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MANAGERIAL INCENTIVES AND MARKET INTEGRATION

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ABSTRACT. This paper develops a new analytical approach to the old question whether market conditions may influence the internal efficiency of firms. The basic textbook model of the firm is slightly extended to incorporate managers' incentives to reduce production costs in an imperfectly competitive product market. This is done without invoking any agency problem or other form of information asymmetry in firms. The analysis extends Marshallian and Hicksian consumer analysis to managers' demand for leisure in imperfectly competitive environments with a fixed number of firms, and free entry, respectively. Conditions are identified under which product market integration enhances the internal efficiency of firms, and it is shown that market integration is Pareto improving under free entry. (*Doc: man.tex*)

1. INTRODUCTION

Policy discussions concerning competition and trade frequently presume that market integration enhances the internal efficiency in the participating firms. In fact, this is one of the basic policy arguments for the European union. The European commission writes that "*...the new competitive pressures brought about by the completion of the internal market can be expected to ... produce appreciable gains in internal efficiency ...*" ([2], p.126). Such a view has support from certain major classics: "*... good management, ... can never be universally established but in consequence of the free and universal competition which forces every body to have recourse to it for the sake of self-defence...* (Adam Smith [18], pp. 163-164).

By contrast, current standard microeconomics text-book treatments do not deal with this issue: all firms are *presumed* to operate at maximal internal efficiency,

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irrespective of market conditions. A monopoly firm is internally just as efficiently organized as is a firm in an oligopolistic or perfectly competitive market. However, recent theoretical studies of this issue have shown that if the basic model of the firm is expanded to include an agency problem then market conditions may indeed influence the internal efficiency of a firm. More specifically, the incentive power of equilibrium contracts between owners and management may depend on external market conditions. As a result, managerial efforts to improve the internal efficiency of the firm may change when market conditions change, see Holmström [8], Hart [3], Nalebuff and Stiglitz [14], Scharfstein [16], Hermalin [6], Martin [12], Horn, Lang and Lundgren [9], and Schmidt [17].

The term "internal efficiency" used in this literature is perhaps unfortunate, and was originally kept undefined. In his seminal paper on this topic, Leibenstein [10] avoided to give a definition but introduced the term *X-inefficiency*. Broadly speaking, the terms "internal efficiency" and "X-efficiency" were used much in the same way as the layman would use the term "good management." Here these terms will be understood in the narrow sense of "low production costs." A firm that operates at a lower cost at all output levels than another firm will be called "internally more efficient." In particular, "internal efficiency" has little, if anything, to do with Pareto efficiency, since it neglects welfare effects on managers. (Pareto efficiency considerations will be introduced later on in this study.)

In contrast to the cited information-based approaches, the present study takes the viewpoint that market conditions may influence managers' incentives to improve internal efficiency even in the absence of agency problems. An owner who operates his or her own firm is likely to face a trade-off between profits and leisure (taken to be the opposite of effort). Exerting more managerial effort - more intense work, longer hours in the office, or less pleasant decisions (such as firing staff) - the owner-*cum*-manager may improve the firm's internal efficiency and thereby its profits. Moreover, this trade-off may depend on market conditions. For instance, under stiffer competition, equilibrium profits may be lower, and the marginal return to increased effort on profit may or may not be higher. The income effect on managerial effort is unambiguously positive in such circumstances if leisure, taken to be the negative of effort, is a normal good. As sir John Hicks put it: "*The best of all monopoly profits is the quiet life.*" (Hicks [7], p. 8). However, the total effect also depends on the substitution effect, and, of course, on what exactly is meant by "stiffer competition." In particular, market integration involves a certain form of "stiffened competition," but represents a more involved comparative statics experiment.

Income and substitution effects of trade barriers on managerial incentives, in markets where all firms are price takers, have been studied in Corden [1] and J. Martin [11]. The present approach can be viewed as an extension of Martin's model to imperfectly competitive markets with and without barriers to entry. The income effect on managerial effort is studied in the context of an agency model in Hermalin [6].

In terms of analytical tools, the present paper suggests a minor extension of the basic microeconomics text-book model of the firm, perhaps the slightest extension that includes managerial effort as a non-traded input. More exactly, instead of treating a firm's production possibilities as exogenous (and known) to every manager, we here endogenize the production possibility set of a firm by letting it in part depend on its manager's efforts.[1] The more such efforts the manager makes, the more production opportunities become available to the firm.

Indeed, it may be argued that an essential part of management's task is precisely to identify production possibilities available to the firm. This view was advocated by Hayek, who saw economic agent's acquisition of knowledge as a fundamental aspect of an economy: "... *knowledge ... is not given to anyone in its totality...*" (Hayek [4], p.321), "*it is only through the process of competition that the facts will be discovered*" (Hayek [5], p.96).

A pioneering work on the present topic is Leibenstein [10]. His treatment was essentially informal and empirical. On the basis of his empirical studies he claimed that "*The simple fact is that neither individuals nor firms work as hard, nor do they search for information as effectively, as they could. The importance of motivation and its association with degree of effort and search arises because the relation between inputs and outputs is not a determinate one.*" *(p. 407)*. He claimed that a significant part of the beneficial effects of competition come about via increased internal efficiency of firms (*X-efficiency*): "*The data suggest that cost reduction that is essentially a result of improvement in X-efficiency is likely to be an important component of the observed residual in economic growth.*" (p. 408). "*Thus we have instances where competitive pressures from other firms or adversity lead to efforts toward cost reduction, and the absence of such pressures tends to cause costs to rise.*" (pp. 408-409).

The present analysis is performed in a simple formal model, restricted to the case of a Cournot market for a homogenous product, and focuses on two polar cases: a given set of firms, and free entry, respectively. When the set of firms in the market is fixed, all managers simultaneously decide on their own level of managerial effort and their firm's output level. In the case of free entry there is a large population of potential entrepreneurs, each of whom first decides whether or not to set up a firm, and thereby become its owner-*cum*-manager, in the studied product market. (All entrepreneurs have access to all production factors and inputs, at fixed and given prices.)

[1] For the sake of simplicity it is assumed that all managers are equally able. In a richer model, the production possibility set of a firm could depend both on its manager's effort and on her ability.

As an alternative to setting up a firm, each entrepreneur has some (unmodeled) outside option. Once the entry decisions have been taken, these decisions become known, and all entering entrepreneurs simultaneously decide how much managerial effort to exert and how much output to produce - just as in the case of a given set of firms.[2] The analysis is focused on the simplest case: identical entrepreneurs/managers, and symmetric market equilibria.

It turns out that the comparative statics analysis of how managers adapt their efforts to changed market conditions is formally parallel to classical models of the price-taking consumer. In the case of a fixed number of firms, the analysis is similar to the Marshallian demand analysis, while under free entry it takes the form of Hicksian demand analysis. In the first case, changed market conditions induce an income and a substitution effect on manager's effort, while in the second case managers are kept at their reservation utility level, and thus the income effect on their effort is "compensated." A qualitative difference, in comparison with classical consumer demand analysis, is the shift from a non-strategic environment to a strategic environment.

Conditions are identified under which an increased number of firms in a given market induces managers to exert more effort. Hence, stiffer competition so defined leads to increased internal efficiency (lower unit cost of production). A central concern of the study is whether market integration (or trade liberalization) induces higher internal efficiency. The thought-experiment is simple: put together two markets - "countries" - that were before completely isolated from each other. Conditions are identified under which such a change in external conditions results in increased internal efficiency, both when the number of firms per market (country) is fixed and unaffected by the change, and when there is free entry and exit of firms before and after the integration. It is as if, in equilibrium, managers' leisure has a higher relative price after market integration.

In both cases, consumers benefit more from trade than in the standard Cournot model: to the pressure on the market price from an increased number of competitors is added the effect from reduced production costs in each firm. According to Leibenstein [10] the second effect is empirically much stronger than the former. Including the welfare effect on entrepreneurs, the net welfare effect of market integration under free entry is unambiguous: consumers' face a lower price of the product and all entrepreneurs remain at their reservation utility level. Market integration is thus a Pareto improvement.

By adopting the present approach, which neglects informational problems, I do

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[2]This is a variant of the usual way how entry in oligopolistic markets is modeled, see e.g. Section 12E in Mas-Colell, Whinston and Green [13].

not suggest that such problems are unimportant for the questions at hand. On the contrary, such problems seem to be of fundamental importance. However, in the spirit of Occam's razor the present study seeks to find a "minimal" collection of assumptions that together allow for the possibility that market conditions influence the internal efficiency of firms. Richer and more complex models, such as those based on asymmetric information between owners and managers, can hopefully be more easily understood and appreciated against the background of such simpler models.

For a discussion of various concepts of efficiency and competition, and the interplay between these, see Vickers [19]. A recent empirical investigation of relations between competition and corporate performance is given in Nickell [15].

The paper is organized as follows. Section 2 presents the model and section 3 the analytical results. Numerical results for a parametric special case are given in Section 4. Section 5 concludes with a brief discussion of potential extensions. Mathematical proofs are relegated to an appendix at the end of the paper.

2. The model

The model is developed in two steps. First, the set of firms participating in the product market in question is taken to be fixed and given. Then entry and exit decisions are introduced. In other words, we first study a "post-entry" subgame of a larger game, then the full game.

2.1. The demand side. Consider a Cournot product market for a homogeneous good, with n identical firms. The market price p is determined by an inverse demand function, $p = P_m(Q)$, where Q is aggregate output, $Q = \sum_{j=1}^{n} q_j$. Here m is an exogenous parameter that will be interpreted as the *number of* (identical) *countries* in a free-trade area for the good in question. With linear demand in each country, $D(p) = 1 - p$, and in the absence of transportation costs, aggregate demand in the free-trade area is $mD(p) = m - mp$. Motivated by this example, the following assumption will be maintained throughout the analysis:

$$P_m(Q) \equiv 1 - Q/m \ , \tag{1}$$

where $m \in \mathbb{R}_{++}$.

The reader is asked to bear with the extreme simplicity of this functional form. The focus of the study will be on other aspects of the model, and it is conjectured that the qualitative results obtained generalize to other demand functions.

2.2. Production costs and managerial effort. Each firm is managed by its owner. The key assumption in this study is that managerial efforts can reduce the firm's production costs. The channel from managerial effort to production cost is thought to go via the firm's production possibility set, or, equivalently, via its family of input requirement sets (one for each output quantity). By definition, an input requirement set consists of those input vectors each of which is sufficient for production of a given output quantity. Here it is assumed that the manager can expand the input requirement set associated with any given output level by exerting more managerial effort. Formally, if $V(q, e)$ is the input requirement set associated with output level q when the manager exerts effort e, then the total cost to produce q is

$$C(q, w, e) = \min_{z \in V(q,e)} w \cdot z \ , \tag{2}$$

where w is the price vector for inputs. The assumption that the manager can expand the input requirement set is technically speaking a monotonicity requirement: $e < e' \Rightarrow V(q, e) \subset V(q, e') \ \ \forall q$.[3] It follows that the production cost $C(q, w, e)$ is non-increasing in managerial effort e.[4]

For the sake of analytical simplicity, the subsequent analysis will be focused on the special case when there is no fixed cost, and, at any given level of managerial effort, the firm's marginal cost is constant. This marginal cost is assumed to be continuously decreasing in managerial effort at a non-increasing rate. The domain of the effort variable is normalized to the unit interval, and the price vector w is notationally suppressed (since this will be held constant):

> **(A)** $C(q, e) \equiv c(e)q$, where $c : [0, 1] \to [0, 1]$ is a twice continuously differentiable function with $c' < 0$, $c'' \geq 0$, $c(0) = 1$ and $c(1) = 0$.

The assumption of a constant marginal cost holds if production exhibits constant returns-to-scale (CRS) in non-managerial inputs. Let $q = f(z, e)$, where z is an input vector and e is managerial effort. If $f(\lambda z, e) = \lambda f(z, e)$ for all z, e and scalars $\lambda > 0$, then (2) gives $C(q, e) \equiv c(e)q$, where $c(e) \equiv \min_{f(z,e) \geq 1} w \cdot z$.

When aggregate output is Q, price is $p = 1 - Q/m$, by equation (1). Subtracting production costs from revenues, we obtain the following expression for the profit to firm i:

$$\pi_i = \left[1 - \frac{Q}{m} - c(e_i)\right] q_i. \tag{3}$$

[3]Equivalently: more managerial effort increases the production possibility set Y of "netput" vectors: $e < e' \Rightarrow Y(e) \subset Y(e')$.

[4]Cost-reducing managerial effort can also be concerned with adaptation to fluctuating input prices, market conditions, or tax rules. Alternatively, managers may exhibit bounded rationality when they seek a cost-minimizing input vector in a known input requirement set. More effort may then lead to less excessive cost (over the minimum cost).

2.3. Managers' preferences. The manager of each firm derives utility $u(\pi, e)$ from her firm's profit π and her managerial effort e. The analysis is restricted to utility functions of the following separable form:[5]

(B) $u(\pi, e) \equiv \varphi(\pi) - \psi(e)$, where

(B1) $\varphi : \mathbb{R} \to \mathbb{R} \cup \{-\infty\}$ is twice differentiable on $\mathbb{R}_{++}$ with $\varphi' > 0$ and relative risk aversion $r_\varphi > \frac{1}{2}$, and $\varphi(\pi) = -\infty$ for $\pi \leq 0$,

(B2) $\psi : [0, 1) \to \mathbb{R}$ is twice differentiable, $\psi', \psi'' > 0$, $\psi(0) = \psi'(0) = 0$, and $\lim_{e \to 1} \psi(e) = +\infty$.

Hence, a manager's utility is increasing in her firm's profit. This is the case if the manager receives a monetary reward that is an increasing function of her firm's profit, granted she does not consume the product in the studied market (the price of which is endogenous). Moreover, the marginal utility of profit, φ', is decreasing at a rate that is sufficiently high to keep the relative risk aversion of the subutility function φ above one half. The disutility of managerial effort increases with effort, from zero to plus infinity, as effort goes from its lower to its upper bound. Also the marginal disutility of effort is increasing with effort.

2.4. Solution concepts. The interaction between the n managers in the product market is modeled as a simultaneous-move game in which each manager i chooses a combination (e_i, q_i) of effort and output in order to maximize her utility $u(\pi_i, e_i)$. The focus will be on *symmetric* Nash equilibria, i.e., Nash equilibria in which all participating managers choose the same output q and the same effort e.

In the case of free entry and exit such product market interaction will be embedded as a "post entry" subgame of a larger game that involves an "entry stage" in which the number of participating firms is determined endogenously. We then imagine a large population of identical entrepreneurs, each of whom may set up a firm - become an owner-*cum*-manager - in the product market in question. The alternative is to take some outside option that yields utility $\bar{u} \in \mathbb{R}$, the same for all entrepreneurs. Let the set of entrepreneurs be $\mathbb{N}$, the set of positive integers.[6]

The "entry game" is played as follows. First, all entrepreneurs simultaneously decide whether to enter the product market or to take the outside option. Secondly, all entrepreneurs are informed of all entry decisions, and those who entered simultaneously choose a combination of effort and output in the product market - a "post entry" subgame as described above. By *equilibrium* is meant a subgame-perfect equilibrium

[5] The relative risk aversion r_φ of the subutility function φ is defined by $r_\varphi(\pi) = -\pi\varphi''(\pi)/\varphi'(\pi)$.

[6] It will become evident from the subsequent analysis that one could just as well assume the set of entrepreneurs to be finite - at the cost of more notation.

in pure strategies that induces a symmetric Nash equilibrium in each post-entry subgame. For the sake of analytical simplicity the equilibrium number of participating firms, n, will be treated as a real, rather than integer, variable. Consequently, the equilibrium utility to all entrepreneurs, those who enter and those who stay out, is the same, $\bar{u}$.

3. Analysis

It turns out to be analytically convenient to make a transformation of variables before one embarks on such an analysis. Instead of using each managers effort e_i as a decision variable, we will use her *effective* effort, defined as $x_i = 1 - c(e_i)$. By condition (A), there is a one-to-one relation between e_i and x_i, such that x_i is strictly increasing from zero to one as e_i increases from zero to one. Hence, it is decision-theoretically (and strategically) immaterial if we use e_i or x_i as part of i:s strategy. For later notational convenience we will write b for the inverse c^{-1} to the marginal-cost function c.[7] Hence $e_i = b(1 - x_i)$, where $b(0) = 1$, $b(1) = 0$, $b' < 0$ and $b'' \geq 0$, by condition (A).[8]

Conditions (A) and (B) will be assumed to hold throughout this section.

3.1. Given number of firms in the market.

As mentioned above, the focus is here on symmetric Nash equilibrium in the interaction between the n managers in the product market. Using the above transformation of variables, a *strategy* to manager i is a pair $s_i = (x_i, q_i) \in (0,1) \times \mathbb{R}_+$, and the *payoff* to manager i, when a strategy profile $s = (s_1, ..., s_n)$ is played, is

$$U_i(s) = \varphi\left[\left(x_i - \frac{1}{m}\sum_{j=1}^{n} q_j\right) q_i\right] - \psi\left[b(1 - x_i)\right] . \quad (4)$$

A strategy profile s is *symmetric* if there exists a pair $(x, q) \in (0,1) \times \mathbb{R}_+$ such that $s_i = (x, q)$ for all i. Such a pair (x, q) is said to *represent* the profile s. A symmetric profile s is *interior* if $q > 0$. Note that a symmetric Nash equilibrium is necessarily interior. For if $q_i = 0$, then no $x_i \in (0,1)$ is optimal: $U_i(s) = \varphi(0) - \psi[b(1 - x_i)]$, a strictly decreasing function of $x_i \in (0,1)$.

A necessary first-order condition for interior Nash equilibrium is, for each $i = 1, ..., n$:

$$\frac{\partial U_i(s)}{\partial q_i} = 0 \quad \Leftrightarrow \quad q_i = \frac{1}{2}\left(m x_i - \sum_{j \neq i} q_j\right) . \quad (5)$$

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[7] The function c is a bijection from the interval $[0,1]$ to itself.

[8] Differentiation of the identity $b(c(e)) \equiv e$ gives $b'(c(e))c'(e) \equiv 1$. Thus $b' < 0$. Differentiation of the latter identity gives $b''(c(e))\left[c'(e)\right]^2 + b'(c(e))c''(e) \equiv 0$. Thus $b'' \geq 0$.

Hence, in a symmetric Nash equilibrium we necessarily have

$$q = \frac{mx}{n+1}\,, \tag{6}$$

a formula familiar from the standard Cournot model with constant marginal cost and linear demand. (Set $m = 1$ and $x = 1-\ c$.) This is not surprising: since profit has positive marginal utility to the manager, she should, at any effort level that she chooses, adapt her firm's output optimally to its marginal cost. In symmetric equilibrium all effort levels, and hence marginal costs, are the same, and equation (6) results. This equation also shows that the more effort managers exert in a symmetric equilibrium, the more output will their firms produce in this equilibrium. The exact relation between effort and output depends on market conditions, here represented by market size, m, and the number of firms, n. Market conditions matter.

Another necessary first-order condition for interior Nash equilibrium is, for each $i = 1, ..., n$:

$$\frac{\partial U_i(s)}{\partial x_i} = 0 \quad \Leftrightarrow \quad q_i\varphi'\left[\left(x_i - \frac{1}{m}\sum_{j=1}^{n} q_j\right) q_i\right] + \psi'\left[b(1-x_i)\right] b'(1-x_i) = 0\,. \tag{7}$$

Hence, in a symmetric equilibrium the following equation in one variable, the effective managerial effort x, holds (we have used (6)):

$$\frac{mx}{n+1}\varphi'\left[m\left(\frac{x}{n+1}\right)^2\right] + \psi'\left[b(1-x)\right] b'(1-x) = 0\,. \tag{8}$$

Increased effort has a direct and an indirect effect on utility. The indirect effect comes about via the induced increase in profit, involving also optimal adaptation of output. The first term above represents this indirect effect of a marginal increase in effort, and the second term (negative) represents the direct effect. This equation plays a key role in the subsequent analysis.

It is not difficult to show that equation (8) has a unique solution:

Lemma 1. *Equation (8) has exactly one solution, x^*, where $x^* \in (0, 1)$.*

In view of this result the question arises whether the found pair (x^*, q^*), with q^* determined from x^* in equation (6), indeed represents a Nash equilibriun. A sufficient condition for this to hold is that the resulting utility to a manager, $u(x^*, q^*)$, exceeds the utility she would obtain when shutting down her firm, $\varphi(0) - \psi(0) = \varphi(0)$.

Proposition 1. *If $u(x^*, q^*) > \varphi(0)$, then there exists exactly one symmetric Nash equilibrium. This is represented by (x^*, q^*).*

Equation (8) permits certain comparative statics observations. One can show that its solution, the equilibrium level x^* of effective managerial effort, rises with the number n of firms, while the equilibrium level q^* of output per firm falls.

Proposition 2. *x^* is strictly increasing in n, and q^* is strictly decreasing in n.*

In this setting consumers benefit more from increased competition - an increased number of firms n - than in the standard Cournot model. On top of the usual beneficial consequence from a larger number of firms operating at given production costs, we here have an incentive effect inside firms that brings down production costs in each firm. Formally, the equilibrium market price, p^*, is given by a convex combination of the marginal cost $c(e^*)$ and 1:

$$p^* = \frac{n}{n+1}c(e^*) + \frac{1}{n+1} \cdot \tag{9}$$

The weight to the unit price is smaller the more firms there are in the market.[9] Since the marginal cost is less than one, the equilibrium price decreases when the number of firms increases, at any fixed level of managerial effort. According to Proposition 2, the marginal cost $c(e^*)$ decreases with the number of firms in the market. Hence, the price effect from an increase in the number of firms is enhanced.

One would expect that managers' utility falls with an increase in the number of firms in the market. This follows readily from Proposition 2. In symmetric equilibrium, the utility to each manager is

$$v(n,m) = \varphi\left[(q^*)^2/m\right] - \psi\left[b\left(1-x^*\right)\right] \tag{10}$$

By Proposition 2, q^* falls and x^* rises with n, so $v(n,m)$ falls with n by monotonicity of the subutility functions φ and ψ.

Corollary 1. *The managers' equilibrium utility, $v(n,m)$, is continuous and strictly decreasing in n.*

In view of the downward pressure from an increased number of firms on the market price we conclude that utility is transferred from managers to consumers as the number of firms in the market rises.[10]

[9] In the limit case $n \to \infty$ of perfect competition, all weight is given to the marginal cost: the market price then equals marginal cost.

[10] It is presumed here that consumers' welfare is decreasing in the market price p.

A key issue, when it comes to the relation between firms' internal efficiency and market conditions, concerns trade liberalization - or market integration. Imagine that two initially isolated identical markets ("countries") are integrated. Let there be k firms in each country, both before and after the market integration, and let the demand function in each country be as in equation (1). Such market integration thus results in a simultaneous doubling of n and m. More generally, let n and m change proportionately. What is the effect on internal efficiency? It is not difficult to show that if there initially is more than one firm in the market, then internal efficiency increases:

Proposition 3. *If $n \equiv km$ for some $k > 0$, then x^* is strictly increasing in m, whenever $n > 1$.*

In other words, trade liberalization is beneficial to the internal efficiency of the participating firms, granted the number of firms per country remains unaffected by the changed market conditions. (Hence, one may think of this as the short run effect.) But what if the number of firms is endogenous? This is the topic of the next subsection.

3.2. Free entry and exit. In equilibrium in the full entry game no active entrepreneur, i.e., an entrepreneur who decided to enter and become the manager-*cum*-owner of a firm, obtains a utility below her reservation utility $\bar{u}$. In this sense, no active entrepreneur has an incentive to exit. Moreover, the number of firms in the reached subgame is such that if one more firm had entered, then the resulting equilibrium utility in that subgame would have fallen below $\bar{u}$. In this sense, no passive entrepreneur has an incentive to enter. Since the number of participating firms will be treated as a real, rather than integer, variable, the equilibrium utility to all entrepreneurs, active and passive alike, is exactly $\bar{u}$: a single equation replaces two inequalities. The subsequent analysis concerns only symmetric equilibria in the product market.

It is assumed in this subsection that the outside option is better than setting up a firm and then shutting it down. Formally: $\bar{u} > \varphi(0)$.

By Corollary 1 above, $v(n, m)$ is continuously and strictly decreasing in n. Hence, for any reservation utility level $\bar{u} \in \mathbb{R}$ there exists at most one (real) number $n \geq 1$ of firms such that $v(n, m) = \bar{u}$, i.e., such that all entrepreneurs are indifferent between market entry and the outside option. If the reservation utility $\bar{u}$ is too high - above a monopolist's utility level - then no such n exists. Likewise, if the reservation utility $\bar{u}$ is too low - below the utility level of a manager of a firm in a perfectly competitive market - then no such n exists. In the first case the number of firms in the market is zero, and in the latter case it is plus infinity. However, the latter case is excluded since in the limit case of infinitely many firms the utility to a manager is at most $\varphi(0)$, a utility level that by assumption is below $\bar{u}$.

Formally, let $n^*(m)$ denote the equilibrium number of firms under free entry and exit. Then $n^*(m) = 0$ if $v(1, m) < \bar{u}$, and otherwise $n^*(m)$ is the unique solution to the equation

$$v(n, m) = \bar{u} \ . \tag{11}$$

Not surprisingly, the number of participating firms under free entry and exit increases with the size m of the market:

Proposition 4. *There exists some minimal market size $m^o > 0$ such that $n^*(m) = 0$ for $m < m^o$, $n^*(m^o) = 1$, and $n^*(m) > 1$ is strictly increasing in m for $m > m^o$.*

The question arises whether this induces more or less managerial effort in the resulting product market equilibrium.[11] Imagine that the product market initially is in a symmetric interior Nash equilibriun, with n active firms (where n may, but need not, equal $n^*(m)$), each producing output quantity q^*, and each manager exerting effective effort x^*. Suppose some manager i contemplates alternative effort/output pairs for herself. If she chooses effort/output pair (x_i, q_i), while all other firms remain at their equilibrium output level, her firm's profit becomes

$$\pi_i = \left[x_i - \frac{n-1}{m} q^* - \frac{1}{m} q_i \right] q_i \ . \tag{12}$$

While profit and effort affect her utility directly, her firm's output matters only indirectly to her, via its effect on her firm's profit. Suppose that manager i, given any effective effort x_i that she contemplates to exert, chooses her firm's output level q_i so that her firm's profit is maximized, conditional on her effective effort x_i and under the hypothesis that the other firms produce their equilibrium output q^*. It is easily verified that she will then choose

$$q_i = \frac{m}{2} \max \left\{ 0, x_i - \frac{n-1}{m} q^* \right\} \ , \tag{13}$$

[11] Another, much simpler question is whether managerial effort increases when a market, that initially has barriers to entry, is opened to free entry. Presuming free exit both before and after the change, the initial number n of firms cannot have exceeded $n^*(m)$. In the case of equality, the answer is that there is no change: neither market size nor the number of firms are affected and so managerial effort remains at the same level, according to the above analysis. In the case of a strict inequality, $n < n^*(m)$, the answer is that managerial effort increases. For m is constant while the number of firms increases, and so managerial effort increases, by Proposition 2.

or equivalently (using (6)),

$$q_i = \frac{m}{2} \max \left\{ 0, x_i - \frac{n-1}{n+1} x^* \right\} . \tag{14}$$

The manager will thus (optimally) choose a higher output level the more effort she has decided to exert. At low level of effort, her firm's marginal cost $c_i = 1 - x_i$ is so high that the optimum output level is zero. From the viewpoint of the resulting utility to the manager, we may without loss of generality assume that she considers only effective effort levels $x_i > \frac{n-1}{n+1} x^*$. Given such choices of effort, and with optimal adaptation of output to effort, the profit to firm i is a convex increasing function of its manager's effective effort:

$$\pi_i = \frac{m}{4} \left(x_i - \frac{n-1}{n+1} x^* \right)^2 . \tag{15}$$

The graph of this function defines the manager's "possibility frontier" in the (π_i, x_i)-plane. These are the best combinations of profit and effective effort available to the manager when all other managers exert their equilibrium effort, i.e., the maximal profit level possible for each level of effective effort. In the same plane we may draw indifference curves for manager i. The optimal effective effort for manager i is x^*, a point of tangency between her possibility frontier and one of her indifference curves. Under free entry this indifference curve is determined by managers' reservation utility, more exactly by the equation $u[\pi_i, b(1 - x_i)] = \bar{u}$, see Figure 1.

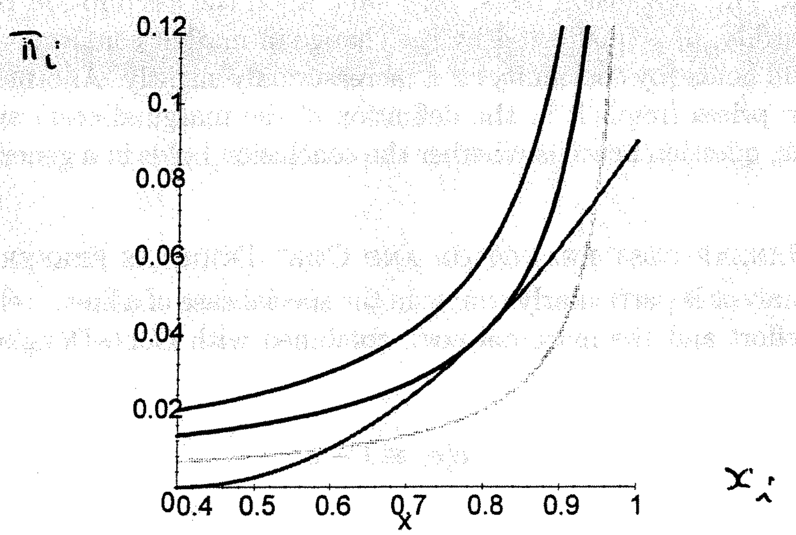

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Figure 1: The equilibrium "possibility frontier" of manager i, and three indiffererence curves. (Parametrization as in Section 4, with $m = \lambda = 1$ and $n = 3$.)

Market integration here means an increase in the parameter m, accompanied by an increase in the number $n^*(m)$ of firms. The reservation utility of managers is assumed to be unaffected, but the equilibrium "possibility frontier" of managers may change. Hence, the effect of market integration is that the tangency point may move. Note that if it moves, then managerial effort and profit per firm move in the same direction: either both increase or both decrease. It turns out that both increase: it is as if managers trade "leisure" for "money" when the market expands. Consequently, market integration under free entry and exit enhances the internal efficiency of firms.

Proposition 5. *The equilibrium managerial effort is strictly increasing in market size m under free entry.*

By construction, the utility of all entrepreneurs remains constant under market integration. However, consumers in the product market benefit in two ways. On top of the well known increased allocative efficiency gain due to the increased number of participating firms, resulting in a lower market price, managers work harder and so firms operate under lower costs, adding to the downward pressure on the market price. Granted that consumers' welfare is decreasing in the price of the product in question, we conclude that market integration is a Pareto improvement: a welfare gain for consumers and no welfare loss for managers.

Remark: This conclusion rests, *inter alia*, upon the assumption that managers' reservation utility, $\bar{u}$, is unaffected by the change in market conditions. The qualitative result still holds (by continuity) if $\bar{u}$ increases only slightly. Another presumption is that factor prices (implicit in the definition of the marginal cost) are unaffected.. An interesting question hence is whether the conclusion holds in a general equilibrium setting.

4. Linear cost reduction and Cobb-Douglas preferences

The above analysis is particularly simple in the special case of a linear relation between managerial effort and the marginal cost, combined with Cobb-Douglas preferences. Let

$$c(e) \equiv 1 - e \tag{16}$$

and

$$u(\pi, e) \equiv \log \pi + \lambda \log (1 - e) \tag{17}$$

for some $\lambda > 0$.[12] The residual $z = 1 - e$ may be interpreted as leisure and λ as the intensity in managers' taste for leisure. It is easily verified that conditions (A) and (B) are met. In this special case effective effort and effort coincide: $x = b(1-e) = 1-(1-e) = e$.

4.1. Fixed number of firms. The first-order condition (8) has the explicit solution

$$e^* = x^* = \frac{n+1}{n+1+\lambda} . \tag{18}$$

A striking feature of this equation is that the parameter for market size, m, is absent. Hence, in this special case equilibrium managerial effort, and hence also the internal efficiency of firms, is independent of market size. The equilibrium effort level is thus a concave increasing function of the number n of firms and hyperbolically decreasing in managers' taste λ for leisure.

The associated market price is a convex decreasing function of the number n of firms and increasing in the managers' taste λ for leisure:

$$p^* = \frac{1+\lambda}{n+1+\lambda} . \tag{19}$$

As the number of firms tend to infinity, the equilibrium managerial effort approaches its upper bound, 1, irrespective of managers' taste $\lambda > 0$ for leisure. Hence, in the limit of perfect competition even the most leisure-loving managers exert maximal effort. Accordingly, the market price then approaches zero - the limit marginal cost when effort is maximal.

In contrast to effort and price, output and profit per firm do depend on market size,

$$q^* = \frac{m}{n+1+\lambda} \text{ and } \pi^* = \frac{m}{(n+1+\lambda)^2} . \tag{20}$$

4.2. Free entry. As expected, the number of firms in the market under free entry is increasing in market size (see Figure 2):

$$n^*(m) = \max\left\{1, \left(m\lambda^\lambda \exp\left(-\bar{u}\right)\right)^{\frac{1}{2+\lambda}} - 1 - \lambda\right\} . \tag{21}$$

[12]Note that the domain of the subutility function φ is here restricted to $\mathbb{R}_{++}$. The domain may be extended to $\mathbb{R}$ by setting $\varphi(\pi) = -\infty$ whenever $\pi \leq 0$, *mutatis mutandis*.

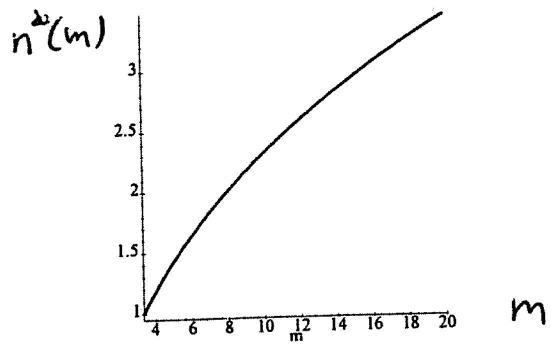

Figure 2: The equilibrium number of firms as a function of market size under free entry, when $\lambda = 1$ and $\exp(\bar{u}) = \frac{1}{8}$, resulting in $n^*(m) = 2(m^{\frac{1}{3}} - 1)$ for $m > m^o = \frac{27}{8}$.

Note the concavity of the function n^*: market integration results in a reduction of the total number of firms. For instance, if two markets of size $m = 8$ is integrated, then the number of firms falls from $2n^{**}(8) = 4$ to $n^{**}(16) \approx 3$ (see Figure 2).

Inserting the expression for $n^*(m)$ in equation (21), one obtains the following expressions for managerial effort, $e^{**}(m)$, under free entry and exit:

$$e^{**}(m) = 1 - \left(\frac{\lambda^2 \exp(\bar{u})}{m}\right)^{\frac{1}{2+\lambda}} \tag{22}$$

As expected, market integration induces managers to exert more effort, see Figure 3.

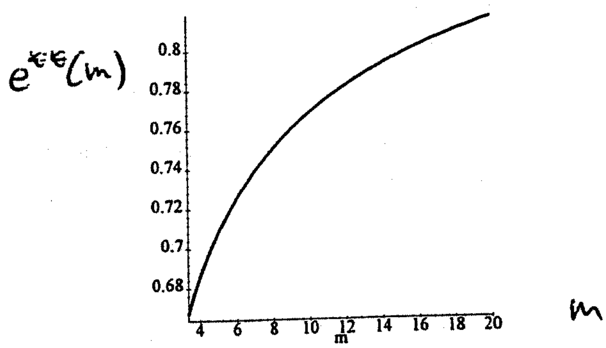

Figure 3: The equilibrium level of managerial effort as a function of market size under free entry, when $\lambda = 1$ and $\exp(\bar{u}) = \frac{1}{8}$, resulting in $e^{**}(m) = 1 - \frac{1}{2}m^{-\frac{1}{3}}$.

Likewise, inserting the expression for $n^*(m)$ in equation (21), one obtains the following expressions for the equilibrium market price, $p^{**}(m)$, under free entry and exit (see Figure 4):

$$p^{**}(m) = (1+\lambda)\left(\frac{\exp(\bar{u})}{\lambda^\lambda m}\right)^{\frac{1}{2+\lambda}} \tag{23}$$

Consequently, market integration increases the welfare of consumers of the product while the welfare of entrepreneurs remains constant (at their reservation level).

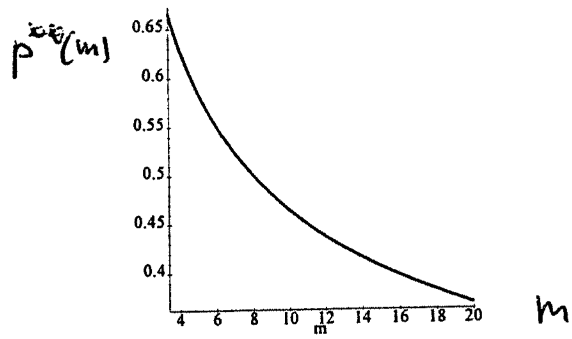

Figure 4: The equilibrium market price as a function of market size under free entry, when $\lambda = 1$ and $\exp(\bar{u}) = \frac{1}{8}$, resulting in $p^{**}(m) = m^{-\frac{1}{3}}$.

4.3. Marshall vs. Hicks. If we think of the residual $z_i = 1 - e_i$ as the manager's *leisure* and π_i as the *upper bound* on her consumption, then the manager's "budget set" $B(n,m)$ in the leisure/consumption space is obtained from equations (15) and (18):

$$\begin{aligned} B(n,m) &= \left\{(z_i,\pi_i) : 0 < z_i \leq \frac{2+\lambda}{n+1+\lambda},\ \pi_i \leq \frac{m}{4}\left(\frac{2+\lambda}{n+1+\lambda} - z_i\right)^2\right\} \cup \\ &\cup \left\{(z_i,\pi_i) : \frac{2+\lambda}{n+1+\lambda} < z_i \leq 1, \pi_i \leq 0\right\} \end{aligned}$$

Figure 5 shows (the interesting part of) this non-convex set, along with a few indifference curves.

First suppose the number n of firms and market size m both are fixed and given. Any change in market conditions, i.e., in these two parameters, results in a shift of the budget set $B(n,m)$, and the effects on managers's demand for leisure - hence supply of managerial effort - can be studied just as in Marshallian demand analysis.

The effect of a small market change can be decomposed into an "income effect," i.e., a parallel shift of the "budget curve," and a "substitution" effect, i.e., a change in the slope of the "budget curve". The total effects of arbitrarily large such changes were reported above.

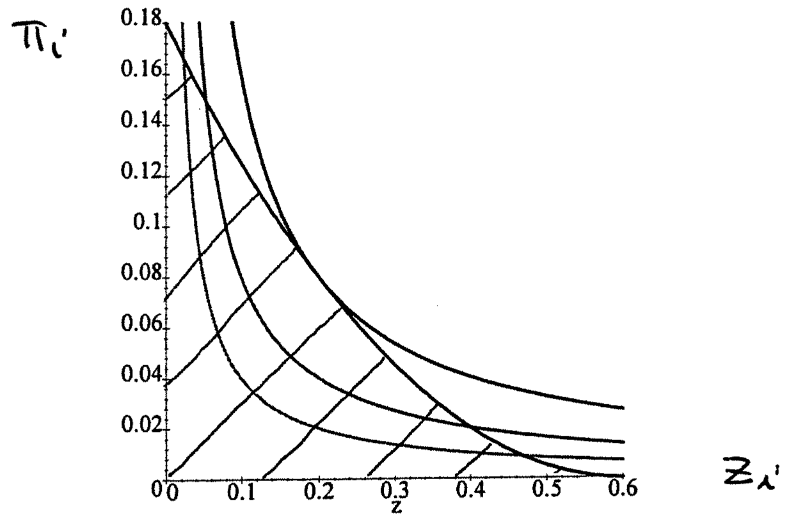

Figure 5: The equilibrium "budget set" of manager i, for $\lambda = 1$, $m = 2$, and $n = 3$, and three indifference curves.

Second, suppose only market size m is fixed and given, while the number of firms is determined by free entry and exit. Hence, the number of firms is $n^*(m)$. A change in market conditions can be studied in the spirit of the Hicksian compensated demand analysis. For any change in m is fully compensated by entry and exit of firms - so that all managers remain at their initial utility level. What changes is only the "relative price" of leisure as against consumption - here given by the "possibility frontier." Figure 6 shows how this frontier changes as the market size m changes under free entry. It is as if market expansion induces a higher (here non-linear) "relative price" of leisure. The total effects of arbitrarily large changes in market conditions were reported above.

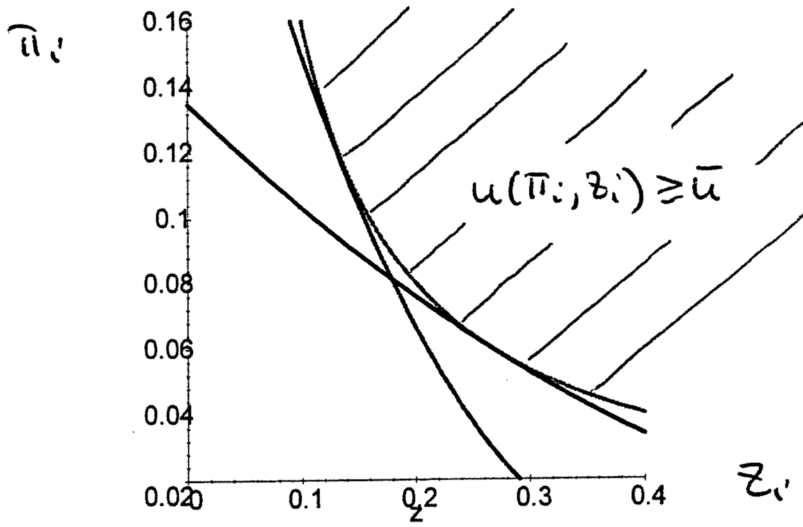

Figure 6: The reservation utility indifference curve, and the "possibility frontier" associated with a small market (the flatter curve), and with a large market (the steeper curve), respectively.

5. Extensions and variations

The above analysis of the old question of whether market conditions may influence the internal efficiency of firms internal efficiency was restricted to a rather special setting. Hence, a variety of extensions are called for before robust conclusions can be claimed. Here is a list of a few possible extensions.

The present study has been restricted to managerial incentives to cut production costs. Incentives to promote product quality and implement useful technical innovations are highly relevant potential extensions. In such a setting the present description of "entrepreneurs" may be enriched.

Transport costs and trade barriers: In the above analysis, changes in trade possibilities were described in of the "all or nothing" variety. In practice, market integration is associated with gradual changes in transport costs or trade tariffs and other barriers. A straight-forward approach to incorporate sizeable transport costs is to let there be a fixed unit cost t, paid by each firm when shipping its produce to a foreign country. Then the product market in each country would consist of domestic firms with marginal cost $c(e)$ and foreign firms with marginal cost $c(e) + t$.

Profit taxes: Unlike in the standard model of the firm, a profit tax can, in the present framework, have effects on managerial incentives and hence on market outcomes. One relevant question is whether taxes may be used to improve managers' incentives.

Demand: The present analysis rests upon the heroic assumption that demand is linear. Are the qualitative results valid under more general demand specifications?

General equilibrium analysis: An important extension of the present model would be to allow for general equilibrium effects. In particular, general equilibrium effects on relevant factor markets and outside options to entrepreneurs may affect some of the results.

Multinationals: What are the incentive effects of market integration in the presence of multinational firms? An analysis of very large firms, as is usually the case with multinationals, calls for a richer model of management than the simple owner-*cum*-manager model adopted here.

6. Appendix: Mathematical Proofs

6.1. Lemma 1. Let $F(x, n, m)$ denote the left-hand side of equation (8). For any $x \in (0, 1)$:

$$\frac{\partial}{\partial x} F(x, n, m) = \frac{m}{n+1} [\varphi' + 2\pi\varphi''] - (b')^2\psi'' - b''\psi' ,$$

where

$$\pi = m \left(\frac{x}{n+1}\right)^2 .$$

By (B), the expression in square brackets is negative at all profit levels, and each of the two other terms, $-(b')^2\psi''$ and $-b''\psi'$, is negative. Hence, $F(x, n, m)$ is strictly decreasing in x and thus (8) has at most one solution. Moreover, by (A) and (B):

$$\lim_{x \downarrow 0} F(x, n, m) = \lim_{x \downarrow 0} \frac{mx}{n+1} \varphi' \left[m \left(\frac{x}{n+1}\right)^2 \right] > 0$$

(the expression is positive and decreasing for all $x > 0$), and

$$\lim_{x \uparrow 1} F(x, n, m) = - \lim_{e \uparrow 1} \psi'(e) = -\infty ,$$

so existence and uniqueness of a solution $x^* \in (0, 1)$ to equation (8) has been established.

6.2. Proposition 1. Assume $u(x^*, q^*) > \varphi(0)$. It follows from the above lemma that (i) there exists at most one symmetric equilibrium, and (ii) this is represented by (x^*, q^*). It thus remains to prove that $(x_i, q_i) = (x^*, q^*)$ is a best reply for manager

i to the strategy profile where all others use strategy (x^*, q^*). To see that this is the case, first note that the payoff to player i, when others play (q^*, x^*), is

$$\varphi\left[\left(x_i - \frac{n-1}{n+1}x^* - \frac{1}{m}q_i\right)q_i\right] - \psi\left[b(1-x_i)\right] .$$

It follows that it is suboptimal for manager i to choose $x_i \leq \frac{n-1}{n+1}x^*$, since in this case $q_i = 0$ is optimal, and hence $U(s_i, s^*_{-i}) = \varphi(0) - \psi\left[b(1-x_i)\right] < \varphi(0) < u(x^*, q^*) = U(s^*)$. We may hence without loss of generality assume $x_i > \frac{n-1}{n+1}x^*$. It is easily verified that, for any such x_i it is optimal for manager i to produce

$$q_i = \frac{m}{2}\left(x_i - \frac{n-1}{n+1}x^*\right) .$$

The resulting payoff to i is

$$R(x_i) = \varphi\left[\pi_i(x_i)\right] - \psi\left[b(1-x_i)\right] ,$$

where

$$\pi_i(x_i) = \frac{m}{4}\left(x_i - \frac{n-1}{n+1}x^*\right)^2 .$$

Thus

$$R''(x_i) = \frac{m}{2}\left[\varphi'(\pi_i) + 2\pi_i\varphi''(\pi_i)\right] - (b')^2\psi'' - b''\psi' ,$$

and so $R''(x_i) < 0$ by conditions (A) and (B). We already know from the lemma that $R'(x^*) = 0$. Hence, R is strictly concave with $x_i = x^*$ as its unique maximum, so $(q_i, x_i) = (q^*, x^*)$ is the unique best reply to (q^*, x^*).

6.3. Proposition 2. To see that x^* is strictly increasing in n it suffices to note that, by (B),

$$\frac{\partial}{\partial n}F(x, n, m) = -\frac{mx}{(n+1)^2}\left[\varphi'(\pi) + 2\pi\varphi''(\pi)\right] > 0 ,$$

where $F(x, n, m)$ is the left-hand side of (8). This implies $\frac{dx^*}{dn} > 0$.

Likewise, by (6), equation (8) can be re-written in terms of output q as $G(q, n, m) = 0$, where

$$G(q, n, m) = q\varphi'\left(\frac{q^2}{m}\right) + \psi'\left[b\left(1 - \frac{n+1}{m}q\right)\right]b'\left(1 - \frac{n+1}{m}q\right) .$$

We have

$$\frac{\partial}{\partial q}G(q,n,m) = \varphi'(\pi) + 2\pi\varphi''(\pi) - \frac{n+1}{m}\left[(b')^2\psi'' + b''\psi'\right] < 0$$

and

$$\frac{\partial}{\partial n}G(q,n,m) = -\frac{q}{m}\left[(b')^2\psi'' + b''\psi'\right] < 0 \, .$$

Hence, $\frac{dq^*}{dn} < 0$.

6.4. Proposition 3. Let $H(x,m)$ denote the left-hand side of equation (8), when $n \equiv km$, i.e.,

$$H(x,m) = \frac{mx}{km+1}\varphi'\left[m\left(\frac{x}{km+1}\right)^2\right] + \psi'\left[b(1-x)\right]b'(1-x) \, .$$

We know from the proof of Lemma 1 that $H(x,m)$ is strictly decreasing in x, so it suffices to show that $H(x,m)$ is strictly increasing in m. But this follows immediately from its definition: the first factor in the first term is an increasing function of m, the marginal subutility of profit is decreasing by (B), and its argument, the profit, is decreasing in m, granted $n = km > 1$.

6.5. Proposition 4. To see that $n^*(m)$ is strictly increasing in m wherever $n^*(m)$ is positive, suppose m and $\bar{u}$ are such that $n^*(m) > 1$. Then $v\left[n^*(m'), m'\right] \equiv \bar{u}$ for all $m' \in \mathbb{R}_+$ in a neighborhood of m. Since $v(n,m)$ is strictly increasing in m (remains to be shown) and strictly decreasing in n, $n^*(m)$ is strictly increasing in m.

6.6. Proposition 5. Let

$$P(x,m) = \frac{m}{4}\left(x - a(m)\right)^2,$$

where

$$a(m) = \frac{n^*(m) - 1}{n^*(m) + 1}x^{**}(m)$$

and $x^{**}(m)$ is the unique solution to equation (8) for $n = n^*(m)$. Hence $P(x,m)$ is the profit to a firm whose manager exerts effective effort x and adapts her firm's output level optimally, granted that all other firms produce their equilibrium output.

Fix m, and study the (π, x)-plane (see Figure 1). The graph of $P(\cdot, m)$ lies below the iso-utility curve $u\left[\pi, b(1-x)\right] = \bar{u}$, with a unique tangency point where $x = x^{**}(m)$. Suppose $m' > m$. Also the graph of $P(\cdot, m')$ lies below the same iso-utility curve, with a unique tangency point where $x = x^{**}(m')$. This implies that

$a(m') > a(m)$, since otherwise the graph of $P(\cdot, m')$ would lie (strictly) above the graph of $P(\cdot, m)$, which would contradict that $P(\cdot, m')$ has a point of tangency with the iso-utility curve. Suppose $P(x, m') \geq P(x, m)$ for some $x < x^{**}(m)$. Then $P(\cdot, m')$ would intersect the indifference curve at some such x, and $x = x^{**}(m)$ would not be optimal for manager i when other managers use their equilibrium strategy and the market size is m, a contradiction. Thus $P(x, m') < P(x, m)$ for all $x < x^{**}(m)$, and hence $x^{**}(m') > x^{**}(m)$.

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