

## STEPWISE PARAMETER ESTIMATION OF A MICRO SIMULATION MODEL

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An intuitive stepwise calibration method has been used so far on the Swedish Micro-Macro Model. This paper codifies this procedure described in the paper on the model already presented<sup>1)</sup> at this conference as a first step towards a more systematic, computer based estimation procedure.

Being recursively specified through-out, the model cannot be solved as simultaneous equations, but is forwarded in time via a simulation scheme. Since a complete set of real micro data has not yet been made ready, we apply fully dynamic simulations and calibrate all blocks simultaneously. It has not yet been possible to fit endogenously simulated micro data to its "correct" values, or to do the same thing partially block by block keeping all other blocks exogenous each time. Exogenization of blocks and partial block by block calibration in fact contradict the essential idea of the whole model. There is so much linkage across blocks, especially in the micro based market processes that exogenization of most blocks involves redesigning the model. Hence exogenization itself should be expected to affect macro behaviour in a not negligible way. Consequently it will not be very helpful in a calibration context.

Once we get a complete set of micro firm data we will also use simulated cross sectional patterns over time to calibrate the model further<sup>2)</sup>. We want to emphasize, however, that

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- 1) G Eliasson: A Micro Simulation model of a National Economy: The case of Sweden, pp 3ff.
  - 2) Some such work can be said to have been done already. For instance, we know roughly the rate at which the correlation between past and current rates of return of individual firms decreases with time. We have checked that model simulations do not contradict this evidence. It should be noted here that there is no randomizing device in the model that sees to it that such results are obtained. In this context the model is deterministic.

the most important empirical test of the model has to do with getting the micro assumptions numerically right. (Cf the discussion on pp. 32-51.) Here we are concerned with "estimating" a very limited number of parameters indirectly where access to direct micro observation has not been possible. This estimation also serves as a complementary check at the macro level that this "other" numerical information, that has gone into the model, is consistent with reality.

It will be obvious from what follows, that this paper is concerned with one side of the estimation procedure only, namely with the practical problem of how to obtain the "best fit" within reasonable computer resource limits. We do not discuss here the important problem of the stochastical properties of the estimates we eventually reach.

The general idea is to first calibrate the model to produce trends for critical endogenous macro variables over the simulation period that are consistent with Swedish post-war development, and then to calibrate the year-to-year historical development. The approach in each of these two phases is to move a selected subset of model parameters within a predetermined range, to get successively better values of an objective function, measuring the closeness of fit. This two-step scheme is made possible by the fact, noted from initial experimentation with the model, that most model parameters can be classified into one of two groups; one largely operating on model trends and the other mostly on short term cyclical behavior.

For each of the two steps, the objective function has been chosen so as to

- a) economize on computer time
- b) allow the inclusion of as much a priori knowledge as possible
- c) lead to an improved numerical specification from the chosen starting point.

The philosophy behind this two-step method is that the complexity of micro simulation models of the Swedish kind

- (1) allows for a multitude of solutions that satisfy the goodness of fit criterion if scanning is unrestricted but

- (2) that a priori considerations (knowledge) allow us to limit the number of choices considerably. We also believe
- (3) that our own intuitive capabilities are superior to mechanical, unlimited scanning when it comes to avoiding non-global optima, but that the mechanical approach is superior when we reach the stage of fine tuning with little risk of going in the wrong direction.

First we define a set of goal variables  $G_i$  and find out by way of sensitivity analysis which parameters work on  $G$  mostly in the long run (Trend =  $GTR_i$ ) and on short run cyclical variations (=  $GCL_i$ ).

Then we define a goodness of fit criterion.

To make the presentation more concrete we introduce the chosen set of goal variables directly from the model version described in Eliasson-Heiman-Olavi (1976)<sup>1)</sup>. There is no practical way whatsoever to perform this estimation on all macro variables and the variables thus have had to be chosen so as to minimize the risk that other variables stray off in undesired directions. We do not, however, explain this choice here.

#### Step one: TRENDS

Goal variables =  $GTR_i$  = Trends for the following macro entities:

Q = (i = 1) = industrial production  
 L (TOT) = (i = 2) = total employment  
 W = (i = 3) = wage costs in industry  
 P = (i = 4) = wholesale price index  
 CPI = (i = 5) = consumer price index  
 SAVH = (i = 6) = household saving

#### Trend criterion

(A) Minimize  $\text{MAX}_i |GTR_i^{\text{model}} - GTR_i^{\text{actual}}$

subject to:

(A1) M, RU, SUM  $\in$  {low, high} through simulation

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1) See Technical specifications of the Swedish micro-macro model version 96 at the end of this conference volume.

(A2) by varying

KSI  $\in \{0.1, 1.0\}$   
 NITER  $\in \{5, 15\}$  (integer)  
 IOTA  $\in \{0.5, 1\}$   
 ALFABW  $\geq 0$   
 BETABW  $\geq 0$   
 DMTEC  $\geq 0.1$   
 MARKET-  
 ITER  $\geq 1$  (integer)  
 MAXDP  $\geq 0$   
 GAMMA  $\geq 0$   
 THETA  $\in \{0.1\}$

Note that we consider correct evolution of firms' profit margins (M), rate of unemployment (RU), and industrial capacity utilization (SUM) to be so important for calibrating the model, that we have chosen to enter them as restrictions rather than to include them in the objective function.

KSI is a parameter that tells to what extent a firm, having performed an unsuccessful raid for new labour on another firm, closes the observed wage difference by increasing its own wage level [5.4.1.8]<sup>1)</sup>. Also see p.44 in this conference volume.

NITER gives the number of interactions (searches) a firm is allowed in the labour market each period (quarter) [5.4.1.2].

IOTA is the fraction of the expected wage increase that a firm chooses to offer directly when entering the labour market in search for people [5.4.1.0].

ALFABW and BETABW the rate of increase in firm (net) borrowing is assumed to depend linearly on the difference between the nominal return to total assets and the borrowing rate. ALFABW is the intercept and BETABW the coefficient [10.6].

1) Numbers refer to algorithms where this parameter appears in the supplement Technical Specifications. See previous footnote.

DMTEC	is the (exogenous) rate of increase in productivity of new equipment invested [4.1.1].
MARKET-ITER	tells the number of producer-household price-volume iterations in the product market [7.3.3].
MAXDP	maximum fraction by which one year's price increases can differ from expected values, as a consequence of excess supply or demand in the product market [7.6.1].
GAMMA	the relative wage improvement a worker demands to move to a new job [5.4.1.8].
THETA	maximum fraction of a firm's labour force that can be lost in <u>one</u> raid [5.4.1.9].

The optimum value of the objective function is of course zero; that is, it should be feasible to track the six trends exactly under the restrictions indicated. However, limited resources (time and money) for the calibration will force us to terminate the iterative process at some point which does not produce the optimum, but a closeness of fit which we have prespecified as satisfactory.

#### Step two: CYCLES

Use the same goal variables  $G_i$  as in step one. Let  $GCL_{ij}$  indicate the value of variable  $G_i$  in year  $j$  of the simulation.

The objective function to be minimized is now, (with an appropriate set ( $w_i$ ) of weights):

$$(B) \quad \sum_i w_i * \sum_j (GCL_{ij}^{\text{model}} - GCL_{ij}^{\text{actual}})^2$$

Restrictions are

(B1)  $M, RU, SUM$  as in (A1)

(B2) Don't let achieved trends suffer more than  $\pm \epsilon$  compared with step 1. Stipulate for each  $GTR_i$  that:

$$|GTR_i^{\text{model}} - GTR_i^{\text{actual}}|_{\text{step 2}} \leq |GTR_i^{\text{model}} - GTR_i^{\text{actual}}|_{\text{step 1}} + \epsilon$$

In step 2, the following model parameters are varied:

SMS	$\in \{0,1\}$
SMP	$\in \{0,1\}$
SMW	$\in \{0,1\}$
SMT	$\in \{0,1\}$
FI	$\in \{0,1\}$
TMSTO	$\geq 0$
TMIMSTO	$\geq 0$
TMX	$\geq 0$
TMIMP	$\geq 0$
SKREPA	$\in \{1,50\}$

SMS, SMP, SMW, SMT	smoothing parameters, used by firms to make each year's trade-off between old and current experiences when forming expectations for sales, prices, wages, and profit targets, respectively [1.1.1, 1.2.1, 1.3.1 and 2.1].
FI	a smoothing parameter, used by firms to make quarterly adjustments of expectations [3.1.2].
TMSTO	a time reaction parameter, used by firms as the time planned for to adjust a deviation of their finished-goods inventories from their optimum level [4.2.2].
TMIMSTO	same as TMSTO, but applied to input-goods inventories.
TMX, TMIMP	time reaction parameters, controlling the rate of change of export/import ratios as a response to foreign-domestic price differentials [6.1.1 and 7.3.1].
SKREPA	a parameter regulating the probability that a recruiting firm will turn to the pool of unemployed (instead of trying to raid other firms), and thus affecting the time pattern of a net increase in total employment.

To be able to calibrate a year-to-year fit, we have been careful to choose, in this step, a parameter set that mainly affects the time response patterns of the model. Compare this with the trend-calibrating step, where we selected

parameters that have a relatively stronger impact on the long-run profitability and growth development of simulated firms, and thus on the long-run behaviour of the entire model.

#### Further considerations

The above is a formalization of our ad-hoc intuitive procedure for estimating critical model parameters. We have also told why we prefer a user-model interaction scheme in a first phase, instead of applying an outright, automatic optimization procedure. In a later phase of the project, when calibrations like this have resulted in reasonable interval estimates of the parameters and the risk of approaching a non-global optimum is smaller, a computer-based algorithm should be appropriate. With any such algorithm, our own interactive scheme would be mechanized into an iterative search process, evaluating for each new step to be taken what changes in the parameter set as we judge them, give the fastest improvement in the closeness-of-fit objective function. However, instead of directly computing the derivatives by way of explicit formula, the algorithm will use trial model simulations at each point, requiring a well-defined algorithm/model interface.

Note that with a computer-based optimization algorithm, the problem formulations in the two steps above might have to be modified to suit the characteristics of the algorithm in question. Integer-restrictions, like NITER and MARKETITER, are awkward to all optimization schemes; and MINMAX formulations often make optimizations very time-consuming. The exact formulations will have to be worked out in concordance with the performance of the chosen algorithm.

Note also, that with the objective function and the restrictions on the allowed parameter combinations, as we have them, we cannot guarantee the convexity of either. This might give rise to problems of finding the correct (global) optimum. Usually this problem is accommodated by running several optimizations, selecting different starting points. That would probably be too resource-consuming in practice in our case. Instead we chose to base our confidence on having found a good starting point for search - from the beginning - namely the parameter set that gave the best fit in the initial, intuitive search procedure.