Collusion-proof yardstick competition*

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Abstract

This paper analyses the incentives for collusion among firms when an industry is regulated by means of yardstick competition. The central assumption is that firms must write collusive side contracts before the revelation of private, correlated information and are unable to communicate later. The analysis shows that collusion is costly to society only if firms can commit to side payments. Third-best, collusion-proof regulation entails more (less) distortion of efficiency for low-productivity (high-productivity) firms than second-best yardstick competition. The benefit of yardstick competition vanishes in the limit as correlation of private information becomes near perfect.

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1. Introduction

Regulated industries such as telecommunications, electricity distribution, water supply and health care consist of multiple firms. Costs typically covary across firms since they produce under similar conditions. Correlation of costs gives the regulatory authority the possibility to establish industry standards against which to measure the performance of each individual firm. High (low) performance firms are rewarded (punished) accordingly. This is known as yardstick competition or relative performance evaluation and is a powerful tool to extract rents and increase production efficiency.[1] Due to its attractive features, Laffont and Tirole (1993, 86) predict "an increased use of yardstick competition in segments of regulated industries such as water and electricity distribution." Examples of current applications of yardstick competition include the UK water industry (Cowan, 1997), Norwegian electricity distribution (Dalen et.al., 1998), the Japanese railway industry (Mizutani, 1997) and the operation of Swedish motor-vehicle inspections (Ylvinger, 1998). The best known example of yardstick competition is probably Medicare's reimbursement of hospital costs in the US, see Shleifer (1985) for a description.[2]

The problem with yardstick competition, as with other forms of competition,

[1] For example, the regulator is able to extract *all* surplus from firms and reach full efficiency if technologies are perfectly correlated. See Baiman and Demski (1982), Holmström (1982), Nalebuff and Stiglitz (1983), Demski and Sappington (1984) and Mookherjee (1984) for general treatments and Shleifer (1985), Auriol and Laffont (1992), Auriol (1993), Dalen (1998) and Sobel (1999) for applications to regulation.

[2] Although this paper focuses on its application to regulation, yardstick competition extends beyond performance comparisons of firms. Many labour contracts have elements of relative performance evaluation (Lazear, 1995), and Rappaport (1999) argues that manager compensation should be related to how the firm performs relative to a "peer index". It has been suggested that public agencies could be divided into subunits in order to facilitate efficiency comparisons. The Review of Commonwealth/State Service Provision (RCSSP) in Australia publishes since 1995 performance indicators on a number of government services such as education, health and justice. One of the intentions is to "foster yardstick competition by promoting greater debate about comparative performance" (RCSSP, 1999, 5). Further, Yaisawarng and Puthucheary (1997) sketch how, within public agencies in New South Wales, resources could be allocated among different operation units as a function of their relative efficiency.

is that it creates incentives for collusion. When regulated firms realise that they are played out against each other, they can be expected to take precautionary measures - to collude. The regulator may be forced to compensate firms for not colluding, which reduces the social value of yardstick competition. This paper studies optimal yardstick competition under the threat of collusion. As such, it adds to the mechanism design literature that emphasises collusion among agents as a source of inefficiency.[3] The paper that lies the closest to this essay is Laffont and Martimort (2000), henceforth abbreviated LM. LM consider, just as I do, collusion among agents with private, correlated information. The key difference is that of timing. LM assume agents to possess private information when they write collusive side contracts, whereas firms in my setting must form their cartel before the arrival of private information.[4] Further, I assume that producers are unable to renegotiate the side contracts. Hence, this paper analyses collusion under symmetric information. LM, on the other hand, consider collusion under asymmetric information.

The assumptions maintained here are justified on the following grounds. The purpose of yardstick schemes such as those in UK water supply and Norwegian electricity distribution is to increase efficiency (reduce slack) in production. This means that collusion would involve collective decisions about the level of efficiency to uphold in each firm. As slack cannot be changed ex post (after the period is over), firms must meet ex ante (in the beginning of the period) to decide on collusion. The running of a firm is a continuous process, which means that firms do not know for sure what their future costs are going to be when they form the

[3] Recent contributions include a sequence of papers by Laffont and Martimort (1997,1998, 2000). See also the references cited therein.

[4] There is a minor difference regarding the stochastic structure of private information. LM assume the matrix of probabilities to be nonsingular, enabling the regulator to implement first-best regulation absent collusion, as in Cremer and McLean (1988). In the current context that matrix is singular, which along with limited liability for firms, calls for second-best regulation absent collusion.

cartel. Shocks to production cost such as changes in factor prices and break-down of machinery, occur regularly. Theoretically, firms could meet every time costs change and renegotiate their collusive contracts. However, this is cumbersome and increases the probability of detection. It therefore seems realistic to assume that collusion takes place in the anticipation that private information will arrive in the future and that this eventuality must be incorporated in the collusive side contract today.

I utilize a framework in which firm productivity is private information and consists of a firm-specific and an industry-specific part (Auriol and Laffont, 1992). The regulator applies yardstick competition to elicit the common, industry-specific productivity parameter. High productivity firms have an incentive to jointly understate industry-specific productivity so as to restore informational rent. However, collusion poses a problem to society only if firms can commit to side payments. Otherwise, the regulator can set up a menu of transfers rewarding firms based on the difference between their reported productivity and the industry average. Under this regulatory contract every firm will profitably deviate from an agreement to understate productivity. This renders collusion impossible. Firms truthfully report industry-specific productivity, rewards need never be paid out, and collusion is avoided at no social cost. If, on the other hand, firms have access to unlimited side payments, any menu of sticks and carrots offered by the regulator can be neutralized by appropriately chosen side payments. The anti-competitive effect of collusion creates a trade-off between efficiency and rent-extraction. In the third-best, *collusion-proof* contract (*i*) efficiency of low productivity producers is distorted below the second-best (yardstick competition) level; (*ii*) high productivity firms produce more efficiently than under second-best regulation. The first effect reflects the possibility of increased rent-extraction from high-productivity firms by a reduction in the efficiency of low-productivity firms. The second appears because nobody mimics a high-productivity firm under collusion, hence there is not much scope for efficiency distortions for those types.

The timing of the collusive side contracting game is crucial to the social cost of collusion. Collusion under asymmetric information, as in the LM model, involves credible sharing of private information within the cartel. LM show how the regulator is sometimes able to exploit internal incentive problems. In particular, the regulator can set transfers so as to make collusion very expensive for the cartel when correlation of information is high. When correlation becomes near perfect, the regulator extracts almost all surplus from firms at infinitesimal cost. Consider in stead the case in which the cartel is formed prior to the arrival of private information, as is the case in the model studied here. Firms do not exchange information, hence there are no incentive problems to be exploited. Contrary to the result in LM, the welfare loss of collusion persists in the almost perfectly correlated environment. In fact, the regulator is unable to extract any surplus at all. Collusion allows firms to behave as a merged entity, fully offsetting the effect of yardstick competition. Consequently, firms would sometimes prefer collusive agreements that do not rely on information exchange.

The remainder of this paper is organised as follows: section 2 formulates the model. In section 3, two benchmark cases are derived. The first is regulation of firms as separate entities, the second is yardstick competition under the assumption that firms are unable to collude. Section 4 studies the incentives for collusion. Collusion-proof regulation is characterized in section 5. Lastly, section 6 concludes. Tedious proofs are collected in the appendix.

2. The model

2.1. Technology and preferences

In order to illustrate the assumptions of the model, I shall assume that the regulated industry is the water industry. The water industry consists of a number of regional natural monopolies that utilize a standardized technology for purifying drinking water and treating sewerage. The fact that firms use similar technologies, plus a minimal scope for direct competition for customers, makes the industry a suitable candidate for yardstick competition (Armstrong et.al., 1994).

Assume that there are two regions ($i = 1, 2$), each with inelastic demand for $S/2$ litres of water. Each region is served by a local water purifying plant that supplies the demanded water at cost:[5]

$$C^i = \beta_i - e_i. \tag{2.1}$$

Cost depends positively on an exogenous parameter β_i and negatively on effort $e_i > 0$ exerted by management. Management's disutility of effort is:

$$\psi(e_i) \text{ with } \psi \geq 0,\ \psi' > 0,\ \psi'' > 0,\ \psi''' \geq 0.$$

Disutility of effort is always non-negative, it is increasing in effort at an increasing rate. Non-negativity of the third derivative simplifies comparative statics analysis. The regulator only observes each producer's aggregate cost C^i and cannot disaggregate β_i and e_i without an appropriate incentive contract. Production cost depends on factors common to both as well as factors unique to each firm. For example, an increase in prices for chemicals for treating sewerage or a decrease in the cost of billing customers affects all producers. A change in local pollution

[5] The assumptions of inelastic demand and monopoly production are inconsequential to the analysis if firms produce at constant marginal cost. In that case the appropriate incentive contracts are independent of the pricing rule, see ch. 3 in Laffont and Tirole (1993) - in particular their proposition 3.4. The most efficient firm produces the lion's share of total output.

affects only the firm operating in that region. I capture these effects by assuming that the efficiency or productivity parameter β_i consists of a common part m (factor prices) and an idiosyncratic part ε_i (local pollution). *Industry-specific (IS) productivity* m is high ($m = m_l$) with probability v and low ($m = m_h$) with probability $1-v$ ($\triangle m = m_h - m_l > 0$). *Firm-specific (FS) productivity* ε_i is low ($\varepsilon_i = \overline{\varepsilon}$) with probability ξ and high ($\varepsilon_i = \underline{\varepsilon}$) with probability $1-\xi$ ($\triangle\varepsilon = \overline{\varepsilon} - \underline{\varepsilon} > 0$). m, ε_1 and ε_2 are all assumed to be stochastically independent. Total productivity is given by the weighted average:

$$\beta_i = \alpha m + (1-\alpha)\varepsilon_i. \tag{2.2}$$

Thus, a firm can be one of four types: $\underline{\beta}_l = \alpha m_l + (1-\alpha)\underline{\varepsilon}$, $\overline{\beta}_l = \alpha m_l + (1-\alpha)\overline{\varepsilon}$, $\underline{\beta}_h = \alpha m_h + (1-\alpha)\underline{\varepsilon}$ and $\overline{\beta}_h = \alpha m_h + (1-\alpha)\overline{\varepsilon}$.[6] The weight α is assumed to be common knowledge. Correlation of productivity is increasing in α and perfect at $\alpha = 1$. To simplify the analysis, assume that $\alpha \in (\underline{\alpha}, 1)$ with $\underline{\alpha} = \triangle\varepsilon/(\triangle m + \triangle\varepsilon)$. This ensures $\underline{\beta}_l < \overline{\beta}_l < \underline{\beta}_h < \overline{\beta}_h$. The regulator filters out the realization of the common parameter m by applying yardstick competition. Suppose firm 2 reports m_l and firm 1 has reported m_h. Since m is perfectly correlated across firms, the regulator knows that one of them is lying. He can punish firms for making such incompatible cost reports, which forces both firms to truthfully report m (unless they are colluding).

By an accounting convention, the firm is fully compensated for its production cost C^i and receives in addition a net transfer t^i for serving its market. Hence, firm i's rent is given by:

$$U^i = t^i - \psi(e_i). \tag{2.3}$$

Welfare equals:

$$W = S + \sum_i U^i - (1+\lambda)\sum_i (t^i + C^i), \tag{2.4}$$

[6] This stochastic structure was first introduced by Auriol and Laffont (1992), but with a continuous distribution of the idiosyncratic shock ε_i.

consumer surplus S plus the sum of firm rent U^i minus the social cost of the gross transfer $(t^i + C^i)$ to each firm. λ is the positive shadow price on public funds. As is now standard (2.1), (2.3) and (2.4) are manipulated to yield:

$$W = S - (1+\lambda)\sum_i(\beta_i - e_i + \psi(e_i)) - \lambda\sum_i U^i. \tag{2.5}$$

2.2. Contracts

The regulator proposes the grand contract $\mathbf{D} = (D_{ll}, D_{lh}, D_{hl}, D_{hh})$. It is modelled as a two-step procedure. First, reports on the common shock m are collected from each firm. A firm is offered the sub contract $D_{ss'}$ if it has reported m_s and the other producer has reported $m_{s'}$ $(ss' \in \{ll, lh, hl, hh\})$. $D_{ss'}$ specifies a menu of transfers and cost targets $(\{\underline{t}_{ss'}, \underline{C}_{ss'}\}, \{\overline{t}_{ss'}, \overline{C}_{ss'}\})$. Given $D_{ss'}$, firms go on to reporting their idiosyncratic part ε_i. A firm reporting $\underline{\varepsilon}$ respective $\overline{\varepsilon}$ obtains the cost/transfer pair $\{\underline{t}_{ss'}, \underline{C}_{ss'}\}$ respective $\{\overline{t}_{ss'}, \overline{C}_{ss'}\}$ under $D_{ss'}$. Firms are regulated separately at the second stage due to stochastic independence of the shocks to firm-specific productivity. Attention is without loss of generality restricted to the class of symmetric contracts since firms are identical ex ante and therefore will be offered an identical menu of contracts in equilibrium.

A side contract $\varpi = \{\mathbf{\Gamma}, \boldsymbol{\varphi}\}$ between firms consists of a reporting rule $\mathbf{\Gamma} = (\boldsymbol{\gamma}_l, \boldsymbol{\gamma}_h)$, with $\boldsymbol{\gamma}_k = (\gamma_k^1, \gamma_k^2)$ for $k \in \{l, h\}$, that specifies which level of industry-specific (IS) productivity $m_{\gamma_k^i}$ firm i should report as a function of realized IS productivity m_k, plus a vector of inter-firm transfers $\boldsymbol{\varphi} = (\varphi_{ss'}^1, \varphi_{s's}^2)$ that depends on the vector of reported IS productivity $(m_s, m_{s'})$. The role of these side payments is to discipline firms to stick with the negotiated reporting rule. For simplicity, a balanced budget constraint $\varphi_{ss'}^1 = -\varphi_{s's}^2$ is imposed.

2.3. Timing

I study a static game in which there is production only once. First, the regulator commits to the regulatory contract **D**. Second, each firm accepts or rejects; rejection leaving a firm with reservation utility 0. If both firms agree to produce, they write a side contract ϖ. Third, productivity is revealed and is the private information of each firm. At this stage firms have the opportunity to turn down the regulatory as well as the side contract by shutting down production and receiving reservation utility 0. Fourth, productivity is reported, first m and then ε_i, transfers are paid out as specified by the regulatory contract, and each firm meets its designated cost target. Finally, side transfers, if any, are made.

The interim participation constraint is added for the sake of realism. Normally firms have limited liability; owners are not liable for more than they have put into the firm. Consequently, an owner cannot be prevented from shutting down her firm if she finds production unprofitable. I assume that consumer surplus S is sufficiently high to guarantee that the regulator always finds it socially optimal to offer a contract that satisfies this constraint. Enforceability of side transfers is normally justified by a dynamic argument; firms who fail to comply are punished in subsequent periods. In the static setting there are no subsequent periods, hence side payments may not be possible. I study collusion under two extreme assumptions about the ability to commit to side payments: (i) firms do not have access to side payments at all; (ii) firms can commit to unlimited side payments. Firms' unlimited access to side payments can be considered a worst-case scenario and defines an upper bound to what firms can achieve by colluding.

3. Regulation in the absence of collusion

In this section I derive two regulatory contracts to serve as benchmarks against which to evaluate the effects of collusion. Under both contracts each firm assumes the other to truthfully report the common part m. Let $U_{ss'}(d_i, \varepsilon_i, m)$ be the rent that accrues to firm i given its type $\beta_i = \alpha m + (1-\alpha)\varepsilon_i$, that it reports common shock m_s and idiosyncratic shock d_i and firm j reports common shock $m_{s'}$. To economize on notation, write $\underline{U}_k = U_{kk}(\underline{\varepsilon}, \underline{\varepsilon}, m_k)$ and $\overline{U}_k = U_{kk}(\overline{\varepsilon}, \overline{\varepsilon}, m_k)$. $\underline{U}_k$ and $\overline{U}_k$ are firm rents contingent on both producers' truthful revelation their type. Denote by $\mathbf{e} = (\underline{e}_l, \overline{e}_l, \underline{e}_h, \overline{e}_h)$ the vector of effort levels and by $\mathbf{U} = (\underline{U}_l, \overline{U}_l, \underline{U}_h, \overline{U}_h)$ the vector of firm rents. The expected social cost of production in a region is given by:

$$\begin{aligned}\mathcal{C}(\mathbf{e}) = (1+\lambda)\{&v[\xi(\underline{\beta}_l - \underline{e}_l + \psi(\underline{e}_l)) + (1-\xi)(\overline{\beta}_l - \overline{e}_l + \psi(\overline{e}_l))] \\ &+(1-v)[\xi(\underline{\beta}_h - \underline{e}_h + \psi(\underline{e}_h)) + (1-\xi)(\overline{\beta}_h - \overline{e}_h + \psi(\overline{e}_h))]\},\end{aligned} \tag{3.1}$$

and expected firm rent by:

$$\square(\mathbf{U}) = v[\xi \underline{U}_l + (1-\xi)\overline{U}_l] + (1-v)[\xi \underline{U}_h + (1-\xi)\overline{U}_h]. \tag{3.2}$$

The regulator maximizes consumer surplus minus the expected social cost of production and firm rent (see expression (2.5)):

$$W(\mathbf{e}, \mathbf{U}) = S - 2\mathcal{C}(\mathbf{e}) - 2\lambda\square(\mathbf{U}), \tag{3.3}$$

subject to the incentive compatibility (IC) constraints ($k \in \{l, h\}$):

$$\underline{U}_k \geq U_{kk}(\overline{\varepsilon}, \underline{\varepsilon}, m_k), \tag{3.4}$$

$$\overline{U}_k \geq U_{kk}(\underline{\varepsilon}, \overline{\varepsilon}, m_k), \tag{3.5}$$

$$\underline{U}_k \geq \max\{U_{-kk}(\underline{\varepsilon}, \underline{\varepsilon}, m_k), U_{-kk}(\overline{\varepsilon}, \underline{\varepsilon}, m_k)\}, \tag{3.6}$$

$$\overline{U}_k \geq \max\{U_{-kk}(\underline{\varepsilon}, \overline{\varepsilon}, m_k), U_{-kk}(\overline{\varepsilon}, \overline{\varepsilon}, m_k)\}, \tag{3.7}$$

and the interim participation (IR) constraints ($k \in \{l, h\}$):

$$\underline{U}_k \geq 0, \tag{3.8}$$

$$\overline{U}_k \geq 0. \tag{3.9}$$

The IC constraints (3.4) and (3.5) state that it must be optimal for a firm to truthfully report the idiosyncratic part ε_i under the sub contract D_{kk} for which both firms truthfully report the common part m_k. Further, IC constraints (3.6) and (3.7) require that each firm prefer its designated subcontract D_{kk} to the subcontract D_{-kk} for which it misrepresents industry-specific productivity (reports m_{-k} when $m = m_k$), given the other firm's truthful report of m. The IR constraints (3.8) and (3.9) require that a firm be willing to produce in equilibrium irrespective of its type.

3.1. Individual regulation of firms

Under this regulatory regime each firm is regulated on a separate basis. Collusion is of no concern because each firm's rent is independent of the other firm's performance. In this case it is as if the firms directly report total productivity $\beta_i \in \{\underline{\beta}_l, \overline{\beta}_l, \underline{\beta}_h, \overline{\beta}_h\}$. (3.4)-(3.7) can be replaced by the set of downward-binding IC constraints:

$$\underline{U}_l \geq \overline{U}_l + \overline{\Phi}(\overline{e}_l), \tag{3.10}$$

$$\overline{U}_l \geq \underline{U}_h + \underline{\Phi}(\underline{e}_h), \tag{3.11}$$

$$\underline{U}_h \geq \overline{U}_h + \overline{\Phi}(\overline{e}_h) \tag{3.12}$$

and the restriction that cost targets be weakly decreasing in productivity [7] :

$$\overline{C}_h \geq \underline{C}_h \geq \overline{C}_l \geq \underline{C}_l. \tag{3.13}$$

[7] See Laffont and Tirole (1993) for a formal proof of this claim for the case with a continuum of types $\beta_i \in [\underline{\beta}, \overline{\beta}]$.

A $\underline{\beta}_k$ type mimicking a $\overline{\beta}_k$ type ($k \in \{l, h\}$) receives the same transfer $\overline{t}_k$ as $\overline{\beta}_k$, but reaches the cost target $\overline{C}_k$ at a lower effort, owing to superior productivity $\overline{\beta}_k - \underline{\beta}_k = (1-\alpha)\triangle\varepsilon > 0$. $\overline{\Phi}(e) = \psi(e) - \psi(e - (1-\alpha)\triangle\varepsilon)$ measures the value of the productivity gap $\overline{\beta}_k - \underline{\beta}_k$. Likewise, $\underline{\Phi}(e) = \psi(e) - \psi(e + (1-\alpha)\triangle\varepsilon - \alpha\triangle m)$ is the value of the productivity gap $\underline{\beta}_h - \overline{\beta}_l$. Noting that rents are increasing in productivity, (3.8) and (3.9) can be replaced by:

$$\overline{U}_h \geq 0. \tag{3.14}$$

Rents are costly to society, hence (3.10)-(3.12) and (3.14) are all binding at the optimum. Manipulate these constraints to obtain expected rent as a function of effort:

$$\Box^I(\mathbf{e}) = v\xi\overline{\Phi}(\overline{e}_l) + v\underline{\Phi}(\underline{e}_h) + (v + \xi(1-v))\overline{\Phi}(\overline{e}_h). \tag{3.15}$$

Either $\overline{C}_h = \underline{C}_h$ or $\underline{C}_h = \overline{C}_l$ may hold in optimum, giving rise to bunching. Substitute (3.15) for (3.2) in (3.3) and construct the Lagrangian:

$$\begin{aligned} \mathcal{L}^I(\mathbf{e}) = S - 2\mathcal{C}(\mathbf{e}) - 2\lambda\Box^I(\mathbf{e}) - 2\Box^I(\overline{e}_h - \underline{e}_h - (1-\alpha)\triangle\varepsilon) \\ -2\gamma^I(\underline{e}_h - \overline{e}_l + (1-\alpha)\triangle\varepsilon - \triangle m). \end{aligned} \tag{3.16}$$

$\Box^I$ and γ^I are the Lagrange multipliers associated with respectively $\overline{C}_h \geq \underline{C}_h$ and $\underline{C}_h \geq \overline{C}_l$. Differentiate (3.16) with respect to $\mathbf{e}$ and manipulate the first order conditions to obtain:

$$\psi'(\underline{e}_l^I) = 1, \tag{3.17}$$

$$\psi'(\overline{e}_l^I) = 1 - \frac{\lambda}{1+\lambda}\frac{\xi}{1-\xi}\overline{\Phi}'(\overline{e}_l^I) + \frac{\gamma^I}{(1+\lambda)(1-\xi)}, \tag{3.18}$$

$$\psi'(\underline{e}_h^I) = 1 - \frac{\lambda}{1+\lambda}\frac{v}{(1-v)\xi}\underline{\Phi}'(\underline{e}_h^I) + \frac{\Box^I - \gamma^I}{(1+\lambda)(1-v)\xi}, \tag{3.19}$$

$$\psi'(\overline{e}_h^I) = 1 - \frac{\lambda}{1+\lambda}\frac{v + \xi(1-v)}{(1-v)(1-\xi)}\overline{\Phi}'(\overline{e}_h^I) - \frac{\Box^I}{(1+\lambda)(1-v)(1-\xi)}. \tag{3.20}$$

(3.17)-(3.20) along with the complementary slackness conditions

$$\gamma^I \geq 0,\ \gamma^I(\underline{e}_h^I - \overline{e}_l^I + (1-\alpha)\triangle\varepsilon - \triangle m) = 0, \tag{3.21}$$

$$\square^I \geq 0,\ \square^I(\overline{e}_h^I - \underline{e}_h^I - (1-\alpha)\triangle\varepsilon) = 0 \qquad (3.22)$$

describe the solution to the problem of individual regulation of firms.[8]

3.2. Yardstick competition

Under individual regulation of firms, the regulator is forced pay informational rents in order to prevent firms from understating industry-specific (IS) productivity ((3.7) is binding for $k = l$). Yardstick competition allows the regulator to costlessly elicit this information. Suppose the regulator refuses to pay transfers in case of incompatible reports on m, i.e. $\underline{t}_{lh} = \overline{t}_{lh} = 0$ and $\underline{t}_{hl} = \overline{t}_{hl} = 0$. Each firm expects the other to behave honestly. Hence, any firm misrepresenting its type by selecting D_{hl} over D_{ll} when its type is m_l or D_{lh} over of D_{hh} when its type is m_h, will run a deficit. Consequently, both firms submit a truthful report on m at no cost to society. The constraints (3.6) and (3.7) are no longer binding.

Given truthful reports on m, the regulator is still faced with the problem of extracting ε_i. The constraints (3.4) and (3.5) are equivalent to:

$$\underline{U}_k \geq \overline{U}_k + \overline{\Phi}(\overline{e}_k), \qquad (3.23)$$

$$\overline{C}_k \geq \underline{C}_k, \qquad (3.24)$$

Inequality (3.23) is the downward-binding IC constraint, and (3.24) is the monotonicity requirement on cost targets. Rent is increasing in productivity, hence (3.8) is no longer binding, but (3.23) and (3.9) are. Use this to obtain an expression for expected firm rent under yardstick competition:

$$\square^{yc}(\mathbf{e}) = v\xi\overline{\Phi}(\overline{e}_l) + \xi(1-v)\overline{\Phi}(\overline{e}_h). \qquad (3.25)$$

[8]The neglected constraint $\overline{C}_l \geq \underline{C}_l$ is satisfied since $\overline{C}_l - \underline{C}_l = (1-\alpha)\triangle\varepsilon + \underline{e}_l^I - \overline{e}_l^I$ and $\underline{e}_l^I > \overline{e}_l^I$.

Substitute (3.25) for (3.2) into (3.3), maximize with respect to $\mathbf{e}$ and manipulate the resulting first order conditions to obtain (for $k \in \{l, h\}$):

$$\psi'(\underline{e}_k^{yc}) = 1, \tag{3.26}$$

$$\psi'(\overline{e}_k^{yc}) = 1 - \frac{\lambda}{1+\lambda}\frac{\xi}{1-\xi}\overline{\Phi}'(\overline{e}_k^{yc}). \tag{3.27}$$

The neglected constraint (3.24) is satisfied by this solution since $\overline{e}_k^{yc} < \underline{e}_k^{yc}$. A comparison of effort levels (3.17)-(3.22) under individual regulation with (3.26)-(3.27) under yardstick competition reveals:

Proposition 3.1. *Yardstick competition requires (strictly) less distortion of effort for firms with low IS productivity ($\underline{e}_h^{yc} > \underline{e}_h^I$ and $\overline{e}_h^{yc} > \overline{e}_h^I$) and (weakly) more distortion of effort for firms with high IS productivity ($\underline{e}_l^{yc} = \underline{e}_l^I$ and $\overline{e}_l^{yc} \leq \overline{e}_l^I$) than individual regulation of firms.*

Proof. Let $\mathbf{x} = (\underline{x}_l, \overline{x}_l, \underline{x}_h, \overline{x}_h)$ be the solution to (3.17)-(3.20) for $\square^I = \gamma^I = 0$. $\underline{e}_l^I = \underline{e}_l^{yc} = e^{fb}$, with e^{fb} denoting first-best effort $\psi'(e^{fb}) = 1$. $\psi'' > 0$, $\underline{\Phi}'' \geq 0$ and $\overline{\Phi}'' \geq 0$ imply (*i*) $\overline{e}_h^I \leq \overline{x}_h < \overline{e}_h^{yc}$ by $\square^I \geq 0$ and $\frac{v+\xi(1-v)}{(1-v)(1-\xi)} > \frac{\xi}{1-\xi}$; (*ii*) $\overline{e}_l^I \geq \overline{x}_l = \overline{e}_l^{yc}$ by $\gamma^I \geq 0$; (*iii*) $\underline{e}_h^I < e^{fb} = \underline{e}_h^{yc}$ for $\square^I = 0$, $\gamma^I \geq 0$; (*iv*) $\underline{e}_h^I = \overline{e}_h^I - (1-\alpha)\triangle\varepsilon < \overline{x}_h < e^{fb} = \underline{e}_h^{yc}$ for $\square^I > 0$, $\gamma^I = 0$. ■

Expected firm rent is lower under yardstick competition than under separate regulation

$$\square^I(\mathbf{e}) - \square^{yc}(\mathbf{e}) = v(\underline{\Phi}(\underline{e}_h) + \overline{\Phi}(\overline{e}_h))$$

due to rent extraction. It is no longer necessary to pay m_l types for not understating productivity to m_h. Consequently, marginal rent-extraction is smaller for $\underline{\beta}_h$ and $\overline{\beta}_h$ types under yardstick competition than under individual regulation. This is why effort is distorted less under yardstick competition than individual regulation for m_h types. Under individual regulation the regulator may not be able to distort effort as much as he wants to for the $\overline{\beta}_l$ type due to the constraint that costs be decreasing in productivity ($\overline{C}_l \leq \underline{C}_h$). This constraint is not binding under yardstick competition, which explains $\overline{e}_l^{yc} \leq \overline{e}_l^I$. $\underline{e}_l^{yc} = \underline{e}_l^I$ follows from the fact that effort is undistorted for the most efficient type $\underline{\beta}_l$ under both types of regulation.

Firm i is able to gain informational rent due to private information about its productivity β_i. Observe from (2.2) that productivity β_i is determined mainly by the common part m for α high and by the idiosyncratic part ε_i for α low. m is costlessly filtered out under yardstick competition. Hence, yardstick competition is particularly useful for extracting informational rent from highly similar firms, i.e. for high α. In the limit, as firms become identical, the regulator extracts all surplus and implements first-best regulation.[9]

4. Collusion

Under yardstick competition firms behave non-cooperatively and truthfully report m. They can do better than this by colluding. Suppose the two producers agree to report m_h irrespective of the true value of m and report ε_i non-cooperatively as before. If industry-specific (IS) productivity happens to be m_h, nothing changes since the common shock m is truthfully reported. Not so if IS productivity is m_l. In this case the regulator is fooled into believing that the common shock is m_h, not m_l. He offers the sub contract D_{hh} instead of the correct one, D_{ll}. This has no effect on transfers, but firms are offered more generous cost targets than before since IS productivity is understated.[10] Collusion on m thus enables firms to *retain earnings at reduced effort*.

[9] To see this, note that $\overline{\Phi}(\overline{e}_k^{yc}) = \psi'(\overline{e}_k^{yc}) - \psi'(\overline{e}_k^{yc} - (1-\alpha)\triangle\varepsilon)$ implies $\overline{\Phi}(\overline{e}_k^{yc}) \rightarrow 0$ and $\overline{\Phi}'(\overline{e}_k^{yc}) \rightarrow 0$ for $\alpha \rightarrow 1$ and, consequently, $\psi'(\overline{e}_k^{yc}) \rightarrow 1$ by (3.27).

[10] Use $t = \psi(e) + U$, $\underline{U}_k^{yc} = \overline{\Phi}(\overline{e}_k^{yc})$ and $\overline{U}_k^{yc} = 0$ to obtain $\underline{t}_k^{yc} = \psi(\underline{e}_k^{yc}) + \overline{\Phi}(\overline{e}_k^{yc})$ and $\overline{t}_k^{yc} = \psi(\overline{e}_k^{yc})$. This, $\underline{e}_h^{yc} = \underline{e}_l^{yc}$ and $\overline{e}_h^{yc} = \overline{e}_l^{yc}$ imply $\underline{t}_h^{yc} = \underline{t}_l^{yc}$ and $\overline{t}_h^{yc} = \overline{t}_l^{yc}$. Further, $\underline{C}_h^{yc} - \underline{C}_l^{yc} = \overline{C}_h^{yc} - \overline{C}_l^{yc} = \alpha\triangle m > 0$.

In order to be able to take advantage of the possibility for collusion, firms must first negotiate the collusive side contract and then make sure they both have an incentive to stick with it. Two restrictions are put on the set of admissible side contracts: *feasibility* and *optimality.* Define

$$Z^i(m_s, m_k, \varepsilon_i) = \max_{d_i \in \{\underline{\varepsilon}, \overline{\varepsilon}\}} \{U_{s\gamma_k^j}(d_i, \varepsilon_i, m_k)\} + \varphi^i_{s\gamma_k^j},$$

firm i's expected rent when reporting common shock m_s, given its true type (m_k, ε_i), contingent on firm j abiding by the side contract $\varpi = \{\boldsymbol{\gamma}, \boldsymbol{\varphi}\}$ and on its own subsequent, optimal report of the idiosyncratic shock ε_i. Let $Z^i_k(\varepsilon_i) = Z^i(m_{\gamma^i_k}, m_k, \varepsilon_i)$ be firm i's rent if both producers stick with ϖ, given i's type (m_k, ε_i).

Definition 4.1. *The side contract ϖ is said to be* feasible *if for both firms ($i \in \{1, 2\}$):*

$$Z^i_k(\varepsilon_i) \geq Z^i(m_s, m_k, \varepsilon_i) \ \forall (s, k, \varepsilon_i) \in \{l, h\}^2 \times \{\underline{\varepsilon}, \overline{\varepsilon}\},$$

$$Z^i_k(\varepsilon_i) \geq 0 \ \forall \{l, h\} \times \{\underline{\varepsilon}, \overline{\varepsilon}\}.$$

Feasibility is equivalent to incentive compatibility and individual rationality of the side contract. If one firm thinks that the other reports according to the negotiated side contract, its best response must be to do the same. Further, production must be preferred to shutting down. In other words, the feasibility requirement states that firms cannot not write contracts one of them may abandon later. Let - be the set of feasible side contracts, and denote by Π_ϖ [$\Pi_{\widehat{\varpi}}$] expected industry rent under the side contract ϖ [$\widehat{\varpi}$].

Definition 4.2. *The side contract ϖ is said to be* optimal *if it is feasible and satisfies:*

$$\Pi_\varpi \geq \Pi_{\widehat{\varpi}} \ \forall \widehat{\varpi} \in - .$$

The optimality requirement is quite obvious. It states that a side contract is admissible only if it is incentive compatible, individually rational and maximises expected industry rent among all feasible side contracts.

5. Collusion-proof regulation

I showed in the previous section that firms benefit from understating industry-specific (IS) productivity so as to obtain the subcontract D_{hh} in stead of the correct one, D_{ll}. By doing so, they are able to retain transfers at lower effort. This section analyses the regulator's optimal response to collusion. Attention is restricted to the class of *collusion-proof* contracts. Define the *truthful side contract* to be the side contract in which no side payments are made and firms agree on truthful revelation of IS productivity. A regulatory contract is said to be collusion-proof if it induces firms to select the truthful side contract and subsequently report truthfully firm-specific productivity.

The interest in collusion-proof regulation stems from the observation that the welfare maximizing regulatory contract can be rewritten as a collusion-proof contract. The intuition behind this *Collusion-proofness Principle* is the same as that underlying the Revelation Principle.[11] Given her knowledge about the collusive agreement, the regulator can tailor transfers and cost targets so as to construct an incentive compatible and individually rational side contract that is truthful and identical to the optimal collusive side contract, hence generates the same expected industry rent. Effort and rent are the same under the new collusion-proof contract as under the original contract, hence expected welfare is the same (the proof is in appendix A.1):

[11] Confer Laffont and Martimort (1997, 1998 and 2000) for other applications of the Collusion-proofness Principle.

Proposition 5.1. *Assume that firms can write and enforce collusive side contracts of the form discussed in section 4. For every welfare maximising regulatory contract, there exists a collusion-proof regulatory contract that generates the same expected welfare.*

Under collusion-proof regulation, the regulator faces the problem of maximizing $W(\mathbf{e}, \mathbf{U})$ subject to the following constraints ($k \in \{l, h\}$):

$$\underline{U}_k \geq U_{kk}(\overline{\varepsilon}, \underline{\varepsilon}, m_k), \tag{5.1}$$

$$\overline{U}_k \geq U_{kk}(\underline{\varepsilon}, \overline{\varepsilon}, m_k), \tag{5.2}$$

$$\underline{U}_k \geq \max\{U_{-kk}(\underline{\varepsilon}, \underline{\varepsilon}, m_k), U_{-kk}(\overline{\varepsilon}, \underline{\varepsilon}, m_k)\}, \tag{5.3}$$

$$\overline{U}_k \geq \max\{U_{-kk}(\underline{\varepsilon}, \overline{\varepsilon}, m_k), U_{-kk}(\overline{\varepsilon}, \overline{\underline{\varepsilon}}, m_k)\}, \tag{5.4}$$

$$\underline{U}_k \geq 0, \tag{5.5}$$

$$\overline{U}_k \geq 0, \tag{5.6}$$

$$2\square(\mathbf{U}) \geq \Pi_{\varpi} \; \forall \varpi \in - \; . \tag{5.7}$$

(5.1) and (5.2) are the IC constraints on the idiosyncratic part ε_i, given the truthful side contract. (5.3)-(5.6) are the feasibility constraints on the truthful side contract, given (5.1) and (5.2). Finally, (5.7) is the optimality constraint, requiring that the truthful side contract maximize expected industry rent among all feasible side contracts.

Two restrictions have been put on the admissible side contract: *i*) that it be feasible, i.e. it must be in each firm's best interest to adhere to the side contract if it expects the other firm to do so; *ii*) that it be optimal, i.e. firms pick a side contract so as to maximize expected industry rent. This means that the regulator can ward off collusion either by rendering every collusive side contract infeasible, or by reassuring that the most profitable side contract is the truthful one. It is shown below that the optimal strategy depends on firms' access to side payments. Two cases are considered. First, I assume that firms cannot commit to side payments at all. This could be relevant, since the model considered here is a one-shot game. It is the best-case scenario, as access to side payments facilitates collusion. At the other extreme is the worst-case scenario allowing firms to commit to unlimited side payments. This case is studied in section 5.2.

5.1. The best-case scenario: no access to side payments

Is it sufficient that firms agree to coordinate on understating productivity? In other words, is it possible to uphold collusive agreements in the absence of side payments? The answer is no. Firms cannot collude unless they can commit to side payments.[12] Suppose firms have agreed to report m_h irrespective of their true type. Suppose also that the regulatory sub contract D_{lh} specifies a reward to a any firm that claims to be of type m_l given that the other firm reportedly is a low (m_h) type. If the reward is sufficiently large, at least one firm will deviate from the collusive agreement and select D_{lh} over D_{hh}. Firms know that this is going to happen when they negotiate the side contract, hence there can be no collusion. Both firms truthfully report their type, the regulator never pays the reward since D_{lh} is never implemented, hence collusion can be costlessly avoided (the proof is in appendix A.2):

Proposition 5.2. *Assume that firms do not have access to side payments. Then there exists a regulatory contract under which truthful revelation of industry-specific productivity is a strictly dominating strategy for both firms and the contract implements second-best effort* $(\underline{e}_l^{yc}, \overline{e}_l^{yc}, \underline{e}_h^{yc}, \overline{e}_h^{yc})$ *at no added social cost.*

[12] However, the absence of observed transfers cannot be taken as evidence that collusion is impossible. In the model side transfers are used as a punishment mechanism to discipline firms. Non-existence of side transfers could imply that firms cannot collude, but it could equally well be explained by disciplined collusive behaviour.

5.2. The worst-case scenario: unlimited access to side payments

If firms can make side payments, collusion is a serious concern, simply because any sticks and carrots offered by the regulator can be neutralised by firms' choice of side transfers:

Proposition 5.3. *Assume that firms have access to unlimited side payments and that the regulator offers the second-best contract. In that case, there exists a feasible collusive side contract that yields strictly higher expected industry rent than the truthful side contract.*

Proof. Consider the side contract ϖ with $\gamma_k^1 = \gamma_k^2 = h$ for $k \in \{l, h\}$ and $\varphi_{lh} = -t_{lh}$, $\varphi_{ss'} = 0$ otherwise. Expected industry rent is larger under ϖ than truthtelling since $\underline{t}_h^{yc} = \underline{t}_l^{yc}$, $\overline{t}_h^{yc} = \overline{t}_l^{yc}$, $\underline{C}_h^{yc} - \underline{C}_l^{yc} = \overline{C}_h^{yc} - \overline{C}_l^{yc} > 0$ (see footnote 6). Yardstick competition is IC and IR, hence ϖ is IC and IR for $k = h$. ϖ is IC and IR for $k = l$ also, since $Z(m_l, m_l, \varepsilon_i) < 0 < Z_l(\varepsilon_i)$. This establishes feasibility of ϖ. ■

Proposition 5.3 states that there exists no collusion-proof regulatory contract that implements second-best effort if firms have access to unlimited side payments. Thus, the regulator is forced to give up rents to firms in order to prevent collusion. To see how this affects regulation, consider again the problem of maximizing $W(\mathbf{e}, \mathbf{U})$ subject to (5.1)-(5.7). As in the non-collusive yardstick competition case (section 3.2), (5.1) and (5.2) are replaced by the downward binding IC constraints

$$\underline{U}_l \geq \overline{U}_l + \overline{\Phi}(\overline{e}_l), \tag{5.8}$$

$$\underline{U}_h \geq \overline{U}_h + \overline{\Phi}(\overline{e}_h), \tag{5.9}$$

and the monotonicity requirements

$$\overline{C}_l \geq \underline{C}_l, \tag{5.10}$$

$$\overline{C}_h \geq \underline{C}_h. \tag{5.11}$$

Constraints (5.3) and (5.4) are taken care of by setting $\underline{t}_{lh} = \underline{t}_{hl} = 0$ and $\overline{t}_{lh} = \overline{t}_{hl} = 0$. Rents are increasing in productivity, hence (5.5) and (5.6) are reduced to:

$$\overline{U}_l \geq 0, \tag{5.12}$$

$$\overline{U}_h \geq 0. \tag{5.13}$$

The relevant collusive incentive compatibility constraint (5.7) is (see appendix A.3):

$$\xi \underline{U}_l + (1-\xi)\overline{U}_l \geq \underline{U}_h + \xi\Lambda(\underline{e}_h) + (1-\xi)\underline{\Phi}(\underline{e}_h). \tag{5.14}$$

Inequality (5.14) states that collusion-proof regulation requires expected industry rent under truth-telling to be at least as high as expected industry rent under a side contract that involves both firms always reporting low industry-specific (IS) productivity, truthfully reporting firm-specific (FS) productivity if IS productivity is low and reporting high FS productivity if IS productivity is high. As in section 3.1, $\underline{\Phi}(\underline{e}_h)$ is the value of the productivity gap $\overline{\beta}_l - \underline{\beta}_h$. $\Lambda(\underline{e}_h) = \psi(\underline{e}_h) - \psi(\underline{e}_h - \alpha\triangle m)$ is the value of the productivity gap $\underline{\beta}_l - \underline{\beta}_h = \alpha\triangle m$. Rents are costly, hence (5.9), (5.13) and (5.14) are binding at the optimum.[13] Use this to obtain an expression for expected firm rent under collusion-proof yardstick competition:

$$\square^{cp}(\mathbf{e}) = v(\xi\Lambda(\underline{e}_h) + (1-\xi)\underline{\Phi}(\underline{e}_h)) + (v + \xi(1-v))\overline{\Phi}(\overline{e}_h). \tag{5.15}$$

The *expected collusive rent* is:

$$\square^{cp}(\mathbf{e}) - \square^{yc}(\mathbf{e}) = v(\xi\Lambda(\underline{e}_h) + (1-\xi)\underline{\Phi}(\underline{e}_h)) + \overline{\Phi}(\overline{e}_h) - \xi\overline{\Phi}(\overline{e}_l)) \tag{5.16}$$

Equation (5.16) shows that the social cost of collusion (firm rent) can be alleviated by reducing the power of the incentive scheme for low-productivity firms

[13] Note that (5.8) and (5.12) binding with (5.14) being slack is impossible. It would bring us back to second-best yardstick competition, which is shown by proposition 5.3 to be non-collusion-proof.

(lowering $\underline{e}_h$ and $\overline{e}_h$) and increasing it for high-productivity firms (increasing $\overline{e}_l$). The regulator has to worry about m_l firms collectively masking as m_h firms under collusion, which is not a problem under non-collusive yardstick competition. By inflating cost targets (distorting effort) and reducing transfers to m_h firms, the regulator is able to extract rents from m_l firms by making collusion less profitable. Conversely, the regulator cannot extract any rent by distorting effort of high productivity (m_l) firms since nobody mimics a m_l firm under collusion. Collusion-proof yardstick competition, therefore, calls for more distortion of effort for low-productivity ($m = m_h$) producers and less distortion of effort for high-productivity ($m = m_l$) producers than regular yardstick competition. The analysis below provides the formal solution to the regulator's problem.

The regulator maximizes $W(\mathbf{e}, \mathbf{U})$ subject to (5.8)-(5.14). (5.9), (5.13) and (5.14) are dealt with by substituting $\square^{cp}(\mathbf{e})$ into the welfare function. (5.14) specifies only expected rent to m_l firms, leaving the regulator with a degree of freedom in the selection of $\underline{U}_l$ and $\overline{U}_l$. Set therefore $\overline{U}_l = 0$. Use this and the fact that (5.14) is binding, to rewrite (5.8) as:

$$\xi\Lambda(\underline{e}_h) + (1-\xi)\underline{\Phi}(\underline{e}_h) + \overline{\Phi}(\overline{e}_h) \geq \xi\overline{\Phi}(\overline{e}_l) \tag{5.17}$$

The remaining constraints are now (5.10), (5.11) and (5.17). Consider the Lagrangian:

$$\begin{aligned}\mathcal{L}^{cp}(\mathbf{e}) = S - 2\mathcal{C}(\mathbf{e}) - 2\lambda\square^{cp}(\mathbf{e}) - 2\square^{cp}(\overline{e}_h - \underline{e}_h - (1-\alpha)\triangle\varepsilon) \\ -2\lambda\gamma^{cp}(\xi\overline{\Phi}(\overline{e}_l) - \xi\Lambda(\underline{e}_h) - (1-\xi)\underline{\Phi}(\underline{e}_h)) - \overline{\Phi}(\overline{e}_h))\end{aligned} \tag{5.18}$$

$\square^{cp}$ and γ^{cp} are the Lagrange multipliers associated with respectively (5.11) and (5.17). Maximizing (5.18) over effort and manipulating the first order conditions yields:

$$\psi'(\underline{e}_l^{cp}) = 1, \tag{5.19}$$

$$\psi'(\overline{e}_l^{cp}) = 1 - \frac{\lambda}{1+\lambda}\frac{\varepsilon}{1-\xi}\frac{\gamma^{cp}}{v}\overline{\Phi}'(\overline{e}_l^{cp}), \tag{5.20}$$

$$\psi'(\underline{e}_h^{cp}) = 1 - \frac{\lambda}{1+\lambda}\frac{v-\gamma^{cp}}{(1-v)\xi}(\underline{\Phi}'(\underline{e}_h^{cp}) + \Lambda'(\underline{e}_h^{cp})) + \frac{\square^{cp}}{(1+\lambda)(1-v)\xi}, \tag{5.21}$$

$$\psi'(\overline{e}_h^{cp}) = 1 - \frac{\lambda}{1+\lambda}\frac{v+\xi(1-v)-\gamma^{cp}}{(1-v)(1-\xi)}\overline{\Phi}'(\overline{e}_h^{cp}) - \frac{\square^{cp}}{(1+\lambda)(1-v)(1-\xi)}, \tag{5.22}$$

which along with the complimentary slackness conditions

$$\square^{cp} \geq 0,\ \square^{cp}(\overline{e}_h^{cp} - \underline{e}_h^{cp} - (1-\alpha)\triangle\varepsilon) = 0, \tag{5.23}$$

$$\gamma^{cp} \geq 0,\ \gamma^{cp}(\xi\Lambda(\underline{e}_h) + (1-\xi)\underline{\Phi}(\underline{e}_h) + \overline{\Phi}(\overline{e}_h) - \xi\overline{\Phi}(\overline{e}_l)) = 0, \tag{5.24}$$

describes third-best, collusion-proof regulation.[14] A comparison of optimal effort (5.19)-(5.24) under collusion-proof regulation with optimal effort (3.26) and (3.27) under regular yardstick competition confirms the expectations:

Proposition 5.4. *Third best, collusion-proof regulation entails more distortion of effort for firms with low industry-specific productivity ($\underline{e}_h^{cp} < \underline{e}_h^{yc}$ and $\overline{e}_h^{cp} < \overline{e}_h^{yc}$) and less distortion of effort for firms with high industry-specific productivity ($\underline{e}_l^{cp} = \underline{e}_l^{yc}$ and $\overline{e}_l^{cp} > \overline{e}_l^{yc}$) than second-best regulation.*

Proof. Note first that $\gamma^{cp} \geq v$ and $\square^{cp} > 0$ as well as $\gamma^{cp} > v$ and $\square^{cp} = 0$ imply $\underline{e}_h^{cp} > e^{fb}$. This cannot be optimal since lowering $\underline{e}_h^{cp}$ would imply reduced rent and increased efficiency. $\gamma^{cp} = v$ and $\square^{cp} = 0$ implies $\underline{e}_k^{cp} = \underline{e}_k^{yc}$ and $\overline{e}_k^{cp} = \overline{e}_k^{yc}$ for $k \in \{l, h\}$, even that an impossibility, since second-best yardstick competition is non-collusion-proof. Thus, $\gamma^{cp} < v$ for all $\square^{cp} \geq 0$. $\underline{e}_l^{cp} = \underline{e}_l^{yc} = e^{fb}$ follows from inspection of (3.26) and (5.19). Convexity of ψ, $\overline{\Phi}$, $\underline{\Phi}$ and Λ implies (i) $\overline{e}_h^{cp} < \overline{e}_h^{yc}$ by $\square^{cp} \geq 0$ and $\frac{v-\gamma^{cp}+\xi(1-v)}{(1-v)(1-\xi)} > \frac{\xi}{1-\xi}$; (ii) $\overline{e}_l^{cp} > \overline{e}_l^{yc}$ by $\frac{\xi}{1-\xi}\frac{\gamma^{cp}}{v} < \frac{\xi}{1-\xi}$; (iii) $\underline{e}_h^{cp} < e^{fb} = \underline{e}_h^{yc}$ for $\square^{cp} = 0$ and (iv) $\underline{e}_h^{cp} = \overline{e}_h^{cp} - (1-\alpha)\triangle\varepsilon < e^{fb} = \underline{e}_h^{yc}$ for $\square^{cp} > 0$. ■

As argued in section 3.2, yardstick competition is particularly useful when productivity is dominated by the common part m, i.e. for α high. Informational

[14] The neglected constraint (5.10) is met by this solution because $\overline{e}_l^{cp} < e^{fb} = \underline{e}_l^{cp}$, which implies $\overline{C}_l > \underline{C}_l$, see previous footnote.

rents are negligible since the regulator extracts m at no social cost. Hence, optimal regulation is close to first-best. This is under the assumption that firms cannot collude. Things change dramatically by allowing for firms to write collusive side contracts. Collusion enables firms to retain almost all earnings when productivity is highly correlated, rendering yardstick competition close to useless.

To see what happens when correlation of information changes, subtract expected rent (5.15) under collusion-proof regulation from expected rent (3.15) under individual regulation of firms:

$$\triangle \Box = \Box^{I}(\mathbf{e}) - \Box^{cp}(\mathbf{e}) = v\xi[\overline{\Phi}(\overline{e}_l) + \underline{\Phi}(\underline{e}_h) - \Lambda(\underline{e}_h)]. \tag{5.25}$$

$\triangle \Box$ is the rent-extraction effect of yardstick competition subject to the constraint that regulation be collusion-proof. Under individual regulation a firm with high IS productivity ($m = m_l$) can credibly mimic a firm with low IS productivity ($m = m_h$). Firms with high IS productivity must therefore be compensated for not understating productivity to m_h under separate regulation, irrespective of the idiosyncratic part ε_i. Under collusion-proof yardstick competition an m_l firm cannot credibly pretend to be an m_h firm. This would involve incompatible reports, for which both firms are punished. However, the regulator must pay firms ex ante for agreeing to not jointly manipulate reports. A deviation from the regulatory contract must be unprofitable in *expectation.* In a sense one can say that collusion-proof yardstick competition allows the regulator to switch from an *ex post* to an *ex ante* incentive constraint for m_l firms. $\triangle \Box$ measures the reduction in expected rent attributed to the change from the ex post to the ex ante incentive constraint.

Proposition 5.5. *The rent-extraction effect $\triangle \Box$ of yardstick competition vanishes when correlation of private information (measured by α) becomes perfect.*

Proof. $\overline{\Phi}(\overline{e}_l) = \psi(\overline{e}_l) - \psi(\overline{e}_l - (1-\alpha)\triangle\varepsilon) \to 0$, $\underline{\Phi}(\underline{e}_h) = \psi(\underline{e}_h) - \psi(\underline{e}_h + (1-\alpha)\triangle\varepsilon - \alpha\triangle m) \to \psi(\underline{e}_h) - \psi(\underline{e}_h - \triangle m)$ and $\Lambda(\underline{e}_h) = \psi(\underline{e}_h) - \psi(\underline{e}_h - \alpha\triangle m) \to \psi(\underline{e}_h) - \psi(\underline{e}_h - \triangle m)$ when $\alpha \to 1$ implies $\triangle\square \to 0$ when $\alpha \to 1$. ■

Rent-extraction vanishes in the limit, hence collusion-proof regulation converges to individual regulation of firms when correlation becomes high. Firms retain *all* rent in a perfectly correlated environment when collusion is possible. Collusion allows producers to act as a merged entity, thereby fully offsetting the rent-extraction effect of yardstick competition.

Firms may be able to form a cartel and write collusive side contracts after private information has been revealed. This is sometimes referred to as *ex post collusion* and is studied by Laffont and Martimort (2000) (henceforth abbreviated LM) for the case with correlated productivity. They find that the social cost of collusion vanishes in the limit when correlation of information becomes near perfect. The regulator is able to exploit informational asymmetries within the cartel to extract almost all rents. Under *ex ante collusion*, the topic of the present paper, the cartel is formed before the arrival of private information and firms never share information.. There are no informational asymmetries for the regulator to exploit, hence informational rent persists. The explanation for the result lies in the restrictions the different modes of collusion put on side transfers. In the LM model the two agents report their true type to a third party who uses the information to manipulate the social planner's mechanism. This puts a lot of restrictions on the side payments. A massive side transfer may be required from agent 2 to agent 1 in order to induce agent 1 to truthfully reveal his type. The huge side transfer may have the implication that it becomes unprofitable for agent 2 to reveal his type or to even participate in the cartel. What LM show, is that although the stakes of collusion increase when correlation of types becomes stronger, there is no way the agents can set up side transfer schemes to reap the benefits. Not so under ex ante collusion. Here, the value of side transfers is measured solely by the pain it inflicts on the potential deviator. Thus firms are able to neutralise any sticks and carrots offered by the regulator, irrespective of the degree of collusion. The two divergent results suggest that firms prefer less information to more when cartels are formed.[15]

Finally, a word on collateral. The first-best regulatory contract is to give each firm a fixed-price contract, which maximises efficiency in production, and pay out transfers sufficient to guarantee that firms' ex ante participation constraint is just met. This is even collusion-proof since each firm is regulated on an individual basis. However, the contract implies that some low-productivity firms run a deficit ex post, hence shut down. The interim participation constraint can be relaxed by the use of collateral. If firms put up sufficient collateral, they are willing to produce at all levels of productivity even with a fixed-price contract. However, firms should realise that rents are decreasing in the amount of collateral they put up, and therefore be reluctant to providing it. A potential method for raising collateral is by means of an auction at which the right to produce goes to the firm with the highest collateral. However, the set of potential bidders may be small and their access to collateral limited. Therefore, the regulator may be forced to apply yardstick competition in regulation. Collateral must be in the form of lump sum payments in order to preserve truth-telling incentives. It can have no effect on firms' joint efforts to manipulate incentive contracts because they retrieve their collateral whether or not they collude. The conclusion is that the use of collateral can be used as an instrument to increase efficiency in production, but not to prevent collusion, unless firms are willing to put up significant amounts of it.

[15] This presupposes that the regulator knows the mode of collusion.

6. Conclusion

This paper has studied the nature of collusion and characterized third-best, collusion-proof regulation when firms have the opportunity to write collusive side contracts prior to the revelation of private, correlated information about productivity. Correlated information enables the regulator to apply relative performance evaluation or yardstick competition in order to elicit information and thereby extract rent from firms. This creates a scope for collusion by which firms jointly understate productivity so as to retain rent.

Collusion has been shown to significantly reduce the benefit of yardstick competition when producers have access to unlimited side payments to sustain manipulative productivity reports. Yardstick competition is near useless if productivity is highly correlated across firms. If yardstick competition in addition is more expensive to administer than a scheme based on individual regulation, e.g. due to stronger requirements on information compilation, it is outright unprofitable. This may account for the fact that yardstick competition is far less widespread than one might expect given its heralded effect on rent-extraction.

The paper prescribes two remedies for the problem of collusion. The first is to offer extreme incentives. Third-best, collusion-proof yardstick competition demands more (less) distortion of effort for low (high) productivity firms than prescribed by regular, second-best yardstick competition. Such a policy increases the value of being a high productivity producer and diminishes the value of being a low productivity producer. The second remedy is to impede side payments. The analysis has shown that collusion poses a problem for regulation only if inter-firm transfers are possible. Otherwise, the regulator can set up a structure of rewards and punishments to break up collusion. Rewards are never paid and punishments never carried out in equilibrium, hence collusion-proof regulation is costless. Presumably, side payments are more difficult to enforce between independent producers than between divisions within a merged company. The authorities should therefore view with a certain amount of scepticism, proposed mergers between firms regulated by means of yardstick competition. This point is well taken by the Office of Water Services (Ofwat) which employs yardstick competition to regulate the water industry in England and Wales. According to its Director General: "Ofwat has always argued against the reduction of independent companies within the water industry, mainly because it is essential for me to be able to compare performance" (Ofwat, 1999). In fact, the 1989 Water Act demands of British competition authorities that they take explicit account of the effect on the ability to make performance comparisons when evaluating the welfare effects of mergers in the water industry.

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A. Appendix

A.1. Proof of proposition 5.1

Write $\mathbf{D}^*$ the welfare maximising grand contract, $D^*_{ss'} = (\{\underline{t}^{i*}_{ss'}, \underline{C}^{i*}_{ss'}\}_{i=1,2}, \{\overline{t}^{i*}_{ss'}, \overline{C}^{i*}_{ss'}\}_{i=1,2})$ $(ss' \in \{ll, lh, hl, hh\})$ a sub contract in $\mathbf{D}^*$, - * the set of feasible side contracts under $\mathbf{D}^*$ and $\varpi = \{\boldsymbol{\Gamma}, \boldsymbol{\varphi}\}$ the corresponding optimal side contract. By the Revelation Principle, $\mathbf{D}^*$ induces truthful revelation of ε_i for $i \in \{1, 2\}$. Define a new grand contract $\mathbf{D}$ with the following properties: $\underline{t}^i_{kk} = \underline{t}^{i*}_{\gamma_k} + \varphi^i_{\gamma_k}$, $\underline{C}^i_{kk} = \underline{C}^{i*}_{\gamma_k}$, $\underline{t}^i_{-kk} = \underline{t}^{i*}_{-\gamma^i_k \gamma^{-i}_k} + \varphi^i_{-\gamma^i_k \gamma^{-i}_k}$, $\underline{C}^i_{-kk} = \underline{C}^{i*}_{-\gamma^i_k \gamma^{-i}_k}$ for $k \in \{l, h\}$. $\{\overline{t}^i_k, \overline{C}^i_k\}$ and $\{\overline{t}^i_{-kk}, \overline{C}^i_{-kk}\}$ are defined correspondingly. Let - be the set of feasible side contracts under D and ϖ^0 the truthful side contract. Denote by $\Pi^D_{\widetilde{\varpi}}$ $[\Pi^{D^*}_{\widetilde{\varpi}}]$ expected industry rent under $\mathbf{D}$ $[\mathbf{D}^*]$ and the side contract $\widehat{\varpi}$ $[\widetilde{\varpi}]$. Suppose $-i$ truthfully reports m under $\mathbf{D}$. Then $\underline{U}^i_k = Z^{i*}(m_k, \underline{\varepsilon})$, $\overline{U}^i_k = Z^{i*}(m_k, \overline{\varepsilon})$ and $\max\limits_{d_i \in \{\underline{\varepsilon}, \overline{\varepsilon}\}} U^i_{-kk}(d_i, \varepsilon_i, m_k) = Z^{i*}(m_{-k}, m_k, \varepsilon_i)$, which implies $\Pi^D_{\varpi^0} = \Pi^{D^*}_{\varpi}$ and $\varpi^0 \in$ - by $\varpi \in$ - *. By the construction of $\mathbf{D}$, there exists for every $(k, k') \in \{l, h\}^2$ a pair $(s, s') \in \{l, h\}^2$ such that $\underline{t}^i_{kk'} = \underline{t}^{i*}_{ss'} + \varphi^i_{ss'}$, $\underline{C}^i_{ss'} = \underline{C}^{i*}_{ss'}$ and correspondingly for $\overline{t}^i_{kk'}$ and $\overline{C}^i_{kk'}$. Thus, any side contract $\widehat{\varpi}$ under D can replicated by a side contract $\widetilde{\varpi}$ under D^*. This implies that for every $\widehat{\varpi} \in$ - $\exists\, \widetilde{\varpi} \in$ - * such that $\Pi^{D^*}_{\widetilde{\varpi}} = \Pi^D_{\widehat{\varpi}}$. Combine the above results to obtain $\Pi^D_{\varpi^0} = \Pi^{D^*}_{\varpi} \geq \Pi^{D^*}_{\widetilde{\varpi}} = \Pi^D_{\widehat{\varpi}}$ $\forall \widehat{\varpi} \in$ - , which establishes optimality of ϖ^0 and thus collusion-proofness of $\mathbf{D}$. Welfare is unchanged since cost targets, expected transfers and expected industry rent remains the same under $\mathbf{D}$ as under $\mathbf{D}^*$. ■

A.2. Proof of proposition 5.2

Consider a regulatory contract with cost targets $\underline{C}_{kk'} = \underline{\beta}_k - \underline{e}_k^{yc}$ and $\overline{C}_{kk'} = \overline{\beta}_k - \overline{e}_k^{yc}$ for all $kk' \in \{ll, lh, hl, hh\}$, with $\underline{e}_k^{yc}$ and $\overline{e}_k^{yc}$ implicitly defined in (3.26) and (3.27). Let the set of transfers be given by:

$$\begin{array}{ll} \underline{t}_{kk} = \psi(\underline{e}_k^{yc}) + \overline{\Phi}(\overline{e}_k^{yc}),\ k \in \{l,h\} & \overline{t}_{kk} = \psi(\overline{e}_k^{yc}),\ k \in \{l,h\} \\ \underline{t}_{lh} = \underline{t}_{ll} + r_{\underline{\varepsilon}} + \delta & \overline{t}_{lh} = \overline{t}_{ll} + r_{\overline{\varepsilon}} + \delta \\ \underline{t}_{hl}^{yc} = \underline{t}_{hh} - (r_{\underline{\varepsilon}} + \delta) & \overline{t}_{hl} = \overline{t}_{hh} - (r_{\overline{\varepsilon}} + \delta) \end{array}, \tag{A.1}$$

with

$$\begin{array}{l} r_{\underline{\varepsilon}} = \psi(\underline{e}_k^{yc} + (1-\alpha)\triangle\varepsilon) - \psi(\underline{e}_k^{yc} + (1-\alpha)\triangle\varepsilon - \alpha\triangle m) \\ r_{\overline{\varepsilon}} = \psi(\overline{e}_k^{yc}) - \psi(\overline{e}_k^{yc} - \alpha\triangle m) \end{array}. \tag{A.2}$$

$\delta > 0$ is a small number. Define by $\triangle U_{kk'}(\varepsilon', \varepsilon) = U_{kk'}(\varepsilon', \varepsilon, m_k) - U_{-kk'}(\varepsilon', \varepsilon, m_k)$ a firm of type $\beta_k(\varepsilon) = \alpha m_k + (1-\alpha)\varepsilon$'s net benefit of truthfully reporting m_k over the manipulation m_{-k}, given the other firm's report $m_{k'}$ of the common shock and given the firm's own subsequent report ε' of the idiosyncratic part. Let $C_k(\underline{\varepsilon}) = \underline{C}_{kk'}$ and $C_k(\overline{\varepsilon}) = \overline{C}_{kk'}$. By invoking (A.1) and (A.2), one obtains

$$\triangle U_{kk'}(\varepsilon', \varepsilon) = \left| \int_{C_l(\varepsilon')}^{C_h(\varepsilon')} \psi'(\beta_k(\varepsilon) - x)dx - x_{\varepsilon'} \right| + \frac{h + l - 2k}{h - l}\delta. \tag{A.3}$$

It is a straightforward (although tedious) task to verify that the first term on RHS of (A.3) is strictly positive for $k = h$ and $\alpha > \underline{\alpha}$. Thus, $\triangle U_{kk'}(\varepsilon', \varepsilon) > 0\ \forall$ $(k, k', \varepsilon', \varepsilon) \in \{l, h\}^2 \times \{\underline{\varepsilon}, \overline{\varepsilon}\}^2$ for δ positive and sufficiently small. Consequently, firm i strictly prefers to truthfully report m_k irrespective of the other firm's report $m_{k'}$ and of the realization of the idiosyncratic part ε_i. In equilibrium both firms truthfully report m_k. $\underline{U}_{kk} = \overline{\Phi}(\overline{e}_k^{yc})$ and $\underline{U}_{kk} = 0$, hence firms truthfully report ε_i, $(\underline{e}_k^{yc}, \overline{e}_k^{yc})$ is implemented and expected firm rent is given by (3.25). ■

A.3. The binding collusive incentive compatibility constraint

This appendix shows that (5.7) is satisfied if (5.8)-(5.14) are satisfied. Note first that $\underline{U}_l > \overline{U}_l \geq 0$ and $\underline{U}_h > \overline{U}_h \geq 0$ imply $2\square(\mathbf{U}) > 0$. (5.7) is trivially met for any side contract ϖ involving $\gamma_k^1 \neq \gamma_k^2$ since $\Pi_\varpi < 0$ owing to the fact that transfers are zero in case of incompatible productivity reports. Thus, $\gamma_k^1 = \gamma_k^2 = \gamma_k$ for $k \in \{l, h\}$. Suppose $\gamma_h = l$. (5.14) merely specifies expected rent to m_l firms, leaving the regulator with a degree of freedom in the selection of $\underline{U}_l$ and $\overline{U}_l$. Set therefore $\overline{U}_l = 0$. Using this, one obtains:

$$U_{ll}(\overline{\varepsilon}, \varepsilon, m_h) = \psi(\overline{e}_l) - \psi(\overline{e}_l + \alpha\triangle m - (1-\alpha)(\overline{\varepsilon} - \varepsilon)), \tag{A.4}$$

$$U_{ll}(\underline{\varepsilon}, \varepsilon, m_h) - U_{ll}(\overline{\varepsilon}, \varepsilon, m_h) = [U_{ll}(\underline{\varepsilon}, \overline{\varepsilon}, m_l) - \overline{U}_l] - \int_{\beta_l}^{\beta_h(\varepsilon)} \int_{\underline{C}_l}^{\overline{C}_l} \psi''(x-y) dy dx \tag{A.5}$$

for $\varepsilon \in \{\underline{\varepsilon}, \overline{\varepsilon}\}$. (A.4) is negative since $\alpha\triangle m > (1-\alpha)\triangle\varepsilon$ and $\psi' > 0$. (A.5) is non-positive by (5.2), $\beta_h(\varepsilon) > \overline{\beta}_l$, $\overline{C}_l \geq \underline{C}_l$ and $\psi'' > 0$. Clearly, $\gamma_h = h$ and a truthful report of ε_i are strictly better than $\gamma_h = l$ for any subsequent report $d_i \in \{\underline{\varepsilon}, \overline{\varepsilon}\}$. This leaves firms with two candidate side contracts, the truthful one $(\gamma_l = l, \gamma_h = h)$ and a collusive one $(\gamma_l = h, \gamma_h = h)$. Denote the collusive side contract by ϖ'. Under this contract firms always report m_h, truthfully report ε_i if their type is m_h (since the regulatory mechanism is IC), and report $\underline{\varepsilon}$ if their type is m_l. To see the latter, observe that:

$$U_{hh}(\underline{\varepsilon}, \varepsilon, m_l) - U_{hh}(\overline{\varepsilon}, \varepsilon, m_l) = [\underline{U}_h - U_{hh}(\overline{\varepsilon}, \underline{\varepsilon}, m_h)] + \int_{\beta_l(\varepsilon)}^{\underline{\beta}_h} \int_{\underline{C}_h}^{\overline{C}_h} \psi''(x-y) dy dx \geq 0 \tag{A.6}$$

for $\varepsilon \in \{\underline{\varepsilon}, \overline{\varepsilon}\}$. (A.6) is non-negative for the same reason that (A.5) is non-positive. It only takes a moment to verify that (5.14) is equivalent to $2\square(\mathbf{U}) \geq \Pi_{\varpi'}$.