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CLEVER AGENTS IN YOUNG'S EVOLUTIONARY BARGAINING MODEL

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Clever agents in Young's evolutionary bargaining model*

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ABSTRACT. In the models of Young (1993a,b), boundedly rational individuals are recurrently matched to play a game, and they play myopic best replies to the recent history of play. It could therefore be an advantage to instead play a myopic best reply to the myopic best reply, something boundedly rational players might conceivably also do. We investigate this possibility in the context of Young's (1993b) bargaining model. It turns out that "cleverness" in this respect indeed does have an advantage in some cases. However, if all individuals are equally informed about past play, in a statistical sense, then the Nash bargaining solution remains the unique long-run outcome when the mutation rate goes to zero.

1. INTRODUCTION

Multiple equilibria is a standard feature of bargaining games. In the early fifties Nash proposed two different approaches to solve the multiplicity problem in bilateral bargaining situations. In a first paper, Nash (1950) showed that the unique solution which satisfies the axioms of invariance, symmetry, Pareto efficiency, and independence of irrelevant alternatives, is the maximizer of the product of the two parties' utility gains. In a second paper Nash (1953) obtains the same bargaining outcome by analyzing a static non-cooperative bargaining model, the Nash Demand Game, in which the agents simultaneously announce demands, which they receive if and only if their demands are compatible. The Nash Demand Game has many Nash equilibria, however. In order to select a single equilibrium, Nash required that an equilibrium be robust to perturbations involving uncertainty about the location of the Pareto frontier of the negotiation set. When the perturbed Demand Game approaches the unperturbed game, for which the Pareto boundary is known with certainty, all the Nash equilibria of the perturbed game converge on the Nash solution (see Binmore 1987a, 1987b).

More recently, Young (1993b) provided a new underpinning of the Nash bargaining solution, based on the unperturbed Nash Demand Game. The approach in Young (1993b) is to instead embed the Nash Demand Game in an evolutionary framework and impose perturbations on individual behaviors in the population. The Nash Demand Game is

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played repeatedly by members from two large populations. In each round, two individuals, one from each population, are randomly selected to play the game. Each of the two drawn individuals simultaneously announce a demand. If their demands are compatible, they obtain these, otherwise they obtain nothing. All individuals have access to a random sample, whose size may differ among them, drawn from the recent history of play. They take their sample as a predictor of the behavior of the individual they face, and almost always play a best reply to the sampled empirical distribution of the opponent population's play. However, occasionally individuals "mutate", and instead make a demand that is not a best reply to any possible sample from the recent history of play.

Young shows that, in the limiting case when the mutation rate goes to zero, the system converges to the Pareto-efficient division that corresponds to the generalized Nash bargaining solution, with the bargaining powers determined by the smallest sample size in each of the two populations.[1] When all individuals' sample size is the same within each population, the better informed population obtains the larger share of the cake. When the sample size is equal in the two populations, Young obtains the (standard) Nash bargaining solution as the unique limiting outcome. Young establishes this result under weak informational assumptions: individuals only know their own preferences and a sample of what happened in the recent past.

We here investigate the robustness of Young's results with respect to the knowledge and rationality of the individuals playing the Nash Demand Game. In particular, we study the effect of letting a population share of the individuals in one of the populations know the opponent population's preferences. Moreover, these "clever" individuals best reply to the opponent population's best replies to the sample that the clever individuals have of their own population's past play. In other words, the clever individuals try to anticipate their opponents' play on the basis of the sample of past play that they themselves have.

Robustness in this respect appears relevant, since it is not clear a priori exactly where to put the bounds on rationality in a model of boundedly rational agents.[2] However, such robustness could be technically difficult to analyze in general games, so it may be useful to first study a particular class of relatively simple games. That is why we have chosen to perform the robustness test in Young's (1993b) bargaining model. In bargaining it may be an advantage to anticipate one's opponent's move, and the Nash Demand Game is

[1]By the "generalized" Nash bargaining solution we mean the the maximizer of the product of the parties utility gains, each gain raised to some power, see section 2. The two power coefficients represent the parties' bargaining power. The "standard" Nash bargaining solution corresponds to the special case when the bargaining powers are equal.

[2]Hurkens (1995) develop a model similar to that of Young (1993a,b) and shows that his own results are robust to the introduction of "clever" individuals of the same type as we study here. The main difference is that his individuals sample past play with replacement, while in Young's model this is done without replacement. Also, the focus of his study is different. He investigates the stability of "curb" sets, product sets of strategies that are "closed under rational behavior." Those results have little bearing on the present context, since any division of the pie constitutes a strict Nash equilibrium, and hence a singleton curb set.

sufficiently simple to allow for a fairly straight-forward analysis.

As described above, the clever agents in our model do not look at the other population's past demands but instead at their own population's past demands, just as the opponent population does. This gives the host population of the clever bargainers an advantage which turns out to affect some of Young's results. However, if the host population uses a sample of at least the same size as the opponent population, then Young's result remains intact. In this case the presence of the new type of agent does not affect the minimum number of mistakes required to displace a convention (an established division of the pie). The effect is equivalent to providing some individuals in the other population with the (larger or equally large) sample size of the host population, and in Young's model it is the smallest sample size in each population that determines the outcome of the process. Therefore, the presence of clever individuals does not affect the outcome in this case. However, if the host population instead uses a smaller sample size than the opponent population, then cleverness does pay off to all individuals in the host population. Their outcome is improved in exactly the same way as if the host population had no clever bargainers, but the sample size of some individuals in the opponent population were reduced to that of the host population. Since it is the smallest sample size in each population that determines the outcome of the process, the long-run outcome now is the (standard) Nash bargaining solution. These results hold for any positive share of clever bargainers below one. Thus, even a "grain of cleverness" in the population exactly compensates the informational disadvantage. In sum, the (standard) Nash bargaining solution is the long-run outcome for any positive share of clever agents below one, both in the case of equal sample sizes in the two populations and in the case of a smaller sample size in the host population. This is our main result (Proposition 2 below).

It turns out, however, that in the extreme case when *all* individuals in one population are clever bargainers; such a population obtains the whole pie (Proposition 4). The intuition for the discontinuity at this end of the spectrum is as follows.[3] First, note that mutations - mistakes or experiments - in one population are favorable for that population's share of the pie in the long-run, in Young's (1993b) model. For if some individuals in population A by mistake ask for *too much*, then it is relatively riskless for individuals in population B to reduce their demands accordingly, since they obtain their new lower demand irrespective of if they meet a "mutant" or "non-mutant." Therefore, relatively few mistakes are needed to cause individuals in B to adapt to such mistakes in A. By contrast, if some individuals in population A by mistake ask for *too little*, then it is relatively risky for individuals in B to increase their demands since they obtain nothing if they happen to meet a "non-mutant." Therefore, relatively many mistakes are needed to cause individuals in B to adapt to mistakes in this direction. Consequently, it takes fewer mistakes in population A to move population B to a more favorable division than to a less favorable division. In other words, the mutations in each population cause a "favorable

[3] For the sake of clarity, we here only discuss the special case when all individuals' samples are of equal size.

drift" for that population. The two drift terms turn out to balance precisely when the division is the Nash bargaining solution. Now suppose all individuals in population A are clever (as in Proposition 4). Then the mutations in population B have no effect at all, since both populations react only to mutations in population A. Therefore B's "favorable drift" is absent. Since A's "favorable drift" is not counter-balanced, the clever individuals in A obtain the whole pie, in the long run. Suppose instead that not all individuals in population A are clever (as in Proposition 2). Then B's "favorable drift" is still present, because the non-clever individuals in A do react to mistakes in B, just as in Young's model. The minimal number of mistakes to upset any convention is the same as in that model. Therefore, the presence of clever individuals, in any population share below one, has no effect on the long-run outcome.

For the purpose of testing the robustness of our results, we also study the case of individuals who are clever in the particular sense of best replying to the actual demands made by their opponents. This case is analytically easier to study. It turns out that clever agents of this type have no effect at all on the outcome in Young's (1993b) model, granted their population share is less than one (Proposition 3). Clever bargainers of this type are closely related to the "responsive agents" in Ellingsen's (1997) evolutionary bargaining model.[4] In Ellingsen's model there is a single population of bargainers who are randomly matched to divide a pie of given size. Agents can be of two types, "obstinate" and "responsive." The first always demand a fix share while the latter correctly identify the obstinate and best reply to their demand. He shows that in all neutrally stable population profiles more than half the population are obstinate agents who demand half the pie. Since all other agents are responsive, the equal split - the Nash bargaining solution - is observed.

The remainder of this study is organized as follows. Section 2 specifies the set-up and states Young's (1993b) main result. Section 3 presents our results and section 4 concludes.

2. Young's Model

Young (1993b) considers two finite populations, A and B, and a finite set $D(\delta)$ of feasible divisions, where $D(\delta) = \{\delta, 2\delta, ..., 1-\delta\}$ and $\delta = 10^{-p}$ for some positive integer p. The parameter δ is called the *precision* of the set of feasible divisions. All individuals in population A have the same concave, strictly increasing and differentiable utility function $u : [0,1] \to \mathbb{R}$, where $u(x)$ represents the von Neumann-Morgenstern utility for an A-individual of obtaining the share x. Likewise, all individuals in population share the concave, strictly increasing and differentiable utility function $v : [0,1] \to \mathbb{R}$, where $v(y)$ is the von Neumann-Morgenstern utility for a B-individual of obtaining the share y. Let $u(0) = v(0) = 0$.

In each period $t = 1, 2, ...$ one individual is drawn at random from each population. These individuals play the Nash Demand Game: the individual from population A demands some share $x \in D(\delta)$, the individual from population B some share $y \in D(\delta)$,

[4]The effect of this type of "clever" agents among "programmed" agents in recurent play of symmetric two-player normal-form games is studied in Banerjee and Weibull (1995).

and they obtain their demanded shares if $x + y \leq 1$, otherwise they obtain nothing. Let the demands in period t be denoted (x_t, y_t), and let the history up to and including period t be denoted $h_t = ((x_1, y_1), ..., (x_t, y_t))$. Suppose individuals $\alpha \in A$ and $\beta \in B$ are drawn to play the Nash Demand Game in period $t + 1$. Individual α then draws a sample without replacement of $k_\alpha = am$ demand pairs (x_s, y_s) from the last m rounds in h_t, and individual β likewise draws a sample without replacement of $k_\beta = bm$ demand pairs (x_s, y_s) from the last m rounds in h_t. The draws of these samples are statistically independent across individuals and periods, and $a, b \in (0, 1)$ are such that k_α and k_β are positive integers. Individual α makes a demand x_{t+1} that maximizes her expected payoff against the sampled distribution of demands from population B, and individual β makes a demand y_{t+1} that maximizes her expected payoff against the sampled distribution of demands from population A.

The process begins at time $t = m$, with some arbitrary initial history h_m up to and including period m. Letting the last m rounds of play be the *state* at any time $t \geq m$, Young (1993b) defines a stationary Markov chain with initial state $s^0 = h_m$ as follows. For any state $s \in S = (D(\delta) \times D(\delta))^m$ let $p_\alpha(x \mid s)$ be the conditional probability that α demands x given that the state is s. Assume that $p_\alpha(x \mid s)$ is positive if and only if x maximizes α's expected utility against *some* sample of size am from s, and similarly for $p_\beta(y \mid s)$. A state s' is a *successor* to a state s if the first $m - 1$ demand pairs in s' coincide with the last $m - 1$ demand pairs in s. The transition probability from any state s to any of its successor states s' is

$$P(s, s') = \sum_{\alpha \in A} \sum_{\beta \in B} \pi(\alpha, \beta) p_\alpha(x \mid s) p_\beta(y \mid s) \ ,$$

where $\pi(\alpha, \beta)$ is the probability for the pair (α, β) of individuals to be drawn to play the game, and (x, y) is the last demand-pair in state s'. Moreover, $P(s, s') = 0$ if s' is not a successor of s.

This is not the end of Young's story. Individuals occasionally make mistakes or experiments. At each time $t = m, m + 1, m + 2, ...$ an individual drawn to play the game either plays a best reply to her sample, as detailed above, or, with probability $\varepsilon\lambda_\alpha$, individual $\alpha \in A$ makes a mistake or experiments (probability $\varepsilon\lambda_\beta$ for $\beta \in B$). In the case of such a "mutation," let $q_\alpha(x \mid s)$ be the conditional probability that α demands the share $x \in D(\delta)$, given the state $s \in S$, and let $q_\beta(y \mid s)$ be likewise defined for individuals β in population B. All mutation probabilities are assumed to be positive for all individuals in all states, and all randomizations are taken to be statistically independent. This addition to the above stationary Markov chain results in a stationary and irreducible Markov chain. Hence, the resulting process is ergodic and thus has a unique stationary distribution μ^ε, which is also stable.

Young defines a *convention* as a state s that consists of m repetitions of one and the same division $(x, 1 - x)$, where $x \in D(\delta)$. A convention s is *stochastically stable* if $\lim_{\varepsilon \to 0} \mu^\varepsilon(s)$ exists and is positive. A division $(x, 1 - x)$ is *generically stable* if the associated convention is stochastically stable for all m such that am and bm are positive integers, all other parameters and functions being fixed.

In a bilateral bargaining situation between two parties with von Neumann-Morgenstern utility functions u and v as specified above, set $\{(x,y) \in \mathbb{R}^2_+ : x + y \leq 1\}$ of feasible agreements, default point $(0,0)$, and bargaining-power coefficients $a, b \in (0,1)$, the *generalized Nash bargaining solution* is the unique division $(x, 1-x)$ that maximizes the product $u^a(x)v^b(1-x)$ subject to $x \in [0,1]$. In the special case $a = b$, the solution will be referred to as the *(standard)* Nash bargaining solution.

An individual in population A is said to be of *type* (a,u) if she samples the fraction a from the last m rounds and has utility function u, and likewise an individual in population B is said to be of *type* (b,v) if he samples the fraction b from the last m rounds and has utility function v.

Young establishes the following main result, Young (Theorem 3, 1993b):

Proposition 1. *Let A and B be homogenous populations composed respectively of types (a,u) and (b,v), where $a, b \leq 1/2$. For every precision $\delta > 0$ there exist at least one and at most two generically stable divisions, and as $\delta \to 0$ they converge to the generalized Nash bargaining solution with bargaining-power indices a and b.*

3. Clever Bargainers

Suppose one of the population contains some individuals who know the preferences of the other population, and who use this knowledge to outsmart their opponents by best replying to the anticipated demands from their opponents. More exactly, these "clever" individuals play a best reply to their opponents' best reply to the sampled history that the clever individuals have of their own population's play. It turns out that if the own population uses a sample that is not smaller than that of the opponent population, then Young's result remains valid. In the opposite case, however, this cleverness improves the outcome for the own population in exactly the same way as if the opponent population's sample size were decreased to that of the own population. Consequently, the outcome in that case is the standard Nash bargaining solution. This result is insensitive to whether or not the clever individuals occasionally mutate (like the non-clever individuals).

Proposition 2. *Let A and B be homogenous populations composed respectively of types (a,u) and (b,v), where $a, b \leq 1/2$. Assume that a share $\lambda < 1$ of population A is replaced by individuals who know B's utility function v, and best reply to a best reply of some sample of size am drawn from the own population history. All other players behave as in Young's model. For every precision δ there exists at least one and at most two generically stable divisions, and as $\delta \to 0$ they converge to the generalized bargaining solution with bargaining-power indices a and b when $a > b$, and to the standard Nash bargaining solution when $a \leq b$.*

Proof: When there are no clever bargainers, the minimum resistance to moving from the convention associated with any division $(x, 1-x) \in D(\delta) \times D(\delta)$ to a state in some other basin is the smallest integer greater than or equal to $mr_\delta(x)$, where

$$r_\delta(x) = \min\left\{r_\delta^A(x), r_\delta^B(x)\right\} , \tag{1}$$

$$r_\delta^A(x) = a \min\left\{1 - \frac{u(x-\delta)}{u(x)}, \frac{u(x)}{u(1-\delta)}\right\} , \tag{2}$$

$$r_\delta^B(x) = b \min\left\{1 - \frac{v(1-x-\delta)}{v(1-x)}, \frac{v(1-x)}{v(1-\delta)}\right\} . \tag{3}$$

(see proof to Lemma 1 in Young (1993b, p. 156). The two terms in $r_\delta^A(x)$ represent mistakes that occur in population B and are drawn by agents in population A, while the two terms in $r_\delta^B(x)$ represent mistakes that occur in A and are drawn by agents in B. Clever agents in A look at their own population history, and best reply to the best reply of the sample they draw. Their resistance to a change of a convention is exactly the same as that of an agent in B who draws a sample of size am. This is the case because a clever agent first figures out how a agent from B would behave had that agent drawn the same sample. Then, the clever bargainer demands a share which is a best reply to this. With clever agents in population A, the factor b in equation (3) is thus replaced by the factor $\min\{a, b\}$.

When $a \geq b$, Young's proof hence applies, since then $\min\{a, b\} = b$. In this case the presence of the new type of agent does not affect the minimum number of mistakes required to displace a convention. The effect is equivalent to providing some individuals in population B with a larger sample size and in Young's model it is the smallest sample size in each population that determines the outcome of the process.

Suppose instead that $a < b$. Then $\min\{a, b\} = a$, so equation (1) becomes

$$r_\delta(x) = a \min\left\{1 - \frac{u(x-\delta)}{u(x)}, \frac{u(x)}{u(1-\delta)}, 1 - \frac{v(1-x-\delta)}{v(1-x)}, \frac{v(1-x)}{v(1-\delta)}\right\} . \tag{4}$$

However, $v(1-x)/v(1-\delta) \geq 1 - u(x-\delta)/u(x)$ for all $x \in D(\delta)$.[5] Equation (4) can thus be simplified to

$$r_\delta(x) = a \min\left\{1 - \frac{u(x-\delta)}{u(x)}, \frac{u(x)}{u(1-\delta)}, 1 - \frac{v(1-x-\delta)}{v(1-x)}\right\} . \tag{5}$$

[5] To see this, note that by concavity of u and v, $v(1-x)/v(1-\delta)$ is greater than or equal to $(1-x)/(1-\delta)$, and δ/x is greater than or equal to $1 - u(x-\delta)/u(x)$, for all $x \geq \delta$. Moreover, since $\delta \leq x \leq 1-\delta$ for all $x \in D(\delta)$, $(1-x)x$ is greater than or equal to $(1-\delta)\delta$, or, equivalently, $(1-x)/(1-\delta) \geq \delta/x$, for all $x \in D(\delta)$.

Let $f : (0,1) \to \mathbb{R}$ be defined by $f(x) = \lim_{\delta\to 0} r_\delta(x)/\delta$, where

$$\lim_{\delta\to 0} r_\delta(x)/\delta = a \min\left\{\frac{u'(x)}{u(x)}, \frac{v'(1-x)}{v(1-x)}\right\} . \tag{6}$$

Since $u'(x)/u(x)$ is decreasing in x and $v'(1-x)/v(1-x)$ is increasing in x, the maximum of f is achieved at the unique point $x \in (0,1)$ where $u'(x)/u(x) = v'(1-x)/v(1-x)$, i.e., at

$$x^* = \arg\max_{x\in[0,1]} u(x)v(1-x) , \tag{7}$$

the standard Nash bargaining solution.[6] **End of proof.**

Next, we briefly consider an alternative and more powerful form of "cleverness" in one of the two populations: individuals who have the ability to actually observe their opponent's demand, and who responds optimally. It turns out that Young's result is robust even with respect to this type of clever agents among his boundedly rational agents, as long as not all agents in the population are this "clever." Also this result is insensitive to whether or not clever individuals occasionally mutate.

Proposition 3. *Let A and B be homogeneous populations composed respectively of types (a,u) and (b,v), where $a,b \leq 1/2$. Assume that a share $\lambda < 1$ of population A is replaced by individuals who observe their opponent's demand, and best reply to it. All other individuals behave as in Young's model. For every precision $\delta > 0$ there exist at least one and at most two generically stable divisions, and as $\delta \to 0$ they converge to the generalized Nash bargaining solution with bargaining-power indices a and b.*

Proof: The proofs of Lemmata 1-3 in Young (1993b) apply to this case. Young's proof is based on assigning to each convention a "resistance" which is the minimum number of mistakes required to displace it. Resistances are not affected by the introduction of clever agents since they only respond to mistakes indirectly by best replying to their opponents' demands. **End of Proof.**

Observe that the presence of this type of clever bargainers improves the social efficiency of the outcomes. Since these individuals always suggest divisions which are Pareto efficient they move the outcome towards the Pareto frontier.

The next and final result considers the extreme case when one of the populations consists entirely of clever individuals of either of the two above types. It turns out that there is a discontinuity at this end of the spectrum: all individuals in the clever population obtain the whole pie as the division precision goes to zero. For this result it is important that the clever individuals occasionally mutate, just like the non-clever individuals do.

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[6]The existence of such a point follows from the assumptions that $u(0)$ and $v(0)$ are zero while $u'(0)$ and $v'(0)$ are positive.

Proposition 4. *Let A and B be homogenous populations composed respectively of types (a,u) and (b,v), where $a,b \leq 1/2$. Assume that all individuals in A either observe their opponent demand and best reply to it, or know B's utility function and best reply to a best reply of some sample of size am drawn from the own population's history. All individuals in population B behave as in Young's model. For every precision $\delta > 0$ there exist at least one and at most two generically stable divisions, and these converge to $(x,y) = (1,0)$ as $\delta \to 0$.*

Proof: Now both populations look only at the history of population A. This implies that the only mistakes which matter are those happening in that population. From (1) we get $r_\delta(x) = r_\delta^B(x)$. Let $f : (0,1) \to \mathbb{R}$ be defined by $f(x) = \lim_{\delta\to 0} r_\delta(x)/\delta$, where

$$\lim_{\delta\to 0} r_\delta(x)/\delta = \min\{a,b\} \frac{v'(1-x)}{v(1-x)} \,. \tag{8}$$

Since $v'(1-x)/v(1-x)$ is increasing in x, the maximum of f is achieved at $x = 1$. **End of Proof.**

For every precision $\delta > 0$, and under the hypotheses of Proposition 4, all individuals in A get at least half the pie in these divisions, for any δ. To see this, first recall from the proof of Proposition 4 that $r_\delta(x) = r_\delta^B(x)$. Second, by concavity of v, $[v(1-x) - v(1-x-\delta)]/\delta$ is less than or equal to $v(1-x)/(1-x)$, for all $x \in (0,1)$. Hence, in view of the monotonicity of the involved function:

$$\arg\max_{x\in D(\delta)} r_\delta(x) \geq \arg\max_{x\in D(\delta)} \left(\min\left\{\frac{\delta}{1-x}, \frac{v(1-x)}{v(1-\delta)}\right\}\right) \,. \tag{9}$$

Again by concavity of v, and using the fact that $1-x \leq 1-\delta$ for all $x \in D(\delta)$, $v(1-x)/(1-x)$ is greater than or equal to $v(1-\delta)/(1-\delta)$ for all $x \in D(\delta)$. In view of the monotonicity of the involved function:

$$\arg\max_{x\in D(\delta)} r_\delta(x) \geq \arg\max_{x\in D(\delta)} \left(\min\left\{\frac{\delta}{1-x}, \frac{1-x}{1-\delta}\right\}\right) \,. \tag{10}$$

Clearly $(1-x)/(1-\delta)$ is decreasing in x, $\delta/(1-x)$ is increasing in x, and these two curves intersect at $x = 1 - \sqrt{\delta(1-\delta)} \geq 1/2$. Therefore,

$$\arg\max_{x\in D(\delta)} r_\delta(x) \geq \max\left\{x \in D(\delta) : x \leq 1 - \sqrt{\delta(1-\delta)}\right\} \tag{11}$$

for all $\delta > 0$.

Remark: If the clever agents never mutate (make mistakes or experiments), and these constitute the whole population A, as in Proposition 4, then mutations in B are disregarded by all individuals in both population. Hence, any division of the pie is then generically stable.

4. Directions for further research

The present study shows that (in important cases) the long-run outcome in Young's (1993b) evolutionary bargaining model is robust to an "invasion" of clever bargainers on one side of the bargaining table. The invasion need not even be small in population terms. What we have not shown, however, is what happens if clever bargainers invade both populations simultaneously. Nor have we shown what happens if clever agents invade some or all populations in the recurrent play of an arbitrary normal-form game, as in the set-up of Young (1993a). However, we hope that our more special results can be helpful for future analyzes of these and related questions.

Such robustness analyses are important not only because it is hard to pin down natural bounds on rationality, but also because they may be helpful in studies of evolutionary selection among alternative forms of boundedly rational behaviors in populations of interacting agents. If a particular form of bounded rationality can be "exploited" by more clever agents, then such clever behaviors may be selected for and such bounded rationality may be selected against, in an evolutionary process. We will accordingly not expect to find much of such boundedly rational behaviors in the long run in environments where such selection pressures operate - which would be bad news for models assuming such behaviors. If instead some form of bounded rationality cannot be exploited, we may expect to find few clever agents but many agents with this form of bounded rationality. In order to study these issues one would have to go beyond our analysis, with fixed population shares of clever and non-clever agents, and study how well clever agents fare among the non-clever.[7] Will their population share increase? Will non-clever agents lose or benefit from the presence of clever agents in another population? Will the long-run outcome be unaffected, as here, or will it be affected via the induced population dynamics? Such further studies appear quite important for the bounded-rationality program in game theory.

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[7] Examples of such approaches, in the context of deterministic population dynamics, are Stahl (1993), Banerjee and Weibull (1995) and Ellingsen (1997).

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