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Transmission Costs, Transmission Capacities and their Influence on Market Power in Wholesale Electricity Markets

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Abstract

The integration of electricity markets around the world has increased the importance of congestion between countries/states and has initiated a discussion of how to harmonize network tariffs. This paper analyzes how the transmission capacity and the transmission cost, such as a transmission tariff, influence bidding behavior in electricity markets. It is shown that transmission costs can have seemingly counter-intuitive effects. Normally, more transmission capacity would improve competition, but this is not necessarily the case when one considers transmission costs. The paper also illustrates that there are cases where increasing transmission costs could have a pro-competitive effect and benefit consumers. In contrast, point of connection tariffs, which are used in the majority of the European countries, always push up electricity prices and always hurt consumers.

KEYWORDS: electricity auctions, wholesale electricity markets, transmission capacity constraints, network tariffs, energy economics.
JEL codes: D43, D44, L13, L94

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1 Introduction

The integration of electricity markets around the world has increased the importance of congestion between countries/states and has initiated a discussion of how to harmonize network tariffs. In general, transmission from regions with low prices to regions with high prices benefits social welfare. In deregulated electricity markets, more transmission would, in addition, normally improve the market competitiveness. However, it is very costly to expand transmission capacity. In order to focus investments to points in the grid where the gains in terms of enhanced market performance will be the largest, one needs a better understanding of how transmission capacity influences competition between spatially distributed producers. The contribution of this paper is to characterize the outcome of an electricity market auction and how it depends on transmission constraints and transmission costs.

The analysis employs a simple duopoly model similar to that in Fabra et al. (2006). In the basic set up, the two suppliers have symmetric production capacities and marginal costs, but are located in two different markets ("North" and "South") that are connected through a transmission line with a limited transmission capacity.[1] Each firm faces a perfectly inelastic demand in each market that is known with certainty when suppliers submit their offer prices. Each supplier must submit a single price offer for its entire capacity[2] in a discriminatory price auction such as those used in the UK wholesale electricity market. The assumption of price-inelastic demand can be justified by the fact that the vast majority of consumers purchase electricity under regulated tariffs that are independent of the prices set in the wholesale market, at least in the short run. The assumption that suppliers have perfect information concerning market demand is reasonable when applied to markets where offers are "short lived", such as in Spain, where there are 24 hourly day-ahead markets each day.

Suppliers pay a monetary charge (tariff) to the network owner when using the grid. The charge is linear and it depends on how much power the suppliers inject into the grid (*point of connection tariff*) or transmit through the grid (*transmission tariff*). The majority of European countries (ENTSO-E, 2013) have point of connection tariffs. With the point of connection tariffs scheme, suppliers pay a linear tariff for the electricity injected into the grid, i.e., the electricity sold in their own market and the one sold in the other market. From the suppliers' point of view, a connection tariff is equivalent to an increase in generation costs. Given that electricity demand is very inelastic, an increase in generation costs is passed through to consumers that face an increase in equilibrium prices in both markets. This is in line with the pass-through literature (Marion and Muehlegger 2011; Fabra and Reguant 2014). For transmission tariffs, electricity suppliers would only pay a linear tariff for the electricity sold to the other market. Hence, similar to a trade model, firms only pay a transport cost for the goods sold in the other market. The analysis indicates that transmission tariffs are better than point of connection tariffs from the consumers' perspective.

[1] The term "transmission capacity constraint" is used throughout this article in the electrical engineering sense: a transmission line is constrained when the flow of power is equal to the capacity of the line, as determined by engineering standards.

[2] Fabra et al. (2006) show that the equilibrium outcome allocation does not change when firms submit single price offers for their entire capacity and when they submit a set of price-quantity offers.

When there are constraints on the possibility to export electricity to another market, the effective size of the market differs for the suppliers. The supplier located in the high-demand market faces a higher residual demand, while the supplier located in the low-demand market cannot sell its entire generation capacity. Therefore, the supplier located in the high-demand market has incentives to submit higher bids than the one located in the low-demand market (size effect). Hence, due to the limited transmission capacity, the equilibrium is asymmetric even if suppliers have identical production costs and production capacities.

Transmission costs also introduce an asymmetry. The supplier located in the low-demand market has to sell a large part of its generation capacity into the other market and thus, it faces a high transmission cost and has incentives to increase its bid. The transmission cost makes the supplier in the high-demand market more efficient in relative terms. In order to exploit its efficiency rent, it has incentives to submit lower bids and, for a sufficiently high transmission cost, the efficient supplier will even try to undercut the exporting supplier (cost effect). Hence, the introduction of transmission tariffs could reduce the bid of the supplier in the high-demand market and there are even cases where consumers would, on average, gain from the introduction of a transmission cost. Point of connection tariffs do not have the pro-competitive cost effect. This suggests that transmission tariffs would, in most cases, be better for market performance and consumers in comparison to point of connection tariffs.

With low transmission tariffs, an increase in the transmission capacity increases the competition between suppliers and the expected bids for both firms decrease. This reduces the profit for the supplier in the high demand market. The profit for the exporting supplier first increases when the transmission capacity increases, because it can export more. But for a sufficiently large transmission capacity, increased competition will dominate and more capacity will reduce the profit also for the exporting supplier. A third effect is that transmission payments will increase if exports increase. If the transmission costs are sufficiently high and fixed per unit exported, an increase in the transmission capacity will increase the bids. Therefore, an increase in transmission capacity could be anti-competitive if the transmission costs are sufficiently high.

A methodological contribution is that this paper is the first to introduce transmission constraints in models with Bertrand competition and capacity constrained production. Kreps and Scheinkman (1983) and Osborne and Pitchik (1986) characterize the equilibrium in a duopoly with production capacity constraints. Deneckere and Kovenock (1996) and Fabra et al. (2006) extend the analysis to include asymmetries in generation capacity and production costs. Hu et al. (2010) extend the analysis to multiple firms, but they have only found a close form solution for the equilibrium when the suppliers are symmetric. Rosenthal (1980) and Janseen and Moraga-González (2004) applied similar techniques to extend the analysis to multiple firms in different sales models. Transmission constraints have been considered in other types of oligopoly models. Borenstein et al. (2000) characterize the equilibrium in an electricity network where suppliers compete in quantities as in a Cournot game. Holmberg and Philpott (2012) solve for symmetric supply function equilibria in electricity networks when demand is uncertain ex-ante, but they do not consider any transmission costs. Escobar and Jofré (2010) analyze the effect of transmission losses, a transmission cost, on equilibrium outcome allocations, but they neglect transmission constraints. Hence, this paper is the first to characterize equilibrium outcomes in networks with both transmission constraints and transmission costs. The paper also shows that the interaction between transmission costs and transmission constraints is non-straightforward.

The results of this paper could also be of relevance for the trade literature. For instance, Krugman (1980), Flam and Helpman (1987), Brezis et al. (1993) and Motta et al. (1997) explain differences in prices and profits in international trade models based on product differentiation or product cost advantages. By introducing transport costs and transport constraints, this paper finds related results, even if the product is homogeneous and suppliers have identical production technologies.

The article proceeds as follows. Section 2 describes the model and characterizes the equilibrium in the presence of transmission capacity constraints. Section 3 characterizes the equilibrium in the presence of transmission capacity constraints and transmission tariffs. Section 4 concludes the paper. The analysis of point of connection tariffs and all proofs are found in the Appendix.

2 The model

Set up of the model. There exist two electricity markets, market North and market South, that are connected by a transmission line with capacity T. When firms transmit electricity through the grid from one market to the other, they face a symmetric linear[3] transmission tariff t. In order to reduce transmission losses,[4] the transmission tariffs in the majority of European countries have a locational and a seasonal component.[5]

There exist two duopolists with capacities k_n and k_s, where subscript n means that the supplier is located in market North and subscript s means that the supplier is located in market South. The suppliers' marginal costs of production are c_n and c_s. In this paper, I analyze the effect of transmission capacity constraints and transmission costs on the equilibrium. In order to focus on this effect, I assume that suppliers are symmetric in capacity $k_n = k_s = k > 0$ and symmetric in costs $c_n = c_s = c = 0$. The level of demand in any period, θ_n in market North and θ_s in market South, is a random variable uniformly distributed that is independent across markets[6] and independent of market price, i.e.,

[3]The transmission tariffs are linear in electricity markets. However, the model can be modified to assume convex costs. When the transmission costs are convex, the existence of the equilibrium is guaranteed by Dixon (1984).

[4]Electricity suppliers pay a linear tariff that depends on the location and the season/period-of-day. The locational component of the tariff penalizes the injection of electricity in points of the grid that generate high flows of electricity. The seasonal/period-of-day component of the tariff penalizes the transmission of electricity when the losses are larger. For a complete analysis of losses in Europe and a complete description of the algorithm implemented to work out power losses, consult the document "ENTSO-E ITC Transit Losses Data Report 2013".

[5]The locational and seasonal component implies that suppliers face asymmetric linear tariffs. However, the model can easily be modified to introduce this type of asymmetries. For a comparison of European tariff systems, check out the document "ENTSO-E Overview of transmission tariffs in Europe: Synthesis 2014".

[6]In the majority of electricity markets, demand in one market is higher than demand in the other market. Moreover, there exists the possibility of some type of correlation between demands across markets. In this paper, I assume uniform distribution and independence of demand. However, the model

perfectly inelastic. In particular, $\theta_i \in [\underline{\theta}_i, \overline{\theta}_i] \subseteq [0, k+T]$ is distributed according to some known distribution function $G(\theta_i)$, $i = n, s, i \neq j$.

The capacity of the transmission line can be lower than the installed capacity in each market $T \leq k$, i.e. the transmission line could be congested for some realization of demands (θ_s, θ_n).

Timing of the game. Having observed the realization of demands $\theta \equiv (\theta_s, \theta_n)$, each supplier simultaneously and independently submits a bid specifying the minimum price at which it is willing to supply up to its capacity, $b_i \leq P$, $i = n, s$, where P denotes the "market reserve price", possibly determined by regulation.[7] Let $b \equiv (b_s, b_n)$ denote a bid profile. On basis of this profile, the auctioneer calls suppliers into operation. If suppliers submit different bids, the capacity of the lower-bidding supplier is dispatched first. Without lost of generality, assume that $b_n < b_s$. If the capacity of supplier n is not sufficient to satisfy total demand $(\theta_s + \theta_n)$ in the case of the transmission line not being congested, or $(\theta_n + T)$ in the case of the transmission line being congested,[8] the higher-bidding supplier's capacity, supplier s, is then dispatched to serve residual demand, $(\theta_s + \theta_n - k)$ if the transmission line is not congested, or $(\theta_s - T)$ if the transmission line is congested. If the two suppliers submit equal bids, then supplier i is ranked first with probability ρ_i, where $\rho_n + \rho_s = 1$, $\rho_i = 1$ if $\theta_i > \theta_j$, and $\rho_i = \dfrac{1}{2}$ if $\theta_i = \theta_j$, $i = n, s$, $i \neq j$. The implemented tie breaking rule is such that if the bids of both suppliers are equal and demand in market i is larger than demand in region j, the auctioneer first dispatches the supplier located in market i.

The output allocated to supplier $i, i = n, s$, denoted by $q_i(\theta, b)$, is given by

$$q_i(b; \theta, T) = \begin{cases} min\,\{\theta_i + \theta_j, \theta_i + T, k_i\} & \text{if } b_i < b_j \\ \rho_i min\,\{\theta_i + \theta_j, \theta_i + T, k_i\} + & \\ \qquad [1 - \rho_i] max\,\{0, \theta_i - T, \theta_i + \theta_j - k_j\} & \text{if } b_i = b_j \\ max\,\{0, \theta_i - T, \theta_i + \theta_j - k_j\} & \text{if } b_i > b_j \end{cases} \tag{1}$$

The output function has an important role in determining the equilibrium and thus, it is explained in detail. Below, I describe the construction of supplier n's output function; the one for supplier s is symmetric.

The total demand that can be satisfied by supplier n when it submits the lower bid $(b_n < b_s)$ is defined by $min\,\{\theta_n + \theta_s, \theta_n + T, k\}$. The realization of (θ_s, θ_n) determines three different areas (left-hand panel in figure 1).

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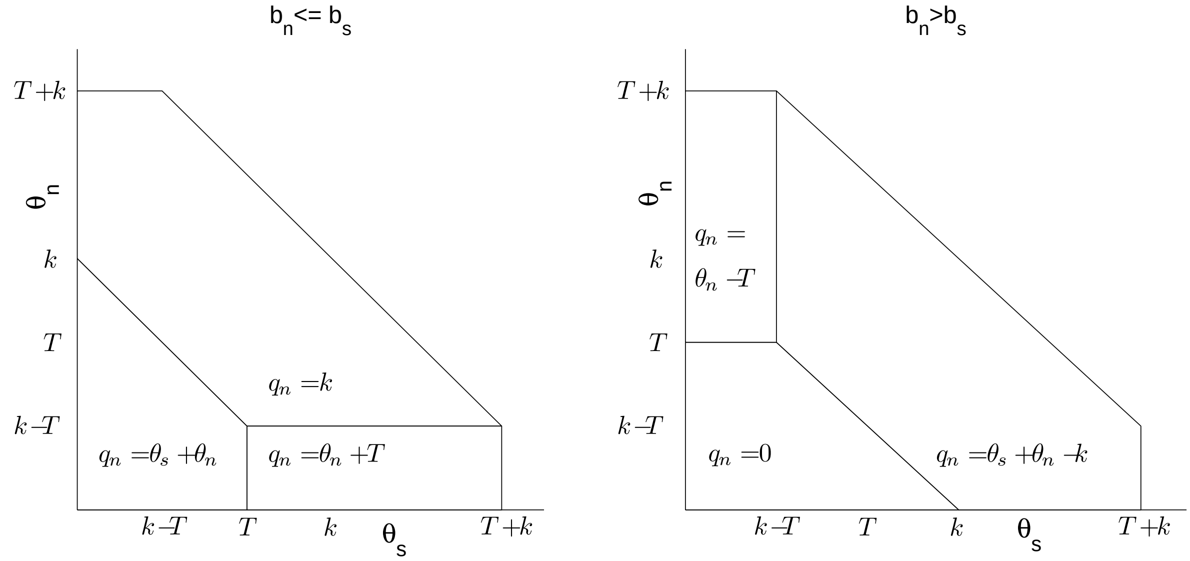
can easily be modified to introduce different distributions of demand and correlation between demands across markets.

[7]P can be interpreted as the price at which all consumers are indifferent between consuming and not consuming, or a price cap imposed by the regulatory authorities. See von der Fehr and Harbord (1993, 1998).

[8]When the demand in market South is larger than the transmission line capacity $\theta_s > T$, supplier n can only satisfy the demand in its own region and T units of demand in region South $(\theta_n + T)$. Below in this section, I explain in detail the total demand and the residual demand that can be satisfied by each supplier.

Figure 1: Output function for supplier n. ($k_n = k_s = 60, T = 40$)



$$min\{\theta_n + \theta_s, \theta_n + T, k\} = \begin{cases} \theta_s + \theta_n & \text{if } \theta_n \leq k - \theta_s \text{ and } \theta_s < T \\ \theta_n + T & \text{if } \theta_n < k - T \text{ and } \theta_s > T \\ k & \text{if } \theta_n > k - \theta_s;\ \theta_s \in [0, T] \\ & \text{or if } \theta_n > k - T;\ \theta_s \in [T, k + T] \end{cases}$$

When demand in both markets is low, supplier n can satisfy total demand $(\theta_s + \theta_n)$. If the demand in market South is larger than the transmission capacity $\theta_s > T$, supplier n cannot satisfy the demand in market South, even when it has enough generation capacity for this; therefore, the total demand that supplier n can satisfy is $(\theta_n + T)$. Finally, if the demand is large enough, the total demand that supplier n can satisfy is its own generation capacity.

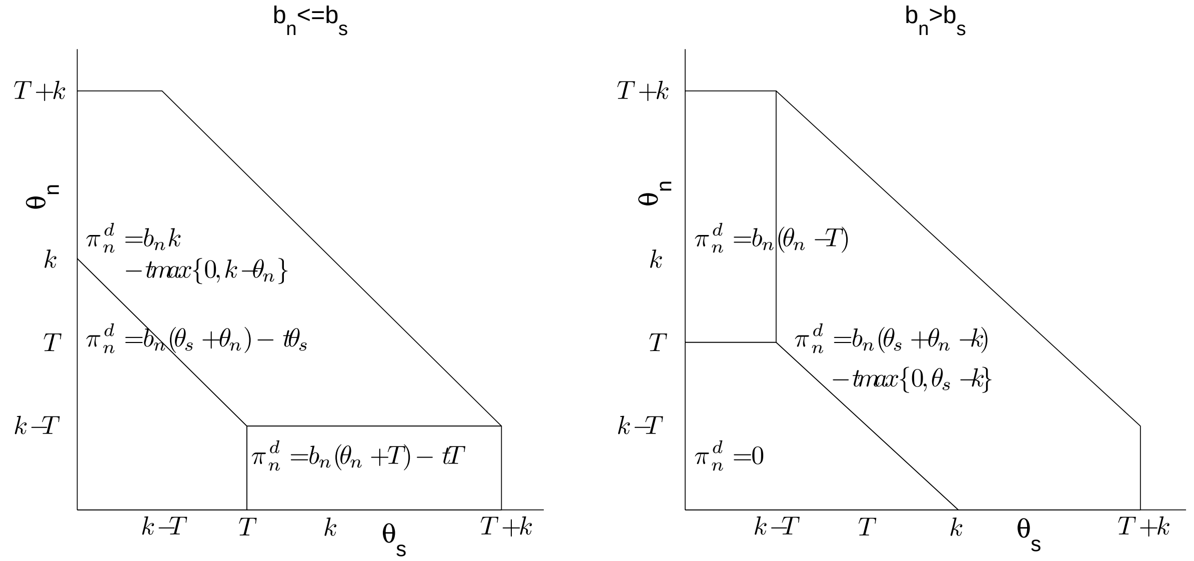
The residual demand that supplier n satisfies when it submits the higher bid $(b_n > b_s)$ is defined by $max\{0, \theta_n - T, \theta_s + \theta_n - k\}$. The realization of (θ_s, θ_n) determines three different cases (right-hand panel in figure 1).

$$max\{0, \theta_n - T, \theta_s + \theta_n - k\} = \begin{cases} 0 & \text{if } \theta_n < T;\ \theta_s \in [0, k - T] \\ & \text{or } \theta_n < k - \theta_s;\ \theta_s \in [k - T, k] \\ \theta_n - T & \text{if } \theta_n > T \text{ and } \theta_s \in [0, k - T] \\ \theta_s + \theta_n - k & \text{if } \theta_n > k - \theta_s;\ \theta_s \in [k - T, T + k] \end{cases}$$

When demand in both markets is low, supplier s satisfies total demand and therefore, the residual demand that remains for supplier n is zero. The total demand that supplier s can satisfy diminishes due to the transmission constraint. As soon as demand in market North is larger than the transmission capacity $(\theta_n > T)$, it cannot be satisfied by supplier s and thus, some residual demand $(\theta_n - T)$ remains for supplier n. When total demand is large enough, supplier s cannot satisfy total demand and some residual demand $(\theta_s + \theta_n - k)$ remains for supplier n.

Finally, the payments are worked out by the auctioneer. When the auctioneer runs a

Figure 2: Profit function for supplier n. ($k_n = k_s = 60, T = 40, t > 0$)



discriminatory price auction,[9] the price received by a supplier for any positive quantity dispatched by the auctioneer is equal to its own bid. Hence, for a given realization of demands $\theta \equiv (\theta_s, \theta_n)$ and a bid profile $b \equiv (b_s, b_n)$, supplier n's profits, $i = n, s$, can be expressed as

$$\pi_i^d(b;\theta,T,t) = \begin{cases} (b_i - c_i)min\left\{\theta_i + \theta_j, \theta_i + T, k\right\} - \\ \quad tmax\left\{0, min\left\{\theta_j, T, k - \theta_i\right\}\right\} & \text{if } b_i \leq b_j \text{ and } \theta_i > \theta_j \\ (b_i - c_i)max\left\{0, \theta_i - T, \theta_i + \theta_j - k\right\} - \\ \quad tmax\left\{0, \theta_j - k\right\} & \text{otherwise} \end{cases}$$

If $b_n \leq b_s$ and $\theta_n \geq \theta_s$, supplier n's payoff function is $\pi_n^d(b;\theta,T) = (b_n - c_n)$ $min\left\{\theta_n + \theta_s, \theta_n + T, k\right\}$. In addition to this expression, due to the transmission costs, supplier n is charged a transmission cost t for the power sold in market South. The transmission costs have four different possible values: $t\theta_s$ when the realization of demand in market North is low and the transmission line is not congested; tT when the realization of demand in market North is low and the transmission line is congested; when the realization of demand in market North is high but lower than its generation capacity, the transmission costs are $t(k - \theta_n)$; finally, when demand in market North is larger than the generation capacity k, supplier n cannot sell any electricity in market South and the transmission costs are zero. Hence, after adding the transmission costs, supplier n's payoff is equal to $\pi_n^d(b;\theta,T,t) = (b_n - c_n)min\left\{\theta_n + \theta_s, \theta_n + T, k\right\} - tmax\left\{0, min\left\{\theta_s, T, k - \theta_n\right\}\right\}$ (left-hand panel, figure 2).

In the rest of the cases, supplier n is dispatched last and satisfies the residual demand. Supplier n's payoff function is $\pi_n^d(b;\theta,T,t) = (b_n - c_n)min\left\{\theta_s + \theta_n, \theta_n + T, k\right\}$.

[9]The aim of this paper is to characterize the equilibrium in an electricity auction in the presence of transmission constraints and transmission costs. I have decided to focus on discriminatory auctions because the equilibrium is unique and therefore, it is easier to make a comparative static analysis. However, using the approach presented in Fabra et al. (2006) and taking into account the allocation of transmission rights (Blázquez, 2014), it is simple to characterize the equilibrium when the auction is uniform.

In addition to this expression, due to the transmission costs, supplier n is charged a transmission cost t for the residual demand satisfied in market South. Therefore, after adding the transmission costs, supplier n's payoff is equal to $\pi_n^d(b;\theta,T) = (b_n - c_n)max\{0,\theta_n - T,\theta_s+\theta_n - k\} - tmax\{0,\theta_s - k\}$ (right-hand panel, figure 2).

3 Effect of transmission capacity constraints

In the presence of transmission capacity constraints, the size of the market differs for both suppliers. The supplier located in the high-demand market faces **a** higher residual demand and the supplier located in the low-demand market cannot sell its entire generation capacity. In this section, I characterize the equilibrium in the presence of transmission capacity constraints and *zero* transmission costs and then I analyze the effect of an increase in transmission capacity.

Lemma 1. When the realization of demands (θ_s, θ_n) is low (area A), the equilibrium is in pure strategies. When the realization of demands (θ_s, θ_n) is intermediate (areas $A1, B1$) or high (area $B2$), a pure strategy equilibrium does not exist (figure 3).

Proof. When the realization of demands (θ_s, θ_n) is low (area A), both suppliers have enough capacity to satisfy total demand in both markets and the transmission line is not congested. Therefore, they compete fiercely to be dispatched first in the auction. Hence, the equilibrium is the typical Bertrand equilibrium where both suppliers submit bids equal to their marginal cost.

When the realization of demands (θ_s, θ_n) is intermediate (areas $A1$, $B1$) or high (area $B2$), at least one of the suppliers faces a positive residual demand. Therefore, a pure strategy equilibrium does not exist. First, an equilibrium such that $b_i = b_j = c$ does not exist because at least one supplier has an incentive to increase its bid and satisfy the residual demand. Second, an equilibrium such that $b_i = b_j > c$ does not exist because at least one supplier has the incentive to undercut the other to be dispatched first. Finally, an equilibrium such that $b_j > b_i > c$ does not exist because supplier i has the incentive to shade the bid submitted by supplier j. $\square$

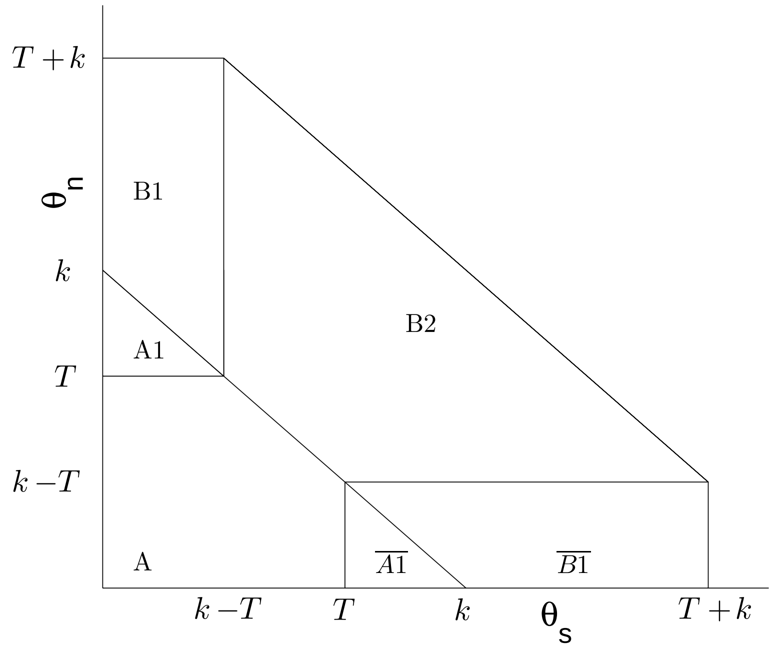
When the realization of demands (θ_s, θ_n) is intermediate or high, a pure strategy equilibrium does not exist. However, the model satisfies the properties[10] established by Dasgupta and Maskin (1986) which guarantee that a mixed strategies equilibrium exists.

Lemma 2. In the presence of transmission constraints. In a mixed strategy equilibrium, no supplier submits a bid lower than bid $(\underline{b}_i)$ such that $\underline{b}_i min\{\theta_i+\theta_j,\theta_i+T,k\} = Pmax\{0,\theta_i - T,\theta_i+\theta_j - k\}$. Moreover, the support of the mixed strategy equilibrium for both suppliers is $S = \left[max\left\{\underline{b}_i,\underline{b}_j\right\},P\right]$.

Proof. Each supplier can guarantee for itself the payoff $Pmax\{0,\theta_i - T,\theta_i+\theta_j - k\}$, because each supplier can always submit the highest bid and satisfy the residual demand. Therefore, in a mixed strategy equilibrium, no supplier submits a bid that generates a payoff equilibrium lower than $Pmax\{0,\theta_i - T,\theta_i+\theta_j - k\}$. Hence, no supplier submits a bid

[10]In annex one, proposition one, I prove that the model satisfies the properties established by Dasgupta and Maskin which guarantee that a mixed strategy equilibrium exists.

Figure 3: Equilibrium areas ($k_n = k_s = k = 60, T = 40, c = 0$)



lower than $\underline{b}_i$, where $\underline{b}_i$ solves $\underline{b}_i min\left\{\theta_i + \theta_j, \theta_i + T, k\right\} = Pmax\left\{0, \theta_i - T, \theta_i + \theta_j - k\right\}$.

No supplier can rationalize submitting a bid lower than $\underline{b}_i, i = n, s$. In the case when $\underline{b}_i = \underline{b}_j$, the mixed strategy equilibrium and the support are symmetric. In the case when $\underline{b}_i < \underline{b}_j$, supplier i knows that supplier j never submits a bid lower than $\underline{b}_j$. Therefore, in a mixed strategy equilibrium, supplier i never submits a bid b_i such that $b_i \in \left(\underline{b}_i, \underline{b}_j\right)$, because supplier i can increase its expected payoff choosing a bid b_i such that $b_i \in \left[\underline{b}_j, P\right]$. Hence, the equilibrium strategy support for both suppliers is $S = \left[max\left\{\underline{b}_i, \underline{b}_j\right\}, P\right]$ $\square$

Using Lemmas one and two, I characterize the equilibrium.

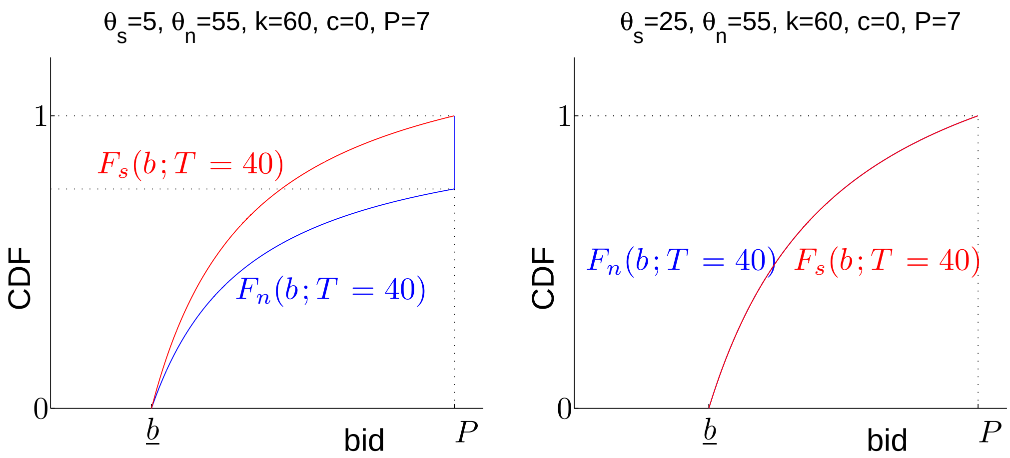
Proposition 1. In the presence of transmission constraints, the characterization of the equilibrium falls into one of the next two categories.

i Low demand (area A). The equilibrium strategy pair is in pure strategies.

ii Intermediate demand (areas $A1$, $B1$) and high demand (area $B2$). The equilibrium strategy pair is in mixed strategies.

When the realization of demands (θ_s, θ_n) is low, suppliers compete fiercely to be dispatched first in the auction and the equilibrium is the typical Bertrand equilibrium in which both suppliers submit bids equal to their marginal cost. When the realization of demands (θ_s, θ_n) is intermediate, due to the scarcity of transmission capacity, the supplier located in the high-demand market faces a higher residual demand and the supplier located in the low-demand market cannot sell its entire generation capacity. Therefore, the equilibrium is an asymmetric mixed strategy equilibrium where the supplier located in the high-demand market randomizes submitting higher bids with a higher probability,

Figure 4: Discriminatory auction. Mixed strategy equilibrium



i.e., its cumulative distribution function stochastically dominates the cumulative distribution function of the supplier located in the low-demand market (left-hand panel, figure 4). Finally, when the realization of demands (θ_s, θ_n) is high, the transmission capacity is not binding, but the generation capacity is. Therefore, both suppliers face the same residual and total demand and the equilibrium constitute a symmetric mixed strategy equilibrium in which both suppliers randomize using the same cumulative distribution function (right-hand panel, figure 4).

In the presence of transmission constraints, there are two relevant constraints that explain the results. When the generation capacity is binding, even when the realization of demands is asymmetric, the equilibrium is symmetric.[11] When the transmission capacity is binding, even when the firms are symmetric in generation capacity and production costs, the equilibrium is asymmetric.

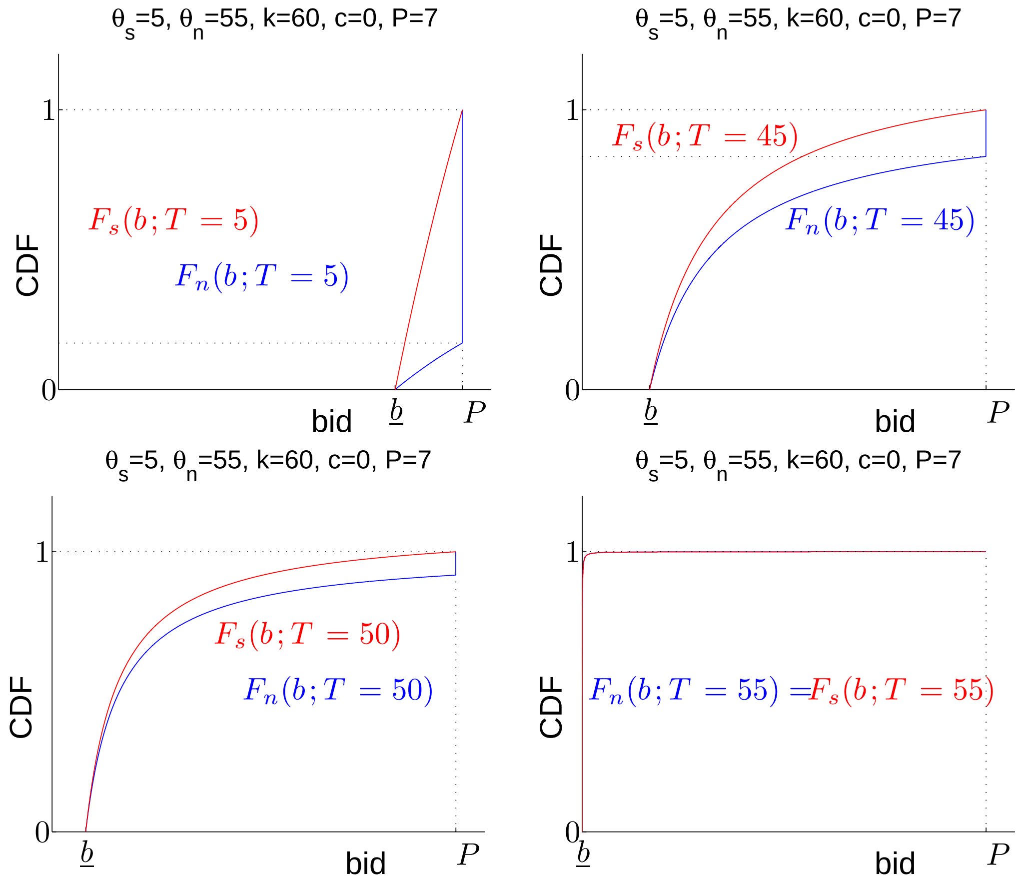
To conclude this section, I analyze the effect of an increase in transmission capacity on equilibrium outcome allocations.

Proposition 2. In the presence of transmission constraints. An increase in transmission capacity $(\triangle T)$ reduces the lower bound of support $\underline{b}$ and reduces the expected bids for both suppliers (an increase in transmission capacity is pro-competitive). Moreover, an increase in transmission capacity reduces the profit of the supplier located in the high-demand market. However, an increase in transmission capacity modifies the profit of the supplier located in the low-demand market in a non monotonic pattern (table 1 and figures 5 and 6).

An increase in transmission capacity modifies the market size as does suppliers' strategies. When the transmission capacity is very low, the supplier located in the high-demand market faces a high residual demand and the supplier located in the low-demand market cannot sell its entire generation capacity. Therefore, the supplier located in the high-demand market submits higher bids than the one located in the low-demand market and

[11] In the next section, I introduce transmission tariffs. In the presence of transmission costs, the realization of demands becomes very relevant because the transmission costs are larger for the supplier located in the low-demand market and the equilibrium is asymmetric.

Figure 5: Increase in transmission capacity $\triangle T$. Cumulative Distribution Function



the cumulative distribution function of the supplier located in the high-demand market stochastically dominates that of the supplier located in the low-demand market (top-left panel, figure 5). When the transmission capacity increases, the supplier located in the high-demand market faces a reduction in its residual demand and the supplier located in the low-demand market faces an increase in the demand that it can satisfy. Therefore, the cumulative distribution function becomes more symmetric (top-right and bottom-left panels). When the transmission capacity is high enough, the transmission line is not congested and the residual and the total demand that both suppliers face are equal; in that case, the equilibrium is symmetric and both suppliers assign probability one to the lower bid (bottom-right panel).

The change in suppliers' strategies induced by an increase in transmission capacity modifies the main variables of the model. An increase in transmission capacity reduces the residual demand and according to lemma two, the lower bound of the support decreases (left-hand panel, figure 6; column two of table 1). A decrease in the lower bound of the support implies that both suppliers randomize submitting lower bids and therefore, the expected bid decreases for both suppliers (right-hand panel, figure 6; columns five and seven of table 1). Finally, an increase in transmission capacity reduces the expected bid and the residual demand of the supplier located in the high-demand market as does its expected profit. In contrast, an increase in transmission capacity reduces the expected bid and increases the total demand of the supplier located in the low-demand market. When the transmission capacity is low, the increase in demand dominates the decrease

Figure 6: Increase in transmission capacity $\triangle T$. Main variables

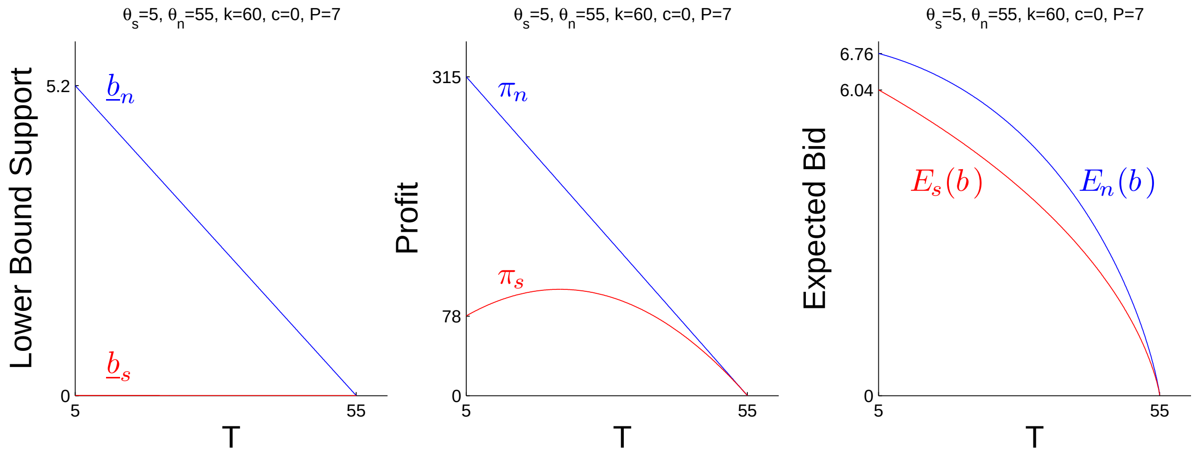


Table 1: Increase in transmission capacity $\triangle T$. Main variables. ($\theta_s = 5$, $\theta_n = 55$, $k = 60$, $c = 0$, $P = 7$)

T	$\underline{b}$	π_n	π_s	$E_n(b)$ Ana.	$E_n(b)$ Sim.	$E_s(b)$ Ana.	$E_s(b)$ Sim.
0	–	385.07	35	7	7	7	7
5	5.835	350.1	58.35	6.8971	6.8963	6.3795	6.3830
15	4.668	280.08	93.36	6.5592	6.5587	5.6770	5.6780
25	3.501	210.06	105.03	5.9264	5.9261	4.8530	4.8532
35	2.335	140.1	93.4	4.8981	4.8981	3.8464	3.8476
45	1.168	70.08	58.4	3.2587	3.2589	2.5102	2.5109
55	0.001	0.06	0.06	0.0089	0.0093	0.0087	0.0093

$E_n(b)$ Ana. and $E_s(b)$ Ana. are the expected values obtained using the analytical expressions presented in proposition one and $E_n(b)$ Sim. and $E_s(b)$ Sim. are the expected values obtained using the simulation explained in detail in Annex 3.

I have assumed that demand in market North (θ_n) is equal to 55.01 to avoid computational problems. This is the reason why the variables in the last row are not exactly equal to zero.

in the expected bid and its expected profit increases. However, when the transmission capacity is large enough, the decrease in bids dominates and its expected profit decreases (central panel, figure 6; columns three and four, table 1.)

Increases in transmission capacity have historically been justified as a way of enhancing competition between markets. However, as I have shown in proposition two, an increase in transmission capacity modifies the profit of the supplier located in the low-demand market and this might increase the competition within a market. For the sake of the argument, imagine that a small hydro-power plant that faces a fixed entry cost would like to install some generation capacity in the low-demand market. When there is no transmission capacity between markets, due to the reduced size of the market, the supplier cannot cover its fixed entry cost. However, if the transmission line increases, the size of the market increases and the supplier could enter the low-demand market. This entry might increase the competition within the low-demand market.

4 Effect of transmission capacity constraints and transmission costs

In the presence of transmission capacity constraints, the size of the market differs for both suppliers. In the presence of transmission costs, the transmission cost differs for both suppliers depending on the realization of the demand. The supplier located in the low-demand market must sell a large part of its generation capacity into the other market and thus, it faces a larger transmission cost than the supplier located in the high-demand market. In this section, I characterize the equilibrium in the presence of transmission capacity constraints and *positive* transmission costs.

Lemma 3. When the realization of demands (θ_s, θ_n) is low (area A) the equilibrium is in pure strategies. When the realization of demands (θ_s, θ_n) is intermediate (area $A1$) and the transmission costs are high, the equilibrium is in pure strategies; otherwise, a pure strategies equilibrium does not exist. When the realization of demands (θ_s, θ_n) is intermediate (areas $B1a, B1b$) or high (area $B2a, B2b$), a pure strategy equilibrium does not exist (figure 7). Moreover, due to the presence of transmission costs, the pure strategy equilibria are asymmetric.

Proof. When the realization of demands (θ_s, θ_n) is low (area A), both suppliers have enough capacity to satisfy total demand and the transmission line is not congested. Therefore, the competition to be dispatched first is fierce. Moreover, the supplier located in the high-demand market (supplier j) faces lower transmission costs. Hence, the equilibrium is the typical Bertrand equilibrium with asymmetries in "costs" [12] where the supplier located in the high-demand market extracts the efficiency rents. The *pure strategies equilibrium* is $b_i = b_j = \dfrac{t\theta_j}{\theta_i + \theta_j}$.

The *equilibrium profit* is:

$$\overline{\pi}_i = (\theta_i + \theta_j)\frac{t\theta_j}{\theta_i + \theta_j} - t\theta_j = 0; \;\; \overline{\pi}_j = (\theta_i + \theta_j)\frac{t\theta_j}{\theta_i + \theta_j} - t\theta_i = t(\theta_j - \theta_i) > 0$$

The *equilibrium price* is $\dfrac{t\theta_j}{\theta_i + \theta_j}$

Electricity flows from the high-demand market to the low-demand market.

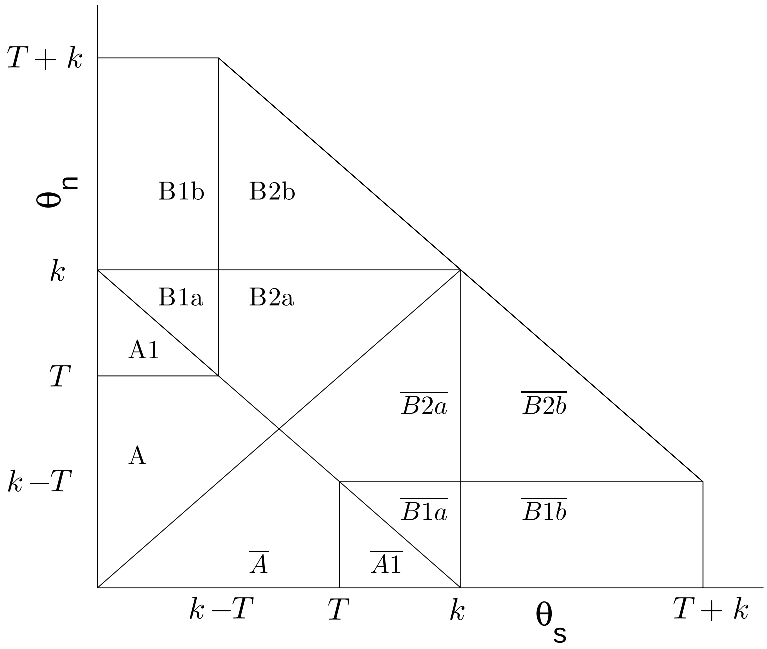
When the demand belongs to area $A1$ (figure 7), the transmission constraint binds for the supplier located in the low-demand market (supplier i); therefore, only the supplier located in the high-demand market can satisfy total demand. The supplier located in the high-demand market prefers to submit a low bid and extract the efficiency rent instead of submitting a high bid and satisfying the residual demand if $(\theta_i+\theta_j)\dfrac{tT}{\theta_i + T} - t\theta_i \geq P(\theta_i - T)$.

In such a case, the pure strategies equilibrium is $b_i = b_j = \dfrac{tT}{\theta_i + T}$.

The *equilibrium profit* is:

[12] It is important to emphasize that the generation costs are symmetric and equal to zero. In this model, the asymmetries in costs are due to the transmission costs.

Figure 7: Equilibrium areas ($k_n = k_s = k = 60, T = 40, c = 0, t > 0$)



$$\overline{\pi}_i = (\theta_i + T)\frac{tT}{\theta_i + T} - tT = 0;\ \overline{\pi}_j = (\theta_i + \theta_j)\frac{tT}{\theta_i + T} - t\theta_i > 0$$

The *equilibrium price* is $\dfrac{tT}{\theta_i + T}$

The electricity flows from the high-demand market to the low-demand market.

In the rest of the cases, a pure strategies equilibrium does not exist and the proof proceeds as in lemma one $\square$

When the realization of demands (θ_s, θ_n) is intermediate or high and the auction is discriminatory, a pure strategy equilibrium does not exist. However, the model satisfies the properties established by Dasgupta and Maskin (1986) which guarantee that a mixed strategy equilibrium exists.

Lemma 4. In the presence of transmission constraints and *positive* transmission costs. In a mixed strategy equilibrium, no supplier submits a bid lower than bid $(\underline{b}_i)$ such that

$$\begin{aligned}&\underline{b}_i min\left\{\theta_i + \theta_j, \theta_i + T, k\right\} - tmax\left\{0, min\left\{\theta_j, T, k - \theta_i\right\}\right\} = \\ &Pmax\left\{0, \theta_i - T, \theta_i + \theta_j - k\right\} - tmax\left\{0, \theta_j - k\right\}.\end{aligned}$$

Moreover, the support for the mixed strategy equilibrium for both suppliers is $S = \left[max\left\{\underline{b}_i, \underline{b}_j\right\}, P\right]$.

Proof. The proof proceeds as in lemma two. $\square$

Using lemmas three and four, I characterize the equilibrium.

Proposition 3. In the presence of transmission constraints and transmission costs, the characterization of the equilibrium falls into one of the next three categories.

i Low demand (area A). The equilibrium strategy pair is in pure strategies.

ii Intermediate demand (area $A1$). When the transmission cost is high, the equilibrium strategy pair is in pure strategies. In contrast, when the transmission cost is low, the equilibrium strategy pair is in mixed strategies.

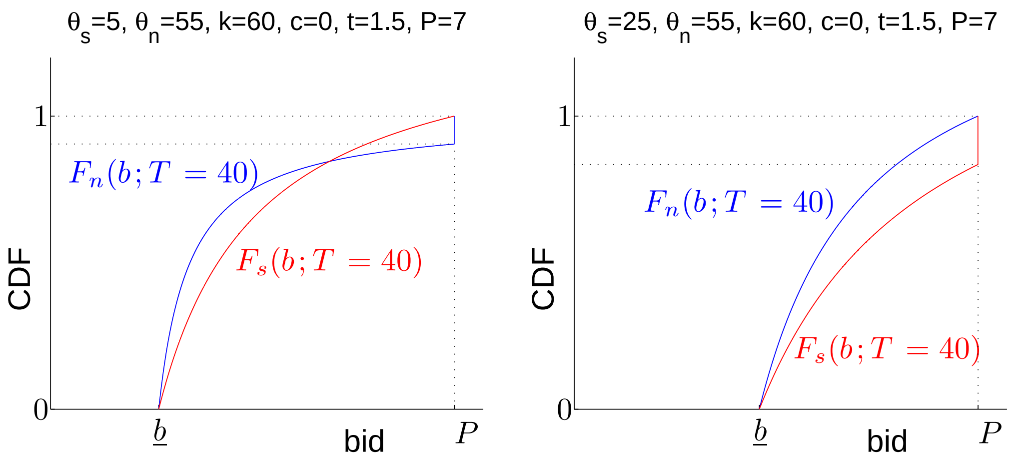
iii Intermediate demand (areas $B1a$, $B1b$) and high demand (areas $B2a$, $B2b$). The equilibrium strategy pair is in mixed strategies.

When the realization of demands (θ_s, θ_n) is low, suppliers compete fiercely to be dispatched first in the auction and the equilibrium is the typical Bertrand equilibrium with asymmetries in costs where the supplier located in the high-demand market extracts the efficiency rents.

When the realization of demands (θ_s, θ_n) belongs to area $A1$ the transmission constraint binds for the supplier located in the low-demand market (supplier i); therefore, only the supplier located in the high-demand market can satisfy total demand. If the transmission costs are high enough, the supplier located in the high-demand market prefers to submit a low bid to extract the efficiency rents. In contrast, when realization of demands (θ_s, θ_n) belongs to area $A1$ and the transmission costs are high or when the realization of demands (θ_s, θ_n) belongs to areas $B1a$ or $B1b$, due to the scarcity of transmission capacity, the supplier located in the high-demand market faces a higher residual demand and the supplier located in the low-demand market cannot sell its entire generation capacity. Therefore, the supplier located in the high-demand market has higher incentives to submit high bids than the one located in the low-demand market (size effect). However, due to the presence of transmission costs, the supplier located in the high-demand market faces lower transmission costs and to exploit its efficiency rents, it has higher incentives than the supplier located in the low-demand market to submit low bids (cost effect). The cost and size effects work in the opposite direction and no stochastic dominance range can be established between the cumulative distribution functions of both suppliers (left-hand panel, figure 8). This is in contrast to the *zero* transmission costs case where only the size effect drives the results and the cumulative distribution function of the supplier located in the high-demand region stochastically dominates the cumulative distribution function of the supplier located in the low-demand market (left-hand panel, figure 4).

When the realization of demands (θ_s, θ_n) is high, the transmission capacity is not binding, but the generation capacity is. Therefore, both suppliers face the same demand. However, due to the transmission costs, the supplier located in the high-demand market faces lower transmission costs and submits lower bids (cost effect). Hence, the cumulative distribution function of the supplier located in the low-demand market stochastically dominates the cumulative distribution function of the supplier located in the high-demand market (right-hand panel, figure 8). This is in contrast to the *zero* transmission costs case where both suppliers randomize using the same cumulative distribution function (right-hand panel, figure 4).

Figure 8: Discriminatory auction. Mixed strategy equilibrium



Finally, when the realization of demands (θ_s, θ_n) is in the diagonal, both suppliers face the same demand and transmission costs. Therefore, the equilibrium is a symmetric mixed strategy equilibrium.

In the rest of this section, I analyze the effect of an increase in transmission capacity on the size and cost effects and thus, on equilibrium outcome allocations (as in the rest of the section, I assume that the suppliers pay a linear tariff only for the electricity sold in the other market).

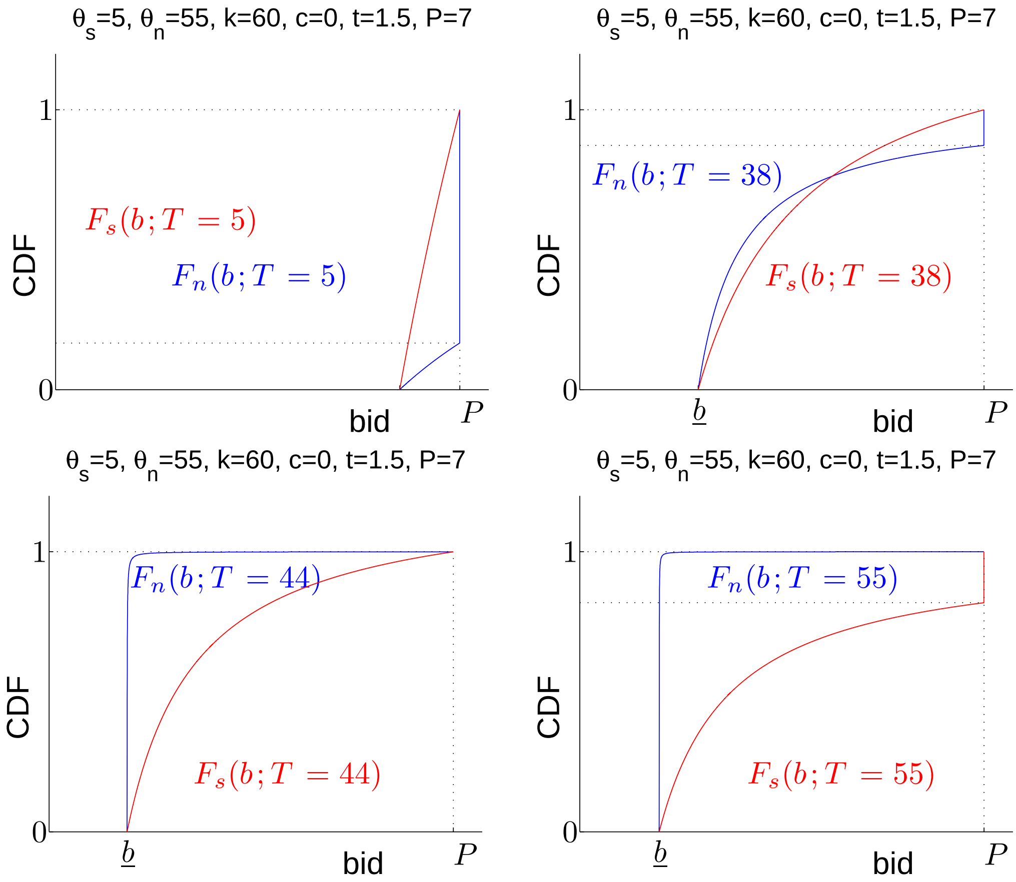
Proposition 4. An increase in transmission capacity ($\triangle T$) reduces the lower bound of the support of the supplier located in the high-demand market and increases the lower bound of the support of the supplier located in the low-demand market (left-hand panel, figure 10).

- When the lower bound of the support of the supplier located in the high-demand market is larger than the lower bound of the support of the supplier located in the low-demand market. An increase in transmission capacity reduces the expected bids of both suppliers (an increase in transmission capacity is pro-competitive), reduces the profit of the firm located in the high-demand market and modifies the profit of the supplier located in the low-demand market in a non-monotonic pattern.

- Otherwise, an increase in transmission capacity increases the expected bids of both suppliers (an increase in transmission capacity is anti-competitive), increases the expected profit of the supplier located in the high-demand market and does not modify the expected profit of the supplier located in the low-demand market (table 2; figures 9 and 10).

An increase in transmission capacity modifies the market size and the transmission costs and thus also suppliers' strategies. When the transmission capacity is very low, the size effect dominates and the cumulative distribution function of the supplier located in the high-demand market stochastically dominates that of the supplier located in the low-demand market (top-left panel, figure 9). When there is an increase in the transmission capacity, no cumulative distribution function stochastically dominates the other (top-right panel, figure 9). When there is a substantial increase in transmission capacity,

Figure 9: Increase in transmission capacity $\triangle T$. Cumulative Distribution Function



there is also an increase in transmission costs, especially for the supplier located in the low-demand market. In that case, the supplier located in the high-demand market submits lower bids than the one located in the low-demand market to extract the efficiency rents and the cumulative distribution function of the supplier located in the low-demand market stochastically dominates that of the supplier located in the high-demand market (bottom-left and bottom-right panels, figure 9).

The change in suppliers' strategies induced by an increase in transmission capacity modifies the main variables of the model. When the transmission capacity is sufficiently low ($T \leq 44$ for the numerical examples in table 2 and figures 9 and 10), the size effect dominates and an increase in transmission capacity induces the same changes in the variables as when the transmission costs are null (proposition two). Hence, an increase in transmission capacity decreases the lower bound of the support and therefore, decreases the expected bid for both suppliers. Hence, an increase in transmission capacity is pro-competitive (right-hand panel, figure 10; columns five and seven, table 2); reduces the expected profit of the supplier located in the high-demand market and modifies in a non-monotonic pattern the profit of the supplier located in the low-demand market (central panel, figure 10; columns three and four, table 2).

When the transmission capacity is high enough ($T > 44$), the cost effect dominates and an increase in transmission capacity increases the lower bound of the support (left-hand panel, figure 10). An increase in the lower bound of the support entailed that both

Figure 10: Increase in transmission capacity $\triangle T$. Main variables

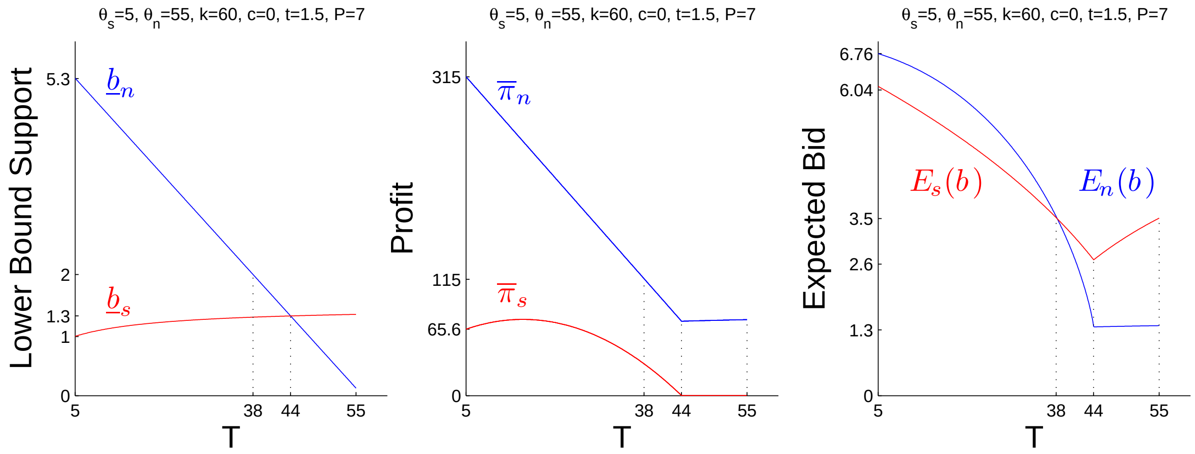


Table 2: Increase transmission capacity $\triangle T$. Main variables. ($\theta_s = 5$, $\theta_n = 55$, $k = 60$, $c = 0$, $t = 1.5$, $P = 7$)

T	$\underline{b}$	$\overline{\pi}_n$	$\overline{\pi}_s$	$E_n(b)$ Ana.	$E_n(b)$ Sim.	$E_s(b)$ Ana.	$E_s(b)$ Sim.
0	–	385.07	35	7	7	7	7
5	5.959	350.05	52.09	6.9079	6.9072	6.4495	6.4483
15	4.793	280.09	73.36	6.5206	6.5201	5.7472	5.7490
25	3.626	210.07	71.28	5.7253	5.7252	4.9294	4.9301
35	2.459	140.05	45.86	4.2942	4.2944	3.9306	3.9307
45	1.351	73.575	0	1.3569	1.3570	2.7299	2.7304
55	1.376	75.075	0	1.3821	1.3825	3.5073	3.5075

Here $E_n(b)$ Ana. and $E_s(b)$ Ana. constitute the expected values obtained using the analytical expressions presented in proposition one and $E_n(b)$ Sim. and $E_s(b)$ Sim. constitute the expected values obtained using the simulation explained in detail in Annex 3.

suppliers randomize submitting higher bids and therefore, the expected bid increases for both suppliers. Hence, an increase in transmission capacity is anti-competitive (right-hand panel, figure 10; columns five and seven, table 2). Finally, an increase in transmission capacity increases the expected profit of the supplier located in the high-demand market because it can exploit the efficiency rents more; in contrast, the expected profit of the supplier located in the low-demand market does not change because the increase in profits derived from an increase in the expected bid is compensated by the increase in transmission costs (central panel, figure 10; columns three and four, table 2).

5 Model comparison and consumer welfare.

As I have shown in the two previous sections, transmission constraints and transmission tariffs have important implications for strategies and equilibrium outcomes. In this section, I compare equilibrium outcome allocations and their effects on consumer welfare in the presence of transmission constraints when the transmission costs are zero and when the suppliers pay a linear transmission tariff. I also compare these results with the equi-

Table 3: Effect of transmission constraint and transmission costs on equilibrium outcome ($\theta_s = 5$, $\theta_n = 55$, $k = 60$, $c = 0$, $P = 7$)

	T	$\underline{b}$	$\overline{\pi}_n$	$\overline{\pi}_s$	$\overline{\pi} = \overline{\pi}_n + \overline{\pi}_s$	$E_n(b)$	$E_s(b)$	$\theta_n E_n(b) + \theta_s E_s(b)$
Model I	40	1.75	105	184	289	4.2	3.2	247
Model II	40	1.87	105	24	129	3.1	3.3	187
Model III	40	2.87	82.5	62	144.5	4.8	4	284

Model I: zero transmission costs. Model II: transmission tariff. Model III: point of connection tariff

librium in the presence of transmission constraints and the point of connection tariffs worked out in annex four.

When the transmission costs are zero and the transmission line is congested (Model I), the supplier located in the high-demand market faces a higher residual demand, while the supplier located in the low-demand market cannot sell its entire generation capacity (size effect). Therefore, the supplier located in the high-demand market has incentives to submit higher bids than the one located in the low-demand market. Given that the majority of consumers are located in the high-demand market, the aggregate cost for consumers is large (second row, column nine; table 3).

When transmission tariffs are implemented (Model II), the supplier located in the low-demand market faces a large increase in transmission costs and thus, its expected bid increases. In contrast, the supplier located in the high-demand market faces a small increase in transmission costs and for high enough transmission tariffs, it can be more profitable to extract the efficiency rents undercutting (in expectation) the supplier located in the low-demand market (cost effect). These changes in equilibrium prices induce a drastic decrease in the total cost that consumers pay for the purchase of electricity[13] (third row, column nine; table 3). Moreover, the presence of transmission costs induces a change in the flow of electricity.

When the suppliers face a point of connection tariff (Model III), they pay the same transmission tariff for the electricity sold in their own market and the one sold in the other market. Therefore, the competitive advantage (cost effect) derived from the location in the high-demand market disappears and equilibrium market outcomes exclusively depend on the size effect. Moreover, given that electricity demand is very inelastic, an increase in generation costs is passed through to consumers that face an increase in equilibrium prices in both markets. Hence, there is a decrease in consumer welfare (row four, column nine; table 3).

The comparison between the three models suggests that the introduction of transmission tariffs increases aggregate consumer welfare. In contrast, the point of connection tariffs always decrease aggregate consumer welfare.

[13] Downward et al. (2014) found that the introduction of a tax on suppliers' profit induces an increase in consumer welfare. However, in their analysis, the reduction in equilibrium prices is not induced by some type of cost effect, but by a change in firms' strategies to avoid the tax.

6 Conclusion

Electricity markets are moving through integration processes around the world. In such a context, there exists an intense debate to analyze the effect of transmission constraints and transmission costs on suppliers' strategies. The contribution of this paper is to characterize the outcome of an electricity market auction and how it depends on transmission constraints and transmission costs.

When there are constraints on the possibility to deliver electricity to a market, the effective size of the market differs for the suppliers. The supplier located in the high-demand market faces a higher residual demand and the one located in the low-demand market cannot sell its entire generation capacity. Therefore, the supplier located in the high-demand market has incentives to submit larger bids than the one located in the low-demand market (size effect). Hence, due to the scarcity of transmission capacity, the equilibrium is asymmetric even when the suppliers are symmetric in generation capacity and costs.

When the suppliers are charged a linear transmission tariff, they face different transmission costs depending on the realization of demand. The supplier located in the high-demand market faces lower transmission costs than the one located in the low-demand market and to exploit its efficiency rents, it has incentives to submit lower bids than the one located in the low-demand market (cost effect). Hence, the introduction of transmission tariffs could lower the bid of the supplier in the high-demand market and there are even cases where consumers would, on average, gain from the introduction of a transmission cost. Point of connection tariffs do not have the pro-competitive cost effect. This suggests that transmission tariffs would, in most cases, be better for market performance and consumers in comparison to point of connection tariffs.

An increase in transmission capacity induces non-monotonic changes in suppliers' profits. The consequences of an increase in transmission capacity depend considerably on whether there are any transmission costs. In the presence of transmission capacity constraints and zero transmission costs, an increase in transmission capacity is always pro-competitive. In the alternative scenario where suppliers pay a linear transmission tariff for the electricity sold in the other market, an increase in transmission capacity could be anti-competitive.

The characterization of the equilibrium in the presence of transmission constraints and transmission costs gives us the opportunity to use the toolbox of the models of competition with capacity constraints to best understand electricity markets. In particular, the model that I have developed in this paper can be used to analyze mergers between suppliers located in different markets and it can be used to analyze investment decisions in generation capacity at different points of the electricity grid.

The size and cost effects described in the paper could appear in models of competition with capacity constraints when the firms face asymmetries in capital and costs as in models presented in Kreps and Scheinkman (1983); Osborne and Pitchik (1986); Deneckere and Kovenock (1996) Fabra et al. (2006). Moreover, due to the size and cost effects, equilibrium firms' cumulative distribution functions do not dominate each other. This characteristic of the equilibrium has important implications for prices and consumer welfare. Our knowledge of these effects is still limited and thus, more study is required to best characterize equilibrium outcome allocations in the presence of some type of "size" and "cost" effects.

This basic model could also be useful to analyze new transmission tariff designs that include seasonal or geographical components.

Annex 1. The effect of transmission capacity constraints

Proposition 1. Characterization of the equilibrium in the presence of transmission constraints.

When demand is low (area A, figure 3): $b_n = b_s = c = 0$, the *equilibrium profit* is zero for both firms. No electricity flows through the grid.

When demand is intermediate (areas $A1$ and $B1$, figure 3) or high (area $B2$, figure 3). As I have proved in lemma one, a pure strategies equilibrium does not exist; however, the model presented in section two satisfies the properties established by Dasgupta and Maskin (1986) which guarantee that a mixed strategy equilibrium exists. In particular, the discontinuities of $\pi_i, \forall i, j$ are restricted to the strategies such that $b_i = b_j$. Furthermore, it is simple to confirm that by reducing its price from a position where $b_i = b_j$, a firm discontinuously increases its profit. Therefore, $\pi_i(b_i, b_j)$ is everywhere left lower semi-continuous in b_i and hence, weakly lower semi-continuous. Obviously, $\pi_i(b_i, b_j)$ is bounded. Finally, $\pi_i(b_i, b_j) + \pi_j(b_i, b_j)$ is continuous because discontinuous shifts in clientele from one firm to another only occur where both firms derive the same profit per customer. Therefore, theorem five in Dasgupta and Maskin (1986) applies and hence, a mixed strategy equilibrium exists.

The existence of the equilibrium is guaranteed by Dasgupta and Maskin (1986). However, they did not provide an algorithm to work out the equilibrium. Nevertheless, using the approach proposed by Karlin (1959), Shapley (1957), Shilony (1977), Varian (1980), Deneckere and Kovenock (1986), Osborne and Pitchik (1986) and Fabra et al. (2006), the equilibrium characterization is guaranteed by construction. I use the approach proposed by this branch of the literature to work out the mixed strategy equilibrium. In particular: first, I work out the general formulas of the *lower bound of the support*, the *cumulative distribution function*, the *probability distribution function*, the *expected equilibrium price* and the *expected profit*; second, I work out the particular formulas associated with each single area[14] in figure 3.

Lower Bound of the Support. The lower bound of the support is defined according to lemma two.

Cumulative Distribution Function.

In the first step, the payoff function for any firm is:

$$\begin{aligned}\pi_i(b) &= b\left[F_j(b)max\left\{0, \theta_i - T, \theta_i + \theta_j - k\right\} + (1 - F_j(b))min\left\{\theta_i + \theta_j, \theta_i + T, k\right\}\right] = \\ &= -bF_j(b)\left[min\left\{\theta_i + \theta_j, \theta_i + T, k\right\} - max\left\{0, \theta_i - T, \theta_i + \theta_j - k\right\}\right] + \\ &\quad bmin\left\{\theta_i + \theta_j, \theta_i + T, k\right\}\end{aligned} \tag{2}$$

In the second step, $\pi_i(b) = \overline{\pi}_i \forall b \in S_i, i = n, s$, where S_i is the support of the mixed strategies. Then,

[14]The general formulas that I will introduce below fully characterize the equilibrium. However, the equilibrium presents specific characteristics in each single area. In order to fully characterize the equilibrium, I have decided to write down the formulas for each single area.

$$\begin{aligned}
\overline{\pi}_i &= -bF_j(b)\left[min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}\right]+\\
&\quad bmin\left\{\theta_i+\theta_j,\theta_i+T,k\right\}\Rightarrow\\
F_j(b) &= \frac{bmin\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-\overline{\pi}_i}{b\left[min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}\right]} \qquad (3)
\end{aligned}$$

The third step, at $\underline{b}$, $F_i(\underline{b})=0\forall i=n,s$. Then,

$$\overline{\pi}_i = \underline{b}min\left\{\theta_i+\theta_j,\theta_i+T,k\right\} \qquad (4)$$

In the fourth step, plugging 4 into 3, I obtain the mixed strategies for both firms.

$$\begin{aligned}
F_j(b) &= \frac{bmin\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-\underline{b}min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}}{b\left[min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}\right]}=\\
&= \frac{min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}}{min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}}\frac{b-\underline{b}}{b} \quad \forall i=n,s \qquad (5)
\end{aligned}$$

For further reference:
$L_i(b)=bmin\left\{\theta_i+\theta_j,\theta_i+T,k\right\}$ and
$H_i(b)=bmax\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}$.

It is easy to verify that equation $F_j(b)\forall i,j$ is indeed a cumulative distribution function. First, in the third step, I have established that $F_j(\underline{b})=0$. Second, $F_j(b)$ is an increasing function in b. At $\underline{b}$, $L_i(\underline{b})=H_i(b)$, for any $b>\underline{b}$, $L_i(\underline{b})<H_i(b)$; moreover, $\frac{\partial L_i(b)}{\partial b}>0$, $\frac{\partial L_i(\underline{b})}{\partial b}=0$ and $\frac{\partial H_i(b)}{\partial b}>0$, therefore, $\frac{\partial\left(L_i(b)-L_i(\underline{b})\right)}{\partial b}>\frac{\partial\left(L_i(b)-H_i(b)\right)}{\partial b}$. Third, $F_j(b)\leq 1\forall b\in S_i$. Finally, $F_j(b)$ is continuous in the support because $L_i(b)-L_i(\underline{b})$ and $L_i(b)-H_i(b)$ are continuous functions in the support.

Probability Distribution Function.

$$\begin{aligned}
f_j(b) &= \frac{\partial F_j(b)}{\partial b}\\
&= \frac{min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}\underline{b}\left(min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}\right)}{b^2\left(min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}\right)^2}\\
&= \frac{min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}\underline{b}}{b^2\left(min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}\right)} \quad \forall i=n,s \qquad (6)
\end{aligned}$$

Expected Equilibrium Bid.

$$\begin{aligned}
E_j(b) &= \int_{\underline{b}}^{P} bf_j(b)\partial b \\
&= \int_{\underline{b}}^{P} \frac{bmin\left\{\theta_i+\theta_j,\theta_i+T,k\right\}\underline{b}}{b^2\left(min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}\right)}\partial b \\
&\quad +P\left(1-F_j(P)\right) \\
&= \frac{min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}\underline{b}}{min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}}\left[ln(b)\right]_{\underline{b}}^{P} \\
&\quad +P\left(1-F_j(P)\right)\ \ \forall i=n,s \qquad (7)
\end{aligned}$$

where $(1-F_j(P))$ in equation 7 is the probability assigned by firm j to the maximum price allowed by the auctioneer.[15]

Expected Profit. The expected profit is defined by equation 4 and is equal to $\overline{\pi}_i = \underline{b}min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}$.

In the rest of the proof, I will work out the *lower bound of the support*, the *cumulative distribution function*, the *probability distribution function*, the *expected equilibrium price* and the *expected profit* for the different possible realization of demands (θ_s,θ_n).

Area A1.

First, I work out the lower bound of the support on the border between areas $B1$ and $B2$, $\theta_s = k-T$. On the border, $\underline{b}_n$ solves $\underline{b}_n min\left\{\theta_n+\theta_s,\theta_n+T,k\right\} = Pmax\left\{0,\theta_n-T,\theta_s+\theta_n-k\right\}$, therefore $\underline{b}_n = \dfrac{P(\theta_n-T)}{k}$ and $\underline{b}_s$ solves $\underline{b}_s min\left\{\theta_n+\theta_s,\theta_s+T,k\right\} = Pmax\left\{0,\theta_s-T,\theta_s+\theta_n-k\right\}$, therefore $\underline{b}_s = \dfrac{P(\theta_n+\theta_s-k)}{\theta_s+T}$. Plugging the value of θ_s on the border between these areas into $\underline{b}_s$ formula, I obtain $\underline{b}_s = \dfrac{P(\theta_n+k-T-k)}{k-T+T} = \dfrac{P(\theta_n-T)}{k} = \underline{b}_n$. Therefore, on the border between these areas, $\underline{b}_s = \underline{b}_n = \dfrac{P(\theta_n-T)}{k}$.

In areas $A1$ and $B1$, $\underline{b}_n > \underline{b}_s$. In area $A1$, taking partial derivatives $\dfrac{\partial \underline{b}_n}{\partial\theta_s} = \dfrac{-P(\theta_n-T)}{(\theta_n+\theta_s)^2} < 0$ and $\dfrac{\partial \underline{b}_s}{\partial\theta_s} = \dfrac{P(k+T-\theta_n)}{(\theta_s+T)^2} > 0$. In area $B1$, taking partial derivatives $\dfrac{\partial \underline{b}_n}{\partial\theta_s} = 0$ and $\dfrac{\partial \underline{b}_s}{\partial\theta_s} = \dfrac{P(k+T-\theta_n)}{(\theta_s+T)^2} > 0$. Therefore, in areas $A1$ and $B1$, $\underline{b}_n > \underline{b}_s$. Hence, $S = \left[max\left\{\underline{b}_n,\underline{b}_s\right\},P\right] = \left[\underline{b}_n,P\right]$. In particular, in area $A1$, $S = \left[\dfrac{P(\theta_n-T)}{(\theta_n+\theta_s)},P\right]$ and

[15] When the transmission line is congested, the mixed strategy equilibrium is asymmetric. In such an equilibrium, the cumulative distribution function for the firm located in the low-demand market is continuous in the upper bound of the support. In contrast, the cumulative distribution function of the firm located in the high-demand region is discontinuous, which means that the firm located in the high-demand market submits the maximum bid allowed by the auctioneer with a positive probability $(1-F_j(P))$. Hence, in order to work out the expected value, in addition to the integral, it is necessary to add the term $P\left(1-F_j(P)\right)$. Figure 4 illustrates these characteristics.

in area $B1$, $S = \left[\frac{P(\theta_n - T)}{k}, P\right]$.

Second, I work out the cumulative distribution function.

$$F_s(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \frac{\theta_n + \theta_s}{\theta_s + T}\frac{b - \underline{b}}{b} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

$$F_n(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \frac{\theta_s + T}{\theta_s + T}\frac{b - \underline{b}}{b} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

Moreover,

$$\begin{aligned} F_s(P) &= \frac{\theta_n + \theta_s}{\theta_s + T}\frac{P - \frac{P(\theta_n - T)}{\theta_n + \theta_s}}{P} = 1 \\ F_n(P) &= \frac{\theta_s + T}{\theta_s + T}\frac{P - \frac{P(\theta_n - T)}{\theta_n + \theta_s}}{P} = \frac{(\theta_s + T)}{(\theta_n + \theta_s)} < 1 \end{aligned}$$

Third, the probability distribution function is equal to:

$$\begin{aligned} f_s(b) &= \frac{\partial F_s(b)}{\partial b} = \frac{\theta_n + \theta_s}{\theta_s + T}\frac{\underline{b}}{b^2} \\ f_n(b) &= \frac{\partial F_n(b)}{\partial b} = \frac{\theta_s + T}{\theta_s + T}\frac{\underline{b}}{b^2} \end{aligned}$$

Fourth, the expected bid is determined by:

$$\begin{aligned} E_s(b) &= \int_{\underline{b}}^{P} b f_s(b_s)\partial b = \int_{\underline{b}}^{P} \frac{\theta_n + \theta_s}{\theta_s + T}\frac{\underline{b}}{b}\partial b = \frac{\theta_n + \theta_s}{\theta_s + T}\underline{b}\left[ln(b)\right]_{\underline{b}}^{P} \\ E_n(b) &= \int_{\underline{b}}^{P} b f_n(b_n)\partial b = \int_{\underline{b}}^{P} \frac{\underline{b}}{b^2}\partial b = \frac{\theta_s + T}{\theta_s + T}\underline{b}\left[ln(b)\right]_{\underline{b}}^{P} + (1 - F_n(P))\, P \end{aligned}$$

Fifth, the expected profit is defined by equation 4 and is equal to $\overline{\pi}_n = \underline{b}(\theta_s + \theta_n)$ and $\overline{\pi}_s = \underline{b}(\theta_s + T)$.

Area $B1$.

First, the lower bound of the support is $S = \left[\frac{P(\theta_n - T)}{k}, P\right]$.

Second, I work out the cumulative distribution function.

$$F_s(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{k}{T+k-\theta_n}\dfrac{b-\underline{b}}{b} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

$$F_n(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{\theta_s+T}{T+k-\theta_n}\dfrac{b-\underline{b}}{b} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

Moreover,

$$\begin{aligned} F_s(P) &= \frac{k}{T+k-\theta_n}\frac{P-\dfrac{P(\theta_n-T)}{k}}{P} = 1 \\ F_n(P) &= \frac{\theta_s+T}{T+k-\theta_n}\frac{P-\dfrac{P(\theta_n-T)}{k}}{P} = \frac{\theta_s+T}{k} < 1 \end{aligned}$$

Third, the probability distribution function is equal to:

$$\begin{aligned} f_s(b) &= \frac{\partial F_s(b)}{\partial b} = \frac{k}{T+k-\theta_n}\frac{\underline{b}}{b^2} \\ f_n(b) &= \frac{\partial F_n(b)}{\partial b} = \frac{\theta_s+T}{T+k-\theta_n}\frac{\underline{b}}{b^2} \end{aligned}$$

Fourth, the expected bid is determined by:

$$\begin{aligned} E_s(b) &= \int_{\underline{b}}^{P} b f_s(b_s)\partial b = \int_{\underline{b}}^{P} \frac{k}{T+k-\theta_n}\frac{\underline{b}}{b}\partial b = \frac{k}{T+k-\theta_n}\underline{b}\left[ln(b)\right]_{\underline{b}}^{P} \\ E_n(b) &= \int_{\underline{b}}^{P} b f_n(b_n)\partial b = \int_{\underline{b}}^{P} \frac{\theta_s+T}{T+k-\theta_n}\frac{\underline{b}}{b}\partial b + \left(1-F_n(P)\right)P \\ &= \frac{\theta_s+T}{T+k-\theta_n}\underline{b}\left[ln(b)\right]_{\underline{b}}^{P} + \left(1-F_n(P)\right)P \end{aligned}$$

Fifth, the expected profit is defined by equation 4 and is equal to $\overline{\pi}_n = \underline{b}k$ and $\overline{\pi}_s = \underline{b}(\theta_s + T)$.

Area B2.

First, the lower bound of the support is $S = \left[max\left\{\underline{b}_n, \underline{b}_s\right\}, P\right] = \left[\dfrac{P(\theta_s+\theta_n-k)}{k}, P\right]$.

Second, I work out the cumulative distribution function.

$$F_i(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{k}{2k-\theta_i-\theta_j}\dfrac{b-\underline{b}}{b} & \text{if } b \in (\underline{b}, P) \quad \forall i = s, n \\ 1 & \text{if } b = P \end{cases}$$

Third, the probability distribution function is equal to:

$$f_i(b) = \frac{\partial F_i(b)}{\partial b} = \frac{k}{2k - \theta_i - \theta_j} \frac{\underline{b}}{b^2} \quad \forall i = s, n$$

Fourth, the expected bid is determined by:

$$E_i(b) = \int_{\underline{b}}^{P} b f_i(b_i) \partial b = \int_{\underline{b}}^{P} \frac{k}{2k - \theta_n - \theta_s} \frac{\underline{b}}{b} \partial b = \frac{k}{2k - \theta_n - \theta_s} \underline{b} \left[ln(b) \right]_{\underline{b}}^{P} \quad \forall i = s, n$$

Fifth, the expected profit is defined by equation 4 and is equal to $\overline{\pi}_n = \overline{\pi}_s = \underline{b}k$.

Proposition 2. The effect of an increase in transmission capacity.

Area A1.

$$\frac{\partial \underline{b}}{\partial T} = \frac{-P}{(\theta_s + \theta_n)} < 0$$

$$\frac{\partial F_n(P)}{\partial T} = \frac{1}{(\theta_s + \theta_n)} > 0$$

$$\begin{aligned}
\frac{\partial E_n(b)}{\partial T} &= \frac{\partial \underline{b}}{\partial T} \left[ln\left(\frac{P}{\underline{b}}\right) \right] + \underline{b} \left[\frac{\underline{b}}{P} \frac{-\frac{\partial \underline{b}}{\partial T} P}{\underline{b}^2} \right] - \frac{\partial F_n(P)}{\partial T} \\
&= \frac{\partial \underline{b}}{\partial T} \left[ln\left(\frac{P}{\underline{b}}\right) - 1 \right] - \frac{\partial F_n(P)}{\partial T} < 0 \Leftrightarrow ln\left(\frac{P}{\underline{b}}\right) > 1
\end{aligned}$$

$$\begin{aligned}
\frac{\partial E_s(b)}{\partial T} &= \frac{\partial \underline{b}}{\partial T} \frac{\theta_s + \theta_n}{\theta_s + T} \left[ln\left(\frac{P}{\underline{b}}\right) \right] - \underline{b} \frac{\theta_s + \theta_n}{(\theta_s + T)^2} \left[ln\left(\frac{P}{\underline{b}}\right) \right] + \underline{b} \frac{\theta_s + \theta_n}{\theta_s + T} \left[\frac{\underline{b}}{P} \frac{-\frac{\partial \underline{b}}{\partial T} P}{\underline{b}^2} \right] \\
&= \frac{\partial \underline{b}}{\partial T} \frac{\theta_s + \theta_n}{\theta_s + T} \left[ln\left(\frac{P}{\underline{b}}\right) - 1 \right] - \underline{b} \frac{\theta_s + \theta_n}{(\theta_s + T)^2} \left[ln\left(\frac{P}{\underline{b}}\right) \right] < 0 \Leftrightarrow ln\left(\frac{P}{\underline{b}}\right) > 1
\end{aligned}$$

$$\frac{\partial \overline{\pi}_n}{\partial T} = -P < 0$$

$$\frac{\partial \overline{\pi}_s}{\partial T} = \frac{-P}{(\theta_s + \theta_n)} (\theta_s + T) + \frac{P(\theta_n - T)}{(\theta_s + \theta_n)} = \frac{P(\theta_n - 2T - \theta_s)}{(\theta_s + \theta_n)} > 0 \Leftrightarrow \theta_n > 2T + \theta_s$$

Area B1.

$$\frac{\partial \underline{b}}{\partial T} = \frac{-P}{k} < 0$$

$$\frac{\partial F_n(P)}{\partial T} = \frac{1}{k} > 0$$

$$\begin{aligned}
\frac{\partial E_n(b)}{\partial T} &= \frac{\partial \underline{b}}{\partial T}\frac{\theta_s + T}{k+T-\theta_n}\left[ln\left(\frac{P}{\underline{b}}\right)\right] + \underline{b}\frac{k+T-\theta_n-\theta_s-T}{(k+T-\theta_n)^2}\left[ln\left(\frac{P}{\underline{b}}\right)\right] \\
&\quad + \underline{b}\frac{\theta_s + T}{k+T-\theta_n}\left[\frac{\underline{b}}{P}\frac{-\frac{\partial \underline{b}}{\partial T}P}{\underline{b}^2}\right] - \frac{\partial F_n(P)}{\partial T} \\
&= \frac{\partial \underline{b}}{\partial T}\frac{\theta_s + T}{k+T-\theta_n}\left[ln\left(\frac{P}{\underline{b}}\right) - 1\right] + \underline{b}\frac{k-\theta_s-\theta_n}{(k+T-\theta_n)^2}\left[ln\left(\frac{P}{\underline{b}}\right)\right] \\
&\quad - \frac{\partial F_n(P)}{\partial T} < 0 \Leftrightarrow ln\left(\frac{P}{\underline{b}}\right) > 1
\end{aligned}$$

$$\begin{aligned}
\frac{\partial E_s(b)}{\partial T} &= \frac{\partial \underline{b}}{\partial T}\frac{k}{k+T-\theta_n}\left[ln\left(\frac{P}{\underline{b}}\right)\right] - \underline{b}\frac{k}{(k+T-\theta_n)^2}\left[ln\left(\frac{P}{\underline{b}}\right)\right] \\
&\quad + \underline{b}\frac{k}{k+T-\theta_n}\left[\frac{\underline{b}}{P}\frac{-\frac{\partial \underline{b}}{\partial T}P}{\underline{b}^2}\right] \\
&= \frac{\partial \underline{b}}{\partial T}\frac{k}{k+T-\theta_n}\left[ln\left(\frac{P}{\underline{b}}\right) - 1\right] \\
&\quad - \underline{b}\frac{k}{(k+T-\theta_n)^2}\left[ln\left(\frac{P}{\underline{b}}\right)\right] < 0 \Leftrightarrow ln\left(\frac{P}{\underline{b}}\right) > 1
\end{aligned}$$

$$\frac{\partial \overline{\pi}_n}{\partial T} = -P < 0$$

$$\frac{\partial \overline{\pi}_s}{\partial T} = \frac{-P}{k}(\theta_s + T) + \frac{P(\theta_n - T)}{k} = \frac{P(\theta_n - 2T - \theta_s)}{k} > 0 \Leftrightarrow \theta_n > 2T + \theta_s$$

Annex 2. The effect of transmission capacity constraints and transmission losses

Proposition 3. Characterization of the equilibrium in the presence of transmission constraints and transmission costs.

For further reference:

$$
\begin{aligned}
H_i(\theta, P, T, t) &= max\{0, \theta_i - T, \theta_j + \theta_i - k\} \\
Ht_i(\theta, P, T, t) &= max\{0, \theta_j - k\} \\
L_i(\theta, P, T, t) &= min\{\theta_i + \theta_j, \theta_i + T, k\} \\
Lt_i(\theta, P, T, t) &= max\{0, min\{\theta_i, T, k - \theta_i\}\}
\end{aligned}
$$

I proceed as in proposition one: first, I work out the general formulas of the *lower bound of the support*, the *cumulative distribution function*, the *probability distribution function*, the *expected equilibrium price* and the *expected profit*; second, I work out the particular formulas associated with each single area in figure 7.

Lower Bound of the Support. The lower bound of the support is defined according to lemma four.

Cumulative Distribution Function.

In the first step, the payoff function for any firm is:

$$
\begin{aligned}
\pi_i(b) &= F_j(b)\left[b\left(H_i(\theta, P, T, t)\right) - t\left(Ht_i(\theta, P, T, t)\right)\right] + \\
&\quad (1 - F_j(b))\left[b\left(L_i(\theta, P, T, t)\right) - t\left(Lt_i(\theta, P, T, t)\right)\right] = \\
&= -F_j(b)\left[b\left(L_i(\theta, P, T, t)\right) - t\left(Lt_i(\theta, P, T, t)\right) - b\left(H_i(\theta, P, T, t)\right) + t\left(Ht_i(\theta, P, T, t)\right)\right] \\
&\quad b\left(L_i(\theta, P, T, t)\right) - t\left(Lt_i(\theta, P, T, t)\right) \qquad (8)
\end{aligned}
$$

In the second step, $\pi_i(b) = \overline{\pi}_i \forall b \in S_i, i = n, s$, where S_i is the support of the mixed strategy. Then,

$$
\begin{aligned}
&= -F_j(b)\left[b\left(L_i(\theta, P, T, t)\right) - t\left(Lt_i(\theta, P, T, t)\right) - b\left(H_i(\theta, P, T, t)\right) + t\left(Ht_i(\theta, P, T, t)\right)\right] \\
&\quad b\left(L_i(\theta, P, T, t)\right) - t\left(Lt_i(\theta, P, T, t)\right) \Rightarrow \\
F_j(b) &= \frac{b\left(L_i(\theta, P, T, t)\right) - t\left(Lt_i(\theta, P, T, t)\right) - \overline{\pi}_i}{b\left[L_i(\theta, P, T, t) - H_i(\theta, P, T, t)\right] - t\left[Lt_i(\theta, P, T, t) - Ht_i(\theta, P, T, t)\right]} \qquad (9)
\end{aligned}
$$

In the third step, at $\underline{b}$, $F_i(\underline{b}) = 0 \forall i = n, s$. Then,

$$
\overline{\pi}_i = b\left(L_i(\theta, P, T, t)\right) - t\left(Lt_i(\theta, P, T, t)\right) \qquad (10)
$$

Fourth step, plugging 10 into 9, I obtain the mixed strategies for both firms.

$$
\begin{aligned}
F_j(b) &= \frac{(b - \underline{b})L_i(\theta, P, T, t)}{b\left[L_i(\theta, P, T, t) - H_i(\theta, P, T, t)\right] - t\left[Lt_i(\theta, P, T, t) - Ht_i(\theta, P, T, t)\right]} = \\
&\quad \forall i = n, s \qquad (11)
\end{aligned}
$$

Probability Distribution Function.

$$
\begin{aligned}
f_j(b) &= \frac{\partial F_j(b)}{\partial b} \\
&= \frac{L_i(\cdot)\left[\underline{b}\left[L_i(\theta,P,T,t)-H_i(\theta,P,T,t)\right]-t\left[Lt_i(\theta,P,T,t)-Ht_i(\theta,P,T,t)\right]\right]}{\left[b\left[L_i(\theta,P,T,t)-H_i(\theta,P,T,t)\right]-t\left[Lt_i(\theta,P,T,t)-Ht_i(\theta,P,T,t)\right]\right]^2} \\
&\quad \forall i = n, s
\end{aligned} \tag{12}
$$

For further reference:

$$
\begin{aligned}
n(\cdot) &= L_i(\cdot)\left[\underline{b}\left[L_i(\theta,P,T,t)-H_i(\theta,P,T,t)\right]-t\left[Lt_i(\theta,P,T,t)-Ht_i(\theta,P,T,t)\right]\right] \\
d_1(\cdot) &= \left[L_i(\theta,P,T,t)-H_i(\theta,P,T,t)\right] \\
d_2(\cdot) &= \left[Lt_i(\theta,P,T,t)-Ht_i(\theta,P,T,t)\right]
\end{aligned}
$$

Expected Equilibrium Bid.

$$
\begin{aligned}
E_j(b) &= \int_{\underline{b}}^{P} b f_j(b)\partial b \\
&= \int_{\underline{b}}^{P} \frac{b\,(n(\cdot))}{\left[b\,(d_1(\cdot)) - t\,(d_2(\cdot))\right]^2}\partial b + P(1-F_j(P)) \;\; \forall i = n, s
\end{aligned}
$$

I solve this equation by substitution of variables. In particular:

$$
\begin{aligned}
U &= \left[b\,(d_1(\cdot)) - t\,(d_2(\cdot))\right] \Rightarrow b = \frac{U + t\,(d_2(\cdot))}{d_1(\cdot)} \\
\frac{\partial U}{\partial b} &= d_1 \Rightarrow \partial b = \frac{\partial U}{\partial d_1}
\end{aligned}
$$

Therefore:

$$
\begin{aligned}
E_j(b) &= \int_{\underline{b}}^{P} \frac{\left(\frac{U + t\,(d_2(\cdot))}{d_1(\cdot)}\right) n(\cdot)}{U^2} \frac{\partial U}{d_1(\cdot)} + P(1-F_j(P)) \\
&= \frac{n(\cdot)}{d_1(\cdot)}\left[\int_{\underline{b}}^{P} \frac{U \partial U}{U^2} + \int_{\underline{b}}^{P} \frac{t\,(d_2(\cdot))\,\partial U}{U^2}\right] + P(1-F_j(P)) \\
&= \frac{n(\cdot)}{d_1(\cdot)^2}\left[ln(U) - \frac{t\,(d_2(\cdot))}{U}\right]_{\underline{b}}^{P} + P(1-F_j(P))
\end{aligned}
$$

Substituting again:

$$
\begin{aligned}
E_j(b) &= \frac{n(\cdot)}{d_1(\cdot)^2} \\
&\quad \left[ln\left(\frac{P\,(d_1(\cdot)) - t\,(d_2(\cdot))}{\underline{b}\,(d_1(\cdot)) - t\,(d_2(\cdot))}\right) - \frac{t\,(d_2(\cdot))}{P\,(d_1(\cdot)) - t\,(d_2(\cdot))} + \frac{t\,(d_2(\cdot))}{\underline{b}\,(d_1(\cdot)) - t\,(d_2(\cdot))}\right] \\
&\quad + P(1-F_j(P))
\end{aligned} \tag{13}
$$

In the rest of the proof, I will work out the *lower bound of the support*, the *cumulative distribution function*, the *probability distribution function*, the *expected equilibrium price* and the *expected profit* for the different possible realizations of demands (θ_s, θ_n) (figure 7).

Area A1.

First, the lower bound of the support is:

$$\begin{aligned} \underline{b}_n\theta_n + \underline{b}_n\theta_s - t\theta_s = P(\theta_n - T) \Rightarrow \underline{b}_n &= \frac{P(\theta_n - T) + t\theta_s}{\theta_n + \theta_s} \\ \underline{b}_s\theta_s + \underline{b}_s T - tT = 0 \Rightarrow \underline{b}_s &= \frac{tT}{\theta_s + T} \end{aligned}$$

Second, I work out the cumulative distribution function.

$$F_s(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{(b - \underline{b})(\theta_n + \theta_s)}{b\left[(\theta_s + \theta_n) - (\theta_n - T)\right] - tmin\left\{\theta_s, k - \theta_n\right\}} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

$$F_n(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{(b - \underline{b})(\theta_s + T)}{b(\theta_s + T) - tT} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

Moreover,

$$\begin{aligned} \text{If } \underline{b}_n \geq \underline{b}_s \Rightarrow F_s(P) &= 1 \\ F_n(P) &= \frac{(P(\theta_s + T) - t\theta_s)(\theta_s + T)}{(P(\theta_s + T) - tT)(\theta_s + \theta_n)} \\ \text{If } \underline{b}_n < \underline{b}_s \Rightarrow F_s(P) &= \frac{(P(\theta_s + T) - tT)(\theta_s + \theta_n)}{(P(\theta_s + T) - t\theta_s)(\theta_s + T)} \\ F_n(P) &= 1 \end{aligned}$$

Third, the probability distribution function is equal to:

$$f_s(b) = \frac{\partial F_s(b)}{\partial b} = \frac{(\theta_n + \theta_s)(\underline{b}(\theta_s + T) - t\theta_s)}{(b(\theta_s + T) - t\theta_s)^2}$$

$$f_n(b) = \frac{\partial F_n(b)}{\partial b} = \frac{(\theta_s + T)(\underline{b}(\theta_s + T) - tT)}{(b(\theta_s + T) - tT)^2}$$

Fourth, the expected bid is determined by:

$$
\begin{aligned}
E_s(b) &= \int_{\underline{b}}^{P} b f_s(b_s)\partial b = \int_{\underline{b}}^{P} b\frac{(\theta_n+\theta_s)(\underline{b}(\theta_s+T)-t\theta_s)}{(b(\theta_s+T)-t\theta_s)^2} + (1-F_s(P))P \\
&= \frac{(\theta_n+\theta_s)(\underline{b}(\theta_s+T)-t\theta_s)}{(\theta_s+T)^2} \\
&\quad \left[ln\left(\frac{P(\theta_s+T)-t\theta_s}{\underline{b}(\theta_s+T)-t\theta_s}\right) - \frac{t\theta_s}{P(\theta_s+T)-t\theta_s} + \frac{t\theta_s}{\underline{b}(\theta_s+T)-t\theta_s}\right] \\
&\quad +(1-F_s(P))P \\
E_n(b) &= \int_{\underline{b}}^{P} b f_n(b_s)\partial b = \int_{\underline{b}}^{P} b\frac{(\theta_s+T)(\underline{b}(\theta_s+T)-tT)}{(b(\theta_s+T)-tT)^2} + (1-F_n(P))P \\
&= \frac{(\underline{b}(\theta_s+T)-tT)}{(\theta_s+T)} \\
&\quad \left[ln\left(\frac{P(\theta_s+T)-tT}{\underline{b}(\theta_s+T)-tT}\right) - \frac{tT}{P(\theta_s+T)-tT} + \frac{tT}{\underline{b}(\theta_s+T)-tT}\right] \\
&\quad +(1-F_n(P))P
\end{aligned}
\tag{14}
$$

In equation 14, I have solved by substituting variables:

$$
\begin{aligned}
U &= b(\theta_s+T)-t\theta_s \Rightarrow b = \frac{U+t\theta_s}{\theta_s+T} \\
\frac{\partial U}{\partial b} &= \theta_s+T \Rightarrow \partial b = \frac{\partial U}{\theta_s+T} \\
\text{and} & \\
U &= b(\theta_s+T)-tT \Rightarrow b = \frac{U+tT}{\theta_s+T} \\
\frac{\partial U}{\partial b} &= \theta_s+T \Rightarrow \partial b = \frac{\partial U}{\theta_s+T}
\end{aligned}
$$

Fifth, the expected profit is defined by equation 10 and is equal to $\overline{\pi}_n = \underline{b}(\theta_s+\theta_n)-t\theta_s$ and $\overline{\pi}_s = \underline{b}(\theta_s+T)-tT$.

Area B1a.

First, the lower bound of the support is:

$$
\begin{aligned}
\underline{b}_n\theta_n + \underline{b}_n(k-\theta_n) - t(k-\theta_n) = P(\theta_n-T) \Rightarrow \underline{b}_n &= \frac{P(\theta_n-T)+t(k-\theta_n)}{k} \\
\underline{b}_s\theta_s + \underline{b}_sT - tT = P(\theta_s+\theta_n-k) \Rightarrow \underline{b}_s &= \frac{P(\theta_s+\theta_n-k)+tT}{\theta_s+T}
\end{aligned}
$$

Second, I work out the cumulative distribution function.

$$F_s(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{(b-\underline{b})k}{b(k+T-\theta_n) - t(k-\theta_n)} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

$$F_n(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{(b-\underline{b})(\theta_s+T)}{b(k+T-\theta_n) - tT} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

Moreover,

$$\begin{aligned} \text{If } \underline{b}_n \geq \underline{b}_s \Rightarrow F_s(P) &= 1 \\ F_n(P) &= \frac{(P(k+T-\theta_n) - t(k-\theta_n))(\theta_s+T)}{(P(k+T-\theta_n) - tT)k} \\ \text{If } \underline{b}_n < \underline{b}_s \Rightarrow F_s(P) &= \frac{(P(k+T-\theta_n) - tT)k}{(P(k+T-\theta_n) - t(k-\theta_n))(\theta_s+T)} \\ F_n(P) &= 1 \end{aligned}$$

Third, the probability distribution function is equal to:

$$f_s(b) = \frac{\partial F_s(b)}{\partial b} = \frac{k(\underline{b}(k+T-\theta_n) - t(k-\theta_n))}{(b(k+T-\theta_n) - t(k-\theta_n))^2}$$
$$f_n(b) = \frac{\partial F_n(b)}{\partial b} = \frac{(\theta_s+T)(\underline{b}(\theta_s+T) - tT)}{(b(\theta_s+T) - tT)^2}$$

Fourth, the expected bid is determined by:

$$\begin{aligned} E_s(b) &= \int_{\underline{b}}^{P} b f_s(b_s)\partial b = \int_{\underline{b}}^{P} b\frac{k(\underline{b}(k+T-\theta_n) - t(k-\theta_n))}{(b(k+T-\theta_n) - t(k-\theta_n))^2} + (1 - F_s(P))P \\ &= \frac{k(\underline{b}(k+T-\theta_n) - t(k-\theta_n))}{(k+T-\theta_n)^2} \\ &\quad \left[ln\left(\frac{P(k+T-\theta_n) - t(k-\theta_n)}{\underline{b}(k+T-\theta_n) - t(k-\theta_n)}\right)\right] \\ &\quad \left[-\frac{t(k-\theta_n)}{P(k+T-\theta_n) - t(k-\theta_n)} + \frac{t(k-\theta_n)}{\underline{b}(k+T-\theta_n) - t(k-\theta_n)}\right] \\ &\quad +(1 - F_s(P))P \\ E_n(b) &= \int_{\underline{b}}^{P} b f_n(b_s)\partial b = \int_{\underline{b}}^{P} b\frac{(\theta_s+T)(\underline{b}(k+T-\theta_n) - tT)}{(b(k+T-\theta_n) - tT)^2} + (1 - F_n(P))P \\ &= \frac{(\theta_s+T)(\underline{b}(k+T-\theta_n) - tT)}{(k+T-\theta_n)^2} \\ &\quad \left[ln\left(\frac{P(k+T-\theta_n) - tT}{\underline{b}(k+T-\theta_n) - tT}\right) - \frac{tT}{P(k+T-\theta_n) - tT} + \frac{tT}{\underline{b}(k+T-\theta_n) - tT}\right] \\ &\quad +(1 - F_n(P))P \end{aligned} \tag{15}$$

In equations 15, I have solved by substituting variables:

$$\begin{aligned}
U &= b(k+T-\theta_n)-t(k-\theta_n) \Rightarrow b = \frac{U+t(k-\theta_n)}{k+T-\theta_n} \\
\frac{\partial U}{\partial b} &= k+T-\theta_n \Rightarrow \partial b = \frac{\partial U}{k+T-\theta_n} \\
\text{and} & \\
U &= b(k+T-\theta_n)-tT \Rightarrow b = \frac{U+tT}{k+T-\theta_n} \\
\frac{\partial U}{\partial b} &= k+T-\theta_n \Rightarrow \partial b = \frac{\partial U}{k+T-\theta_n}
\end{aligned}$$

Fifth, the expected profit is defined by equation 10 and is equal to $\overline{\pi}_n = \underline{b}k - t(k-\theta_n)$ and $\overline{\pi}_s = \underline{b}(\theta_s+T) - tT$.

Area B1b.

First, the lower bound of the support is:

$$\begin{aligned}
\underline{b}_n k = P(\theta_n - T) \Rightarrow \underline{b}_n &= \frac{P(\theta_n-T)}{k} \\
\underline{b}_s\theta_s + \underline{b}_s T - tT = P(\theta_s+\theta_n-k) - t(\theta_n-k) \Rightarrow \underline{b}_s &= \frac{P(\theta_s+\theta_n-k)+t(k+T-\theta_n)}{\theta_s+T}
\end{aligned}$$

Second, I work out the cumulative distribution function.

$$F_s(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{(b-\underline{b})k}{b(k+T-\theta_n)} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

$$F_n(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{(b-\underline{b})(\theta_s+T)}{b(k+T-\theta_n)-t(T+k-\theta_n)} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

Moreover,

$$\begin{aligned}
\text{If } \underline{b}_n \geq \underline{b}_s \Rightarrow F_s(P) &= 1 \\
F_n(P) &= \frac{P(k+T-\theta_n)(\theta_s+T)}{(P-t)(k+T-\theta_n)k} \\
\text{If } \underline{b}_n < \underline{b}_s \Rightarrow F_s(P) &= \frac{(P-t)(k+T-\theta_n)k}{P(k+T-\theta_n)(\theta_s+T)} \\
F_n(P) &= 1
\end{aligned}$$

Third, the probability distribution function is equal to:

$$f_s(b) = \frac{\partial F_s(b)}{\partial b} = \frac{\underline{b}k}{b^2(k+T-\theta_n)}$$
$$f_n(b) = \frac{\partial F_n(b)}{\partial b} = \frac{(\underline{b}-t)(\theta_s+T)}{(b-t)^2(k+T-\theta_n)}$$

Fourth, the expected bid is determined by:

$$\begin{aligned}
E_s(b) &= \int_{\underline{b}}^{P} b f_s(b_s)\partial b = \int_{\underline{b}}^{P} b\frac{\underline{b}k}{b^2(k+T-\theta_n)} + (1-F_s(P))P \\
&= \frac{\underline{b}k}{(k+T-\theta_n)}\left[ln\left(\frac{P}{\underline{b}}\right)\right] + (1-F_s(P))P \\
E_n(b) &= \int_{\underline{b}}^{P} b f_n(b_s)\partial b = \int_{\underline{b}}^{P} b\frac{(\underline{b}-t)(\theta_s+T)}{(b-t)^2(k+T-\theta_n)} + (1-F_n(P))P \\
&= \frac{(\underline{b}-t)(\theta_s+T)}{(k+T-\theta_n)}\left[ln\left(\frac{P-t}{\underline{b}-t}\right) - \frac{t}{P-t} + \frac{t}{\underline{b}-t}\right] + (1-F_n(P))P \qquad (16)
\end{aligned}$$

In equations 16, I have solved by substituting variables:

$$\begin{aligned}
U &= b-t \Rightarrow b = U+t \\
\frac{\partial U}{\partial b} &= 1 \Rightarrow \partial b = \partial U
\end{aligned}$$

Fifth, the expected profit is defined by equation 10 and is equal to $\overline{\pi}_n = \underline{b}k$ and $\overline{\pi}_s = \underline{b}(\theta_s+T) - tT$.

Area B2a.

First, the lower bound of the support is:

$$\begin{aligned}
\underline{b}_n\theta_n + \underline{b}_n(k-\theta_n) - t(k-\theta_n) = P(\theta_s+\theta_n-k) \Rightarrow \underline{b}_n &= \frac{P(\theta_s+\theta_n-k)+t(k-\theta_n)}{k} \\
\underline{b}_s\theta_s + \underline{b}_s(k-\theta_s) - t(k-\theta_s) = P(\theta_s+\theta_n-k) \Rightarrow \underline{b}_s &= \frac{P(\theta_s+\theta_n-k)+t(k-\theta_s)}{k}
\end{aligned}$$

Second, I work out the cumulative distribution function.

$$F_s(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{(b-\underline{b})k}{b(2k-\theta_n-\theta_s)-t(k-\theta_n)} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$
$$F_n(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{(b-\underline{b})k}{b(2k-\theta_n-\theta_s)-t(k-\theta_s)} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

Moreover,

$$\begin{aligned}
F_s(P) &= \frac{P(2k-\theta_n-\theta_s)-t(k-\theta_s)}{P(2k-\theta_n-\theta_s)-t(k-\theta_n)} \\
F_n(P) &= 1
\end{aligned}$$

Third, the probability distribution is equal to:

$$\begin{aligned}
f_s(b) = \frac{\partial F_s(b)}{\partial b} &= \frac{k(\underline{b}(2k-\theta_n-\theta_s)-t(k-\theta_n))}{(b(2k-\theta_n-\theta_s)-t(k-\theta_n))^2} \\
f_n(b) = \frac{\partial F_n(b)}{\partial b} &= \frac{k(\underline{b}(2k-\theta_n-\theta_s)-t(k-\theta_s))}{(b(2k-\theta_n-\theta_s)-t(k-\theta_s))^2}
\end{aligned}$$

Fourth, the expected bid is determined by:

$$\begin{aligned}
E_s(b) &= \int_{\underline{b}}^{P} bf_s(b_s)\partial b = \int_{\underline{b}}^{P} b\frac{k(\underline{b}(2k-\theta_n-\theta_s)-t(k-\theta_n))}{(b(2k-\theta_n-\theta_s)-t(k-\theta_n))^2} + (1-F_s(P))P \\
&= \frac{k(\underline{b}(2k-\theta_n-\theta_s)-t(k-\theta_n))}{(b(2k-\theta_n-\theta_s)-t(k-\theta_n))^2} \\
&\quad \left[ln\left(\frac{P(2k-\theta_n-\theta_s)-t(k-\theta_n)}{\underline{b}(2k-\theta_n-\theta_s)-t(k-\theta_n)}\right)\right] \\
&\quad \left[-\frac{t(k-\theta_n)}{P(k+T-\theta_n)-t(k-\theta_n)} + \frac{t(k-\theta_n)}{\underline{b}(2k-\theta_n-\theta_s)-t(k-\theta_n)}\right] \\
&\quad +(1-F_s(P))P \\
E_n(b) &= \int_{\underline{b}}^{P} bf_n(b_s)\partial b = \int_{\underline{b}}^{P} b\frac{k(\underline{b}(2k-\theta_n-\theta_s)-t(k-\theta_s))}{(b(2k-\theta_n-\theta_s)-t(k-\theta_s))^2} + (1-F_n(P))P \\
&= \frac{k(\underline{b}(2k-\theta_n-\theta_s)-t(k-\theta_s))}{(b(2k-\theta_n-\theta_s)-t(k-\theta_s))^2} \\
&\quad \left[ln\left(\frac{P(2k-\theta_n-\theta_s)-t(k-\theta_s)}{\underline{b}(2k-\theta_n-\theta_s)-t(k-\theta_s)}\right)\right] \\
&\quad \left[-\frac{t(k-\theta_s)}{P(k+T-\theta_n)-t(k-\theta_s)} + \frac{t(k-\theta_s)}{\underline{b}(2k-\theta_n-\theta_s)-t(k-\theta_s)}\right] \\
&\quad +(1-F_n(P))P
\end{aligned} \tag{17}$$

where in equation 17, I have solved by substituting variables:

$$\begin{aligned}
U &= b(2k-\theta_n-\theta_s)-t(k-\theta_n) \Rightarrow b = \frac{U+t(k-\theta_n)}{2k-\theta_n-\theta_s} \\
\frac{\partial U}{\partial b} &= 2k-\theta_n-\theta_s \Rightarrow \partial b = \frac{\partial U}{2k-\theta_n-\theta_s} \\
&\text{and} \\
U &= b(2k-\theta_n-\theta_s)-t(k-\theta_s) \Rightarrow b = \frac{U+t(k-\theta_s)}{2k-\theta_n-\theta_s} \\
\frac{\partial U}{\partial b} &= 2k-\theta_n-\theta_s \Rightarrow \partial b = \frac{\partial U}{2k-\theta_n-\theta_s}
\end{aligned}$$

Fifth, the expected profit is defined by equation 10 and is equal to $\overline{\pi}_n = \underline{b}k - t(k - \theta_n)$ and $\overline{\pi}_s = \underline{b}k - t(k - \theta_s)$.

Area B2b.

First, the lower bound of the support is:

$$\begin{aligned} \underline{b}_n k = P(\theta_s + \theta_n - k) \Rightarrow \underline{b}_n &= \frac{P(\theta_s + \theta_n - k)}{k} \\ \underline{b}_s \theta_s + \underline{b}_s (k - \theta_s) - t(k - \theta_s) = & \\ P(\theta_s + \theta_n - k) - t(\theta_n - k) \Rightarrow \underline{b}_s &= \frac{P(\theta_s + \theta_n - k) + t(2k - \theta_n - \theta_s)}{k} \end{aligned}$$

Second, I work out the cumulative distribution function.

$$F_s(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{(b - \underline{b})k}{b(2k - \theta_n - \theta_s)} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

$$F_n(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{(b - \underline{b})k}{(b - t)(2k - \theta_n - \theta_s)} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

Moreover,

$$\begin{aligned} F_s(P) &= \frac{P(2k - \theta_n - \theta_s) - t(2k - \theta_n - \theta_s)}{P(2k - \theta_n - \theta_s)} \\ F_n(P) &= 1 \end{aligned}$$

Third, the probability distribution function is equal to:

$$\begin{aligned} f_s(b) = \frac{\partial F_s(b)}{\partial b} &= \frac{\underline{b}k}{b^2(2k - \theta_n - \theta_s)} \\ f_n(b) = \frac{\partial F_n(b)}{\partial b} &= \frac{(\underline{b} - t)k}{(b - t)^2(2k - \theta_n - \theta_s)} \end{aligned}$$

Fourth, the expected bid is determined by:

$$\begin{aligned} E_s(b) &= \int_{\underline{b}}^{P} b f_s(b_s) \partial b = \int_{\underline{b}}^{P} b \frac{\underline{b}k}{b^2(2k - \theta_n - \theta_s)} + (1 - F_s(P))P \\ &= \frac{\underline{b}k}{(2k - \theta_n - \theta_s)} \left[ln \left(\frac{P}{\underline{b}} \right) \right] + (1 - F_s(P))P \\ E_n(b) &= \int_{\underline{b}}^{P} b f_n(b_s) \partial b = \int_{\underline{b}}^{P} b \frac{(\underline{b} - t)k}{(b - t)^2(2k - \theta_n - \theta_s)} + (1 - F_n(P))P \\ &= \frac{(\underline{b} - t)k}{(2k - \theta_n - \theta_s)} \left[ln \left(\frac{P - t}{\underline{b} - t} \right) - \frac{t}{P - t} + \frac{t}{\underline{b} - t} \right] + (1 - F_n(P))P \qquad (18) \end{aligned}$$

where in equations 18, I have solved by substituting variables:

$$\begin{aligned} U &= b - t \Rightarrow b = U + t \\ \frac{\partial U}{\partial b} &= 1 \Rightarrow \partial b = \partial U \end{aligned}$$

Fifth, the expected profit is defined by equation 10 and is equal to $\overline{\pi}_n = \underline{b}k$ and $\overline{\pi}_s = \underline{b}k - t(k - \theta_s)$.

Proposition 4. Effect of an increase in transmission capacity.

In the presence of transmission capacity constraints and transmission costs, the "size" and "cost" mechanisms determine the equilibrium. These two mechanisms work in opposite directions which has important implications for equilibrium outcome allocations. Hence, an increase in transmission capacity modifies the relevant model variables (lower bound of the support, expected bids and expected profits) in a non-monotonic pattern. Therefore, no clear conclusions can be obtained through the analysis of the partial derivatives.

In this section, I present the static comparative in order to illustrate the difficulties to obtain a formal analysis from the analytical solutions. I present the results for area $A1$, the analysis is the same for the rest of the areas.

Area $A1$.

$$\begin{aligned} \frac{\partial \underline{b}_n}{\partial T} &= \frac{-P}{(\theta_s + \theta_n)} < 0 \\ \frac{\partial \underline{b}_s}{\partial T} &= \frac{t(\theta_s + T) - tT}{(\theta_s + T)^2} = \frac{t\theta_s}{(\theta_s + T)^2} > 0 \end{aligned}$$

$$\begin{aligned} \frac{\partial F_n(P)}{\partial T} = {} & \frac{(2P(\theta_s + T) - t\theta_s)\left((P(\theta_s + T) - tT)(\theta_n + \theta_s)\right)}{\left((P(\theta_s + T) - tT)(\theta_n + \theta_s)\right)^2} + \\ & \frac{t(\theta_n + \theta_s)(P(\theta_s + T) - t\theta_s)(\theta_s + T)}{\left((P(\theta_s + T) - tT)(\theta_n + \theta_s)\right)^2} > 0 \end{aligned}$$

$$
\begin{aligned}
\frac{\partial E_n(b)}{\partial T} &= \frac{\frac{\partial \underline{b}}{\partial T}(\theta_s+T)+(\underline{b}-t)(\theta_s+T)-\underline{b}(\theta_s+T)+tT}{(\theta_s+T)^2} \\
&\left[ln\left(\frac{P(\theta_s+T)-tT}{\underline{b}(\theta_s+T)-tT}\right)-\frac{tT}{P(\theta_s+T)-tT}+\frac{tT}{\underline{b}(\theta_s+T)-tT}\right]+ \\
&\frac{\underline{b}(\theta_s+T)-tT}{\theta_s+T} \\
&\left[\frac{\underline{b}(\theta_s+T)-tT}{P(\theta_s+T)-tT}\right] \\
&\left[\frac{(P-t)(\underline{b}(\theta_s+T)-tT)-\left(\frac{\partial \underline{b}}{\partial T}(\theta_s+T)+\underline{b}-t\right)(P(\theta_s+T)-tT)}{(\underline{b}(\theta_s+T)-tT)^2}\right]+ \\
&\frac{\underline{b}(\theta_s+T)-tT}{\theta_s+T}\left[-\frac{t(P(\theta_s+T)-tT)-(P-t)tT}{(P(\theta_s+T)-tT)^2}\right]+ \\
&\frac{\underline{b}(\theta_s+T)-tT}{\theta_s+T}\left[\frac{t(\underline{b}(\theta_s+T)-tT)-\left(\frac{\partial \underline{b}}{\partial T}(\theta_s+T)+\underline{b}-t\right)tT}{(\underline{b}(\theta_s+T)-tT)^2}\right]
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E_s(b)}{\partial T} &= \frac{\frac{\partial \underline{b}}{\partial T}(\theta_s+T)^3(\theta_s+\theta_n)+\underline{b}(\theta_n+\theta_s)(\theta_s+T)^2-2(\theta_s+T)\left[(\theta_s+\theta_n)(\underline{b}(\theta_s+T)-t\theta_s)\right]}{(\theta_s+T)^4} \\
&\left[ln\left(\frac{P(\theta_s+T)-t\theta_s}{\underline{b}(\theta_s+T)-t\theta_s}\right)-\frac{t\theta_s}{P(\theta_s+T)-t\theta_s}+\frac{t\theta_s}{\underline{b}(\theta_s+T)-t\theta_s}\right]+ \\
&\frac{(\theta_n+\theta_s)(\underline{b}(\theta_s+T)-tT)}{(\theta_s+T)^2} \\
&\left[\frac{(\underline{b}(\theta_s+T)-t\theta_s)}{P(\theta_s+T)-t\theta_s}\right] \\
&\left[\frac{P(\underline{b}(\theta_s+T)-t\theta_s)-\left(\frac{\partial \underline{b}}{\partial T}(\theta_s+T)+\underline{b}\right)(P(\theta_s+T)-t\theta_s)}{(\underline{b}(\theta_s+T)-t\theta_s)^2}\right]+ \\
&\frac{(\theta_n+\theta_s)(\underline{b}(\theta_s+T)-t\theta_s)}{(\theta_s+T)^2}\left[-\frac{Pt\theta_s}{(P(\theta_s+T)-t\theta_s)^2}\right]+ \\
&\frac{(\theta_n+\theta_s)(\underline{b}(\theta_s+T)-t\theta_s)}{\theta_s+T}\left[\frac{-\underline{b}t\theta_s-\left(\frac{\partial \underline{b}}{\partial T}(\theta_s+T)t\theta_s\right)}{(\underline{b}(\theta_s+T)-t\theta_s)^2}\right]
\end{aligned}
$$

$$
\frac{\partial \pi_n}{\partial T} = -P < 0
$$

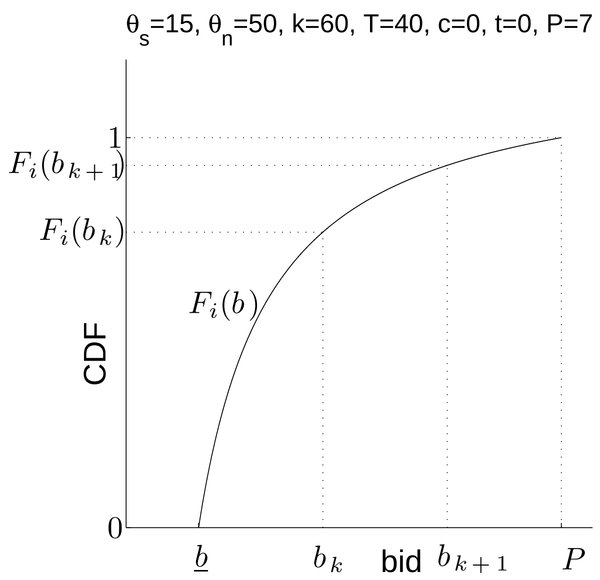
$$\frac{\partial \overline{\pi}_s}{\partial T} = \frac{-P}{(\theta_s+\theta_n)}(\theta_s+T) + \frac{P(\theta_n - T) + t\theta_s}{(\theta_s+\theta_n)} - t$$
$$= \frac{P(\theta_n - 2T - \theta_s) - t\theta_n}{(\theta_s+\theta_n)}$$

Annex 3. Expected equilibrium price: Simulation

Propositions one and three fully characterize the equilibrium. However, due to the complexity of calculations and to ensure that I did not make any algebra mistake, I work out the expected bid for both firms using the algorithm presented in this annex. The algorithm is based on the cumulative distribution function that is the mixed strategies equilibrium from which the rest of the variables of the model are derived.

As can be observed in tables 1 and 2, the differences between the expected bid using the analytical formulas from propositions one and three and using the algorithm proposed here are almost null.[16]

Figure 11: Expected bid. Simulation.



Algorithm: (figure 11)

1. I split the support of the mixed strategies equilibrium into K grid values (where K is a large number e.g., 5000 or 10000). I call each of these values $b_i(k)$ $\forall i = s, n$.

2. For each $b_i(k)$, I work out $F_i(b_i(k))$ using the formulas obtained in propositions one and three.

[16] I have applied this algorithm to work out the expected value for any realization of demand (all areas) and I have compared this with the analytical values and the results are almost identical.

3. The probability assigned to $p_i(b_i(k))$ equals the difference in the cumulative distribution function between two consecutive values $F_i(b_i(k+1)) - F_i(b_i(k))$. Therefore, $p(b_i(k)) = F_i(b_i(k+1)) - F_i(b_i(k))$. It is important to remark that one observation is lost during the process to work out the probabilities.

4. The expected value is the sum of each single bid multiplied by its probability: $E_i(b) = \sum_{k=0}^{K-1} b_i(k) p_i(b_i(k)) \ \forall i = s, n$

Annex 4. Characterization of the Nash Equilibrium when firms pay a point of connection tariff

In this paper, I assume that suppliers face transmission constraints and they are charged by a linear transmission tariff for the electricity sold in the other market. Under this assumption, I show that suppliers' strategies are affected by the "size" and the "cost" mechanisms that work in the opposite direction and determine equilibrium outcome allocations. However, when suppliers face transmission constraints and they are charged on basis of the total electricity that they inject in the grid (point of connection tariff), the suppliers pay the same transmission tariff for the electricity sold in their own market and the one sold in the other market. Therefore, the competitive advantage (cost effect) derived from the location in the high-demand market disappears and equilibrium market outcomes exclusively depend on the size effect. Moreover, given that electricity demand is very inelastic, an increase in generation costs is passed through to consumers that face an increase in equilibrium prices in both markets. This result is in line with the pass through literature (Marion and Muehlegger 2011; Fabra and Reguant 2014). Hence, a change in the design of transmission tariffs from the one used in the majority of the countries to the one proposed in this article could induce a large improvement in consumer welfare.

The general formulas of the *lower bound of the support*, the *cumulative distribution function*, the *probability distribution function*, the *expected equilibrium price* and the *expected profit* can be worked out using the same approach as that in annexes one and two. In this annex, I only work out the particular formulas associated with each single area (figure 3). Once that I characterize the equilibrium, I analyze the effect of an increase in transmission capacity on the main variables of the model. Finally, I compare the equilibrium outcome of the three model specifications: transmission constraints and *zero* transmission costs (model I); transmission constraints and *positive* transmission cost for the electricity sold in the other market (model II) and finally transmission constraints and *positive* transmission cost for the entire generation capacity (model III).

Area A1.

First, I work out the lower bound of the support. Using the same approach as in annex one, it is straightforward to show that in areas $A1$ and $B1$, $\underline{b}_n > \underline{b}_s$. Hence, $S = [max\{\underline{b}_n, \underline{b}_s\}, P] = [\underline{b}_n, P]$. Therefore, it is enough to work out $\underline{b}_n$. $\underline{b}_n$ can be derived from the next equation $(\underline{b}_n - t)(\theta_n + \theta_s) = (P - t)(\theta_n - T)$. Therefore, in area $A1$, $S = \left[t + \dfrac{(P-t)(\theta_n - T)}{(\theta_n + \theta_s)}, P\right]$.

Second, I work out the cumulative distribution function.

$$F_s(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \frac{\theta_n + \theta_s}{\theta_s + T}\frac{b - \underline{b}}{b - t} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

$$F_n(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \frac{\theta_s + T}{\theta_s + T}\frac{b - \underline{b}}{b - t} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

Moreover,

$$\begin{aligned} F_s(P) &= \frac{\theta_n + \theta_s}{\theta_s + T}\frac{P - t - \frac{(P-t)(\theta_n - T)}{\theta_n + \theta_s}}{P - t} = 1 \\ F_n(P) &= \frac{\theta_s + T}{\theta_s + T}\frac{P - t - \frac{P - t(\theta_n - T)}{\theta_n + \theta_s}}{P - t} = \frac{(\theta_s + T)}{(\theta_n + \theta_s)} < 1 \end{aligned}$$

Third, the probability distribution function is equal to:

$$\begin{aligned} f_s(b) &= \frac{\partial F_s(b)}{\partial b} = \frac{\theta_n + \theta_s}{\theta_s + T}\frac{\underline{b} - t}{(b-t)^2} \\ f_n(b) &= \frac{\partial F_n(b)}{\partial b} = \frac{\theta_s + T}{\theta_s + T}\frac{\underline{b} - t}{(b-t)^2} \end{aligned}$$

Fourth, the expected bid is determined by:

$$\begin{aligned} E_s(b) &= \int_{\underline{b}}^{P} bf_s(b_s)\partial b = \int_{\underline{b}}^{P} b\frac{\theta_n + \theta_s}{\theta_s + T}\frac{(\underline{b} - t)}{(b-t)^2}\partial b = \\ &\quad \frac{\theta_n + \theta_s}{\theta_s + T}(\underline{b} - t)\left[ln\left(\frac{P - t}{\underline{b} - t}\right) - \frac{t}{P - t} + \frac{t}{\underline{b} - t}\right] \\ E_n(b) &= \int_{\underline{b}}^{P} bf_n(b_n)\partial b = \int_{\underline{b}}^{P} b\frac{\underline{b} - t}{(b-t)^2}\partial b + (1 - F_n(P))\,P = \\ &\quad (\underline{b} - t)\left[ln\left(\frac{P - t}{\underline{b} - t}\right) - \frac{t}{P - t} + \frac{t}{\underline{b} - t}\right] + (1 - F_n(P))\,P \qquad (19) \end{aligned}$$

In equation 19, I have solved by substituting variables:

$$\begin{aligned} U &= b - t \Rightarrow b = U + t \\ \frac{\partial U}{\partial b} &= 1 \Rightarrow \partial b = \partial U \end{aligned}$$

Fifth, the expected profit is defined by $\overline{\pi}_n = (\underline{b} - t)(\theta_s + \theta_n)$ and $\overline{\pi}_s = (\underline{b} - t)(\theta_s + T)$.

*Area B*1.

First, the lower bound of the support is $S = \left[t + \frac{(P-t)(\theta_n - T)}{k}, P\right]$.

Second, I work out the cumulative distribution function.

$$F_s(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \frac{k}{T+k-\theta_n}\frac{b-\underline{b}}{b-t} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

$$F_n(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \frac{\theta_s + T}{T+k-\theta_n}\frac{b-\underline{b}}{b-t} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

Moreover,

$$\begin{aligned} F_s(P) &= \frac{k}{T+k-\theta_n}\frac{(P-t) - \frac{(P-t)(\theta_n - T)}{k}}{P-t} = 1 \\ F_n(P) &= \frac{\theta_s + T}{T+k-\theta_n}\frac{(P-t) - \frac{(P-t)(\theta_n - T)}{k}}{P-t} = \frac{\theta_s + T}{k} < 1 \end{aligned}$$

Third, the probability distribution function is equal to:

$$\begin{aligned} f_s(b) &= \frac{\partial F_s(b)}{\partial b} = \frac{k}{T+k-\theta_n}\frac{\underline{b}-t}{(b-t)^2} \\ f_n(b) &= \frac{\partial F_n(b)}{\partial b} = \frac{\theta_s + T}{T+k-\theta_n}\frac{\underline{b}-t}{(b-t)^2} \end{aligned}$$

Fourth, the expected bid is determined by:

$$\begin{aligned} E_s(b) &= \int_{\underline{b}}^{P} b f_s(b_s)\partial b = \int_{\underline{b}}^{P} b\frac{k}{T+k-\theta_n}\frac{\underline{b}-t}{(b-t)^2}\partial b = \\ &\quad \frac{k}{T+k-\theta_n}(\underline{b}-t)\left[ln\left(\frac{P-t}{\underline{b}-t}\right) - \frac{t}{P-t} + \frac{t}{\underline{b}-t}\right] \\ E_n(b) &= \int_{\underline{b}}^{P} b f_n(b_n)\partial b = \int_{\underline{b}}^{P} b\frac{\theta_s + T}{T+k-\theta_n}\frac{\underline{b}-t}{(b-t)^2}\partial b + (1 - F_n(P))\,P \\ &= \frac{\theta_s + T}{T+k-\theta_n}(\underline{b}-t)\left[ln\left(\frac{P-t}{\underline{b}-t}\right) - \frac{t}{P-t} + \frac{t}{\underline{b}-t}\right] + (1 - F_n(P))\,P \quad (20) \end{aligned}$$

In equation 20, I have solved by substituting the variables:

$$\begin{aligned} U &= b - t \Rightarrow b = U + t \\ \frac{\partial U}{\partial b} &= 1 \Rightarrow \partial b = \partial U \end{aligned}$$

Table 4: Effect of transmission constraint and transmission costs on equilibrium outcome ($\theta_s = 5$, $\theta_n = 55$, $k = 60$, $c = 0$, $P = 7$)

	T	$\underline{b}$	$\overline{\pi}_n$	$\overline{\pi}_s$	$\overline{\pi}$	$E_n(b)$	$E_s(b)$	$\theta_n E_n(b) + \theta_s E_s(b)$
Model I	40	1.75	105	184	289	4.2	3.2	247
Model II	40	1.87	105	24	129	3.1	3.3	187
Model III	40	2.87	82.5	62	144.5	4.8	4	284

Fifth, the expected profit is defined by $\overline{\pi}_n = (\underline{b} - t)k$ and $\overline{\pi}_s = (\underline{b} - t)(\theta_s + T)$.

Area B2.

First, the lower bound of the support is $S = [max\left\{\underline{b}_n, \underline{b}_s\right\}, P] = \left[t + \frac{(P-t)(\theta_s + \theta_n - k)}{k}, P\right]$.

Second, I work out the cumulative distribution function.

$$F_i(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{k}{2k - \theta_i - \theta_j} \dfrac{b - \underline{b}}{b - t} & \text{if } b \in (\underline{b}, P) \quad \forall i = s, n \\ 1 & \text{if } b = P \end{cases}$$

Third, the probability distribution function is equal to:

$$f_i(b) = \frac{\partial F_i(b)}{\partial b} = \frac{k}{2k - \theta_i - \theta_j} \frac{\underline{b} - t}{(b-t)^2} \quad \forall i = s, n$$

Fourth, the expected bid is determined by:

$$\begin{aligned} E_i(b) &= \int_{\underline{b}}^{P} b f_i(b_i) \partial b = \int_{\underline{b}}^{P} b \frac{k}{2k - \theta_n - \theta_s} \frac{\underline{b} - t}{(b-t)^2} \partial b = \\ & \frac{k}{2k - \theta_n - \theta_s} (\underline{b} - t) \left[ln\left(\frac{P - t}{\underline{b} - t}\right) - \frac{t}{P - t} + \frac{t}{\underline{b} - t}\right] + (1 - F_n(P))\, P \end{aligned} \quad (21)$$

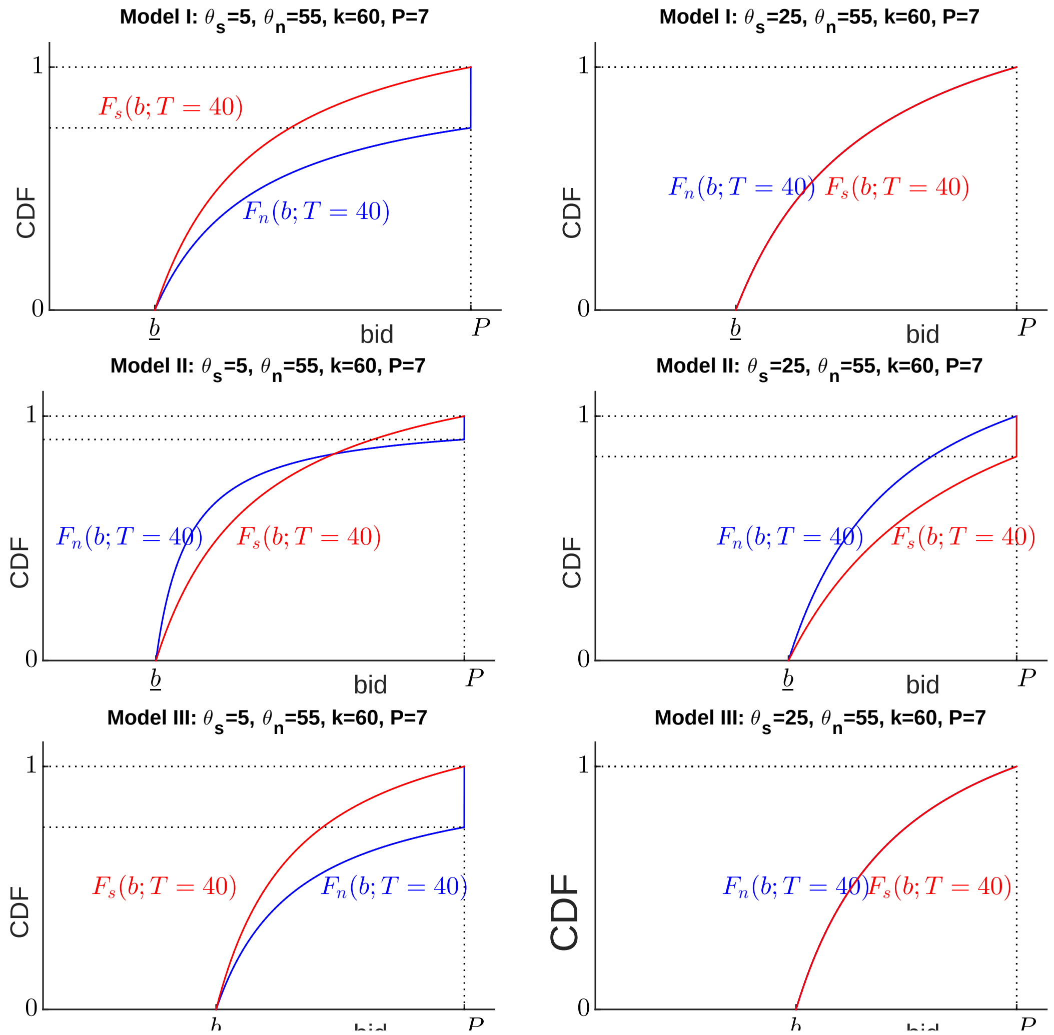
In equation 21, I have solved by substituting variables:

$$\begin{aligned} U &= b - t \Rightarrow b = U + t \\ \frac{\partial U}{\partial b} &= 1 \Rightarrow \partial b = \partial U \end{aligned}$$

Fifth, the expected profit is defined by $\overline{\pi}_n = \overline{\pi}_s = (\underline{b} - t)k$.

It is straightforward to show that an increase in transmission capacity induces the same changes in equilibrium outcome as when the transmission costs are *zero* (proposition two).

Figure 12: Cumulative Distribution Functions of models I, II and III.



Model comparison

In the last part of the annex, I compare the equilibrium outcome of the three different model specifications: transmission constraints and *zero* transmission costs (model I); transmission constraints and *positive* transmission costs for the electricity sold in the other market (model II) and, finally, transmission constraints and *positive* transmission costs for the entire generation capacity (model III).

The tree different model specifications affect suppliers' strategies in very different ways as can be observed in figure 12. The diversity of strategies induces important changes on the most relevant variables of the model (table 4).

I have discussed the three models in detail in section four (pages. 17-18). I refer the reader to those pages to follow the analysis.

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