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by

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Abstract

In this paper we extend models of "search market equilibrium" to incorporate general equilibrium considerations. The model we treat is one with a single product market and a single labor market. Imperfectly informed individuals follow optimal strategies in searching for a suitably low price and high wage. For any distribution of price and wage offers across firms these optimal strategies generate product demand and labor supply schedules. Firms then choose prices and wages to maximize expected profits taking these schedules as given, and the resulting profits are paid out to individuals as dividends.

An equilibrium distribution of prices and wages is one which results from optimal price and wage setting behavior by firms given individuals' optimal search strategies. There are two possible equilibrium configurations, a degenerate equilibrium in which all firms charge the same price and wage and a price and wage dispersion equilibrium. We show that there exists a degenerate equilibrium at the monopoly-monopsony price-wage combination. We also show some of the properties of a price-wage dispersion equilibrium, conditional on existence.

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1. Introduction

This paper analyzes a model of a simple general equilibrium economy with a single product and a single factor of production (labor). The model has two crucial features. The first is that prices and wages are set by firms, ie, there is no Walrasian auctioneer. The second is that individuals have incomplete information in the sense that if prices and wages follow non-degenerate distribution functions, then individuals know the forms of those distribution functions but don't know which firms are charging what prices and wages. These two features correspond to two fundamental (and related) problems of economic theory, namely, the logical foundations of competitive analysis and of search theory.

In competitive analysis individuals and firms are assumed to regard prices as exogenous. Demands and supplies are then treated as functions of the exogenously given prices, and equilibrium is determined by a vector of prices that equates supply and demand on all markets. This equilibrium should be locally stable if it is to be of any interest; that is, if prices are close to their equilibrium values, then the system should have a tendency to approach equilibrium. The usual way to ensure local stability is to assume a price adjustment mechanism. If there is excess demand for a good, then its price must rise; likewise excess supply must lead to a price decrease.

The idea of price adjustment in response to excess demand or supply is appealing since firms do in reality adjust prices in response to perceived profit opportunity. Unfortunately, this intuitive justification of the price adjustment mechanism faces a logical problem in the context of competitive analysis. (The classic statement of this problem is given in Arrow(1959).) To derive a competitive equilibrium it is assumed on the one hand that firms regard prices as exogenously given while on the other hand the local stability of that equilibrium is ensured by a price adjustment mechanism that is intuitively justified by a story in which firms are active price-setters. Either firms set prices or they do not; they cannot be price-takers and price-makers simultaneously.

Of course the standard way to plug this logical hole is to introduce the fiction of the Walrasian auctioneer. Given the existence of the auctioneer, firms can be regarded as price-takers both in the derivation of equilibrium and in the analysis of the local stability of that equilibrium. The problem with this device is that it is so blatantly false. Almost no markets exhibit institutional arrangements that could be thought of as even remotely corresponding to the auctioneer. A much more satisfactory approach would thus be to assume from the beginning that prices are set by firms themselves.

What sort of equilibrium might one expect in a model with price-setting firms? If the market power of any one firm vis a vis other firms is negligible and if individuals are not completely ill-informed, then one might expect to find an equilibrium tolerably close to the one produced by competitive analysis. In that case one could accept the notion of equilibrium prices determined as if they were set by the auctioneer.

Unfortunately, there exist no well-formulated models with price-setting firms that generate the competitive outcome. On the contrary, a variety of models (Diamond(1971), Hey(1974), Axell(1977) and Burdett and Judd(1979)) have produced the monopoly outcome. More precisely, these models have shown in a single-market, partial equilibrium setting that if an equilibrium exists in which all firms charge the same price, then that price will be the one that would be charged by a monopolist controlling the entire market.

An even more interesting equilibrium possibility to consider is one in which not all firms charge the same price. The existence of such a dispersion equilibrium is of course essential for the logical foundations of search theory. This point has been forcefully made by Rothschild (1973). In that well-known survey paper the model in which consumers search from a known distribution of prices (or job-seekers search from a known distribution of wages) was criticized as being "partial-partial." The first "partial" refers to the fact that only one side of the market is analyzed; ie, the price-setting behavior of firms that presumably generated the distribution from which individuals are searching is left untreated. The second "partial" refers to the fact that one market is analyzed in isolation. Consumer demand (or labor supply) is taken as given, usually at the level of one "unit" per period of analysis, which is equivalent to ignoring linkages between markets.

The problem of removing the first "partial" was addressed by Axell (1977) using a model in which each individual searches for one unit of a homogeneous good. His approach was to postulate a density function for prices, say $f(p)$, and a density function for consumer search costs, say $\gamma(c)$. Assuming that individuals follow an optimal sequential search rule, one can use the two postulated densities to derive the density function of reservation prices and of actual purchase or "stopping" prices, say $\omega(p)$. Next, he argues that a firm's expected demand will be proportional to $\omega(p)/f(p)$; then for a constant marginal cost function, he derives $\Pi(p)$, ie, expected profits as a function of price. A price dispersion equilibrium is defined as a non-degenerate density, $f(p)$, such that $\Pi(p)$ is constant for all p in the support of $f(p)$. The basic result derived is a set of necessary and sufficient conditions on $\gamma(c)$ that ensure the existence of a price dispersion equilibrium. These are that $\gamma(c)$ must not be bounded away from zero, that $\gamma(c)$ must be decreasing and convex, and that the "degree of convexity" must satisfy certain conditions.

There are several other models of equilibrium price dispersion in the literature. Although almost none of these are based on the optimal sequential search strategy that is the essence of mainstream search theory, they are nonetheless supportive of the idea that the "law of one price" is quite capable of violation. (A model of equilibrium price dispersion that does use the optimal sequential strategy is Reinganum (1979), but even that model is not completely satisfactory from a search-theoretic point of view since in equilibrium each individual necessarily searches only once.)

The current state of research on search market equilibria, ie, equilibria in markets characterized by incomplete information and the absence of an external price-setting authority, can thus be broadly summarized as follows. In a single-market setting the equilibrium outcome of competition among firms will be either a degenerate equilibrium at the monopoly price or a price dispersion equilibrium. (See Hey(1979), Ch 25 for a good survey.)

In this paper we extend models of search market equilibrium to incorporate general equilibrium considerations. The motive for such an extension is of course to investigate whether the extremely anti-competitive (alternatively, mildly pro-search theoretic) results of the existing literature are a partial equilibrium artifact. Simply stated, our results indicate that they are not.

The basic idea of our model can be introduced as follows. We consider the simplest general equilibrium economy with a product market and a labor market. There are u individuals and n firms in this economy. Both u and n are arbitrarily large, and $\mu \equiv u/n$ is also arbitrarily large.

Denote the distribution functions of prices and wages by F(p) and M(w), respectively. Assume that individuals are following optimal search strategies (in a sense to be made precise below) given F and M. Then, conditional on F and M, each firm faces a product demand schedule q(p) and a labor supply schedule $\ell(w)$.

Assume each firm sets p and w to maximize expected profits. This maximization proceeds subject to the constraint the offered wage elicits sufficient labor supply to produce the product demand induced by the offered price. Assume the simplest linear production function,

$$q(p) = \ell(w). \tag{1}$$

Then the firm's decision problem is to choose p,w to maximize

$$\Pi(p,w) = pq(p) - w\ell(w) \tag{2}$$

subject to the production constraint (1). Assume that the profits earned by firms are paid out to individuals as dividends.

We want to characterize the Nash equilibria in this model. This means that we want to find distribution functions F and M such that:

(1) Each individual is following an optimal search strategy given F and M;

(2) Each firm is setting (p,w) to maximize $\Pi(p,w)$ subject to the production constraint, where the optimal choice is taken conditional on F and M;

(3) The outcome of firms' optimal choices of p and w generates the distribution functions F and M.

There are two possible types of equilibria to consider in this model:

(1) Degenerate equilibria, ie, an equilibrium in which all firms charge the same price, p', and the same wage, w';

(2) Dispersion equilibria in which both prices and wages follow non-degenerate distribution functions.

The organization of this paper is as follows. In sections 2 and 3 we describe the optimal search strategy for individuals and derive the product demand and labor supply schedules faced by firms. In section 4 we use these results to derive the degenerate equilibrium, and in sections 5 and 6 we discuss dispersion equilibria. We have not yet been able to prove the existence of dispersion equilibrium; however, we are able to establish some of the properties that a dispersion equilibrium must have if it exists.

2. Individuals' Search

In each period there are u individuals in the economy. The individual's decision problem is to search in an optimal fashion for a suitably low price and high wage.

The individual is assumed to die with probability τ at the end of each period. This "constant death risk" assumption is a convenient means of combining the tractability of the "infinite horizon search model with discounting" with the introduction of a steady flow of new searchers into the economy.

The individual is assumed to decide whether or not to search based on the criterion of maximizing expected future lifetime consumption. Thus, if at the end of period t he faces the decision of whether or not to search, he chooses that alternative which maximizes the sum of expected consumptions over periods t+1, t+2,...

During each period of his existence the individual is endowed with a non-wage (dividend) income of θ. θ is assumed to be the same for all individuals and closes the economy (all profit is distributed to individuals as dividends).

At the beginning of an individual's existence he draws a "doubleton" price-wage offer, ie, a price drawn at random from one firm and a wage drawn at random from another firm; and so long as he continues to search, he continues to draw a random price-wage offer at the end of each period. We assume that the individual's consumption during any period of search is θ divided by the price drawn at the end of that period. The crucial point is that while engaged in search the individual consumes only out of non-wage income.

Suppose the individual has drawn a price-wage offer of (p,w). If he accepts (p,w), then he goes to work at the wage w and consumes $(w+\theta)/p$ per period so long as he continues to survive. Having accepted (p,w), the probability of surviving one period is $1-\tau$, of surviving two periods is $(1-\tau)^2$, etc, so the expected future lifetime consumption from an accepted (p,w) offer is $\frac{1-\tau}{\tau}\cdot\frac{w+\theta}{p}$.

The statistic $(w+\theta)/p$ summarizes everything of relevance about the pair (p,w) for an individual with a non-wage income of θ. The optimal search strategy of such an individual can be stated as a "reservation rule". If the observed value of $(w+\theta)/p$ exceeds a critical value, k, terminate search; otherwise continue search. The reservation income k thus describes the optimal search strategy of individuals given the distribution of prices and wages offered by firms.

Proposition: The optimal reservation income k satisfies

$$\frac{1-\tau}{\tau}\int_k^{\infty}(y-k)\xi(y)dy = k - \theta E(\frac{1}{p}) \tag{3}$$

where

$$\xi(y) = \int_k^{\infty}\int_{\theta/y}^{\infty} yzdF(z)dM(zy-\theta) \tag{4}$$

Proof:

Let V(k) be expected future lifetime consumption if a reservation value of k is chosen. Then the optimal k is chosen to maximize V(k). But

$$V(k) = \frac{1-\tau}{\tau}E(\frac{w+\theta}{p}|\frac{w+\theta}{p} \geq k)\cdot Pr(\frac{w+\theta}{p} \geq k) + (1-\tau)V(k)Pr(\frac{w+\theta}{p} < k) + \theta E(\frac{1}{p}).$$

To evaluate this expression use the change of variable $y=(w+\theta)/p$ and $z=p$, with inverse $w=zy-\theta$ and $p=z$. We want to allow $0<y<\infty$; therefore, $(\theta/y)<z<\infty$. The Jacobian of the transformation is z, so the joint density of y and z is $zf(z)m(zy-\theta)$; $y>0$, $(\theta/y)<z<\infty$. (Here f and m are the density functions of prices and wages, respectively.) Thus,

$$V(k) = \frac{1-\tau}{\tau} E(y|y\geq k)\cdot Pr(y\geq k) + (1-\tau)V(k)Pr(y<k) + \theta E(\frac{1}{p}).$$

But $E(y|y\geq k)\cdot Pr(y\geq k) = \int_k^\infty \int_{\theta/y}^\infty yzdF(z)dM(zy-\theta) \equiv \int_k^\infty y\xi(y)dy$

and $Pr(y<k) = 1 - \int_k^\infty \xi(y)dy.$

Thus

$$V(k) = \frac{1-\tau}{\tau}\int_k^\infty y\xi(y)dy + (1-\tau)V(k)(1-\int_k^\infty \xi(y)dy) + \theta E(\frac{1}{p})$$

$$= \frac{1}{\tau + (1-\tau)\int_k^\infty \xi(y)dy} (\frac{1-\tau}{\tau} \int_k^\infty y\xi(y)dy + \theta E(\frac{1}{p})).$$

Setting $dV(\cdot)/dk = 0$ shows that k must satisfy

$$-\frac{1-\tau}{\tau} k\cdot\xi(k)(\tau+(1-\tau)\int_k^\infty \xi(y)dy) + (1-\tau)\xi(k)(\frac{1-\tau}{\tau} \int_k^\infty y\xi(y)dy) + \theta E(\frac{1}{p}) = 0$$

ie, $\frac{1-\tau}{\tau} (\int_k^\infty y\xi(y)dy - k\int_k^\infty \xi(y)dy) = k - \theta E(\frac{1}{p})$

ie, $\frac{1-\tau}{\tau}\int_k^\infty (y-k)\xi(y)dy = k - \theta E(\frac{1}{p})$. QED

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3. Product Demand and Labor Supply Schedules

The crucial first step in the derivation of product demand and labor supply schedules is to compute the fraction of the u individuals in the economy who will search during any period. Recall that each individual faces a constant death risk of τ. Therefore in a steady state τu individuals will enter and exit the system each period.

Let h denote the probability that a randomly drawn (p,w) offer will be acceptable, ie,

$$h \equiv \Pr\left(\frac{w+\theta}{p} \geq k\right) = \int_k^\infty \xi(y)dy. \tag{5}$$

Then to compute the number of searching individuals in the economy in any period t, reason as follows. There are τu individuals entering the system at time t. There are $\tau u(1-\tau)(1-h)$ who entered at t-1 and neither died nor found their initial offer acceptable. There are $\tau u(1-\tau)^2(1-h)^2$ who entered at t-2 and who neither died nor found either of their first two offers acceptable, etc, etc. The allocation of searchers across firms is completely random; thus, each firm can expect to encounter $(\frac{u}{n})\tau/(1-(1-\tau)(1-h)) \equiv \mu s$ searchers per period, where

$$s \equiv \tau/(1-(1-\tau)(1-h)) \tag{6}$$

is the fraction of individuals in the economy who are searching in any given period. Each of these μs searchers will buy a quantity θ/p; thus, the <u>expected demand from searchers</u> for a firm charging p is <u>$\mu s\theta/p$</u>.

Among the μs searchers contacting a firm charging a price p in any given period, a fraction $1-M(kp-\theta)$ will terminate search and accept p and the wage offer they simultaneously receive. In period t the firm will have $(1-\tau)\mu s(1-M(kp-\theta))$ employed customers who terminated search at the end of period t-1, $(1-\tau)^2\mu s(1-M(kp-\theta))$ who terminated search at the end of period t-2, etc. The expected demand from each of these is

$$\frac{1}{p}\int_{kp-\theta}^{\infty}(w+\theta)dM(w)/(1-M(kp-\theta)).$$

Thus, the <u>expected demand from employed consumers</u> for a firm charging p is $\frac{(1-\tau)\mu s}{\tau p}\int_{kp-\theta}^{\infty}(w+\theta)dM(w)$.

Adding together the expected demands from searchers and employed customers gives the <u>firm's expected demand schedule</u>:

$$q(p) = \frac{\mu s}{p}\left(\theta + \frac{1-\tau}{\tau}\int_{kp-\theta}^{\infty}(w+\theta)dM(w)\right) \tag{7}$$

Likewise, among the μs searchers contacting a firm offering a wage w in any given period, a fraction $F((w+\theta)/k)$ will terminate search and become employees. In period t the firm will have $(1-\tau)\mu sF((w+\theta)/k)$ employees who terminated search at the end of period t-1, $(1-\tau)^2\mu sF((w+\theta)/k)$ who terminated search at the end of period t-2, etc. Each of these employees provides one unit of labor per period. Thus, the <u>firm's expected labor supply schedule</u> is:

$$\ell(w) = \frac{1-\tau}{\tau}\mu sF\left(\frac{w+\theta}{k}\right). \tag{8}$$

4. Degenerate Equilibrium

We begin by considering the possibility of a degenerate equilibrium. Suppose all firms are charging a common price p' and offering a common wage w'. The combination (p',w') is a degenerate equilibrium if (i) each firm's production constraint is satisfied and (ii) no firm can increase its profits subject to its production constraint by deviating from (p',w').

If all firms are offering (p',w'), then $s = \tau$, ie, the only searchers in the market are new entrants. Furthermore $F(\frac{w'+\theta}{k}) = 1$ since with the common (p',w') the reservation income will be such that $\frac{w'+\theta}{k} > p'$. Thus equations (7) and (8) reduce to

$$q(p') = \frac{\mu}{p'}(\theta + (1-\tau)w')$$

$$\ell(w') = \mu(1-\tau).$$

Note that the production constraint thus implies $p' = w' + \theta/(1-\tau)$.

Next, consider the consequences of a deviation from (p',w'). Given that all other firms are offering (p',w'), the individual firm faces the schedules

$$q(p) = \frac{\mu}{p}(\theta + (1-\tau)w') \qquad \text{for p "sufficiently close" to p'}$$

$$\ell(w) = \mu(1-\tau) \qquad \text{for w "sufficiently close" to w'.}$$

Thus, $\Pi(p,w) = \mu(\theta + (1-\tau)(w'-w))$

and the production constraint is

$$p = w' + \theta/(1-\tau).$$

The firm has no possibility to increase its price but it has both the possibility and the incentive to decrease its wage. Thus, (p',w') cannot be a degenerate equilibrium.

However, consider the case of w'=0. If all firms are charging p'>0, no individual will ever accept a negative wage since to do so would decrease expected lifetime consumption below that attainable through continued search. That is, no firm has the possibility to decrease w below w'=0. Thus, w'=0 combined with any arbitrary p'>0 (where $\theta=(1-\tau)p'$) constitutes a degenerate equilibrium.

This degenerate equilibrium is similar to the degenerate equilibrium at the monopoly price derived in a partial equilibrium setting. Imagine a single firm controlling both the labor market and the product market. Acting as a monopsonist on the labor market this firm would exploit the zero elasticity of labor supply (by employed workers) to drive the wage as low as possible, ie, to w'=0. However, once w'=0, the firm faces a demand schedule which has a constant unitary elasticity, implying the firm is indifferent as to the price it charges.

5. Existence of Dispersion Equilibrium

The firm's decision problem is to set (p,w) to maximize

$$\Pi(p,w) = pq(p) - w\ell(w)$$

subject to $q(p) = \ell(w)$. However, the production constraint together with equations (7) and (8) shows that the wage a firm must offer is given as a non-increasing function of the price it offers; namely,

$$w(p) = kF^{-1}\left(\frac{1}{p}\left(\frac{\theta\tau}{1-\tau} + \int_{kp-\theta}^{\infty} (w+\theta)dM(w)\right)\right) - \theta. \tag{9}$$

The firm's decision problem may therefore be expressed as one of choosing p to maximize

$$\Pi(p) = (p-w(p))q(p). \tag{10}$$

This formulation gives some intuition as to why the existence of a dispersion equilibrium is likely. By examining (10) one can see the plausibility of the simultaneous existence of "high-price" firms and "low-price" firms in equilibrium. A given level of profit earned by a high-price firm via a high margin (ie, $p-w(p)$) combined with a low volume ($q(p)$) could also be earned by a low-price firm via a low margin combined with a high volume. And one can easily imagine that the given level of profit could be earned by various intermediate-price policies.

Moving from this intuition to a formal existence proof of dispersion equilibrium is the task that remains for this paper. The basic problem can be explained as follows.

Recall that a Nash equilibrium is defined as a set of actions by all agents in the economy such that (i) each individual agent is taking an optimal action conditional on the actions taken by all other agents and (ii) the actions of all agents are mutually consistent. In the present context the set of actions taken by firms are given by the distribution functions F and M and the actions of individuals are given by the reservation income k. However, F, M and k are interrelated. In particular, the production constraint imposes a necessary relationship between F and M; namely, the probability that a randomly selected firm will be offering a price of p or less must equal the probability that a randomly selected firm will be offering a wage of w(p) or more. That is, F and M are related according to

$$F(p) = 1 - M(w(p)). \tag{11}$$

Furthermore, given F and M, equations (3) and (4) determine k.

Now consider an interval of prices $[p^0,p^1]$. Suppose all firms are charging a price in $[p^0,p^1]$, and let F describe the distribution of firms over that interval. Given F (and therefore M and k), the form of the function $\Pi(p)$ is completely determined, ie, $\Pi(p) = \Pi(p;F)$. The <u>question of dispersion equilibrium</u> can then be posed as follows: Does there exist a non-degenerate distribution function F defined on some interval $[p^0,p^1]$ such that $\Pi(p;F)$ is constant and maximum for all $p\varepsilon[p^0,p^1]$ where F is increasing?

Since all profits are returned to individuals as dividends, average profits per firm must equal $\mu\theta$. This implies that the requirement that profits be equal in dispersion equilibrium may be written as

$$(p-w(p))q(p) = \mu\theta.$$

The functions q(p) and w(p) are determined by F(p) and M(w(p)), so this equilibrium condition in fact involves four interrelated functions of p; namely, q, w, F and M.

Consider the following system of four equations:

$$(p-w(p))q(p) = \mu\theta \tag{12}$$

$$\frac{-k^2}{\gamma} pm(kp-\theta) = q(p) + pq'(p) \ , \text{ where } \gamma\equiv\tau/\mu s(1-\tau) \tag{13}$$

$$F(p) = 1 - M(w(p)) \tag{14}$$

$$F\left(\frac{w(p)+\theta}{k}\right) = \gamma q(p) \tag{15}$$

The two new equations in this system are derived from the expressions for $q(p)$ and $\ell(w)$ (eqns (7) and (8)) and the constraint that $q(p)=\ell(w)$. What this system of equations shows is that the question of whether there exists an F generating the equal profit condition on $[p^0,p^1]$ could just as well be posed in terms of the existence of a suitable function M, q or w.

It seems from this system that the question is most conveniently posed in terms of the function $q(p)$. Working through (12)-(15), one can derive

$$q(p) = \frac{1}{\gamma} + \frac{1}{k} \int_{p^0}^{A(v(q(p)))} \frac{q(\zeta) + \zeta q'(\zeta)}{\zeta} \, d\zeta \tag{16}$$

where

$$v(q(p)) \equiv \frac{(p+\theta)q(p) - \mu\theta}{kq(p)} \tag{17}$$

and $$A(v) \equiv \frac{q(v)(v+\theta) - \mu\theta}{kq(v)} \tag{18}$$

Formidable as (16) may appear, it shows that the question of whether there exists a suitable function $q(p)$ is "simply" a fixed point problem. That is, can there exist a function $q(p)$ defined on some interval $[p^0,p^1]$ which, when the operations implied by the RHS of (16) are applied to it, returns itself?

There are, however, two complications connected with this approach that should be mentioned. The first is that some restrictions need to be placed on $q(p)$. In particular, the functions F and M can be derived from q, and F and M must be distribution functions. For example, eqn (13) implies $q(p)+pq'(p)\leq 0$ must hold. The second complication is that (16) was derived treating k, γ, and θ as given constants. Assuming that the fixed point problem posed in (16) has an affirmative answer, ie, that a function $q(p)$ does exist for some values of k, γ, and θ, one must then go back and check that there is no inconsistency in the generation of k, γ, and θ.

6. Properties of Dispersion Equilibrium

Conditional on existence, it is easy to establish some of the properties of a price-wage dispersion equilibrium.

<u>Property 1</u>: The wage distribution is truncated to the right and the price distribution is truncated to the left. Furthermore, the minimum price exceeds the maximum wage.

<u>Proof</u>: In equilibrium $\Pi(p) = (p-w(p))q(p) = \mu\theta > 0$, implying that the price offer of any firm exceeds its wage offer. Therefore there is a highest wage, say w^1, and a lowest price, say p^0; ie, the wage distribution is truncated to the right and the price distribution is truncated to the left. Since w(p) is non-increasing (cf, eqn (9)) a firm offering p^0 must also be offering w^1, ie, $p^0 > w^1$.

<u>Property 2</u>: The wage distribution is truncated to the left.

<u>Proof</u>: For any distributions F and M there is a corresponding reservation income k. The existence of k and the minimum price p^0 implies no individual will ever accept a wage below w^0, where $k=(w^0+\theta)/p^0$. But, no firm will ever offer a wage below this minimum "reservation wage" since to do so would elicit zero labor supply, implying zero profits.

<u>Property 3</u>: The price distribution is truncated to the right.

<u>Proof</u>: The existence of a reservation income k and a maximum wage w^1 implies that no individual will ever accept a price above p^1 as a "permanent" price, where $k=(w^1+\theta)/p^1$. The implication that no firm will offer a price above p^1 is, however, not immediately obvious. Even though no individual will become a permanent (ie, employed) customer at a price above p^1, such a price still generates demand from searchers.

However, for $p > p^1$, $q(p) = \mu s\theta/p$; ie, $\Pi(p) = \frac{\mu s\theta}{p}(p-w(p))$. Since $w(p)$ is non-increasing, $\Pi(p)$ is non-decreasing for $p > p^1$. But $\lim_{p\to\infty} \Pi(p) = \mu s\theta$ is therefore the maximum profit to be earned by offering a price above p^1. Finally, $0 < s < 1$; ie, $\mu s\theta < \mu\theta$, implies no firm will offer $p > p^1$.

Property 4: If any firm offers the maximum "reservation price" p^1, then the distributions of p and w must have "mass points" at p^0 and w^1, respectively.

Proof: If any firm is offering p^1, then $\Pi(p^1) = \mu\theta$. But $\Pi(p)<\mu s\theta$ for all $p > p^1$; therefore $\Pi(p)$ must be discontinuous at p^1. This is turn implies that

$$q(p) = \frac{\mu s}{p}(\theta + \frac{1-\tau}{\tau}\int_{kp-\theta}^{\infty}(w+\theta)dM(w))$$

must be discontinuous at p^1. As $p\to p^1$, $kp-\theta\to kp^1-\theta=w^1$; thus, the density of w must exhibit a "jump" or "mass point" at w^1. Finally, the relationship $F(p) = 1 - M(w(p))$ implies the corresponding mass point at p^0.

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