

THE METHOD OF ISOLATION IN ECONOMIC STATICS -- A PEDAGOGICAL NOTE

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Summary

The paper analyzes under what conditions partial equilibrium analysis by a submodel of a larger system gives exactly the same results as total equilibrium analysis in the context of the entire system. The conditions when this is the case are called "isolation requirements". The next step in the paper is to give examples from various types of wellknown economic models--classical as well as keynesian--where these isolation requirements have in fact, explicitly or implicitly, been assumed.

The purpose of the paper is mainly educational.

I. Introduction¹

All economic analyses are *partial* in the sense that the economist always has to restrict his analysis to a given part of reality. Certain phenomena in the world are selected and analyzed separately, as if their relation to the rest of the world were of no relevance for the analysis. We shall call this procedure the *method of isolation*.

The smaller the part of the world which the economist isolates, the more partial is the analysis, of course. The problem may be clarified if we think of the procedure as consisting of two consecutive steps. The first step is to decide which variables to include in the analysis, and which ones to leave outside. In other words, the economist selects and isolates certain entities from the "world model" which he has in the back of his mind. The second step is to define the relations between the entities he has selected, i.e. to specify the properties of his "economic model". If the "economic model" is characterized by complete interdependence between the variables, so that all variables are simult-

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¹ The purpose of this paper is mainly educational. I am indebted to Mr. Karl Göran Målsjö and docent Jan Odhnoff for helpful comments and suggestions.

Let the unknowns be solved out by Cramer's rule. For instance, $\partial x_1 / \partial a_1$, the effects of the shift in a_1 on the solution value of the variable x_1 , is

$$\frac{\partial x_1}{\partial a_1} = \frac{\partial f_1}{\partial a_1} \begin{vmatrix} \dots & -(\partial f_1 / \partial a_1) & \partial f_1 / \partial x_2 \dots \partial f_1 / \partial x_n \\ \dots & -(\partial f_2 / \partial a_1) & \partial f_2 / \partial x_2 \dots \partial f_2 / \partial x_n \\ \dots & \dots & \dots \\ \dots & -(\partial f_n / \partial a_1) & \partial f_n / \partial x_2 \dots \partial f_n / \partial x_n \\ \partial f_1 / \partial x_1 & \partial f_1 / \partial x_2 \dots \partial f_1 / \partial x_n \\ \partial f_2 / \partial x_1 & \partial f_2 / \partial x_2 \dots \partial f_2 / \partial x_n \\ \dots & \dots & \dots \\ \partial f_n / \partial x_1 & \partial f_n / \partial x_2 \dots \partial f_n / \partial x_n \end{vmatrix} \quad (3)$$

This kind of analysis is conventionally called *total* equilibrium analysis, because the effects on x_1 are calculated on the total model. Thus $\partial x_1 / \partial a_1$ is the *total effects* on x_1 .

Now, let us make an isolated analysis for a subset of system (1); say that the subset consists of the first two equations and the first two variables. The entities x_3, \dots, x_n then have to be considered as parameters; (therefore they will be denoted by bars). The model may now be written,

$$\left. \begin{aligned} f_1(x_1, x_2, \bar{x}_3, \dots, \bar{x}_n; a_1, \dots, a_m) &= 0 \\ f_2(x_1, x_2, \bar{x}_3, \dots, \bar{x}_n; a_1, \dots, a_m) &= 0. \end{aligned} \right\} \quad (4)$$

The effects on the solution values of x_1 and x_2 of a parameter change are calculated by implicit differentiation of the new, smaller model. For instance, the effects on x_1 of a change in a_1 are,

$$\left(\frac{\partial x_1}{\partial a_1} \right)_{x_3, \dots, x_n} = \left(\frac{\partial x_1}{\partial a_1} \right)_s \begin{vmatrix} -(\partial f_1 / \partial a_1) & \partial f_1 / \partial x_2 \\ \dots & \dots \\ -(\partial f_2 / \partial a_1) & \partial f_2 / \partial x_2 \\ \partial f_1 / \partial x_1 & \partial f_1 / \partial x_2 \\ \partial f_2 / \partial x_1 & \partial f_2 / \partial x_2 \end{vmatrix} \quad (5)$$

This is a partial equilibrium analysis of equation (1). The derivative $(\partial x_1 / \partial a_1)_s$, therefore, may be called the *partial equilibrium effects* of a_1 on x_1 . This derivative should not be confused, of course, with the partial derivative $\partial f_1 / \partial a_1$, which is computed with *all* the variables x_1, \dots, x_n constant. $\partial f_1 / \partial a_1$ measures the "shift" in function f_1 , due to a change in the parameter a_1 . It may therefore be called the *direct effect*, or the

impact effect, of a_1 on function f_1 . To compute $\partial f_1/\partial a_1$ is to make a *partial disequilibrium* analysis.¹

We now want to find out when an isolated analysis for a subset is legitimate in the sense that it gives exactly the same result as a total analysis. This is the case, of course, when equation (3) collapses into equation (5). This may occur in a number of different cases. However, it is easily shown by the rules of developing determinants that three *alternative* sufficient conditions are:²

$$\text{Case 1: } \quad \partial f_j/\partial x_1 = \partial f_j/\partial x_2 = \partial f_j/\partial a_1 = 0 \quad (j = 3, \dots, n),$$

i.e. when the variables inside the subset, as well as the shift parameter, do not affect the functions, and hence the variables, outside the subset.

$$\text{Case 2: } \quad \partial f_1/\partial x_j = \partial f_2/\partial x_j = 0 \quad (j = 3, \dots, n),$$

i.e. when the variables outside the subset have no influence on the functions, and hence the variables, inside the subset.

$$\text{Case 3: } \quad \frac{\partial f_j}{\partial x_r} = 0 \quad \text{when} \quad \frac{\partial f_1}{\partial x_r} = \frac{\partial f_2}{\partial x_r} = 0 \quad \text{and}$$

$$\frac{\partial f_j}{\partial x_1} = \frac{\partial f_j}{\partial x_2} = \frac{\partial f_j}{\partial a_1} = 0 \quad (j = 3, \dots, p) \quad (r = p+1, \dots, n)$$

i.e. when functions outside the subset that are affected do not affect variables outside, that can in turn influence variables inside the subset.

The common property of all three cases is that they guarantee that there are no effects from the shift parameter or from the variables in the subset to variables outside the subset *and back again*. An isolated analysis is legitimate where at least one of these three conditions is fulfilled (sufficient conditions); the conditions will be called *isolation requirements*.

We may expect that the isolation requirements are very seldom exactly fulfilled in the real world, where “everything depends on everything”. However, as an approximation, it is natural to *assume* that they are fulfilled in cases where the partial derivatives in the isolation requirements are small. Of course, the less exacting analysis we require, the more the isolated sector can be narrowed down.

¹ In Don Patinkin's terminology, we would say $\partial x_1/\partial a_1$ and $(\partial x_1/\partial a_1)_z$ describe “market experiments”, whereas $\partial f_1/\partial a_1$ describes an “individual experiment.”

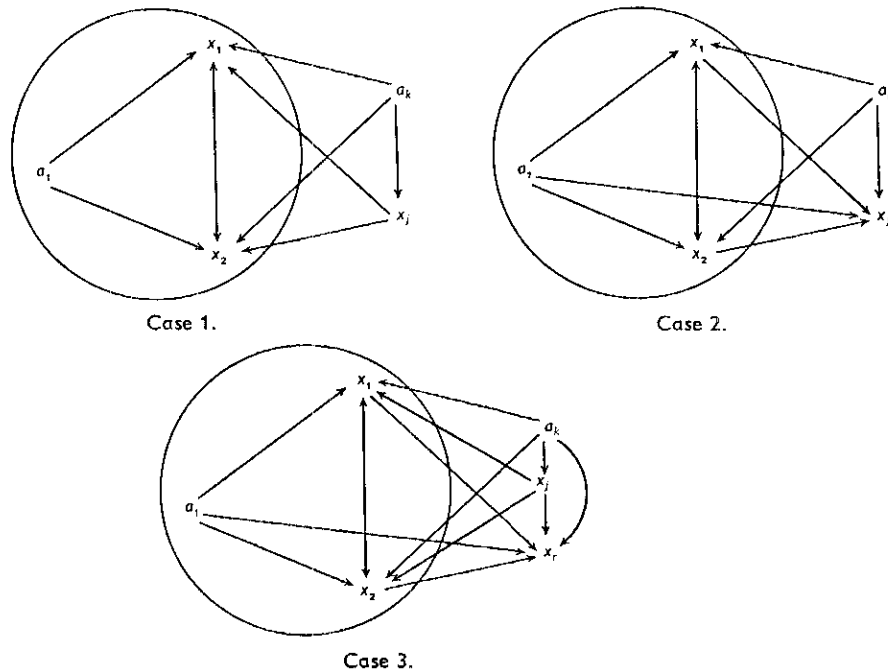
² For a more detailed analysis, see appendix at the end of the paper.

To assume that condition 1 is fulfilled is to assume that everything outside the subset is unaffected by changes in the shift parameter and in the variables inside the subset. This, of course, is the familiar *ceteris paribus* clause. To assume, by contrast, that condition 2 is fulfilled means that the *variables* outside the subset are assumed not to affect variables inside. (Thus, *parameters* outside the subset may very well affect variables inside the subset.) Since this means that the values of variables outside the subset are impertinent to the analysis of variables inside, the assumption will be called a *ceteris impertinentibus* clause. Both clauses are frequently implied in economic analyses, even though the *ceteris paribus* clause alone has been discussed extensively, and even honored by a name of its own, in the literature. Case 3, finally, will not be baptized in latin here.¹

The methodology of isolation can, alternatively, be described with the help of directed graphs or "arrow schemes". Therefore, let every variable and parameter be represented by a point in a plane. The existence of a *direct* effect of the variable x_r on the variable x_i is then indicated by a line segment between x_r and x_i , with an arrow pointing towards x_i . We say, in graphtheoretical language, that there exists a *path* in one step from x_r to x_i . If there is also a direct effect of x_i on a third variable, x_p , we say that x_r effects x_i *indirectly* in two steps. Indirect effects in m steps are defined analogously. If a path in m steps both starts and ends at the same variable, x_p , we shall say that there exists a "feed back", or a "cycle", in m steps on x_p . If there are no "feed backs", the effects of a parameter change on the variables can be computed one by one. In this case the model is *recursive*. Otherwise it is *interdependent*. Alternatively, we may say that the model is recursive if the directed graph of the model can be written in such way that all arrows point in the same direction, whereas it is interdependent if the graph cannot be written in this way. This latter definition of recursivity has been used by Ragnar Bentzel and Bent Hansen.²

¹ Actually, case 3 could be formally included as a special version of case 1, if we adopt the convention of always including in the isolated submodel all variables that are affected by the shift parameter or by the analyzed variables, and that do not influence variables outside the subset that have effects back on the subset.

² Ragnar Bentzel and Bent Hansen, "On Recursiveness and Interdependency in Economic Models", *Review of Economic Studies*, no. 59, 1954, pp. 153-68.



- Case 1. *Ceteris paribus* (no arrows out from the subset).
 Case 2. *Ceteris impertinentibus* (no arrows in from variables outside the subset).
 Case 3. No arrows from variables influenced by the subset (x_r) to variables that influence the subset (x_j).

Diagram 1.

The *ceteris paribus* clause now implies that there are no paths from the variables in a subset, and from the shift parameter, to variables outside the subset ("no arrows out"). The *ceteris impertinentibus* clause, by contrast, means that there are no paths from variables outside the subset to variables inside ("no arrows in" from variables outside). Case 3, finally, means that there are no paths from variables where arrows from the subset end to variables from which arrows point to the subset. In this case the "circuit" between the variables is broken *outside* the submodel. The issues are illustrated in Diagram 1, where the three different isolation requirements are depicted.¹

¹ In Herbert Simon's terminology, x_j in case 1 form a *minimal self-contained subset*. The same holds for x_1 and x_2 in case 2, as well as for x_j in case 3. x_1 and x_2 in case 1 form a *derived structure of the first order* in Simon's terminology. The same holds for x_j in case 2, as well as for x_1 and x_2 in case 3. x_r in case 3 form a *derived structure of the second order*. Herbert Simon, "Causal Ordering and Identifiability", *Studies in Econometric Method*, ed. by W. M. C. Hood and T. C. Koopmans 1953, Chap. 3.

Directed graphs will be frequently used in the subsequent exposition to describe the method of isolation in different economic theories. (As there is a one-to-one correspondence between directed graphs and matrices containing only 0's and 1's as elements—such matrices are often called “relations matrices”—the arguments could alternatively be recasted in matrix form.)

III. Examples of the Method of Isolation

A. *The Lausanne school*

Even in the most “general” types of economic theory, such phenomena as Wants, Nature, and Technology are regarded as exogenous, “non-economic” entities. As a rule, politics and population are treated in the same way. By regarding all these phenomena as unaffected by entities in the economic model, a *ceteris paribus* clause is in fact introduced into the “world model” in the back of the theorist's mind. On the other hand, no *ceteris impertinentibus* clause is applied, as changes in the “non-economic” factors are often assumed to influence the variables of the economic model.

However, only the most general kind of economic theories, such as the so-called “total equilibrium theory” of the Lausanne school, stop the isolation process at this stage. Other examples from the main stream of economic theory will illustrate more far-reaching uses of the method of isolation.

B. *Single-market models*

Let us interpret f_i and a_i in the system (1) as the excess demand function and the price, respectively, in the i th market. Equation (3) then describes the total effects on the price in market 1 of a change in parameter a_1 , when demand and supply in each market is a function of all commodity prices. The partial equilibrium effects in a single commodity model, by contrast, are obtained by differentiating the system when prices in all markets but one, for instance the first, are assumed constant. This gives,

$$\left(\frac{\partial x_i}{\partial a_1}\right)_{x_2, \dots, x_n} = \left(\frac{\partial x_i}{\partial a_1}\right)_p = -\frac{(\partial f_i / \partial a_1)}{\partial f_1 / \partial a_1}. \quad (6)$$

To the extent that the methodology of isolation has been discussed in the literature, it has been primarily in connection with this type of partial equilibrium analysis à la Cournot and Marshall.¹ The common opinion in the literature seems to be that the justification for analyses of this kind lies in the fact that the analyzed market is very small in comparison to the rest of the economy.² This means that a parameter shift in this market, for instance in the form of a change in consumer taste, has only negligible effects on national income and other aggregate determinants for the demand and supply in the analyzed market. A *ceteris paribus* clause for aggregate entities is therefore natural. However, if the subdivision of markets goes very far, other types of relations may appear, namely *price relations* between competing and complementary commodities. This means that the smallness of the market is not necessarily a sufficient justification for the use of the method of isolation. This fact is considered when two-commodity or multi-commodity analyses are made for competing and complementary commodities, such as investigations of the markets for butter and margarine. Thus, isolated markets have to be small enough so that effects via national income will not arise, and large enough to prevent substantial price relations with markets outside the model.

¹ The following works, *inter alia*, are relevant for the methodology of partial equilibrium analysis for single markets: A. Marshall, "A Note on Method in Economic Study", *Industry and Trade*, 1927, pp. 677-88; H. L. Moore, *Synthetic Economics*, 1929, pp. 27-28; A. Cournot, *The Mathematical Principles of the Theory of Wealth* (English translation), 1927, Chap. 11; Henry Schultz, *Statistical Laws of Demand and Supply*, 1928, p. 14 ff.; Joseph Schumpeter, *Das Wesen und der Hauptinhalt der theoretischen Nationalökonomie*, 1908, pp. 469-70; M. Panteleoni, "Una visione cinematografica del progresso della scienza economica", *Errotimi di Economia*, Vol. 1, 1925, pp. 201-02; Erik Lundberg, *Studies in the Theory of Economic Expansion*, 1937, Chap. 1; Friedrich von Wieser, *Theorie der Gesellschaftlichen Wirtschaft*, in *Grundriss der Sozialökonomie* I, II Teil, 2 Aufl., 1924, p. 10 ff.; V. Pareto, *L'economie mathématique*, 1911, pp. 594-620, and 627-28; Otto Weinberger, *Mathematische Volkswirtschaftslehre*, 1930, p. 123 ff.; F. Y. Edgeworth, *On some Theories due to Pareto*, Zawadzky, W. E. Johnson and others, *Papers Relating to Political Economy*, Vol. II, 1925, pp. 453 ff.; Oskar Lange, "Die Allgemeine Interdependenz der Wirtschaftsgrößen und die Isolierungsmethode", *Zeitschrift für Nationalökonomie*, Band IV, 1933; Friedrich Zeuthen, "Der Wirtschaftliche Zusammenhang — ein Netz von Teilzusammenhängen", *Weltwirtschaftliches Archiv*, Band 58, Sept. 1943, Heft 2, pp. 175-205.

² Erik Lundberg, for instance, describes a partial equilibrium analysis as an analysis "... in which we investigate the relations within so small a part of the whole economic system that changes in the total system can be considered given". Erik Lundberg, *op. cit.* p. 1.

When the analysis is short-run, it is also assumed that the capital stock in the analyzed market is not affected by changes in the price and quantity in the market. An assumed time lag for capital formation is the justification for this *ceteris paribus* clause.¹

C. Classical "total" analyses

The method of isolation is also frequently used in analyses of larger parts of the economy, even in so-called "total economic analyses". Of course, even in such models, a great number of phenomena are "isolated away" simply by not being included in the model. A less trivial application of the method of isolation is found in the internal structure of the economic models themselves: one or several subsets of variables are isolated from the rest of the model. This is the case for both "classical" and Keynesian analyses.

The most striking application of the method of isolation in classical "total" analyses is the dichotomy between a real and a monetary sector. This dichotomy exists in the disaggregate as well as in the aggregate version of the classical model.

The *disaggregate version* contains a real sector in the form of excess demand functions for a great number of commodities, and a monetary sector in the form of the equation of exchange. The excess demand functions for commodities are homogeneous in degree zero in absolute prices. If d_i and f_i ($i = 1, \dots, n - 1$) are the demand and excess demand functions, respectively, for $n - 1$ commodities, p_i the commodity prices, $\pi_i = (p_i/p_{n-1})$ the relative prices, m the quantity of money, and k a constant (the reciprocal of the velocity of money), the model may be written:

$$f_i(\pi_1, \dots, \pi_{n-2}) = 0 \quad i = 1, \dots, n - 1 \quad \text{the real sector} \quad (7)$$

$$k \sum_{i=1}^{n-1} p_i d_i = m \quad \text{the monetary sector} \quad (8)$$

¹ Marshall tried to justify partial equilibrium analysis with a distinction between "direct" and "indirect" effects, the indirect being neglected for approximative purposes. Cournot made the same distinction, referring to the indirect effects as magnitudes of the "second order". Schumpeter argued that we could confine our analysis in the neighborhood of the point where the disturbance occurs, whereas repercussion far away from this point could be neglected. Pantaleoni used the same argument, but he also pointed out that time lags may be a rational ground for partial equilibrium analysis. (*Op. cit.*, footnote 1, p. 155.)

By adding Say's law,

$$\sum_{i=1}^{n-1} p_i f_i \equiv 0, \quad (9)$$

relative prices can, if certain mathematical properties are fulfilled, be solved out from the real sector. Absolute prices are thereafter computed from the monetary sector.¹ As variables in the real sector influence the monetary sector, but not *vice versa*, the real sector is isolated by a *ceteris impertinentibus* clause and the monetary sector by a *ceteris paribus* clause.

One of the essential behaviour assumptions in this model is that demand and supply of commodities are unaffected by the quantity of money. As Patinkin has demonstrated, this dichotomy between the real and the monetary sector breaks down if, for instance, the real value of cash balances is introduced as an argument in the excess demand function for one or several commodities. (It is also a well known fact that the classical model, as it appears above, is caught in logical inconsistencies if the equation of exchange is interpreted as an equilibrium condition for the money market, expressing the condition that excess demand for money balances should be zero. However, this is a problem outside the scope of this paper.)

In the *aggregate version* of the classical model, aggregation is performed not only over individuals but also over commodities. The model is usually written in the following way.²

- | | |
|---|------------------------------------|
| (c.1) $N_s = N_s(W^*)$ | labor supply function |
| (c.2) $dY^*/dN = W^*$ or $N_d = N_d(W^*)$ | labor demand function |
| (c.3) $N_s = N_d = N$ | labor market equilibrium equation |
| (c.4) $Y^* = Y^*(N)$ | production function |
| (c.5) $m = k \cdot Y$ or $Y = m \cdot v$ | equation of exchange |
| (c.6) $Y^* = Y/P$ | definition of real national income |
| (c.7) $W^* = W/P$ | definition of real wage rate, |

¹ (8) gives $k \sum_{i=1}^{n-1} \pi_i p_{n-1} d_i(\pi_1, \dots, \pi_{n-2})$, where p_{n-1} is now the only unknown, which can be solved out. Then all p_i may be solved out from the relation $p_i = \pi_i p_{n-1}$.

² For expositions of the classical aggregate model, see for instance, L. R. Klein, *The Keynesian Revolution*, 1950, pp. 200-06; Challis Hall and James Tobin, "Income Taxation, Output, and Prices", *Economia Internazionale*, Vol. VII, no. 4, Nov. 1955, pp. 751-55; Richard Musgrave, *The Theory of Public Finance*, 1959, pp. 407-11; and Gardner Ackley, *Macroeconomics*, 1961, Chaps. 5-9.

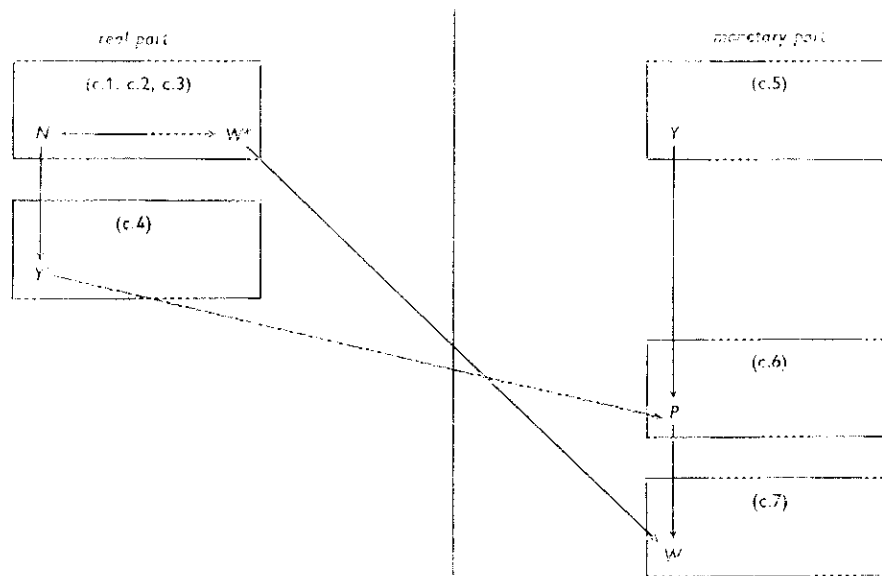


Diagram 2.

where W = index of money wage rates, P = index of product prices, W^* = index of real wage rates, Y = money national income, Y^* = real national income (or output), N_s = labor supply, N_d = labor demand, N = employment, m = quantity of money, v = income velocity of money, and $k = 1/v$. Variables are denoted by capital letters, parameters by small letters.

It is easily seen that the three labor market equations (c.1–3) form an isolated subset, which determines employment and real wages, whereas the equation of exchange (c.5) forms another isolated subset, which determines money national income. The structure of the model is illustrated by the directed graphs in Diagram 2.¹

The diagram shows that changes in the real part of the model (c.1–c.4) influence both real variables and monetary variables, whereas changes in the monetary part (c.5–c.7) only influence monetary variables. Thus, also in this version of the classical model, the monetary part is isolated from the rest of the model in the *ceteris paribus* sense, whereas the real part is isolated in the *ceteris impertinentibus* sense

¹ In Simon's terminology, c.1–3 and c.5 each form a *minimal self-contained subset*, whereas c.4 is a *derived structure of the first order*, c.6 a *derived structure of the second order* and c.7 a *derived structure of the third order*.

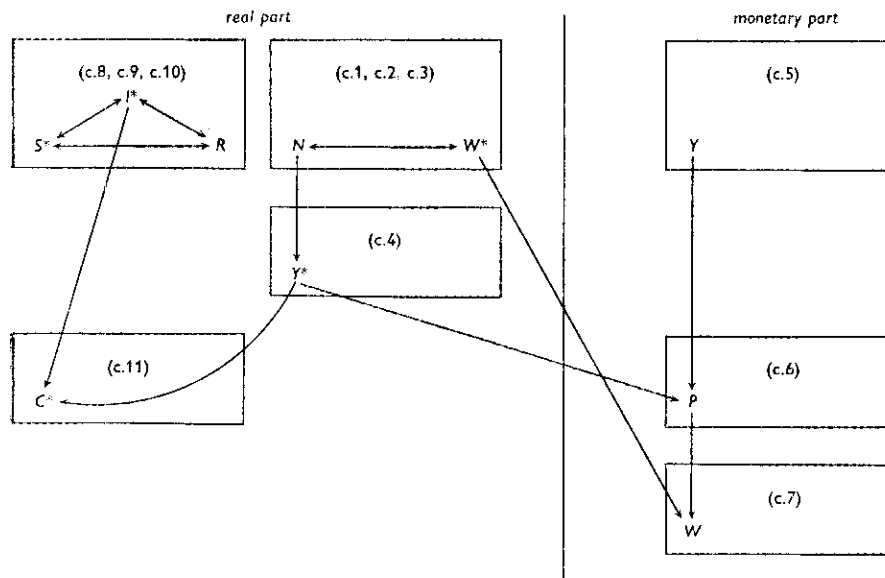


Diagram 3.

If a traditional saving-investment analysis is added to the “pure” classical model, a new isolated subset is in fact added to the original system. This subset is isolated from the rest of the model in both the *ceteris paribus* and *ceteris impertinentibus* sense, and it simply determines the breakdown of national output on investment and consumption. Letting S^* denote real saving, I^* real investment, and R interest rates, the additional subset may be written,

- (c.8) $S^* = S^*(R)$ saving function
- (c.9) $I^* = I^*(R)$ investment function
- (c.10) $S^* = I^*$ saving-investment equilibrium equation.

Real consumption, C^* , becomes a residual, which can be solved from the national income identity,

(c.11) $Y^* = C^* + I^*$ national output identity.

Diagram 3 illustrates the analytical structure of the enlarged model.

As in the disaggregate version of the classical model, commodity demand is insensitive to money balances. Similarly, commodity demand is unaffected by national income; in other words, there is no keynesian spending function in the model. The *real* value of national income is

in fact, for a given production function, determined by the demand and supply functions on the labor market, whereas the value of *money* national income is a function of the quantity of money: a given quantity of money supports a given *money* national income.

D. Keynesian aggregate analysis

Keynesian aggregate theory is another example of the method of isolation. Using the same symbols as earlier, let a simple Keynesian model be defined by the following equations, where the price level is assumed to be constant.

- (k.1) $C = C(Y, R)$ consumption function
 (k.2) $I = I(Y, R)$ investment function
 (k.3) $Y = C + I + g$ equilibrium condition for the commodity market
 (k.4) $L = L(Y, R)$ liquidity preference function
 (k.5) $m = L$ equilibrium condition for the money market,

where g is autonomous government expenditures on goods and services and L the liquidity preference function.¹

As it stands, the model is completely integrated in the sense that no isolated subset exists. However, it is usual in Keynesian analysis to select the three first equations and use them as an isolated "multiplier model" determining C , Y , and I . This means that the commodity markets are isolated from the money market. The isolation requirements can be derived by implicit differentiation of the system. Let us therefore differentiate with respect to g and solve out the total effects $\partial Y/\partial g$, on national income.

$$\frac{\partial Y}{\partial g} = \frac{\begin{vmatrix} 1 & -(\partial C/\partial R + \partial I/\partial R) \\ 0 & \partial L/\partial R \end{vmatrix}}{\begin{vmatrix} (1 - \partial C/\partial Y - \partial I/\partial Y) & -(\partial C/\partial R + \partial I/\partial R) \\ \partial L/\partial Y & \partial L/\partial R \end{vmatrix}}$$

$$= \frac{1}{(1 - \partial C/\partial Y - \partial I/\partial Y) (\partial C/\partial R + \partial I/\partial R) \partial L/\partial Y + \partial L/\partial R}$$

¹ In larger Keynesian models the price level is introduced as an endogenous variable, and a production function as well as labor market relations are added to the model.

It is easily seen that $\partial Y/\partial g$, which may be called the *general multiplier*, simplifies to the usual, simple multiplier, $\frac{1}{1 - (\partial C/\partial Y + \partial I/\partial Y)}$, if at least one of the following three conditions is satisfied.¹

1. $\left(\frac{\partial C}{\partial R} + \frac{\partial I}{\partial R}\right) = 0$, i.e. spending insensitive to the interest rate
2. $\frac{\partial L}{\partial Y} = 0$ i.e. demand for money insensitive to the national income
3. $\frac{\partial L}{\partial R} = \infty$ i.e. the liquidity preference function infinitely elastic at the prevailing interest rate (“liquidity trap”).

In case 1 the multiplier model is isolated from the rest of the system by a *ceteris impertinentibus* clause, whereas it is isolated by a *ceteris paribus* clause in case 2. In case 3, finally, the multiplier model is isolated from the rest of the system by the isolation requirements in the “unbaptized” case 3 on page 151; demand for money may be affected by national income, and spending may be affected by the interest rate, but a shift in the demand for money due to a change in income has no influence on the interest rate. Thus the condition $\partial L/\partial R = \infty$ means that $dR/dY = 0$, where dR/dY is calculated on the function $L(Y, R) = m$, m being a constant.

Sometimes, the liquidity preference theory is used as a partial theory for the credit market. In other words, equations (k.4) and (k.5) are isolated from the rest of the system to determine the interest rate. This is what people do when they say that the interest rate is determined by the demand and supply for money. The isolation requirements are derived by implicit differentiation of the model with respect to m and by solving out the total effects $\partial R/\partial m$

¹ For discussions of the relation between the simple multiplier model and the complete Keynesian system, see Tord Palander, “Keynes’ allmänna teori och dess tillämpning inom ränte-, multiplikator- och pristeorin”. *Ekonomisk Tidskrift*, no. 4, XLIV, 1942, pp. 233–72; and Erich Schneider, *Einführung in die Wirtschaftstheorie*, III. Teil (1959), pp. 175–85.

$$\frac{\partial R}{\partial m} = \frac{\begin{vmatrix} (1 - \partial C/\partial Y - \partial I/\partial Y) & 0 \\ \partial L/\partial Y & 1 \end{vmatrix}}{\begin{vmatrix} (1 - \partial C/\partial Y - \partial I/\partial Y) & -(\partial C/\partial R + \partial I/\partial R) \\ \partial L/\partial Y & \partial L/\partial R \end{vmatrix}}$$

$$= \frac{1}{\partial R + \frac{\partial L/\partial Y (\partial C/\partial R + \partial I/\partial R)}{1 - \partial C/\partial Y - \partial I/\partial Y}}$$

The *partial* equilibrium effects in an isolated money market analysis, by contrast, are

$$\left(\frac{\partial R}{\partial m}\right)_Y = \frac{1}{\partial L/\partial R}$$

Obviously, the isolated analysis gives the same result as the total analysis in two alternative cases,

1. $\left(\frac{\partial C}{\partial R} + \frac{\partial I}{\partial R}\right) = 0$ i.e. spending insensitive to the interest rate
2. $\frac{\partial L}{\partial Y} = 0$ i.e. demand for money insensitive to the national income.

Case 1 means isolation of the money market by a *ceteris paribus* clause, case 2 by a *ceteris impertinentibus* clause.

There are several phenomena in the real world that may justify an isolated multiplier analysis or an isolated liquidity preference analysis. For instance, if it is true, as many economists seem to believe, that spending is quite interest-inelastic, isolated analyses will give results rather similar to those obtained by the "total" keynesian model. There may be an additional reason for an isolated liquidity preference analysis: it may be argued that reactions in the credit market are much more rapid than reactions in other markets. Consequently, the credit market may reach a temporary, short-run equilibrium position after a parameter change, before other markets have been considerably affected. There may also exist situations, such as during the thirties in the United States, when a "liquidity trap" may be an approximate description of the situation in the credit market. This would, in the context of keynesian theory, justify an isolated multiplier analysis of changes in national income. Situations may also exist in which the demand for

money balances is not very sensitive to changes in national income, for instance in periods of easy monetary policy. Such situations would justify both isolated multiplier analyses and isolated liquidity preference analyses.

Therefore, as isolated analyses may often give approximately the same result as total analyses with the keynesian model, it may be argued that the main weakness of the isolated multiplier model and the isolated liquidity preference model may not be that they neglect other parts of the "total" keynesian model, but rather that the "total" keynesian model itself neglects many important relations from the real world. Among these neglected relations, we may mention the effects of investment on the capacity of the economy; as is well known, investment is only looked upon as a *demand* factor in the static keynesian model. A time-lag between investment expenditure and capacity increase is often regarded as a justification for this assumption. Another possible justification could be that in a relatively short period, such as a year or less, investment is very small in comparison to the total capital stock, so that the effects on capacity can, for approximate reasons, be neglected.¹

IV. Final Remarks

The purpose of this paper has been to derive general requirements for isolation. An attempt has also been made to indicate some circumstances in the real world that may justify the use of the method of isolation in different situations. The existence of autonomous, "non-economic" factors is one example of such circumstances. The smallness of a sector in relation to the entire economy is another factor that may justify the method (c.g. in partial equilibrium analysis for a one-commodity model). The existence of time-lags is a third factor (e.g. a capital formation lag as in short-run statics, or a time-lag for monetary factors to influence "real" markets as in isolated liquidity preference analysis).

¹ Psychological effects of changes in government spending on private consumption or investment, asset effects on spending, the importance of the availability of credit and "internal funds" for investment, are a few examples of other neglected factors in the total keynesian system. However, some of these factors may easily be added to the system, and have in fact been added by various economists.

A fourth factor, finally, is general insensitivity between variables (e.g. inelastic commodity demand with respect to money balances as in "total" classical analyses, and interest-inelasticity of spending as in the isolated Keynesian multiplier model). Most applications of the method of isolation in economics may be classified under one of these four headings. A fifth case may be added: pedagogical isolation. This is isolation *for expository reasons*: we consciously neglect an important factor, possibly for the purpose of introducing it later on in the analysis.

Appendix

Consider a linear model

$$(I) \quad Ay = b, \text{ where } A \text{ has the inverse } A^{-1}.$$

$$A = \begin{bmatrix} a_{11} & \dots & a_{1k} & \dots & a_{1n} \\ \vdots & & \vdots & & \vdots \\ a_{k1} & \dots & a_{kk} & \dots & a_{kn} \\ \vdots & & \vdots & & \vdots \\ a_{n1} & \dots & a_{nk} & \dots & a_{nn} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ \vdots & \vdots \\ A_{21} & A_{22} \end{bmatrix}$$

$$y = \begin{bmatrix} y_1 \\ \vdots \\ y_k \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} y' \\ y'' \end{bmatrix} \quad b = \begin{bmatrix} b_1 \\ \vdots \\ b_k \\ \vdots \\ b_n \end{bmatrix} = \begin{bmatrix} b' \\ b'' \end{bmatrix}$$

Equation (I) can be interpreted as equation (2) in the text, with

$$a_{ij} = \frac{\partial f_i}{\partial x_j} \quad b_i = \dots = \frac{\partial f_i}{\partial \alpha_1} \quad y_i = \frac{\partial x_i}{\partial \alpha_1}.$$

The solution to (I) is

$$(II) \quad \begin{bmatrix} y' \\ y'' \end{bmatrix} = A^{-1} \begin{bmatrix} b' \\ b'' \end{bmatrix}.$$

Let us define a submodel consisting of the first k variables y' and the first k equations. The solution of the submodel is

$$(III) \quad y' = A_{11}^{-1} b',$$

assuming A_{11} to be nonsingular.

We now want to investigate when y' in (III) is a component of a solution

$$\begin{bmatrix} y' \\ y'' \end{bmatrix} \text{ to (I).}$$

Consider system (I) in the form of the components:

$$(IV') \quad A_{11}y' + A_{12}y'' = b',$$

$$(IV'') \quad A_{21}y' + A_{22}y'' = b''.$$

From equations (IV') and (IV'') it is seen that a necessary and sufficient condition for y' in (III) to be a component of the solution to (I) is that

$$(V) \quad A_{12}y'' = 0$$

and (IV'') holds.

By substituting (III) in (IV'') we see that the last condition is equivalent to

$$(VI) \quad A_{21}A_{11}^{-1}b' + A_{22}y'' = b''.$$

These necessary and sufficient conditions hold if y'' belongs to the null-space of A_{12} and y'' satisfies (VI). Let us distinguish between three different sufficient conditions for this to hold

α $A_{12} = 0$. y'' can then always be solved from (VI) if A_{22} is non-singular. This is identical to case 2 in the text.

β $y'' = 0$. From (VI) it is seen that a sufficient condition for this to hold is that $A_{21} = 0$ and $b'' = 0$. This is identical to case 1 in the text.

γ $A_{12}y'' = 0$, without $A_{12} = 0$ or $y'' = 0$, and (VI) holds.

This may happen in a number of different cases depending on the elements in the matrix A and the vector b . One case which is of particular interest from the point of view of economic model-building is the following, where A_{12} , A_{21} , A_{22} and b'' have been partitioned, so that (I) can be written

$$(XI) \quad \begin{bmatrix} A_{11} & A_{12}^{(1)} & A_{12}^{(2)} \\ A_{21}^{(1)} & A_{22}^{(1)} & A_{22}^{(2)} \\ A_{21}^{(2)} & A_{22}^{(3)} & A_{22}^{(4)} \end{bmatrix} \begin{bmatrix} y' \\ y''_{(1)} \\ y''_{(2)} \end{bmatrix} = \begin{bmatrix} b' \\ b''_{(1)} \\ b''_{(2)} \end{bmatrix}$$

where $A_{12}^{(2)} = 0$, $A_{21}^{(2)} = 0$, $A_{22}^{(3)} = 0$, $b''_{(2)} = 0$ and $A_{22}^{(4)}$ is non-singular.

The last equation in (XI) is then $A_{22}^{(4)}y''_{(2)} = 0$, which gives $y''_{(2)} = 0$. The first equation in (XI) now implies

$$(III) \quad y' = A_{11}^{-1}b',$$

which is the solution of the submodel. The assumptions in (XI) correspond to case 3 in the text.

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