilibrium in the post-entry game. Thus it enters market $t$ and entry deterrence is not possible.

**Proposition 2** *If condition S is satisfied and condition D is violated, then entry deterrence is not possible in* $\Gamma_n^1$.

**Proof.** ($S$ and $\neg D \Rightarrow \neg$ entry deterrence). First, note that $\overline{x}_t^m$ is player $m$'s highest output level in a subgame with entry in market $t$. From (S), $\pi\left(x_t^e, x_t^m\right)$ is monotonically decreasing in $x_t^m$ and reaches its minimum at $\overline{x}_t^m$. If $\neg D$, i.e. $v\left(\overline{x}^t, \overline{x}_t^m\right) - c\overline{x}^t - A > 0$, player $t$ could ensure a positive profit, if entering market $t$. ■

After these two qualitative results, a more precise result can be established, characterizing the disadvantage of multi-market competition on entry-deterrence. If firms compete in strategic substitutes and the necessary deterrence condition is satisfied, the incumbent must install $k > n\widetilde{x}$ to deter entry in the $n$–market game $\Gamma_n^1$.

Consider for instance the two-market game. Why is twice the deterrence level, $\widetilde{x}$, not enough to deter entry in two markets? The main reason is that if one potential competitor enters and the other stays out, the incumbent has an incentive to redistribute capacity to the monopoly market.

In the last period, the remaining capacity is $k - x_1^m$. If condition D is satisfied, $k - x_1^m \geq \widetilde{x}$ will deter entry. Working backwards to period 1, there are two subgames. If player 1 stays out, the incumbent will split the capacity equally in both markets. If the potential competitor enters, the marginal incentive to use capacity in market 1 and 2 must be equal:

$$\frac{\partial \pi}{\partial x_1^m} \left(x_1^m, \beta^e \left(x_1^m\right)\right) = \frac{\partial \pi}{\partial x_2^m} \left(k - x_1^m, 0\right) \tag{4.6}$$

It follows from strategic substitutes that $k - x_1^m > x_1^m$. Thus, if $k = 2\widetilde{x}$, then $x_1^m < \widetilde{x}$ and entry is not deterred in the first market. More specifically,

**Proposition 3** *If D and S are satisfied in the first version of the n-market game, $\Gamma_n^1$, then the multi-market incumbent installs capacity $n\widetilde{x} < \overline{k}_n^1 \leq \widetilde{x} + (n-1)\overline{\overline{x}}^0$ to deter entry.*

**Proof.** Appendix A ■

## 4.4 Simultaneous Competition from Local Entrants

Consider a market situation similar to the first version of the multi-market game. In this version, the incumbent owns a global patent expiring at the same time in all markets and potential competitors can enter the local markets simultaneously. If a potential competitor challenges the established firm in a local market, the incumbent and the entrant choose outputs simultaneously and the market will clear as duopoly. If the potential entrant stays out, monopoly will prevail.

The rules of the second version of the multi-market game are defined as follows. The game, $\Gamma_n^2$, has $n+1$ players, player $m$ and player $1, ..., n$ $(n \geq 1)$. The game is played over two periods. In the first period, the incumbent must choose a pre-entry capacity, $k$. At the beginning of the second period, player $t = 1, ..., n$ must simultaneously decide to enter or stay out of market $t$. Player $t$'s decision is immediately announced to all other players. If player $t$ decides to enter market $t$, then the incumbent and the entrant choose $x_t^m$ and $x_t^e$ simultaneously. At the end of the second period, all markets clear and payoffs are distributed to the incumbent and players $1, ..., n$. Player $m$'s payoff is given by eq. (4.1) and player $t$'s payoff by eq. (4.2).

The analysis in the second version of the multi-market game is similar to the analysis in the first version. If players compete in strategic substitutes and the necessary deterrence condition is satisfied, entry can also be deterred in the second version of the game. To deter entry, the established firm must install $k > n\widetilde{x}$ in the $n$–market game.

Consider, for instance, the two-market case. There are four subgames in the last stage of the two-market game. In two of the four subgames, one potential competitor enters, and the other stays out. To see why twice the single market deterrence capacity does not suffice, consider the profit maximizing conditions when $k = 2\widetilde{x}$:

$$\frac{\partial \pi}{\partial x_1^m}\left(x_1^m, \beta^e\left(x_1^m\right)\right) = \frac{\partial \pi}{\partial x_2^m}\left(2\widetilde{x} - x_1^m, 0\right) \tag{4.7}$$

Strategic substitutes imply that the output in the duopoly market is strictly lower than the deterrence level, i.e. $x_1^m < \widetilde{x}$. Thus, entry would not be deterred.

**Proposition 4** *If $D$ and $S$ are satisfied in the second version of the $n$-market game, $\Gamma_n^2$, then the incumbent installs capacity $n\widetilde{x} < \overline{k}_n^2 \leq \widetilde{x} + (n-1)\overline{\overline{x}}^0$ to deter entry.*

**Proof.** Appendix B ■

The first and the second version of the multi-market game differ in one important respect. If the incumbent installed enough capacity to deter simultaneous entry by all potential competitors but not enough to deter unilateral entry by one potential competitor, then the potential entrants would face a coordination problem in the second version of the game. This coordination problem does not occur in the first version where player 1 enters and player 2 stays out. In the second version, both potential competitors wish to enter if they are the only entrant, but not otherwise. [8]

The coordination problem in the second version of the game remains unsolved, since both Nash equilibria are strict. This problem will not be further dealt with, since we are mainly interested in the conditions on entry deterrence. In a real market situation, however, the coordination problem may affect the entrants' decisions and, possibly, facilitate entry-deterrence.

## 4.5 Competition from a Multi-Market Entrant

Once more, a multi-market firm has advertised and meets demand for its product in $n$ markets. In the third version of the multi-market game, a

[8] This is a version of the "chicken" game.

single potential competitor, another multi-market company, considers entry in all markets selling the same product as the established firm. The incumbent's global patent expires at the same time in all markets and the potential competitor may enter all local markets simultaneously. Entry in each market is associated with a fixed sunk cost, which can be considered an advertising cost. The multi-market entrant remains unknown in all markets where it does not advertise. If the second multi-market firm enters the local market, the incumbent and the entrant choose their output simultaneously and the market will clear as a duopoly.

The rules of the third version of the game are defined as follows. The game, $\Gamma_n^3$, has two players, called player $m$ and player $e$. The game is played over a sequence of two periods. In the first period, the established firm must choose a pre-entry capacity, $k$. At the beginning of the second period, the potential competitor must decide to enter or stay out in $n$ separate markets called $t = 1, ..., n$. Player $e$'s decision is immediately announced to player $m$. If player $e$ decides to enter market $t$, the players will choose $x_t^m$ and $x_t^e$ simultaneously. If player $e$ decides to stay out, monopoly will prevail in that market. At the end of the second period, all markets clear and payoffs are distributed to player $m$ and player $e$.

The incumbent's payoff is given by eq. (4.1). Entry in market $t$ is associated with a market-specific fixed cost $A > 0$ for player $e$. Let $E$ be the set of all markets that player $e$ will enter. Player $e$'s partial revenue, in a market it enters, is $v(x_t^e, x_t^m)$. The per-unit capital cost is $c > 0$. The objective of player $e$ is to maximize its total payoff:

$$\pi^e(x_1^e, .., x^n, x_1^m, .., x_n^m) = \sum_{t \in E} (v(x_t^e, x_t^m) - cx_t^e - A) \qquad (4.8)$$

Inequality D is also a sufficient condition on entry deterrence in the third version of the multi-market game. If the incumbent invests in a sufficiently large capacity, which makes the capacity constraint non-binding in every subgame, the optimal output in every market can be independently determined. The potential competitor chooses its optimal strategy in each market separately, and the best reply functions in all markets are identical. The unique Nash-equilibrium output in every market is $\{\overline{x}_t^m, \overline{x}_e^t\}$. Thus, player $e$'s partial revenue does not cover the fixed and variable costs in any market and the total payoff is negative.

In fact, the strategic interaction in the second and third versions of the multi-market game is identical, except for the coordination problem in the second version of the game. Two factors make the strategic decisions in the two games identical with respect to entry deterrence. First, the strategic variables $x_1^e, ..., x_n^e$ are independent to the entrant in the third version of the multi-market game and it will choose its optimal strategy in each market separately. Thus, player $e$'s best reply function in market $t$ is identical to player $t$'s best reply function in the second version of the multi-market game.

Second, since the fixed cost $A$ is the same in all markets, the revenue in each market the potential competitor enters must cover the variable and fixed costs. Player $e$ would only enter a market where the expected payoff is positive, which exactly resembles the condition on entry for a local competitor in $\Gamma_n^2$. The analysis of the second version of the game therefore also applies to the third version. Player $m$ must install $k > n\widetilde{x}$ to deter entry in the $n$–market game $\Gamma_n^3$.

**Proposition 5** *If $D$ and $S$ are satisfied in the third version of the two-market game, $\Gamma_n^3$, then the incumbent installs capacity $n\widetilde{x} < \overline{k}_n^3 \leq \widetilde{x} + (n-1)\overline{\overline{x}}^m$ to deter entry.*

**Proof.** Appendix B ■

In the previous sections, the difficulties of entry deterrence in the first, second and third versions of the multi-market game have been characterized. It takes more capacity than $n$ times the deterrence level $\widetilde{x}$ to deter entry of many potential competitors in a sequential or simultaneous market structure. More specifically, the established firm installs exactly the same capacity to deter entry in $\Gamma_n^1$, $\Gamma_n^2$ and $\Gamma_n^3$. Thus, the unique optimal deterring capacity is independent of the market situation, as described in the first, second and third versions of the multi-market game.

**Proposition 6** *If conditions $D$ and $S$ are satisfied, the global capacity required to deter entry in the $n$-market game is independent of the timing of the game, i.e. sequential or simultaneous entry of potential competitors, and the size of the potential entrant.*

**Proof.** Appendix C. ■

This proposition is interesting for two reasons. First, it might be difficult for the incumbent to obtain information about potential entrants *ex ante,* but our results suggest that such information might not be necessary. The result implies that an incumbent does not need information about the timing and the number of potential entrants to determine its entry-deterring strategy. The results of the model apply to several different situations, for example both to a situation with one large competitor and to a situation with competition from a series of local competitors.

Not surprisingly, it also follows that the difference between the single-market game and the multi-market game increases with the number of markets in the multi-market game.

If condition D holds with equality, the entry-deterring capacity per market in an $n$-market game increases in the number of markets and converges to $\overline{\overline{x}}^m$, as $n$ goes to infinity.

The intuition for this result is that unilateral entry in a single market is harder to deter as partial exit to the remaining $n-1$ markets becomes increasingly attractive. As the number of monopoly-markets increases, the alternative to fight entry in a single market looks less and less attractive, in comparison to using the capacity in the remaining monopoly markets. It should, however, be noted that per-market profits are less affected by unilateral entry in a single market, if the number of markets is large.

## 4.6 Market Commitments

In this section, I extend the analysis and let the incumbent first determine the organization of its production, either with a global capacity, referred to as the global strategy, or with a combination of a global capacity and local capacities that can be used in specific markets only, referred to as the local strategy. The local strategy can be regarded as a vertically integrated production process, where the production process is split into two vertical stages. It will be shown that if sufficient conditions apply, then local capacities can be assigned to local markets and successfully deter entry.

We study a three-stage game similar to the two-stage game in the previous sections. In the first stage, the multi-market firm can choose a global or a local strategy. The local strategy, i.e. assigning a local capacity to each local market, is associated with an extra fixed cost $G$ in each market.

We can now describe the rules of the fourth version of the game. The game, $\Gamma_n^4$, has two players, player $m$ and player $e$. The game is played over a sequence of three stages. In the first stage, the incumbent must begin by choosing a local or global strategy. In the second stage, the incumbent must choose local capacities in each market, $k_t$, and a multi-market capacity, $k$. Unlike the global capacity, it is assumed that local capacities can be increased in the third stage. All decisions of the established firm is immediately announced to the potential competitor. At the beginning of the third stage, player $e$ must decide to enter or stay out in $n$ separate markets called $t = 1, ..., n$. Player $e$'s decision is announced to the incumbent. If player $e$ decides to enter market $t$, player $m$ and player $e$ will choose $x_t^m$ and $x_t^e$ simultaneously. Finally, all markets clear and payoffs are distributed to player $m$ and player $e$.

If the incumbent chooses a local strategy, the unit-cost of local capacity is $c_1 > 0$, and the unit-cost of multi-market capacity is $c_2 > 0$. Moreover, each local assignment is associated with a fixed cost $G > 0$. If the incumbent chooses a global strategy, the cost of capacity is $c$. For simplicity, we assume that the total unit-cost is independent of the strategy, i.e. $c_1 + c_2 = c$. The incumbent's payoff is given by:

$$\pi^m (x_t^m, x_t^e) = \begin{cases} \sum_{t=1}^{n} v(x_t^m, x_t^e) - ck & \text{global} \\ \sum_{t=1}^{n} [v(x_t^m, x_t^e) - c_1 q_t] + c_2 k - nG & \text{local} \end{cases} \tag{4.9}$$

where $q_t = \max\{x_t^m, k_t\}$. The potential competitor must incur a market-specific fixed cost $A > 0$ to enter market $t$. Let $E$ be the set of all markets that player $e$ will enter. Player $e$'s revenue is $v(x_t^e, x_t^m)$. The marginal capital cost is $c > 0$ and additive. The objective of player $e$ is to maximize its payoff given by eq. (4.8).

We shall call $k_t$ a *market commitment,* if this part of the total capacity in a multi-market firm is assigned to market $t$ and cannot profitably be used for production of goods sold in other local markets. A sufficient condition for market commitments is that the marginal cost to increase local capacity is larger than the marginal incentive to increase the output in a monopoly market at the deterring level $\widetilde{x}$. We refer to this condition as (C). More precisely,

$$\text{(C)} \qquad c_1 > \frac{\partial v}{\partial x_t^m} (\widetilde{x}, 0)$$

Condition C simply guarantees that it is not profitable for player $m$ to redistribute capacity to a monopoly market, if entry occurs in other markets. If condition C is satisfied and condition D is satisfied with equality, it is sufficient for player $m$ to install a local capacity equal to the deterrence level $k_t = \widetilde{x}$ and a multi-market capacity $k = n\widetilde{x}$, to deter entry.

**Proposition 7** *If conditions C, D and S are satisfied in the fourth version of the n-market game, $\Gamma_n^4$, local capacities $\overline{k}_t = \widetilde{x}$ and global capacity $\overline{k}_n^4 = n\widetilde{x}$ is sufficient to deter entry.*

**Proof.** Entry deterrence is possible in $\Gamma_n^4$, due to (D). Player $m$ will choose a local strategy and installs capacity $\overline{k}_n^4 = n\widetilde{x}$ and $\overline{k}_t = \widetilde{x}$ for $t = 1, .., n$. If player $e$ enters all markets, symmetric incentives imply that $x_t^m = \widetilde{x}$ and D implies that the profit of player $e$ is not positive. If player $e$ enters one market (w.l.o.g. market 1) and stays out of all other markets, the following inequality must hold for the incumbent to deter entry

$$\frac{\partial v}{\partial x_1^m} (\widetilde{x}, \beta^e(\widetilde{x})) + \left( c_1 - \frac{\partial v}{\partial x_t^m} (\widetilde{x}, 0) \right) \geq 0 \tag{4.10}$$

for $t = 2, ..., n$. The first part of the LHS is equal to zero and from (C), the second part is positive. Thus the inequality holds. Equal parts of the total capacity should be assigned to each market, i.e. $\overline{k}_n^4/n$. $\widetilde{x}$ deters entry in market $t$, hence $n\widetilde{x}$ is enough to deter entry in all markets. ■

The incumbent installs strictly less capacity with market commitments compared to the capacity needed to deter entry, if the capacity is not assigned to specific markets. The difference in the established firm's profit, if C is satisfied in $\Gamma_n^4$, between the local and the global strategy is called the commitment premium, denoted $\Delta\pi$. Working backwards, the multi-market firm will choose a local strategy if the commitment premium minus the cost of assignment is positive.

**Proposition 8** *If C is satisfied in* $\Gamma_n^4$ *the multi-market firm will choose a local strategy to deter entry i.f.f.* $\Delta\pi - nG > 0$.

**Proof.** Follows immediately from the definition of the commitment premium and the cost of a local strategy. ■

It follows from this proposition that a local strategy is more likely, the lower the assignment cost. Thus, the organization of production within the multi-market firm is primarily determined by the relationship between economies of scale at the local level and the commitment premium.

Another important issue is what factors determine the incumbent's opportunities to make market commitments. These factors can be exogenous, e.g. different national standards or trade regulations. A more interesting case, however, is when the incumbent chooses to induce market segmentation endogenously.

First, firms can bundle their tradable products with locally produced and consumed nontradables, e.g. services. If the product cannot be used without local services, the capacity is assigned to the local market provided that the marginal cost to expand the service capacity is sufficiently high. In this case, a global strategy would correspond to the manufacturing of a sophisticated product, which can be used without services. A local strategy, on the other hand, would be to produce a less sophisticated product which must be consumed with some local support or services.

Second, strategic market segmentation can occur in a horizontally differentiated product space.[9] If consumers in the local markets have preferences for local products, capacities can be assigned to the domestic market. The local strategy is manufacturing of goods adapted to local preferences, i.e. products which can be used by consumers in a specific market only, and the

[9] This case is analyzed in detail in a parametric model presented in Ganslandt (1996).

global strategy is production of a standardized good, which can be used by consumers in all markets. If the cost of adjusting the adapted products in the post-entry game is sufficiently high, the local strategy can successfully deter entry.

Third, market commitment can be induced by network lock-ins. The producer can introduce local standards, which assign capacities to a specific market. In this case, a global strategy is a standard common to all markets.

Thus, the model of endogenously determined multi-market production potentially applies to many different market conditions.

## 4.7 Price Competition in Differentiated Goods

Having shown that multi-market competition obstructs the incumbent's possibilities to deter entry if firms compete in strategic substitutes, we will now show that strategic complements give the same result, if sufficient conditions apply.

An incumbent commit to a global capacity for two markets in the first stage. A potential entrant in each market, called player $t$, observes the incumbent's capacity and then chooses to enter or stay out. If player $t$ enters market $t$, the incumbent and the entrant both choose prices for their respective variety of the differentiated good.

We use the Shubik (1980) system of demand functions where the demand for variety $i$ in market $t$ is given by

$$x_t^i = \frac{1}{n}\left[a - b\left(p_t^i + g\left(p_t^i - \overline{p}_t\right)\right)\right], \tag{4.11}$$

where $n$ is the total number of active firms in the local market, $\overline{p}_t$ is the average price in the local market and $g$ is a measure of substitutability between products. Assume that the parameters of the model satisfy some restrictions, $a \geq b \geq c$, and that the degree of substitutability is not too large, $g \leq 2$.

Consider a situation where entry deterrence is possible in the single-market game and the entrant makes zero profit in a subgame with a non-binding capacity constraint for the incumbent. It can be shown that twice the capacity needed to deter entry in a single market game does not suffice to deter entry in the multi-market game. For this purpose, let $k$ be exactly twice the capacity needed to deter entry in a single market game. Capacity $k/2$ in a market without entry results in a price which is strictly higher than the price the incumbent would set as a monopolist, if the capacity constraint was not binding. If unilateral entry in market 1 occurs, profit maximization under the binding capacity constraint requires

$$\frac{d\pi_1^m(p_1^m, p_1^e)}{dp_1^m} = \frac{d\pi_2^m(p_2^m)}{dp_2^m}. \tag{4.12}$$

It is not satisfied, however, if the capacity is evenly distributed between the markets. In this case, the RHS is strictly negative and the incumbent will increase its profit by setting a lower price in its monopoly market and move some productive capacity to this market.[10] Accordingly, the resulting price in market 1 is higher. But firms compete in strategic complements and a price increase by the incumbent is followed by a price increase by the entrant, which increases the profit of the entrant in equilibrium and, therefore, entry is not deterred. Hence, as in the case of strategic substitutes, the multi-market incumbent must install more capacity to deter entry in the multi-market game.

## 4.8 Applications

### *(i) Franchising and Strategic Delegation*

Franchising is a long-term vertical contract between a franchisor (the incumbent) and a franchisee. Through the contract, the franchisor collects revenues from a franchise fee as well as from the wholesale markup. The contract allows the incumbent to strategically design the terms of the contract in order to overcome its own incentives in the future.[11] Hadfield (1991) shows that in a model of horizontal product differentiation, strategically designed franchise contracts can deter entry.

Following Hadfield (1991), we can analyze market commitments through strategic delegation in our model. Consider a franchise contract which is a standard-form, long-term-duration contract designed by the incumbent and offered to potential franchisees. The contract consists of a franchise fee, $F$, a wholesale price scheme, $w(x_t)$, and an exclusive territory, $t$. The contract obliges a franchisee to sell the product to customers in its own market only, i.e. exporting the product to another territory is either prohibited or associated with an additional fee, $c$. The contract also specifies that violations of the contract are associated with damages, $V$.

The incumbent can then design a contract with the following terms; the wholesale price is zero up to a quantity equal to one $n$:th of the incumbent's global capacity and infinite thereafter, the exclusive territory is a local

[10] It can be shown that the equality is $(1/2)\,a-(1/2)\,(2+g)\,bp_1^m+(1/4)\,bgp_1^e = a-2bp_2^m$

[11] This idea of strategic delegation was first suggested by Schelling (1980).

market, $t$, and the franchise fee is the expected revenue for a monopolist in that market, minus the assignment cost, i.e. $F = v\,(k/n, 0) - G$.

Under this contract, the independent franchisee in each market has an incentive to produce and sell its full capacity and entry is successfully deterred. The market assignment cost is identical to the profit of the franchisee and it is determined by the relative bargaining power of the franchisee and the incumbent. Hence, the profitability of franchising for the manufacturing firm is determined by the outcome of the bargaining between the franchisor and the franchisees.

### *(ii) Strategic Investment and Multinational Production*

Multinational production and strategic foreign direct investment constitute another natural application of the model.[12]

Consider a modified version of the game. The incumbent firm has incurred the market-specific fixed costs and meet demand for its product in all markets. In the first stage, the incumbent has two options: either to concentrate production in a single plant, i.e. an export strategy, or to install local plants, i.e. a multinational strategy. If it is choosing the former strategy, the incumbent must choose a global pre-entry capacity, whereas, if it is choosing the latter strategy, the incumbent must choose a global capacity and local capacities assigned to each of the plants. In the second stage, a potential competitor considers entry in the local markets. If it enters, it must also decide whether to establish one or several plants.

In this game a multi-market incumbent can choose a multinational or export strategy to deter entry. The multinational strategy requires less total capacity, while the export strategy requires fewer plants. For some parameter values the multinational strategy is a more profitable strategy to deter entry, for other values the export strategy is more profitable.

However, if the firms must incur a firm-specific cost, $F$, as well as plant-specific costs, $G$, the current specification adds a new dimension to the problem. The firm-specific cost results in economies of scale at the firm level and the plant-specifc cost in economies of scale at the plant level. An entrant can use these assets in all markets, which makes single-market entry less profitable compared to multi-market entry. Hence, single-market

[12] In models with variable trade costs, Smith (1987) and Horstmann and Markusen (1987), show that an incumbent has an incentive to make a foreign direct investment to deter entry. Multinational production reduces variable costs and makes the incumbent more aggressive. A more aggressive play will reduce the revenues of potential entrants and, thus, entry is deterred. If monopoly rents outweigh any costs associated with installing an additional plant, the first-mover would choose this strategy.

entry can be a strictly dominated strategy. But this is not the case in all situations. If scale-economies at the firm and plant level are not too large, the potential entrant will consider single-market entry rather than multi-market entry. Ganslandt (1997) shows that multinational production is more likely if the plant-specific setup cost is low, market-specific costs are high and the total fixed cost contains a large share of market-specific costs. The intuition is that the multinational strategy becomes relatively less expensive for the incumbent and single-market entry becomes more attractive, for the potential entrant.

### *(iii) Mergers*

A conglomerate merger is a union of assets from two firms which were previously active in two separate markets.[13] Correspondingly, an international merger is a union of assets from two firms previously active in two distinct geographic markets. The multi-market model in this paper can be used for analyzing the effect of these types of mergers.

Consider a situation where two firms have separately entered two local markets and successfully deterred further entry. Each firm is active in one market only. Local production is associated with a fixed cost, $G$. If the firms choose to merge, they will reduce their fixed costs. If capacity can be used in all markets, the merged firm is obliged to install more capacity and expand its output to successfully deter entry in the post-merger equilibrium. If the firm cannot expand its capacity to deter entry, the result is local entry in one of the markets. In both cases, production is expanded and the monopoly distortion is reduced. Hence, the merger is clearly pro-competitive.

## 4.9 Conclusions

Multi-market competition without market commitment makes the incumbent's possibilities to exploit first-mover advantages more difficult. A firm's opportunity in one market influences its possibility to successfully commit to its optimal strategy in a second market. The incumbent must install a higher level of global capacity to successfully deter entry in all markets. If exogenous or endogenous factors allow the incumbent to assign parts of its capacity to local markets, multi-market production can be profitable, even under increasing returns to scale at the global level. The results suggest

[13] Scherer and Ross (1990) give some empirical evidence for this being the most common type of mergers in the US.

that local investments can be regarded as market commitments, in order to restrict or prevent competition in specific markets.

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# Appendix A. Sequential Entry

**Proof.** *Step 1.* Start in period $n$. Let the remaining capacity be $k_n = k_{n-1} - x^m_{n-1}$. There are two subgames; either player $n$ enters or stays out of market $n$. In the subgame with entry, the unique Nash equilibrium is $\{\widehat{x}^m_n(k_n), \widehat{x}^e_n\}$, where $\widehat{x}^m_n(k_n) = \min\{k_n, \overline{x}^m\}$ and $\widehat{x}^e_n = \beta^e(\widehat{x}^m_n(k_n))$. If player $n$ decides to stay out of market $n$, we have the following limit Nash equilibrium $\{\overline{\overline{x}}^m, 0\}$, where $\frac{\partial \pi}{\partial x^m_n}(\overline{\overline{x}}^m, 0) = 0$. The unique Nash equilibrium in the subgame with no entry is $\{\widehat{x}^m_n(k), 0\}$, where $\widehat{x}^m_n(k_n) = \min\{k_n, \overline{\overline{x}}^m\}$. From S, it follows that $\overline{\overline{x}}^m > \overline{x}^m$.

*Step 2.* Player $n$ would enter if $k_n < \widetilde{x}$ and stay out as long as $k_n \geq \widetilde{x}$. To deter entry, player $m$ would need $k_n \geq \widetilde{x}$. Now, assume that enough unused capacity remains to deter entry. Rewrite the equilibrium output of player $m$ in period $n$ as a function of $k_{n-1}$ and $x^m_{n-1}$, i.e. $x^m_n(k_{n-1}, x^m_{n-1}) = \min\{k_{n-1} - x^m_{n-1}, \overline{\overline{x}}^m\}$.

*Step 3.* Working backwards to period $n-1$, we have two subgames; either player $n-1$ enters or stays out of market $n-1$. First, capacity $k_{n-1}$ would ensure a successful commitment by player $m$ in market $n-1$ to an output $\widetilde{x}^m_{n-1}$, if and only if:

$$\frac{\partial v}{\partial x^m_{n-1}}\left(\widetilde{x}^m_{n-1}, \beta^1\left(\widetilde{x}^m_{n-1}\right)\right) + \frac{\partial x^m_n}{\partial x^m_{n-1}} \cdot \frac{\partial v}{\partial x^m_n}\left(x^m_n\left(k, \widetilde{x}^m_{n-1}\right), 0\right) \geq 0 \quad (4.13)$$

Now, $\frac{\partial x^m_n}{\partial x^m_{n-1}} = -1$ if $k \leq \overline{\overline{x}}^m + \widetilde{x}^m$ and $\frac{\partial x^m_n}{\partial x^m_{n-1}}(k, \widetilde{x}^m_{n-1}) = 0$ if $k > \overline{\overline{x}}^m + \widetilde{x}^m_{n-1}$. To deter entry, player $m$ has to commit to $\widetilde{x}$ in the subgame with entry. The following inequality must be satisfied:

$$\frac{\partial v}{\partial x^m_{n-1}}(\widetilde{x}, \beta^m(\widetilde{x})) = \frac{\partial v}{\partial x^m_n}(x^m_n, 0) \quad (4.14)$$

If $x^e_{n-1} > 0$, it follows from (S) that $x^m_n > \widetilde{x} \Rightarrow k_{n-1} > 2\widetilde{x}$. If (D) holds with equality, i.e. $\widetilde{x} = \overline{x}^m$, then the LHS of equality [4.14] is equal to zero and the equality is satisfied if and only if $k_{n-1} - \widetilde{x} = \overline{\overline{x}}^m \Rightarrow k_{n-1} = \widetilde{x} + \overline{\overline{x}}^m$.

*Step 4.* Working backwards to period $n-2$, we have two subgames; either player $n-2$ enters or stays out of market $n-2$. First, capacity $k_{n-2}$ would deter entry if:

$$\frac{\partial v}{\partial x^m_{n-2}}(\widetilde{x}, \beta^m(\widetilde{x})) = \frac{\partial v}{\partial x^m_{n-1}}\left(x^m_{n-1}, 0\right) = \frac{\partial v}{\partial x^m_n}(x^m_n, 0) \quad (4.15)$$

From (S), we have $x^m_n = x^m_{n-1} > \widetilde{x} \Rightarrow k_{n-2} > 3\widetilde{x}$. If (D) holds with equality, i.e. $\widetilde{x} = \overline{x}^m$, then the LHS of equality [4.14] equals to zero and the equality is satisfied if and only if $k_{n-2} - \widetilde{x} = 2\overline{\overline{x}}^m \Rightarrow k_{n-1} = \widetilde{x} + 2\overline{\overline{x}}^m$. Work in the same way inductively to period 1. In period 1, we have $x_n^m = x_{n-1}^m = .. = x_2 > \widetilde{x} \Rightarrow k > n\widetilde{x}$ and as (D) holds with equality $k_{n-1} = \widetilde{x} + (n-1)\overline{\overline{x}}^m$. The entry deterring capacity $\overline{k}^1$ is implicitly defined by

$$\frac{\partial v}{\partial x_1^m}\left(\widetilde{x}, \beta^1\left(\widetilde{x}\right)\right) = \frac{\partial v}{\partial x_t^m}\left(\frac{\overline{k}^1 - \widetilde{x}}{n-1}, 0\right) \text{ for } t = 2, .., n \tag{4.16}$$

and we conclude that $\overline{k}^1 \in \left(n\widetilde{x}, \widetilde{x} + (n-1)\overline{\overline{x}}^m\right]$.

*Step 5.* In a subgame without entry

$$\frac{\partial v}{\partial x_1^m}\left(x_1^m, 0\right) - \frac{\partial v}{\partial x_t^m}\left(\frac{\overline{k}^1 - x_1^m}{n-1}, 0\right) = 0 \tag{4.17}$$

and $x_t^m = k/n < \overline{\overline{x}}^m$ for all $t = 1, ..., n$. Hence, the entire capacity will be used in an equilibrium without entry. No capacity is left idle.

*Step 6.* Working backward to period 0, the incumbent would install $\overline{k}^1$ to deter entry in all markets. ∎

# Appendix B. Simultaneous Entry

This proof is valid for the main result in the second and third versions of the multi-market game.

**Proof.** *Step 1.* Begin in stage two. The objective of player $m$ in the second stage is to solve the following program:

$$\begin{array}{l} \max\ v\left(x_1^m, x_1^e\right) + v\left(x_2^m, x_2^e\right) + .. + v\left(x_n^m, x_n^e\right) \\ \text{s.t. } x_1^m + x_2^m + .. + x_n^m \leq k \end{array}$$

If $x_1^m + x_2^m + .. + x_n^m < k$, then $\partial v\left(x_t^m, x_t^e\right)/\partial x_t^m = 0$ for $t = 1, ..., n$. If $x_1^m + x_2^m = k$, then $\partial v\left(x_1^m, x_1^e\right)/\partial x_1^m = \partial v\left(x_2^m, x_2^e\right)/\partial x_2^m = .. = \partial v\left(x_n^m, x_n^e\right)/\partial x_n^m$.

*Step 2.* In the last stage there are $2^n$ subgames. First, if entry does not occur in any market and $k > n\overline{\overline{x}}^m$, then $\partial v\left(x_t^m, 0\right)/\partial x_t^m = 0$ for all $t = 1, ..., n \Rightarrow x_t^m = \overline{\overline{x}}^m$ for all $t$. If $k \leq n\overline{\overline{x}}^m$, then $\partial v\left(x_1^m, x_1^e\right)/\partial x_1^m = \partial v\left(x_2^m, x_2^e\right)/\partial x_2^m = .. = \partial v\left(x_n^m, x_n^e\right)/\partial x_n^m \Rightarrow x_t^m = \frac{k}{n}$ for all $t$.

*Step 3.* Second, if one player enters (w.l.o.g. player 1) and $k > \overline{x}^m + (n-1)\overline{\overline{x}}^m$, then $\partial v\left(x_1^m, x_1^e\right)/\partial x_1^m = 0$ and $\partial v\left(x_t^m, 0\right)/\partial x_t^m = 0 \Rightarrow x_1^m = \overline{x}^m$ and $x_t^m = \overline{\overline{x}}^m$ for $t = 2, .., n$. If $k \leq \overline{x}^m + (n-1)\overline{\overline{x}}^m$, then from (S) $\partial v\left(x_1^m, x_1^e\right)/\partial x_1^m = \partial v\left(x_t^m, 0\right)/\partial x_t^m$ for $t = 2, ..., n \Rightarrow x_1^m < k/n$ and $x_t^m > k/n$. To deter the entry of a single entrant while $n-1$ players stays out, the incumbent must install

$$\frac{\partial v\left(\widetilde{x}^m, \widetilde{x}^e\right)}{\partial x_1^m} = \frac{\partial v\left(\frac{k-\widetilde{x}}{n-1}, 0\right)}{\partial x_t^m} \tag{4.18}$$

and from (S) $k > n\widetilde{x}$.

*Step 4.* Next, if capacity $k$ deters the entry of a single entrant, $k$ deters the entry of more than one player, which is shown with induction. Assume $k$ deters the entry of $t$ players. Then

$$\partial v\left(\widetilde{x}^m, \widetilde{x}^e\right)/\partial x_i^m - \partial v\left(\frac{\left(k - t\widetilde{x}^m\right)}{(n-t)}, 0\right)/\partial x_j^m \geq 0 \tag{4.19}$$

where entry occurs in $i$ and no entry occurs in market $j$. If $t+1$ players enter, deterrence is credible if

$$\partial v\left(\widetilde{x}^m, \widetilde{x}^e\right)/\partial x_i^m - \partial v\left(\frac{\left(k - (t+1)\,\widetilde{x}^m\right)}{(n-t-1)}, 0\right)/\partial x_j^m \geq 0 \tag{4.20}$$

where entry occurs in $i$ and no entry occurs in market $j$. The last inequality holds as long as $k > n\widetilde{x}$. Hence, we have shown that if capacity $k$ deters the entry of a single entrant, then $k$ deters the entry of more than one entrant.

*Step 5.* If (D) holds with equality, i.e. $\widetilde{x} = \overline{x}^m$, then $\partial\pi\left(\widetilde{x}^m, \widetilde{x}^e\right)/\partial x_1^m = \partial\pi\left(\frac{k-\widetilde{x}}{n-1}, 0\right)/\partial x_t^m$ and the LHS is zero and, therefore, the entry-deterring capacity is $k = \overline{x}^m + (n-1)\overline{\overline{x}}^m$.

*Step 6.* Working backwards to the first stage. Now, the incumbent capacity is $\overline{k}^2 \in \left(n\widetilde{x}, \overline{x}^m + (n-1)\overline{\overline{x}}^m\right]$, where $\overline{k}^2$ is determined by equation (4.18). ■

# Appendix C. Equivalence

**Proof.** $\overline{k}_n^1, \overline{k}_n^2$ and $\overline{k}_n^3$ are all implicitly defined by

$$\frac{\partial v}{\partial x_1^m}\left(\widetilde{x}, \beta^e\left(\widetilde{x}\right)\right) - \frac{\partial v}{\partial x_t^m}\left(\frac{\overline{k} - \widetilde{x}}{n-1}, 0\right) = 0 \tag{4.21}$$

for $t = 2, .., n$. Hence, the implicit conditions are identical for all three versions of the multi-market game and the entry-deterring capacity is the same. ■

# 5

# Scale Economies and Arbitrage in International Markets

## 5.1 Introduction

Modelling the interaction between firms located in different countries must be at the centre of the theory of trade under imperfect competition; yet most of the literature in the field has concentrated on two rather extreme cases - market integration and market segmentation.

The integrated market assumption implies that producers set a single quantity or price at the world level and let arbitrageurs determine the distribution of sales to national markets (e.g. Markusen, 1981; Helpman and Krugman, 1985). At the opposite extreme, with segmented market behavior, firms choose strategic variables in each market separately (e.g. Brander, 1981; Brander and Krugman, 1983). In the case of integrated markets, price differentials are bounded by transportation costs and, thus, nationality has no systematic effect on prices for otherwise identical products. In the case of segmented markets, however, prices can differ more than the marginal cost of physically moving the goods from one location to another.

While recent research has made progress in measuring and explaining market power in international markets, the sources of international market segmentation are still not well understood. What factors make arbitrage costly and thus enable substantial price differentials in international markets?

This is the main topic of this paper. Market linkages and conditions for international arbitrage will be studied. For this purpose, a simple oligopoly model of international competition is presented. The model builds on two crucial components: economies of scale in the transportation technology and market-specific access costs.

Two arbitrage conditions are introduced: the importing firms' arbitrage condition and the individual arbitrage condition. Each condition can hold with equality or inequality. If the conditions hold with equality, the role of price linkages in international markets is unambiguous, i.e. a lower world market price implies a lower domestic price. However, if both arbitrage conditions hold with strict inequality, there is no link between world market prices and domestic prices.

Large economies of scale in the transportation technology gives the importing firms market power. Thus, prices in the domestic market can be strictly higher than the world market price, even if the variable transportation cost is zero. Free entry of importing firms yields a price equal to the average cost of market access and transportation. International prices differ more or less, not because markets are assumed to be segmented or integrated, but because arbitrage is more or less profitable.[1] In this respect, our analysis has implications for different aspects of economic integration.

First, the results suggest that even under the assumption of international arbitrage, the generally assumed implications of the law of one price may not hold. The law of one price asserts that prices of homogenous and identical goods should be equalized internationally, when adjusted for transportation costs and expressed in a common currency. The argument is that arbitrage should work to eliminate any price difference that is due to country of origin. The simple model in this paper suggests that international price differentials can be large, if the transportation technology exhibits economies of scale. In principle, we expect the effect of international arbitrage to be both country- and industry-specific. This result is also supported by empirical evidence suggesting that deviations from absolute price equalization are often large and the variation between industries is substantial (Goldberg and Knetter, 1997).

Second, the results have strong implications for the assumption of transmission of economic disturbances. In our model, high market-specific barriers work as a buffer and may prevent international transmission of economic shocks. Furthermore, the results indicate that while there is a strong microfoundation for macroeconomic price convergence in the form of purchasing power parity under constant returns to scale in transportation technology and low barriers to entry in the domestic market, price convergence need not occur when these conditions are violated.

Finally, an old insight in the trade literature is that international trade increases competition in the domestic market (cf. Caves, 1985, and Levinsohn, 1993 and 1996). Our analysis shows, however, that the effects of trade liberalization on the domestic market structure and the domestic equilibrium price may be small, when the market-specific barriers are high.

This suggests that imports do not automatically discipline domestic mar-

[1] In Venables (1990), Ben-Zvi and Helpman (1992) and Ganslandt (1998) some strategic variables are determined at the global level, while others are determined at the local level. In particular, production capacity is a global strategic variable, while price and output decisions are local. The approach taken in this paper is complementary to these contributions as we establish some structural foundations for the fundamental assumptions about market segmentation.

ket concentration, even if the trade barriers are reduced.[2] This result will be generalized in the rest of this paper.

The paper is organized as follows. Section 2 introduces a general two-country oligopoly model. In section 3, the autarchy equilibrium is studied and it is shown that small countries have high prices compared to large countries. Section 4 analyzes how different barriers affect individual arbitrage and importing firms' arbitrage. It is shown that either of the two arbitrage conditions binds or both of them hold with strict inequality. In the former case, imports can discipline domestic market concentration, in the latter case it cannot. In section 5, a linear demand model is introduced and it is shown that the results hold when firms compete in quantities. The linear model is also used to show that international price differentials can be substantial, if market-specific barriers are high. International prices can differ more than the average trade cost and, therefore, the law of one price does not hold. It is also shown that individual arbitrage can equalize international prices, if the technology exhibits moderate economies of scale in transportation. Section 6 discusses our main findings and section 7 illustrates a few extensions. Finally, section 8 concludes.

## 5.2 The Basic Model

There are two countries, $h$ (the home market) and $w$ (the rest of the world). The home country is small and the rest of the world is very large. Each economy has an imperfectly competitive sector, producing a homogenous good $X$ and a competitive, numeraire sector producing $Y$. Our analysis focuses on the imperfectly competitive sector in the home market, which has two types of firms: domestic producers (type $m$) and importing firms (type $a$). In the subsequent sections superscripts refer to the type of firm and subscripts refer to the markets.

Domestic producers are manufacturing firms which produce $X$, with identical technologies and sell it in the local market. To begin producing $X$, a firm must incur the once-for-all sunk cost $G$, which is a production-specific sunk cost. The production-specific cost is expenses on research and development required to create a correct specification of the product, as

[2] This has important policy implications. Smith and Venables (1988) suggest that substantial welfare effects are expected as a result of the European integration. The largest effect is obtained if markets are completely integrated, i.e. prices are completely equalized. Baldwin and Venables (1995) summarize several studies with similar results. Our analysis suggest that these predictions might be exaggerated unless market-specific barriers are dismantled at the same time as trade costs are reduced.

well as fixed costs to install a manufacturing plant. The variable cost of production in the imperfectly competitive sector is assumed to be constant and equal to $c$.

There are also barriers to entry in the local market. Entry in the home market is associated with a fixed market access cost $A$.[3] In order to obtain demand for its products in the local market, the firm must incur some fixed advertising costs and costs of installing and maintaining some means for distribution of its output to the consumers. The market-specific costs also include costs of providing local services and after-sales. If the firm incurs the market-specific fixed cost $A$, all consumers in the local market are informed about the firm's products and the organization of distribution allows the firm to deliver its good to each consumer in the market. Combining these components, we obtain the following cost function for a domestic producer:

$$C^m (X_i^m) = G + A + c \cdot X_i^m, \tag{5.1}$$

when the firm produces $X_i^m$ units of the good.

Importing firms are the second type of firms in the imperfectly competitive sector in the home market. They buy the product in the world market at some price $p_w$ and ship it to the home market. Furthermore, the home country is assumed to be small compared to the rest of the world. Hence, each importing firm can take the world market price as given. In the subsequent analysis it is, for simplicity, assumed that $p_w = c$.

In order to import the good, the buyer must incur a variable trade cost $t$. Moreover, the transportation technology exhibits increasing returns to scale. The buyer must incur a fixed cost of transportation, $T$. Consequently, an importing firm in the home country has the following cost function:

$$C^a (X_i^a) = A + T + (p_w + t) \cdot X_i^a, \tag{5.2}$$

when the firm imports $X_i^a$ units of the good in the imperfectly competitive sector.

A consumer in the home market is also allowed to import the homogenous good from the world market without intermediaries. Superscript $i$ refers to individual arbitrage. The cost of personal imports of the homogenous good from the world market is

[3] The fixed cost will be exogenous in this model. In models with exogenous sunk costs market concentration converges to zero as the market size increases to infinity. If sunk costs are endogenously determined, the results will be different in some respects. Sutton (1991) shows that, in models with endogenous sunk costs, market concentration has a lower bound strictly different from zero.

$$C^i(q) = T + (p_w + t) \cdot q, \tag{5.3}$$

where $q$ is the private consumption of the homogenous good.

Competition is modelled as a two-stage game. In the first stage, $n$ firms choose to incur sunk costs and enter the home market. In the second stage, firms non-cooperatively choose quantities or prices. Markets clear.

In the basic model, we make some assumptions about the equilibrium prices and quantities in the last stage of the game instead of modelling demand explicitly. Let $p(n,r)$, $x(n,r)$ denote the $n$-firm last stage equilibrium price and per-capita quantity for each firm with marginal cost, $r$. The last stage equilibrium output of a representative firm of a specific type is $X = S \cdot x(n,r)$, where $S$ is the number of consumers in the market. Moreover, $\pi^m$ denotes the profit of a manufacturing firm and $\pi^a$ the profit of an importing firm.

It is assumed that the non-cooperative equilibrium price and per-capita quantity per firm is decreasing in the number of firms. A manufacturing monopolist is also assumed to make a non-negative profit.

(C1) $p'(n) < 0$

(C2) $x'(n) < 0$

(C3) $\pi^m \geq 0$, for $n = 1$

These assumptions are sufficient for the total equilibrium profit of $n+1$ firms to be less than the total profit of $n$ firms, e.g. for a monopolist to make more profit than two non-cooperative duopolists.[4] It is also assumed that the equilibrium per-capita quantity is a decreasing and non-concave function in the marginal cost of the firm, i.e. $\partial x/\partial r < 0$ and $\partial^2 x/\partial r^2 \geq 0$. Consumers are price-takers.

The following section considers the autarchy equilibrium in the home market. In section 4, arbitrage is introduced to analyze under what conditions imports can discipline market concentration in the home market.

## 5.3 Autarchy

Trade costs are prohibitive in the autarchic equilibrium. Thus, importing firms will not enter and no individual arbitrage occurs. In the first stage of

[4] It can be shown that, for instance, price or quantity competition in a model with linear demand for differentiated goods satisfy assumptions C1, C2 and C3 (Appendix A).

the game $n$ domestic producers enter, where $n \in \mathbb{R}$. We assume that the equilibrium profit after the second stage can be written as the sum of the revenues net variable costs from $S$ consumers minus fixed costs:

$$\pi^m(n) = S \cdot x_h^m(n) \cdot (p(n) - c) - \sigma^m, \tag{5.4}$$

where $x_h^m(n)$ is the per capita supply from each firm, $p(n)$ is the equilibrium price and $\sigma^m$ is the total fixed cost of a domestic producer, i.e. $\sigma^m \equiv G + A$. Let $p_h^\infty$ denote the resulting equilibrium price in the domestic market under the autarchy regime.

Working backwards to the first stage of the game, we find the solution to the number of firms in equilibrium. The number of firms, $n \in \mathbb{R}$, is determined by the free-entry condition

$$x_h^m(n) \cdot (p(n) - c) \leq \frac{\sigma^m}{S}, \tag{5.5}$$

which ensures that no additional firm can enter the market with strictly positive profit. From the free-entry condition, it is clear that the number of firms in equilibrium will depend on the total fixed cost of starting domestic production, but not the relative share of market access costs or production-specific setup costs. More precisely, the free-entry condition can be used to show the following results:

**Proposition 1** *The number of firms in the autarchic equilibrium is (i) increasing in the size of the domestic market, S, and (ii) decreasing in the total fixed cost to enter as a domestic producer,* $\sigma^m$.

The result follows directly from the free-entry condition and is here shown with implicit differentiation. For this purpose, let the number of firms in equilibrium be a function of the size of the market and the total fixed cost, i.e. $n = n(S, \sigma^m)$.

**Proof.** To prove (i), we differentiate the free-entry condition implicitly with respect to market size S:

$$\frac{\partial n}{\partial S}\left(\frac{\partial x_h^m}{\partial n}(p - c) + \frac{\partial p}{\partial n}x_h^m\right) = -\frac{\sigma^m}{S^2} \tag{5.6}$$

where the expression in the brackets is negative as we have assumed that the equilibrium price (C1) and the per-capita quantity (C2) decreases in the number of firms. Thus, $\partial n / \partial S > 0$. Similarly, to prove part (ii) we differentiate the free-entry condition implicitly with respect to total fixed cost $\sigma^m$ :

$$\frac{\partial n}{\partial \sigma^m}\left(\frac{\partial x_h^m}{\partial n}(p - c) + \frac{\partial p}{\partial n}x_h^m\right) = \frac{1}{S}, \tag{5.7}$$

where, once again, the expression within the brackets is negative. Therefore, $\partial n / \partial \sigma^m < 0$, which concludes the proof. ■

From the proposition and assumption (C1) that the equilibrium price will fall in the number of firms, it immediately follows that the equilibrium price decreases in the market size and increases in fixed costs.

**Corollary 2** *The equilibrium price under the autarchy regime, $p_h^\infty$, (i) decreases in the market size, $S$, and (ii) increases in the total fixed cost of entering as a domestic producer, $\sigma^m$.*

These are the fundamental reasons for price differentials in our model. In particular, the home market has a high autarchy price compared to world market, if the home market is small. Accordingly, the price differential between the home and world market will decrease in the size of the home market. If the countries are of similar size, the price differential will diminish.[5]

## 5.4 Arbitrage

In the previous section, it was established that the autarchy price in the home market can be high compared to the world market price, due to differences in market size. The purpose of this section is to analyze under what conditions imports can discipline market concentration in the home market and, thus, result in a reduced international price differential.[6]

More specifically, two issues related to the profitability and the effects of arbitrage will be highlighted. First, the effect of fixed and variable costs in transportation is analyzed. Second, the impact of arbitrage in the presence of market-specific barriers is studied.

For this purpose, two types of arbitrage are modelled; individual arbitrage and importing firms' arbitrage. Each type of arbitrage results in an arbitrage condition. If the autarchy price satisfies both arbitrage conditions, it is "arbitrage free" and potential imports from the world market do not discipline market concentration in the home market.

First, an individual consumer in the domestic market has an indirect

[5]It is worth noting that large price differentials can also be obtained in models of third-degree price discrimination (see Chapter 6 in this thesis). In the model presented here, however, there will be no scope for price discrimination. Free entry in the manufacturing and trading sector imply average cost pricing and, thus, no third degree price discrimination can occur in equilibrium.

[6]For the empirical relevance of this test, see Levinsohn (1993) and (1996).

utility function, $v(p, I)$, where $p$ is the price of the homogenous goods and $I$ is the income, which is continuous for $p \geq 0$ and $I \geq 0$. The good produced in the oligopolistic sector is normal and a substitute for the numeraire good, i.e. $v_p(p, I) < 0$. The marginal utility of income is positive, i.e. $v_I(p, I) > 0$, and $v(p, 0) = 0$.

The individual arbitrage condition guarantees that a representative consumer in the home market has at least as high utility from consuming in the home market as he would have obtained from consumption in the world market. More precisely, the individual arbitrage condition is

$$\text{(A1)} \qquad v(p_w + t, I - T) - v(p_h, I) \leq 0,$$

where the first part is the utility for the consumer, if he buys the goods in the world market and the second part is the utility of home market consumption.

Next, the importing firms' arbitrage condition is a free-entry condition. If the importing firms' arbitrage condition is satisfied, no importing firms can enter the home market with a non-negative profit. More precisely, the importing firms' arbitrage condition is

$$\text{(A2)} \qquad S \cdot x^a \cdot (p_h - p_w - t) - \sigma^a \leq 0,$$

where $\sigma^a$ is the fixed cost of an importing firm. It is assumed that the fixed cost of production is higher than the per-capita income, i.e. $G > I$, and that the market-specific barrier is strictly positive, $A > 0$. For simplicity, it is assumed that $p_w = c$. Equilibrium quantities are determined by the first order conditions in the non-cooperative game. First order conditions combined with the arbitrage condition (A2) gives the number of importing firms.

The price $p_h$ is *arbitrage free* if arbitrage is unprofitable for individuals and importing firms. In this case, imports do not discipline market concentration in the home market. More precisely, conditions (A1) and (A2) can be used to define arbitrage free prices.

**Definition 1** *$p_h$ is an arbitrage free price if conditions (A1) and (A2) are satisfied.*

The aim is to characterize the arbitrage free prices at different levels of fixed and variable transportation costs. For this purpose, let $\overline{T}^i(t)$ and $\overline{T}^a(t)$ denote the arbitrage blocking barriers at the variable trade cost $t$, for individuals and importing firms, respectively. For some variable transportation cost, $\widetilde{t}$, the arbitrage blocking barriers are identical, i.e. $\overline{T}^i(\widetilde{t}) = \overline{T}^a(\widetilde{t})$.

The following proposition shows that importing firms' arbitrage is profitable for a wider range of fixed costs in the transportation technology. In the case of low variable costs, the domestic price is arbitrage free, if the fixed cost is sufficiently large to block entry of importing firms.

Individual arbitrage, on the other hand, is profitable for a wider range of variable costs. If the variable cost is high, the autarchy price is arbitrage free when individual arbitrage is unprofitable.

Hence, individual arbitrage does not discipline home market concentration, if transportation incurs a large fixed cost. Correspondingly, importing firms' arbitrage does not discipline market concentration, if the variable transportation cost is large. Thus, for relatively high fixed and variable costs, the autarchy price is arbitrage free.

**Proposition 3** *$p_h^\infty$ is an arbitrage free price if and only if $T \geq \overline{T}^a(t)$ for $t < \widetilde{t}$ and $T \geq \overline{T}^i(t)$ for $t > \widetilde{t}$.*

To prove this proposition, the following five lemmas are useful. The first lemma shows that for every variable transportation cost, there is a unique fixed cost of transportation such that condition (A1) is satisfied with equality.

**Lemma 4** *For every $t < p_h^\infty - p_w$ there exists an arbitrage blocking barrier $\overline{T}^i(t) \in (0, I)$, such that (A1) is satisfied if and only if $T \geq \overline{T}^i(t)$.*

**Proof.** Let $t < p_h^\infty - p_w$. If $T = 0$ then $v(p_w + t, I) > v(p_h^\infty, I)$. If $T = I$ then $v(p_w + t, 0) < v(p_h^\infty, I)$. $v$ is continuous and we conclude that $\exists \overline{T}^i(t) \in (0, I)$, such that $v(p_w + t, I - T) = v(p_h^\infty, I)$. For each $t$, the utility is increasing in $I$ and, thus, $\partial v / \partial T < 0$. Hence, (A1) is satisfied if and only if $T \geq \overline{T}^i(t)$. ■

Second, it can be shown that importing firms arbitrage is unprofitable for large variable transportation costs.

**Lemma 5** *For $T = 0$, there exists a $\overline{t} \in (0, p_h^\infty - p_w)$, such that $S \cdot x^a \cdot (p_h^\infty - p_w - t) - \sigma^a < 0$ if and only if $t > \overline{t}$.*

**Proof.** Let $T = 0$. Note that $\sigma^a < \sigma^m$. If $t = 0$ then $S \cdot x^a \cdot (p_h^\infty - p_w) - \sigma^a > 0$. If $t = p_h^\infty - p_w$ then $S \cdot x^a \cdot (p_h^\infty - p_w - t) - \sigma^a < 0$. The profit is continuous in $t$ and, thus, $\exists \overline{t} \in (0, p_h^\infty - p_w)$, such that $S \cdot x^a \cdot (p_h^\infty - p_w - \overline{t}) - \sigma^a = 0$. Finally, $S \cdot x^a \cdot (p_h^\infty - p_w - t) - \sigma^a$ is decreasing in $t$, which concludes the proof. ■

Next, we can show that for every variable transportation cost, there is a unique fixed cost of transportation, such that condition (A2) is satisfied with equality. In other words, it is shown that the arbitrage blocking barrier for importing firms is unique and takes values on $(0, G]$.

**Lemma 6** *For every $t < \bar{t}$, there exists an arbitrage blocking barrier $\overline{T}^a(t) \in (0, G]$, such that (A2) is satisfied if and only if $T \geq \overline{T}^a(t)$.*

**Proof.** Let $t < \bar{t}$ . If $T = 0$ then $S \cdot x^a \cdot (p_h^\infty - p_w - t) - \sigma^a > 0$. If $T > G$, the free-entry condition of manufacturing firms and $\partial x / \partial r < 0$ implies $S \cdot x^a \cdot (p_h^\infty - p_w - t) - \sigma^a < 0$. The profit is continuous in $T$ and we conclude that $\exists \overline{T}^a(t) \in (0, G)$, such that $S \cdot x^a \cdot (p_h^\infty - p_w - t) - \sigma^a = 0$. Moreover, the profit is decreasing in the fixed cost of transportation and, thus, (A2) is satisfied if and only if $T \geq \overline{T}^a(t)$. ■

Moreover, without variable transportation costs, $t = 0$, the arbitrage blocking barrier for importing firms is strictly higher than for individuals.

**Lemma 7** $\overline{T}^a(0) > \overline{T}^i(0)$.

**Proof.** Let $t = 0$ and $T = I$. (A1) is satisfied as $v(p_w, 0) < v(p_h^\infty, I)$. (A2) is not satisfied as $S \cdot x^a \cdot (p_h^\infty - c) - \sigma^a > 0$ for $T < G$ and, by assumption, $I < G$. Hence, $\overline{T}^a(0) > \overline{T}^i(0)$. ■

Finally, both arbitrage blocking barriers decrease in the variable transportation cost. More precisely, the arbitrage blocking barriers are decreasing and convex functions in $t$.

**Lemma 8** *$\overline{T}^a(t)$, $\overline{T}^i(t)$ are decreasing, convex and continuous functions.*

**Proof.** The arbitrage blocking barrier for individuals is decreasing in variable trade costs

$$\frac{d\overline{T}^i(t)}{dt} = -q^*(p_w + t) < 0, \tag{5.8}$$

where $q^*(p_w + t)$ is the consumed quantity at the price $p_w + t$. The second-order derivative is clearly positive. Moreover, the arbitrage blocking barrier for importing firms is decreasing in variable trade costs:

$$\frac{d\overline{T}^a(t)}{dt} = -Sx^a + S\frac{\partial x^a}{\partial t}(p_h^\infty - p_w - t) < 0, \tag{5.9}$$

where, by assumption, $\partial x^a / \partial t < 0$ and the second-order derivative is strictly positive. ■

It is now established that for each $t$ there is a unique arbitrage blocking barrier for individuals and a unique arbitrage blocking barrier for importing firms. The barriers are decreasing, convex functions in the variable transportation cost and the importing firms arbitrage blocking barrier begins above the individual arbitrage blocking barrier and ends below. This information is sufficient to prove the main proposition.

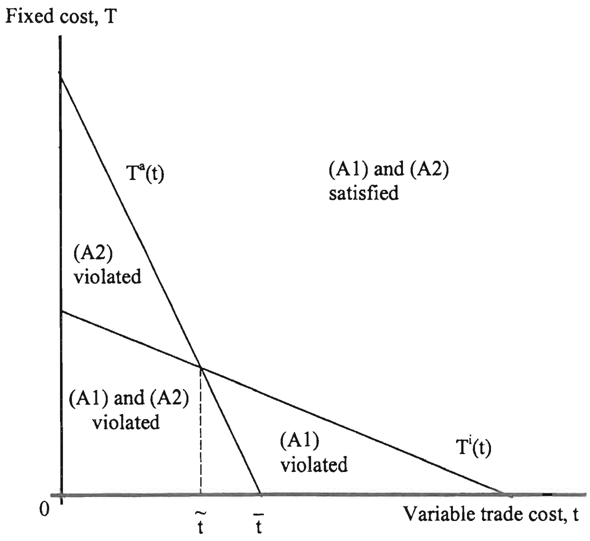


FIGURE 5.1. Arbitrage Blocking Barriers

**Proof.** (of the proposition) It has be shown that $\overline{T}^a(0) > \overline{T}^i(0)$ (lemma 7) and $\overline{T}^a(\bar{t}) < \overline{T}^i(\bar{t})$ (implicit from lemma 5 and lemma 8). The arbitrage blocking barriers are unique (lemma 4 and lemma 6) and decreasing, non-concave and continuous functions (lemma 8). Hence, there is a unique $\widetilde{t}$, such that $\overline{T}^a(\widetilde{t}) = \overline{T}^i(\widetilde{t})$ ,where $\widetilde{t} < \bar{t} < p_h^{\infty} - p_w$. The proposition follows immediately. ■

The two arbitrage conditions can now be illustrated in a figure with fixed and variable trade costs. We have illustrated all possible cases in Figure 5.1 First, both types of arbitrage are profitable at autarchy prices for small variable and small fixed transportation costs. Second, for intermediate fixed costs and small variable costs, the arbitrage of importing firms is profitable and individual arbitrage unprofitable. Third, individual arbitrage is profitable for small fixed costs and intermediate variable costs and importing firms can not enter profitably. Fourth, both types of arbitrage is unprofitable for large variable and fixed costs.

Next, we proceed to analyze the relative profitability of individual arbitrage and importing firms' arbitrage. If the individual arbitrage condition (A1) binds, price differences between different countries are determined by trade costs only. Prices in different countries are completely equalized as trade costs diminish. For instance, with constant returns to scale in the transportation technology, individual arbitrage results in complete market integration as variable trade costs go to zero. This is a special case, however. Individual arbitrage is not profitable for large fixed costs of transportation. In particular, for $T \geq \overline{T}^i(t)$ individual arbitrage is unprofitable. In this case only importing firms' arbitrage can be profitable and this is the situation to be analyzed next.

Let $t = 0$ and consider an equilibrium with importing firms. In the last stage of the game, $n$ importing firms have entered and the profit of each importing firm is:

$$\pi^a(n) = S \cdot x(n)(p(n) - p_w) - \sigma^a, \tag{5.10}$$

where $\sigma^a$ is the total fixed cost to set up an importing firm, i.e. $\sigma^a \equiv T + A$. Each importing firm is symmetric to a domestic producer in terms of demand and the variable costs. It follows that the equilibrium quantity and price must be identical to the quantity and price chosen by a domestic producer.

Importing firms and domestic producers will enter, as long as it is possible to enter with a positive profit. In the first stage importing firms will enter until

$$x(n)(p(n) - p_w) \leq \frac{\sigma^a}{S} \tag{5.11}$$

and domestic producers until

$$x(n)(p(n) - c) \leq \frac{\sigma^m}{S}, \tag{5.12}$$

where $n$ is the total number of firms. Note that the left-hand side is identical in both expressions as we replace $p_w$ with $c$. In a situation where the fixed cost of a manufacturing firm and the fixed cost of an importing firm are different, only one condition holds with equality and the other with strict inequality. Hence, in all generic cases, there is only one type of firms active in equilibrium, either importing firms or manufacturing firms.

Consider a case when importing firms' arbitrage is profitable. If the market access cost is strictly positive, $A > 0$, the equilibrium prices in the home and world market will differ more than the average trade costs. Hence, the price differential is not bounded by transportation costs. More precisely,

**Proposition 9** *If $A > 0$ and $\overline{T}^i(t) < T < \overline{T}^a(t)$, the difference between the domestic and the world market price is larger than the average trade costs.*

**Proof.** $T \geq \overline{T}^i(t)$ and individual arbitrage is not profitable. Importing firms' arbitrage is profitable. The free entry condition binds and

$$p_h - p_w = t + \frac{T + A}{Sx^a}, \tag{5.13}$$

which is clearly larger than the average cost of transportation, i.e.. $p_h - p_w > t + T/Sx^a$. ■

Furthermore, we can find a relationship between the autarchy price and the fixed costs of domestic production, if studying the production-specific and market-specific fixed costs as relative shares of the total fixed costs for a manufacturing firm. We define a relative market-specific barrier $\lambda$, such that $A = \lambda\sigma^m$. Accordingly, the fixed cost of an importing firm is $\sigma^a = \lambda\sigma^m + T$. If we consider an equilibrium with importing firms, we can study the equilibrium price as a function of $\lambda$.

**Proposition 10** *Let* $\overline{T}^i(t) < T < \overline{T}^a(t)$. *The price in the home market is increasing in* $\lambda$.

**Proof.** Let $n$ be a function of $\lambda$. Now, $T < \overline{T}^a(t)$ and the free-entry condition for the importing firms hold with equality. We implicitly differentiate the free-entry condition with respect to $\lambda$ to obtain

$$\frac{\partial n}{\partial \lambda}\left(\frac{\partial x_h^a}{\partial n}(p - p_w) + \frac{\partial p}{\partial n}x_h^a\right) = \frac{\sigma^m}{S}, \tag{5.14}$$

where the expression in the brackets is strictly negative. Therefore, the number of importing firms in equilibrium is decreasing in $\lambda$. Using the assumption that the equilibrium price is higher if the number of firms is smaller, concludes the proof. ■

This proposition is interesting as it suggests that the equilibrium price changes, when variable transportation costs are reduced, if the market access cost is a relatively small share of the total fixed cost of starting domestic production. The effect is small if the opposite holds. Hence, trade liberalization is very pro-competitive in a small market, if the scale economies in production are large, market access barriers are low and the fixed cost of transportation is relatively small. However, if the market-specific barriers are high and the fixed cost of transportation is large, the pro-competitive effect of trade liberalization on the equilibrium price will be small. In this case, the pro-competitive effect can be reinforced if market barriers are dismantled while trade costs are reduced.

## 5.5 A Parametric Example

In this section, it is assumed that firms produce a homogeneous good and compete in quantities à la Cournot. Let the utility function of the representative consumer be separable and linear in the numeraire good:

$$U(q, y) = a \cdot q - \frac{b}{2} q^2 + y, \tag{5.15}$$

where $q$ is consumption of goods from the $X$-sector and $y$ is the consumption of a composite good. In this case, the inverse demand function is reduced to:

$$p(X) = a - \frac{b}{S} X, \tag{5.16}$$

where $S$ is the size of the domestic market and $X$ is aggregate supply.

The autarchy equilibrium is derived with the assumption that variable transportation costs are prohibitive. Each manufacturing firm maximizes its profit:

$$\pi^m (X_i^m, X_{-i}) = \left( a - \frac{b}{S} X \right) X_i^m - c \cdot X_i^m - \sigma^m, \tag{5.17}$$

where $X_i^m$ is the output of firm $i$ and $X_{-i}$ is a vector of the output of the competitors. The equilibrium is determined by $n^m + 1$ equilibrium conditions. Optimal quantities are determined by $n^m$ first order conditions and the equilibrium number of firms by the free-entry condition. All firms are symmetric. In equilibrium, $X_i^m = X^m$ for all $i$ and the equilibrium conditions can be reduced to a single first order condition and free-entry condition:

$$a - \frac{b}{S} \cdot (n^m + 1) X^m - c \leq 0 \qquad (X^m) \tag{5.18}$$

$$a - \frac{b}{S} n^m X^m - c - \frac{\sigma^m}{X^m} \leq 0 \qquad (n^m) \tag{5.19}$$

which is a complementary slackness problem. Complementary variables are indicated in brackets. Next, we can solve for $X^m$ and $n^m$ and use the free-entry condition to obtain the equilibrium price

$$p_h^{\infty} = c + \sqrt{\frac{b}{S} (G + A)}, \tag{5.20}$$

which converges to the marginal cost of production, $c$, as the market size increases to infinity. The equilibrium can also be derived in the usual backward fashion as shown in Appendix B.

Next, consider a situation where trade occurs with importing firms as trading entities. An importing firm maximizes its profit:

$$\pi^a \left(X_i^a, X_{-i}\right) = \left(a - \frac{b}{S}X\right) X_i^a - (p_w + t) \cdot X_i^a - \sigma^a \tag{5.21}$$

where $X_i^a$ is the number of units of the homogenous good the importing firm chooses to buy in the world market and $X_{-i}$ is a vector of the output of the competitors. All importing firms are symmetric and $X_i^a = X^a$ for all importing firms. Correspondingly, all manufacturing firms are symmetric and $X_i^m = X^m$ for all manufacturing firms. The equilibrium is determined by the first order condition and the free-entry condition of manufacturing firms as well as the first order condition and the free-entry condition of importing firms:

$$a - \frac{b}{S} \cdot (n^m + 1) X^m - \frac{b}{S} \cdot n^a X^a - c \leq 0 \qquad (X^m) \quad (5.22)$$

$$a - \frac{b}{S} \cdot n^m X^m - \frac{b}{S} \cdot (n^a + 1) X^a - p_w - t \leq 0 \qquad (X^a) \quad (5.23)$$

$$a - \frac{b}{S} n^m X^m - \frac{b}{S} n^a X^a - c - \frac{\sigma^m}{X^m} \leq 0 \qquad (n^m) \quad (5.24)$$

$$a - \frac{b}{S} n^m X^m - \frac{b}{S} n^a X^a - p_w - t - \frac{\sigma^a}{X^a} \leq 0 \qquad (n^a), \quad (5.25)$$

which is a complementary slackness problem. Complementary variables are indicated in brackets next to the condition. Solving for $X^m$ and $X^a$, the free-entry conditions can be reduced to

$$p_h \leq c + \sqrt{\frac{b}{S}(G + A)} \qquad (n^m) \quad (5.26)$$

$$p_h \leq p_w + t + \sqrt{\frac{b}{S}(T + A)} \qquad (n^a), \quad (5.27)$$

with the associated complementary variables within brackets. The latter inequality can also be derived as a subgame perfect equilibrium in a two-stage game as shown in Appendix C. Except for non-generic cases, only one condition is satisfied with equality and the other condition holds with strict inequality. One type of firms is thus normally inactive in equilibrium.

It follows that a sufficient condition for no importing firms in equilibrium is that fixed cost of transportation are higher than the fixed cost of production. Interestingly, that implies that no importing firms will enter if the total fixed cost of setting up a local manufacturing firm entirely consist of market-access costs. Thus, if the share of market-specific fixed costs is sufficiently large we cannot expect arbitrage by importing firms to reduce the domestic prices. More specifically, the arbitrage blocking barrier for importing firms can be obtained from the two free-entry conditions:

$$\overline{T}^{a}(t) = G - 2t\sqrt{\frac{\sigma^{m} S}{b}} + \frac{S}{b}t^{2}. \tag{5.28}$$

However, the possibility of individual arbitrage must also be considered. If the transportation technology exhibits small economies of scale, personal arbitrage undertaken by individuals would restrict domestic market concentration and result in a reduced international price differential. More precisely, a consumer is indifferent to consuming the homogenous good bought in the home market and the same good bought in the world market, if the indirect utility is identical in both cases, i.e.

$$v(p_h, I) = v(p_w + t, I - T), \tag{5.29}$$

where $I$ is the per capita income and $I - T$ is the per capita income net fixed cost of transportation. The consumer prefers domestic consumption to world market consumption if:

$$p_h \leq a - \sqrt{(a - p_w - t)^2 - 2bT}, \tag{5.30}$$

where $a \geq p_w + t$ (for details see Appendix D). If there is no fixed cost in the transportation technology, i.e. $T = 0$ , the individual arbitrage condition implies that the home market price is the world market price plus the variable trade cost, $p_h = p_w + t$, which is the traditional "law of one price".

Combining the price in the autarchic equilibrium and the individual arbitrage condition, we can derive a level of fixed costs of transportation, at which individual arbitrage is blocked:

$$\overline{T}^{i}(t) = \frac{1}{2b}(a - p_w - t)^2 - \frac{1}{2b}(a - p_h^{\infty})^2 \tag{5.31}$$

In all generic cases, one type of arbitrage is more profitable than the other. Let $\overline{T}^{c}(t)$ denote the level of fixed such that a representative consumer in the home market is indifferent between consuming at world market prices and consuming at importing firms' prices. Above this level importing firms' arbitrage results in an equilibrium price which makes individual arbitrage unprofitable. $\overline{T}^{c}(t)$ is implicitly defined by

$$p_w + t + \sqrt{\frac{b}{S}\left(\overline{T}^{c}(t) + A\right)} = a - \sqrt{(a - p_w - t)^2 - 2b\overline{T}^{c}(t)} \tag{5.32}$$

The arbitrage blocking barriers $\overline{T}^{a}(t)$ and $\overline{T}^{i}(t)$ as well as $\overline{T}^{c}(t)$ are illustrated in Figure 5.2. For relatively small fixed costs of transportation,

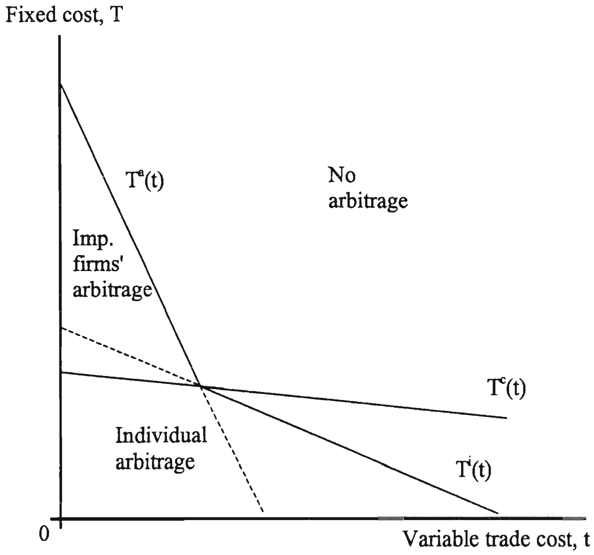


FIGURE 5.2. Importing Firms' and Individual Arbitrage

individual arbitrage is more profitable than importing firms' arbitrage, independently of the level of variable trade costs. For intermediate fixed costs, however, only importing firms' arbitrage is profitable.

With this background, it is interesting to study the effects of reduced variable transportation costs, when the fixed cost of transportation and the market-specific barrier take different values. Indeed, different combinations of $A$ and $T$ result in arbitrage by importing firms, arbitrage by individuals, arbitrage by both or arbitrage by none of these as the variable transportation cost varies. There are four cases of interest.

In the first case, the fixed cost in the transportation technology is very small and the market-specific barrier is relatively large (Figure 5.3). In this case, individual arbitrage can discipline the market concentration in the home market and result in a reduced international price differential when variable trade costs are reduced. Importing firms can not enter with positive profits. The individual arbitrage condition binds, if $t \leq \bar{t}^i$, where $\bar{t}^i$ is defined by

$$\bar{t}^i = a - p_w - \sqrt{\left(a - r - \sqrt{\frac{b}{S}(G + A)}\right)^2 + 2bT}, \qquad (5.33)$$

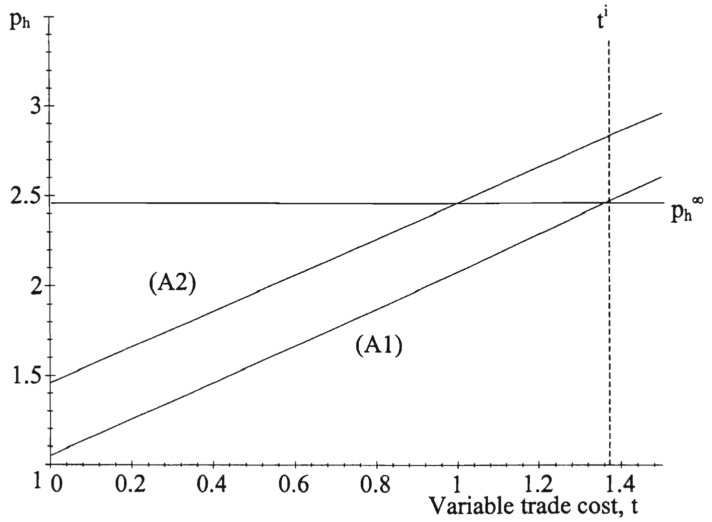


FIGURE 5.3. $T$ is very small, $A$ is relatively large

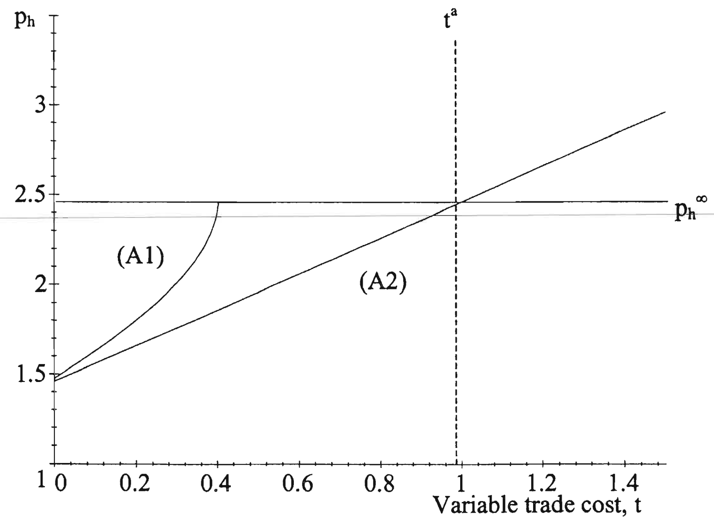


FIGURE 5.4. $T$ is moderately large, $A$ is moderately low

which is larger than zero if $T$ is sufficiently small.

In the second case, the fixed cost of transportation is moderately large and the market-specific barrier is low (Figure 5.4). In this case, the transportation technology exhibit large economies of scale and the individual arbitrage condition would not imply any restrictions on the equilibrium price. However, at some levels of trade costs, the importing firm' arbitrage condition binds and importing firms can discipline the market concentration in the home market, which results in a reduced international price differential. Let $\bar{t}^a$ denote the variable transportation cost at which importing firms begin to enter the home market:

$$\bar{t}^a = r - p_w + \sqrt{\frac{b}{S}} \left( \sqrt{G+A} - \sqrt{T+A} \right) \tag{5.34}$$

and at every variable transportation cost $t < \bar{t}^a$, there will exist some importing firms in the home market. On the other hand, if the variable transportation cost is above the threshold level, no importing firms can enter profitably and the autarchic equilibrium price will remain.

In the third case, the fixed cost of transportation is relatively small and the market-specific barrier is moderately high (Figure 5.5). In this case, the individual arbitrage condition binds for some values of $t$ and the importing firms' arbitrage condition binds for other values. In this case individual arbitrage can discipline the market concentration in the home market and result in a reduced international price differential at low levels of $t$. However, at trade costs above $\bar{t}^c$ individual arbitrage is no longer profitable and importing firms can enter. More precisely:

$$\bar{t}^c = a - p_w - \frac{1}{2}\sqrt{\frac{b}{S}(G+A)} - \frac{T\sqrt{bS}}{\sqrt{A+T}}. \tag{5.35}$$

At variable trade costs in the range of 0 to $\bar{t}^c$, the individual arbitrage condition binds and in the range of $\bar{t}^c$ and $\bar{t}^a$ the importing firms' arbitrage condition binds and importing firms serve to reduce international price differentials.

In the fourth case, the fixed cost of transportation is moderately high and the market-specific barrier is relatively high (Figure 5.6). In this case, none of the arbitrage conditions bind and the autarchy price is unaffected by changes in the variable trade cost.

Next, some comparative statics can be performed on the results previously obtained. In particular, it is interesting to analyze how the equilibrium price and the likelihood for arbitrage change as we change the market-specific costs and the size of the market.

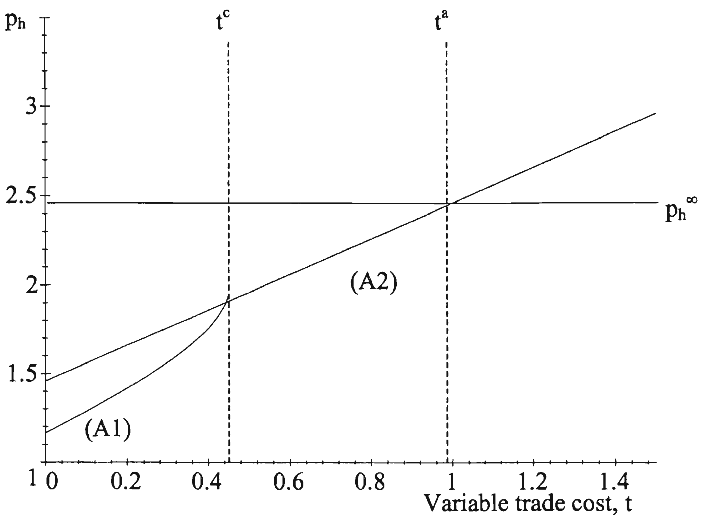


FIGURE 5.5. $T$ is relatively small, $A$ is relatively large

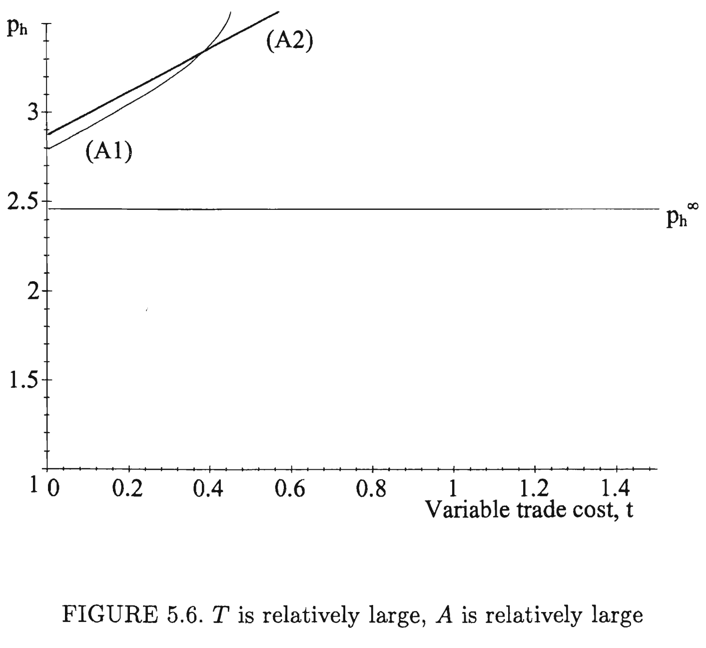


FIGURE 5.6. $T$ is relatively large, $A$ is relatively large

First, consider different levels of $\lambda$ when individual arbitrage is unprofitable. Take the partial derivative of the home market price, $p_h$, and the threshold level, $\bar{t}^a$, with respect to $\lambda$ to obtain:

$$\frac{\partial p_h}{\partial \lambda} = \begin{cases} \frac{1}{2}\frac{\sigma^m\sqrt{c}}{\sqrt{T+\lambda\sigma^m}} > 0 & ,t < \bar{t}^a \\ 0 & ,t \geq \bar{t}^a \end{cases} \tag{5.36}$$

$$\frac{\partial \bar{t}^a}{\partial \lambda} = -\frac{1}{2}\frac{\sigma^m\sqrt{b}}{\sqrt{ST+S\lambda\sigma^m}} < 0. \tag{5.37}$$

The equilibrium price in an equilibrium with importing firms is high when the market-specific cost is a large share of the total fixed cost of a manufacturing firm. Moreover, importing firms are less likely, if the market-specific cost is a large share of the total fixed cost of a manufacturing firm.

Second, consider the effect of differences in market size. Take the partial derivative of the equilibrium price and the threshold levels with respect to the size of the home market to obtain:

$$\frac{\partial p_h}{\partial S} = \begin{cases} -\frac{\sqrt{b}}{2S^{3/2}}\sqrt{T+\lambda\sigma^m} < 0 & ,t < \bar{t}^a \\ -\frac{\sqrt{c}}{2S^{3/2}}\sqrt{\sigma^m} < 0 & ,t \geq \bar{t}^a \end{cases} \tag{5.38}$$

$$\frac{\partial \bar{t}^a}{\partial S} = -\frac{\left(\sqrt{b\sigma^m} - \sqrt{(T+\lambda\sigma^m)\,b}\right)}{2S^{3/2}} < 0 \tag{5.39}$$

The absolute value of the partial derivative of the price w.r.t. $S$ is larger for variable transportation costs above the threshold level $\bar{t}^a$.[7] The price falls more quickly in market size, if trade costs admit importing firms to enter the domestic market and, thus, importing firms are less likely if the market is large. Moreover, the importing firms' arbitrage condition shifts much faster than the individual arbitrage condition as the market size increases:

$$\frac{\partial \bar{t}^c}{\partial S} = -\sqrt{\frac{b}{S}}\left(\frac{2ST-(A+T)}{4S\sqrt{(A+T)}}\right) < 0. \tag{5.40}$$

and individual arbitrage is less likely relative to importing firms' arbitrage, if the home market is large. The main reason is that importing firms' arbitrage becomes relatively more profitable compared to individual arbitrage as the market size increases.

Third, consider the effects of a higher market-specific barrier. Take the

[7] $T+\lambda\sigma^m < \sigma^m$ or there would be no entry of importing firms and $\bar{t}^a \leq 0$.

partial derivatives with respect to the market-specific cost to obtain:

$$\frac{\partial p_h}{\partial A} = \begin{cases} \frac{1}{2}\sqrt{\frac{b}{S(T+\lambda\sigma^m)}} > 0 & , t < \bar{t}^a \\ \frac{1}{2}\sqrt{\frac{b}{S\sigma^m}} > 0 & , t \geq \bar{t}^a \end{cases} \tag{5.41}$$

$$\frac{\partial \bar{t}^a}{\partial A} = -\sqrt{\frac{b}{S}}\left(\frac{\sqrt{\sigma^m} - \sqrt{T+\lambda\sigma^m}}{2\sqrt{\sigma^m}\sqrt{T+\lambda\sigma^m}}\right) < 0 \tag{5.42}$$

The absolute value of the partial derivative of the price w.r.t. to $A$ is larger if the variable transportation cost is below the threshold level. Thus, the equilibrium price in the home market increases more quickly in the market-specific fixed cost, if the variable transportation cost admits entry of importing firms. Moreover, importing firms are more likely, if the market-specific barriers are low.

## 5.6 Main Results

This section will sum up the results from the analysis. We can start the discussion with some of the main properties of the model. The main features can be illustrated in four different cases.

In our model, the transportation technology can exhibit large or small economies of scale. If there are no fixed cost of transportation, there will exist no importing firms in equilibrium. In this case individual arbitrage can discipline market concentration and result in a lower home market price. There exists a one-to-one relationship between price variability in the world market and the domestic market.

A second case of international price equalization occurs, when the fixed cost of transportation is large for a single consumer, but relatively small for an importing firm. If production exhibits large economies of scale and the market-access cost is relatively low, importing firms will approximately equalize international prices as the variable trade cost is reduced.

A third case occurs, when the fixed cost of transportation is relatively small and the market-specific cost is moderately high. In this case the home market concentration is disciplined by individual arbitrage at low variable transportation costs and disciplined by importing firms' arbitrage at intermediate levels of the variable transportation cost.

A fourth case occurs, if there are high market-specific barriers, no scale-economies in production, but economies of scale in transportation. In this case, the arbitrage conditions hold with strict inequality and no arbitrage is undertaken. The transportation technology does not allow for any individual arbitrage. Moreover, the firm-specific fixed costs are higher for importing firms than domestic producers and no trade will occur in equilibrium. In this case international prices can be widely dispersed.

In the first, second and third case there is a link between international prices. If the world market price changes, it immediately affects the domestic price. On the other hand, in the fourth case there is no immediate link and domestic prices will be unaffected by changes in the world market. In this case the barriers to entry in the local market, measured as the size of the market-specific cost, work as a buffer to international disturbances.

After this presentation of the general properties of the model, the results of the comparative statics will be summarized.

First, the *relative* size of the market-specific barrier affects the equilibrium. At any level of the variable transportation cost admitting importing firms in equilibrium, the equilibrium price is high when the share of market-specific fixed cost is large. Moreover, importing firms are less likely.

Second, the effect of changes in the *level* of the market-specific fixed cost is unambiguous. Higher market-specific setup costs would result in a higher equilibrium price. The effect on the price level is stronger when importing firms have entered the home market.

Third, the *size* of the domestic market also influences the equilibrium in two ways. Importing firms is less likely and the equilibrium price is strictly lower, if the domestic market is large compared to a smaller market.

Finally, if the transportation technology exhibits relatively small fixed costs, then individual arbitrage would result in a reduced international price differential. Individual arbitrage is less likely when the home market is large.

## 5.7 Extensions

We consider two extensions. First, the timing of the game is extended to allow for an unexpected change in the variable trade cost. Second, we consider an ad valorem tariff, rather than per-unit variable transportation costs.

### *(i) An Unexpected Change in the Variable Trade Cost*

The results in the previous parametric example are robust to an extended theoretical experiment, e.g. an unexpectedly reduced variable transportation cost. The experiment is conducted in the following way:

First, the variable transportation cost is prohibitive and does not admit any importing firms or any individual arbitrage. $n^m$ producing firms enter the home market. The domestic producers expect the high variable transportation cost to remain. The domestic producers compete in quantities in a homogeneous good. In equilibrium, no manufacturing firms can enter the domestic market profitably.

Second, the trade policy is changed, i.e. the variable trade cost $t$ is reduced and $n^a$ importing firms enter the home market. Since the firm-specific costs incurred by the domestic producers are sunk, all $n^m$ producing firms remain in the market if operating profits are positive. The importing and producing firms choose optimal quantities. The domestic market clears.

All manufacturing firms remain in the market as the operating profit is positive, $p_h \geq c$, and importing firms enter until the free-entry condition binds

$$p_h \leq p_w + t + \sqrt{\frac{b}{S}(T + A)}, \tag{5.43}$$

which is the same equilibrium price as in the standard case. The number of importing firms, however, will be smaller and more fixed costs are used as the manufacturing firms operate at prices below average total cost.

## *(ii) Ad Valorem Tariffs*

The cost function for importing firms could easily be extended to allow for other specifications. One alternative is an ad valorem tariff. In this case an importing firm maximizes its profit:

$$\pi^a\left(X_i^a, X_{-i}\right) = \frac{1}{\tau}\left(a - \frac{b}{S}X\right)X_i^a - p_w \cdot X_i^a - \sigma^a, \tag{5.44}$$

where $\tau \geq 1$ is a measure of the ad valorem tariff. The resulting first order condition and the free-entry condition:

$$\frac{1}{\tau}\left(a - \frac{b}{S} \cdot n^m X^m - \frac{b}{S} \cdot (n^a + 1) X^a\right) - p_w \leq 0 \qquad (X^a) \tag{5.45}$$

$$\frac{1}{\tau}\left(a - \frac{b}{S}n^m X^m - \frac{b}{S}n^a X^a\right) - p_w - \frac{\sigma^a}{X^a} \leq 0 \qquad (n^a), \tag{5.46}$$

which is a complementary slackness problem. Complementary variables are indicated in brackets next to the condition. Solving for $X^a$ the free-entry condition can be reduced to

$$p_h \leq \tau \cdot p_w + \sqrt{\frac{\tau \cdot b}{S}(T + A)} \qquad (n^a), \tag{5.47}$$

and the difference between the domestic and the world market price is larger than the average trade cost, if $A > 0$.

## 5.8 Conclusions

The price differential between the domestic market and the world market can be large in the autarchic equilibrium, either due to a high market-specific barriers or to a large scale-economies in production. The effects of a reduced variable trade cost depends crucially on the relation between the fixed costs. If high domestic prices in autarchy is due to large scale-economies in production, the effects of a lower variable transportation cost can be expected to result in more substantial changes in the domestic price. If the high domestic price is due to high barriers to entry in the domestic market, the effect of a reduced variable transportation is small. In the latter case, the pro-competitive effect can be reinforced if market barriers are dismantled while trade costs are reduced.

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# Appendix A. The Linear Model

Let the utility function of the representative consumer be separable and linear in the numeraire good (an extended version of Dixit, 1979; see also Singh and Vives, 1984):

$$U(x,y) = a\sum_{k=1}^{n} x_k - \frac{b}{2}(1-g)\left(\sum_{k=1}^{n} x_k^2\right) - \frac{b}{2}g\left(\sum_{k=1}^{n} x_k\right)^2 + y \qquad (5.48)$$

where $n$ is the number of varieties of the differentiated good and $y$ is the consumption of a composite good. Let $a > c$. The degree of product differentiation is measured by $1/g^2$. We assume that $0 < g \leq 1$, i.e. the differentiated goods are substitutes. Product differentiation increases as $g$ decreases.

The price of the numeraire good is $w$ and $p_k$ is the price of variety $k$ in the $X$ sector. The budget constraint for each consumer is $\sum_{k=1}^{n} p_k x_k + wy \leq I$. Maximizing the utility with respect to the budget constraint and normalizing the relative prices in terms of the composite good ($w = 1$) gives the following set of inverse demand functions:

$$p_i(x_i, x_{-i}) = a - bx_i - bg\sum_{k\neq i} x_k \qquad (5.49)$$

for $i = 1, ..., n$, where $x_i$ is the consumption of good $i$ and $x_{-i}$ is a vector of consumption of the $(n-1)$ other goods. The individual demand for each variety is

$$x_i(p_i, p_{-i}) = \frac{a(1-g) - (1-2g+ng)p_i + g\sum_{k\neq i} p_k}{b(1-g)(1-g+ng)}, \qquad (5.50)$$

and the aggregate demand for each variety is $X_i(p_i, p_{-i}) = S \cdot x_i(p_i, p_{-i})$, where $p_{-i}$ is a vector of the prices of the $n-1$ competitors.

### Price competition

In the price competition game, products are assumed to be differentiated, i.e. $0 < g < 1$. In the last stage of the game, firms choose prices to maximize their profit, holding all the other players' strategies constant. In the last stage, $n$ firms each choose a price, $p_i$, maximizing their individual profit

$$\pi(p_i, p_{-i}) = p_i \cdot X_i(p_i, p_{-i}) - c \cdot X_i(p_i, p_{-i}) - \sigma, \qquad (5.51)$$

where $p_{-i}$ is a vector of prices for the $n-1$ competitors. The subgame equilibrium for $n$ firms is solved as a system of equations (i.e. reaction functions). As each firm is symmetric, the subscript $i$ can be dropped. The equilibrium price is

$$p = \frac{a(1-g) + r + (n-2)gc}{2 + (n-3)g} \tag{5.52}$$

for each variety. First, take the derivative of the price with respect to the number of firms to obtain

$$\frac{dp}{dn} = -\frac{(a-c)(1-g)g}{(2+(n-3)g)^2} < 0 \tag{5.53}$$

and, hence, C1 is satisfied. Substitute the equilibrium price in the individual inverse demand function to obtain the equilibrium quantity. Take the derivative of the equilibrium quantity with respect to the number of firms and use the previous inequality to obtain:

$$\frac{dx}{dn} = \frac{-\frac{dp}{dn}(b - bg + nbg) - (a-p)gb}{(b - bg + nbg)^2} < 0 \tag{5.54}$$

for all $n \geq 2$ and C2 is satisfied.

### Quantity Competition

Next, consider competition in quantities. Products can either be differentiated or homogeneous. It is assumed that in the last stage of the game, firms choose quantities to maximize their profits, holding the strategies of all the other firms fixed. In the last stage, $n$ firms each choose a quantity, $x_i$, maximizing their individual profit.

$$\pi(X_i, X_{-i}) = p_i(X_i, X_{-i}) \cdot X_i - c \cdot X_i - \sigma \tag{5.55}$$

where $X_{-i}$ is a vector of quantities for the $n-1$ competitors. The subgame equilibrium for $n$ firms is solved as a system of equations. As each firm is symmetric, the subscript $i$ can be dropped. The equilibrium quantity per capita is

$$x = \frac{a-c}{(2+(n-1)g)b} \tag{5.56}$$

for each variety. Now, insert the equilibrium quantity in the individual inverse demand function to obtain the equilibrium price

$$p = \frac{a-(n-1)gc}{2+(n-1)g} \tag{5.57}$$

Now, we can show that all assumptions in section 2 are satisfied. First, take the derivative of the price with respect to the number of firms

$$\frac{dx}{dn} = -\frac{(a-c)\,g}{(2+(n-1)\,g)^2\,b} < 0, \tag{5.58}$$

where $a > c$. Correspondingly, we can take the derivative of the equilibrium price with respect to the number of firms to obtain:

$$\frac{dp}{dn} = -\frac{(a+2c)\,g}{(2+gn-g)^2} < 0 \tag{5.59}$$

for all $n \geq 2$.

# Appendix B. The Autarchy Equilibrium

Begin with $n^m$ domestic producers in the last subgame. Solving the $n^m$ first-order conditions, we obtain the equilibrium quantities

$$X^m = \frac{S\,(a-c)}{b\,(n^m+1)} \tag{5.60}$$

and insert the equilibrium quantities in the variable profit function. Working backward to stage one, a firm would enter if the fixed costs were covered:

$$G + A \leq \frac{S}{b}\left(\frac{a-c}{n^m+1}\right)^2 \tag{5.61}$$

which is more likely to be satisfied the lower are the fixed costs. The number of domestic producers in equilibrium is:

$$n^m = (a-c)\sqrt{\frac{S}{G+A}} - 1 \tag{5.62}$$

and, finally, inserting equilibrium quantities and the equilibrium number of domestic producers in the inverse demand function, we obtain the autarchy equilibrium price:

$$p_h = c + \sqrt{\frac{b}{S}\,(G+A)} \tag{5.63}$$

which converges to the marginal cost of production $c$, as the market size $S$ increases.

## Appendix C. Importing Firms' Arbitrage

Consider the arbitrage regime. In the last stage, $n^a$ importing firms and $n^m$ manufacturing firms have entered. Combining the reaction functions of the importing firms with the reaction functions of the domestic producers, we obtain the equilibrium quantity:

$$X^a = \frac{a - (\overline{n}^m + 1)(p_w + t) + \overline{n}^m \cdot c}{c(\overline{n}^m + n^a + 1)} \tag{5.64}$$

for each importing firm. Each domestic producer will choose a quantity:

$$X^m = \frac{a - (n^a + 1)c + n^a \cdot (p_w + t)}{c(\overline{n}^m + n^a + 1)}, \tag{5.65}$$

where $\overline{n}^m$ is fixed. We note that the new quantity is slightly different from the quantity of a domestic producer in the autarchy equilibrium. Now, subgame equilibrium quantities are determined for both importing firms and domestic producers, at the last stage of the game. We can combine the equilibrium quantities to obtain the equilibrium price at the second stage of the game

$$p_h = \frac{a + \overline{n}^m \cdot c + n^a (p_w + t)}{\overline{n}^m + n^a + 1}, \tag{5.66}$$

where $\overline{n}^m$ domestic producers have entered and $n^a$ importing firms have entered in the last stage subgame. Working backward we can determine the equilibrium number of firms. An importing firm $j$ will enter if the fixed costs are covered and the equilibrium number of importing firms is

$$n^a = \frac{(a - (\overline{n}^m + 1)(p_w + t) + \overline{n}^m \cdot c)\sqrt{S}}{\sqrt{b(A + T)}} - \overline{n}^m - 1, \tag{5.67}$$

which is determined by the exogenous parameters. The equilibrium number of firms can be inserted in the price equation to give the equilibrium price, if the fixed costs allow for the entry of importing firms:

$$p_h = p_w + t + \sqrt{\frac{b}{S}(A + T)} \tag{5.68}$$

However, the last stage includes subgames, where the tariff does not admit profitable entry of any importing firms. In this case, the price will equal the autarchy price.

# Appendix D. Individual Arbitrage

Utility maximization gives that each consumer would buy

$$q = \frac{a-p}{b} \tag{5.69}$$

units of the homogenous good at price $p$. If he chooses to buy the good in the world market he would have to incur the fixed transportation cost $T$ and he would get:

$$v\left(p_w + t, I - T\right) = \frac{1}{2b}\left(a - p_w - t\right)^2 + I - T \tag{5.70}$$

and the utility obtained when the homogeneous good is bought in the world market is exclusively given by exogenous parameters. At any level of transportation costs, $t$ and $T$, the utility level is fixed. On the other hand, if the consumer buys the good in the home market he would get the utility:

$$v\left(p_h, I\right) = \frac{1}{2b}\left(a - p_h\right)^2 + I \tag{5.71}$$

where $I$ is income and $p_h$ is the price in the home market. Next, set

$$v\left(p_w + t, I - T\right) = v\left(p_h, I\right) \tag{5.72}$$

and we solve for $p_h$ to obtain the arbitrage free price.

# 6

# Third-Degree Price Discrimination and Arbitrage in International Trade

## 6.1 Introduction

This paper contributes to the discussion of the causes and consequences of arbitrage in international trade.

Arbitrage in imperfectly competitive markets often takes the form of parallel imports, or "gray-market" imports, that is, genuine products imported by unauthorized retailers.[1] A common situation is that one firm owns the national trademarks in several countries, and each trademark confers the exclusive distribution right in that country. Another party then obtains the product in one market and exports it to another, without the authorization of the trademark holder. The basic question in this situation is: Should a manufacturing firm be entitled to enforce exclusive distribution territories?

Opponents of arbitrage argue that international arbitrage is mainly profitable because arbitrage firms free-ride on the investments of authorized distributors at various levels of the distribution chain.[2] Proponents of arbitrage, on the other hand, argue that parallel import is primarily a response to the international price discrimination that a supplier tries to sustain, via exclusive distribution territories over different national markets and that arbitrage undermines such, potentially welfare-reducing, discrimination.[3]

There now exists a large literature on third-degree price discrimination as well as many studies of local investments in the presence of spillovers.

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[1] "Gray-market imports" are defined by the U.S. Customs Service as those products bearing genuine trademarks that are imported into the United States by unauthorized distributors, when the authorized distributor is related to the foreign trademark holder.

[2] see e.g. Lexecon (1985), DeMuth (1990) and Chard and Mellor (1989)

[3] There is now extensive empirical evidence of price discrimination in international oligopolies. Phlips (1983) gives numerous examples of spatial price discrimination. Flam and Nordström (1994) and Verboven (1996) show that firms have discretion in setting different automobile prices in different European countries. The comprehensive literature on pricing to market also supports this conjecture (see, for instance Krugman and Baldwin, 1987, Feenstra, 1989, and Knetter, 1989. For a survey, see Froot and Rogoff, 1995).

The effects of price-discrimination in the presence of both arbitrage and market-specific investments, however, yet remain to be analyzed. In this paper, this problem is modeled as a multi-stage game. In the first stage, the manufacturing firm makes market-specific investments. Second, barriers to entry in the arbitrage sector are determined. Third, arbitrage firms enter the market. Fourth, the manufacturing firm chooses output. Finally, arbitrage firms ship some goods between markets and markets clear.

Generally, the total welfare effect of price-equalization is a trade-off between the benefits of equalized marginal valuations in all markets and the cost of arbitrage. In the long-run, however, it is also a trade-off between reduced monopoly distortions and insufficient market-specific investments, due to externalities and free-riding.

My starting point is the well-known result that if price discrimination does not result in higher output, the total welfare will be higher under uniform pricing, since marginal valuations in all markets are equalized (a result originally due to Robinson, 1933, and formally proved by Schmalensee, 1981, Varian, 1985, and Schwartz, 1990). This result, however, is static and depends crucially on costless price-equalization. Therefore, I introduce costly arbitrage and endogenous market-specific investments and ask the following three questions: What is the welfare effect of a reduced international price differential when arbitrage uses real resources? Does obstructive behavior by the price discriminating firm change the total effect on welfare? Will the incentives for market-specific investments be too weak if arbitrage is permitted?

First, by explicitly modelling arbitrage undertaken by importing firms, we can study the effects of real resources being used in arbitrage activities. The results show that if arbitrage uses real resources, the positive welfare effect of a reduced price differential is moderated. The effect is especially large at intermediate barriers to entry in the arbitrage sector. This result is interesting, since several studies suggest that international market integration would result in substantial welfare gains for the integrated economies.[4] The analysis in this paper suggests, however, that resources used in arbitrage activities can moderate these gains substantially.

It should also be noted that a monopolist has an incentive to discourage arbitrage, if possible, by raising the barriers to entry in the arbitrage sector. In this case, the cost of arbitrage is endogenously determined and my results show that neither laissez-faire nor restricting arbitrage is an optimal policy.

[4]Smith and Venables (1988) suggest that substantial welfare effects are expected as a result of the European integration. The largest effect is obtained if markets are completely integrated, i.e. prices are completely equalized. Baldwin and Venables (1995) summarize several studies with similar results.

Instead, the optimal policy in the short-run is to prevent the manufacturing firm from raising the barrier in the arbitrage sector, while arbitrage is, at the same time, permitted.

Moreover, the incentives for market-specific investments have to be considered.[5] Free-riding on market-specific investments may result in some redistribution of investments from markets with a large consumption to markets with a lower consumption. This effect can moderate and even reverse the positive welfare effect of uniform pricing.

The rest of the paper is organized as follows. In section 2 we present a simple two-country model with a single manufacturing firm. Section 3 focuses on the segmented equilibrium without arbitrage. The manufacturing firm chooses a level of local services in each market, which raises the value of the product for all consumers in that market. Next, it chooses output levels in each market. Section 4 introduces arbitrage. Arbitrage firms buy the goods in the low price market and ship them to the high price market. In the short-run equilibrium the manufacturing firm can change its output in the two markets but local services remain at the same level as in the segmented equilibrium. As an intermediate step, section 5 considers the manufacturing firm's incentive to raise its rivals' costs. In the medium-run equilibrium the manufacturing firm can make a costly investment to raise the barriers to entry for arbitrage firms in order to increase the scope for price discrimination. Finally, section 6 derives the long-run equilibrium in which the manufacturing firm can change all its strategic variables, including the level of local services. To discourage arbitrage, the manufacturing firm can reduce the level of local services in the high price market and increase the level of services in the low price market. Section 7 presents the welfare analysis, section 8 discusses the policy implications and section 9 concludes.

## 6.2 The Model

Consider two countries, referred to as markets 1 and 2. Let $x$ denote the consumption of goods produced in a sector with imperfect competition. Assume that each consumer has a valuation, $v$, for one unit of the good. The product is sold with a market specific, non-traded complementary service. In market $i$, there is a continuum of $e_i$ consumers with valuations

[5] Posner (1981), Mathewson and Winter (1984), (1986) discuss in some detail how vertical territorial arrangements (closed territory distribution) can enhance local investments in the presence of externalities.

uniformly distributed on $[0, e_i]$.[6] Without loss of generality, it is assumed that market 1 is the large market with high average valuation compared to market 2, i.e. $e_1 > e_2$. Now, the surplus for a single consumer from consumption of one unit of the good is

$$u = v - p_i + s_i, \tag{6.1}$$

where $p_i$ is the price of the good and $s_i$ is the level of services in market $i$. For simplicity, it is assumed, that a higher level of services is a perfect substitute for price reduction. The complementary service provided in market $i$ is market-specific, consumption non-rivalry and free for all consumers in that market, i.e. services are a market-specific public good. A consumer in market $i$ buys a unit of the good if the total value, $v + s_i$, is higher than the price, $p_i$. Thus, demand in market $i$ is

$$x_i^c = e_i + s_i - p_i, \tag{6.2}$$

which is linear in prices and services.[7] In this paper it is assumed that the difference in $e_i$ is moderate and it is always profitable to serve both markets.

On the supply-side, there are two types of firms; manufacturing firms (type $m$) and arbitrage firms (type $a$).

First, consider the manufacturing firm. It can choose the level of services $s_i$ in each market. The cost is $c(s_i)$, which is assumed to be quadratic, i.e. $c(s_i) = s_i^2$. It can also choose the barrier to entry for arbitrage firms, $\sigma^a$. The cost of raising the fixed cost of arbitrage firms is $h(\sigma^a) = r\sigma^a$. Finally, it chooses output for each market $x_i^m$. Hence, the manufacturing firms' profit is:

$$\pi^m = \sum_{i=1}^{2} [p_i x_i^m - c(s_i)] - h(\sigma^a). \tag{6.3}$$

Second, consider arbitrage firms. The arbitrage firms buy the goods in the low price market and sell them in the high price market. In order to enter the high price market the importing firm must incur a fixed cost $\sigma^a$.

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[6] It should be noted that the same analysis applies if both markets are equally large and the distribution with a higher average is truncated from below.

[7] The assumption of linear demand functions is certainly restrictive. However, uniform pricing is normally welfare enhancing if demand functions are linear, this therefore is a natural point of departure, if the aim is to study counteracting effects (an assumption adopted from Malueg and Schwartz, 1994). Even if prices and marginal valuations are equalized, total output will not be reduced, as long as all markets are served. Hence, negative effects must stem from other sources.

Hence, the profit of an arbitrage firm is

$$\pi^a = (p_1 - p_2)\, x^a - \sigma^a \tag{6.4}$$

and arbitrage firms will only enter if the profit is non-negative.

Competition is modelled as a multi-stage game with sequential actions. Investments are irreversible and, thus, associated with sunk costs.

In the first stage, the manufacturing firm chooses the level of services in market $i$, i.e. $s_1$ and $s_2$. The manufacturing firm incurs a sunk cost $c\,(s_i)$ in market $i$ and the level of services in both markets is made public. In the second stage, the manufacturing firm can raise the fixed cost, $\sigma^a$, which is made public. In the third stage, $n$ arbitrage firms enter and incur sunk costs, $\sigma^a$. All firms are informed about $n$. In the fourth stage, the manufacturing firm chooses an output for market $i$, i.e. $x_1^m$ and $x_2^m$. Arbitrage firms are informed about the manufacturing firm's output decision. In the fifth and last stage, arbitrage firms choose quantities $x^a$, shipped from market 2 to market 1. Finally, markets clear.

## 6.3 The Segmented Equilibrium

Exogenous differences in the willingness to pay for the product in the two countries make third-degree price discrimination profitable. The monopolist will choose a higher price in a market with higher marginal valuation.

The cost of market-specific investment in services is symmetric, but the exogenous difference in the willingness to pay generates incentives for the monopolist to make a larger investment in the market with higher valuations, due to a leverage effect. Output will be larger in the market with higher valuations and therefore, it is more profitable to use resources to marginally raise the level of services in this market, rather than in the market with a lower willingness to pay. Thus, the market-specific investments will further increase the difference in optimal prices.

Before we proceed to analyze the manufacturing firm's strategies in presence of arbitrage the segmented equilibrium will be studied. No arbitrage occurs in the segmented equilibrium and the manufacturing firm is free to optimize, without restrictions on the price differential between the markets. More precisely,

**Proposition 1** *In the segmented equilibrium* $x_i^m = 2e_i/3$, $p_i = 2e_i/3$ *and* $s_i = e_i/3$ *for* $i = 1, 2$.

**Proof.** In the segmented equilibrium the manufacturing firm maximizes its profit

$$\pi^m = \sum_{i=1}^{2} (e_i + s_i - x_i^m)\, x_i^m - s_i^2 \tag{6.5}$$

choosing optimal quantities, $x_i^m = (e_i + s_i)/2$ and optimal investment, $s_i = e_i/3$, for $i = 1, 2$. Prices are obtained from the inverse demand functions. ■

In our model, a monopolist has incentives to price discriminate between two markets, when there are different marginal valuations in the two countries. This difference stems from exogenous factors in the two countries and is further increased, due to a leverage effect on market-specific investments.[8]

## 6.4 Short-Run Equilibrium with Arbitrage

In this section, we consider free entry of arbitrage firms and derive the short-run equilibrium. In the short-run equilibrium, $s_1$ and $s_2$ are fixed. The short-run profit of the manufacturing firm is

$$\pi^m = p_1 (x_1^m + X^a) \cdot x_1^m + p_2 (x_2^m - X^a) \cdot x_2^m, \tag{6.6}$$

where $X^a$ is the (total) quantity chosen by arbitrage firms, $x_1^c = x_1^m + X^a$ the quantity consumed in market 1, and $x_2^c = x_2^m - X^a$ the quantity consumed in market 2. Correspondingly, the profit of an arbitrage firm is:

$$\pi^a = p_1 (x_1^m + X^a) \cdot x^a - p_2 (x_2^m - X^a) \cdot x^a - \sigma^a. \tag{6.7}$$

To simplify the notation it is useful to define two new parameters, $\psi \equiv (\sigma^a/2)^{1/2}$ and $\omega_i \equiv e_i + s_i$. For any vector of real numbers $(z_1, ..., z_n)$, let $\overline{z} = \frac{1}{n} \sum z_i$, and for $z_1, z_2 \in \mathbb{R}_+$ such that $z_1 > z_2$, let $\Delta z = z_1 - z_2$.

The equilibrium is derived in the usual backward fashion. We solve for optimal quantities of importing firms in the last stage; taking the output of the manufacturing firm as given. Then, we use the total output of arbitrage firms to obtain an optimal quantity for the manufacturing firm in

[8] Exogenous differences in willingness to pay for the goods is not the only reason to price discriminate in international markets. In other models of spatial price discrimination, a monopolist must incur a variable trade cost to export a product to a foreign market and the pass-through to consumers is only partial. The f.o.b. prices are lower for foreign consumers than for domestic ones and third-degree price discrimination occurs (for a survey, see e.g. Varian, 1988, or Tirole, 1988). A similar result is obtained in oligopoly models of intra industry trade, where a firm's domestic price is higher than its f.o.b. price for foreign consumers (see Brander, 1981, and Brander and Krugman, 1983).

the preceding stage. Next, we use the optimal quantities and the free-entry condition to determine the number of arbitrage firms.[9]

**Proposition 2** *In the short-run equilibrium,* $x_i^m = \frac{1}{2}\omega_i$ *for* $i = 1, 2$, $x^a = \psi$, $n = \frac{1}{4\psi}(\omega_1 - \omega_2) - 1$ *and the equilibrium prices are* $p_1 = \overline{\omega}/2 + \psi$ *and* $p_2 = \overline{\omega}/2 - \psi$.

**Proof.** In the last stage, there are $n$ (where $n \geq 1$) first-order conditions for arbitrage firms

$$\frac{d\pi^a}{dx^a} = (\omega_1 - x_1^m - (n+1)\,x^a) - (\omega_2 - x_2^m + (n+1)\,x^a) = 0, \tag{6.8}$$

which yield the optimal quantities

$$x^a = \frac{1}{2\,(n+1)}\left((\omega_1 - x_1^m) - (\omega_2 - x_2^m)\right), \tag{6.9}$$

which can be inserted into profit function of the manufacturing firms, to obtain first-order conditions for the market $i = 1, 2$:

$$\frac{d}{dx_i^m}\left(p_1\left(x_1^m, x_2^m, n\right) x_1^m + p_2\left(x_1^m, x_2^m, n\right) x_2^m\right) = 0, \tag{6.10}$$

with optimal quantities $x_i^m = \omega_i/2$ for $i = 1, 2$. Next, we use the free-entry condition of the arbitrage firms

$$(\omega_1 - x_1^m - nx^a) - (\omega_2 - x_2^m + nx^a) - \frac{\sigma^a}{x^a} = 0 \tag{6.11}$$

to derive the output of a single arbitrage firm, the total output of arbitrage firms and the number of arbitrage firms. More precisely:

$$x^a = \psi, X^a = \frac{\omega_1 - \omega_2}{4} - \psi, n = \frac{\omega_1 - \omega_2}{4\psi} - 1, \tag{6.12}$$

and equilibrium prices are $p_1 = \overline{\omega}/2 + \psi$ and $p_2 = \overline{\omega}/2 - \psi$. ■

The proposition shows that - at the same levels of market-specific investments as in the segmented equilibrium - the manufacturing firm will

[9] The equilibrium price and quantities derived in this game can also be supported as an equilibrium in a game with a slightly different timing. If the manufacturing firm chooses quantitities before arbitrage firms enter and choose quantities, the prices and quantities derived here will also be a subgame perfect equilibrium. However, a different timing incurs the problem of multiplicity of equilibria, which is avoided in the current game.

choose the same quantities for markets 1 and 2, as in the equilibrium without arbitrage and it will be left to the arbitrage firms to reduce the price differential.

The total cost of arbitrage activities in the short-run equilibrium is

$$n \cdot \sigma^a = \frac{\psi}{2}(\omega_1 - \omega_2) - 2\psi^2, \tag{6.13}$$

which is a concave function with a maximum at intermediate levels of $\psi$. The intuition is straightforward. There is no arbitrage at the entry-blocking barrier and the cost of arbitrage is consequently zero. Correspondingly, if the barrier is zero, the total cost of arbitrage is zero. At intermediate levels, on the other hand, the cost is strictly positive. The function is continuous and, thus, takes a maximum in the interior.

Finally, in the short-run equilibrium, the manufacturing firm's equilibrium profit is

$$\pi^m = \left(\frac{\overline{\omega}}{2} + \psi\right)\frac{\omega_1}{2} + \left(\frac{\overline{\omega}}{2} - \psi\right)\frac{\omega_2}{2}, \tag{6.14}$$

which is increasing in $\psi$. A higher barrier for arbitrage firms admits more extensive price discrimination and, consequently, results in higher profits for the manufacturing firm. Hence, the manufacturing firm has an incentive to raise the barrier or obstruct arbitrage, which is the topic of the next section.

## 6.5 Arbitrage with Endogenous Barriers

From the short-run analysis in the previous section, it is clear that the manufacturing firm benefits from higher barriers in the arbitrage sector. Hence, in this section, we consider the possibility for the manufacturing firm to determine the barriers to entry for arbitrage firms.[10]

The barriers can be changed in the medium-run and we model the strategic interaction as a multi-stage game similar to the short-run analysis. However, the entry of arbitrage firms is preceded by a stage where the manufacturing firm determines the barrier, $\sigma^a$. The last three stages of the game are identical to the short-run game in the previous section.

The prospects for the manufacturing firm of preventing arbitrage, differs from industry to industry and must be evaluated on a case-by-case basis. There are numerous cases where the manufacturing firm can obstruct arbitrage, even if arbitrage is formally permitted. The manufacturing firm

[10] For a mor general treatment of the incetives for raising rivals' costs, see for instance Salop and Scheffman (1983).

might directly try to discourage arbitrage, through legal actions against arbitrage firms, or indirectly, through measures on either the supply or the demand-side.

The most straightforward action for preventing arbitrage is to control deliveries in the low-price market. In that way, the manufacturing firm can change the conditions for the supply to arbitrage firms. For instance, the manufacturing firm can stop, delay or reduce deliveries to buyers identified as arbitrage firms, thereby forcing arbitrage firms to use middlemen (e.g. decoys). This, in turn would increase the cost and the uncertainty for arbitrage firms.

Moreover, the manufacturing firm is the only producer and it controls the standard of goods shipped to the two markets. Potentially, the manufacturing firm can diversify the products for the different local markets, for instance by using different packaging, user manuals, brand names or local standards. These actions obliges the arbitrage firms to make costly adjustments of the goods for the local market and, thus, arbitrage is discouraged.

Generally, actions taken by the manufacturing firm do not prevent arbitrage but they affect the entry conditions for arbitrage firms. More specifically, the barriers to entry in the arbitrage sector is raised and the profit of the manufacturing firm is

$$\pi^m = p_1\left(x_1^m + X^a\right) \cdot x_1^m + p_2\left(x_2^m - X^a\right) \cdot x_2^m - 2r\psi^2, \tag{6.15}$$

where the quantities are obtained from the last three stages of the game. Working backwards, it is possible to solve for the optimal barrier. If the marginal cost for raising the barrier is low ($r < 1/2$), the manufacturing firm will raise the barrier to an entry-deterring level. If $r \geq 1/2$, entry-deterrence is unprofitable and we have the following result:

**Proposition 3** *In the medium-run equilibrium,* $\sigma^a = \frac{1}{32r^2}\left(\omega_1 - \omega_2\right)$ *and* $\Delta p = \frac{1}{4r}\left(\omega_1 - \omega_2\right)$ *for* $r \geq 1/2$.

**Proof.** We obtain optimal quantities and prices from the short-run equilibrium. Working backwards

$$\pi^m = \left(\frac{\omega_1 + \omega_2}{4} + \psi\right)\frac{\omega_1}{2} + \left(\frac{\omega_1 + \omega_2}{4} - \psi\right)\frac{\omega_2}{2} - 2r\psi^2 \tag{6.16}$$

and the first order condition

$$\frac{d\pi^m}{d\psi} = \frac{1}{2}\left(\omega_1 - \omega_2\right) - 4r\psi = 0 \tag{6.17}$$

with a unique solution

$$\psi^* = \frac{1}{8r}\left(\omega_1 - \omega_2\right). \tag{6.18}$$

Inserting $\psi^*$ in the difference $\Delta p = 2\psi$ concludes the proof. ■

With a linear cost for the manufacturing firm of raising the barriers for arbitrage firms, the optimal level of the barrier is increasing in the difference of services and willingness to pay in the two markets. The intuition for this is that a larger difference in services and willingness to pay gives the manufacturing firm incentives for further price discrimination and, thus, preventing arbitrage and price-equalization becomes more profitable.

Next, the last proposition can be used for deriving the total cost for the manufacturing firm to raise the barrier in optimum. It can be shown that the total cost as well as the profit of the manufacturing firm in optimum are decreasing in the marginal cost of raising the barrier. Formally, we have the following result:

**Proposition 4** *In medium-run equilibrium, $d\pi^m/dr < 0$ and $dh/dr < 0$.*

**Proof.** Taking the derivative of the profit of the manufacturing firm in the medium-run equilibrium yields

$$\frac{d\pi^m}{dr} = -\frac{(\omega_1 - \omega_2)^2}{32r^2} \tag{6.19}$$

and, after inserting the solution $\psi^*$ into the cost function, we obtain

$$\frac{dh}{dr} = -\frac{(\omega_1 - \omega_2)^2}{32r^2} \tag{6.20}$$

both of which are strictly negative. ■

The first part of the proposition simply repeats the result that the manufacturing firm's profit is increasing in the barriers to entry for arbitrage firms. With a low marginal cost $r$, the manufacturing firm chooses a high barrier in optimum and the resulting profit is higher. The second part is more interesting, since it shows that a higher marginal cost reduces the total cost incurred by the manufacturing firm to raise the barrier in equilibrium.

## 6.6 Spillovers and Market-Specific Investment

In the long run, we must consider the manufacturing firm's opportunity to change the level of services for the goods in markets 1 and 2. In the long run, the manufacturing firm will respond strategically to the reduced price differential between markets. It will not be as profitable to invest in the high price market as in the segmented equilibrium. Instead, the manufacturing firm will increase its investments in the low-price market and reduce its investment in the high-price market, which will reduce the quantity of arbitrage.

Thus, in the long run, entry in the arbitrage sector will be profitable for a much smaller range of fixed costs compared to the situation in the short-run equilibrium. More specifically, entry will be profitable for some arbitrage firms in the long-run equilibrium, as long as fixed costs are $\sigma^a \in [0, \underline{\sigma}]$, and we have the following result:

**Proposition 5** *If $\sigma^a < \underline{\sigma}$, then $s_1 = \frac{1}{3}\overline{e} + \frac{1}{4}\psi$ and $s_2 = \frac{1}{3}\overline{e} - \frac{1}{4}\psi$ in the long-run equilibrium.*

**Proof.** Rewrite the first-order conditions (using $s_i = \omega_i - e_i$) and solve

$$\frac{d}{d\omega_i}\left(\left(\frac{\overline{\omega}}{2}+\psi\right)\frac{\omega_1}{2}+\left(\frac{\overline{\omega}}{2}-\psi\right)\frac{\omega_2}{2}-\sum_{i=1}^{2}(\omega_i-e_i)^2\right)=0 \qquad (6.21)$$

to obtain the optimal solution

$$\omega_1 = \frac{7e_1+e_2}{6}+\frac{\psi}{4} \qquad (6.22)$$

$$\omega_2 = \frac{7e_2+e_1}{6}-\frac{\psi}{4} \qquad (6.23)$$

and $s_i = \omega_i - e_i$ gives the market-specific investments. ■

We can use this proposition and proposition 2 to derive the arbitrage blocking barrier:

$$\underline{\sigma} = \frac{8}{49}\left(e_1 - e_2\right)^2. \qquad (6.24)$$

The proposition shows that in the long-run equilibrium, the manufacturing firm will reduce the investment in the high-valuation market and increase the investment in the low-valuation market, compared to the segmented equilibrium. To understand the intuition for this result, we focus on the specific case with no barriers to entry in the arbitrage sector. In this case, prices between markets will be completely equalized. A marginal increase in the market-specific investment in market 1 will raise total demand for the manufacturing firm's product, to exactly the same effect as a marginal increase in the market-specific investment in market 2. Since prices are identical in both markets, the marginal revenue of a marginal increase in the investment in either market is equal in both markets. In optimum, the marginal revenue and marginal cost of an extra investment must be equal and, as we have just shown, equal in both markets. Marginal costs, however, are only equal in both markets, if investments are equal, which gives the intuition..

This concludes the analysis of the different games. Next, we proceed to analyze welfare in the short-, medium- and long-run equilibria and discuss some policy implications of the results.

## 6.7 Welfare Analysis

Welfare is the total consumer surplus plus the net profits of firms. Consumer surplus in market $i$ is

$$CS_i^j = \omega_i x_i^c - \frac{1}{2}\left(x_i^c\right)^2 - p_i x_i^c \tag{6.25}$$

and total producer surplus

$$PS^j = \sum_{i=1}^{2}\left(p_i x_i^c - s_i^2\right) - 2\psi^2\left(n+r\right) \tag{6.26}$$

and the sum of producer surplus and consumer surplus in the two markets is the following general formula

$$W^j = \sum_{i=1}^{2}\left(\omega_i x_i^c - \frac{1}{2}\left(x_i^c\right)^2 - s_i^2\right) - 2\psi^2\left(n+r\right), \tag{6.27}$$

where $n$ is the number of importing firms and the last term is the resources used in arbitrage activities. Superscript $j$ refers to the equilibrium evaluated. In the subsequent analysis, $SEG$ refers to the segmented equilibrium, $SR$ to the short-run equilibrium, $MR$ to the medium-run equilibrium and $LR$ to the long-run equilibrium. In some cases it is interesting to compare the welfare level in different equilibria to the welfare level when arbitrage does not use any real resources, denoted $W^{PQ}$. More generally, it is interesting to compare the welfare level when arbitrage is permitted and when it is prohibited.

First, in the short-run equilibrium, the welfare under the arbitrage regime is always higher than the segmented welfare for any given willingness to pay for the goods. However, the welfare gain from arbitrage is substantially lower if arbitrage uses real resources.

**Proposition 6** *If arbitrage uses real resources, then (i)* $CS_1^{SR} > CS_1^{SEG}$, *(ii)* $CS_2^{SR} < CS_2^{SEG}$, *(iii)* $PS^{SR} < PS^{SEG}$ *and (iv)* $W^{SR} > W^{SEG}$.

**Proof.** To prove part (i), compute the consumer surplus in market 1 in the short-run equilibrium and in the segmented equilibrium and take the difference:

$$CS_1^{SR} - CS_1^{SEG} = \frac{(5e_1 - e_2)(e_1 - e_2)}{18} - \left(e_1 - \frac{1}{3}e_2 - \frac{1}{2}\psi\right)\psi, \quad (6.28)$$

where the expressions within the first two brackets are positive and the last term is smaller than the first term for fixed costs below the arbitrage blocking barrier. Second, to prove part (ii), compute the consumer surplus in market 2 in the short-run equilibrium and in the segmented equilibrium and take the difference:

$$CS_2^{SR} - CS_2^{SEG} = -\frac{(5e_2 - e_1)(e_1 - e_2)}{18} + \left(e_2 - \frac{1}{3}e_1 + \frac{1}{2}\psi\right)\psi, \quad (6.29)$$

where the expressions in the first two brackets are positive, the product negative and the last term is smaller than the first product for fixed costs below the arbitrage blocking barrier. Third, in a similar way, we compute the difference in producer surplus

$$PS^{SR} - PS^{SEG} = -\frac{2}{9}(e_1 - e_2)^2 + \frac{2}{3}(e_1 - e_2)\psi, \quad (6.30)$$

which is negative for fixed costs in the relevant range. Finally, the difference in welfare is the sum of the previous three differences

$$W^{SR} - W^{SEG} = \frac{1}{9}(e_1 - e_2)^2 - \frac{2}{3}(e_1 - e_2)\psi + \psi^2, \quad (6.31)$$

which is strictly positive at $\psi = 0$. The difference is a decreasing function in $\psi$ and equals zero at $\psi = \frac{1}{3}(e_1 - e_2)$, which is the arbitrage-blocking barrier. ■

The intuition for this proposition is that arbitrage firms redistribute some goods from consumers with a lower valuation in market 2 to consumers with a higher valuation in market 1. The manufacturing firm chooses the same output for markets 1 and 2 in the short-run equilibrium as in the segmented equilibrium and, consequently, the aggregate output is the same. As more consumers in the high-valuation market can buy the goods at a price below their valuation, the consumer surplus in this market increases. Every unit transported from market 2 to market 1, however, means lower consumption in the low-valuation market and the consumer surplus decreases in this market.

The price difference corresponds to the difference in valuation between the markets and arbitrage firms enter as long as the price difference covers the average cost of transportation. The remaining price differential times

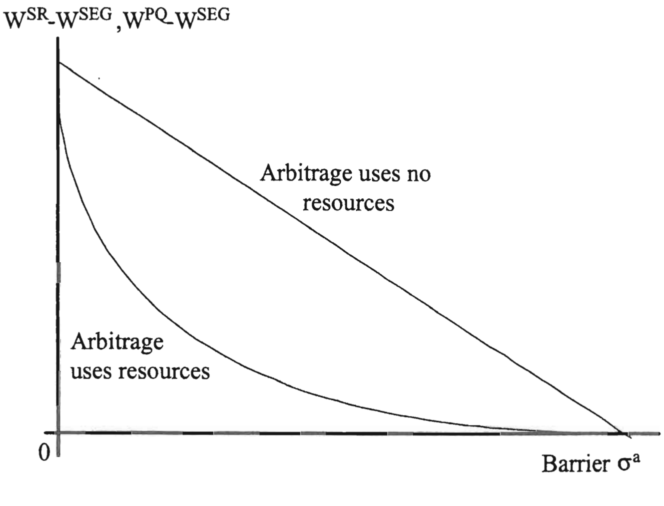


FIGURE 6.1. Welfare in the Short-Run and Segmented Equilibrium

the volume of arbitrage in the short-run equilibrium is the revenues of arbitrage firms, which must cover the total cost of arbitrage activities. However, except for the marginal unit transported between the two markets, the difference in valuation is larger than the price differential in the short-run equilibrium and, thus, arbitrage is welfare-enhancing in the short run. Finally, less price-discrimination reduces the manufacturing firm's profit in the short-run equilibrium. The arbitrage firms make no profit. Consequently, producer surplus is lower in the short-run equilibrium compared to the segmented equilibrium.

The result is illustrated in Figure 6.1. As expected, the difference converges to zero as we approach the threshold at which entry in the arbitrage sector is no longer profitable. This result is similar to the result in Brander and Krugman's (1983) that cross-hauling can have negative effects on welfare, despite a pro-competitive effect if too much resources are used in trade activities. The total effect, however, will always be positive if there is free entry in the imperfectly competitive sector. In Brander and Krugman's model, new entry of firms has a positive effect on welfare, while trade activities use real resources. In our model, however, the aggregate effect is a trade-off between reduced monopoly distortions and resources being used in arbitrage activities.

From a policy-perspective, it is also interesting to study how the welfare effects of arbitrage relates to a regulation defining a maximum price differential between markets. In other words, it is interesting to analyze how the cost of arbitrage relates to the positive effect of price equalization. For this purpose, we compute the welfare in an equilibrium at barrier $\psi$, but with no real resources used in the arbitrage activities:

$$W^{PQ} = \frac{2}{3}e_1^2 + \frac{2}{3}e_2^2 - \frac{2}{9}e_1 e_2 - \psi^2, \tag{6.32}$$

which is clearly positive for all fixed costs below the arbitrage-blocking barrier. We can consider this welfare level as an upper bound on what can be achieved in terms of welfare improvements in the short run at this level of the barrier. Next, we can use it to study two problems. First, it is interesting to compute how much of the potential welfare gain from a reduced price differential can be realized if arbitrage uses real resources. More specifically, we compute

$$\frac{W^{SR} - W^{SEG}}{W^{PQ} - W^{SEG}} = \frac{\Delta e - 3\psi}{\Delta e + 3\psi}, \tag{6.33}$$

which is a decreasing function in the level of the barrier. It shows that arbitrage is more equivalent to price regulation at lower levels of the barrier in the arbitrage sector. Higher barriers in the arbitrage sector absorb a large part of the potential welfare gain from price equalization. It does not, however, illustrate to what extent arbitrage activities use resources. In order to illustrate how much resources that are used in arbitrage activities we compute

$$W^{SR} - W^{PQ} = -\frac{2\psi}{3}\Delta e + 2\psi^2, \tag{6.34}$$

which is a convex function with a unique minimum. In other words, arbitrage activities are most costly at intermediate levels of the barrier. If there are no barriers, arbitrage is costless. At the entry-blocking barrier, no arbitrage occurs and, accordingly, it uses no resources. At intermediate levels, the total cost of arbitrage is first increasing and then decreasing, reaching a maximum in the interior. This suggest that arbitrage activities are particularly expensive at intermediate barriers, compared to price regulation.

Next, we analyze welfare in the medium-run equilibrium. It turns out that the cost of arbitrage activities will be of even greater importance for welfare results in the medium run. If the barrier is sufficiently high in optimum, welfare is lower in the medium-run equilibrium compared to the segmented equilibrium. The critical level is

$$\widehat{\sigma} = 0.032 \cdot (e_1 - e_2)^2 , \tag{6.35}$$

and we have the following proposition:

**Proposition 7** *If* $\sigma > \widehat{\sigma}$, $W^{MR} < W^{SEG}$.

**Proof.** We compute the welfare in the medium-run equilibrium and take the difference to the welfare in the segmented equilibrium

$$W^{MR} - W^{SEG} = \frac{1}{9}(e_1 - e_2)^2 - (e_1 - e_2)\psi + \psi^2, \tag{6.36}$$

which is strictly positive for $\psi = 0$. Now, the derivative of $W^{MR} - W^{SEG}$ w.r.t. $\psi$ is strictly negative for the relevant range of fixed costs. Moreover, $W^{MR} - W^{SEG} = 0$ at $\widehat{\sigma}$. The difference is a continuous function in $\psi$ and, thus, $W^{MR} - W^{SEG} < 0$ i.f.f. $\sigma > \widehat{\sigma}$. ■

In the medium run, the consumer surplus is the same as in the short-run equilibrium at the same level of barriers in the arbitrage sector, but the producer surplus is different. In the short-run equilibrium, we think of the barrier for arbitrage firms as an exogenously given parameter of the model. In the medium-run, however, the manufacturing firm will use some real resources to raise the barriers for arbitrage firms. At any given level of the barrier, it will have a negative effect on producer surplus. Accordingly, the producer surplus in the medium-run equilibrium is lower than the producer surplus in the short-run equilibrium, which also constitutes the main basis for the second part of the proposition.

If the barrier is sufficiently high, corresponding to a sufficiently low marginal cost of raising the barrier, the resources used to raise the barrier and resources used in arbitrage activities might dominate the positive effect of a reduced price-difference. The total effect of arbitrage can then be negative compared to the segmented equilibrium.

Consider, for instance, a low $r$ which allows the manufacturing firm to raise the barrier to an arbitrage-blocking level. In that case, the consumer surplus is the same as in the segmented equilibrium as no price equalization occurs, but the cost of raising the barrier has a negative effect on producer surplus. Thus, the total welfare is lower in the medium-run compared to the segmented equilibrium.

Next, consider a very high $r$. In this case, the positive effect of price equalization is considerable and the total cost of raising the barrier is relatively low. Thus, welfare is high compared to the welfare level in the segmented equilibrium. The result is illustrated in Figure 6.2.

The result suggests that a policy that raises the marginal cost for the manufacturing firm to raise the barrier for arbitrage firms enhances welfare.

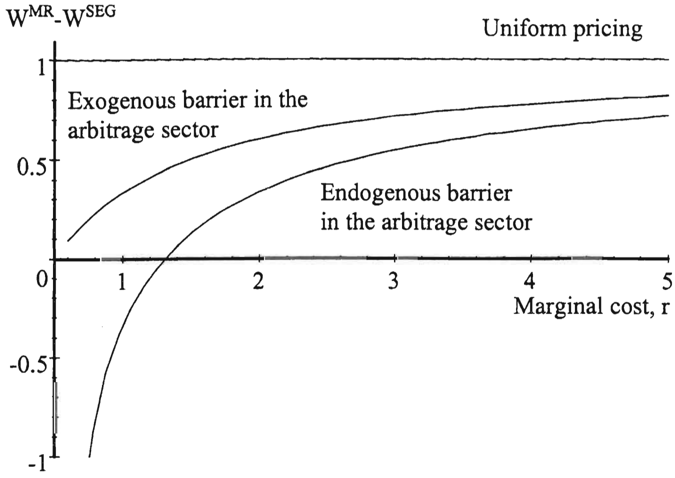


FIGURE 6.2. Welfare in the Medium-Run and Segmented Equilibrium

In optimum, less resources are used for raising the barriers in the arbitrage sector and in arbitrage activities, which is positive for total welfare.

Next, we proceed to study the long-run equilibrium. In the long run, the welfare effects are determined by the levels of market specific investments. The manufacturing firm chooses a lower market-specific investment in the high-valuation market and a higher market-specific investment in the low-valuation market compared to the short-run equilibrium. Thus, consumer surplus is negatively affected in the former and positively affected in the latter market, compared to the short-run equilibrium.

**Proposition 8** *(i)* $CS_1^{LR} < CS_1^{SR}$, *(ii)* $CS_2^{LR} > CS_2^{SR}$ *and (iii)* $PS^{LR} > PS^{SR}$.

**Proof.** To prove part (i), compute the consumer surplus in market 1 in the long-run and the short-run equilibrium and take the difference:

$$CS_1^{LR} - CS_1^{SR} = \frac{(e_2 - e_1)(11e_1 - 3e_2)}{72} + \left(\frac{9e_1 - 5e_2}{24} - \frac{7\psi}{32}\right)\psi, \quad (6.37)$$

where the expressions in the first bracket are negative and the ones in the second, positive, the product is negative and the last term is smaller than the first term. Second, to prove part (ii), compute the consumer surplus in market 2 and take the difference:

$$CS_2^{LR} - CS_2^{SR} = \frac{(11e_2 - 3e_1)(e_1 - e_2)}{72} + \left(\frac{5e_1 - 9e_2}{24} - \frac{7\psi}{32}\right)\psi, \quad (6.38)$$

where the expressions with in the two first brackets are positive, the product positive and the last term is smaller than the first product for fixed costs below the arbitrage blocking barrier. Third, in a similar way, we compute the difference in producer surplus

$$PS^{LR} - PS^{SR} = \frac{1}{18}(e_1 - e_2)^2 + \frac{1}{3}(e_1 - e_2)\psi - \frac{13}{8}\psi^2, \tag{6.39}$$

which is positive for fixed costs in the relevant range. ■

If prices are completely equalized, this result is easily understood. In the long-run equilibrium, the manufacturing firm invests less in the high-valuation market and more in the low-valuation market. With the same prices in the short-run and long-run equilibrium, the changes in market-specific investments result in lower consumption in market 1 and higher consumption in market 2. Lower consumption at lower service levels, creates a lower consumer surplus in market 1 and more consumption at higher service levels, creates a larger consumer surplus in market 2.

Finally, we have noticed that prices are the same in the short-run and long-run equilibria and that the total output is the same. Thus, the revenue of the manufacturing firm under complete price-equalization must be identical. In the long run, however, the manufacturing firm invests equal amounts in both markets and saves costs. Moreover, less arbitrage occurs. Therefore, total producer surplus is higher in the long-run compared to the short-run equilibrium. The three effects illustrated in this proposition also constitute the mechanism behind the next result.

The welfare level in the long-run equilibrium is lower than the welfare level in the segmented equilibrium. If prices are completely equalized, welfare is strictly lower.

**Proposition 9** *If* $\sigma^a < \underline{\sigma}$, $W^{LR} < W^{SEG}$.

**Proof.** To prove this proposition, compute the welfare in the long-run equilibrium and the segmented equilibrium and take the difference:

$$W^{LR} - W^{SEG} = -\frac{1}{36}(e_1 - e_2)^2 + \frac{1}{4}(e_1 - e_2)\psi - \frac{17}{16}\psi^2 < 0, \tag{6.40}$$

where the first part is negative and the sum of the two last terms smaller than the first part. ■

In the long run, the manufacturing firm reduces the level of investment in the high-valuation market and increases its investment in the low-valuation market, due to free-riding in the high-valuation market. Consumption in the high-valuation market is higher than consumption in the low valuation market and, therefore, a smaller number of consumers benefit from the increase in market-specific investments in market 2 than the number of consumers loosing from the reduced investment in market 1. Thus, the negative effect on consumer surplus in market 1 dominates the positive effect in market 2. Moreover, producer surplus is lower in the long-run equilibrium, compared to the segmented equilibrium. Hence, total welfare is lower in the long-run equilibrium compared to the segmented equilibrium.

## 6.8 Policy Discussion

The welfare analysis has important policy implications. The short-run analysis shows that arbitrage can improve welfare, while trade costs moderate the positive effects of price equalization. The medium-run analysis illustrates that endogenous barriers can reverse the positive welfare effects and, thus, it has some important implications for European policy on parallel imports. The long-run analysis suggests that free-riding by importing firms may result in an equilibrium where total welfare is lower than in the segmented equilibrium. These issues will now be discussed in some detail.

First, the short-run analysis highlights the importance of reducing the barrier to entry for arbitrage firms to a very low level, in order to obtain large welfare improvements. While the total consumer surplus increases considerably if high barriers are marginally reduced, the cost of arbitrage almost cancels this effect. When barriers are gradually dismantled, the positive marginal effect on total consumer surplus diminishes as the difference in marginal valuation is reduced. However, the total cost of arbitrage reaches a maximum at intermediate levels of the barrier in the arbitrage sector. At the arbitrage blocking barrier, the relative welfare improvement of arbitrage, compared to price-equalization, is zero and increasing to one as the barriers are completely dismantled. This suggests that barriers must be dismantled to a very low level to obtain most of the positive effects of price equalization through arbitrage activities. If this is impossible to achieve, arbitrage can be a very costly method to reduce international price differentials.

Second, the manufacturing firm has incentives to raise the barriers to entry for arbitrage firms. This can reverse the positive effect of price-equalization. If raising the barrier is costly and the resulting optimum is a relatively high barrier, the negative effect of resources used in arbitrage activities dominates the positive effects on consumer surplus. In this case, welfare is higher in the segmented equilibrium. Total welfare in the latter equilibrium is, however, lower than the total welfare in an equilibrium where the barriers to entry for arbitrage firms are very low. At any given level of the endogenous barrier, preventing the manufacturing firm from raising the barrier $\sigma$ is welfare enhancing. Total welfare increases for two reasons. The resulting optimal barrier is lower, which has a positive effect on welfare, since the price differential is reduced and arbitrage uses less real resources. Moreover, the total cost of raising the barrier is reduced, which also has a positive effect on welfare. Thus, the government can improve welfare above the level in the segmented equilibrium, if adopting a policy which discourages the manufacturing firm from raising the barriers for arbitrage firms. The local analysis shows that welfare is always marginally improved, if the cost of raising barriers is increased, while the global analysis shows that the cost must be sufficiently high for welfare to be higher in the medium-run equilibrium than in the segmented equilibrium.

This result is particularly interesting in the European context. According to European case law, arbitrage firms are allowed to adjust genuine products to local standards, in order to facilitate sales of goods imported from other countries. Arbitrage firms are allowed to correct for strategies creating an artificial partitioning of the internal market. For instance, products can be re-packaged and instructions translated even if such activities are, in normal cases, considered as violations of intellectual property rights.

It is worth noting that our analysis suggests that this is not necessarily an optimal policy. The medium-run analysis shows that the total welfare would be substantially higher if price-discrimination was banned and, accordingly, price-equalization achieved without costly arbitrage. An equivalent policy is to dismantle the barriers in the arbitrage sector entirely or to reduce the incentives of the manufacturing firms to raise barriers in the arbitrage sector completely.. For practical reasons, however, it may not be possible to adopt any of these policies in practice.

Hence, we must consider the current policy as a second-best solution to the problem of price-equalization. While this might be true for some cases, our analysis shows that it is not generally true. The medium-run analysis suggests that if the resulting endogenous barrier in the arbitrage sector is high, then welfare is higher in the segmented equilibrium. Thus, in some cases, preventing arbitrage is a second-best solution. For political and institutional reasons, however, it might not be a desirable policy.

Now, taking the endogenous barriers and arbitrage as a given policy the current European policy is, indeed, an optimal solution. The short-run analysis shows that when the cost of raising the barriers is reduced, welfare is higher in the short-run equilibrium compared to the segmented equilibrium. Thus, our conclusion is that European case law is not necessarily optimal from an economic perspective, but at least a third-best policy in the short and medium run.

Next, consider the trade-off between static efficiency and dynamic efficiency. The long-run analysis shows that a reduced price-differential results in weaker incentives for market-specific investments in the high-valuation market. If arbitrage is primarily free-riding on market-specific investments by the manufacturing firm, it is not necessarily welfare improving. In this case, it might be a better policy to reduce the barriers to entry for new products, thereby, promoting the competition between varieties rather than trade in the same variety between markets.

Finally, the distribution of surplus is a problem particular to international trade and international politics. International agreements normally need a unanimous vote to be implemented. However, consumers in the high-valuation market gain from price-equalization, while producers and consumers in the low-valuation market suffer. In the short-run, total welfare increases as the former effect dominates the latter two. In the short-run, consumers in the high-valuation country can transfer some of their surplus to the low-valuation country. In the long-run equilibrium, however, welfare is lower than in the segmented equilibrium. Hence, no transfer can be made to compensate consumers in the low-valuation market. A unanimous vote for the arbitrage regime is, therefore, very unlikely.

## 6.9 Conclusions

It has previously been argued that arbitrage ultimately results in uniform pricing and, therefore, equalization of marginal willingness to pay for one unit of a traded product. This effect increases welfare under arbitrage, if output is at least as large as under third-degree price discrimination.

In this paper, I have shown that the positive welfare effects of uniform pricing can be reversed for two reasons particular to international trade. First, if arbitrage make extensive use of real resources, total welfare is higher in the segmented equilibrium than under the arbitrage regime. Second, in the long run, the manufacturing firm will adjust to price equalization by redistribution of market-specific investments and the welfare result of uniform pricing is reversed. As the market-specific investment in the market with lower output is increased at the expense of the market with higher output, welfare is strictly lower under the arbitrage regime than in the segmented equilibrium.

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