The competitive effects of linking electricity markets across space: Online appendix*

Thomas P. Tangerås† Fank A. Wolak‡

February 29, 2024

Abstract

This appendix contains the analysis and proofs of formal statements that extend results in Tangerås and Wolak (2024). The appendix analyzes (*i*) general cost and inverse demand functions; (*ii*) an arbitrary number of asymmetric local markets; (*iii*) the case where producers own generation capacity and exercise market power in multiple local markets; (*iv*) oligopoly in the local short-term market; (*v*) multiple trading periods in the forward market.

Key words: Electricity markets, equity, market design, market power, regional markets

JEL: C72, D43, G10, G13, L13

1 Introduction

This appendix contains the analysis and proofs of formal statements in Section 5 of Tangerås and Wolak (2024). Section 2 allows more general cost and inverse demand functions than the linear specifications analyzed in the main text. Section 3 extends the analysis to an arbitrary number of asymmetric local markets. Section 4 generalizes the analysis to the case where producers may own generation capacity in multiple local markets instead of just in one local market. Section 5 considers oligopoly in each local short-term market. Finally, Section 6 compares the equilibrium in a spatially independent market design if there are two compared to one trading period in the forward market.

*This research was conducted within the "Sustainable Energy Transition" research program at IFN. Financial support from Jan Wallanders och Tom Hedelius stiftelse (Tangerås), the Swedish Competition Authority (Tangerås) and the Swedish Energy Agency (Tangerås and Wolak) is gratefully acknowledged.

†Research Institute of Industrial Economics (IFN), PO. Box 55665, SE-10215 Stockholm, Sweden. E-mail: thomas.tangeras@ifn.se.

‡Program on Energy and Sustainable Development and Department of Economics, Stanford University, 579 Serra Mall, Stanford, CA 94305-6072. E-mail: wolak@stanford.edu.

2 General cost and inverse demand functions

There are two symmetric local markets with $H \geq 1$ retailers, one large producer with market power, and a competitive fringe in each local market. The producer has total production cost $C(q) \geq 0$, $q \in \mathbb{R}$. The total demand from each large consumer is constant and equal to $\frac{D}{H} > 0$. Consumers value electricity usage at $v > 0$ per MWh. Each consumer has an additional benefit $B(k_h) \in \mathbb{R}$ of trading in the forward market. In Tangerås and Wolak (2024), $B(k_h) = -\psi \times (\frac{D}{H} - k_h)$, where $\psi \geq 0$. The competitive fringe supplies the residual demand net of the producer's supply, $D - q \in \mathbb{R}$, at upward-sloping linear marginal cost $MC(D - q) = P(q) \in \mathbb{R}$. All functions are twice continuously differentiable.

We assume that the underlying cost and demand functions yield regular demand and profit functions in the following sense: (i) the spatially independent market design and the one where markets are linked through a regional forward contract each feature a symmetric interior equilibrium; (ii) the demand $K_m^{RI}(f_m, f_{-m})$ for forward contracts in a regional forward market is strictly decreasing in the own forward price f_m; (iii) the forward demand function $K_m^{RI}(f, f)$ is strictly decreasing in the symmetric forward price f; (iv) the producer's marginal profit function $\frac{\partial \Pi_m^{RI}(f,f)}{\partial f_m}$ is strictly decreasing in the symmetric forward price f.

The rest of this section is organized as follows. Section 2.1 characterizes the equilibrium in a spatially independent market, and Section 2.2 does the same under the assumption that local markets are linked through a regional forward contract. Section 2.3. compares the equilibrium forward quantities under the two different designs.

2.1 Spatially independent markets

Short-term markets and forward markets are both local under this market design.

Equilibrium in the short-term market The profit of the producer is

$$(f - P(q))k + P(q)q - C(q). \tag{1}$$

The first term measures the forward profit if the producer has sold contracts for k MWh electricity in the forward market at a price of f per MWh, and the firm produces q MWh electricity. The two other terms represent the profit in the short-term market. Maximizing (1) over q yields the first-order condition

$$-P'(q)k + P(q) - c + P'(q) = 0 \tag{2}$$

for the production of the monopoly in the short-term market. This yields profit maximizing quantity $q(k)$ and short-term price $p(k) = P(q(k))$. Both functions are continuously differentiable.

Substituting $q(k)$ into (1) delivers the profit

$$\Pi(k, f) = (f - P(q(k)))k + P(q(k))q(k) - C(q(k)) \tag{3}$$

of the monopolist written as a function of the forward quantity k and the forward price f.

The demand for local forward contracts The profit of consumer h is

$$\Omega_h^I(k_h, k_{-h}, f) = -(f - p(k))k_h + (v - p(k))\frac{D}{H} + B(k_h) \tag{4}$$

The first term is the forward market deficit. It is equal to the forward price f minus the exercise price $p(k)$ of the forward contract, multiplied by the forward quantity k_h. The second term is consumer h's profit in the short-term market. It is equal to the value v of consumption minus the price $p(k)$ of electricity in the short-term market, multiplied by the individual demand $\frac{D}{H}$ for electricity. The third term is the additional benefit of forward contracting.

All consumers move simultaneously and independently in stage 2 of the game. The marginal effect on profit of increasing the forward quantity k_h is

$$\frac{\partial \Omega_h^I}{\partial k_h} = -(f - p(k)) - p'(k)(\frac{D}{H} - k_h) + B'(k_h)$$

A marginal increase in the demanded forward quantity k_h has a direct effect on consumer h profit by increasing the forward market deficit. This is the first term on the right-hand side of the marginal profit expression. An increase in demand also reduces the short-term price of electricity, $p'(k) < 0$. The marginal value of this indirect, pro-competitive effect of forward contracting, is measured by the second term. The final term is the marginal benefit of reducing the imbalance between consumption and the forward quantity.

By setting $\frac{\partial \Omega_h^I}{\partial k_h} = 0$ and using symmetry, $k_h = \frac{k}{H}$ for all h, we get

$$f = p(k) - p'(k)\frac{D-k}{H} + B'(\frac{k}{H}) \tag{5}$$

as the implicit solution to the demand $k = K^I(f)$ for forward contracts.

Differentiating the above equilibrium condition yields the marginal effect

$$K^{I\prime}(f) = \frac{1}{p'(k)\frac{H+1}{H} - p''(k)\frac{D-k}{H} + \frac{1}{H}B''(\frac{k}{H})} \tag{6}$$

on demand of an increase in the forward price.

The price of local forward contracts Substitute $K^I(f)$ into $\Pi(k,f)$ defined in (3) to get the producer profit

$$\Pi^I(f) = [f - P(q(K^I(f)))]K^I(f) + P(q(K^I(f)))q(K^I(f)) - C(q(K^I(f)))$$

as a function of the forward price f. The marginal effect of a small increase in the forward price is

$$\frac{\partial \Pi^I}{\partial f} = k + [f - p(k)]K^{I\prime}(f) + [-P'(q)k + P(q) + P'(q)q - C'(q)]q'(k)K^{I\prime}(f)$$

By way of the first-order condition (2) in the short-term market, the equilibrium forward price f^I solves

$$k^I + [f^I - p(k^I)]K^{I\prime}(f^I) = 0,$$

where $k^I = K^I(f^I)$. If we use (5) to get rid of f^I, then we can alternatively write the equilibrium condition for the forward quantity k^I as

$$\frac{k^I}{K^{I\prime}(f^I)} = p'(k^I)\frac{D - k^I}{H} - B'(\frac{k^I}{H}). \tag{7}$$

2.2 Linking forward markets across space

This extension of the model is similar to Section 3.2 in Tangerås and Wolak (2024), where the settlement price in the forward market is the average, $\frac{1}{2}(p_1 + p_2)$, of the short-term prices in the two markets.

Equilibrium in the short-term market The third-stage profit of the producer in local market m equals

$$[f_m - \frac{1}{2}(P(q_1) + P(q_2))]k_m + P(q_m)q_m - C(q_m), \tag{8}$$

Maximization of the profit of the monopoly producer yields the first-order condition

$$-P'(q_m)\frac{k_m}{2} + P(q_m) + P'(q_m)q_m - C'(q_m) = 0. \tag{9}$$

In particular, the equilibrium production satisfies $q_m(k_m) = q(\frac{k_m}{2})$, and the short-term price is $p_m(k_m) = p(\frac{k_m}{2})$.

By substituting these expression into (8), we obtain the profit function

$$\Pi_m(k_m, k_{-m}, f_m) = [f_m - \frac{1}{2}(P(q(\frac{k_1}{2})) + P(q(\frac{k_2}{2})))]k_m + P(q(\frac{k_m}{2}))q(\frac{k_m}{2}) - C(q(\frac{k_m}{2})) \tag{10}$$

of producer m as a function of its own forward quantity k_m, the forward quantity k_{-m} in the other local market, and the forward price f_m.

The demand for regional forward contracts Consumer h in local market m maximizes

$$\Omega^{RI}_{hm}(k_{hm}, k_{-hm}, k_{-m}, f_m) = -[f_m - \frac{1}{2}(p(\frac{k_1}{2}) + p(\frac{k_2}{2}))]k_{hm} + [v - p(\frac{k_m}{2})]\frac{D}{H} + B(k_{hm})$$

over k_{hm}. Solving the first-order condition yields

$$f_m = \frac{1}{2}(p(\frac{k_1}{2}) + p(\frac{k_2}{2})) - \frac{1}{2}p'(\frac{k_m}{2})(\frac{D}{H} - \frac{1}{2}\frac{k_m}{H}) + B'(\frac{k_m}{H}), \tag{11}$$

in local market m, where $k_m = K_m^{RI}(f_m, f_{-m})$. Total differentiation of the two first-order conditions yield the comparative statics results

$$\frac{\partial K_m^{RI}}{\partial f_m} = \frac{4Z_m}{Z_1 Z_2 - p'(\frac{k_1}{2})p'(\frac{k_2}{2})}, \quad \frac{\partial K_{-m}^{RI}}{\partial f_m} = \frac{-4p'(\frac{k_{-m}}{2})}{Z_1 Z_2 - p'(\frac{k_1}{2})p'(\frac{k_2}{2})} \tag{12}$$

where

$$Z_m = p'(\frac{k_m}{2})\frac{H+1}{H} - p''(\frac{k_m}{2})(\frac{D}{H} - \frac{1}{2}\frac{k_m}{H}) + \frac{4}{H}B''(\frac{k_m}{H}). \tag{13}$$

These comparative statics results will be useful later.

The price of regional forward contracts Inserting $K_1^{RI}(f_1, f_2)$ and $K_2^{RI}(f_2, f_1)$ into $\Pi_m(k_m, k_{-m}, f_m)$ defined in (10) delivers the first-stage profit

$$\begin{aligned} \Pi_m^{RI}(f_m, f_{-m}) \quad = \quad & [f_m - \frac{1}{2}\{P(q(\frac{K_1^{RI}(f_1, f_2)}{2})) + P(q(\frac{K_2^{RI}(f_2, f_1)}{2}))\}]K_m^{RI}(f_m, f_{-m}) \\ & + P(q_m(\frac{K_m^{RI}(f_m, f_{-m})}{2}))q_m(\frac{K_m^{RI}(f_m, f_{-m})}{2}) - C(q_m(\frac{K_m^{RI}(f_m, f_{-m})}{2})) \end{aligned}$$

of producer m. Invoking the first-order condition (9) from the short-term market delivers producer m's marginal profit

$$\frac{\partial \Pi_m^{RI}}{\partial f_m} = [1 - \frac{1}{4}p'(\frac{k_{-m}}{2})\frac{\partial K_{-m}^{RI}}{\partial f_m}]k_m + [f_m - \frac{1}{2}(p(\frac{k_1}{2}) + p(\frac{k_2}{2}))]\frac{\partial K_m^{RI}}{\partial f_m}. \tag{14}$$

An interior symmetric equilibrium $f_1^{RI} = f_2^{RI} = f^{RI}$ solves $\frac{\partial \Pi_m^{RI}(f^{RI}, f^{RI})}{\partial f_m} = 0$.

2.3 Comparison of market designs

The purpose of this section is to show that $k^{RI} \geq 2k^I$ if $B(k_m) = -\psi \times (\frac{D}{H} - k_{hm})$, and where the inequality is strict if $\psi > 0$. Rewrite the marginal profit expression (14) as

$$\frac{1}{\frac{\partial K_m^{RI}}{\partial f_m}}\frac{\partial \Pi_m^{RI}}{\partial f_m} = B'(\frac{k_m}{H}) + [1 - \frac{1}{4}p'(\frac{k_{-m}}{2})\frac{\partial K_{-m}^{RI}}{\partial f_m}]\frac{k_m}{\frac{\partial K_m^{RI}}{\partial f_m}} - \frac{1}{2}p'(\frac{k_m}{2})(\frac{D}{H} - \frac{1}{2}\frac{k_m}{H})$$

after invoking the equilibrium condition (11) from the demand for forward contracts. Next, evaluate this marginal profit expression at the symmetric forward price $f_1 = f_2 = \hat{f}$ for which $K_m^{RI}(\hat{f}, \hat{f}) = 2k^I$:

$$\frac{1}{\frac{\partial K_m^{RI}(\hat{f},\hat{f})}{\partial f_m}}\frac{\partial \Pi_m^{RI}(\hat{f}, \hat{f})}{\partial f_m} = B'(\frac{2k^I}{H}) + [1 - \frac{1}{4}p'(k^I)\frac{\partial K_{-m}^{RI}(\hat{f}, \hat{f})}{\partial f_m}]\frac{2k^I}{\frac{\partial K_m^{RI}(\hat{f},\hat{f})}{\partial f_m}} - \frac{1}{2}p'(k^I)\frac{D - k^I}{H}$$

Substitute in (7) to get

$$\frac{1}{\frac{\partial K_m^{RI}(\hat{f},\hat{f})}{\partial f_m}}\frac{\partial \Pi_m^{RI}(\hat{f},\hat{f})}{\partial f_m} \begin{aligned}[t] &= B'(\tfrac{2k^I}{H}) - \tfrac{1}{2}B'(\tfrac{k^I}{H}) \\ &+\tfrac{1}{2}\{[1-\tfrac{1}{4}p'(k^I)\tfrac{\partial K_{-m}^{RI}(\hat{f},\hat{f})}{\partial f_m}]\tfrac{4K^{I\prime}(f^I)}{\frac{\partial K_m^{RI}(\hat{f},\hat{f})}{\partial f_m}} - 1\}\tfrac{k^I}{K^{I\prime}(f^I)} \end{aligned} \tag{15}$$

Apply symmetry to (12) and (13) to get the partial derivatives

$$\frac{\partial K_m^{RI}(\hat{f},\hat{f})}{\partial f_m} = \frac{4[p'(k^I)\frac{H+1}{H} - p''(k^I)\frac{D-k^I}{H} + \frac{4}{H}B''(\frac{2k^I}{H})]}{[p'(k^I)\frac{H+1}{H} - p''(k^I)\frac{D-k^I}{H} + \frac{4}{H}B''(\frac{2k^I}{H})]^2 - [p'(k^I)]^2}$$

$$\frac{\partial K_{-m}^{RI}(\hat{f},\hat{f})}{\partial f_m} = \frac{-4p'(k^I)}{[p'(k^I)\frac{H+1}{H} - p''(k^I)\frac{D-k^I}{H} + \frac{4}{H}B''(\frac{2k^I}{H})]^2 - [p'(k^I)]^2}$$

of the demand for regional forward contracts evaluated at $f_1 = f_2 = \hat{f}$. Substitute these expressions and

$$K^{I\prime}(f^I) = \frac{1}{p'(k^I)\frac{H+1}{H} - p''(k^I)\frac{D-k^I}{H} + \frac{1}{H}B''(\frac{k^I}{H})}$$

from (6) into the expression in curly brackets on the second row of the marginal profit expression (15) and simplify to

$$[1-\frac{1}{4}p'(k^I)\frac{\partial K_{-m}^{RI}(\hat{f},\hat{f})}{\partial f_m}]\frac{4K^{I\prime}(f^I)}{\frac{\partial K_m^{RI}(\hat{f},\hat{f})}{\partial f_m}} - 1 = [4B''(\frac{2k^I}{H}) - B''(\frac{k^I}{H})]\frac{K^{I\prime}(f^I)}{H}.$$

Insert this expression into (15) to write the marginal profit of the producer with market power in local market m as

$$\frac{\partial \Pi_m^{RI}(\hat{f},\hat{f})}{\partial f_m} = \{B'(\frac{2k^I}{H}) - \frac{1}{2}B'(\frac{k^I}{H}) + \frac{1}{2}[4B''(\frac{2k^I}{H}) - B''(\frac{k^I}{H})]\frac{k^I}{H}\}\frac{\partial K_m^{RI}(\hat{f},\hat{f})}{\partial f_m} \tag{16}$$

evaluated at $f_1 = f_2 = \hat{f}$. The expression within curly brackets on the right-hand depends entirely on the properties of $B(x)$. In Tangerås and Wolak (2024), $B(k_{hm}) = -\psi \times (\frac{D}{H} - k_{hm})$, $\psi \geq 0$, in which case (16) simplifies to

$$\frac{\partial \Pi_m^{RI}(\hat{f},\hat{f})}{\partial f_m} = \frac{\psi}{2}\frac{\partial K_m^{RI}(\hat{f},\hat{f})}{\partial f_m} \leq 0.$$

The inequality follows from the regularity assumption that $\frac{\partial K_m^{RI}}{\partial f_m} < 0$. By the assumed monotonicity of $\frac{\partial \Pi_m^{RI}(f,f)}{\partial f_m}$ in f, we get $\frac{\partial \Pi_m^{RI}(f,f)}{\partial f_m} < 0$ for all $f > \hat{f}$. By necessity, all symmetric equilibria $f_1 = f_2 = f^{RI}$ satisfy $f^{RI} \leq \hat{f}$, with strict inequality if $\psi > 0$. Since $K_m^{RI}(f,f)$ is strictly decreasing by assumption, we get

$$k^{RI} = K_m^{RI}(f^{RI}, f^{RI}) \geq K_m^{RI}(\hat{f},\hat{f}) = 2k^I,$$

with strict inequality if $\psi > 0$. We collect our findings in the following result:

Proposition 1 *Linking two symmetric markets through a regional forward contract is pro-competitive relative to a spatially independent market design, $k^{RI} \geq 2k^I$ with strict inequality if $\psi > 0$, for any pair $\{C(q), P(q)\}$ of twice continuously differentiable cost and inverse demand functions that yield regular demand and profit functions.*

The right-hand side of (16) is strictly negative even for other formulations of $B(k_{hm})$ than $B(k_{hm}) = -\psi \times (\frac{D}{H} - k_{hm})$, $\psi > 0$. We give two examples. If $B(k_{hm}) = -e^{\psi(\frac{D}{H} - k_{hm})}$, $\psi > 0$, then

$$\begin{aligned} & B'(\frac{2k^I}{H}) - \frac{1}{2}B'(\frac{k^I}{H}) + \frac{1}{2}[4B''(\frac{2k^I}{H}) - B''(\frac{k^I}{H})]\frac{k^I}{H} \\ = \quad & \frac{\psi}{2}e^{\psi\frac{D-k^I}{H}}[2e^{-\psi\frac{k^I}{H}} - 1 - \frac{\psi}{2}(4e^{-\psi\frac{k^I}{H}} - 1)\frac{k^I}{H}]. \end{aligned}$$

The expression inside the square brackets on the right-hand side converges to 1 as $\psi \to 0$, so the left-hand side is strictly positive for ψ sufficiently close to zero.

If $B(k_{hm}) = -\frac{\psi}{1+\rho}(\frac{D}{H} - k_{hm})^{1+\rho}$, $\psi > 0$, $\rho > 0$, then

$$\begin{aligned} & B'(\frac{2k^I}{H}) - \frac{1}{2}B'(\frac{k^I}{H}) + \frac{1}{2}[4B''(\frac{2k^I}{H}) - B''(\frac{k^I}{H})]\frac{k^I}{H} \\ = \quad & \frac{\psi}{2}(\frac{D-k^I}{H})^{\rho-1}[(2(\frac{D-2k^I}{D-k^I})^{\rho} - 1)\frac{D-k^I}{H} - \rho(4(\frac{D-k^I}{D-2k^I})^{1-\rho} - 1)\frac{k^I}{H}]. \end{aligned}$$

The expression inside the square brackets on the right-hand side converges to $\frac{D-k^I}{H} > 0$ as $\rho \to 0$, so the left-hand side is strictly positive for ρ sufficiently close to zero.

3 Multiple asymmetric local markets

This section generalizes the model to an arbitrary number $M \geq 2$ of local markets that are heterogeneous in terms of demand characteristics and production costs. We maintain the assumption of one producer with market power and $H \geq 1$ large consumers in each local market.

Index local markets (and individual producers with market power) by $m \in \mathcal{M} = \{1, ..., M\}$. In the first stage, each producer m sets a forward price $f_m \geq 0$ per MWh at which it is willing to sell an unlimited forward quantity. In the second stage, each large consumer $h \in \{1, ..., H\}$ in local market m purchases forward quantity $k_{hm} \in \mathbb{R}$ from producer m. Denote by $k_m = \sum_h k_{hm}$ the total forward quantity sold by producer m. In the third stage, each producer m decides how much electricity, $q_m \in \mathbb{R}$, to produce for the short-term market at constant marginal cost $c_m \geq 0$. Each large consumer in m uses $\frac{D_m}{H} > 0$ MWh electricity, so that the total demand for electricity in local market m equals D_m. Let $D = \frac{1}{M}\sum_m D_m$ be the average demand for electricity across the M local markets. The residual demand $D_m - q_m \in \mathbb{R}$ in each local market is covered by a local competitive fringe that supplies electricity at linear marginal cost $b_m(D_m - q_m)$, $b_m > 0$. The inverse demand curve facing the producer in the short-term market m can then be written as $p_m = P_m(q_m) = a_m - b_m q_m$, where $a_m = b_m q_m$.

3.1 Spatially independent markets

Assume that all forward contracts sold by producer m settle against the local spot price p_m. The analysis is qualitatively the same as in Section 3.1 of Tangerås and Wolak (2024) with adjustment for notation. The equilibrium forward quantity and short-term price equal

$$k_m^I = \frac{D_m + 2H\frac{\psi}{b_m}}{H+2},\ p_m^I = \frac{a_m + a_m}{2} - \frac{1}{2}\frac{a_m + 2H\psi}{H+2} \tag{17}$$

in short-term market m.

3.2 Linking forward markets across space

Let all forward contracts settle against the same quantity-weighted average, $\frac{1}{M}\sum_m p_m$, of the spot prices in all M local markets. The producer in local market m receives and consumers in local market$m$ pay the local short-term price p_m for their production and consumption, respectively. The analysis of the spot market is qualitatively the same as in Section 3.2 of Tangerås and Wolak (2024). In particular, the production by producer m and the spot price in local market m equal

$$q_m(k_m) = \frac{1}{2}\frac{a_m - c_m}{b_m} + \frac{1}{2}\frac{D_m}{D}\frac{k_m}{M},\ p_m(k_m) = \frac{a_m + c_m}{2} - \frac{b_m}{2}\frac{D_m}{D}\frac{k_m}{M}$$

if producer m has sold k_m MWh electricity in the forward market.

Turning next to the demand for forward contracts, each large consumer h in local market m purchases forward quantity k_{hm} from producer m to maximize profit

$$\Omega_{hm}^{RI}(\mathbf{k}, f_m) = -[f_m - \frac{1}{M}\sum_{n\in\mathcal{M}}\frac{D_n}{D}p_n(k_n)]k_{hm} + [v - p_m(k_m)]\frac{D_m}{H} - \psi(\frac{D_m}{H} - k_{hm}),$$

taking the aggregate forward quantity k_{-hm} by all other large consumers in local market m, the aggregate forward quantities $\mathbf{k}_{-m} = (k_1, ..., k_{m-1}, k_{m+1,...,}k_M)$ in all other local markets, and the forward price f_m as given. We can use the M first-order conditions

$$f_m - \frac{1}{M}\sum_{n\in\mathcal{M}}\frac{D_n}{D}p_n(k_n) = \frac{b_m}{2M}(\frac{D_m}{D})^2\frac{MD - k_m}{MH} + \psi \tag{18}$$

for consumers' profit-maximization (where we have applied $p_m'(k_m) = -\frac{b_m}{2M}\frac{D_m}{D}$ and symmetry, $k_{hm} = \frac{k_m}{H}$) to solve for the demand function

$$K_m^{RI}(\mathbf{f}) = \frac{M}{b_m}(\frac{D}{D_m})^2\frac{(MH+1)\frac{D_m}{D}a_m + MHc + 2MH(\psi - f_m + H\sum_{n\in\mathcal{M}}(f_n - f_m))}{MH+1},$$

for forward contract in local market m as a function of the forward prices $\mathbf{f} = (f_1, ..., f_M)$ of all M producers with market power.[1] In this demand expression, $c = \frac{1}{M}\sum_{n\in\mathcal{M}}\frac{D_n}{D}c_n$ measures the

[1] Strict concavity with respect to k_{hm} in consumer h's profit function, $\partial^2\Omega_{hm}^{RI}/\partial k_{hm}^2 = -b_m(\frac{D_m}{MD})^2 < 0$, implies

quantity-weighted average of the marginal production costs of all producers with market power across the M local markets. The demand for forward contracts is linearly decreasing in the own forward price and linearly increasing in the forward prices of the producers in the other local markets:

$$\frac{\partial K_m^{RI}}{\partial f_m} = -\frac{2M^2H}{b_m}(\frac{D}{D_m})^2\frac{(M-1)H+1}{MH+1}, \quad \frac{\partial K_n^{RI}}{\partial f_m} = \frac{2M^2H}{b_n}(\frac{D}{D_n})^2\frac{H}{MH+1}, \; n \neq m. \tag{19}$$

At the first stage of the game, each local producer m chooses its forward price f_m to maximize profit

$$\Pi_m^{RI}(\mathbf{f}) = [f_m - \frac{1}{M}\sum_{n\in\mathcal{M}}\frac{D_n}{D}p_n(K_n^{RI}(\mathbf{f}))]K_m^{RI}(\mathbf{f}) + [p_m(K_m^{RI}(\mathbf{f})) - c_m]q_m(K_m^{RI}(\mathbf{f})),$$

taking the forward prices of the other suppliers as given. The first term in producer m's marginal profit expression

$$\frac{\partial \Pi_m^{RI}(\mathbf{f})}{\partial f_m} = [1 - \frac{1}{M}\sum_{n\notin m}\frac{D_n}{D}p_n'(k_n)\frac{\partial K_n^{RI}}{\partial f_m}]k_m + [f_m - \frac{1}{M}\sum_{n\in\mathcal{M}}\frac{D_n}{D}p_n(k_n)]\frac{\partial K_m^{RI}}{\partial f_m}$$

measures the marginal benefit of an increase in the forward premium, and the second term represents the marginal cost of a decrease in the demand for forward contracts. Substituting the forward premium (18) and the marginal demand effects (19) into producer m's marginal profit function and setting the expression to zero, enables us to solve for the forward quantity

$$k_m^{RI} = \frac{MD + 2M^2H(\frac{D}{D_m})^2\frac{\psi}{b_m}}{H+2}$$

sold by producer m in equilibrium.[2] This expression generalizes k^{RI} characterized in Section 3.2 of Tangerås and Wolak (2024) to the case of $M \geq 2$ asymmetric markets. The corresponding equilibrium price in spot market m equals

$$p_m^{RI} = p_m(k_m^{RI}) = \frac{a_m + c_m}{2} - \frac{1}{2}\frac{a_m + 2MH\frac{D}{D_m}\psi}{H+2} \tag{20}$$

3.3 Comparison of market designs

A comparison of the quantity-weighted equilibrium prices p_m^I characterized in (17) in the spatially independent market design with the quantity-weighted equilibrium prices p_m^{RI} characterized in (20) in a market design in which local markets are linked through a regional forward contract, yields:

Proposition 2 *Consider an electricity market with $M \geq 2$ local markets. Let there be one*

that the individual demand $k_{hm} = \frac{K_m^{RI}(\mathbf{f})}{H}$ indeed represents an equilibrium best-response.

[2] Strict concavity, $\partial^2\Pi_m^{RI}/\partial f_m^2 = [2 + H\frac{(M-2)H+H-1}{MH+1}]\frac{\partial K_m^{RI}}{\partial f_m} < 0$, of producer m's profit function implies that the solution to the first-order condition $\partial \Pi_m^{RI}(\mathbf{f})/\partial f_m = 0$ indeed represents an equilibrium.

producer with market power in each local market, and assume that each producer is active only in one local market. Linking the M local markets through a regional forward contract with a settlement price equal to the quantity-weighted average of the shot-term prices in those M markets, increases competition in the short-term markets by reducing the quantity-weighted average of the short-term prices,

$$\frac{1}{M}\sum_{m\in\mathcal{M}}\frac{D_m}{D}(p_m^I - p_m^{RI}) = \frac{M-1}{H+2}H\psi \geq 0, \tag{21}$$

compared to the benchmark of spatially independent markets. The inequality is strict if $\psi > 0$.

An alternative formulation of this proposition is that consumers' total spot market purchases across the M local markets are cheaper under a regional forward contract, compared to a design with M local forward markets. The proposition also speaks to the efficiency of bundling local forward markets through a regional forward contract. The average pro-competitive effect is stronger when more markets are linked because the right-hand side of (21) is increasing in M.

To derive a formal result, consider a collection $\mathcal{O}$ of O local markets indexed by o. Assume that $\mathcal{O}$ initially is partitioned into two regional forward markets, $\mathcal{M}$ and $\mathcal{N}$. The first regional forward market encompasses M local markets indexed by m, and the other consists of N local markets indexed by n. Let the average electricity demand per local market be equal to D_M in $\mathcal{M}$ and D_N in $\mathcal{N}$. Using (20) we get the total spot market expenditures

$$\sum_{m\in\mathcal{M}} D_m p_m^{RI} = \sum_{m\in\mathcal{M}} D_m \frac{(H+1)a_m + (H+2)c_m}{2(H+2)} - \frac{H}{H+2}M^2 D_M \psi$$

across the M local markets contained in region $\mathcal{M}$ under a regional forward contract that settles against the quantity-weighted average of the spot market prices in those M local markets. A corresponding expression exists in the N local markets contained in region $\mathcal{N}$. Summing up across all markets yields the total spot market expenditures

$$\sum_{m\in\mathcal{M}} D_m p_m^{RI} + \sum_{n\in\mathcal{M}} D_n p_n^{RI} = \sum_{o\in\mathcal{O}} D_o \frac{(H+1)a_o + (H+2)c_o}{2(H+2)} - \frac{H}{H+2}(M^2 D_M + N^2 D_N)\psi$$

when there are two regional forward markets. In a single regional forward market that spans all $\mathcal{O} = \mathcal{M} \cup \mathcal{N}$ local markets, the total spot market expenditures are instead

$$\sum_{o\in\mathcal{O}} D_o p_o^{RI} = \sum_{o\in\mathcal{O}} D_o \frac{(H+1)a_o + (H+2)c_o}{2(H+2)} - \frac{H}{H+2}O^2 D_O \psi,$$

where $D_O = \frac{1}{O}\sum_{o\in\mathcal{O}} D_o$. Subtracting the spot market expenditures under the two different regional market designs yields:

Corollary 1 *Merging two regional forward markets $\mathcal{M}$ and $\mathcal{N}$ into a larger regional forward market $\mathcal{O} = \mathcal{M} \cup \mathcal{N}$ reduces consumers' total spot market expenditures on electricity across the $O = M + N$ short-term markets that constitute the geographical footprint of the enlarged regional* *forward market by*

$$\sum_{m\in\mathcal{M}} D_m p_m^{RI} + \sum_{n\in\mathcal{M}} D_n p_n^{RI} - \sum_{o\in\mathcal{O}} D_o p_o^{RI} = \frac{HMN}{H+2}(D_M + D_N)\psi \geq 0,$$

with strict inequality if $\psi > 0$.

By this corollary it would be globally efficient to link all local markets through one global forward market, although doing so would not necessarily reduce spot prices in all local markets.

4 Producers active in multiple markets

Assume that there are $M \geq 2$ asymmetric local markets with one local monopoly in each market. Let there be $1 \leq S \leq M$ producers. Every regional monopoly producer s is active (owns generation capacity) in a subset $\mathcal{M}_s$ of the M local markets. We denote such a company a regional monopoly. Denote by $M_s \geq 1$ the cardinality of $\mathcal{M}_s$. By this construction, $\sum_{s=1}^{S} M_s = M$.

4.1 Linking forward markets across space

Regional ownership of generation assets does not matter under spatial independence by the assumption that local markets are functionally independent. Hence, we only consider the case when local markets are linked through a regional forward contract.

Equilibrium in the short-term market The third-stage profit of producer s equals

$$[f_s - \frac{1}{M}\sum_{n=1}^{M}\frac{D_n}{D}P_n(q_n)]k_s + \sum_{m\in\mathcal{M}_s}[P_m(q_m) - c_m]q_m,$$

where $k_s = \sum_{m\in\mathcal{M}_s} k_m$ is the total forward quantity sold by producer s. Maximizing over q_m, $m \in \mathcal{M}_s$, yields the first-order condition

$$-\frac{1}{M}\frac{D_m}{D}P'_m(q_m)k_s + P_m(q_m) - c_m + P'_m(q_m)q_m = 0$$

We can then solve for the quantity produced and the short-term price

$$q_m(k_s) = \frac{1}{2}\frac{a_m - c_m}{b_m} + \frac{1}{2}\frac{D_m}{D}\frac{k_s}{M},\ p_m(k_s) = P(q_m(k_s)) = \frac{a_m + c_m}{2} - \frac{b_m}{2}\frac{D_m}{D}\frac{k_s}{M} \qquad (22)$$

in $m \in \mathcal{M}_s$.

The demand for regional forward contracts The consumer in market $m \in \mathcal{M}_s$ maximizes

$$\Omega_{hm}^{RI}(k_{hm}, k_{-hm}, \mathbf{k}_{-m}, f_m) = -[f_s - \frac{1}{M}\sum_{t=1}^{S}\sum_{n\in\mathcal{M}_t}\frac{D_n}{D}p_n(k_t)]k_{hm} + [v - p_m(k_s)]\frac{D_m}{H} - \psi(\frac{D_m}{H} - k_{hm})$$

over k_{hm}. Differentiating with respect to k_{hm} yields the first-order condition

$$f_s - \frac{1}{M}\sum_{t=1}^{S}\sum_{n\in\mathcal{M}_t}\frac{D_n}{D}p_n(k_t) = \frac{b_m(\frac{D_m}{D})^2 MD - z_s k_m}{2M^2 H} + \psi,\ z_s = \sum_{n\in\mathcal{M}_s} b_n(\frac{D_n}{D})^2.$$

The maximization problem is strictly concave by $\frac{\partial^2\Omega_{hm}^{RI}}{\partial(k_{hm}^{RI})^2} = -\frac{z_s}{M^2} < 0$. Summing up the first-order conditions across all those local markets in which s owns generation capacity, produces the forward premium

$$f_s - \frac{1}{M}\sum_{t=1}^{S}\sum_{n\in\mathcal{M}_t}\frac{D_n}{D}p_n(k_t) = \frac{z_s}{M_s}\frac{MD - k_s}{2M^2 H} + \psi. \tag{23}$$

By implication,

$$f_s - \frac{z_s}{M_s}\frac{MD - k_s}{2M^2 H} = f_t - \frac{z_t}{M_t}\frac{MD - k_t}{2M^2 H},$$

for all pairs of regional monopolies s and t, which we can use to solve for the forward quantity k_t

$$\frac{k_t}{M} = 2MH(f_s - f_t)\frac{M_t}{z_t} + (\frac{z_t}{M_t} - \frac{z_s}{M_s})\frac{M_t}{z_t}D + \frac{M_t}{z_t}\frac{z_s}{M_s}\frac{k_s}{M},$$

as a linear function of k_s. We can then derive the average price in the region controlled by t

$$\begin{aligned}\frac{1}{M}\sum_{n\in\mathcal{M}_t}\frac{D_n}{D}p_n(k_t) &= \frac{1}{M}\sum_{n\in\mathcal{M}_t}\frac{D_n}{D}(\frac{a_n + c_n}{2} - \frac{b_n}{2}\frac{D_n}{D}\frac{k_t}{M}) \\ &= \frac{1}{M}\sum_{n\in\mathcal{M}_t}\frac{D_n}{D}\frac{c_n}{2} - H(f_s - f_t)M_t + \frac{M_t}{M}\frac{z_s}{M_s}\frac{MD - k_s}{2M}\end{aligned}$$

as a linear function of k_s. Now sum up over all local monopolies to solve for the average short-term price

$$\frac{1}{M}\sum_t\sum_{n\in\mathcal{M}_t}\frac{D_n}{D}p_n(k_t) = \frac{1}{M}\sum_n\frac{D_n}{D}\frac{c_n}{2} - H\sum_{t\neq s}(f_s - f_t)M_t + \frac{z_s}{M_s}\frac{MD - k_s}{2M}.$$

We can then substitute this expression back into (23) and solve for the demand

$$K_s^{RI}(\mathbf{f}) = MD + \frac{2M^2 H}{MH+1}\frac{M_s}{z_s}\frac{1}{M}\sum_n\frac{D_n}{D}\frac{c_n}{2} + \frac{M_s}{z_s}\frac{2M^2 H}{MH+1}[\psi - f_s + H\sum_{t\neq s}M_t(f_t - f_s)]$$

for forward contracts from producer s.The demand for forward contracts is linearly decreasing in the own-forward price, and linearly increasing in the forward price of the other producers:

$$\frac{\partial K_s^{RI}}{\partial f_s} = -\frac{M_s}{z_s}\frac{2M^2 H}{MH+1}[1 + H(M - M_s)],\ \frac{\partial K_t^{RI}}{\partial f_s} = \frac{M_t}{z_t}\frac{2M^2 H}{MH+1}HM_s. \tag{24}$$

The price of regional forward contracts Let us now solve for the forward price f_s that maximizes the profit

$$\begin{aligned}\Pi_s^{RI}(\mathbf{f}) &= [f_s - \frac{1}{M}\sum_{t\neq s}\sum_{n\in\mathcal{M}_t}\frac{D_n}{D}p_n(k_t) - \frac{1}{M}\sum_{m\in\mathcal{M}_s}\frac{D_m}{D}P_m(q_m(k_s))]k_s \\ &+ \sum_{m\in\mathcal{M}_s}[P(q_m(k_s)) - c_m]q_m(k_s)\end{aligned}$$

of producer s subject to $k_t = K_t^{RI}(\mathbf{f})$ for all t. The marginal profit of charging a higher forward price f_s equals

$$\begin{aligned}\frac{\partial \Pi_s^{RI}}{\partial f_s} &= [1 - \frac{1}{M}\sum_{t\neq s}\sum_{n\in\mathcal{M}_t}\frac{D_n}{D}p_n'(k_t)\frac{\partial K_t^{RI}}{\partial f_s}]k_s + [f_s - \frac{1}{M}\sum_{t}\sum_{n\in\mathcal{M}_t}\frac{D_n}{D}p_n(k_t)]\frac{\partial K_s^{RI}}{\partial f_s} \\ &+ \sum_{m\in\mathcal{M}_s}[-\frac{1}{M}\frac{D_m}{D}P_m'(q_m)k_s + P_m(q_m) - c_m + P_m'(q_m)q_m]q_m'(k_s)\frac{\partial K_s^{RI}}{\partial f_s}\end{aligned}$$

The expression on the second-row is only of second-order importance and therefore vanishes. The second-order condition is met by

$$\frac{\partial^2 \Pi_s^{RI}}{\partial f_s^2} = \frac{(2M - M_s)H + 2 + M_s H^2(M - M_s)}{MH + 1}\frac{\partial K_s^{RI}}{\partial f_s} < 0$$

We can therefore proceed to solve for the first-order condition.

Substitute in the marginal demand effects (24) and the forward premium (23) into the marginal profit expression and solve the first-order condition $\frac{\partial \Pi_s^{RI}}{\partial f_s} = 0$ for the equilibrium forward quantity sold by regional monopoly s:

$$k_s^{RI} = \frac{MD + 2M^2 H M_s \frac{\psi}{z_s}}{M_s H + 2}.$$

This expression generalizes k_m^{RI} defined in equation (43) in Tangerås and Wolak (2024) to the case where firm s has monopoly power in $M_s \geq 1$ short-term markets.

We can then calculate the price in local market $m \in \mathcal{M}_s$

$$p_m^{RI} = p_m(k_s^{RI}) = \frac{a_m + c_m}{2} - \frac{b_m}{2}\frac{D_m}{D}\frac{D + 2MHM_s\frac{\psi}{z_s}}{M_s H + 2}$$

and then aggregate across all $m \in \mathcal{M}_s$ to get the quantity-weighted average spot price

$$\frac{1}{M}\sum_{m\in\mathcal{M}_s}\frac{D_m}{D}p_m^{RI} = \frac{1}{M}\sum_{m\in\mathcal{M}_s}\frac{D_m}{D}\frac{a_m + c_m}{2} - \frac{1}{2M}\frac{z_s D + 2MHM_s\psi}{M_s H + 2}$$

charged by producer s.

4.2 Mergers

Consider a merger between two firms s and t to form the single entity u in a regional forward market. Specifically, the merged entity has monopoly power in the subset $\mathcal{M}_u = \mathcal{M}_s \cup \mathcal{M}_t$ of the local markets. Observe also that $M_u = M_s + M_t$. Absent any merger,

$$\frac{1}{M}\sum_{m\in\mathcal{M}_u}\frac{D_m}{D}p_m^{RI} = \frac{1}{M}\sum_{m\in\mathcal{M}_u}\frac{D_m}{D}\frac{a_m+c_m}{2} - \frac{1}{2M}\frac{z_sD+2MHM_s\psi}{M_sH+2} - \frac{1}{2M}\frac{z_tD+2MHM_t\psi}{M_tH+2}$$

characterizes the average price in region M_u. After the merger, the average regional price becomes

$$\frac{1}{M}\sum_{m\in\mathcal{M}_u}\frac{D_m}{D}\hat{p}_m^{RI} = \frac{1}{M}\sum_{m\in\mathcal{M}_u}\frac{D_m}{D}\frac{a_m+c_m}{2} - \frac{1}{2M}\frac{z_uD+2MHM_u\psi}{M_uH+2}$$

The difference is

$$\frac{1}{M}\sum_{m\in\mathcal{M}_s\cup\mathcal{M}_t}\frac{D_m}{D}(\hat{p}_m^{RI}-p_m^{RI}) = \frac{1}{M_uH+2}\frac{H}{2M}[M_t\frac{z_sD+2MHM_s\psi}{M_sH+2}+M_s\frac{z_tD+2MHM_t\psi}{M_tH+2}] > 0.$$

We collect our findings in the following result:

Proposition 3 *Merging two regional producers s and t into one larger unit u, increases the quantity-weighted average of spot prices in the subset $\mathcal{M}_u$ of short-term markets if local markets are linked by a regional forward contract, but has no implications for prices in a spatially independent market.*

Under a regional monopoly, so that there is only one producer, we have

$$\frac{1}{M}\sum_m\frac{D_m}{D}p_m^{RI} = \frac{1}{M}\sum_m\frac{D_m}{D}\frac{a_m+c_m}{2} - \frac{1}{2M}\frac{\sum_m\frac{D_m}{D}a_m+2M^2H\psi}{MH+2}$$

In a spatially independent market:

$$\frac{1}{M}\sum_m\frac{D_m}{D}p_m^{I} = \frac{1}{M}\sum_m\frac{D_m}{D}\frac{a_m+c_m}{2} - \frac{1}{2M}\frac{\sum_m\frac{D_m}{D}a_m+2MH\psi}{H+2}$$

Hence,

$$\frac{1}{M}\sum_m\frac{D_m}{D}(p_m^{RI}-p_m^{I}) = \frac{H(M-1)\frac{1}{M}\sum_m(\frac{D_m}{D}a_m-4\psi)}{2(MH+2)(H+2)}$$

which is positive if and only if $\frac{1}{M}\sum_m(\frac{D_m}{D}a_m-4\psi) > 0$. Under symmetry, we assumed $a-c > 4\psi$. Under similar assumptions, merger to monopoly is anti-competitive.

5 Multiple producers with market power

Assume that there are $M \geq 2$ symmetric local markets with $L \geq 2$ large producers that exercise market power in each local short-term market. Assume that every producer with market power is active in one market.

5.1 Spatially independent markets

We solve the game by backward induction, as usual.

Equilibrium in the short-term market Producer l has profit

$$[f_l - P(q_l + Q_{-l})]k_l + [P(q_l + Q_{-l}) - c]q_l$$

where Q_{-l} is the output of all large producers except l. The first-order condition is

$$-P'(Q)k_l + P(Q) - c + P'(Q)q_l = 0.$$

Sum up over all producers and solve for total supply

$$Q(k) = \frac{L}{L+1}\frac{a-c}{b} + \frac{k}{L+1},$$

where k is the total forward quantity sold by all producers. The corresponding short-term price equals

$$p(k) = P(Q(k)) = \frac{a + Lc}{L+1} - \frac{bk}{L+1}. \tag{25}$$

We can also solve for the individual supply

$$q_l(k_l, k_{-l}) = \frac{1}{L+1}\frac{a-c}{b} + \frac{L}{L+1}k_l - \frac{k_{-l}}{L+1}$$

to the short-term market of producer l and the residual supply

$$Q_{-l}(k_l, k_{-l}) = \frac{L-1}{L+1}\frac{a-c}{b} - \frac{L-1}{L+1}k_l + \frac{2}{L+1}k_{-l}$$

of all large producers other than l.

The demand for local forward contracts Forward quantities are perfect substitutes because the spot price only depends on the aggregate forward quantity k of all producers with market power. Therefore, all consumers want to buy from the producer with the lowest forward price. Define this price as $f = \min_l f_l$. Then consumer h has profit

$$\Omega_h^I(k_h, k_{-h}, f) = -[f - p(k)]k_h + [v - p(k)]\frac{D}{H} - \psi(\frac{D}{H} - k_h).$$

The marginal profit is

$$\frac{\partial \Omega_h^I}{\partial k_h} = -[f - p(k)] - p'(k)(\frac{D}{H} - k_h) + \psi$$

The profit function is strictly concave by $\partial^2 \Omega_h^I / \partial k_h^2 = -\frac{2b}{L+1} < 0$. We can then use symmetry to solve for the demand

$$K^I(f) = \frac{(H+1)a + LHc + (L+1)H(\psi - f)}{b(H+1)} \tag{26}$$

for forward contracts in a spatially independent market with L large producers.

The price of local forward contracts We finally solve for the equilibrium forward price $f^I = \min_l f_l^I$ and the associated forward quantity $k^I = K^I(f^I)$. By way of Bertrand competition, we need to solve the equilibrium also by other means than through differentiation of profit functions, which complicates the analysis.

Consider first a general situation in which producer l has charged the minimum price, $f_l = f$, and all other producers a higher price, $f_{-l} > f$. Producer l then has a monopoly in the forward market with associated profit

$$\pi_l^-(f) = [f - P(q(K^I(f)))]K^I(f) + [P(q(K^I(f))) - c]q_l(K^I(f), 0).$$

The marginal effect on profit of a slight increase in f equals:

$$\begin{aligned}\frac{\partial \pi_l^-}{\partial f} &= k + [f - p(k)]K^{I\prime}(f) + P'(Q)(q_l - k)\frac{\partial Q_{-l}}{\partial k_l}K^{I\prime}(f) \\ &\quad +[-P'(Q)k + P(Q) - c + P'(Q)q_l]\frac{\partial q_l}{\partial k_l}K^{I\prime}(f).\end{aligned}$$

Notice the strategic effect on the competing producers in the short-term market. An increase in the forward price reduces the demand for forward contracts from producer l. This reduction causes producer l to produce less and the other producers in local market m to produce more. The strategic effect is negative if $q_l > k$. The term on the second line is of second-order importance, so we can write the marginal profit as

$$\frac{\partial \pi_l^-}{\partial f} = k - [f - p(k)]\frac{H}{b}\frac{L+1}{H+1} - (q_l - k)H\frac{L-1}{H+1}$$

after invoking the marginal demand effect $K^{I\prime}(f) = -\frac{H}{b}\frac{L+1}{H+1} < 0$ from the forward market and the marginal price, $P'(Q) = -b$, and supply effect, $\frac{\partial Q_{-l}}{\partial k_l} = -\frac{L-1}{L+1}$, from the short-term market. The profit function $\pi_l^-(f)$ is strictly concave,

$$\begin{aligned}\frac{\partial^2 \pi_l^-}{\partial f^2} &= [1 + p'(k)\frac{H}{b}\frac{L+1}{H+1} - (\frac{\partial q_l}{\partial k_l} - 1)H\frac{L-1}{H+1}]K^{I\prime}(f) \\ &= -[1 + H\frac{L-1}{L+1}]\frac{L+1}{(H+1)^2}\frac{H}{b} < 0,\end{aligned}$$

so we can derive the forward quantity $\tilde{k}$ that maximizes π_l^- by solving producer l's first-order condition. Substitute the forward premium

$$f - p(k) = \frac{b}{L+1}\frac{D-k}{H} + \psi$$

and the explicit expression for $q_l(k,0)$ into the first-order condition $\frac{\partial \pi_l^-}{\partial f} = 0$ to get

$$\tilde{k} = \frac{(L+1)D + H(L+1)^2\frac{\psi}{b} + H(L-1)(D-\frac{c}{b})}{(H+2)(L+1)+H(L-1)} > 0 \tag{27}$$

after simplification. Observe in particular that

$$K^I(c) - \tilde{k} = \frac{(H+1)^2(L+1)(a-c) + (L+1+H(L-1))(c+H(L+1)\psi)}{b(H+1)((H+2)(L+1)+H(L-1))} > 0.$$

The demand $K^I(f)$ for forward contracts is strictly decreasing in f, so the forward price $\tilde{f}$ that maximizes $\pi_l^-(f)$ satisfies $\tilde{f} > c$.

From the above results, we can draw a number of conclusions. First, $f^I \in [0,\tilde{f}]$ in any equilibrium. For if $f^I > \tilde{f}$, then producer l could deviate to $\tilde{f}$ and earn the monopoly profit $\pi_l^-(\tilde{f})$, which is strictly higher than any profit it could achieve at the forward price $f_l^I \geq f^I > \tilde{f}$. Our second conclusion is $k^I = K^I(f^I) \geq \tilde{k} > 0$ since $f^I \leq \tilde{f}$ and the demand for forward contracts is strictly decreasing in f.

We next demonstrate that $f^I \geq c$ in any equilibrium. This is trivially true if $c = 0$ by our assumption that the forward price must be non-negative. Assume therefore that $c > 0$. Suppose $f^I \in [0,c)$ and that a total of $L^I \geq 1$ producers charge the forward price f^I. Assume that forward quantities are uniformly distributed across all those producers, so that producer l earns $\pi_l^I(L^I, f^I)$ in equilibrium if $f_l^I = f^I$, where

$$\pi_l^I(L, f^I) = (f^I - p^I)\frac{k^I}{L} + (p^I - c)q_l(\frac{k^I}{L}, \frac{L-1}{L}k^I)$$

and $p^I = p(k^I)$. Suppose $L^I \geq 2$, and consider an upward deviation by l to $f_l > f^I$. Producer l earns the profit

$$\pi_l^+ = (p^I - c)q_l(0, k^I) \tag{28}$$

by pricing itself out of the forward market because some producer(s) other than l sell the k^I forward contracts even if l does not sell any. The deviation net profit equals

$$\pi_l^+ - \pi_l^I(L^I, f^I) = (p^I - c)(q_l(0,k^I) - q_l(\frac{k^I}{L^I}, \frac{L^I-1}{L^I}k^I)) - (f^I - p^I)\frac{k^I}{L^I} = (c - f^I)\frac{k^I}{L^I} > 0. \tag{29}$$

Hence, $f^I \in [0,c)$ only if $L^I = 1$. Suppose $L^I = 1$, and $f_l^I = f^I$. By strict concavity of the monopoly profit function $\pi_l^-(f)$, and $\tilde{f} > c$, we have $\frac{\partial \pi_l^-}{\partial f}|_{f=f^I} > 0$ for all $f^I \in [0,c)$. Therefore, $f^I \in [0,c)$, $L^I = 1$, cannot be an equilibrium, either. We conclude that $f^I \geq c$ in any equilibrium.

We next demonstrate that $f^I \leq c$ in any equilibrium. Suppose $f^I > c$ and that $f_l^I > f^I$ for some producer l. This producer earns π_l^+ defined in (28) as it is priced out of the market in equilibrium. Consider instead a downward deviation to $f_l^I = f^I$. Producer l would earn

$\pi_l^I(L^I+1, f^I)$ under this deviation. The deviation net profit is strictly positive:

$$\pi_l^I(L^I+1, f^I) - \pi_l^+ = (f^I - p^I)\frac{k^I}{L^I+1} + (p^I - c)(q_l(\frac{k^I}{L^I+1}, \frac{L^I}{L^I+1}k^I) - q_l(0, k^I)) = (f^I - c)\frac{k^I}{L^I+1} > 0.$$

Hence, $f^I > c$ only if $f_l^I \leq f^I$ for all L producers with market power. Of course, $f_l^I < f^I$ for some producer l would contradict f^I as an equilibrium forward price. Hence, $f^I > c$ implies $f_l^I = f^I$ for all L, and therefore $L^I = L \geq 2$. Producer l could then undercut f^I by ε and earn approximately

$$\pi_l^-(f^I) = (f^I - p^I)k^I + (p^I - c)q_l(k^I, 0)$$

as a monopoly seller of forward contracts. The net profit of this deviation is approximately equal to

$$\pi_l^-(f^I) - \pi_l^I(L, f^I) = (f^I - p^I)\frac{L-1}{L}k^I + (p^I - c)(q_l(k^I, 0) - q_l(\frac{k^I}{L}, \frac{L-1}{L}k^I)) = (f^I - c)\frac{L-1}{L}k^I > 0.$$

We conclude that $f^I \leq c$ in any equilibrium.

On the basis of the above results, $f^I = c$ is the only possible equilibrium forward price. We finally show that $f^I = c$ is indeed sustainable as an equilibrium. Suppose all L producers with market power charge the forward price $f_l^I = c$. A unilateral upward deviation by producer l to $f_l > c$ then is weakly unprofitable by $\pi_l^+ - \pi_l(L, c) = 0$; see (29). A unilateral downward deviation by l to $f_l < c$, $c > 0$, yields monopoly profit $\pi_l^-(f)$. This profit function is strictly concave with a global maximum at $\tilde{f} > c$, as we have already established. Hence, all unilateral downward deviations are also strictly unprofitable.

These results deliver $f^I = c$ as the unique equilibrium forward price in a spatially independent market with multiple producers $L \geq 2$ with market power. By implication

$$k^I = K^I(c) = \frac{D + (L+1)H\frac{\psi}{b} + H(D - \frac{c}{b})}{H+1} \tag{30}$$

measures the quantity of forward contracts traded in equilibrium in a spatially independent market.

For completeness, we can calculate the forward premium

$$f^I - p^I = \frac{c + H(L+1)\psi}{(L+1)(H+1)} > 0$$

Seeing as $f^I = c$, this means that $p^I < c$.

The profit if producers do not sell any forward contracts equals

$$\pi^0 = (p(0) - c)\frac{Q(0)}{L} = \frac{1}{b}\frac{(a-c)^2}{(L+1)^2}$$

The difference

$$\begin{aligned}\pi^0 - \pi^I &= \frac{(H+1)(a-c)^2 - (c+H(L+1)\psi)^2}{b(H+1)(L+1)^2} \\ &= \frac{(\sqrt{H+1}(a-c) - c - H(L+1)\psi)^2(\sqrt{H+1}(a-c) + c + H(L+1)\psi)}{b(H+1)(L+1)^2}\end{aligned}$$

is positive if and only if

$$\frac{a-c}{c+H(L+1)\psi} > \frac{1}{\sqrt{H+1}}$$

Under plausible circumstances, generation owners are collectively worse off if they sell forward contracts in a forward market with Bertrand competition.

5.2 Linking forward markets across space

There are M local markets with L producers with market power in each local market.

Equilibrium in the short-term market Producer l in local market m has profit

$$[f_{lm} - \frac{1}{M}\sum_n P(Q_n)]k_{lm} + [P(Q_m) - c]q_{lm}$$

The first-order condition is

$$-\frac{1}{M}P'(Q_m)k_{lm} + P(Q_m) - c + P'(Q_m)q_{lm} = 0$$

Sum up over all producers and solve for the total production

$$Q_m(k_m) = Q(\frac{k_m}{M}) = \frac{L}{L+1}\frac{a-c}{b} + \frac{1}{L+1}\frac{k_m}{M},$$

where k_m is the total forward quantity sold by all L producers. The corresponding short-term price equals

$$p_m(k_m) = p(\frac{k_m}{M}) = \frac{a+Lc}{L+1} - \frac{b}{L+1}\frac{k_m}{M}. \tag{31}$$

We can also solve for the individual

$$q_{lm}(k_{lm}, k_{-lm}) = \frac{1}{L+1}\frac{a-c}{b} + \frac{L}{L+1}\frac{k_{lm}}{M} - \frac{1}{L+1}\frac{k_{-lm}}{M}$$

and residual

$$Q_{-lm}(k_{lm}, k_{-lm}) = \frac{L-1}{L+1}\frac{a-c}{b} - \frac{L-1}{L+1}\frac{k_{lm}}{M} + \frac{2}{L+1}\frac{k_{-lm}}{M}$$

supply to the short-term market.

The demand for regional forward contracts Forward quantities are perfect substitutes within local market m because the spot price only depends on the aggregate k_m. Therefore, all consumers want to buy from the producer with the lowest forward price in local market m.

Define this price as $f_m = \min_l f_{lm}$. Then consumer h has profit

$$\Omega^{RI}_{hm}(k_{hm}, k_{-hm}, \mathbf{k}_{-m}, f_m) = -[f_m - \frac{1}{M}\sum_n p_n(k_n)]k_{hm} + [v - p_m(k_m)]\frac{D}{H} - \psi(\frac{D}{H} - k_{hm}).$$

The solution to the first-order condition yields the forward premium

$$f_m - \frac{1}{M}\sum_n p_n(k_n) = \frac{b}{L+1}\frac{MD - k_m}{M^2 H} + \psi. \tag{32}$$

The consumer's profit function is strictly concave by the negative second-derivative $\frac{\partial^2 \Omega^{RI}_{hm}}{\partial k^2_{hm}} = -\frac{1}{M}\frac{2b}{L+1} < 0$. We can then compare first-order conditions to obtain forward quantity

$$\frac{b}{L+1}\frac{k_n}{M} = \frac{b}{L+1}\frac{k_m}{M} + MH(f_m - f_n)$$

in local market n as a function of the forward quantity k_m in local market m and the price differences in the two markets. Summarizing across all local markets yields

$$\frac{1}{M}\sum_n p_n(k_n) = \frac{a + Lc}{L+1} - \frac{b}{L+1}\frac{k_m}{M} + H\sum_n (f_m - f_n).$$

We can then insert this expression into the forward premium (32) and solve for the demand

$$K^{RI}_m(\mathbf{f}) = \frac{M}{b}\frac{(MH+1)a + LMHc + (L+1)MH[\psi - f_m + H\sum_{n\neq m}(f_n - f_m)]}{MH+1}$$

for forward contracts in local market m. This demand is linearly decreasing in the own forward price and linearly increasing in the forward price in the other local markets even in this context:

$$\frac{\partial K^{RI}_m}{\partial f_m} = -\frac{M}{b}MH(L+1)\frac{1 + H(M-1)}{MH+1}, \ \frac{\partial K^{RI}_n}{\partial f_m} = \frac{M}{b}MH(L+1)\frac{H}{MH+1}, \ n \neq m.$$

The profit-maximizing forward price We finally solve for the profit-maximizing forward price. The method of proof is similar to the proof in a spatially independent market. Assume that all local markets n other than m are in equilibrium, $f_n = f^{RI}_n$, $n \neq m$, where $f^{RI}_n = \min_l f^{RI}_{\ln}$. We start out by considering the properties of the monopoly profit

$$\pi^-_{lm}(\mathbf{f}) = [f_m - \frac{1}{M}\sum_n p_n(K^{RI}_n(\mathbf{f}))]K^{RI}_m(\mathbf{f}) + [p_m(K^{RI}_m(\mathbf{f})) - c]q_{lm}(K^{RI}_m(\mathbf{f}), 0)$$

of producer l in local market m. The associated marginal profit is

$$\begin{aligned}\frac{\partial \pi_{lm}^{-}}{\partial f_m} &= [1-\frac{1}{M}\sum_{n\neq m} p_n'(k_n)\frac{\partial K_n^{RI}}{\partial f_m}]k_m + [f_m - \frac{1}{M}\sum_n p_n(k_n)]\frac{\partial K_m^{RI}}{\partial f_m} \\ &+P'(Q_m)[q_{lm}(k_m,0)-\frac{k_m}{M}]\frac{\partial Q_{-lm}}{\partial k_{lm}}\frac{\partial K_m^{RI}}{\partial f_m} \\ &+[-P'(Q_m)\frac{k_{lm}}{M}+P(Q_m)-c+P'(Q_m)q_{lm}]\frac{\partial q_{lm}}{\partial k_{lm}}\frac{\partial K_m^{RI}}{\partial f_m}\end{aligned}$$

The first term on the first row measures the marginal effect on the forward premium of an increase in the forward price f_m. The second is the marginal effect on the demand for forward contracts. The term on the second row is the strategic effect. The terms of the third row are of second-order importance for marginal profit in the forward contracting stage. The second-derivative is

$$\begin{aligned}\frac{\partial^2 \pi_{lm}^{-}}{\partial f_m^2} &= [2-\frac{2}{M}\sum_{s\neq m} p'(k_s)\frac{\partial K_s^{RI}}{\partial f_m} - \frac{1}{M}p'(k_m)\frac{\partial K_m^{RI}}{\partial f_m} \\ &+P'(Q_m)[\frac{\partial q_{lm}}{\partial k_{lm}}-\frac{1}{M}]\frac{\partial Q_{-lm}}{\partial k_{lm}}\frac{\partial K_m^{RI}}{\partial f_m}]\frac{\partial K_m^{RI}}{\partial f_m},\end{aligned}$$

which we can simplify to

$$\frac{\partial^2 \pi_{lm}^{-}(\mathbf{f})}{\partial f_m^2} = -\frac{2}{b}\frac{M^2H}{(MH+1)^2}(HL+L+1)(1+H(M-1))^2 < 0.$$

Hence, $\pi_{lm}^{-}(\mathbf{f})$ is strictly concave in f_m. We can then derive the forward quantity k_m^* that maximizes $\pi_{lm}^{-}(\mathbf{f})$ by solving the first-order condition $\frac{\partial \pi_{lm}^{-}(\mathbf{f})}{\partial f_m} = 0$:

$$k_m^* = \frac{M}{2b}\frac{MH(L+1)^2}{HL+L+1}[\frac{(L+1)a+H(L-1)(a-c)}{MH(L+1)^2}+\psi].$$

There is no guarantee that this forward quantity can be sustained at a non-negative forward price f_m^*. We therefore define the monopoly quantity as $\tilde{k}_m = \min\{k_m^*; K_m^{RI}(0,\mathbf{f}_{-m}^{RI})\}$, and let $\tilde{f}_m \geq 0$ be the forward price that generates this demand.

Any equilibrium forward price must feature $f_m^{RI} \leq \tilde{f}_m$ because producer l could implement its (constrained) monopoly profit by deviating to $f_{lm} = \tilde{f}_m < f_m^{RI} \leq f_{lm}^{RI}$ otherwise. By implication, $k_m^{RI} \geq \tilde{k}_m > 0$ as $K_m^{RI}(\mathbf{f})$ is a decreasing function of f_m, and k_m^* and $K_m^{RI}(0,\mathbf{f}_{-m}^{RI})$ are both strictly positive.

The expression

$$X_m(f_m) = f_m - \frac{1}{M}\sum_n p_n(K_n^{RI}(f_m,\mathbf{f}_{-m}^{RI})) + \frac{1}{M}(p(K_m^{RI}(f_m,\mathbf{f}_{-m}^{RI}))-c) \qquad (33)$$

will be important to characterize the equilibrium. Seeing as

$$X'_m(f_m) = 1 - \frac{1}{M}\sum_{n\neq m} p'_n(k_n)\frac{\partial K_n^{RI}}{\partial f_m} = (H+1)\frac{1+H(M-1)}{MH+1} > 0,$$

we can define $g_m \in \mathbb{R}$ by $X_m(g_m) = 0$. By inserting the forward premium (32) into $X_m(f_m)$ and also the explicit expression for $p_m(k_m)$ from (31), we obtain an alternative useful formulation

$$X_m(f_m) = \frac{1}{MH}\frac{(H+1)a - Hc}{L+1} + \psi - \frac{b}{L+1}\frac{1}{M}\frac{H+1}{H}\frac{K_m^{RI}(f_m, \mathbf{f}_{-m}^{RI})}{M}. \tag{34}$$

In particular, we can set (34) to zero and solve for the forward quantity

$$K_m^{RI}(g_m, \mathbf{f}_{-m}^{RI}) = \frac{M}{b}\frac{(H+1)a - Hc + MH(L+1)\psi}{H+1} > 0. \tag{35}$$

Armed with these definitions, we can identify necessary conditions for an equilibrium. At the equilibrium price, $f_{lm}^{RI} = f_m^{RI}$, producer l in local market m earns $\pi_{lm}^{RI}(L_m^{RI}, f_m^{RI})$, where

$$\pi_{lm}^{RI}(L_m, f_m^{RI}) = [f_m^{RI} - \frac{1}{M}\sum_n p_n^{RI}]\frac{k_m^{RI}}{L_m} + (p_m^{RI} - c)q_{lm}(\frac{k_m^{RI}}{L_m}, \frac{L_m - 1}{L_m}k_m^{RI}),$$

if $L_m^{RI} \geq 1$ producers in local market m charge f_m^{RI}, and forward sales are uniformly distributed across those L_m^{RI} producers.

We first show that $f_m^{RI} \geq \max\{g_m; 0\}$. This property follows directly from non-negativity constraint $f_m^{RI} \geq 0$ on the forward price if $g_m \leq 0$. Assume therefore that $g_m > 0$, and suppose $f_m^{RI} \in [0, g_m)$. If $L_m^{RI} \geq 2$, then producer l could deviate upward to $f_{lm} > f_{lm}^{RI} = f_m^{RI}$ and obtain profit

$$\pi_{lm}^{+} = (p_m^{RI} - c)q_{lm}(0, k_m^{RI}). \tag{36}$$

This deviation would be strictly profitable by

$$\begin{aligned}\pi_{lm}^{+} - \pi_{lm}^{RI}(L_m^{RI}, f_m^{RI}) &= (p_m^{RI} - c)(q_{lm}(0, k_m^{RI}) - q_{lm}(\frac{k_m^{RI}}{L_m^{RI}}, \frac{L_m^{RI} - 1}{L_m^{RI}}\frac{k_m^{RI}}{L_m^{RI}})) - [f_m^{RI} - \frac{1}{M}\sum_n p_n^{RI}]\frac{k_m^{RI}}{L_m^{RI}} \\ &= -X_m(f_m^{RI})\frac{k_m^{RI}}{L_m^{RI}} > 0\end{aligned}$$

since $X_m(f_m) < 0$ for all $f_m < g_m$. Hence, $f_m^{RI} < g_m$ implies $L_m^{RI} = 1$. The single producer l that charges $f_{lm}^{RI} = f_m^{RI}$ then earns $\pi_{lm}^{-}(\mathbf{f}^{RI})$ as the monopoly supplier of forward contracts. Seeing as

$$\begin{aligned}K_m^{RI}(g_m, \mathbf{f}_{-m}^{RI}) - k_m^* &= \frac{M}{2b}\frac{(L+1)(H+1)^2(a-c) + ((H+1)(L-1)+2)c}{(H+1)(HL+L+1)} \\ &+ \frac{M}{2b}\frac{H(L-1)+L+1}{(H+1)(HL+L+1)}MH(L+1)\psi\end{aligned}$$

is strictly positive, strict monotonicity of $K_m^{RI}(\mathbf{f})$ in f_m implies $\tilde{f}_m > g_m$. Strict concavity of

$\pi^-_{lm}(f_m, \mathbf{f}^{RI}_{-m})$ in f_m then implies $\frac{\partial \pi^-_{lm}}{\partial f_m}|_{f_m = f^{RI}_m} > 0$ for all $f^{RI}_m < g_m$, which violates the assumption of profit maximization. Hence, $f^{RI}_m \in [0, g_m)$ and $L^{RI}_m = 1$ cannot be an equilibrium, either. Therefore, $f^{RI}_m \geq g_m$ if $g_m > 0$.

We next show that $f^{RI}_m \leq \max\{g_m; 0\}$. Suppose $f^{RI}_m > \max\{g_m; 0\}$ and $f^{RI}_{lm} > f^{RI}_m$ for some large producer. The equilibrium profit of producer l would then be equal to π^+_{lm} characterized in (36). Producer l would instead earn $\pi^{RI}_{lm}(L^{RI}_m + 1, f^{RI}_m)$ by deviating to $f_{lm} = f^{RI}_m$. This deviation would be strictly profitable by

$$\pi^{RI}_{lm}(L^{RI}_m + 1, f^{RI}_m) - \pi^+_{lm} = X_m(f^{RI}_m)\frac{k^{RI}_m}{L^{RI}_m + 1} > 0$$

since $X_m(f_m) > 0$ for all $f_m > g_m$. Hence, $f^{RI}_m > \max\{g_m; 0\}$ implies $f^{RI}_{lm} \leq f^{RI}_m$ for all L producers with market power in local market m. Of course, $f^{RI}_{lm} < f^{RI}_m$ for some producer l would violate f^{RI}_m as the equilibrium forward price in local market m. Hence, $f_m > \max\{g_m; 0\}$ implies $f^{RI}_{lm} = f^{RI}_m$ for all L. Producer l would then earn $\pi^{RI}_{lm}(L, f^{RI}_m)$ in equilibrium. Alternatively, producer l could reduce its forward price by ε and earn approximately $\pi^-_{lm}(\mathbf{f}^{RI})$ as the monopoly seller of forward contracts. Such a deviation would be strictly profitable by

$$\pi^-_{lm}(\mathbf{f}^{RI}) - \pi^{RI}_{lm}(L, f^{RI}_m) = X_m(f^{RI}_m)\frac{L-1}{L}k^{RI}_m > 0.$$

We conclude that $f^{RI}_m \leq \max\{g_m; 0\}$.

The above results deliver $f^{RI}_m = \max\{g_m; 0\}$ as the only feasible equilibrium forward price. We next show that this price indeed can be sustained as an equilibrium. Assume that all producers with market power in local market m sell forward contracts at the same price $f^{RI}_m = \max\{g_m; 0\}$. A unilateral upward deviation by producer l is unprofitable, $\pi^+_{lm} - \pi^{RI}_{lm}(L, f^{RI}_m) = -X_m(f^{RI}_m)\frac{k^{RI}_m}{L} \leq 0$, by $X_m(f^{RI}_m) = X_m(\max\{g_m; 0\}) \geq X_m(g_m) = 0$. Unilateral downward deviations are infeasible if $g_m \leq 0$ because then $f^{RI}_m = 0$. Assume therefore that $g_m > 0$, and consider a downward deviation by l to $f_{lm} = f_m < g_m = f^{RI}_m$. Strict concavity of the profit function $\pi^-_{lm}(f_m, \mathbf{f}^{RI}_{-m})$ and a profit-maximizing forward price $\tilde{f}_m > g_m$ implies that such unilateral downward deviations are strictly unprofitable.

We are now equipped with all the results we need to derive the quantity of forward contracts sold in symmetric equilibrium. The above results deliver $f^{RI}_m = \max\{g_m; 0\}$ as the equilibrium forward price in local market m. If $g_m \geq 0$, then the associated equilibrium forward quantity equals

$$k^{RI}_m = K^{RI}_m(\mathbf{f}^{RI}) = K^{RI}_m(g_m, \mathbf{f}^{RI}_{-m}) = \frac{M}{b}\frac{(H+1)a - Hc + MH(L+1)\psi}{H+1}. \tag{37}$$

by (35). If $g_m < 0$, then $f^{RI}_m = 0$ and therefore $k^{RI}_m = K^{RI}_m(0, \mathbf{f}^{RI}_{-m})$. However, this is not a complete characterization. Impose symmetry across all markets to obtain

$$k^{RI}_m = K^{RI}_m(\mathbf{0}) = \frac{M}{b}\frac{(MH+1)a + LMHc + (L+1)MH\psi}{MH+1} \tag{38}$$

in this case. Which of these two equilibria is sustainable depends on the underlying parameters.

Solve for the symmetric forward price

$$f^* = \frac{(H+1)(MH+1)}{2MH(HL+L+1)}a + \frac{ML((2L+1)H+2(L+1)) - MH + L - 1}{2M(L+1)(HL+L+1)}c - \frac{(L+1)(MH+1) - 2(HL+L+1)}{2(HL+L+1)}\psi$$

that generates forward demand characterized by (37). This price is non-negative, for instance, if ψ is sufficiently small. Otherwise, sellers give away forward contracts for free in symmetric equilibrium.

5.3 Comparison of market designs

Straightforward comparison of (37) and (30) delivers

$$K_m^{RI}(g_m, \mathbf{f}_{-m}^{RI}) - MK^I(c) = M(M-1)H\frac{L+1}{H+1}\frac{\psi}{b} \geq 0,$$

with strict inequality if $\psi > 0$. Subtracting (30) from (38) produces

$$K_m^{RI}(\mathbf{0}) - MK^I(c) = MH\frac{MH+1+LM(H+1)}{(MH+1)(H+1)}\frac{c}{b} + MH\frac{(M-1)(L+1)}{(MH+1)(H+1)}\frac{\psi}{b} \geq 0,$$

which is strictly positive if either $c > 0$ or $\psi > 0$. We collect these findings in the following result:

Proposition 4 *Consider an electricity market with $M \geq 2$ symmetric local markets. Let there be $L \geq 2$ producers with market power in each local market, and assume that each producer is active in one local market. Linking the M local electricity markets through a regional forward contract that has an exercise price equal to the average of the short-term prices in those M markets, increases the symmetric equilibrium forward quantity k^{RI} sold in each local market by at least a factor M, compared to the benchmark k^I of spatially independent markets, $k^{RI} \geq Mk^{RI}$. The inequality is strict if $\psi > 0$.*

6 Multiple trading periods for forward contracts

Consider the spatially independent electricity market. Assume that forward contracts are sold sequentially over two trading periods. Let k_τ be the forward quantity sold and f_τ the linear forward price in period $\tau = 1, 2$. Let $k = k_1 + k_2$ be the total forward quantity sold over both periods. Let $k_{h\tau}$ be consumer h's purchased forward quantity in trading period τ, and denote by $k_h = k_{h1} + k_{h2}$ its total forward quantity. We consider a five-stage game:

1. The monopoly producer commits to a linear price f_1 at which to sell forward contracts in trading period 1.

2. Each consumer h decides the forward quantity k_{h1} to purchase in trading period 1.

3. The monopoly producer commits to a linear price f_2 at which to sell forward contracts in trading period 2.

4. Each consumer h decides the forward quantity k_{h2} to purchase in trading period 2.

5. The monopoly producer supplies q to the short-term market.

We solve the game by backward induction.

Equilibrium in the short-term market The monopoly firm has profit

$$[f_1 - P(q)]k_1 + [f_2 - P(q)]k_2 + [P(q) - c]q$$

in the short-term market as a function of its production q. The first-order condition

$$-P'(q)k + P(q) - c + P'(q)q = 0$$

yields the monopoly production and spot price

$$q(k) = \frac{1}{2}\frac{a-c}{b} + \frac{1}{2}k,\ p(k) = P(q(k)) = \frac{a+c}{2} - \frac{b}{2}k.$$

The demand for forward contracts in trading period 2 Consider now the demand for forward contracts by firm h in the second trading period. Consumer h has total profit

$$-[f_1 - p(k)]k_{h1} - [f_2 - p(k)]k_{h2} + [v - p(k)]\frac{D}{H} - \psi(\frac{D}{H} - k_h)$$

The second-derivative of this profit function with respect to k_{h2} equals $-b < 0$. We can thus solve the first-order condition and apply symmetry, $k_h = \frac{k}{H}$, to get the forward premium

$$f_2 - p(k) = \frac{b}{2}\frac{D-k}{H} + \psi$$

in trading period 2. We can then solve for the demand

$$K_2^I(f_2, k_1) = \frac{(H+1)a + Hc + 2H(\psi - f_2)}{b(H+1)} - k_1$$

for forward contracts in period 2 as a function of the forward price f_2 that period and the forward quantity k_1 sold in trading period 1. We can also plug this expression into marginal profit to solve for the individual forward demand

$$K_{h2}^I(f_2, k_{h1}) = \frac{(H+1)a + Hc + 2H(\psi - f_2)}{bH(H+1)} - k_{h1}$$

of consumer h in trading period 2.

The forward price in trading period 2 We next solve for the profit-maximizing forward price in trading period 2. The monopoly firm maximizes its profit

$$[f_1 - P(q(k_1 + K_2^I(f_2, k_1)))]k_1 + [f_2 - P(q(k_1 + K_2^I(f_2, k_1)))]K_2^I(f_2, k_1)$$
$$+[P(q(k_1 + K_2^I(f_2, k_1))) - c]q(k_1 + K_2^I(f_2, k_1))$$

over f_2. Differentiation yields the marginal profit

$$K_2^I(f_2, k_1) + [f_2 - p(k_1 + K_2^I(f_2, k_1))]\frac{\partial K_2^I}{\partial f_2}$$

The second-derivative of the producer's profit function is

$$[2 - p'(k)\frac{\partial K_2^I}{\partial f_2}]\frac{\partial K_2^I}{\partial f_2} = \frac{H+2}{H+1}\frac{\partial K_2^I}{\partial f_2} < 0.$$

Using the first-order condition, we can solve for the monopoly's profit-maximizing forward price

$$F_2^I(k_1) = \max\{\frac{(H+1)^2 a + H(H+2)c + 2H\psi - (H+1)^2 b k_1}{2H(H+2)}; 0\}$$

in trading period 2.

The demand for forward contracts in trading period 1 Consider now the demand for forward contracts by firm h in the first trading period. Consumer h has total profit

$$-[f_1 - p(k_1 + K_2^I(f_2, k_1))]k_{h1} - [f_2 - p(k_1 + K_2^I(f_2, k_1))]K_{h2}^I(f_2, k_{h1})$$
$$+[v - p(k_1 + K_2^I(f_2, k_1))]\frac{D}{H} - \psi(\frac{D}{H} - k_{h1} - K_{h2}^I(f_2, k_{h1})).$$

Recall also $f_2 = F_2^I(k_1)$. Then the marginal profit of increasing demand in the first trading period can be decomposed into

$$-[f_1 - p(k)] - p'(k)(\frac{D}{H} - k_h) + \psi$$
$$-\frac{\partial F_2^I}{\partial k_1}k_{h2} - p'(k)(\frac{D}{H} - k_h)[\frac{\partial K_2^I}{\partial f_2}\frac{\partial F_2^I}{\partial k_1} + \frac{\partial K_2^I}{\partial k_1} - \frac{\partial K_{h2}^I}{\partial f_2}\frac{\partial F_2^I}{\partial k_1} - \frac{\partial K_{h2}^I}{\partial k_{h1}}]$$
$$-[f_2 - p(k) + p'(k)(\frac{D}{H} - k_h) - \psi][\frac{\partial K_{h2}^I}{\partial f_2}\frac{\partial F_2^I}{\partial k_1} + \frac{\partial K_{h2}^I}{\partial k_{h1}}]$$

The terms on the first row constitute the direct effect in the first trading period. The terms on the second row represent the strategic effect on the forward price in trading period 2 and the other large consumers' trade in the second trading period. The terms on the final row are the effect on firm h's trade in the second trading period, which is only of second-order importance for marginal profit.

Let us first see if we can evaluate the strategic effect on the second row at the interior forward price $F_2^I(k_1) > 0$:

$$\frac{\partial K_2^I}{\partial f_2}\frac{\partial F_2^I}{\partial k_1} + \frac{\partial K_2^I}{\partial k_1} = \frac{H+1}{H+2} - 1, \ \frac{\partial K_{h2}^I}{\partial f_2}\frac{\partial F_2^I}{\partial k_1} + \frac{\partial K_{h2}^I}{\partial k_{h1}} = \frac{1}{H}\frac{H+1}{H+2} - 1.$$

These marginal effects jointly imply

$$\frac{\partial K_2^I}{\partial f_2}\frac{\partial F_2^I}{\partial k_1} + \frac{\partial K_2^I}{\partial k_1} - \frac{\partial K_{h2}^I}{\partial f_2}\frac{\partial F_2^I}{\partial k_1} - \frac{\partial K_{h2}^I}{\partial k_{h1}} = \frac{H-1}{H}\frac{H+1}{H+2} \geq 0$$

An increase in demand for forward contracts by consumer h in period 1 causes all other large consumers to increase their demand for forward contracts in period 2 because of the reduction in the forward price in period 2,

$$\frac{\partial F_2^I}{\partial k_1} = -\frac{b(H+1)^2}{2H(H+2)},$$

at an interior forward price.

Differentiation of consumer h's marginal profit expression yields

$$-\frac{b}{H}\frac{(H+1)^2}{H+2}\frac{H(H+2)-1}{H(H+2)} < 0.$$

Hence, the consumer objective functions are strictly concave also at this stage of the game. Inserting the functional forms above into consumer h's first-order condition allows us to solve for the demand for forward contracts in trading period 1:

$$K_1^I(f_1) = \frac{(H+1)^2(H+3)a + H(H+2)^2c + 2(H(H+2)+2(H+1))\psi - 2H(H+2)^2 f_1}{b(H+1)^2(H+3)}$$

This demand is linearly decreasing in the forward price in trading period 1.

The forward price in trading period 1 We finally solve for the profit-maximizing forward price in trading period 1. The monopoly then has profit

$$\begin{aligned}&[f_1 - P(q(k_1 + K_2^I(F_2^I(k_1),k_1)))]k_1 + [F_2^I(k_1) - P(q(k_1 + k_2(F_2^I(k_1),k_1)))]K_2^I(F_2^I(k_1),k_1)\\ &+[P(q(k_1 + K_2^I(F_2^I(k_1),k_1))) - c]q(k_1 + K_2^I(F_2^I(k_1),k_1)),\end{aligned}$$

where $k_1 = K_1^I(f_1)$. Differentiation yields the marginal profit

$$\begin{aligned}&k_1 + [f_1 - p(k)]\frac{\partial K_1^I}{\partial f_1} + [f_2 - p(k)]\frac{\partial K_2^I}{\partial k_1}\frac{\partial K_1^I}{\partial f_1}\\ &+[k_2 + [f_2 - p(k)]\frac{\partial K_2^I}{\partial f_2}]\frac{\partial F_2^I}{\partial k_1}\frac{\partial K_1^I}{\partial f_1}\\ &+[-P'(q)k + P(q) - c + P'(q)q]q'(k)[1 + \frac{\partial K_2^I}{\partial f_2}\frac{\partial F_2^I}{\partial k_1} + \frac{\partial K_2^I}{\partial k_1}]\frac{\partial K_1^I}{\partial f_1},\end{aligned}$$

The terms on the second and last row have a second-order effect on marginal profit in trading period 1. Hence, the marginal profit expression becomes

$$k_1 + [f_1 - p(k) + (f_2 - p(k))\frac{\partial K_2^I}{\partial k_1}]\frac{\partial K_1^I}{\partial f_1} = k_1 + (f_1 - f_2)\frac{\partial K_1^I}{\partial f_1}$$

The second-derivative is

$$[2 - \frac{\partial F_2^I}{\partial k_1}\frac{\partial K_1^I}{\partial f_1}]\frac{\partial K_1^I}{\partial f_1} = -\frac{H+4}{H+3}\frac{2H(H+2)^2}{b(H+1)^2(H+3)} < 0,$$

so we can solve for the interior equilibrium forward price. However, it is easier to solve for the equilibrium forward quantity in period 1. By inverting the demand function $K_1^I(f_1)$ for forward contracts and using $F_2^I(k_1)$ we can write the difference in forward prices as:

$$f_1^I - f_2^I = \frac{H+1}{H+2}\frac{(H+1)a + 4\psi - (H+1)bk_1^I}{2H(H+2)}$$

We can then solve for the equilibrium forward quantity

$$k_1^I = \frac{(H+1)D + 4\frac{\psi}{b}}{(H+1)(H+4)} > 0. \tag{39}$$

in trading period 1. Let us calculate the forward price

$$f_1^I = \frac{(H+1)^2(H+3)^2a + H(H+2)^2(H+4)c + 2(H(H+2)(H+4) + 2(H+1))\psi}{2H(H+2)^2} > 0$$

in trading period 1 and

$$f_2^I = \frac{(H+1)^2(H+3)a + H(H+4)(H+2)c + 2(H^2 + 2(H-1))\psi}{2H(H+2)(H+4)} > 0$$

The forward price is decreasing over time along the equilibrium path:

$$\begin{aligned} f_1^I - f_2^I &= (H+1)^2(H+3)\frac{(H+3)(H+4) - (H+2)}{2H(H+4)(H+2)^2}a + \frac{H+3}{2}c \\ &\quad + \frac{(H+2)(H^2(H+7) + 14H + 2) + 2(H+1)(H+4)}{H(H+4)(H+2)^2}\psi \end{aligned}$$

The total forward quantity is

$$k^I = \frac{D + 2H\frac{\psi}{b}}{H+2} + \frac{H+1}{H+2}bk_1^I > \frac{D + 2H\frac{\psi}{b}}{H+2},$$

which implies that sequential contracting increases efficiency compared to the case of one single trading period. We collect these results in the following statement:

Proposition 5 *Assume that there is one producer with market power in each local market. Under a spatially independent market design, more electricity is traded in the forward market if there are two trading periods for forward contracts instead of just one.*

Let us now explore the mechanisms behind the result. The first question is how an increase in the number of trading periods affects the producer. Define the monopoly profit

$$\begin{aligned}\pi^I_{(2)} &= [f^I_1 - p^I_{(2)}]k^I_1 + [f^I_2 - p^I_{(2)}]k^I_2 + [p^I_{(2)} - c]q^I_{(2)} \\ &= (f^I_1 - f^I_2)k^I_1 + (\frac{b}{4}\frac{2D - (H+2)k^I_{(2)}}{H} + \psi)k^I_{(2)} + \pi_0,\end{aligned}$$

where subscript (2) identifies the equilibrium with two trading periods. To get the expression on the second row, we have substituted in the profit $\pi_0 = (p(0) - c)q(0)$ when the producer does not sell any forward contracts and the forward premium $f^I_2 - p^I_{(2)} = \frac{b}{2}\frac{D-k^I_{(2)}}{H} + \psi$ in the second trading period. If there is only one trading period, then the corresponding monopoly profit equals

$$\pi^I_{(1)} = [f^I_{(1)} - p^I_{(1)}]k^I_{(1)} + [p^I_{(1)} - c]q^I_{(1)} = (\frac{b}{4}\frac{2D - (H+2)k^I_{(1)}}{H} + \psi)k^I_{(1)} + \pi_0,$$

where subscript (1) identifies the equilibrium with two trading periods. By invoking

$$k^I_{(2)} = k^I_{(1)} + \frac{H+1}{H+2}k^I_1 \tag{40}$$

we can write the profit difference as

$$\pi^I_{(2)} - \pi^I_{(1)} = [f^I_1 - f^I_2 + \frac{H+1}{H+2}(\frac{b}{4}\frac{2D - (H+2)(2k^I_{(1)} + \frac{H+1}{H+2}k^I_1)}{H} + \psi)]k^I_1,$$

This expression simplifies to

$$\pi^I_{(2)} - \pi^I_{(1)} = \frac{H+1}{4H(H+2)^2}[(H+1)a + 4\psi]k^I_1 > 0,$$

after substituting in

$$f^I_1 - f^I_2 = \frac{H+1}{(H+2)^2}\frac{(H+1)a + 4\psi - (H+1)bk^I_1}{2H},$$

$k^I_{(1)} = \frac{D+2H\frac{\psi}{b}}{H+2}$ and k^I_1 from (39). The monopoly producer can always implement the equilibrium $k^I_{(1)}$ in trading period 2 by charging a forward price $\tilde{f}_1$ in trading period 1 such that $K^I_1(\tilde{f}_1) = 0$. Revealed preference for $f^{Ri}_1 \neq \tilde{f}_1$ and strict concavity of the producer's profit function implies $\pi^I_{(2)} > \pi^I_{(1)}$. By similar intuition, the monopoly producer probably benefits from adding additional trading periods in the forward market.

Consider next the expected profit of the H consumers. Each consumer earns

$$\begin{aligned}\omega^I_{(2)} &= -[f^I_1 - p^I_{(2)}]\frac{k^I_1}{H} - [f^I_2 - p^I_{(2)}]\frac{k^I_2}{H} + [v - p^I_{(2)}]\frac{D}{H} - \psi\frac{D - k^I_{(2)}}{H} \\ &= -(f^I_1 - f^I_2)\frac{k^I_1}{H} + \frac{b}{2}(\frac{k^I_{(2)}}{H})^2 + \omega_0\end{aligned}$$

To get the expression on the second row, we have substituted in the profit $\omega_0 = [v - p(0)]\frac{D}{H} - \psi\frac{D}{H}$ when the consumer does not purchase any forward contracts and the forward premium in the second trading period. If there is only one trading period, then the corresponding monopoly profit equals

$$\omega_{(1)}^I = -[f_{(1)}^I - p_{(1)}^I]\frac{k_{(1)}^I}{H} + [v - p_{(1)}^I]\frac{D}{H} - \psi\frac{D - k_{(1)}^I}{H} = \frac{b}{2}(\frac{k_{(1)}^I}{H})^2 + \omega_0.$$

where subscript (1) is used in order to identify the equilibrium with one trading period. Substitute in (40) into $\omega_{(2)}^I$ to get the net benefit

$$\omega_{(2)}^I - \omega_{(1)}^I = (\frac{b}{2}\frac{H+1}{H+2}\frac{2k_{(1)}^I + \frac{H+1}{H+2}k_1^I}{H} - (f_1^I - f_2^I))\frac{k_1^I}{H}$$

to the consumer of introducing a second trading period. Substitute in the relevant expressions from above and simplify to

$$\omega_{(2)}^I - \omega_{(1)}^I = \frac{1}{2H(H+4)}\frac{H+1}{(H+2)^2}[4(H^2+3H-2)\psi - (H^2+H-6)a]\frac{k_1^I}{H}$$

This expression is non-negative if either $H = 1$ or $H = 2$. If $H \geq 3$, then $\omega_{(2)}^I \geq \omega_{(1)}^I$ is equivalent to

$$\frac{4\psi}{a} \geq \frac{H^2+H-6}{H^2+3H-2}.$$

Under plausible assumptions, $H \geq 3$ and $\frac{a}{\psi}$ is above a threshold, then $\omega_{(1)}^I > \omega_{(2)}^I$, in which case consumers jointly lose from introducing multiple trading rounds for forward contracts.

References

Tangerås, Thomas and Frank A,Wolak (2024): The Competitive Effects of Linking Electricity Markets Across Space. Unpublished Manuscript. February 29 revision. Available at www.ifn.se/thomast.