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by

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Abstract:

Within the framework given by the theory of optimal income taxation this paper investigates the progressivity of the Swedish income tax. On the assumption that taxes distort labour leisure choice some tax reforms are designed that improve social welfare while keeping tax revenues unchanged. The instrument used in the analysis is an extended version of a model for simulation of the Swedish system of personal income taxation earlier developed by the authors.

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Introduction

In this paper we will investigate what kind of implications the theory of optimal income taxation yields for the graduation of the income tax schedule in Sweden.

The optimal income tax problem is one way of formalizing the trade-off between equality and efficiency, that the authorities (should) bear in mind when deciding on the progressiveness of the income tax schedule. The trade off problems considered in the litterature are of two kinds:

(i) between equity and efficiency losses due to distortions of labour-leisure choice. (See e.g. Diamond [4], Mirrlees [9], Phelps [11].)

(ii) between equity and distortions of the incentives to invest in human capital. (See e.g. Atkinson [2], Phelps [11], and Sheshinski [13].)

So far there are few works where these trade-offs have been studied in connection with an actual tax system.[1)]

We will, however, study the first mentioned trade-off problem in connection with the Swedish system of personal income taxation. Even though we cast the problem into an optimal taxation mould, we do not intend to find the optimal tax schedule. Instead we will try to find welfare improving tax reforms.

The instrument used in this analysis is an extended version of

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1) The only examples we know of are Bruno and Habib [3] and Rosen [13]. None of these works however did primarily investigate the rate structure of the tax system.

the model for simulation of the Swedish system of personal income taxation first presented in Jakobsson & Normann [7]. The original simulation model belongs to a class of models with explicit public parameters that by now is quite common.[2)] This article might be seen as an attempt to indicate how these models can be extended to include behavioural relations, which opens up the possibility of using them for a broader range of problems than today.

The first section of the article is a description of the model used. We start by presenting the original simulation model by which tax revenues at the individual and aggregate levels can be computed. The original model provides us with one of the essential features of the optimal tax problem, namely a tax function defined on individual income. This model is then extended to encompass the other main ingredients of the optimal income tax problem as posed by Mirrlees [9]. These are individual utility functions defined on consumption and leisure, a skill distribution, a social welfare function defined on individual utilities, and a production relation. We give a fairly detailed description of how this extension is made in the last part of section 1.

To find the optimal tax system, the social welfare function is maximized subject to two constraints. The first is that the individual maximizes his utility subject to his income constraint. The second is that the total labour supplied can produce the total quantity of goods demanded. Welfare improving tax reforms will analogously be tax changes that improve social welfare subject to these two constraints. Sections 2 and 3 of the article are devoted to finding that kind of tax changes,

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2) For early examples or models of this type see Pechman [10]; Rechtenwald [12].

where the present Swedish tax system is the initial state. This is done by simulation in the extended tax model.

We find that under the assumptions usually made in the litterature on optimal income taxation progression in the Swedish income tax should be decreased. The most striking result is that all statutory marginal tax rates should be diminished in brackets above 30000 Sw Cr (c:a 7500 $) which is well below the median income. The main explanation for this turns out to be a "perverse revenues" effect. Revenues will actually be increased when marginal tax rates are diminished. The extra revenues could be used for introducing a lump sum transfer. This combination of parameter changes will obviously increase the utility for everybody. Therefore the specification of the social welfare function is not important for the result mentioned, as long as we restrict ourselves to Paretian functions.

What is important, however, is the labour supply response to a change in marginal tax rates, since this response obviously is crucial for the "perverse revenue effect." In section 4 we investigate how sensitive this effect is to different assumptions on the elasticity of substitution (σ) between consumption and leisure in the individual utility function. It is found that this effect appears in most rate brackets for $\sigma \geq 0.4$.

In the last section we briefly discuss what kind of conclusions can be drawn from our results.

1. Model description

The original model consists of two parts namely a micro part and an aggregative part. The former part is constructed to compute the tax for a random individual. The individuals were partitioned in ten categories such that all individuals in a category are treated at least approximately equal by the tax laws. The categories are of the type single persons (age 17-66) without children, married men (age 17-66) and so on. An individual is in the model characterized not only by the category he belongs to but also by the level of his income before tax. Thus the micro model is an algorithm that for a given set-up of public parameters computes the tax for an individual on the basis of two pieces of information of him, namely: (1) the individual's level of income before tax; (2) the category the individual belongs to.

As can be seen from figure 1 the micro model is the place where the public parameters are introduced. Jakobsson & Normann [7] give a short description[3)] of how the tax laws were formalized and to some extent simplified so that they could be integrated in the model.

If we consider a specific category a condensed description of the micro-model is given by:

$$t = F(y; P) \tag{1}$$

t = individual tax payments

y = individual income before tax[4)]

P = set of deduction and tax parameters

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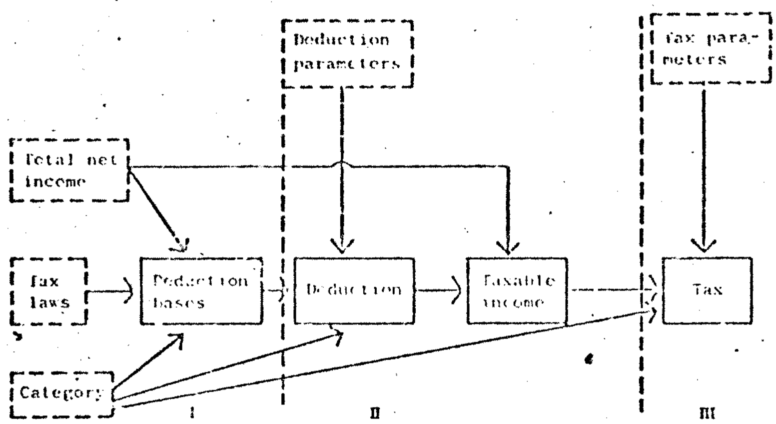
3) For a full description, see Jakobsson & Normann [8].

4) The income before tax concept used here is total net income (sammanraknad nettoinkomst). Our choice of this concept that is defined by the tax law has been dictated by the existing data on income distribution.

Figure 1. Chart of the micro-model.



To get from (1) to a macro-relation between income and taxes an aggregation procedure is introduced. The one we have used relies on knowledge of the income distributions in different categories. Still considering a specific category the total tax (T), paid by the category is given by

$$T = N \cdot \int_{y_{min}}^{y_{max}} F(y;P) \cdot \Psi'(y)dy \qquad (2)$$

where N = number of persons in the category, $\Psi'(y)$ = density function of incomes in the category.

In this simulation model it is possible to distinguish and compare the effects on e.g. revenues and income distribution after tax of different specified changes in the parameter set. The level and distribution of income before taxes also appear explicitly so the built-in-flexibility of the tax system can be investigated. An important limitation of the model, however, is that income before tax is exogenous. By introducing, in the micro-model, utility maximizing choice between labour and leisure on part of individuals, this assumption is relaxed in the present version of the model.

1.1. Individual behavior

The assumptions on individual behaviour made here are those of standard labour leisure analysis. We will thus assume that individuals have identical preferences and try to maximize their utility. It is also assumed that consumptions of goods (C) and consumption of leisure (L) enter a utility function U(C;L). Each individual makes his choice (C;L) in light of his budget constraint which can be written

$$C = f(wH;P) \equiv wH - F(wH;P) \qquad (3)$$

H = hours worked (H = Q-L; Q = hours available)

w = wage rate

f represents the function from income before tax to income after tax.

The formulation of the budget constraint implies two assumptions, both common in the optimal tax litterature:

(i) Savings are ignored

(ii) Other income than wage income is ignored, i.e. y = wH.

In order to make a quantitative analysis it is necessary to be more specific on the form of the individual utility function. We have here chosen the standard assumption that the utility function is of the Cobb-Douglas type. In a special section we will discuss how sensitive our basic results are to this assumption.

On the assumption that the individual tries to maximize his utility, he will face the following optimum problem:

$$\mathrm{Max}_U = C^{\alpha}(Q-H)^{1-\alpha} \qquad \text{subject to } C = f(wH;P) \tag{4}$$

The optimal labour supply of the individual will be

$$H = \frac{Q \cdot e \cdot \alpha}{1 - \alpha + e \cdot \alpha} \qquad (5)^{5)}$$

where

$$e = \frac{f_1(wH;P) \cdot wH}{f(wH;P)} = \text{residual progression}^{6)}.$$

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5) Q stands for maximal labour supply. Supposing that there is a limit at 16 hours per day every day, we get for a full year Q = 5,840. To get realistic values on labour supply we have chosen $\alpha = 0.33$. Experimentation with different values on α indicates that our results are not sensitive to changes in α.

6) For a discussion of this concept see Jakobsson [6].

If we suppose that the wage rate (w) for each individual is given exogenously then (5) in principle can be solved for H, provided that f is completely specified. Furthermore it is clear that to each specific set of public parameters (P) we get a related solution for H. So (t) defines a function from (w;P) to H or

$$H = g^1(w;P) \tag{6}$$

By (6), the budget-restriction (3), and the utility function we get

$$U = g^2(w;P) \tag{7}$$

Since we are assuming that y = wH, we also get by (6) and (1) individual tax payments

$$t = g^3(w;P) \tag{8}$$

1.w. Aggregation over wage rates

A basic difference between the micro-model defined by (1) and that defined by the preceding equations is that the wage rate is exogeneous in the latter while income is exogeneous in the original model. From the empirical point of view this represents a difficulty since the only information we have got on individuals is the distribution of income. In order to aggregate the model (6)-(8) it is therefore necessary to relate individual income in the initial position to wage rates. This is done by (5). At the existing tax system we can observe income distribution before tax. Formula (5) then relates each income to a specific value on H. Since y = wH, we there also get a specific wage rate associated with each income level in the initial stage. Thereby we get from the observed income distribution a distribution of wage rates that is exogenously given in the model and constant throughout the experiments carried out here. By this device we get for a specific category aggregate tax payments as

$$T = \int_{w_{min.}}^{w_{max.}} g^3(w;\ ;P)\ \Phi'(w)dw \tag{9}$$

where $\Phi'(w)$ is the "derived" distribution of wages. We will assume that this distribution is equivalent to the skill distribution in the optimal income tax problem. Concerning production we adopt the assumption that the production of each worker equals his wage.

1.3 The social welfare function

A central element for the whole concept of an optimal tax schedule is an interpersonal comparison of utilities. The valuation of utilities for different persons is made by a social welfare function. The proper specification of this function is of course a very difficult problem. We have, however, chosen the form most commonly used in the literature on optimal taxation, namely addition of individual utilities raised to the power of $1-\varepsilon$, where ε could be interpreted as social inequality aversion (Atkinson [1]) $(\frac{U^{1-\varepsilon}}{1-\varepsilon}\ ;\ \varepsilon \geq 0;\ \varepsilon \neq 1)$. By this function we have social welfare

$$W = \frac{1}{1-\varepsilon} \int_{w_{min.}}^{w_{max.}} [g^2(w;\ ;P)]^{(1-\varepsilon)}\ \phi'(w)dw \tag{10}$$

Restricted as this form might seem it still allows for a wide range of social preference orderings. Included are the strictly utilitarian approach (ε=0) and the Rawlsian welfare function, maxi-min, ($\varepsilon \to \infty$). This illustrates the well-known fact that the sensitivity of the function W to changes in different parts of the distribution is affected by the value of the parameter ε. The higher the value of ε the larger is the weight given to changes in the lower part of the distribution. A higher value does also increase the general sensitivity for inequality.

By (10) our extended simulation model is complete and it will now be used to investigate what effects we get when public parameters are changed. By simulations with the model we compute partial derivatives of H, U, t (individual level), W and T (aggregate level) with respect to specific public parameters P_j.

2. Simulation results

All simulations are restricted to the category married men (wife not assessed) in active ages. Important for our analysis is that in this category a very high fraction of total income is wage income. Table 1 gives for this category average pre-tax income in each income class (1975) and corresponding average and marginal effective tax rates in the 1975 tax system.

The policy instruments we are going to consider are the statutory marginal tax rates at national taxation, the local tax rates and the basic tax deduction. In addition to these existing parameters we consider the effects of the introduction of a lump-sum transfer equal to all persons in the distributions.

2.1 Effects on the individual

On the individual level we can for a specific wage rate, according to (6)-(8) compute $\frac{\partial H}{\partial P_i}$; $\frac{\partial U}{\partial P_i}$; $\frac{\partial t}{\partial P_i}$ etc. Before we report on the results

Table 1. Tax rates and income distribution for married (wife not assessed) in 1975

Income class	Relative frequency of tax payers %	Pre-tax mean income Sw.Cr.	Average tax rate %	Marginal tax rate %	Residual progression[a]
1	2.2	118	0	0	1.00
2	0.1	2 801	0	0	1.00
3	2.0	9 076	0	31	0.69
4	4.1	14 411	9	31	0.76
5	5.7	20 259	16	36	0.76
6	8.2	25 598	20	41	0.74
7	13.6	31 416	24	46	0.71
8	17.6	36 634	28	52	0.66
9	14.6	42 373	31	52	0.69
10	15.1	49 323	35	57	0.65
11	6.6	61 274	40	62	0.63
12	5.7	74 882	44	72	0.50
13	2.1	98 365	51	72	0.57
14	2.3	161 158	61	80	0.51

a) Elasticity of income after tax with respect to income before tax.

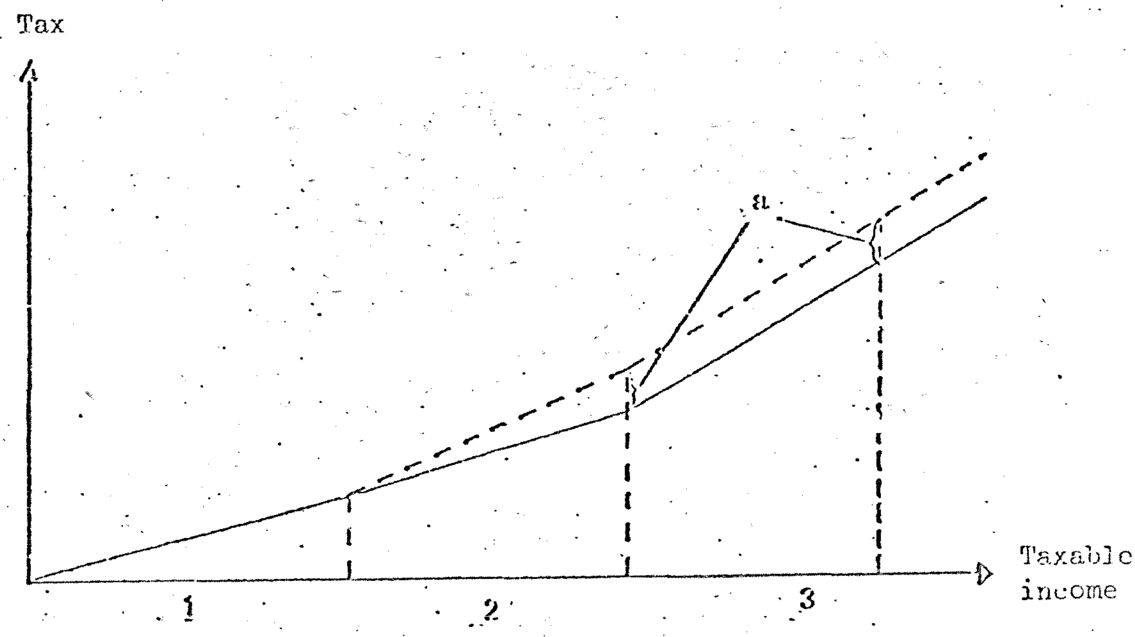
of these computations we shall indicate the nature of the different parameter changes and the kind of individual response we might expect under the assumptions made.

The effect on individual labour supply from a tax change can be divided in an income effect and a substitution effect. The income effect is positive, which in this connection means that an isolated increase in the average tax rate will increase labour supply. The negative substitution effect implies that an isolated increase in the marginal tax rate will lead to a diminished labour supply. For a given tax schedule a specific revenue is collected from the individual.

The tax schedule in the Swedish tax system can be described as an increasing step-wise linear function from income to tax payments. The general shape of the function is determined by the statutory marginal tax rates at national taxation and the so called basic tax deduction. Figure 2 illustrates an increase of the statutory marginal tax rate within a specific bracket (bracket 2 in the figure). Obviously, people below this bracket will not be affected by the change. Everybody in bracket 2 and above will have their utility levels diminished. An individual within the bracket gets his marginal tax rate as well as his average tax rate increased, so the effect on labour supply is in principle undetermined and so is the revenue effect. If the effect on labour supply is positive, the revenue effect will of course also be positive. A negative supply effect might, however, diminish the tax-base enough to offset the effect on revenue from the upward shift in the tax schedule.

As the tax increase in bracket 3 and above is of the same nature as an additional lump sum tax labour supply in these brackets will be greater than before and so will revenues collected. Utility levels, however, will

Figure 2. Increase of the statutory marginal tax rate within a specific bracket.



of course be diminished.

If we now go to the local tax it could mainly be seen as a linear tax with constant marginal tax rate which is equal to the local tax rate. It is clear that an increase in this tax rate will for the whole range of income give rise to exactly the same effects as we met within bracket 2 in the preceding paragraph.

The qualitative effects of changes in the other two instruments (basic tax deduction, lump-sum transfer) are obvious since they do not affect marginal tax rates and therefore only give rise to income effects.

Results on the micro level for changes of the statutory marginal tax rates in brackets 0-10 000 Sw.cr and 30 000-40 000 Sw.cr., can be seen in Table 2. Each of these parameters has been increased by one percentage unit. In the table the resulting changes in percent of initial values are given for tax payments, hours worked and individual utilities at different income levels. To pick an example we can in row 8, column 9, read the value of $(\partial t/\partial P_j)/(t) \cdot 100$ at income level $\sim$ 36 600, where P_j stands for the marginal tax rate in the bracket 30 000-40 000 Sw.Cr.

As could be expected, utilities are decreased for all individuals affected by the tax increase. Furthermore, those individuals that get their tax rates increased with unchanged marginal tax rates will increase their hours worked. The amount of tax collected from these people will, of course, also increase. These results do not depend on our specific choice of utility function for the individual. The Cobb-Douglas assumption is, however, important in the brackets where marginal tax rates are increased. Here we get a decrease in labour supply. For individuals with taxable income in the bracket 30 000-40 000 this effect is strong enough to produce a negative overall effect on their tax payments.

Table 2. Effects of parameter changes on the individual at different income levels.

Income class	Pre-tax mean income before tax change (1)	Increase of statutory marginal tax rate in taxable income bracket 0-10 000 Sw.Cr. Marginal tax rate (2)[a]	Work effort (3)[b]	Utility (4)[b]	Tax payment (5)[b]	30 000-40 000 Sw.Cr. Marginal tax rate (6)[a]	Work effort (7)[b]	Utility (8)[b]	Tax payment (9)[b]
1	118	0	0	0	0	0	0	0	0
2	2 801	0	0	0	0	0	0	0	0
3	9 076	+1	-1.1	-0.1	0.0	0	0	0	0
4	14 411	+1	-0.6	-0.2	4.8	0	0	0	0
5	20 259	0	0.5	-0.2	5.5	0	0	0	0
6	25 598	0	0.4	-0.2	2.3	0	0	0	0
7	31 416	0	0.4	-0.1	2.1	0	0	0	0
8	36 634	0	0.4	-0.1	1.6	+1	-2.1	-0.0	-3.7
9	42 373	0	0.3	-0.1	1.3	+1	-1.8	-0.1	-2.5
10	49 323	0	0.3	-0.1	1.1	0	0.3	-0.1	1.1
11	61 274	0	0.3	-0.1	0.8	0	0.3	-0.1	0.8
12	74 882	0	0.3	-0.1	0.8	0	0.3	-0.1	0.8
13	98 865	0	0.3	-0.1	0.4	0	0.2	-0.1	0.4
14	161 158	0	0.2	-0.1	0.3	0	0.2	-0.1	0.3

a) Change given in percentage units.
b) Change given in percent of initial value.

Table 3. Aggregate effects of parameter changes on social welfare and tax revenue.

	Parameters		Change of parameter	Effect on: Tax revenue mill. Sw.Cr.	Effect on: Social welfare[a)]		
	National income tax schedule			$\frac{\partial T}{\partial P_i}$	$\frac{\partial W}{\partial P_i}$		
	Taxable income bracket. Thousands of Sw.Cr.	Initial statutory marginal tax rate %			$\varepsilon=0.8$	$\varepsilon=3.0$	$\varepsilon=6.0$
P1	0-15	7	+1 p.u[b)]	99	-4.21 10^{-1}	-8.9 10^{-3}	3.3 10^{-5}
P2	15-20	12	"	27	-1.41 "	-2.7 "	-.8 "
P3	20-25	17	"	22	-1.27 "	-2.3 "	-.6 "
P4	25-30	22	"	10	-1.03 "	-1.7 "	-.4 "
P5	30-40	28	"	-19	-.89 "	-1.3 "	-.3 "
P6	40-45	33	"	-30	-.25 "	-.3 "	-.1 "
P7	45-65	38	"	-0	-.44 "	-.4 "	-.0 "
P8	65-100[c)]	43	"	-14	-.46 "	-.39 "	-.0 "
P9	100-[d)]	52	"	-33	-.68 "	-.49 "	-.0 "
P10	Lump sum transfer[e)]		+100 Sw.Cr.	-71	1.0	1.0	1.0
P11	Basic tax deduction[f)]		+100 Sw.Cr.	-35	1.73 10^{-1}	3.4 10^{-3}	1.2 10^{-5}
P12	Local income tax[g)]		+1 p.u	33	-6.40 "	-12.9 "	-4.5 "

Notes to table 3.

a) These effects are normalized so that the effect of the introduction of a lump-sum transfer by 100 Sw.Cr. is equal to one.

b) Percentage unit.

c) Two brackets put together. The statutory marginal tax rate is 48% in the subbracket 70 000-100 000 Sw.Cr.

d) Cf. c). The statutory marginal tax rate is 56% in the subbracket 150 000-.

e) This parameter does not exist in the actual tax system.

f) Presently 4 500 Sw.Cr.allowed to all income earners subject to the restriction that taxable income should not become negative.

g) Flat rate of approximately 26% applied to taxable income.

This negative effect is essential for the results we will give later on. Some readers might find it so extreme that it would rule out any form of the individual utility function producing this effect. However, as soon as any incentive effects at all are admitted, a perverse revenue effect does not seem to be too far fetched which might be clear from the following.

Example

Consider a full time worker supplying 2 000 hours/year at a wage rate of 22.5 Sw.Cr/hour. This gives a yearly wage of 45 000 Sw.Cr and a taxable income of approximately 40 000 Sw.Cr. Tax payments are roughly 12 000 Sw.Cr. Now let the marginal tax rate in the brackets above the taxable income 30 000 Sw.Cr be increased by one percentage unit. At a taxable income of 40 000 this gives an initial tax increase on 100 Sw.Cr or 0.8 percent of taxes paid. By how much must hours worked be diminished in order to offset this positive revenue effect? Since the elasticity of tax payments with respect to income in this bracket is roughly equal to 2, an adjustment in hours worked by 0.4%, or 8 hours per year, would be sufficient to give a zero revenue effect. Higher adjustments than 8 hours per year will consequently give negative revenue effects.

2.2 Aggregate effects

From the aggregative part of the model (e.g. (9)-(10)) we can investigate the effects of specific parameter changes on tax revenues and the social welfare function. Table 3 gives computed values of $\frac{\partial T}{\partial P_i}$ and $\frac{\partial W}{\partial P_i}$ for different parameters.

The most striking result of the table is that the perverse revenue effects we could observe on the micro-level in certain cases give rise to similar effects on the macro-level. Take e.g. the bracket 30-40 000 Sw.Cr.

From the table we can see that a rise of the marginal tax-rate in this bracket by 1 percentage unit will decrease the aggregate tax revenues by 19 million Sw.Cr. From the micro-simulations (Table 2) it is clear that this figure is the net effect of diminished revenues from people within the bracket getting their marginal tax rate increased and revenue increases from people above the bracket, where the average tax rate is increased while the marginal tax rate is unchanged.

The interpretation of the perverse revenue effects for certain brackets is that the tax schedule in these brackets is not Pareto-optimal under the assumptions on individual behaviour made here. Lowered marginal tax rates would increase utilities for the persons affected at the same time as total revenues would be increased.

We can also observe that the effect on social welfare of introducing a lump-sum transfer, with one exception is much greater than any other welfare effect. The exception is the rate of the regressive local tax. For ε=0.8 it would not increase social welfare to finance an increased lump-sum transfer with an increase in the local tax rate.

For higher values of ε the welfare effect of other parameter changes become almost negligible compared to the welfare effect of a change in the lump-sum transfer.

3. Welfare improving policies under a fixed budget-constraint

We are now equipped to answer the question of which parameter changes to choose in order to increase social welfare. As we do not consider other branches of public policy than personal income taxation it is natural to restrict the changes in the tax schedule to leave total net revenues constant. This restriction is under the assumptions made here equivalent to the restriction that changes in consumption shall be equal to changes in production (see Stern [15]). By the help of Table 3 it is easy to design policies, i.e.

combinations of parameter changes that improve social welfare keeping total revenues constant.

In terms of our previous notation our task is to find combinations of parameter changes $dP_k; dP_c$ such that

$$dW = \frac{\partial W}{\partial P_k} \cdot dP_k + \frac{\partial W}{\partial P_c} \cdot dP_c > 0$$

$$dt = \frac{\partial T}{\partial P_k} \cdot dP_k + \frac{\partial T}{\partial P_c} \cdot dP_c = 0 \qquad (11)$$

In Table 4 we give a selection of combined parameter changes that fulfills (11). The results are in accordance with those reached by Mirrlees [9] and Phelps [11]. Both authors present results indicating that the optimal marginal tax rates should be falling at higher income levels. Here it is clear that marginal tax rates in brackets above 30 000 should be lowered. In Table 4, II and III are examples of such policies. It should also be mentioned that these two policies are of special interest since they as well as policy VI represent Pareto improvements.

We have introduced the possibility of a lump-sum transfer in the tax system. Our results strongly indicate that such an element should be included in the actual tax system. This is of course also in accordance with the results reached in the theoretical literature.

In our analysis this result can be explained by the heavy weight attached to income in the lowest part of the distribution, already by the utilitarian sum of utilities. This tendency is reinforced by the social welfare function. It should also be pointed out that the financing of such policies is comparatively easy in the category married men since it has few persons in the bottom (see Table 1).

Table 4. Combination of parameter changes improving social welfare under a fixed revenue constraint.

	I		II		III	
Parameters involved	P1[1)] marginal tax rate bracket 0'-15' Sw.Cr.	P10[2)] lump-sum transfer	P6[1)] marginal tax rate bracket 40'-45' Sw.Cr.	P10[2)] lump-sum transfer	P6[1)] marginal tax rate bracket 40'-45' Sw.Cr.	P1[1)] marginal tax rate bracket 0'-15' Sw.Cr.
Parameter changes	+0.71	+1	-2.3	+1	-3.3	-1

	IV		V*		VI	
Parameters involved	P1[1)] marginal tax rate bracket 0'-15' Sw.Cr.	P3[1)] marginal tax rate bracket 20'-25'	P12[1)] local tax rate	P10[2)] lump-sum transfer	P12[1)] local tax rate	P1[1)] marginal tax rate bracket 0'-15' Sw.Cr.
Parameter changes	-1	+4.5	+2.2	+1	-3	+1

*) For $\varepsilon = 0.8$ the indicated combination of changes in local tax rate (P12) and lump-sum transfer (P10) leads to a decreased value of the social welfare function.

1) Change given in percentage units.

2) Change given in hundreds of Sw.Cr.

Another general conclusion from the results is that the valuation of different policies do not change much with the value of ε. For the piecemeal policy analysis done here, it is in most cases indifferent if ε is equal to zero (the strictly utilitarian approach) or if we let ε tend to infinity (the Rawlsian criterion). A related point is that utility changes in the higher income classes mostly could be neglected. What is important here is the revenue effect. Therefore the assumptions made on disincentives in these classes are important for the results we will get.

From Table 3 it is seen that an increase in the local tax rate combines a low revenue effect[1)] with a high welfare loss. Policies V and VI in Table 4 are both encompassing a change in the local tax rate (P12). When it is used to finance an increased lump-sum transfer we get a welfare increase only when ε is greater than 0.8. This increase is much less than the one we get when the local tax rate is lowered in combination with an increase in the marginal tax rate in the lowest bracket (policy VI).

4. Disincentives and the revenue effect

A clear cut result of our previous analysis is that, under the assumptions made, marginal tax rates should be decreased in all brackets above 30 000 Sw.Cr. This result depends crucially on the fact that in these brackets a decreased marginal tax rate leads to an increase in aggregate tax revenues (T).

It is important to check how sensitive this result is to changes in the elasticity of substitution between consumption of goods and consumption of leisure. We have done this by letting the individual's labour supply be governed by a utility function of the CES-type.[7)] By simulating the response

7) $U = \left[\alpha C^{-\mu} (1-\alpha)(T-H)^{-\mu}\right]^{1/\mu} \left(= \frac{1}{1+\mu}\right)$. If U is maximized subject to the

of hours worked and revenues for different values of σ for a change in the marginal tax rates in each one of the brackets above 30 000 Sw.Cr. we get an indication of the range of σ where the disincentive effect is strong enough to create a perverse revenue effect.

From Table 5 it is seen that in the two highest brackets there is quite a wide range of values on σ that will give a perverse aggregate revenue effect. For the lower brackets, however, we get a picture that is a bit more mixed. Still, the Cobb-Douglas-assumption does not seem to be essential for our results. An interesting result in this connection is provided by Stern [15] who calculated implied elasticities of substitution from the estimated supply curves by Ashenfelter and Heckman [ ]. This calculation gives σ=.4, which indicates that the range of σ in table 5 for most brackets contains realistic values.

6. Concluding remarks

A clear-cut conclusion of our analysis is that the graduation of the Swedish income tax differs greatly from what would be prescribed by the theory of optimum income taxation with its usual assumptions. One may then take either the position that the tax system should be changed or the position that the assumptions in the theory of optimal income taxation need re-examination.

Certainly one would like to have more empirical evidence on individual behavior before using our results for policy prescriptions. The analysis

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budget constraint the number of hours worked will be determined implicitly by the following equation

$$\frac{\alpha}{1-\alpha}\left(\frac{C}{T-H}\right)^{-(\mu+1)} = \left(\frac{1}{f'(wH)w}\right)/(1-M)$$

In lack of data on hourly wage rates we have computed values on w from yearly incomes on the assumption that everybody initially is working 2 000 hours/year.

Table 5. Least value on σ in the CES-function where an increased marginal tax rate produces diminished aggregate tax revenues

	P 5	P 6	P 7	P 8	P 9
Bracket of the tax schedule (thousands of Sw.Cr.)	30-40	40-45	45-65	65-100	100-
Revenues will be diminished for $\sigma \geq$	0.8	0.4	1.0	0.4	0.3

made has highlighted the crucial importance of the labour supply response to tax changes. Therefore one objection against the results reached might be that the assumptions on disincentives have little empirical support. Econometric work in this area indicates that labour force participation and average hours of adult men are affected relatively little by changes in tax rates. As we could see in section 4, calculations made by Stern [15] indicate that the elasticity of substitution between labour and leisure among adult men still is high enough to produce the "perverse revenue effect" in a wide range of tax brackets. A more important fact, however, is that there is a downward bias in the estimates of these studies since they only are concerned with one dimension of labour supply, namely hours of work, while more important dimensions are left out, like work effort, choice of job, demand for education.

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