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by  
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for Labor Market Policy Analysis

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# EMPIRICAL WAGE DISTRIBUTIONS: A NEW FRAMEWORK FOR LABOR MARKET POLICY ANALYSIS

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The object is to specify a labor market model of search where the behavior of firms is taken into account. Within the context of the model developed, it is shown there is a unique market equilibrium. At such an equilibrium distribution of wage offers is non-degenerate. A major objective is to show how this equilibrium wage distribution changes as either market parameters or policy parameters are changed. A new and potentially fruitful approach to policy analysis results.

## 1. INTRODUCTION

Although search theory is now a central feature of labor economics, its major weakness is well-known; it only considers the supply side of a labor market, holding constant (or, less politely, ignoring) the demand side of the story. In spite of this, the predictions of search theory have become the focus of much empirical work (see, for example, Kiefer and Neumann (1979), Nickell (1979), Flinn and Heckman (1982), Lancaster and Chesher (1983) and Devine and Kiefer (1988)). The purpose of this study is to question if the predictions of job search theory hold in a labor market model of search where both sides of the market are taken into account? It will be shown in the labor market model of search developed here, where both sides of the market are taken into account, a much richer body of predictions accrue some of which are in sharp contrast to those previously presented in job search theory.

There have been a few previous studies which have considered a market approach to search behavior (see, for example, Diamond (1971), Axell (1977), Butters (1977), Wilde and Schwartz (1979), Reinganum (1979), Burdett and Judd (1983), and Albrecht and Axell (1984)). These studies, however, have typically only considered the conditions required for the existence of an equilibrium non-degenerate distribution of wages (or prices, if a consumer market was considered). Unlike previous work in the area, the model presented here has a unique non-degenerate equilibrium distribution of wages which can be fully described. More particularly, it is shown how this distribution depends on the market and policy parameters. Testable predictions and a new approach to policy analysis are consequences.

The market structure used here can be briefly described as follows. At any given time each employed worker is attached to one of the firms and repeatedly sells labor services to it. From time to time workers (both employed and unemployed) receive wage offers. Unlike previous studies it is assumed that offers arrive at a fixed rate which cannot be influenced by worker behavior. An unemployed worker accepts any wage offer greater than the income flow from unemployment, whereas if an employed

worker receives a wage offer greater than that currently faced, he or she changes employer. This process continues until, at some uncertain date, a worker leaves the market for good.

Although the life of each worker is of uncertain duration, the total number of workers is fixed. Workers leaving the market are instantaneously replaced by new ones. These new workers, however, are initially unemployed. Given the wage offers of firms remain constant through time, the flow of workers from firm to firm (and in and out of the market) can be calculated. Further, as time passes such flows will settle down to their steady-state levels (where the flow of workers into each firm equals the outflow). Each firm is assumed to choose its own wage offer to maximize its steady-state profit flow subject to its beliefs about the offers of other firms. At an equilibrium the beliefs of firms are confirmed.

There exists a unique market equilibrium in the model developed. A non-degenerate distribution of wages is a characteristic of this equilibrium. Indeed, as it is shown how the particular form of this distribution (and other equilibrium characteristics) depend on the parameters, the equilibrium consequences of a change in any parameter can be investigated in detail. To illustrate the suitability of the model for policy analysis, particular attention is paid to the consequences of changes in unemployment insurance payments and changes in a legal minimum wage.

Within the context of a standard search model, an increase in unemployment insurance (UI) payments is predicted to (a) increase the duration of unemployment, and (b) increase the expected acceptable wage. These conclusions follow as an increase in UI payments leads to an increase in an unemployed worker's reservation wage, given the distribution of wages is held constant throughout. In the model developed in this study an increase in UI payments induces firms to increase their offers such that, after adjusting for the change, an unemployed worker's probability of accepting an offer remains the same as before the change. Thus, a change in UI payments is predicted not to change the expected duration of unemployment although it is expected to increase the expected acceptable wage offer. As firms make greater offers on average after an increase in UI payments, such a change is predicted to lower the profit rate of firms.

Standard job search theory typically has little to say about changes in a legal minimum wage; the analysis of such a change is usually presented within the context of a competitive labor market. Within such a framework, a legal minimum wage is either irrelevant or causes unemployment if it is above the market equilibrium wage. When such an analysis is performed within the framework developed here significantly different issues become the focus of attention. Will, for example, an increase in a minimum wage increase or decrease the matching rate of workers and firms? At first blush, one would expect an increase in the minimum wage to have an ambiguous effect on the matching rate as it increases the desire of unemployed workers to contact firms but reduces the desire of firms to hire workers. Within the context of the model developed in the present study, unemployed workers cannot increase the rate at which they contact firms by assumption. Further, each firm makes a strictly positive profit from each worker it employs at an equilibrium. Thus, as long as the increase in the minimum wage does not make this profit per employed worker negative, firms will employ the same amount of workers after the change. These results imply a change in the legal minimum wage will not change the unemployment rate. Nevertheless, will be shown that an increase in a minimum wage changes significantly the distribution of wages and the profit rate of firms.

It should be noted that the model used has been deliberately kept as simple as possible. The objective is to develop a new framework and with this goal in mind some realism has been sacrificed.

## 2. THE MODEL

Suppose a large number of homogeneous workers and a large number of essentially identical firms participate in a labor market;  $n$  denotes the measure of participating workers and  $m$  the measure of firms. Although the number of workers is fixed there is turnover. Specifically,  $\delta h$  indicates the probability any worker leaves the market for good in small time interval  $h$ . Any worker who leaves the market for good is instantly replaced by a new one. On the other hand, firms last forever. Although firm behavior is considered in detail later, two facts about them are assumed to hold when analyzing worker behavior. First, suppose firms do not change the wage rates they offer through time. Second, different firms may offer different wage rates. These assumptions guarantee the distribution of wages offered by firms at any particular time,  $F$ , is well specified.

On entering this labor market a worker is initially unemployed. As time passes, however, job offers are received. An offer is accepted if and only if the wage offered is at least as great as income flow when unemployed,  $y$  (which includes the dollar value of his or her leisure flow and any unemployment compensation the worker receives). As all workers reject any offer less than  $y$ , assume all firms offer a wage at least as great as  $y$ .

At a moment in time each employed worker is matched with one (and only one) of the firms in the market. While matched with a firm offering wage rate  $w$  a worker repeatedly sells his or her labor services at the rate of  $s(\cdot)$  units per instant. Hence,  $s(\cdot)$  describes how a worker's work intensity changes with the wage rate changes. Assume  $s(\cdot)$  is a bounded differentiable function such that  $s(w) > 0$  and  $s'(w) \geq 0$  for all  $w \geq y$ .

Employed as well as unemployed workers receive job offers. Assume the arrival rate of new job offers to any given individual (whether employed or unemployed) can be characterized by a Poisson process with parameter  $\lambda$ . Thus,  $\lambda h$  denotes the probability any given worker receives a job offer in small period of time  $h$ . It should be stressed that workers cannot influence this arrival rate. When an employed worker is offered a job at a higher wage rate than that currently faced, he or she changes employer and becomes matched with the other firm. Of course, an employed who receives an offer no greater than that currently faced will ignore it. This process continues until the worker leaves the market for good.

If a worker receives a job offer, which firm makes the offer? In what follows it is assumed a worker is equally likely to be offered a job by any firm in the market, i.e., a job offer is envisaged as a random draw from the distribution of wage offers. Hence, given a worker has obtained a job offer,  $F(w)$  denotes the probability this worker learns the wage offer of a firm who is offering wage no greater than  $w$ . For obvious reasons, random matching technology is said to hold when this restriction is satisfied. For a discussion of other possible matching technologies and their consequences see Burdett and Vishwanath (1988).

Given the arrival rate of offers faced by each worker and the random matching technology described above, the flows of workers between states can be calculated. First, consider the flow of workers in and out of unemployment. At time  $t$  the change in the unemployment stock can be defined as

$$(1) \quad \partial U(t)/\partial t = n\delta - \lambda U(t) - \delta U(t)$$

The flow of workers into and out of the market is  $\delta n$ , by assumption. As all workers

are initially unemployed,  $\delta n$  is the flow of new workers into unemployment. There are two ways to leave unemployment; by getting a job, or by leaving the market altogether. The flow of unemployed workers who get a job at time  $t$  is indicated by  $\lambda U(t)$ , whereas the flow of unemployed out of the market altogether is denoted by  $\delta U(t)$ . Obviously, the only steady-state level of unemployment,  $U^*$  is given by

$$(2) \quad U^* = n/[1 + k]$$

where  $k = \lambda/\delta$ . The flow of employed workers between firms is now considered in the special case where the unemployment level is held constant at  $U^*$ . It is straightforward to consider more general dynamics but little is gained.

For any particular distribution of wages offered,  $F$ , the distribution of wages paid to employed workers at any time  $t$  can be calculated. For a particular initial allocation of workers to firms, let  $G(w|t, F, U^*)$  denote the probability a randomly chosen employed worker at time  $t$  is currently matched with a firm whose wage is no greater than  $w$ , where  $F$  indicates the distribution of wages offered and  $U^*$  denotes the constant number of unemployed specified in (2). The number of workers receiving a wage greater than  $w$  at time  $t$  is  $[1 - G(\cdot|t, F, U^*)][nk/(1+k)]$ , i.e., the fraction of employed workers receiving a wage greater than  $w$  multiplied by the total number of employed workers. The time derivative of this number can be expressed as

$$(3) \quad \begin{aligned} \frac{\partial [1 - G(w|t, F, U^*)]}{\partial t} [nk/(1+k)] &= n\lambda[1 - F(w)]/[1+k] \\ &+ \lambda[1 - F(w)]G(w|t, F, U^*)[nk/(1+k)] - \delta[1 - G(w|t, F, U^*)][nk/(1+k)] \end{aligned}$$

for any  $w$  on the support of  $F$ . The term  $n\lambda[1 - F(w)]/[1+k]$  specifies the flow of unemployed workers into firms offering a wage greater than  $w$ , whereas  $\delta[1 - G(w|t, F, U^*)][nk/(1+k)]$  is the flow of workers attached to these firms who leave the market for good at time  $t$ . The second term on the right hand side of (3),  $\lambda[1 - F(w)]G(w|t, F, U^*)[nk/(1+k)]$ , indicates the flow of workers at time  $t$  from firms offering a wage no greater than  $w$  into firms offering a wage greater than  $w$ .

As time passes the distribution of wages paid to employed workers will converge to its steady-state. Let  $G(w|F, U^*) = \lim_{t \rightarrow \infty} G(w|t, F, U^*)$  as  $t \rightarrow \infty$  denote the steady-state distribution of wages paid to employed workers. Manipulating (3) it can be seen that this steady-state distribution can be written as

$$(4) \quad G(w|F, U^*) = \frac{F(w)}{[1 + k(1 - F(w))]}$$

for any  $w$  on the support of  $F$  and for any particular initial allocation of workers to firms. Thus, (4) presents the steady-state distribution of wages paid to employed workers given the distribution of wages offered is indicated by  $F$ . Inspection of (4) establishes that  $G$  has the same support as  $F$ , is increasing where  $F$  is increasing, and is discontinuous at any point where  $F$  is discontinuous. Indeed,  $G$  first-order stochastically dominates  $F$  as  $F(w) - G(w|F, U^*) \geq 0$  for all  $w$ . The steady-state fraction of employed workers matched with the firms offering a wage in the interval  $w \in (w, w + \epsilon]$ , given  $F$ , can be written as

$$(5) \quad G(w+\epsilon|F,U^*) - G(w|F,U^*) = \frac{(1+k)[F(w+\epsilon) - F(w)]}{[1+k(1-F(w+\epsilon))][1+k(1-F(w))]}$$

As distribution functions are right continuous, (5) can be used to specify the steady-state number of employed workers per firm offering wage  $w$ ,  $n(w|F)$ , given the distribution of wage offers,  $F$ . In particular, for any  $w$  in the support of  $F$

$$(6) \quad n(w|F) = \lim_{\epsilon \rightarrow \infty} \frac{[G(w+\epsilon|F,U^*) - G(w|F,U^*)] (nk/(1+k))}{\{F(w+\epsilon) - F(w)\}m}$$

$$= \frac{bk}{[1+k(1-F(w))]^2}$$

where  $F$  is continuous and  $b = n/m$ . Furthermore, it follows directly from (4) that

$$(7) \quad n(w\#|F) = \frac{bk}{[1+k(1-F(w\#-))][1+k(1-F(w\#))]}$$

where  $w\#$  is a mass point of  $F$  such that  $F(w\#) = F(w\#-) + \nu(w\#)$ , where  $F(w\#-)$  is the limit of  $F(w\#-e)$  as  $e$  approaches zero from above and  $\nu(w\#)$  is the mass at  $w\#$ . Above the steady-state number of workers per firm was shown to be well defined for all  $w$  on the support of  $F$ . Nevertheless, as shown below, firms will be interested the steady-state number of matched workers per firm at any wage  $w$ ; even those wages not on the support of the distribution of wage offers,  $F$ . To construct  $n(\cdot|F)$  for those wages not on the support of  $F$  assume firms have expectations that are self-fulfilling, i.e., if a firm has correct expectations about the distribution of wage offers,  $F$ , then its belief about  $n(\cdot|F)$  will also be correct.

Suppose a firm contemplates offering a wage less than any other firm (with probability one), i.e., where  $F(w) = 0$ . From (5) and (6) (or (5) and (7) if the infimum of the support is a mass point) it follows that such a firm will have  $bk/[1+k][1+k]$  attached workers in a steady-state. This is, of course, the number of workers attached to a firm in a steady-state when any of its employees leave after receiving another offer. Hence,

$$(8a) \quad n(w|F) = bk/[1+k]^2,$$

for any  $w$  such that  $F(w) = 0$ . By similar reasoning, it can be shown that a firm offering a wage greater than any other in the market will have  $bk$  attached workers in a steady-state, i.e.,

$$(8b) \quad n(w|F) = bk,$$

for any  $w$  such that  $F(w-e) = 1$  for some  $e > 0$ . Note that (8b) describes the number of workers attached to a firm whose job offers are accepted by any participating worker who receive one. Finally, it is straightforward to show a firm will expect the number of its attached workers in a steady-state not to change on any interval off the support of  $F$ , i.e.  $n(\cdot|F)$  is a constant on any open interval not on the support of  $F$ . The following claim summarizes the above analysis and thus no proof is given.

### PROPOSITION 1

For any given  $F$ ,  $n(\cdot|F)$  is increasing in  $w$ . In particular,  $n(\cdot|F)$  is such that

- (a) it is continuous and strictly increasing in  $w$  where  $F$  is continuous and strictly increasing;
- (b) it is discontinuous where  $F$  is discontinuous,
- (c) it is equal to constant on any open interval off the support of  $F$ ;
- (d) it satisfies the equations of (8).

It should be stressed that the above results about steady-state behavior depend to a large extent on the information structure and the matching technology. Very few restrictions have yet been imposed on firm behavior. For example, a firm charging the highest price in the market will have at most  $bk$  attached workers in a steady-state. Whether it desires this number depends on the restrictions imposed on the profit function of such firms.

### 3. STEADY-STATE EQUILIBRIUM

We now turn to the profit flows of firms. For simplicity, suppose each firm only uses labor in production. In particular, assume each firm's production function is linear such that its flow of output equals the number currently employed multiplied by the labor services supplied by each worker per instant. Thus, a firm's profit flow per employed worker when offering wage rate  $w$ ,  $\pi(w)$ , can be written as

$$(9) \quad \pi(w) = s(w)[p - w],$$

where  $p$  indicates the output price faced by each firm in the market. The restrictions imposed on  $s(\cdot)$  in the previous section imply  $\pi(\cdot)$  is a bounded differentiable function of  $w$  such that

$$> 0, \text{ if } w \in [y, p)$$

$$(10) \quad \pi(w) =$$

$$= 0, \text{ if } w = p$$



Even though  $s'(\cdot) \geq 0$  by assumption, the sign of  $\pi'(\cdot)$  is indeterminate as

$$(11) \quad \pi'(w) \begin{matrix} > \\ < \end{matrix} 0 \text{ as } s'(w)[p-w] \begin{matrix} > \\ < \end{matrix} s(w)$$

The situation where  $\pi'(w) > 0$  for some  $w$  is, of course, the focus of attention in the prototype efficiency wage models. In such an interval a small increase in a firm's wage results in such a large increase in work intensity of the attached workers that the firm increases its steady-state profit flow per worker. Such situations will not be ruled out in what follows. Furthermore, the wage offers of firms may be constrained by a legal minimum wage, MW. If MW is less than the utility flow of leisure, then obviously it will play no role. Let

$$x = \max\{y, MW\}$$

and assume all firms make wage offers at least as great as  $x$ . To rule out the trivial, assume the given output price faced by firms,  $p$ , is strictly greater than  $x$ .

As the focus of attention is on steady-state behavior, assume each firm maximizes its steady-state profit flow subject to its beliefs about the wages offered by other firms, i.e., its beliefs about  $F$ . Note that a firm's steady-state profit flow when charging wage  $w$  ( $x \leq w < p$ ) is strictly positive as both  $n(w|F)$  and  $\pi(w)$  are strictly positive in this interval. A **steady-state equilibrium** (sse) is defined by  $(F^*, v^*, U^*)$  such that

- (a)  $\pi(w)n(w|F^*) = v^*$ , if  $w$  is on the support of  $F^*$ , and
- (b)  $\pi(w)n(w|F^*) \leq v^*$ , otherwise.
- (c)  $U^* = n/(1+k)$

Thus, at a sse firms maximize their steady-state profit flows and the beliefs of firms are self-fulfilling.

To establish the existence of a unique sse four claims are first stated. These claims present a characterization of any sse, if one exists.

#### CLAIM 1

If a sse exists, the distribution of wage offers,  $F^*$ , is continuous.

#### PROOF

Suppose a firm chooses a wage  $w\#$ , where  $w\#$  is a mass point of  $F$ . If  $w\# < p$ , then by offering a wage slightly greater than  $w\#$  the firm will change its profits per matched worker hardly at all, as  $\pi(\cdot)$  is continuous, but gain a significant number of matched workers, as  $n(\cdot|F)$  is discontinuous at  $w\#$ . Hence, there exists an  $\epsilon > 0$  such that the expected profit at  $w\# + \epsilon$  is strictly greater than at  $w\#$  (note this implication is independent of the sign of  $\pi'(w\#)$ ). If  $w\# = p$ , then  $\pi'(w\#) < 0$  as  $\pi(p) = 0$  and  $\pi(w) > 0$  when  $x \leq w < p$ . Thus, if  $w\# = p$ , a firm offering  $w\#$  can increase its profit flow above zero by reducing its wage. The above implies a sse cannot involve a

discontinuous wage offer distribution which completes the proof.

CLAIM 2

If a sse exists, the rate of profit,  $v^*$ , is defined by

$$(12) \quad v^* = \frac{\pi(w(1))kb}{(1+k)^2} = \frac{\pi(w(2))kb}{[1+k(1-F(w))]} = \frac{\pi(w)kb}{[1+k(1-F(w))]}$$

for all  $w$  in the support of  $F^*$  (let  $S(F^*)$  denote the support of  $F^*$ ), where  $w(1) = \inf S(F^*)$  and  $w(2) = \sup S(F^*)$ .

PROOF

As any sse distribution of wages offered is continuous, it follows from Proposition 1 that the number of workers matched with the firm offering the lowest wage is  $bk/[1+k][1+k]$  in a steady-state, whereas the number matched with the firm offering the highest wage is  $bk$ , i.e.,  $n(w(1)|F) = bk/[1+k][1+k]$  and  $n(w(2)|F) = bk$ , when  $F$  is continuous. The equal profit condition defining a sse and (6) guarantees (12) holds. This completes the proof.

CLAIM 3

If a sse exists, the infimum of  $S(F^*)$ ,  $w(1)$ , is the largest wage in the interval  $(x,p)$  which satisfies

$$(13) \quad \pi(w(1)) \geq \pi(w) \text{ for all } w \in (x,p)$$

PROOF

Suppose the firm offering the lowest wage offers a wage  $w\#$  which does not satisfy (13). First, assume  $\pi(w\#) < \pi(w(1))$ , where  $w(1)$  satisfies (13). As the firm offering the lowest wage has  $bk/[1+k][1+k]$  workers at a sse, the firm offering  $w\#$  can obtain a greater profit flow from  $w(1)$  as such an offer yields more profit per matched worker and at least the same number of matched workers. Thus, such a situation cannot be part of a sse. Second, assume  $\pi(w\#) = \pi(w(1))$ , where  $w(1) - w\# > e$  for some  $e > 0$ . In this case, as the equilibrium distribution is continuous, any firm offering a wage  $w(1)$  will have more attached workers in a steady-state than the firm offering  $w\#$  and thus a greater profit flow. In either case we have a contradiction as the firm offering the lowest wage cannot be maximizing its steady-state profit flow. Hence, such a situation cannot be part of any sse. This completes the proof.

CLAIM 4

If a sse exists, the distribution of wage offers,  $F^*$ , can be written as

$$(14) \quad F^*(w) = \frac{(1+k)}{k} [1 - \Omega(w)^{1/2}]$$

for all  $w$  on  $S(F^*)$ , where

$$(15) \quad \Omega(w) = \pi(w)/\pi(w(1))$$

PROOF

The proof of this claim follows from the definition of a sse and manipulation of (6) and (8).

The existence of a unique sse is now readily established.

PROPOSITION 2

There exists a unique sse,  $(v^*, F^*, U^*)$ , which satisfies Claims 1–4 and where  $U^*$  is defined by (2).

PROOF

As already established,  $\pi(\cdot)$  is a continuous bounded function that is positive for all  $w \in [x, p)$  and  $\pi(p) = 0$ . Given the properties of  $\pi(\cdot)$  and Claims 1–4 it is straightforward to establish there is a unique triple  $(v^*, F^*, U^*)$  which satisfies the conditions for a sse. This completes the proof.

From (14) it can be seen that the equilibrium wage offer density function and its slope can be written as

$$(16a) \quad F^{*'}(w) = - [(1+k)\Omega(w)^{-1/2}/2k]\Omega'(w)$$

$$(16b) \quad F^{*''}(w) = - [(1+k)\Omega(w)^{-1/2}/2k][\Omega''(w) - (1/2)\Omega'(w)^2/\Omega(w)]$$

The equations of (16) and Figure 1 can be used to illustrate the relationship between the profit per employee function and the equilibrium wage offer density. Assume there is no other wage which yields as great a profit per employee as  $z$  in Figure 1. As the firm offering the lowest wage always has the same number of employees it will offer  $z$ . As can be seen in (16a) only those wage offers where  $\pi'(w) < 0$  are possible candidates to be on the support of the equilibrium wage offer distribution. However, all wage offers  $w \in (w(3), w(4))$  yield a smaller profit per worker than either  $w(3)$ , or  $w(4)$ . Hence, such wage offers will not be on the support of the equilibrium wage offer distribution. The highest wage in the market  $w(2)$  is defined by (12). As the profit per employee function is drawn concave on the support of  $F^*$  in Figure 1, it can be seen from (16b) that the density function must be increasing.

Thus, as long as the profit per worker function is not strictly convex, i.e., given  $\pi''(w) \leq 0$  holds for all  $w$  in the relevant range, the sse distribution of wages paid has a

strictly increasing density function on its support. The following claim strengthens this observation somewhat. Let a worker's elasticity of labor supply,  $e(w)$ , be defined by  $e(w) = s'(w)w/s(w)$ .

### PROPOSITION 3

If a representative worker's elasticity of labor supply is non-increasing, then both the sse distribution of wages paid and sse distribution of wages offered have their support on the interval  $[w(1), w(2)]$ . Further, the densities of these distribution functions are strictly increasing on their common support.

### PROOF

The relationship between the profit per worker and a worker's elasticity of supply can be written as

$$(17a) \quad \pi'(w) = s(w)\{[e(w)(p-w)/w] - 1\}$$

$$(17b) \quad \pi''(w) = s'(w)\{[e(w)(p-w)/w] - 1\} + e'(w)s(w)[(p-w)/w] - s(w)e(w)p/w$$

As  $\pi'(w) < 0$  for all  $w$  on the support of  $F^*$  and  $s(w) > 0$ ,  $1 > [e(w)(p-w)/w]$  for all  $w$  on the support of  $F^*$ . Thus, when  $e'(w) \leq 0$  for all  $w$ ,  $\pi''(w) < 0$ . The claim made about the sse distribution of wages paid follows directly from the equations of (18). The claims made about the sse distribution of wages offered follow from the above argument and the first two derivatives of the sse distribution given in (16). This completes the proof.

The unique sse can be characterized in even greater detail than presented in Claims 1-4. So far only the equilibrium distribution of wage offers has been derived. Given these results, however, it is straightforward to construct the sse distribution of wages paid

$$(18) \quad G(w|F^*, U^*) = \frac{1 - \Omega(w)^{1/2}}{k\Omega(w)^{1/2}}$$

for all  $w$  on the support of  $F^*$ . Further, the first two derivatives can be written as

$$(19a) \quad G'(w|F^*, U^*) = -[\Omega(w)^{1/2}/2k]\Omega'(w)$$

and

$$(19b) \quad G''(w|F^*, U^*) = -[\Omega(w)^{-1/2}/2k][\Omega''(w) - (3/2)(\Omega(w)^2/\Omega(w))]$$

for all  $w$  on the support of  $F^*$ .

Using equations (16), (17), and (19) it is possible to make very specific statements about the form of the sse distributions once a particular assumption is made about the form of the supply of labor function. For example, assuming worker's have a constant elasticity of supply, or always supply one unit of labor per instant, leads to particular results. Such results lead to specific predictions about the form of the wage offer distributions (or wage paid distributions) to be found in actual market. Such interesting issues will not be pursued further here.

#### 4. POLICY ANALYSIS AND PARAMETER CHANGES

As the unique sse is simple to describe it is possible to investigate in detail how changes in policy and market parameters influence the equilibrium. To illustrate the consequence of such a change, an increase in  $k$  is first considered. Such an increase will be the result of either an increase in the arrival rate of job offers,  $\lambda$ , or a decrease in the turnover rate of workers,  $\delta$ .

As  $dU^*/dk = -n/[1+k]^2 < 0$ , an increase in  $k$  reduces the steady-state number unemployed. Further, from Claim 3 it follows that any change in  $k$  does not change  $w(1)$  (the infimum of the sse distribution). Somewhat surprisingly, however, an increase in  $k$  can, in general, increase or decrease the steady-state profit flow,  $v^*$ , as (12) implies

$$(20) \quad dv^*/dk = [\pi(w(1))b/(1+k)^3][1-k]$$

The reason for this ambiguity can be explained as follows. As shown in (12), a firm offering the lowest wage in the market defines the equilibrium profit rate. As an increase in  $k$  does not change  $w(1)$ , such a firm's profit rate will increase or decrease as the change in  $k$  increases or decreases the number of its employed workers in a steady-state. Further, although an increase in  $k$  reduces the proportion of employed workers attached to such a firm, it increases the total number employed in the market in a steady-state. Thus, an increase in  $k$  has, in general, an indeterminate effect on the steady-state numbers attached to the firm offering the lowest wage. Nevertheless, as can be seen in (20), the change in the number employed by the firm offering the lowest wage will increase as  $k$  increases as long as  $1-k < 0$ , i.e.,  $\lambda > \delta$ , or the arrival rate of job offers is at least as great as the turnover rate of labor. As such a restriction appears eminently reasonable, it will be assumed to hold in what follows. Given  $1-k < 0$ , it follows from (16) that the equilibrium profit rate declines as  $k$  increases. This restriction also implies the supremum of the sse distribution increases as  $k$  increases, i.e.,  $dw(2)/dk > 0$ .

The change in the sse distribution of wages offered in the market due to an increase in  $k$  can be readily seen as (14) implies

$$(21) \quad dF^*(w)/dk = -[1/k^2]\{1 - \Omega(w)^{1/2}\} < 0$$

for all  $w$  in the support of  $F^*$ .

In the thought experiment performed above the consequences of an increase in  $k$  on

the sse distribution of wage offers was considered. The implications of such a parameter change on the sse distribution of wages paid,  $G(.|F^*,U^*)$ , follows directly from the above analysis and (19). Specifically, the sign predictions due to an increase in  $k$  on  $G(.|U^*,F^*)$  are the same as those for  $F^*$ .

An increase in  $k$  can be usefully thought of as an increase in information received during an average worker increase in the "competitiveness" of the market, or as decrease in market "frictions". Considering a sequence of sse distributions as  $k$  goes to infinity, the minimum wage offered is always  $x$  although a greater and greater percentage of firms offer high wages. Further, from (16) it can be seen that as  $k$  goes to infinity  $G(.|F^*,U^*)$  converges to a mass point at the competitive wage, i.e., at any given time only a minutely small number of workers are paid a wage  $w \bullet [x,p)$ . Of course, as  $k$  increases to infinity, the equilibrium rate of profit  $v^*$  declines to zero.

Suppose now there is an increase in the legal minimum wage, MW. Three cases can be readily identified.

Case A: Suppose the increase in the MW makes the MW greater than the output price faced by each firm. Such a change will obviously destroy the labor market altogether in that none of the firms will have a positive demand labor after the change.

Case B: Suppose the MW is initially smaller than the lowest wage in the market, i.e.,  $MW < w(1)$ . In this case as long as the resulting MW is still smaller lower than the lowest wage in the market, an increase in the MW will have no effect on the sse. So far it has been assumed the MW is greater than worker's income flow when unemployed. If this were not the case workers would prefer not to work when offered the MW. In such a situation, the smallest wage offered in the market in a sse must be at least a worker's income flow when unemployed. If an increase in the MW is such that it is still less than the worker's income flow when unemployed, then there will be no change in the sse.

Case C: Suppose the increase in MW makes the infimum of the support of  $F^*$  infeasible, although the MW is still below the output price of each firm. In this case the infimum of the support of the new sse distribution of wage offers must increase as it would be illegal to now offer a wage rate less than the new MW. Claim 2 establishes that the sse profit rate per worker and hence the steady-state profit rate of each firm,  $v^*$ , must decline. These consequences, in turn, imply the supremum,  $w(2)$ , of the support increases.

Obviously, as a change in MW does not change the rate at which workers contact firms or the rate at which workers leave the market, the steady-state unemployment rate will not be influenced by a change in the MW (given case (a) does not hold). The major reason for this result is that firms are in a sense always quantity constrained within the model described above in that any firm would employ an extra worker if it could contact them – it is the information flows and matching technology which constrains them. Thus, an increase in the MW, as long as it is small enough, does not lower any firm's desire to hire labor although it does lower the equilibrium profit rate. Note that an increase in the MW in Case C is unambiguously a good thing for the workers as it does not change the unemployment rate but the new distribution of wages offered (as well as wages paid) first-order stochastically dominates the initial distribution.

An increase in a worker's income when unemployed can be envisaged as either an increase in UI payments, or an increase in a representative worker's utility of leisure. Here a change in UI payments is considered because of its policy relevance.

Note as(1) establishes that workers cannot influence the arrival rate of offers by assumption, any change in UI payments will not change the unemployment rate (with the exception of when  $y$  is raised above  $p$ ). It is instructive to compare this prediction with that obtained from the standard "partial-partial" job search model. Given the distribution of wage offers is fixed, an increase in UI payment increases a worker's reservation wage (and possibly reduces search intensity). Thus, within the context of a standard search model with fixed search intensity, a change in UI payments induces a "reservation wage effect" in that it reduces the probability of accepting a job. Suppose the income from unemployment is binding in the sense that it defines the minimum wage offered in the market, i.e.,  $y > MW$  and  $y = w(1)$ , where  $w(1)$  is as defined above. Utilizing similar arguments to those used when considering changes in the MW, it can be seen that an increase in UI payments induces a change in the sse distribution of wage offers. Such changes imply the probability an unemployed worker accepts an offer remains constant (at one). Thus, the "reservation wage effect" disappears when changes in both sides of the market are taken into account. When firm behavior is taken into account a change in UI payments changes unemployment only to the extent that search intensity changes.

A change in the demand for labor can be characterized as a change in the output price faced by firms. A change in the given output price faced by the firms in the market increases the sse profit rate of firms as well as shifting the distribution of wage offers to the right such that it first-order stochastically dominates the previous sse offer distribution. Hence, an increase is unambiguously good for both workers and firms although it does not change the steady-state unemployment rate.

An increase in  $b$  can be caused by an increase in the number of participating workers, or a reduction in the number of firms. Such a change has very few consequences on a sse; it changes the equilibrium profit rate of firms but nothing else. Although an increase in the number of participating workers increases the number of unemployed, it does not change the sse rate of unemployment. The consequences of the parameter changes stated above are summarized in Table 1.

Table 1 illustrates how parameter changes will influence the equilibrium values of the endogenous parameters and functions. Although not attempted here it is a relatively simple task to consider the consequences of different labor market policies than those discussed above such as wage rate subsidies and employment subsidies. Again, definite sign predictions can be made.

TABLE 1

	$dU^*$	$dw(1)$	$dv^*$	$dw(2)$	$dF^*(w)$
$d\lambda^1$	(-)	(0)	(-)	(+)	(-)
$d\delta^1$	(+)	(0)	(+)	(-)	(+)
$dMW^2$	(0)	(+)	(-)	(+)	(-)
$dp$	(0)	(+)	(+)	(+)	(+)
$dn$	(+) <sup>3</sup>	(0)	(+)	(0)	(0)
$dm$	(0)	(0)	(-)	(0)	(0)

- 1: Given  $\lambda > \delta$
- 2: Given Case C discussed above.
- 3: Although the number of unemployed increases the unemployment rate is a constant.

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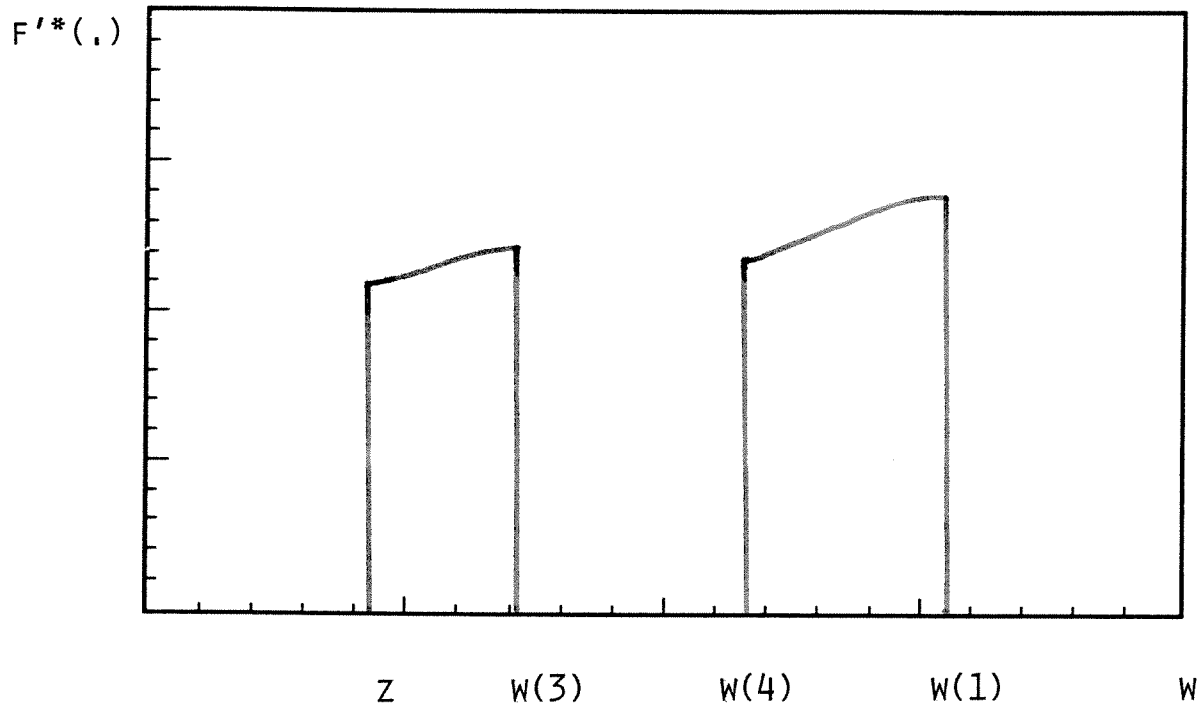
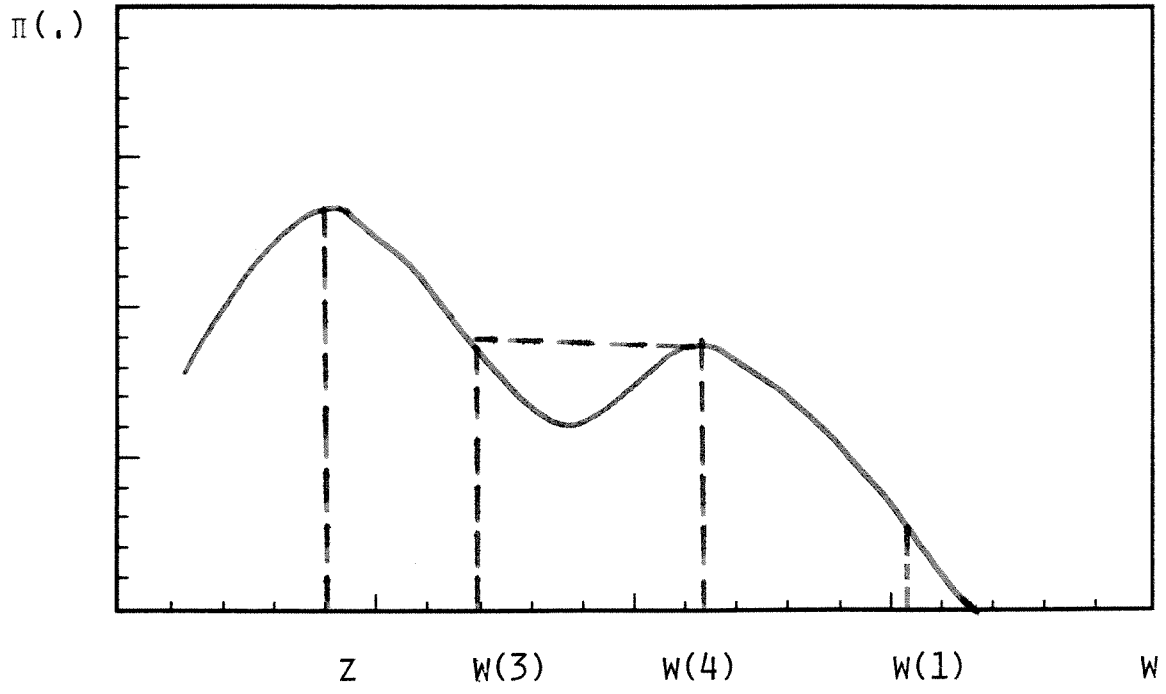


FIGURE 1