A complete list of Working Papers
on the last pages

No. 33, 1980

THE DEMAND FOR ENERGY IN SWEDISH MANUFACTURING

by

Joyce M. Dargay

This work has been carried out for the "Kran-project" within the Energy System Research Group at the University of Stockholm. The paper is intended for later publication, along with other similar studies in the Kran-report: Energy in Swedish Manufacturing.

Revised Augusti 1982

1. INTRODUCTION

Understanding the rôle of energy in the structure of production is an essential prerequisite for many energy and industrial policy decisions. The formulation and evaluation of energy conservation measures, the analysis of the effects of energy price rises or the question of reducing dependence on imported oil require knowledge of the characteristics of energy demand and the interaction between energy utilisation and economic relationships.

A fundamental feature of energy demand is its derived nature. Energy demand arises from the utility derived from its use as light, heat and motive power. In industry, energy is essential for the operation of capital equipment - machines and plant. In production, capital and energy are combined with the inputs of labour, raw materials etc. all of these being inputs in the production process. The demand for energy can be explained by the same mechanism that determines the demand for other factors of production: by production level, relative factor prices and the substitution possibilities amongst inputs.

Analysis of industrial energy demand must, therefore, simultaneously consider the complexity of relationships among all inputs in the production process. This study represents a first attempt to estimate these relationships for Swedish manufacturing. We examine the substitution possibilities between energy and other factors of production as well as interfuel substitution possibilities. Our approach, similar to that employed in a number of recent studies of energy demand, is to consider energy as one of a series of inputs in the production process. The theoretical basis of our study stems from Neoclassical production theory: the existence of a production function relating output to various inputs, cost-minimising behaviour on the part of firms and duality between production and cost functions.

We begin by specifying a cost function which relates production costs to the prices of aggregated production factors: energy, capital, labour and intermediate goods. For a given level of production and given factor prices, it is assumed that firms choose that input-mix which corresponds to minimum production costs. The theory of duality between production and cost allows us to derive demand equations for energy and the remaining inputs from the cost function and assures that these are consistent with the substitution possibilities inherent in the underlying technology. Energy demand is thus modeled as a part of an interrelated system of equations relating factor demand to production level and relative factor prices. Estimation of the model results in estimates of price elasticities of demand for each production factor as well as estimates of the substitution relationships amongst them.

Our approach allows us not only to study the price-sensitivity of energy demand but also to explain this response in terms of the substitution relationships between energy and other production factors. If these possibilities for substitution are substantial, higher energy prices could be absorbed with minimal effects on production. On the other hand, if substitution possibilities are limited, adjustment by industry to higher energy prices will be difficult.

The nature of the substitution relationships has obvious implications for economic growth and employment. If energy is a substitute for both capital and labour, then higher energy prices will tend to accelerate investment and increase employment. In this case, the effects on economic growth will be minimal. If on the other hand, a complementary relationship exists between energy and capital and/or labour, then higher energy prices will reduce investment and/or increase unemployment. The effects on individual industries and on the economy could be serious indeed.

The need for distinguishing between effects in different time perspectives is evident. The substitution possibilities between energy and other inputs or among different energy forms are certainly greater in the long-run than in the short-run. In the short-run, physical capital - machinery and plant - is given and only limited possibilities exist for reducing energy usage. In the longer run, industry is no longer bound to a given production process or product-mix. Energy conserving production processes can be introduced, thereby reducing energy utilisation by the additional inputs of other factors of production. Shifts can occur towards less energy-intensive products. The introduction of alternative technologies and changes in product composition entail investment in new capital equipment. The time that is required for the complete adjustment is thus dependent on the technical life-span of physical capital, relative factor prices and the competitive conditions and technological development within the particular industry.

A thorough analysis of energy demand should, then, not only describe factor substitution relationships, but should also distinguish between short- and long-run factor demand responses. Our study, as the majority of others to date, falls short of this. Our model is not dynamic in the sense that it distinguishes between short- and long-run demand relationships. The results presented here should therefore be viewed as only a first step towards a consistent analysis of industrial energy demand.

A technical description of our model is presented in section 2. This includes a brief summary of the underlying economic theory of cost and production, a presentation of the translog cost function and the derived factor demand functions as well as formal definitions of elasticity measures employed in the remainder of the paper. The statistical model and estimation procedure are presented in section 3. Both these sections are highly summaric and readers not familar with production theory or econometric methods may wish to move directly on to section 4, where a rather detailed - and hopefully accessible - discussion of the empirical results is presented. In section 5, we compare our findings with those of other energy demand studies in different countries. A discussion of the questions raised by our analysis and suggestions for further research concludes the paper.

2. THE MODEL

Our study of energy demand begins with an analysis of the total demand for energy in various manufacturing subsectors. A derivation of the model for the demand for aggregate inputs is given in section 2.1 below. In the next phase of our study, we extend our model to include the demand for individual energy forms - electricity, oil products and solid fuels. The two-stage model used in this analysis is presented in section 2.2.

2.1 The demand for aggregate inputs

In order to explore the substitution possibilities between energy and other production factors certain assumptions must be made regarding the structure of production. We begin by assuming that technology can be represented by a production function which relates gross production (Q) to the input of aggregated production factors: energy (E), capital (K), labour (L) and intermediate goods (M).

$$Q = q\ (E,K,L,M) \tag{1}$$

This specification implicity assumes that the production function is weakly separable in the E, K, L and M aggregates, that is to say, the marginal rates of substitution between individual energy forms (or types of K, L and M) are independent of the quantities of the remaining inputs demanded.

Further we assume that the producers minimise the costs of production and that factor prices and output level are exogenously determined. According to the theory of duality between producion and cost, the production structure (1) can, under certain regularity conditions, alternatively be described by a cost function relating total production costs (C) to the level of output (Q) and factor prices (P_i):

$$C = c(Q, P_E, P_K, P_L, P_M). \tag{2}$$

For purposes of empirical implementation it is necessary to specify an explicit functional form for c. It is desirable to chose a functional form which places minimal a priori restrictions on the characteristics of the production function, and in particular on the elasticities of substitution. Several functional forms fulfilling these requirements have been proposed recently; among these are the translog, generalised Leontief, generalised Cobb-Douglas and generalised square root quadratic.[1] All of these forms provide a local approximation to an arbitrary cost function, but their global properties are not generally known and there are no theoretical grounds for choosing among them.[2] In the present study we have chosen the translog form because it reduces to fairly simple demand relationships which are comparatively easy to work with.[3]

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[1] The generalised Leontief, Cobb-Douglas and square-root quadratic forms have been introduced by Diewert (1971, 1973, 1974) and the translog by Christensen, Jorgenson and Lau (1973).

[2] The choice of flexible functional forms has been the subject of a number of recent articles. Berndt and Khaled (1979) and Appelbaum (1979) estimate a generalised Box-Cox functional form which provides a statistical basis for choosing among the translog, generalised Leontief and the square-root quadratic forms. Estimating cost functions for U.S. manufacturing, Berndt and Khaled find the generalised Leontief form to be the preferred whereas the Appelbaum study supports the square-root quadratic.

[3] A recent Monte Carlo study by Guilkey and Lovell (1980) indicates that the translog model provides adequate estimates of quite complex technologies. The accuracy of the estimates decreases, however, when the elasticities of substitution differ greatly from unity.

The translog cost function can be interpreted as a second-order approximation to an arbitrary cost function. Denoting factor prices as P_i and assuming Hicks neutral technical change, the translog function has the following form

$$\text{Ln } C = \alpha_o + \alpha_q \ln Q + \sum_i \alpha_i \ln P_i + | \gamma_{qq} (\ln Q)^2 \qquad (3)$$

$$+ | \sum_i \sum_j \gamma_{ij} \ln P_i \ln P_j + \sum_i \gamma_{qi} \ln Q \ln P_i + \lambda T$$

where $\gamma_{ij} = \gamma_{ji}$.[1] The time trend T is included in the cost function to allow for the effects of neutral technical change on total production costs. This specification assumes that technological change affects the demand for all factors equally without altering cost-minimising factor proportions.[2] In order to assure that the underlying production function is well-behaved, the cost function must be homogeneous of degree one in input prices. That is, for a given level of output a proportionate increase in all factor prices results in a proportionate increase in total production costs. This implies the following relationships among the parameters:[3]

$$\sum_i \alpha_i = 1 \qquad (4)$$

$$\sum_i \gamma_{ij} = \sum_j \gamma_{ij} = 0$$

$$\sum_i \gamma_{iq} = 0.$$

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[1] γ_{ij} and γ_{ji} are the cross partial derivatives $\partial^2 \ln c / \partial \ln P_i \partial \ln P_j$ and $\partial^2 \ln c / \partial \ln P_j \partial \ln P_i$. These are necessarily equal.

[2] The model can be extended to the more general case of biased technological change. See, for example, Stevenson (1980).

[3] See, for example, Berndt and Christensen (1973).

Without any further restrictions on the parameters, the cost function as specified in (3) allows for non-homotheticity and non-constant returns to scale. The translog approximation is homothetic if it could be written as a separable function of output and factor prices, that is if $\gamma_{iq} = 0$ for all i. In terms of the cost function, homotheticity implies that the cost-minimising input-mix is determined solely by input prices and is independent of the level of production. Further, a homothetic cost function is homogeneous if the elasticity of cost with respect to output is constant, i.e. if $\gamma_{qq} = 0$. Given the above restrictions,the degree of homogeneity of the cost function is determined by the coefficient α_q. Thus, if $\alpha_q = 1$, the cost function is linearly homogeneous and the underlying technology is characterised by constant returns to scale.

Although it is, in principle, possible to analyse the structure of production by estimating the cost function directly, the number of parameters to be estimated is quite large and multicollinearity among exogenous variables may be a problem, resulting in imprecise parameter estimates. It is common practice, therefore, to base empirical studies of substitution possibilities not on the cost function itself, but on the derived demand equations.

The input demand functions are derived from the cost function using a result first noted by Hotelling and formally established by Shephard.[1] This result, commonly known as Shephard's lemma, states that the cost function is related to the cost-minimising demand functions through its partial derivatives with respect to input prices. Further,

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[1] Hotelling (1932), Shephard (1953).

since total cost (C) is equal to the sum of the costs for the individual factor inputs ($\Sigma X_i P_i$), we have $\partial \ln c / \partial \ln P_i = P_i X_i / C = S_i$, where S_i is the share of the ith input in total costs. Thus, the factor demand functions in terms of cost shares follow from partial logarithmic differentiation of the cost function (3) with respect to factor prices. We have

$$S_i = \alpha_i + \sum_j \gamma_{ij} \ln P_j + \gamma_{iq} \ln Q \qquad i,j = E,K,L,M \qquad (5)$$

where $\Sigma S_i = 1$.

Comparing the cost share equations with the cost function (3) we see that the majority of the parameters of the cost function can be determined by estimation of the system of equations given in (5), with the constraints $\gamma_{ij} = \gamma_{ji}$ and restrictions implied by (4). The parameters α_o, α_q, γ_{qq} and λ, and thus the returns to scale of the cost function and the influence of technical change are, however, not identified unless the cost function is estimated directly.

Our particular interest, however, lies in the structure of factor substitution and price responsiveness. The most commonly used measure of factor substitution is the Allen partial elasticity of substitution.[1] This measures the percentual change in the relationship between two production factors which results from a 1 % change in their relative prices, all other inputs being allowed to adjust to their cost minimising levels. For the cost function, the Allen partial elasticities of substitution between inputs i and j are given by[2]

$$\sigma_{ij} = \frac{C(\partial^2 C / \partial P_i \partial P_j)}{(\partial C / \partial P_i)(\partial C / \partial P_j)} \qquad (6)$$

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[1] Allen, R.G.D. (1959)

[2] Uzawa, H. (1962)

For the translog cost function these measures can be calculated as[1]

$$\sigma_{ij} = (\gamma_{ij} + \hat{S}_i \hat{S}_j)/\hat{S}_i \hat{S}_j \qquad i \neq j$$

$$\sigma_{ii} = (\gamma_{ii} + \hat{S}_i^2 \; / \; \hat{S}_i)/\hat{S}_i^2 \tag{7}$$

where $\hat{S}_i$ are the predicted cost shares.

As shown in Allen[2], the partial elasticities of substitution are related to the price elasticities of demand for factor inputs (η_{ij}) according to

$$\eta_{ij} = \hat{S}_j \, \sigma_{ij} \; . \tag{8}$$

It should be noted that the translog function does not constrain these elasticities to be constant. As they as functions of the cost shares, they are dependent on the level of factor prices, and for the non-homothetic cost function, even on production level. Thus, the estimated elasticities are allowed to vary over the observation period.

A disadvantage of the translog function is that one cannot test for zero substitution between factor pairs directly from the estimated demand functions. It is clear from expression (7) that the elasticity of substitution between factors i and j is equal to unity if $\gamma_{ij} = 0$. Thus if all $\gamma_{ij} = 0$, the translog cost function corresponds to a Cobb-Douglas production structure. We can test this hypothesis [3]

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[1] Berndt and Wood (1975)

[2] Allen, R.G.D. (1959)

[3] or similarly the hypothesis of homotheticity, $\gamma_{iq} = 0$ for all i.

using a simple likelihood ratio test. The appropriate test statistic is

$$-2\ \ln(L_R/L_U) \tag{9}$$

where L_R and L_U are the maximum likelihood values for the restricted and unrestricted models respectively. This statistic is asymptotically distributed as Chi-square under the null-hypothesis of the more restrictive model with degrees of freedom equal to the number of parameters being tested.

2.2 The demand for individual energy forms - the two-stage model

Next we extend our model to encompass the substitution possibilities among individual energy types. Ideally, we would like to estimate a model that places minimal a priori restrictions on the substitution relationships not only between individual energy forms but also among the individual energy forms and other production factors. In principle, this can be achieved by specifying the production function (1) with total energy, E, disaggregated into its constituent fuel types and deriving the corresponding cost function. Estimation of the many-input case, however, poses computational problems. Not only do the number of share equations increase, but multicollinearity among the price variables is likely to be a problem. In order to minimise estimation problems, we chose a somewhat more simplified model.

Our approach, similar to that introduced by Fuss[1], is to specify the demand for energy as a two-stage process. First, the structure of energy demand is determined by choosing the

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[1] Fuss (1977)

fuel-mix that minimises energy costs. This provides an analysis of interfuel substitution and allows us to construct a consistent aggregate price index for energy. Secondly, overall energy demand is optimised in conjunction with the inputs of capital, labour and intermediate goods, providing estimates of substitution possibilities between aggregate energy and each of the three non-energy inputs.

Although the two-stage procedure facilitates estimation of a cost function with many inputs, it does impose restrictions on the structure of production. Specifically, it requires that the cost function is weakly separable in the energy aggregate, that is to say, that the cost-minimising energy-mix is independent of the prices and level of capital, labour and intermediate goods.[1] Thus the relationship between the individual energy components and the remaining production factors are determined solely through the energy aggregate.

The first stage of the analysis involves the specification and estimation of an energy submodel for electricity (e), oil (o) and solid fuels (s). The total cost of energy, C_E, is represented by a translog cost function with constant returns to scale. Under these conditions, the unit cost function for the energy aggregate follows directly from the cost function, providing an aggregate price index for energy:

$$\ln P_E = \ln \frac{C_E}{Q} = \alpha_0 + \sum_i \alpha_i \ln P_{Ei} + \frac{1}{2} \sum_i \sum_j \gamma_{ij} \ln P_{Ei} \ln P_{Ej} \qquad (10)$$

$i,j = e,o,s$

where P_{Ei} represent the prices of the energy components.

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[1] This assumption is also implied by the aggregate model presented in Section 2.1. Weak separability is in fact a prerequisite for the existence of aggregates.

As in the previous section, we derive the share equations implied by this cost function

$$S_{Ei} = \alpha_i + \sum_j \gamma_{ij} \ln P_{Ej} \qquad (11)$$

$i,j = e,o,s.$

Again, the properties of production require the restrictions $\gamma_{ij} = \gamma_{ji}$, $\Sigma\alpha_i = 1$ and $\sum_i \gamma_{ij} = \sum_j \gamma_{ij} = 0$ for all i, j.

Estimation of the system of cost shares allows us to calculate the partial own- and cross-price elasticities for the three energy forms. These elasticities are partial in the sense that they reflect substitution among the fuel types within the energy aggregate, given that total energy utilisation remains constant.

By substituting the estimated coefficients α_i and γ_{ij}, $i,j = e,o,s$ into (10) we are able to construct a price index, P_E, for the energy aggregate.[1] This index is then used as an instrumental variable for the price of energy in the second stage of the analysis, which entails estimation of the translog cost share equations (5) for the E, K, L and M aggregates. In addition to providing information concerning the substitution relationships between the energy aggregate and the remaining inputs, this permits calculation of the total price elasticities of demand for each energy form. Since a change in the price of an energy component also changes $\hat{P}_E$, it results in a substitution between energy and other inputs, affecting the demand for aggregate energy and thereby the demand for each energy component. This effect combined with those of interfuel substitution form

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[1] Since the price indices for the individual energy forms are normalised to 1 for 1975, a similarly normalised price index for the energy aggregate is calculated by setting α_0 to 0 in (10).

the total price elasticity of demand for each fuel. This is given by

$$\eta^{T}_{ij} = \frac{d\ln X_{Ei}}{d\ln P_{Ej}} = \left[\frac{\partial \ln X_{Ei}}{\partial \ln P_{Ej}}\bigg|_{X_E} + \frac{\partial X_{Ei}}{\partial X_E}\,\frac{\partial X_E}{\partial P_E}\,\frac{\partial P_E}{\partial P_{Ej}}\right]\frac{\partial P_{Ej}}{\partial X_{Ei}} \qquad (12)$$

$i,j = e,o,s$

where the X_{Ei} are the quantities of each fuel demanded, X_E the total quantity of energy demanded and P_E is the price index for energy. Since P_E is given by (10) and since the energy cost function is homogeneous this reduces to

$$\eta^{T}_{ij} = \eta^{P}_{ij} + \eta_{EE}\, S_{Ej} \qquad (13)$$

$i,j = e,o,s$

where the η^{P}_{ij} are the partial price elasticities obtained from the energy submodel and η_{EE} is own-price elasticity for the energy aggregate.

Finally a few words should be said about the properties of the translog cost function in relation to neoclassical production theory. In general, a cost function is well behaved, that is, satisfies the requirements of cost-minimising demand theory, if it is concave in input prices and if its input demand functions are strictly positive. The translog function does not satisfy these requirements globally,[1] that is to say, for all possible values of factor prices. It is therefore necessary to test for positivity and concavity at each observation. Positivity is satisfied if all fitted cost shares are positive. A necessary condition for concavity is that all own-price elasticities are negative, while a necessary and sufficient condition is the negative semidefiniteness of the Hessian matrix[2] based on the estimated parameters.

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[1] Nor do any of the other generalised functional forms mentioned earlier.

[2] The matrix of second-order partial derivatives of the cost function with respect to factor prices.

3. Estimation Procedure

Characterisation of the structure of production entails estimation of the input demand equations (5) subject to the restrictions imposed by linear homogeneity in prices (4).[1] The stochastic model includes the specification of additive disturbances for each of the share equations. These disturbances may be interpreted alternatively as random errors in cost-minimising behaviour or as the random influence of unspecified explanatory variables. In either case, it is probable that these factors are related for the share equations, and allowance should be made for non-zero contemporaneous correlation across equations.

The stochastic specification of (5) takes the following form

$$S_i = \alpha_i + \sum_i \gamma_{ij} \ln P_j + \gamma_{iq} \ln Q + \varepsilon_i \qquad i,j = E,K,L,M \tag{14}$$

Letting $\tilde{\varepsilon}_t$ denote the vector of error terms for the four share equations we assume that $\tilde{\varepsilon}_t$ is joint normally distributed with zero mean and variance-covariance matrix Σ, that is

$$\tilde{\varepsilon}_t \sim N(0, \Sigma) \qquad \text{for all } t \tag{14a}$$

such that

$$E(\tilde{\varepsilon}_t \tilde{\varepsilon}'_s) = \delta_{ts}\, \Sigma \qquad \delta_{ts} \begin{array}{l} = 1 \text{ if } t=s \\ = 0 \text{ if } t \neq s. \end{array} \tag{14b}$$

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[1] Since the input shares must sum to unity, these restrictions are equivalent to $\gamma_{ij} = \gamma_{ji}$ $i \neq j$. Thus the validity of the assumption of homogeneity of degree one in input prices is directly testable through the symmetry conditions.

This specification implies that the error terms ε_i have a constant variance-covariance matrix and allows for non-zero correlation between contemporaneous error terms of the share equations. In (14b) we assume zero intertemporal correlations between all error terms.[1]

Similarly, the stochastic specification of the energy submodel (10) includes additive disturbances for each energy component share equation

$$S_{Ei} = \alpha_i + \sum_j \gamma_{ij} \ln P_{Ej} + u_i \qquad i,j = e,o,s \tag{15}$$

where, as above,

$$\tilde{u}_t \sim N(0,\Omega) \qquad \text{for all } t \tag{15a}$$

and

$$E(\tilde{u}_t \tilde{u}'_s) = \delta_{st}\Omega \qquad \delta_{ts} = \begin{matrix} 1 \text{ if } t = s \\ 0 \text{ if } t \neq s \end{matrix} \tag{15b}$$

Estimation of the two-stage model requires specification of the relationship between error terms in (14) and (15). For the sake of simplicity we assume that the error term vectors $\tilde{\varepsilon}_t$ and $\tilde{u}_t$ are uncorrelated so that the distribution for

$\binom{\tilde{\varepsilon}_t}{\tilde{u}_t}$ is given by

$$\binom{\tilde{\varepsilon}_t}{\tilde{u}_t} \sim N\{0, \begin{bmatrix} \Sigma & 0 \\ 0 & \Omega \end{bmatrix}\} \tag{16}$$

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[1] Ideally one would like to estimate a stochastic specifica-ion which in addition allows for non-zero intertemporal correlations. This, however, would further complicate the the estimation procedure and could not easily be done with the programs available.

Further, since the share equations must sum to unity, the estimated disturbance covariance matrix is singular. The most common method of dealing with this problem is to delete one equation from the system and choose an estimation to which equation is deleted. In this study we employ a full information maximum likelihood estimation procedure.[1]

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[1] The computer program was written by L. Jansson, and entails maximisation of the concentrated likelihood function. For a formulation of this, see Barten (1969).

4. The empirical results

Various versions of the models described in the previous sections were estimated for total manufacturing, excluding energy production sectors, and for 12 manufacturing subsectors. The subsector miscellaneous manufacturing is excluded from individual analysis, but is included in total manufacturing. The sector divisions and sector numbers correspond to those used in the long-term economic surveys prepared by the Swedish Ministry of Finance.[1] Comparison with ISIC nomenclature is given in the appendix. A description of data sources and the construction of the cost and price series is also contained in the appendix.

First, we analyse the demand for aggregate inputs - energy, capital, labour and intermediate goods. This gives us information regarding the substitution possibilities between energy and other factors of production and the price elasticity of demand for aggregate energy. The results are presented and discussed in section 4.1 below.

The second stage of our study, presented in section 4.2, involves an analysis of interfuel substitution. Three energy forms are considered: electricity, oil products and solid fuels.

4.1 The Aggregate Demand for Energy

The demand for aggregate production factors is analysed by estimating the system of share equations given in (14). In accordance with the discussion in section 3 the equation for intermediate goods is dropped from the estimation procedure, and the coefficients for that equation are calculated from

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[1] so called LU-(långtidsutredningen)sectors.

the identities given in (4). The data for each sector include annual observations on costs and prices for labour, capital, energy and intermediate goods and production volume for the period 1952-1976. All price indices and production volume are normalised to unity for 1975.

Both homothetic and non-homothetic versions of the cost function are estimated. This allows us to statistically test for the more restrictive assumption of separability between prices and production level (homotheticity) and to compare the estimated elasticities for the two specifications.

Homothetic specification

The first results presented here are based on the assumption that the cost-function is homothetic, that is, we estimate equation system (14) under the constraints that $\gamma_{iq} = 0$ for all $i = K,L,E,M$. The estimated parameters for the fitted translog share equations along with their estimated standard errors, R^2 and the maximum likelihood value for each system of equations are shown in table A 1 in the appendix.

The majority of slope-coefficients (γ_{ij}) are significantly different from zero at normal confidence levels, suggesting that the variation in cost shares is at least partially explained by changes in relative factor prices. As mentioned in section 2 above, we can test the hypothesis that all corresponds to a Cobb-Douglas production structure. The likelihood ratio test statistics, which are given in the first column of table A3 in the appendix, fall in the interval 92-217. For all branches, the test statistic is clearly significant at the 1 % level, so that the hypothesis of unitary elasticities of substitution between all factor pairs can be rejected.

In order to analyse price-responsiveness and factor substitution possibilities we compute the Allen partial elasticities of substitution (σ_{ij}) and the price elasticities (η_{ij}) for all cost-share observations according to equations (7) and (8). Although the resulting elasticities vary somewhat over the time period analysed, no significant trends are discernable. We therefore present the elasticities calculated at the mean values of the exogenous variables as representative results.

The own-price elasticities of demand for energy, capital, labour and intermediate goods are shown in table 1 along with their asymptotic standard errors.[1] These elasticities measure the percentage change in the use of a given input resulting from a 1 % change in its price. In accordance with cost minimising principles we would expect these elasticities to be negative. For example, a rise in the price of energy in relation to other production factors should lead to a substitution away from energy and thus decrease its use in production.

From table 1 we see that the majority of the estimated own price elasticities of demand are negative[2] and with few exceptions significantly so at least at the 5 % level. Furthermore, the estimated own-price elasticities of demand are less than unity for all inputs and for all sectors, indicating that input demand is inelastic. Although the elasticities do vary somewhat for the individual industries,

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[1] Approximate standard errors are calculated at mean input shares under the assumption that these are non-stochastic.

[2] The fitted shares were positive for all observations and all sectors insuring the positivity of the cost function. Although the estimated own-price elasticities are negative for the overwhelming majority of observations, a few sign reversals did occur in some sectors, indicating a local departure from concavity. More rigorous tests for concavity have, however, not been carried out.

a few general trends are apparent. First, we find that the own-price elasticities for capital and labour are rather similar for the majority of branches. For total manufacturing, as well as for at least half of the subsectors, labour appears to be the most price-sensitive production factor with an elasticity generally on the order of -0.5. Capital, on the other hand, exhibits the most inelastic demand, with an average elasticity around -0.25. Although the results for intermediate goods show somewhat more variation, the elasticities are generally rather low.

Our prime concern, however, is with the price sensitivity of energy demand. Here, the elasticities show a far wider range of variation. Although the own-price elasticities for energy generally fall in the interval -0.4 to -0.6, the extremes range from non-significance to nearly -1.0. It is worth noting that of the four subsectors that show positive and/or non-significant energy price elasticities two of these - Wood, pulp and paper (8) and Primary metals (14) - are the most energy intensive Swedish industries.

In the case of Wood, pulp and paper (8) the large standard errors of the estimated elasticities make it impossible to reject the null hypothesis that energy demand is insensitive to price changes. This result may partially be due to a misspecification of the cost share for energy in this sector. Our measure of energy costs includes only expenditures for fuels purchased from outside the establishment so that the use of internal energy supplies - for example, of wood fuels - is omitted from the cost function. Wood fuels constitute an important energy source in paper and pulp production, and the omission of a large proportion of these fuels may have some effect on the estimated elasticities. In view of this specification error, it would be rash to draw any conclusions concerning the price elasticity for energy in this sector.

Table 1. Own-Price Elasticities for Energy, Capital, Labour and Intermediate goods. Homothetic cost function.

	Sector	Energy	Capital	Labour	Intermediate goods
4	Sheltered food	-.13 (0.16)	-.14 (0.02)	-.54 (0.02)	-.06
5	Import-competing food	-.47 (0.09)	-.18 (0.03)	-.66 (0.02)	-.13
6	Beverage and tobacco	-.15 (0.20)	-.16 (0.04)	-.74 (0.04)	-.24
7	Textiles and clothing	-.98 (0.16)	-.26 (0.05)	-.53 (0.01)	-.28
8	Wood, pulp and paper	.02 (0.11)	-.28 (0.03)	-.63 (0.03)	-.19
9	Printing	-.54 (0.11)	-.40 (0.06)	-.43 (0.01)	-.48
10	Rubber products	-.52 (0.17)	-.18 (0.09)	-.43 (0.01)	-.21
11	Chemicals	-.26 (0.12)	-.24 (0.03)	-.54 (0.02)	-.32
13	Non-metallic mineral products	-.41 (0.10)	-.29 (0.05)	-.62 (0.01)	-.66
14	Primary metals	.33 (0.14)	-.25 (0.05)	-.65 (0.03)	-.28
15	Engineering	-.64 (0.12)	-.24 (0.05)	-.57 (0.01)	-.41
16	Shipbuilding	-.56 (0.09)	-.15 (0.04)	-.65 (0.03)	-.33
	Total Manufacturing	-.25 (0.09)	-.28 (0.01)	-.57 (0.05)	-.28

Note: Approximate asymptotic standard errors are in parenthesis. As the share equation for intermediate goods was excluded from the estimation, standard errors are not readily available.

For the Primary metal industry, on the other hand, we find a significant positive energy price elasticity.[1] This, of course, is economic nonsense and must be rejected. A possible explanation to this spurious relationship may lie in the model formulation, and particularly in its inability to capture the effects of technological development. This is of utmost importance in the primary metal industry where factors such as the development of blast furnaces and the increased use of oxygen converters have lead to a considerable decrease in specific energy usage since the beginning of the 60's.[2] The gradual introduction of new techniques has been contemporaneous with falling real energy prices. One can thus suspect that the positive estimated price elasticity reflects an energy-saving technical change that has not been specified in our model.

The results for these two highly energy intensive industries illustrate the weakness of our model and suggest the need of further model development, particularly towards an explicit specification of non-neutral technological change.

Finally, our results indicate that energy is less price-elastic for aggregate manufacturing than it is for 8 out of 12 of the manufacturing subsectors. This is perhaps not surprising considering that two of the industries with positive elasticities account for nearly 2/3 of energy utilisation in the manufacturing sector. It should be pointed out, however, that estimates based on aggregate manufacturing partially reflect the changes in relative production shares among the individual industries that have

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[1] The calculated own-price elasticities were positive for nearly all the observations.

[2] Carling, Dargay, Oettinger, Sohlman (1978)

occurred under the 1952-1976 time period.[1] These elasticities are therefore are not directly comparable with those obtained for the disaggregated sectors.

Next, we turn to an examination of the substitution possibilities among inputs. For this purpose the Allen-Uzawa elasticity of substitution is calculated for each input pair. For a given factor pair, this elasticity measures the percentage change in the input ratio that results from a 1 % change in their relative prices. A negative value denotes that the factors are complements, that is to say, that a relative increase in the price of one factor leads to a decrease in the use of the other. A positive value denotes substitutability: a relative increase in the price of one factor leads to a relative increase in the use of the other.

These elasticities are shown in table 2 together with their asymptotic standard errors. Of particular interest for energy policy are the substitution possibilities between energy-capital and between energy-labour. In six subsectors (5,7,8,9,15,16) we may conclude that energy and capital are complements.[2] Only one sector, sheltered food (4), exhibits capital-energy substitutability. In the remaining sectors, all of which show negative elasticities, the standard errors make it impossible to reject the hypothesis that the elasticity is 0. The predominance of energy-capital complementarity in the individual industries is consistant with the results obtained for total manufacturing. We see, however, that the aggregate measure over-estimates the degree of complementarity for all but 2 subsectors. The results for the substitution relationship between energy and labour

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[1] A description of the development of the composition of industrial production in Sweden under the period 1965-75 and a discussion of the effects of changes in branch structure on energy utilisation can be found in Östblom (1980).

[2] These parameters are significant at the 5 % level.

Table 2. Substitution Elasticities for Energy (E), Capital (K), Labour (L) and Intermediate goods (M). Homothetic Cost function.

	Sector	E-K	E-L	E-M	K-M	L-M	
4	Sheltered food	2.81	0.33	-0.03	1.60	-0.09	0.56
		(1.27)	(0.01)	(0.14)	(0.12)	(0.03)	(0.03)
5	Import-competing food	-2.11	1.06	0.62	0.28	0.22	0.80
		(0.64)	(0.19)	(0.14)	(0.14)	(0.05)	(0.02)
6	Beverage and tobacco	-0.18	-1.26	0.84	1.50	-0.36	1.00
		(0.48)	(0.21)	(0.27)	(0.26)	(0.06)	(0.03)
7	Textiles and clothing	-3.73	0.31	2.08	1.11	-0.06	0.75
		(0.99)	(0.14)	(0.34)	(0.07)	(0.12)	(0.02)
8	Wood, pulp and paper	-0.59	0.02	0.08	1.19	0.06	0.79
		(0.28)	(0.10)	(0.19)	(0.08)	(0.06)	(0.03)
9	Printing	-1.82	1.06	0.74	0.42	0.56	0.90
		(0.48)	(0.15)	(0.39)	(0.10)	(0.19)	(0.06)
10	Rubber products	-0.07	0.46	0.75	0.78	-0.17	0.63
		(0.87)	(0.14)	(0.43)	(0.08)	(0.21)	(0.04)
11	Chemicals	-0.11	-0.21	0.55	-0.08	0.46	0.96
		(0.20)	(0.19)	(0.16)	(0.07)	(0.06)	(0.05)
13	Non-metallic mineral products	-0.32	-0.24	1.34	0.31	0.50	1.41
		(0.36)	(0.12)	(0.34)	(0.10)	(0.14)	(0.04)
14	Primary metals	-0.66	-0.61	-0.17	0.61	0.29	1.10
		(0.42)	(0.21)	(0.29)	(0.10)	(0.14)	(0.06)
15	Engineering	-0.91	0.02	1.30	0.21	0.34	1.02
		(0.47)	(0.06)	(0.26)	(0.09)	(0.10)	(0.02)
16	Shipbuilding	-0.60	0.37	0.85	0.54	-0.02	1.02
		(0.32)	(0.09)	(0.20)	(0.10)	(0.10)	(0.06)
	Total Manufacturing	-1.43	0.12	0.66	0.66	0.24	0.84
		(0.49)	(0.10)	(0.20)	(0.09)	(0.08)	(0.01)

Note: Approximate asymptotic standard errors are in parenthesis.

are quite the opposite. In 6 of the Swedish manufacturing industries (4,5,7,9,10,16), the elasticities are significantly positive, indicating substitutability, while only two sectors (6,13) exhibit energy-labour complementarity at normal significance levels. Finally, in the remaining two sectors the high standard errors preclude any conclusions concerning energy-labour relationships. The statistically significant elasticities between energy and labour fall in a rather wide region, ranging from strong complementarity - nearly -1.3 in sector (6) - to a degree of substitutability somewhat greater than +1.0 in sectors (5) and (9). Because of these divergences in the sign and magnitude of the elasticities of substitution across the individual industries, the estimates based on aggregate manufacturing could be quite missleading. Our results for total manufacturing indicate that energy and labour are rather weak substitutes.

The relationship between energy and materials is, in all statistically significant cases, positive, indicating that these factors are substitutes. The elasticities range from about 0.6 to somewhat over 2.0.

With regard to non-energy inputs, we see that capital and labour are substitutes in all but the Chemical industry (11) where the elasticity is not statistically significant. In four sectors (4,6,7,8) we find the elasticity to be somewhat greater than unity while in others (5,13,15) the substitution possibilities are rather small ($\sigma_{kl} = +.2$ to $+0.3$).

Finally, we see that capital and intermediate goods are statistically significant, but weak substitutes in six industries, while a weak complementary relationship exists in two. The large standard errors of the remaining 3 estimates do not allow rejection of the hypothesis of zero substitution between these inputs. In general, the results are indicative of a more or less independent relationship between capital and intermediate goods. This is strikingly contrary to the results obtained for labour-materials. As is seen, all industries exhibit a high degree of substitutability between labour and intermediate goods.

Non-homothetic specification

The results presented above are based on the assumption that the cost function is homothetic, that is to say that the cost-minimising input shares are independent of the level of production. For the sake of comparison, we now examine what happens when this restriction is relaxed, by estimating equation system (14) with the γ_{iq} no longer constrained to zero. This allows us to empirically test for homotheticity by the likelihood ratio test.

The estimated coefficients for the non-homothetic specification along with their asymptotic standard errors, R^2 and maximum likelihood values are given in table A2 in the appendix. In particular, two results are worth noting. First, we find that production volume generally has a significant influence on the factor demand shares, which suggests that the cost function is non-homothetic. This is also supported by the likelihood ratio test statistics which are given in the second column of table A3 in the appendix. The null-hypothesis of homotheticity is strongly rejected for all sectors, with the exception of the Non-metallic mineral products industry (13). Secondly, a strong negative relationship exists between labours' cost share and production level for the majority of the industries. According to the assumptions of our model this is indicative of an output elasticity of labour demand that is less than unity.[1] However, as production volume increases over time, it is exceedingly difficult to separate scale effects from, for example, the effects of biased technological change. It may be that the output variable is partially capturing the effects of a labour-saving technical development,[2] which is not specified in our model. We therefore consider it unwarranted to attempt to interpret our results in terms of scale effects, until an explicit allowance is made for non-neutral technological progress.

The own-price elasticities and the elasticities of substitution for energy, capital, labour and intermediate goods implied by the non-homothetic cost function are shown in tables 3 and 4. We see that resulting own-price elasticities are quite similar to those obtained from the homothetic specification (compare table 1). In seven out of the twelve subsectors the own-price elasticity for energy falls in the interval from -0.4 to -0.7. For the remaining sectors, the large standard errors do not allow us to reject the hypothesis of zero price-responsiveness. Again, we find positive, although non-significant, price elasticities in the two most energy intensive branches: Wood, pulp and paper (8) and Primary metals (14).

Further, we find that the own-price elasticities for capital are more or less identical to those presented earlier. The major differences between the homothetic and non-homothetic

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[1] The output elasticity of demand for factor i is given by $\eta_{iq} = \frac{\gamma_{iq}}{Si} + \alpha_q + \gamma_{qq} \ln Q + \sum_j \gamma_{jq} \ln P_j$, $i = K,L,E,M$. It can be computed only if α_q and γ_{qq} are known, that is, by estimating the cost function directly.

[2] Evidence of a labour-saving technological development in Swedish industrial sectors is noted in the capital-labour production function studies of Bergström and Melander (1979) and Eriksson, Jakobsson and Jansson (1976).

Table 3. Price elasticities of demand for energy, capital, labour and intermediate goods. Non-homothetic cost function.

	Sector	Energy	Capital	Labour	Intermediate goods
4	Sheltered food	-0.16 (0.12)	-0.15 (0.02)	-0.15 (0.06)	-0.00
5	Import-competing food	-0.44 (0.03)	-0.19 (0.02)	-0.18 (0.02)	-0.01
6	Beverage and tobacco	0.05 (1.68)	-0.11 (0.04)	-0.05 (0.01)	-0.07
7	Textiles and clothing	-0.67 (0.14)	-0.15 (0.04)	-0.54 (0.02)	-0.21
8	Wood, pulp and paper	0.08 (0.08)	-0.24 (0.03)	-0.02 (0.08)	-0.08
9	Printing	-0.55 (0.12)	-0.25 (0.05)	-0.16 (0.05)	-0.15
10	Rubber products	-0.63 (0.18)	-0.26 (0.06)	-0.14 (0.03)	-0.09
11	Chemicals	-0.19 (0.12)	-0.23 (0.03)	0.06 (0.02)	0.03
13	Non-metallic mineral products	-0.46 (0.11)	-0.30 (0.06)	-0.50 (0.05)	-0.52
14	Primary metals	0.29 (0.16)	-0.26 (0.03)	-0.22 (0.11)	-0.06
15	Engineering	-0.57 (0.15)	-0.24 (0.05)	-0.18 (0.07)	-0.12
16	Shipbuilding	-0.47 (0.14)	-0.16 (0.05)	-0.28 (0.07)	-0.12
	Total Manufacturing	-0.10 (0.08)	-0.21 (0.03)	-0.25 (0.09)	-0.12

Note: Approximate asymptotic standard errors are in parenthesis. As the share equation for intermediate goods was excluded from the estimation, standard errors are not readily available.

specifications lie in the resulting price elasticities of demand for labour and intermediate goods. In nearly all branches, these elasticities decrease considerably when the homotheticity constraints are relaxed. The estimated price elasticity of demand for labour is under -0.2 for all but three sectors - Textiles (7), Non-metallic minerals (13) and Shipbuilding (16). This result is quite different from that implied by the homothetic model, which indicated labour to be the most price-sensitive production factor with an average elasticity on the order of -0.5. Finally, intermediate goods show very little price-responsiveness with elasticities of demand very near zero in the majority of industries.

Regarding the substitution relationships among inputs, our estimates show the same general pattern as that obtained for the homothetic specification. A few changes in sign do occur, but these estimates are generally non-significant in both cases. The most notable exception is the relationship between energy and capital in total manufacturing, where the relationship switches from strong complementarity ($\sigma_{EK} = -1.4$) to a positive, but non-statistically significant value. Again, energy and capital are seen to be complements in most significant cases, while substitutability predominates between energy and labour and between energy and intermediate goods. The magnitudes of these relationships are, however, somewhat different than those obtained when homotheticity is imposed. The general trend seems to be towards weaker energy-capital complementarity and greater energy-labour substitutability. The most substantial difference between the two specifications is the elasticity of substitution between labour and intermediate goods. With the assumption of homotheticity, this elasticity is greater than +0.5 for all sectors, whereas relaxing this assumption reduces it to insignificance in well over half the cases.

Table 4. Elasticities of substitution among energy (E), capital (K) and labour (L) and intermediate goods (M). Non-homothetic cost function.

	Sector	E-K	E-L	E-M	K-L	K-M	L-M
4	Sheltered food	1.98	4.88	-0.59	0.66	0.06	0.08
		(0.91)	(1.28)	(0.12)	(0.40)	(0.06)	(0.07)
5	Import-competing food	-1.70	4.18	0.03	1.19	0.07	0.05
		(0.64)	(1.57)	(0.30)	(0.37)	(0.04)	(0.11)
6	Beverage and tobacco	0.27	-3.40	1.32	0.57	-0.07	0.14
		(0.45)	(1.06)	(0.28)	(0.24)	(0.08)	(0.13)
7	Textiles and clothing	-0.66	0.65	0.91	1.34	-0.48	0.72
		(0.85)	(0.10)	(0.31)	(0.06)	(0.10)	(0.03)
8	Wood, pulp and paper	-0.06	-0.36	0.01	0.71	0.14	0.01
		(0.15)	(0.52)	(0.21)	(0.12)	(0.06)	(0.10)
9	Printing	-1.23	0.65	1.05	0.40	0.19	0.25
		(0.38)	(0.61)	(0.91)	(0.08)	(0.11)	(0.14)
10	Rubber products	-1.22	0.06	1.57	0.68	-0.13	0.09
		(0.73)	(0.56)	(0.57)	(0.18)	(0.08)	(0.01)
11	Chemicals	0.14	0.38	0.14	0.52	0.15	-0.23
		(0.20)	(0.25)	(0.04)	(0.08)	(0.05)	(0.04)
13	Non-metallic mineral products	-0.52	-0.17	1.47	0.53	0.39	1.01
		(0.49)	(0.40)	(0.39)	(0.18)	(0.14)	(0.14)
14	Primary metals	0.14	-0.20	-0.47	0.51	0.25	0.28
		(0.46)	(0.90)	(0.32)	(0.26)	(0.10)	(0.18)
15	Engineering	-0.70	0.48	0.85	0.42	0.19	0.26
		(0.47)	(0.49)	(0.51)	(0.24)	(0.10)	(0.12)
16	Shipbuilding	-0.93	1.07	0.36	0.53	-0.00	0.37
		(0.42)	(0.26)	(0.23)	(0.30)	(0.15)	(0.11)
	Total Manufacturing	0.33	0.17	0.03	0.26	0.21	0.36
		(0.52)	(0.82)	(0.45)	(0.21)	(0.08)	(0.14)

Note: Approximate asymptotic standard errors are in parenthesis.

In conclusion, our results indicate that the majority of the elasticities are at least slightly sensitive to the specification of homotheticity.[1] Some of these, and especially those pertaining to labour, are highly so. The discrepancies are mainly in the magnitude of the estimated elasticities, whereas the pattern of substitution possibilities is largely in agreement for the two specifications. Of most relevance for the purposes of our study, however, is the result that the own-price elasticities for energy and the substitution relationships between energy and other production factors are quite robust to differences in homotheticity assumptions.

4.2 Interfuel substitution

Thus far our analysis has concentrated on aggregate energy and the substitution relationships between total energy and other factors of production. Our next task is to extend our analysis to encompass the substitution possibilities among individual energy types.

Three energy subgroups are considered: electricity (e), oil products (o) and solid fuels (s).[2] Oil products include fuel oil, gas oil and motor gasoline while solid fuels include coal, coke and wood fuels. We have chosen to aggregate all oil products and all solid fuels because of the similar price development of the individual fuels within each group. A further disaggregation of fuel types would only increase

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[1] The sensitivity of the elasticities to the specification of homotheticity was also observed by Denny, May and Pinto (1978) for Canadian manufacturing. They found that the imposition of homotheticity decreases the elasticities of substitution. In particular, energy-capital complementarity was reduced and strong energy-labour substitutability was reversed to complementarity.

[2] Because of their limited usage, gases are excluded from the analysis.

multicollinearity problems and reduce the precision of the estimates. It should be noted that fuel oils account for the greatest part of the oil aggregate for all industries. Regarding solid fuels, coal predominates in all but the Wood, pulp and paper industry (8) and Primary metals (14) where the major solid fuels used are, respectively, wood fuels and coke.

In addition to total manufacturing, we limit our analysis to five subsectors that account for approximately 90 % of energy usage in manufacturing and 70 % of manufacturing production. Four of these are the most energy intensive Swedish industries: Wood, pulp and paper (8), Primary Metals (14), Chemicals (11) and Non-metalic mineral products (13). The last, Engineering (15), is the largest in terms of production and employment.

The share equations for the two-stage model are estimated using annual data over the period 1962-1976. The pre-1962 time period is excluded from the estimation to eliminate the remaining effects of the substitution away from solid fuels that had begun in the previous decade. This substitution of liquid for solid fuels cannot be explained solely in terms of the energy price relationships specified in our model.

The energy submodel

The first stage of our analysis involves the estimation of the energy submodel given in equation system (15) under the constraints implied by linear homogeneity in prices. The equation for solid fuels is deleted from the estimation procedure.

The estimated coefficients for the energy submodel, along with their estimated standard errors, R^2 and maximum likelihood values are given in table A4 in the appendix. We test the hypothesis that the slope coefficients (γ_{ij}) are all zero, i.e., that the cost shares for the individual fuels are independent of relative fuel prices. The likelihood ratio test statistics, which are given in the last column of the table, are significant at the 0.5 % level, so that this hypothesis can be rejected.

The partial price elasticities and elasticities of substitution corresponding to these parameter estimates are shown in table 5.[1] It should be held in mind that these elasticities are derived under the constraint that total energy input remains constant. Thus they represent only the effects of interfuel substitution, i.e. η_{ij}^{P} in (13).

One observes a high degree of similarity in the results for the different industries. Firstly, we see that the own price elasticities for oil and electricity clearly fall in the inelastic range. Of all energy components, electricity appears to be the least sensitive to price changes. This elasticity is less than .2 in absolute value for all industries, but nevertheless is found to be significantly different from zero. Oil products are somewhat more price-sensitive, with an elasticity on the order of -0.25. On the other hand, we find solid fuels to be highly price-sensitive, with elasticities of demand greater than 1.0 in absolute value in four out of the five manufacturing subsectors. The most significant exception is the low price elasticity obtained for solid fuels in the Primary metal industry (14). This can be attributed to the fact that the solid fuel component in this sector is primarly comprised of coke, which is used not as a source of energy but as a

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[1] The elasticities are calculated at the mean values of the exogenous variables. Little variation was found over the 62-76 time period. Fitted shares were positive and own-price elasticities negative for all observations included in the sample.

Table 5. Partial Price and Substitution Elasticities for the Energy Subcomponents: Electricity (e), Oil products (o) and Solid Fuels (s).

	Sector	Own Price Elasticity			Substitution Elasticity		
		e	o	s	e-o	e-s	o-s
8	Wood, pulp and paper	-0.12	-0.24	-1.39	0.22	1.00	2.28
		(0.03)	(0.34)		(0.08)	(0.21)	(0.33)
11	Chemicals	-0.09	-0.15	-1.80	-0.23	1.54	3.29
		(0.04)	(0.17)		(0.17)	(0.33)	(1.44)
13	Non-metallic mineral products	-0.12	-0.25	-1.42	-0.24	1.40	1.91
		(0.03)	(0.06)		(0.10)	(0.41)	(0.46)
14	Primary metals	-0.12	-0.26	-0.14	0.24	0.18	0.39
		(0.06)	(0.07)		(0.12)	(0.29)	(0.20)
15	Engineering	-0.20	-0.27	-1.02	0.36	1.11	1.05
		(0.03)	(0.06)		(0.07)	(0.42)	(1.07)
	Total Manufacturing	-0.16	-0.26	-0.60	0.21	0.55	0.96
		(0.03)	(0.06)		(0.06)	(0.17)	(0.30)

reduction agent. The limited possibility of replacing coke with other fossil fuels is similarly reflected in the comparatively low elasticity of substitution between solid fuels and oil products in this sector.

In the majority of the remaining subsectors, as well as in total manufacturing, we find that the most important substitution possibilities exist between oil products and solid fuels. These elasticities are particularly high for those industries in which solid fuels account for a significant proportion of total energy supply, e.g. Paper and Pulp (8) and Non-metallic minerals (13).

Regarding the relationship between oil and electricity, our results suggest marginal, but generally non-zero, substitution possibilities. The only exceptions are two cases of complementarity between electricity and oil products in the Chemical industry (11) and the Non-metallic mineral products industry (13). Although a strict complementary relationship is highly unlikely, there are reasons for expecting minimal substitutability between oil and electricity in these industries. In the Chemical industry, a large proportion of electricity is used for electrolysis and as such is indispensible. In the production of non-metallic mineral products - cement, lime etc - oil is the dominant source of thermal energy whereas electricity is chiefly a source of motive power.

Finally, our results suggest a surprisingly high degree of substitutability between electricity and solid fuels. Considering the nature of the usage of these energy forms, this result seems highly unlikely. Although there is some scope for substitution between electricity and solid fuels, we hardly expect these possibilities to outweigh those between electricity and oil products. One explanation for these results may be that the trend towards increased mechanization - and thereby electricity use - has coincided with the substitution away from solid fuels.

Aggregate energy demand

The estimates of the energy cost function for each of the subsectors are now used to generate the corresponding aggregate price indicies for energy. These, in turn, serve as instrumental variables for the price of energy in the estimation of the total (E,K,L,M) cost functions. The estimated parameters and the resulting price and substitution elasticities are shown in table A5-6 in the appendix. As homotheticity is clearly rejected for all sectors, only the results for the non-homothetic specification are presented.

The estimated demand relationships provide little information in addition to the results discussed in section 4.1, so only a few comments need to be made. Firstly, we find that the elasticity estimates for capital, labour and intermediate goods are in agreement with those presented earlier (Tables 1-4) for the 1952-1976 time period. There are, however, considerable discrepancies in the estimated own-price elasticities for energy, as well as in the magnitude of substitutability/complementarity between energy and the remaining production factors. As noted previously, an overall pattern of energy-capital complementarity and energy-labour substitutability is suggested, but again, the high standard errors of the estimates do not allow rejection of the null-hypothesis in the majority of cases.

A comparison of the aggregate energy price elasticities obtained from the two-stage estimation with those presented in section 4.1 for the non-homothetic model is given in the first two columns of table 6.[1] We find that for Primary metals (14) the elasticities are in agreement, but these must be rejected on the basis of sign in both cases, whilst for Wood, pulp and paper (8) and Total manufacturing the elasticities are not statistically significant. It is apparent, however, that the resulting elasticities for the remaining three sectors (11, 13 and 15) differ considerably for the alternative estimations.

The explanation for these discrepancies cannot be found solely on the basis of these results since the estimates are based not only on different time periods, but also on different price indices for aggregate energy. Although a thorough sensitivity analysis has not yet been carried out, our findings thus far seem to suggest that choice of observation period is the determining factor for the resulting estimates, while construction of the price index is of minor importance.[2]

A plausible explanation for the sensitivity of the estimates of the energy elasticities to estimation period may be the drastic energy price-rises from 1974 onwards. These have a greater influence on the estimates based on 1962-1976 than on those based on the longer time period. It is not obvious precisely what effects the relative up-weighting of the post-1974 time period has on the estimates and it is possible that the mere inclusion of this period has a

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[1] The elasticities are calculated at the mean values of the exogenous variables for each sample. This, however, has no relevance for the comparison since the calculated elasticities vary only slightly over time in both cases.

[2] The aggregate energy price indices based on the energy submodel estimates are, in fact, nearly identical to those constructed as simple weighted averages of the individual energy forms.

significant influence on the estimated parameters.[1] As mentioned earlier, the translog function provides only a local approximation to the underlying cost function. It is possible that the validity of this approximation may be weakened by fitting a single cost function to a period that is characterised by so vastly divergent factor prices. The effects, if any, of including the post-1974 time period could be determined by reestimating the model excluding this data and comparing the resulting parameter estimates with those obtained when the post-1974 data are included. Until a thorough investigation into the causes of the sensitivity of the elasticity estimates is carried out, our results must be interpreted with utmost caution.

With this in mind, we proceed, mainly for illustrative purposes, to calculate the total price elasticities for the individual energy components on the basis of the two-stage model. The results are presented in table 6 along with the mean cost shares for each fuel type. The partial price elasticities for the energy components presented previously are also given for the sake of comparison.

We recall that according to the assumptions of our model the total price-sensitivity (η_{ij}^{T}) of an individual fuel is determined in a bi-level adjustment process. Firstly, a change in the price of an energy component results in interfuel substitution. This effect on demand is measured in the partial price elasticities η_{ij}^{P}. Secondly, the price change affects the aggregate price index for energy, resulting in substituion between energy and other production factors. The resultant change in aggregate energy demand in turn affects the demand for the energy component. The magnitude of this effect is determined by the energy components' share of total energy costs (S_i).

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[1] The sensitivity of the estimates to the inclusion of years with rapid price changes (1972-74) is also noted by Berndt, Fuss and Waverman (1979).

From table 6 we see that for the majority of the subsectors, as well as for total manufacturing, the total price elasticities are only marginally greater than the partial. This clearly follows from the "inelasticity" of aggregate energy demand in these sectors. The total price-sensitivity of the individual fuels is therefore attributed primarly to the effects of interfuel substitution. For the Chemical industry (11), on the other hand, substitution between energy and other production factors plays a substantial role. The effect on electricity demand is particularly large due to electricity's high cost share. For solid fuels, which account for a very small part of total energy costs, the effects are minimal.

These results are meaningful, of course, only if we accept the estimates of the two-stage model. It is obvious that the estimates of aggregate elasticities based on the 1952-76 time period (column 1) would lead to somewhat different conclusions for at least three subsectors (11, 13, 15).

Table 6. Own-price elasticities for aggregate energy (η_{EE}), mean cost-shares (S_i) and partial (η_{ii}^P) and total (η_{ii}^T) own-price elasticities for the energy components. Non-homothetic total cost function.

	Aggregate Energy (η_{EE})		Electricity			Oil Products			Solid Fuels		
Sector	1952-76	Two Stage 1962-76	S_e	η_{ee}^P	η_{ee}^T	S_o	η_{oo}^P	η_{oo}^F	S_s	η_{ss}^P	η_{ss}^F
8 Wood, pulp and paper	0.08 (0.08)	-0.13 (0.08)	.62	-0.12	-0.20	.34	-0.24	-0.28	.04	-1.39	-1.40
11 Chemicals	-0.19 (0.12)	-0.57 (0.11)	.68	-0.09	-0.48	.23	-0.15	-0.28	.09	-1.80	-1.85
13 Non-metallic mineral products	-0.46 (0.11)	-0.05 (0.05)	.32	-0.12	-0.14	.51	-0.25	-0.28	.17	-1.42	-1.43
14 Primary metals	0.29 (0.16)	0.29 (0.11)	.38	-0.12	-0.12*	.18	-0.26	-0.26*	.44	-0.14	-0.14
15 Engineering	-0.57 (0.15)	-0.17 (0.14)	.57	-0.28	-0.30	.37	-0.27	-0.33	.06	-1.02	-1.03
Total manufacturing	-0.10 (0.08)	-0.09 (0.07)	.51	-0.16	-0.21	.33	-0.26	-0.29	.16	-0.60	-0.60

* Calculated with the price elasticity for aggregate energy set to 0.

5. Comparison of Results with other studies

As mentioned in the introduction, the analysis of energy demand and of the relationships between energy and other production factors has been the topic of a large number of econometric studies. It can be of interest to compare our results for Sweden with those of other studies of energy demand. For this purpose, we present in the following tables a survey of estimates of energy demand elasticities obtained by other authors and for other countries. The estimates shown in the tables are all based on cost-minimising multifactor demand models similar to those estimated for Sweden. Choice of functional form, separability assumptions, observation period and data construction vary, however, from study to study. The majority of these studies, as our own, are based on static models[illegible] some using time-series data for individual countries, others using a combination of time series and cross-section data for a number of countries or regions. The study by Denny, Fuss and Waverman (i) is based on a dynamic adjustment model which allows estimation of both short- and long-run elasticities.

In table 7 we present a comparison of estimates of the own-price elasticity for aggregate energy and the elasticities of substitution between energy and other aggregate production factors: capital (K), labour (L) and intermediate goods (M). Estimates of the elasticity of substitution between capital and labour are also shown. It is not within the scope of this paper to thoroughly discuss this enormous wealth of results, much less to analyse the apparent discrepancies amongst them. Our comments will only be brief and the interested reader is refered to the original articles for a complete description of model formulation and empirical results.

Table 7. Comparison of estimates of the own-price elasticity for energy and the elasticities of substitution between energy (E) and capital (K), labour (L) and intermediate goods (M).

Source	Country	Data				Energy own-price elasticity	Elasticity of substitution			
							E-K	E-L	E-M	K-L
a)	USA	1947-71	TS	TM		-0.47	-3.22	0.65	0.70	1.01
b)	Canada	1941-70	TS	TM	H	-	0.60	-1.28	0.37	2.26
					NH	-0.50	-11.91	4.86	0.12	5.46
c)	Canada	1961-71	CSTS	TM		-0.49	-	+	-	0.80 to 0.86
d)	Netherlands	1950-76	TS	TM		-0.16	-2.30	1.25	...	0.30
This study	Sweden	1952-76	TS	TM	H	-0.25	-1.43	0.12	0.66	0.66
					NH	-0.10	0.33	0.17	0.03	0.26
e)	9 countries	1955-69	CSTS	TM		-0.77 to -0.82	1.02 to 1.07	0.80 to 0.87	...	0.06 to 0.52
f)	10 countries	1963-73	CSTS	TM		-0.83 to -0.87	0.36 to 1.77	0.03 to 1.23	...	0.64 to 1.43
g)	USA	1971	CS	10MS		-0.54 to -1.65	-3.80/2.09[1]	+	...	+
h)	Belgium	1960-75	TS	4MS		-0.08 to -0.15	-	+	-	0.99
This study	Sweden	1952-76	TS	12MS	H	0 to -0.98	-3.73 to 2.81	-1.26 to 1.06	-0.17 to 2.08	0 to 1.60
					NH	0 to -0.67	-1.70 to 1.98	-3.40 to 4.88	-0.59 to 5.08	0.42 to 1.34
i)	USA	1948-71	TS	18MS	SR	0 to -1.09		-6.28 to 3.54	na	
					LR	-0.01 to -1.10	-22.40 to 8.04	-6.40 to 10.93		-22.40 to 8.05
i)	Canada	1962-75	CSTS	18MS	SR	0 to -1.46		-2.16 to 5.86	na	-9.00 to 18.6
					LR	-0.03 to -2.86	-9.00 to 18.60	-4.71 to 5.08		

Note: Key to table follows after table 8.
1) Physical capital and working capital respectively.

Studies (a-d) analyse factor demand relationships in total manufacturing for individual countries. Studies (a, b, d) are based on national time-series data, while (c) is based on time-series data for Canadian provinces. These results can be compared with our own for total Swedish manufacturing. Our estimates based on the homothetic specification (H) seem in closest agreement with those of the other studies, which, with the exception of (b), all assume homotheticity. We see that the results of the Canadian study (b) show a substantial sensitivity to homotheticity assumptions. As in the case with Sweden, the homothetic model is rejected on the basis of the statistical tests. Further, we find that our estimate of the own-price elasticity for energy is lower than in studies (a-c), but quite similar to that obtained for the Netherlands (d), which also includes post-1973 data in the estimation.

Studies (e, f) are based on a combination of time-series and cross-section data for total manufacturing in a sample of industrialised countries. The most striking difference between the results of the international studies and those for individual countries is that the former find energy and capital to be substitutes in total manufacturing rather than complements. The resulting own-price elasticities for aggregate energy are also somewhat higher in the international studies. The authors argue that observations across countries capture long-run adjustments whereas time-series data reflect short-run effects. Thus, they conclude that although energy and capital may be complements in the short run, they will be substitutes in the long run. These results should, however, be interpreted with care, as there may be other explanations for the contradictory findings regarding energy-capital relationships. Berndt and Wood (1979) suggest that they may be due to the fact that different elasticities are being measured. The international studies, in contrast to studies (a-c) and to our own study, omit intermediate goods (M) from the estimation, thereby assuming that intermediate goods are weakly separable from the remaining (KLE) inputs. As shown by Berndt and Wood substitution between energy and capital in the three factor (KLE) subset does not necessarily rule out overall energy-capital complementarity when the substitution relationships amongst all factors (KLEM) are considered. Even if the assumption of separability is valid, these two elasticities are equivalent only if the substitution possibilities between intermediate goods and the remaining inputs are zero.[1]

Study (g) is an attempt to resolve the controversy regarding the relationship between energy and capital. The authors maintain that an explanation to the contradictory results noted in previous studies lies in the differences in definition of capital. Studies b, e and f use a value-added approach in estimating the cost of capital, in which capital costs are defined as value-added minus labour costs. Studies a and c - as well as our own - use a service price approach, in which capital costs are defined as physical capital x service price. The value-added definition includes more than the cost of physical capital, and the authors argue that it is the difference between them, which they term the contribution of "working capital", that is the cause of the divergent results. To investigate this, the authors disaggregate the "capital" component of value added into costs for physical and working capital. The results obtained for ten manufacturing subsectors suggest that physical capital and energy are complements whereas substitutability exists between energy and working capital.

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[1] This is the case because the Allen Elasticity of substitution is a partial elasticity and is dependent on factor grouping.

The remaining studies are also based on disaggregated manufacturing subsectors. The Belgian study (h) covers only the most energy-intensive industries: Primary metals, Non-ferrous metals, Chemicals and Building materials. The low energy price elasticities obtained for these industries are not vastly different from our own findings. As in the majority of the other studies, they find that capital-energy complementarity and energy-labour substitutability predominate.

The final study (i) employs a dynamic partial-adjustment model to explain the intertemporal relationship between factor prices and input-mix. Briefly, the firm is assumed to minimise the present value of future production costs. Lags in adjustment to factor price changes are explained by the increasing marginal costs that would be incurred during rapid adjustment of the capital stock. By specifying the adjustment mechanism, both short- and long-run responses are estimated. It is difficult to adequately summarise their results. As shown in the table, both the short- and long-run elasticities fall in a wide range for the industries studied. The results indicate that, on average, in the first year after a factor price rise, firms adjust about 30-40 % of the difference between their new desired stock and the existing capital stock at the beginning of the period. The price elasticity of energy demand is less than 1 in absolute value even in the long run and differences between short- and long-run price elasticities of energy demand are generally rather small. Regarding the substitution relationship between energy-capital and energy-labour, they find a wide variety of responses across industries within each country as well as across the two countries studied.

Table 8 gives a comparison of partial price elasticities for individual energy forms. With the exception of study (i), which is based on a dynamic model formulation, all of the elasticity estimates shown are based on static models similar to the energy submodel employed in this study. The elasticities are thus partial and represent the price response due to interfuel substitution only. The breakdown of total energy differs somewhat in the various studies many include gases and some further disaggregate oil products and/or solid fuels.

Although the magnitude of the price-responses varies from study to study as well as across individual industries, a number of conclusions are evident. The general consensus seems to be that solid fuels are most sensitive to relative price changes, whereas oil and electricity are considerably less responsive to changes in relative prices. Secondly, although the results are not shown here, all studies indicate substitution possibilities between the majority of energy forms, with the most substantial substitution generally existing between solid and liquid fuels.

Table 8. Comparison of Partial Price elasticities for individual fuels

Source	Country	Data	Electricity	Oil	Solid Fuels	Gas
c)	Canada	1961-71 CSTS TM	-0.52	-1.22, -1.56[1]	-1.41	-1.21
f)	10 countries	1959-73 CSTS TM	-0.07 to -0.16	-0.08 to -0.72	-1.04 to -2.17	-0.33 to -2.31
h)	Belgium	1960-75 TS 4MS	-0.33 to -1.07	-0.57 to -1.19	-0.66 to -5.19[2] -0.33 to -2.91	-0.91 to -2.10
j)	USA	1974-75 CSTS TM	-0.13 to -0.88	-0.08 to -0.70	-0.34 to -1.91	-0.13 to -0.88
i)	Canada[3]	1962-75 CSTS 18MS	LR -0.01 to -1.77	-0.05 to -1.26	-0.64 to -2.18	-0.81 to -1.97
This study	Sweden	1962-76 TS TM	-0.16	-0.26	-0.60	...
		1962-76 TS 5MS	-0.12 to -0.20	-0.15 to -0.27	-0.14 to -1.80	...

1) fuel oil and motor gasoline respectively
2) coal and coke respectively
3) total elasticities based on a dynamic model

Key to tables 7 and 8

TS = time series data, CS = cross-section data, TM = total manufacturing, x MS = x manufacturing sectors, ... = not included in the estimation, na = included but estimates not available, H = homothetic specification, NH = non-homothetic specification, SR = short-run, LR = long-run

Sources: a) Berndt and Wood (1975)
b) Denny, May and Pinto (1978)
c) Fuss (1977)
d) Magnus (1979)
e) Griffen and Gregory (1976)
f) Pindyck (1979)
g) Field and Grebenstein (1980)
h) Bossier, Duwein and Gouzée (1979)
i) Denny, Fuss and Waverman (1980)
j) Uri (1978)

6. Summary and conclusions

In the preceeding sections we have presented demand models designed to study the interaction between energy and other aggregate production factors and to analyse interfuel substitution possibilities. Empirical implementation of these models has resulted in estimates of price and substitution elasticities for individual energy forms, aggregate energy and other aggregate production factors for total Swedish manufacturing and disaggregated manufacturing sectors. It is impossible to adequately summarise the results there is a variety of responses across industries and a number of questions concerning the sensitivity and interpretation of the estimates remain unanswered. A few tentative conclusions are, however, evident. Those most relevant to energy demand are the following:

- It is important to disaggregate manufacturing into its component industries. The magnitude and even the nature of the demand and substitution responses vary according to the production structure of the individual industry.

- Energy demand is at least somewhat sensitive to changes in its own price. The own-price elasticity is less than unity but the magnitude of response varies from industry to industry.

- Complementary relationships prevail between energy and capital, while substitutability predominates between energy-labour and energy-intermediate goods.

- Regarding the partial elasticities of the energy subcomponents, solid fuels appear to be highly price-sensitive, while the demands for petroleum products and electricity seem to be less sensitive to price variations.

- The elasticities between energy types generally indicate substitution possibilities, with the most substantial substitution existing between petroleum products and solid fuels.

Although the results presented in this study provide an insight into the complicated relationships that govern energy demand, they also illustrate the difficulties involved in estimating and interpreting these relationships. For example, the experiences with varying the homotheticity assumptions and the observation period for the aggregate demand estimations produce a number of interesting, although in some cases disconcerting, results. Regarding homotheticity, we find, on the basis of statistical tests, that the non-homothetic specification is the preferred. Although this suggests that the cost-minimising input-mix is dependent on the level of production, we feel that our model is far too simplified to justify interpreting these results very strictly. The results indicate that the estimated elasticities - and particularly those pertaining to labour - are sensitive to the specification of homotheticity. We find, however, that the price elasticities for energy and the nature of substitution relationships between energy and other aggregate production factors are, with few exceptions, quite robust to homotheticity assumptions.

Far more problematic for the analysis of energy demand is the sensitivity of the estimated energy elasticities to choice of observation period. Significant differences are found particularly for the own-price elasticity of energy as estimated on the basis of the 1952-1976 contra the 1962-1976 time periods. These results clearly emphasise the need of analysing the sensitivity of the estimates to variations in sample periods and in particular, of investigating the effects of including the drastic energy price-rises of the post 1973 period in the estimation.

Another question which requires further investigation, and which has only been touched upon in this study, is that of technological development. Although our model does allow for neutral technical change, its influence on production costs and factor demand has not been estimated. This can be done by estimating the cost function simultaneously with the share equations, thus identifying the effects on production costs of increased efficiency of factor use.

A further improvement would be to extend our model to allow for biased technical change. This is most likely a more realistic specification in view of the long time period under consideration, and would be more consistent with the results of other Swedish production function studies which indicate a significant labour-saving technical change.

The most serious shortcoming of the majority of multi-factor demand studies is the inability of the models to distinguish between short- and long-run responses and to specify the adjustment path over time. As discussed in the previous section these studies have traditionally been based on static cost-minimisation models, which are derived under the assumption that production technique is fully optimised with respect to the prevailing factor price relationships. Estimation of the model requires, therefore, a data sample that includes combinations of production techniques and factor prices which represent long-run equilibria. Historic data on actual techniques and prices hardly fulfil this requirement. Because of this, the results obtained by estimation of static models based on such data are exceedingly difficult to interpret. A strict implementation of static models on the basis of time-series data is equivalent to assuming that all inputs fully adjust to their long-run equilibrium levels within one time period (in our case, one year). As this nearly "instantaneous" adjustment is highly unrealistic - particularly in the case of physical capital - the resulting elasticities can hardly be considered to represent long-run relationships.

The necessity of incorporating intertemporal adjustment mechanisms in energy demand models is apparent. Only on the basis of such dynamic models can the adjustment process from short- to long-run be determined. The recent advances in the specification and estimation of dynamic interrelated factor demand models form an obvious point of departure for further research into the characteristics of energy demand in Swedish industry.

APPENDIX

Manufacturing subsector classification

LU	Sector	Swedish industry Nomenclature 1952-1967	ISIC(SNI)
4	Sheltered food	7a-c,e,f	3111-3112,3116-3118
5	Import-competing food	7d,g-k	3113-3115,3119 3121-3122
6	Beverages and tobacco	8	313-314
7	Textiles	9a-d,f-r 10a-d,i	32
8	Wood, pulp and paper	4, 5	33, 341
9	Printing	6	342
10	Rubber products	10g,h	355
11	Chemicals	11a-d,g-m,9e	351,352,356
13	Non-metallic mineral products	3d-k	36
14	Primary metals	2a,b	37
15	Engineering	2c-e,g-i,l,m	38 excl.3841
16	Shipbuilding	2k	3841

Data Sources

(1) Energy variables

Quantities and costs of energy consumed in Swedish manufacturing subsectors are taken from the Official Statistics of Sweden: Manufacturing, annual reports 1952-58 (Board of Trade) and 1959-1976 (National Central Bureau of Statistics-SCB). The data include quantities (1952-1976) and costs (1962-1976) for individual fuels: motor gasoline, fuel oils, gas oil, coal, coke and wood fuels, costs (1952-1976) for aggregate fuels and quantities and costs (1952-1976) for electricity. Fuels and electricity produced and used at the same plant are not included. Most data pertain to establishments with five or more persons employed.

Expentitures for electricity and total fuels consumed, in current and constant prices, were also supplied by the National Accounts Department of the National Central Bureau of Statistics (SCB). These are based on the Manufacturing statistics above, but in addition include information on establishments with less than five employees.

Prices for electricity and each fuel are calculated for each subsector on the basis of the costs and quantities obtained from the manufacturing statistics. Since costs for individual fuels were not available for the pre-1962 time period, the subsector fuel prices for these years were constructed using average fuel prices for industrial consumers. This was done assuming that the relationship between sector price and average price for each fuel noted for the 1962-1970 time period was the same for 1952-1961.

Sources for average energy prices are:

Oils and motor gasoline - Swedish Petroleum Institute: *En bok om olja* (1970).

Coal, coke and wood fuels - implicit import prices calculated from the Official Statistics of Sweden: *Foreign Trade*, annual reports 1950-1976 (SCB).

The prices for energy aggregates in each sector are calculated as a weighted average of the prices of the individual energy forms.

(2) Non-energy variables

Data on labour, capital, material inputs and production volume were provided by the Industrial Institute for Economic and Social Research.

Labour

Total labour costs are taken from the National Accounts of Sweden (SCB). Total labour costs include wages plus social security charges, wage fees paid by employers, etc. The price of labour is taken as the total wage cost, including the above benefits.

Capital

Data on capital stock in current and constant prices are taken from the National Accounts of Sweden (SCB). The user cost of capital p^K is calculated[1] as

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[1] A detailed discussion of procedures used in constructing the expected rate of return is found in Bergström (1976).

$p^K = p^I(r+\delta)$

where

p^I = price of investment goods (branch specific)

r = expected rate of return

δ = depreciation rate (branch specific)

Capital costs are obtained by multiplying capital stock by the user price of capital.

Intermediate goods

Data on costs for goods and services in each sector in current and constant prices are taken from the National Accounts of Sweden (SCB). Costs for intermediate goods are obtained by subtracting energy costs. Implicit price indices for intermediate goods are formed by using the current and constant price data adjusted for energy inputs.

Production volume

Data on gross production in producers' prices for each sector are obtained from the National Accounts of Sweden (SCB). Output indices are defined as production in constant prices.

Table A1. Translog Cost Function Parameter Estimates. Homothetic Cost Function. Estimation period 1952-1976.

Sector	α_E	γ_{EE}	γ_{EK}	γ_{EL}	γ_{EM}	α_K	γ_{KK}	γ_{KL}	γ_{KM}	α_L	γ_{LL}	γ_{LM}	α_M	γ_{MM}	R^2_E	R^2_K	R^2_L	LogL
4 Sheltered food	.0150	.0110	.0010	-.0010	-.0110	.0469	.0350	.0030	-.0390	.1544	.0400	-.0420	.7837	.0920	.80	.97	.87	372.36
	(.0005)	(.0020)	(.0007)	(.0005)	(.0015)	(.0007)	(.0009)	(.0006)	(.0014)	(.0033)	(.0030)	(.0030)						
5 Import-competing food	.0139	.0062	-.0029	.0001	-.0036	.0583	.0572	-.0073	-.0471	.1549	.0278	-.0206	.7729	.0711	.71	.94	.86	343.11
	(.0003)	(.0012)	(.0006)	(.0003)	(.0013)	(.0015)	(.0027)	(.0014)	(.0030)	(.0022)	(.0022)	(.0020)						
6 Beverage and tobacco	.0207	.0242	-.0042	-.0171	-.0029	.1183	.0856	.0152	-.0966	.2520	.0025	-.0006	.6090	.1001	.90	.93	.20	283.20
	(.0008)	(.0058)	(.0017)	(.0016)	(.0048)	(.0026)	(.0046)	(.0025)	(.0045)	(.0061)	(.0063)	(.0050)						
7 Textiles and clothing	.0153	.0001	-.0072	-.0041	.0111	.0819	.0551	.0031	-.0510	.3639	.0483	-.0473	.6349	.0871	.58	.90	.79	315.41
	(.0007)	(.0029)	(.0015)	(.0008)	(.0035)	(.0018)	(.0044)	(.0019)	(.0056)	(.0046)	(.0049)	(.0036)						
8 Wood, pulp and paper	.0467	.0431	-.0086	-.0098	-.0246	.1061	.0731	.0053	-.0699	.2443	.0330	-.0285	.6029	.1230	.84	.94	.51	296.50
	(.0007)	(.0048)	(.0015)	(.0010)	(.0052)	(.0018)	(.0036)	(.0021)	(.0044)	(.0047)	(.0057)	(.0044)						
9 Printing	.0073	.0034	-.0029	.0002	-.0008	.1040	.0627	-.0355	-.0243	.4827	.0540	-.0187	.4061	.0484	.27	.78	.91	319.76
	(.0024)	(.0008)	(.0005)	(.0005)	(.0012)	(.0047)	(.0081)	(.0063)	(.0106)	(.0026)	(.0061)	(.0112)						
10 Rubber products	.0250	.0116	-.0038	-.0047	-.0038	.1279	.0955	-.0104	-.0812	.4037	.0775	-.0624	.4434	.1468	.46	.85	.93	273.66
	(.0011)	(.0042)	(.0031)	(.0012)	(.0031)	(.0039)	(.0124)	(.0039)	(.0145)	(.0045)	(.0045)	(.0061)						
11 Chemicals	.0454	.0320	-.0059	-.0142	-.0120	.0854	.0753	-.0321	-.0371	.2882	.0527	-.0371	.5810	.0550	.82	.96	.68	293.15
	(.0005)	(.0057)	(.0011)	(.0022)	(.0043)	(.0019)	(.0039)	(.0020)	(.0044)	(.0055)	(.0060)	(.0072)						
13 Non-metallic mineral products	.0675	.0389	-.0174	-.0321	.0106	.1264	.0929	-.0401	-.0354	.3505	.0151	.0570	.4556	-.0311	.72	.80	.55	262.40
	(.0030)	(.0078)	(.0047)	(.0032)	(.0106)	(.0053)	(.0083)	(.0056)	(.0099)	(.0044)	(.0046)	(.0060)						
14 Primary metals	.0830	.0949	-.0185	-.0273	-.0491	.1178	.0891	-.0128	-.0578	.2366	.0275	.0126	.5626	.0942	.72	.94	.43	260.10
	(.0027)	(.0106)	(.0047)	(.0034)	(.0123)	(.0024)	(.0072)	(.0033)	(.0114)	(.0052)	(.0073)	(.0071)						
15 Engineering	.0129	.0053	-.0025	-.0052	.0025	.0582	.0570	-.0236	-.0308	.3736	.0258	.0031	.5553	.0252	.92	.91	.60	319.50
	(.0025)	(.0019)	(.0006)	(.0003)	(.0021)	(.0023)	(.0041	(.0026)	(.0050)	(.0040)	(.0049)	(.0043)						
16 Shipbuilding	.0103	.0047	-.0015	-.0022	-.0010	.0535	.0653	-.0124	-.0514	.3275	.0110	.0036	.6087	.0487	.85	.95	.01	316.69
	(.0003)	(.0010)	(.0003)	(.0003)	(.0013)	(.0022)	(.0027)	(.0027)	(.0052)	(.0074)	(.0096)	(.0109)						
Total Manu-facturing	.0296	.0209	-.0071	-.0076	-.0062	.0819	.0610	-.0086	-.0452	.3006	.0420	-.0263	.5879	.0777	.70	.95	.81	332.10
	(.0009)	(.0026)	(.0012)	(.0009)	(.0035)	(.0018)	(.0029)	(.0021)	(.0039)	(.0033)	(.0039)	(.0024)						

Note: Asymptotic standard errors are given in parenthesis. As the equation for intermediate goods was excluded from the estimation, standard errors for α_M and γ_{MM} and $R^2{}_M$ are not readily available.

Table A2. Translog Cost Function Parameter Estimates. Non-homothetic Cost Function. Estimation period 1952-1976.

Sector	α_E	γ_{EE}	γ_{EK}	γ_{EL}	γ_{EM}	α_K	γ_{KK}	γ_{KL}	γ_{KM}	α_L	γ_{LL}	γ_{LM}	α_M	γ_{MM}	γ_{QE}	γ_{QK}	γ_{QL}	γ_{QM}	R^2_E	R^2_K	R^2_L	LogL
4 Sheltered food	.0155	.0107	.0005	-.0057	-.0170	.0463	.0348	-.0017	-.0336	.1589	.0843	-.0884	.7793	.1390	-.0330	.0210	-.2100	-.2220	.93	.98	.95	391.36
	(.0003)	(.0015)	(.0005)	(.0019)	(.0013)	(.0006)	(.0009)	(.0020)	(.0020)	(.0021)	(.0073)	(.0067)			(.0078)	(.0093)	(.0339)					
5 Import-competing food	.0144	.0066	-.0025	.0050	-.0091	.0595	.0564	.0020	-.0559	.1615	.0904	-.0975	.7645	.1624	-.0070	-.0135	-.0877	.1082	.89	.94	.76	247.17
	(.0004)	(.0012)	(.0006)	(.0025)	(.0028)	(.0055)	(.0028)	(.0038)	(.0025)	(.0159)	(.0115)	(.0116)			(.0034	(.0055)	(.0166)					
6 Beverage and tobacco	.0218	.0303	-.0026	-.0333	.0056	.1208	.0926	-.0130	-.0762	.2382	.1764	-.1302	.6192	.2008	.0301	.0589	-.3526	.2636	.92	.94	.57	298.61
	(.0009)	(.0072)	(.0016)	(.0078)	(.0056)	(.0026)	(.0050)	(.0074)	(.0057)	(.0043)	(.0024)	(.0204)			(.0134)	(.0156)	(.0474)					
7 Textiles and clothing	.0180	.0056	-.0025	-.0021	-.0010	.0878	.0643	.0094	-.0711	.3612	.0448	-.0521	.5331	.1241	-.0216	-05379	.0174	.0579	.90	.97	.76	329.07
	(.0005)	(.0025)	(.0013)	(.0006)	(.0032)	(.0014)	(.0036)	(.0017)	(.0048)	(.0067)	(.0074)	(.0065)			(.0033)	(.0107)	(.0444)					
8 Wood, pulp and paper	.0464	.0458	-.0058	-.0137	-.0264	.1060	.0777	-.0081	-.0639	.2448	.1585	-.1367	.6029	.2271	.0054	.0204	-.1819	.1561	.86	.95	.85	321.53
	(.0007)	(.0035)	(.0013)	(.0052)	(.0058)	(.0017)	(.0036)	(.0033)	(.0048)	(.0028)	(.0180)	(.0147)			(.0071)	(.0076)	(.0247)					
9 Printing	.0073	.0033	-.0023	-.0019	.0001	.1064	.0842	-.0370	-.0449	.4889	.1747	-.1365	.3974	.1813	.0040	.0576	-.2241	.1625	.44	.87	.94	338.19
	(.0002)	(.0009)	(.0004)	(.0021)	(.0028)	(.0033)	(.0072)	(.0051)	(.0060)	(.0024)	(.0234)	(.0264)			(.0030)	(.0177)	(.0386)					
10 Rubber products	.0242	.0089	-.0079	-.0081	.0072	.1284	.0834	-.0151	-.0605	.4215	.1764	-.1533	.4259	.2066	.0062	.0143	-.1401	.1197	.25	.82	.94	296.00
	(.0014)	(.0047)	(.0026)	(.0049)	(.0072)	(.0041)	(.0079)	(.0086)	(.0058)	(.0042)	(.0115)	(.0123)			(.0068)	(.0122)	(.0162)					
11 Chemicals	.0441	.0349	-.0046	-.0072	-.0230	.0823	.0770	-.0142	-.0581	.2641	.2028	-.1814	.6095	.2626	-.0090	-.0207	-.1695	.1992	.88	.96	.99	340.57
	(.0006)	(.0057)	(.0011)	(.0029)	(.0053)	(.0019)	(.0036)	(.0023)	(.0035)	(.0014)	(.0051)	(.0063)			(.0026)	(.0030)	(.0057)					
13 Non-metallic mineral products	.0673	.0353	-.0200	.0302	.0150	.1281	.0909	-.0272	-.0437	.3569	.0560	.0014	.4477	.0273	-.0052	-.0213	-.0522	.0787	.72	.81	.45	266.19
	(.0034)	(.0085)	(.0064)	(.0104)	(.0123)	(.0055)	(.0111)	(.0105)	(.0102)	(.0044)	(.0182)	(.0202)			(.0163)	(.0206)	(.0273)					
14 Primary metals	.0823	.0918	-.0096	-.0202	-.0620	.1197	.0869	-.0160	-.0613	.2466	.1250	-.0888	.5514	.2121	-.0053	.0031	-.1058	.1080	.99	.94	.74	276.99
	(.0029)	(.0125)	(.0052)	(.0152)	(.0135)	(.0024)	(.0051)	(.0089)	(.0083)	(.0038)	(.0256)	(.0227)			(.0152)	(.0095)	(.0257)					
15 Engineering	.0127	.0063	-.0022	-.0028	-.0013	.0577	.0574	-.0176	-.0376	.3620	.1638	-.1435	.5676	.1824	-.0024	-.0059	-.1350	.1433	.92	.91	.71	330.77
	(.0004)	(.0022)	(.0006)	(.0026)	(.0042)	(.0020)	(.0046	(.0073)	(.0050)	(.0039)	(.0242)	(.0237)			(.0026)	(.0078)	(.0244)					
16 Shipbuilding	.0100	.0057	-.0018	.0003	-.0041	.0519	.0648	-.0126	-.0504	.3065	.1290	-.1166	.6316	.1711	-.0030	-.0021	-.1424	.1475	.92	.95	.62	328.49
	(.0003)	(.0015)	(.0004)	(.0009)	(.0015)	(.0023)	(.0042)	(.0083)	(.0078)	(.0055)	(.0229)	(.0209)			(.0011)	(.0100)	(.0272)					
Total Manufacturing	.0301	.0256	-.0019	-.0067	-.0170	.0827	.0683	-.0197	-.0466	.3009	.1309	-.1046	.5863	.1682	-.0008	.0175	-.1291	.1124	.80	.96	.89	343.96
	(.0006)	(.0026)	(.0015)	(.0065)	(.0080)	(.0013)	(.0027)	(.0056)	(.0047)	(.0027)	(.0240)	(.0236)			(.0092)	(.0081)	(.0334)					

Note: Asymptotic standard errors are given in parenthesis. As the equation for intermediate goods was excluded from the estimation, standard errors for α_M, α_{MM} and γ_{QM} and $R^2{}_M$ are not readily available.

Table A3. Likelihood Ratio Test Statistics

		Hypothesis	
	Sector	Cobb-Douglas Production Structure $H_o: \gamma_{ij}=0$ K,L,E,M	Homotheticity $H_o: \gamma_{iq}=0$ K,L,E,M
4	Sheltered food	217.66	38.00
5	Import-competing	143.82	28.70
6	Beverage and tobacco	129.44	30.82
7	Textiles and clothing	137.98	27.32
8	Wood, pulp and paper	142.92	50.06
9	Printing	96.02	36.86
10	Rubber products	143.94	44.68
11	Chemicals	150.80	94.84
13	Non-metallic mineral products	122.04	7.58
14	Primary metals	116.12	33.78
15	Engineering	126.10	22.54
16	Shipbuilding	134.12	23.60
	Total Manufacturing	171.78	23.72
	degrees of freedom:	6	3
	χ^2 values		
	degrees of freedom:	6	3
	significance level:		
	.005	18.55	12.84
	.01	16.81	11.34
	.05	12.59	7.81

Table A4. Two-stage translog cost function parameter estimates: energy submodel. Estimation period 1962-1976.

Sector	α_e	γ_{ee}	γ_{eo}	γ_{es}	α_o	γ_{oo}	γ_{os}	α_s	γ_{ss}	R^2_e	R^2_o	LogL	$-2(L_R/L_U)$
8 Wood, pulp and paper	.5349 (.0090)	.1627 (.0152)	-.1627 (.0180)	-.0001 (.0055)	-.4183 (.0107)	.1438 (.0217)	-.0192 (.0064)	.0468	.0193	.87	.82	102.93	46.68
11 Chemicals	.6154 (.0110)	.1585 (.0249)	-.1925 (.0261)	.0340 (.0206)	.3055 (.0114)	.1430 (.0408)	.0495 (.0309)	.0791	-.0835	.82	.84	86.36	33.22
13 Non-metallic mineral products	.2439 (.0037)	.1811 (.0071)	-.2030 (.0141)	.0218 (.0157)	.5982 (.0089)	.1243 (.0384)	.0787 (.0434)	.1579	-.1005	.97	.88	89.70	64.44
14 Primary metals	.2970 (.0103)	.1885 (.0216)	-.0513 (.0082)	-.1373 (.0233)	.2086 (.0038)	.0990 (.0129)	-.0477 (.0164)	.4944	.1850	.86	.90	93.92	51.70
15 Engineering	.5111 (.0077)	.1315 (.0136)	-.1351 (.0155)	.0035 (.0123)	.4322 (.0081)	.1340 (.0325)	.0011 (.0263)	.0567	-.0046	.87	.82	86.92	31.34
Total Manufacturing	.4346 (.0058)	.1692 (.0120)	-.1326 (.0106)	-.0366 (.0132)	.3878 (.0052)	.1347 (.0216)	-.0021 (.0204)	.1776	.0387	.94	.91	96.76	47.88

Note: Asymtotic standard errors are given in parenthesis. As the equation for solid fuels was excluded from the estimation, standard errors for α_s and γ_{ss} and $R^2{}_s$ are not readily available.

1) likelihood ratio test statistics for Ho: $\gamma_{ij} = 0$ for all e,o,s

$\chi^2 = 12.84$ for significance level 0.005 and 3 degrees of freedom.

Table A5 Two-stage translog cost function parameter estimates: total cost function

Non-homothetic specification. Estimation period 1962-1976

Sector	α_E	γ_{EE}	γ_{EK}	γ_{EL}	γ_{EM}	α_K	γ_{KK}	γ_{KL}	γ_{KM}	α_L	γ_{LL}	γ_{LM}	α_M	γ_{MM}	γ_{QE}	γ_{QK}	γ_{QL}	γ_{QM}	R^2_E	R^2_K	R^2_L	LogL
8 Wood, pulp	.0482	.0350	-.0064	-.0108	-.0179	.1070	.0732	-.0034	-.0633	.2394	.1444	-.1301	.6055	.2110	.0090	.0181	-.1972	.1701	.97	.96	.83	212.76
and paper	(.0004)	(.0039)	(.0011)	(.0032)	(.0069)	(.0018)	(.0038)	(.0039)	(.0029)	(.0019)	(.0094)	(.0092)			(.0051)	(.0078)	(.0135)					
11 Chemicals	.0445	.0171	-.0065	-.0109	.0003	.0809	.0712	-.0143	-.0504	.2664	.1832	-.1580	.6082	.2080	-.0022	-.0195	-.1543	.1760	.94	.96	.98*	214.11
	(.0004)	(.0049)	(.0011)	(.0026)	(.0056)	(.0017)	(.0044)	(.0036)	(.0029)			(.0074)	(.0013)	(.0095)	(.0024)	(.0041)		(.0068)				
13 Non-metallic	.0802	.0575	-.0127	.0031	-.0479	.1361	.0706	-.0465	-.0465	.3592	.0526	-.0444	.4245	.1388	-.0008	-.0152	-.1194	.1354	.92	.74	.16	191.32
mineral products	(.0011)	(.0041)	(.0023)	(.0049)	(.0066)	(.0021)	(.0075)	(.0043)	(.0043)	(.0029)	(.0134)	(.0126)			(.0099)	(.0321)	(.0379)					
14 Primary metals	.0888	.0848	-.0063	-.0134	-.0651	.1235	.0760	-.0327	-.0369	.2382	.1009	-.0549	.5496	.1569	.0084	.0410	-.1444	.0950	.93	.96	.91*	202.15
	(.0017)	(.0077)	(.0034)	(.0077)	(.0078)	(.0020)	(.0041)	(.0037)	(.0054)			(.0113)	(.0025)	(.0173)	(.0093)	(.0075)		(.0130)				
15 Engineering	.0133	.0113	-.0015	-.0018	-.0080	.0654	.0499	-.0225	-.0259	.3525	.1303	-.1060	.5688	.1399	-.0023	.0186	-.1474	.1311	.96	.97	.91	237.15
	(.0002)	(.0019)	(.0005)	(.0021)	(.0041)	(.0011)	(.0023	(.0035)	(.0037)	(.0019)	(.0158)	(.0174)			(.0018)	(.0042)	(.0151)					
Total Manu-	.0321	.0242	-.0032	-.0098	-.0112	.0859	.0628	-.0190	-.0407	.2927	.1400	-.1113	.5892	.1632	.0090	.0329	-.1829	.1410	.99	.98	.95	244.72
facturing	(.0002)	(.0008)	(.0005)	(.0023)	(.0028)	(.0009)	(.0021)	(.0032)	(.0036)	(.0012)	(.0152)	(.0159)			(.0033)	(.0054)	(.0021)					

* R^2 for equation for intermediate goods.

Note: Asymptotic standard errors are given in parenthesis. As the equation for labour in sectors 11 and 14 and that for intermediate goods in the remaining sectors were excluded from the estimation, the respective standard errors and R^2 are not readily available.

Table A6. Own-price elasticities and elasticities of substitution for energy (E), capital (K), labour (L) and Intermediate goods (M). Two-stage non-homothetic specification

	Own-Price Elasticities				Elasticities of Substitution					
Sector	E	K	L	M	E-K	E-L	E-M	K-L	K-M	L-M
8 Wood, pulp	-0.13	-0.28	-0.16	-0.05	-0.24	-0.08	0.29	0.88	0.14	0.09
and paper	(0.08)	(0.03)		(0.02)	(0.21)	(0.34)	(0.26)	(0.12)	(0.04)	(0.06)
11 Chemicals	-0.57	-0.23	-0.06	-0.06	-0.35	0.11	1.01	0.51	0.18	-0.01
	(0.11)	(0.04)		(0.01)	(0.21)	(0.24)	(0.23)	(0.14)	(0.05)	(0.04)
13 Non-metallic	-0.05	-0.38	-0.50	-0.25	-0.28	1.14	-0.69	0.78	0.30	0.71
mineral products	(0.05)	(0.05)	(0.04)		(0.20)	(0.17)	(0.20)	(0.17)	(0.07)	(0.08)
14 Primary metals	0.29	-0.32	-0.32	-0.16	0.35	0.17	-0.68	0.58	0.53	0.58
	(0.11)	(0.03)		(0.03)	(0.31)	(0.50)	(0.21)	(0.08)	(0.60)	(0.08)
15 Engineering	-0.17	-0.26	-0.28	-0.20	-0.48	0.64	-0.06	0.18	0.37	0.47
	(0.14)	(0.04)	(0.05)		(0.47)	(0.42)	(0.28)	(0.13)	(0.09)	(0.07)
Total Manu-	-0.09	-0.24	-0.23	-0.13	-0.23	0.25	0.30	0.30	0.27	0.35
facturing	(0.03)	(0.02)	(0.05)		(0.29)	(0.29)	(0.17)	(0.11)	(0.06)	(0.09)

Note: Approximate asymptotic standard errors are given in parenthesis. As the share equation for labour in sectors 11 and 14 and the share equation for intermediate goods in the remaining sectors were excluded from the estimation, standard errors are not readily available.

Table A7. Likelihood Ratio Test Statistics. Two-stage model. Total Cost function.

Sector	Cobb-Douglas Production Structure $H_o: \gamma_{ij}=0$ K,L,E,M	Homotheticity $H_o: \gamma_{iq}=0$ K,L,E,M
8 Wood, pulp and paper	41.87	22.53
11 Chemicals	44.58	30.05
13 Non-metallic mineral products	45.76	11.87
14 Primary metals	38.71	34.13
15 Engineering	52.07	17.39
Total Manufacturing	57.55	23.14
Degrees of freedom	6	3
χ^2 values		
Significance level:		
.005	18.55	12.84
.01	16.81	11.34
.05	12.59	7.81

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