Competition for flexible distribution resources in a 'smart' electricity distribution network: Online appendix*

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October 3, 2023

Abstract

This appendix extends Tangerås (2023) in two directions. First, a large generation owner exercises market power in the real-time market. Most results remain unchanged compared to Tangerås (2023) where all suppliers bid their marginal cost. Contrary to Tangerås (2023), a competitive effect of vertical integration arises when the large generator also aggregates flexible distribution resources (FDRs). A second extension derives aggregator market shares in the FDR market as equilibrium outcomes in a model of horizontal differentiation. These market shares are exogenous in Tangerås (2023).

Key words: Aggregator, balancing market, distribution system operator, market power, regulation, smart grid

JEL codes: H41; L12; L51; L94

*This work was conducted within the "Sustainable Energy Transition" research program at IFN. Financial support from the Swedish Energy Agency (2015-002474) is gratefully acknowledged. Data sharing is not applicable to this appendix as no datasets were generated or analysed during the current study

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1 Introduction

The analysis in Tangerås (2023) is conducted under the assumption that generation owners supply their capacity to the local real-time (flexi) market at marginal production cost. This appendix considers exercise of market power in electricity production in Section 2.

The fixed fees and market shares of aggregators in the market for flexible distribution resources (FDRs) are treated as exogenous and symmetric in Tangerås (2023). Section 3 of this appendix derives such fees as an equilibrium outcome of a multi-stage model. Horizontally differentiated aggregators first compete for market shares in the FDR market in a pre-stage and thereafter purchase electricity in the real-time market.

Section 4 draws conclusions about the robustness of the results in Tangerås (2023) concerning the assumptions of strategic interaction in the real-time market.

2 Market power in generation

This section considers exercise of market power in the production of flexible generation. The first subsection presents the theoretical model with producer market power. It also characterizes the efficient consumption that goes into the production of the household energy service, conditional on generator market power. The second subsection characterizes the equilibrium with $A \geq 1$ independent aggregators. The third subsection analyzes the mixed market structure with one integrated DSO/aggregator that controls dispatch and $A-1 \geq 1$ independent aggregators. The fourth subsection considers the case with one integrated generator/aggregator that exercises market power one both sides of the market.

2.1 Efficient consumption

The model Consider a two-period model of consumption and production within a local distribution area. In each period $i = 1, 2$, there is exogenous demand for $x_i > 0$ megawatt hours (MWh) electricity that is entirely unresponsive to price changes. Period 1 is the *peak demand period* and 2 the *off-peak demand period* if $x_1 > x_2$. The peak and off-peak definitions are reversed if $x_2 > x_1$. Both x_1 and x_2 are known entities at the start of period 1. There is a continuum of households of measure one that consumes an energy service in amount $\bar{s} > 0$. The energy service is produced by withdrawing $s_1 = s \in [0, \bar{s}]$ MWh electricity from the grid in period 1 and $s_2 = \bar{s} - s$ MWh in period 2.

The total demand $x_1 + s$ in period 1 and $x_2 + \bar{s} - s$ in period 2 must be covered by local electricity production. One large firm produces $y_1 \geq 0$ in period 1 and $y_2 \geq 0$ in period 2. The cost function $\Psi(y)$ of this producer is the same in both periods, smooth and strictly increasing and weakly convex.

The residual demand $q_i = x_i + s_i - y_i$ is supplied at marginal cost by a competitive fringe. The cost function $C(q)$ of the competitive fringe is the same in both periods, smooth and strictly increasing. The marginal cost function $C'(q)$ is strictly increasing and weakly convex. The price of electricity in period i is $p_i = P(q_i) = C'(q_i)$.

I assume that the share of total electricity consumption that goes into the production of the energy service is small, in the sense that $\bar{s} < \min\{x_1; x_2\}$, $\Psi'(\bar{s}) < \min\{P(x_1); P(x_2)\}$ and $P(\bar{s}) < \min\{\Psi'(x_1); \Psi'(x_2)\}$. To ensure that the model is well-behaved, I also impose a regularity condition on the cost function $C(q)$:

$$\text{If } \alpha > \theta \geq 0 \text{ and } \frac{C''(\alpha-\theta)\theta}{C'(\alpha-\theta)} < 1, \text{ then } \frac{C'''(\alpha-\theta)\theta}{C''(\alpha-\theta)} < 1 + \min\{\frac{C''''(\alpha-\theta)\theta}{C'''(\alpha-\theta)}; 0\}. \tag{1}$$

The cost function $C(q) = bq^{2+\sigma}$, $b > 0$, $\sigma \geq 0$ is smooth, strictly increasing, strictly convex and has non-negative third derivative. It also meets condition (1). Substituting

$$1 - \frac{C''(\alpha-\theta)\theta}{C'(\alpha-\theta)} = 1 - (1+\sigma)\frac{\theta}{\alpha-\theta}$$

into (1) yields

$$1 - \frac{C'''(q)\theta}{C''(q)} = 1 - \frac{C''(\alpha-\theta)\theta}{C'(\alpha-\theta)} + \frac{\theta}{\alpha-\theta} > 0,$$

and

$$1 - \frac{C'''(\alpha-\theta)\theta}{C''(\alpha-\theta)} + \frac{C''''(\alpha-\theta)\theta}{C'''(\alpha-\theta)} = 1 - \frac{C''(\alpha-\theta)\theta}{C'(\alpha-\theta)} + \sigma\frac{\theta}{\alpha-\theta} > 0.$$

Condition (1) implies that the profit function

$$U(x_i + s_i, y_i) = P(x_i + s_i - y_i)y_i - \Psi(y_i)$$

of the large firm is strictly quasi-concave. The unique optimum $Y(x_i + s_i) > 0$ is characterized by

$$P(x_i + s_i - Y(x_i + s_i))) - P'(x_i + s_i - Y(x_i + s_i))Y(x_i + s_i) = \Psi'(Y(x_i + s_i)). \tag{2}$$

The optimum satisfies $Y(x_i + s_i) < x_i$ because the marginal profit evaluated at $y_i = x_i$ is negative: $P(s_i) - P'(s_i)x_i - \Psi'(x_i) < 0$.

I let $Q(x_i + s_i) = x_i + s_i - Y(x_i + s_i) > 0$ be the residual supply of the competitive fringe. The supply $Y_i = Y(x_i + s_i)$ of the flexible generator with market power and the competitive supply $Q_i = Q(x_i + s_i)$ are both strictly increasing in total demand:

$$Y'(x_i+s_i) = \frac{C''(Q_i) - C'''(Q_i)Y_i}{\Psi''(Y_i) + 2C''(Q_i) - C'''(Q_i)Y_i} > 0, \; Q'(x_i+s_i) = \frac{\Psi''(Y_i) + C''(Q_i)}{\Psi''(Y_i) + 2C''(Q_i) - C'''(Q_i)Y_i} > 0.$$

System cost The total system cost equals

$$Sys(s) = \Psi(Y(x_1 + s)) + C(Q(x_1 + s)) + \Psi(Y(x_2 + \bar{s} - s)) + C(Q(x_2 + \bar{s} - s)) \tag{3}$$

as a function of the amount s of electricity withdrawn from the grid in the first period to produce the energy service. A small marginal increase in the amount of electricity s withdrawn in the first period has the following effect on total system cost

$$Sys'(s) = \Psi'(Y_1)Y_1' + C'(Q_1)Q_1' - \Psi'(Y_2)Y_2' - C'(Q_2)Q_2',$$

where $Y_i' = Y'(x_i + s_i)$, and $Q_i' = Q'(x_i + s_i)$, $i = 1, 2$.

Assume that total demand in period 2 initially is larger than total demand in period 1, $z_2 = x_2 + \bar{s} - s > x_1 + s = z_1$. This condition is equivalent to $s < \frac{1}{2}(x_2 + \bar{s} - x_1) = s^*$. A marginal increase in s shifts demand from period 2 to period 1. The resulting reallocation appears to be efficient because generators initially produce less electricity in period 1, $Y_1 < Y_2$ and $Q_1 < Q_2$. However, it is also necessary to account for the marginal effect on output to gauge the overall effect on system cost. In principle, the marginal effects Q_1' and Y_2' could be close to one, and $C'(Q_1) > \Psi'(Y_2)$, in which case the marginal effect on the system cost would be positive. Still, the direct effect dominates by way of assumption (1). To see this, rewrite

$$\begin{aligned}\Psi'(Y)Y' + C'(Q)Q' &= \Psi'(Y)[Y' + Q'] + [C'(Q) - \Psi'(Y)]Q' \\ &= \Psi'(Y) + [P(Q) - \Psi'(Y)]Q' \\ &= \Psi'(Y) + C''(Q)YQ'.\end{aligned}$$

On the second row, I have used $Q' = 1 - Y'$ and $P = C'$. On the third row, I have used the first-order condition (2) and $P' = C''$. Define

$$H(z) = C''(Q(z))Y(z)Q'(z).$$

Substitute

$$\frac{Q''}{Q'} = \frac{\Psi'''Y' + C'''Q'}{\Psi'' + C''} - \frac{\Psi'''Y' + 2C'''Q' - C''''YQ' - C'''Y'}{\Psi'' + 2C'' - C'''Y}$$

into

$$\frac{H'}{H} = \frac{C'''}{C''}Q' + \frac{Y'}{Y} + \frac{Q''}{Q'}$$

and rearrange expressions to get

$$\begin{aligned}\frac{H'}{H} &= [1 - (\frac{C'''}{C''} - \frac{C''''}{C'''})Y + \frac{\Psi''}{C''} + \frac{C''}{\Psi'' + C''}]\frac{C'''(Q')^2}{\Psi'' + C''} \\ &+ [\frac{1}{Y} + \frac{C'''Q'}{\Psi'' + C''} + \frac{\Psi''' + C'''Q'}{\Psi'' + C''}Y']Y'.\end{aligned}$$

By the assumptions on the cost functions, each term on the right-hand side is non-negative, and some are strictly positive. Hence, $H'(z) > 0$, and therefore $[H(z_1) - H(z_2)][z_1 - z_2] > 0$ for all $z_1 \neq z_2$. It then follows that $Sys'(s)(s^* - s) < 0$ for all $s \neq s^*$. Strict quasi-convexity of $Sys(s)$ implies:

Proposition 1 *The most efficient way to allocate consumption s is to equalize total demand across periods, if possible. If price equalization is not possible, then all electricity consumed in the production of the energy service should be withdrawn in the off-peak demand period. Hence, the efficient consumption satisfies $s = s^{fb}$.*

The electricity used in the production of the energy service is withdrawn from the grid in such a way as to smooth out all variations in marginal production costs across periods if the local distribution area is resource unconstrained, $|x_1 - x_2| \leq \bar{s}$. If the local distribution area is resource constrained, $|x_1 - x_2| > \bar{s}$, then the generator with market power and the competitive fringe both produce more output in the peak than the off-peak period. First,

$$[Y(x_1 + s^{fb}) - Y(x_2 + \bar{s} - s^{fb})](x_1 - x_2) \geq 0$$

follows from

$$(x_1 + s^{fb} - x_2 - \bar{s} + s^{fb})(x_1 - x_2) = 2(s^{fb} - s^*)(x_1 - x_2) \geq 0$$

and $Y' > 0$. The inequalities are strict if $x_1 - x_2 > \bar{s}$ because then $s^* < 0$ and if $x_2 - x_1 > \bar{s}$ because then $s^* > \bar{s}$.

A comparison between production under efficient withdrawal s^{fb} relative to passive consumption $\frac{1}{2}\bar{s}$ yields

$$[Y(x_1 + \frac{1}{2}\bar{s}) - Y(x_1 + s^{fb})](x_1 - x_2) > 0 \text{ for all } x_1 \neq x_2$$

because $(\frac{1}{2}\bar{s} - s^{fb})(x_1 - x_2)$ for all $x_1 \neq x_2$. Likewise

$$[Y(x_2 + \frac{1}{2}\bar{s}) - Y(x_2 + \bar{s} - s^{fb})](x_2 - x_1) > 0 \text{ for all } x_1 \neq x_2$$

Analogous arguments can be used to establish the properties of $Q(x_1+s^{fb})$ versus $Q(x_2+\bar{s}-s^{fb})$, $Q(x_1 + \frac{1}{2}\bar{s})$ versus $Q(x_1 + s^{fb})$ and $Q(x_2 + \frac{1}{2}\bar{s})$ versus $Q(x_2 + \bar{s} - s^{fb})$.

Proposition 1 and the properties of flexible generation under efficient electricity withdrawal s^{fb} imply that Lemma 1 and Proposition 1 of Tangerås (2023) extend to the case where one large generator exercises market power in the real-time market.

2.2 Independent aggregators

Assume that $A \geq 1$ independent aggregators supply the energy service to all households by purchasing electricity in the real-time (flexi) market. Let all aggregators have the same market share $\frac{1}{A}$. Each aggregator a supplies a price-independent bid to purchase $s_a \in [0, \frac{1}{A}\bar{s}]$ in period 1 and $\frac{1}{A}\bar{s} - s_a$ in period 2. The large generator simultaneously and independently submits price-independent bids to supply y_1 in period 1 and y_2 in period 2. The competitive fringe bids its marginal cost curve in both periods.

The first period joint electricity consumption s_A by the A aggregators is characterized by

$$-P(Q(x_1+s^A)) - \frac{1}{A}P'(Q(x_1+s^A))s^A + P(Q(x_2+\bar{s}-s^A)) + \frac{1}{A}P'(Q(x_2+\bar{s}-s^A))(\bar{s}-s^A) = 0 \quad (4)$$

in interior equilibrium, $s^A \in (0, \bar{s})$. Differentiation of this equilibrium condition yields the marginal effect

$$\frac{\partial s^A}{\partial x_1} = \frac{-(C_1''A + C_1'''s^A)Q_1'}{C_1''(1 + AQ_1') + C_1'''s^AQ_1' + C_2''(1 + AQ_2') + C_2'''(\bar{s} - s^A)Q_2'} < 0$$

of an increase in first-period demand $x_1 = x_1$ on the aggregators' first-period consumption s^A. Seeing as $s^A = \frac{1}{2}\bar{s}$ if $x_1 = x_2$, it follows that $(\frac{1}{2}\bar{s} - s^A)(x_1 - x_2) > 0$ for all $x_1 \neq x_2$. Hence, aggregators allocate most of their consumption to the off-peak demand period.

Evaluated at $s^A = s^* = \frac{1}{2}(\bar{s} + x_2 - x_1)$, the left-hand side of (4) simplifies to

$$\frac{1}{A}C''(Q(\frac{1}{2}(\bar{s} + x_1 + x_2)))(x_1 - x_2).$$

By implication, $(s^A - s^{fb})(x_1 - x_2) \geq 0$, with strict inequality for all $s^{fb} \in (0, 1)$ and $x_1 \neq x_2$. Hence, aggregators allocate too much of their consumption to the peak demand period.

Next,

$$\frac{\partial s^A}{\partial A} = \frac{p_2^A - p_1^A}{C_1''(1 + AQ_1') + C_1'''s^A Q_1' + C_2''(1 + AQ_2') + C_2'''(\bar{s} - s^A)}, \; s^A \in (0, 1)$$

implies that electricity consumption is closer to s^{fb} when there are more aggregators.

I finally demonstrate that s^A converges to s^{fb} as the number of aggregators becomes large. Assume that $x_1 > x_2$ and $s^A > s^{fb}$. I can then write the equilibrium condition for aggregation as

$$\frac{1}{A}[P'(Q(x_2 + \bar{s} - s^A))(\bar{s} - s^A) - P'(Q(x_1 + s^A))s^A] = P(Q(x_1 + s^A)) - P(Q(x_2 + \bar{s} - s^A))$$

The term in square brackets on the left-hand side of this expression is bounded, so the left-hand side converges to zero when A goes to infinity. If $\lim_{A\to\infty} s^A > s^{fb}$, then the right-hand side is strictly positive in the limit, which is a contradiction. By a similar argument $\lim_{A\to\infty} s^A < s^{fb}$ is a contradiction if $x_1 < x_2$. It is straightforward to verify that $s^A = s^* = s^{fb}$ if $x_1 = x_2$. I summarize these results as:

Proposition 2 *$A \geq 1$ independent aggregators with symmetric market shares withdraw more electricity from the grid in the off-peak than the peak demand period, but withdraw too much electricity in the off-peak period from an efficiency viewpoint. Efficiency increases if the number of aggregators increases, and the inefficiency vanishes in the limit.*

An integrated DSO/aggregator behaves equivalently to one single independent aggregator. Based on the above analysis, I therefore conclude that Proposition 2 and Proposition 3 in Tangerås (2023) extend to the case where a large generator exercises market power in the real-time market.

The total system cost equals $Sys(\frac{1}{2}\bar{s})$ in the benchmark case of passive consumers. Quasi-convexity of $Sys(s)$ implies that it is then efficient to reallocate consumption from the peak demand to the off-peak demand period. Such intertemporal substitution of demand is precisely what occurs when independent aggregators activate flexible distribution resources, and more so when there are more aggregators. The total system cost therefore is smaller compared to when there is no market for FDRs and consumers are passive, $Sys(\frac{1}{2}\bar{s}) > Sys(s^A)$. The efficiency ranking (i) versus (ii) and (ii) versus (iv) in Proposition 6 therefore still are valid even under imperfect competition in the production of electricity.

2.3 Mixed market structure

Assume that aggregators bid their capacity $\bar{s}$, the large producer bids capacity $\bar{y}$ and that the competitive fringe bids the marginal cost curve $C'(q)$ into the market prior to period 1. The DSO then allocates this consumption and production capacity across the two periods to maximize the joint profit of its DSO and aggregator operations. The DSO withdraws electricity $\tilde{s}^{DSO} \in [0, \bar{s}^{DSO}]$ from the grid in period 1 and $\bar{s}^{DSO} - \bar{s}^{DSO}$ in period 2 to satisfy the electricity demand of the own aggregator, where $\bar{s}^{DSO} = L^{DSO}\bar{s}$. It withdraws $\tilde{s}^A \in [0, \bar{s}^A]$ from the grid in period 1 and $\bar{s}^A - \tilde{s}^A$ in period 2 to satisfy the electricity demand of the independent aggregators, where $\bar{s}^A = \bar{s} - \bar{s}^{DSO}$. The DSO allocates $\tilde{y} \in [0, \bar{y}]$ of the capacity supplied by the large generator to period 1 and the rest, $\bar{y} - \tilde{y}$ to period 2. The DSO covers residual demand, $q_1 = x_1 + \tilde{s}^{DSO} + \tilde{s}^A - \tilde{y}$ in period 1 and $q_2 = x_2 + \bar{s} - \tilde{s}^{DSO} - \tilde{s}^A - \bar{y} + \tilde{y}$ in period 2, by the production of the competitive fringe. Hence, the real-time price of electricity in period i equals $p_i = P(q_i) = C'(q_i)$. The profit of the integrated DSO/aggregator equals $F + L^{DSO}t^{DSO} + \tilde{\Omega}(\tilde{s}^{DSO}, \tilde{s}^A - \tilde{y})$, where

$$\tilde{\Omega}(\tilde{s}^{DSO}, \tilde{s}^A - \tilde{y}, \bar{y}) = -P(x_1 + \tilde{s}^{DSO} + \tilde{s}^A - \tilde{y})\tilde{s}^{DSO} - P(x_2 + \bar{s} - \tilde{s}^{DSO} - \tilde{s}^A - \bar{y} + \tilde{y})(\bar{s}^{DSO} - \tilde{s}^{DSO}).$$

I impose the regularity assumption

$$\frac{C'''(q_i)s_i^{DSO}}{C''(q_i)} < 1 + \frac{C'''(q_1)s_1^{DSO} + C'''(q_2)s_2^{DSO}}{C''(q_1) + C''(q_2)}, \; \forall s_i^A \in [0, \bar{s}^A], \; s_i^{DSO} \in [0, \bar{s}^{DSO}], \; i = 1, 2 \quad (5)$$

on the cost function. For instance, $C(q) = \frac{1}{2}\phi q^2$ satisfies this assumption. The proof of the following result is in the Appendix:

Proposition 3 *Consider a mixed market structure in which a DSO/aggregator and $A-1$ independent aggregators compete in the market for flexible distribution resources. Assume that the DSO allocates $(\bar{s}^A, \bar{s}^{DSO}, \bar{y})$ across the two periods to maximize the DSO/aggregator profit.*
(i) The DSO allocates all electricity consumption of the independent aggregators to the peak demand period and all production of the large generator to the off-peak demand period [$s^A = \bar{s}^A$ and $y = 0$ if $x_1 > x_2$, $s^A = 0$ and $y = \bar{y}$ if $x_1 < x_2$].
(ii) More electricity is withdrawn from the grid in the peak relative to the off-peak demand period to produce the household energy service if the DSO controls half or less of the market for flexible distribution resources [$(s^{DSO} + s^A - \frac{1}{2}\bar{s})(x_1 - x_2) \geq 0$ if $L^{DSO} \leq \frac{1}{2}$].

By pursuing this strategy, the integrated DSO/aggregator can minimize the cost of electricity by allocating most of its electricity consumption to the off-peak demand period.

The marginal profit

$$\begin{aligned} \frac{\partial \tilde{\Omega}(\tilde{s}^{DSO}, \bar{s}^A, \bar{y})}{\partial \tilde{s}^{DSO}}|_{\tilde{s}^{DSO} = \frac{1}{2}\bar{s}^{DSO}} &= -[P(x_1 + \frac{1}{2}\bar{s}^{DSO} + \bar{s}^A) - P(x_2 + \frac{1}{2}\bar{s}^{DSO} - \bar{y})] \\ & \quad -[P'(x_1 + \frac{1}{2}\bar{s}^{DSO} + \bar{s}^A) - P'(x_2 + \frac{1}{2}\bar{s}^{DSO} - \bar{y})]\frac{1}{2}\bar{s}^{DSO}. \end{aligned}$$

of the integrated DSO/aggregator is negative if $x_1 > x_2$ by $x_1 + \bar{s}^A > x_2 - \bar{y}$. Hence, the integrated DSO/aggregator aggregates most of its own consumption to the off-peak demand period, $(\frac{1}{2}\bar{s}^{DSO} - S^{DSO}(s^A, \bar{y}))(x_1 - x_2) > 0$ for all $x_1 \neq x_2$. Proposition 3 and the result about $S^{DSO}(s^A, \bar{y})$ thus confirm that Proposition 4 in Tangerås (2023) also holds under the assumption of market power in generation.

Let $x_1 > x_2$, and consider the optimal allocation of the consumption of the integrated DSO/aggregator in more detail. If

$$P(x_1 + \bar{s}^A) \geq P(x_2 + \bar{s}^{DSO}) + P'(x_2 + \bar{s}^{DSO})\bar{s}^{DSO},$$

then $S^{DSO}(s^A, \bar{y}) = 0$ for all $\bar{y}$. The above inequality is met, for instance, if $\bar{s}^{DSO}$ is sufficiently small. Let $\hat{y} = 0$ in that case. Otherwise, define $\hat{y} > 0$ by

$$P(x_1 + \bar{s}^A) = P(x_2 + \bar{s}^{DSO} - \hat{y}) + P'(x_2 + \bar{s}^{DSO} - \hat{y})\bar{s}^{DSO}.$$

By way of this definition, $\frac{\tilde{\Omega}(\bar{s}^{DSO}, \bar{s}^A, \bar{y})}{\partial \bar{s}^{DSO}}|_{\bar{s}^{DSO}=0} \leq 0$ and therefore $S^{DSO}(s^A, \bar{y}) = 0$ for all $\bar{y} \geq \hat{y}$. If $\hat{y} > 0$, then

$$\begin{aligned} & P(x_1 + S^{DSO} + \bar{s}^A) + P'(x_1 + S^{DSO} + \bar{s}^A)S^{DSO} \\ = \; & P(x_2 + \bar{s}^{DSO} - S^{DSO} - \bar{y}) + P'(x_2 + \bar{s}^{DSO} - S^{DSO} - \bar{y})(\bar{s}^{DSO} - S^{DSO}) \end{aligned}$$

characterizes $S^{DSO}(s^A, \bar{y}) \in (0, \frac{1}{2}\bar{s}^{DSO})$ for all $\bar{y} \in [0, \hat{y})$. In interior optimum, the integrated DSO/aggregator allocates more consumption to the off-peak demand period when flexible generation capacity is larger:

$$\frac{\partial S^{DSO}(\bar{s}^A, \bar{y})}{\partial \bar{y}} = -\frac{C''(Q_2^{DSO}) + C'''(Q_2^{DSO})(\bar{s}^{DSO} - S^{DSO})}{2C''(Q_1^{DSO}) + C'''(Q_1^{DSO})S^{DSO} + 2C''(Q_2^{DSO}) + C'''(Q_2^{DSO})(\bar{s}^{DSO} - S^{DSO})} < 0,$$

where $Q_1^{DSO} = x_1 + S^{DSO} + \bar{s}^A$ and $Q_2^{DSO} = x_2 + \bar{s}^{DSO} - S^{DSO} - \bar{y}$ measure production by the competitive fringe in each period.

Generator market power The DSO exacerbates the resource constraints by allocating all generation capacity bid into the market by the large generator to the off-peak demand period. This aspect of centralized dispatch affects the incentives for the generator to supply capacity to the real-time market. In particular, the large generator takes into account the effect of capacity $\bar{y}$ on $S^{DSO}(s^A, \bar{y})$. The large generator has profit

$$P(x_2 + \bar{s}^{DSO} - S^{DSO}(\bar{s}^A, \bar{y}) - \bar{y})\bar{y} - \Psi(\bar{y})$$

if $x_1 > x_2$ because the DSO dispatches $\bar{y}$ in the off-peak demand period.

Let $\bar{y}^M$ be the equilibrium dispatch by the large generation owner. The aggregator profit equals $U(x_2 + \bar{s}^{DSO}, \bar{y}^M)$ if $\bar{y}^M \geq \hat{y}$ because then $S^{DSO}(\bar{s}^A, \bar{y}) = 0$. Optimal production in this case equals $\bar{y}^M = Y(x_2 + \bar{s}^{DSO})$. If $\bar{y}^M < \hat{y}$, then the generator accounts for the effect on

$S^{DSO}(s^A,\bar{y})$, in which case $\bar{y}^M>0$ solves

$$P(q_2^M)-P'(q_2^M)\bar{y}^M(1+\frac{\partial S^{DSO}(\bar{s}^A,\bar{y}^M)}{\partial\bar{y}})-\Psi'(\bar{y}^M)=0,$$

where $q_2^M=x_2+\bar{s}^{DSO}-S^{DSO}(\bar{s}^A,\bar{y}^M)-\bar{y}^M$. The marginal profit of an independent monopoly aggregator is negative evaluated at $y_2=\bar{y}^M$,

$$\frac{\partial U(x_2+\bar{s}^{DSO}-S^{DSO}(\bar{s}^A,\bar{y}^M),\bar{y})}{\partial\bar{y}}|_{\bar{y}=\bar{y}^M}=P'(q_2^M)\bar{y}^M\frac{\partial S^{DSO}(\bar{s}^A,\bar{y}^M)}{\partial\bar{y}}<0,$$

and therefore $Y(x_2+\bar{s}^{DSO}-S^{DSO}(\bar{s}^A,\bar{y}^M))<\bar{y}^M$ for all $S^{DSO}(\bar{s}^A,\bar{y}^M)>0$. The explanation is that an increase in $\bar{y}$ increases demand for electricity in the off-peak period, a strategic effect which drives up supply in the off-peak demand period. Since $\partial S^{DSO}(\bar{s}^A,\bar{y})/\partial\bar{y}>-1$, it also follows that $\bar{y}^M<Y^*(x_2+\bar{s}^{DSO})$, where $Y^*(z)$ defines the dispatch that equates marginal generation costs at demand z: $C'(z-Y^*(z))=\Psi'(Y^*(z))$.

Under robust assumptions, it is also the case that $\bar{y}^M<2Y(x_2+\bar{s}^{DSO}-S^{DSO}(\bar{s}^A,\bar{y}^M))$. This always holds if $\bar{y}^M\geq\hat{y}$. Assume that $\bar{y}^M<\hat{y}$, and consider the quadratic cost functions, $C(q)=\frac{1}{2}\phi q^2$, $\phi>0$, $\Psi(y)=\frac{1}{2}\psi y^2$, $\psi>0$. In his case, $\frac{\partial S^{DSO}(\bar{s}^A,\bar{y})}{\partial\bar{y}}=-\frac{1}{4}$ yields

$$\bar{y}^M=\frac{4\phi}{7\phi+4\psi}(x_2+\bar{s}^{DSO}-S^{DSO}),$$

whereas

$$Y(x_2+\bar{s}^{DSO}-S^{DSO})=\frac{\phi}{2\phi+\psi}(x_2+\bar{s}^{DSO}-S^{DSO}).$$

Hence,

$$2Y(x_2+\bar{s}^{DSO}-S^{DSO})-\bar{y}^M=\frac{2\phi(3\phi+2\psi)}{(2\phi+\psi)(7\phi+4\psi)}(x_2+\bar{s}^{DSO}-S^{DSO})>0.$$

System costs The system cost in period i equals

$$Sys_i(x_i+s_i,y_i)=\Psi(y_i)+C(x_i+s_i-y_i),$$

which is strictly convex in y_i. Then $Y^*(x_i+s_i)$ defined above measures the efficient production by the large generator in period i given total demand x_i+s_i. Observe that

$$\frac{\partial Sys_i(x_i+s_i,y_i)}{\partial y_i}|_{y_i=s_i}=\Psi'(s_i)-C'(x_i)<0$$

by the assumption that $\Psi'(\bar{s})<\min\{C'(x_1);C'(x_2)\}$. Hence, $Y^*(x_i+s_i)>s_i$. Observe also that

$$\frac{\partial Sys_i(x_i+s_i,y_i)}{\partial y_i}|_{y_i=x_i}=\Psi'(x_i)-C'(s_i)>0$$

by the assumption that $C'(\bar{s})<\min\{\Psi'(x_1);\Psi'(x_2)\}$. Hence, $Y^*(x_i+s_i)<x_i$.

The total system cost under the mixed market structure is given by

$$Sys^M = Sys_1(x_1 + s^A + s^{DSO}, y^M) + Sys_2(x_2 + \bar{s} - s^A - s^{DSO}, \bar{y}^M - y^M)$$

if the DSO dispatches $y^M \in [0, \bar{y}^M]$ of the generation capacity in period 1 and the rest, $\bar{y}^M - y^M$, in period 2.

The corresponding system cost equals

$$Sys(s^A+s^{DSO}) = Sys_1(x_1+s^A+s^{DSO}, Y(x_1+s^A+s^{DSO}))+Sys_2(x_2+\bar{s}-s^A-s^{DSO}, Y(x_2+\bar{s}-s^A-s^{DSO}))$$

if there is one single independent aggregator who allocates the same amount $s^A + s^{DSO}$ of the total consumption to period 1 and the rest to period 2.

The difference in system cost between a market structure where an integrated DSO/aggregator dispatches all capacity and a situation without any FDR market and passive consumers, can be written as

$$Sys^M - Sys(\frac{1}{2}\bar{s}) = Sys^M - Sys(s^A + s^{DSO}) + Sys(s^A + s^{DSO}) - Sys(\frac{1}{2}\bar{s})$$

Assume throughout that $L^{DSO} \leq \frac{1}{2}$, so that $\bar{s}^{DSO} \leq \bar{s}^A$. The term $Sys(s^A+s^{DSO})-Sys(\frac{1}{2}\bar{s}) \geq 0$ measures the distortion associated with inefficient use of the flexible distribution resource because $(s^{DSO}+s^A-\frac{1}{2}\bar{s})(x_1-x_2) \geq 0$ if $L^{DSO} \leq \frac{1}{2}$. Specifically, the DSO allocates to much consumption to the peak demand period.

The term $Sys^M - Sys(s^A + s^{DSO})$ measures the effect on system cost associated with DSO dispatch of the generation capacity of the large producer. Assume from now on that $x_1 > x_2$; the case with $x_1 < x_2$ is analogous and omitted. If $x_1 > x_2$, then $s^A = \bar{s}^A$ and $y^M = 0$ by Proposition 3. Hence,

$$Sys^M = Sys_1(z_1, 0) + Sys_2(z_2, \bar{y}^M),$$

whre $z_1 = x_1 + \bar{s}^A + S^{DSO}$, $z_2 = x_2 + \bar{s}^{DSO} - S^{DSO}$, and $z_1 > z_2$ by the assumption that $\bar{s}^A \geq \bar{s}^{DSO}$. The difference in system cost can be written as

$$\begin{aligned} Sys^M - Sys(\bar{s}^A + S^{DSO}) \quad = \quad & \int_0^{\bar{y}^M - Y_2} \int_\theta^{\theta + Y_2} \frac{\partial^2 Sys_2(z_2, y)}{\partial y^2} dy d\theta \\ & - \int_0^{\bar{y}^M - Y_2} \int_{z_2}^{z_1} \frac{\partial^2 Sys_1(z, y)}{\partial z \partial y} dz dy \\ & - \int_{\bar{y}^M - Y_2}^{Y_1} \frac{\partial Sys_1(z_1, y)}{\partial y} dy \end{aligned}$$

The term on the first row of the right-hand side is strictly positive by $\bar{y}^M > Y_2$ and convexity $\frac{\partial^2 Sys_2}{\partial y^2} = \Psi''(y_2) + C''(z_2 - y_2) > 0$. The term on the second row is strictly positive by $z_1 > z_2$ and $\frac{\partial^2 Sys_1}{\partial z \partial y} = -C''(z_1 - y_1) < 0$. The term on the third row is non-negative if $Y_1 \geq \bar{y}^M - Y_2$ because $\frac{\partial Sys_1(z_1,y)}{\partial y} < 0$ for all $y \leq Y_1 < Y_1^*$. Seeing as $Y_1 > Y_2$ by $z_1 > z_2$, the inequality is fulfilled if $\bar{y}^M \leq 2Y_2$. I demonstrated above that this inequality holds for robust specifications of the cost function. Hence, DSO control of the dispatch of the capacity of the large generator can by itself distort efficiency.

Since $Sys^M > Sys(\frac{1}{2}\bar{s})$ if $L^{DSO} \leq \frac{1}{2}$ in the above analysis, I conclude that the efficiency ranking (iv) versus (v) in Proposition 6 of Tangerås (2023) is valid in robust circumstances also under imperfect competition in the production of electricity.

2.4 Integrated generator/aggregator

Assume that the large generator also is a monopoly aggregator in the market for flexible distribution resources. This integrated generator/aggregator supplies (y_1, y_2, s) to maximize its profit

$$\tilde{\Pi}^I(y_1, y_2, s) = \bar{t} + P(x_1 + s - y_1)(y_1 - s) + P(x_2 + \bar{s} - s - y_2)(y_2 - \bar{s} + s) - \Psi(y_1) - \Psi(y_2).$$

I solve the problem in two stages. The firm first maximizes profit over y_1 and y_2 for given $s \in [0, \bar{s}]$. It then maximizes profit over s.

Profit-maximizing generation Maximizing $\tilde{\Pi}^I(y_1, y_2, s)$ over y_i returns the production $Y^I(x_i, s_i)$ of the large firm in period i as a solution to the first-order condition

$$P(Q^I(x_i, s_i)) - P'(Q^I(x_i, s_i))(Y^I(x_i, s_i) - s_i) = \Psi'(Y^I(x_i, s_i)). \tag{6}$$

In this expression, $Q^I(x_i, s_i) = x_i + s_i - Y^I(x_i, s_i)$ measures the production of the competitive fringe. The marginal profit

$$\frac{\partial \tilde{\Pi}^I}{\partial y_i} = P(x_i + s_i - y_i) - P(x_i) + P'(x_i + s_i - y_i)(s_i - y_i) + \Psi'(\bar{s}) - \Psi'(y_i) + P(x_i) - \Psi'(\bar{s})$$

is strictly positive for all $y_i \leq s_i$ by the assumption that $P(x_i) > \Psi'(\bar{s})$. Hence, $Y^I(x_i, s_i) > s_i$. Moreover,

$$\frac{\partial \tilde{\Pi}^I}{\partial y_i} = P(x_i + s_i - y_i) - P(s_i) - P'(x_i + s_i - y_i)(y_i - s_i) + \Psi'(x_i) - \Psi'(y_i) + P(s_i) - \Psi'(x_i)$$

is strictly negative for all $y_i \geq x_i$ by the assumption that $x_i > s_i$ and $P(\bar{s}) < \Psi'(x_i)$. Hence, $Y^I(x_i, s_i) < x_i$. Condition (1) guarantees that the first-order condition (6) has a unique solution and that the supply $Y^I(x_i, s_i)$ of the integrated generator/aggregator is strictly increasing in x_i and s_i:

$$\frac{\partial Y_i^I}{\partial x_i} = \frac{C''(Q_i^I) - C'''(Q_i^I)(Y_i^I - s_i)}{\Psi''(Y_i^I) + 2C''(Q_i^I) - C'''(Q_i^I)(Y_i^I - s_i)} > 0, \quad \frac{\partial Y_i^I}{\partial s_i} = \frac{2C''(Q_i^I) - C'''(Q_i^I)(Y_i^I - s_i)}{\Psi''(Y_i^I) + 2C''(Q_i^I) - C'''(Q_i^I)(Y_i^I - s_i)} > 0.$$

In this expression, $Y_i^I = Y^I(x_i, s_i)$ and $Q_i^I = Q^I(x_i, s_i)$.

An integrated generator/aggregator has a stronger incentive to supply generation to the market than an independent generator because the associated reduction in the price of electricity reduces the aggregator's cost of supplying the energy service to households. This mechanism is formally equivalent to the well-known pro-competitive effect of producers selling forward contracts (e.g. Wolak, 2007) or when there is vertical integration between generation and retail (Bushnell et al., 2008). Formally, $Y^I(x_i, s_i) \geq Y(x_i + s_i)$, with strict inequality if $s_i > 0$. To establish this result, notice first that profit is strictly larger if the generator/aggregator produces $Y_1 = Y(x_1 + s)$ compared to $y < Y_1$ in period 1:

$$\begin{aligned}\tilde{\Pi}^I(Y_1, y_2, s) - \tilde{\Pi}^I(y, y_2, s) &= P(x_1 + s - Y_1)Y_1 - \Psi(Y_1) - P(x_1 + s - y)y \\ &\quad +\Psi(y) + [P(x_1 + s - y) - P(x_1 + s - Y_1)]s \\ &> 0.\end{aligned}$$

This is because Y_1 uniquely maximizes $P(x_1 + s - y_1)y_1 - \Psi(y_1)$, and because $P(x_1 + s - y) > P(x_1 + s - Y_1)$ if $y < Y_1$. Hence, $Y^I(x_1, s) \geq Y(x_1 + s)$. The inequality is strict if $s > 0$ because then $\frac{\partial \Pi^I}{\partial y_1}|_{y_1 = Y_1} = P'(x_1 + s - Y_1)s > 0$. By an analogous argument, $Y^I(x_2, \bar{s} - s) \geq Y_2$, with strict inequality if $s < \bar{s}$.

Profit-maximizing consumption Consider next the optimal choice of s. Define the profit of the integrated generator/aggregator

$$\Pi^I(s) = \tilde{\Pi}^I(Y^I(x_1, s), Y^I(x_2, \bar{s} - s), s).$$

The marginal profit of increasing s equals

$$\begin{aligned}\Pi^{I\prime}(s) &= P(Q^I(x_2, \bar{s} - s)) - P'(Q^I(x_2, \bar{s} - s))(Y^I(x_2, \bar{s} - s) - \bar{s} + s) \\ &\quad -P(Q^I(x_1, s)) + P'(Q^I(x_1, s))(Y^I(x_1, s) - s)\end{aligned}$$

The indirect effects working through the changes to production $Y^I(x_1, s)$ and $Y^I(x_2, \bar{s} - s)$ are of second-order importance. Rewrite the marginal profit expression as

$$\Pi^{I\prime}(s) = \Psi'(Y^I(x_2, \bar{s} - s)) - \Psi'(Y^I(x_1, s))$$

by invoking (6). The corresponding second-derivative of the profit function is

$$\Pi^{I\prime\prime}(s) = -\Psi''(Y_2^I)\frac{\partial Y_2^I}{\partial s_2} - \Psi''(Y_1^I)\frac{\partial Y_1^I}{\partial s_1} < 0.$$

Hence, the solution to the first-order condition identifies the profit-maximizing consumption s^I. The integrated generator/aggregator chooses flexible production y_1 and y_2 to maximize its profit in the flexi market and then allocates s across periods to minimize its total production cost.

Next, I derive the properties of s^I. Compare first s^I with $\frac{1}{2}\bar{s}$. By $\partial Y_i^I/\partial x_i > 0$, $[Y^I(x_1, \frac{1}{2}\bar{s}) -$

$Y^I(x_2, \frac{1}{2}\bar{s})](x_1 - x_2) > 0$ for all $x_1 \neq x_2$. Hence,

$$(x_1 - x_2)\Pi^{I\prime}(s)|_{s=\frac{1}{2}\bar{s}} = -[\Psi'(Y^I(x_1, \frac{1}{2}\bar{s})) - \Psi'(Y^I(x_2, \frac{1}{2}\bar{s}))](x_1 - x_2) < 0$$

if $x_1 \neq x_2$. The integrated generator/aggregator therefore withdraws more electricity from the grid in the off-peak than the peak demand period, $(\frac{1}{2}\bar{s} - s^I)(x_1 - x_2) > 0$ if $x_1 \neq x_2$.

Second, compare s^I with s^{fb}. If $x_1 \geq \bar{s} + x_2$, then $s^{fb} = 0$, in which case $(s^I - s^{fb})(x_1 - x_2) = s^I(x_1 - x_2) \geq 0$. If $x_2 \geq \bar{s} + x_1$, then $s^{fb} = \bar{s}$, in which case $(s^I - s^{fb})(x_1 - x_2) = (\bar{s} - s^I)(x_2 - x_1) \geq 0$. If $|x_1 - x_2| < \bar{s}$, then $s^{fb} = s^* \in (0, \bar{s})$. Observe that

$$(x_1 - x_2)\frac{\partial \tilde{\Pi}^I(y_1, y_2, s^*)}{\partial y_1}|_{y_1 = Y^I(x_2, \bar{s} - s^*)} = -P'(Q^I(x_2, \bar{s} - s^*))(x_1 - x_2)^2 < 0 \text{ for all } x_1 \neq x_2.$$

Hence, $[Y^I(x_2, \bar{s} - s^*) - Y^I(x_1, s^*)](x_1 - x_2) > 0$ if $x_1 \neq x_2$. By implication,

$$(x_1 - x_2)\Pi^{I\prime}(s)|_{s=s^*} = [\Psi'(Y^I(x_2, \bar{s} - s^*)) - \Psi'(Y^I(x_1, s^*))](x_1 - x_2) > 0$$

if $s^* \in (0, \bar{s})$ and $x_1 \neq x_2$. It then follows that $(s^I - s^{fb})(x_1 - x_2) > 0$ if $s^{fb} \in (0, 1)$ and $x_1 \neq x_2$. The integrated generator/aggregator generally withdraws too much electricity in the peak demand period compared to the efficient withdrawal s^{fb}. I summarize these results as follows

Proposition 4 *An integrated generator/aggregator that has market power in generation and a monopoly in the market for flexible distribution resources, withdraws more electricity from the grid in the off-peak relative to the peak demand period, but withdraws too much electricity in the off-peak period from an efficiency viewpoint.*

The comparison with a market structure in which one independent aggregator has a monopoly in the market for flexible distribution resources ($A = 1$) is less straightforward. To gain some insights, I add more structure to the model. Specifically, cost functions are quadratic: $C(q) = \frac{1}{2}\phi q^2$ and $\Psi(y) = \frac{1}{2}\psi y^2$, where $\phi > 0$ and $\psi > 0$.

If $s^I = 0$, then $x_1 > x_2$, in which case $(s^A - s^I)(x_1 - x_2) = s^A(x_1 - x_2) \geq 0$. If $s^I = \bar{s}$, then $x_2 > x_1$, in which case $(s^A - s^I)(x_1 - x_2) = (\bar{s} - s^A)(x_2 - x_1) \geq 0$. If $s^I \in (0, \bar{s})$, then I first apply the functional form assumptions to (6) and solve for the optimal production

$$Y^I(x_i, s_i) = \frac{\phi}{2\phi + \psi}(x_i + 2s_i),\ Q^I(x_i, s_i) = \frac{(\phi + \psi)x_i + \psi s_i}{2\phi + \psi}$$

of the integrated generator/aggregator. Armed with these expressions, I use $\Pi^{I\prime}(s^I) = 0$ to solve for

$$s^I = \frac{1}{2}\bar{s} + \frac{x_2 - x_1}{4}.$$

Consider now the incentives of an independent aggregator. Apply the functional form assumptions to (2) and solve for production

$$Y(x_i+s_i) = \frac{\phi}{2\phi+\psi}(x_i+s_i),\ Q(x_i+s_i) = \frac{\phi+\psi}{2\phi+\psi}(x_i+s_i)$$

when generation and aggregation are bid independently into the market. The marginal profit of the independent aggregator then equals

$$\begin{aligned} & P(Q(x_2+\bar{s}-s^I)) + P'(Q(x_2+\bar{s}-s^I))(\bar{s}-s^I) - P(Q(x_1+s^I)) - P'(Q(x_1+s^I))s^I \\ = \ & \frac{\phi}{2\phi+\psi}[\frac{\phi+\psi}{3\phi+2\psi}(x_2-x_1)+\bar{s}-2s^I] = \frac{1}{2}\frac{\phi}{2\phi+\psi}\frac{\phi}{3\phi+2\psi}(x_1-x_2) \end{aligned}$$

evaluated at $s=s^I$. By implication, $(s^A-s^I)(x_1-x_2)\geq 0$ also for $s^I\in(0,1)$, with strict inequality if $x_1\neq x_2$.

The integrated generator/aggregator withdraws electricity more efficiently from the grid that an independent aggregator under a quadratic parametrization of cost functions. A market structure with one independent aggregator is formally equivalent to one with an integrated DSO/aggregator. Hence, the comparison between an integrated generator/aggregator and an integrated DSO/aggregator becomes $(s^{DSO}-s^I)(x_1-x_2)\geq 0$ in the parametric case.

System costs Despite the pro-competitive effect of vertical integration between production and aggregation, market power still distorts flexible generation downwards:

$$\frac{\partial\tilde{\Pi}^I}{\partial y_i}|_{y_i=Y^*(x_i+s_i)} = -P'(x_i+s_i-Y^*(x_i+s_i))(Y^*(x_i+s_i)-s_i) < 0$$

implies $Y^I(x_i,s_i) < Y^*(x_i+s_i)$.

The total system cost with an integrated generator/aggregator is $Sys^I(s^I)$, where

$$Sys^I(s) = \Psi(Y^I(x_1,s)) + C(Q^I(x_1,s)) + \Psi(Y^I(x_2,\bar{s}-s)) + C(Q^I(x_2,\bar{s}-s)). \tag{7}$$

A comparison with the benchmark case of passive consumption yields

$$Sys(\frac{1}{2}\bar{s}) - Sys^I(s^I) = Sys(\frac{1}{2}\bar{s}) - Sys(s^I) + Sys(s^I) - Sys^I(s^I).$$

The $Sys(\frac{1}{2}\bar{s}) - Sys(s^I)$ term measures the effect on system cost of the activation of flexible distribution resources for balancing purposes. It is positive by strict quasi-concavity of $Sys(s)$ and because s^I is closer than $\frac{1}{2}\bar{s}$ to s^{fb}. The $Sys(s^I) - Sys^I(s^I)$ term measures the competitive effect of vertical integration. More generally,

$$\begin{aligned} Sys(s) - Sys^I(s) &= \sum_{i=1,2}[Sys_i(x_i+s_i, Y(x_i+s_i)) - Sys_i(x_i+s_i, Y^I(x_i,s_i))] \\ &= -\sum_{i=1,2}\int_{Y(x_i+s_i)}^{Y^I(x_i,s_i)} \frac{\partial Sys_i(x_i+s_i,y_i)}{\partial y_i}dy_i > 0. \end{aligned}$$

Either $Y^I(x_1, s) > Y(x_1 + s)$ or $Y^I(x_2, \bar{s} - s) > Y(x_2 + \bar{s} - s)$ because of the positive effect of aggregation on output. Moreover, $\frac{\partial Sys_i}{\partial y_i} < 0$ for all $y_i \leq Y^I(x_i, s_i)$ because of convexity of Sys_i in y_i, and $Y^I(x_i, s_i) < Y^*(x_i + s_i)$.

A market structure with an integrated generator/aggregator has a double efficiency benefit compared to a structure without any market for FDR resources and passive consumption. First, the household energy service is produced more efficiently because the aggregator allocates more consumption to the off-peak demand period. Second, aggregation increases production efficiency by replacing high-cost generation of the competitive fringe with low cost production of the generator with market power. Hence, the efficiency ranking (*iii*) versus (*iv*) in Proposition 6 is valid even under imperfect competition in the production of electricity.

The comparison with system costs in a market structure with an independent aggregator ($A = 1$) is more complicated:

$$Sys(s^A) - Sys^I(s^I) = Sys(s^A) - Sys(s^I) + Sys(s^I) - Sys^I(s^I).$$

It still holds that $Sys(s^I) > Sys^I(s^I)$, but s^I is ambiguous compared to s^A. However, $(s^A - s^I)(x_1 - x_2) \geq 0$ under quadratic production costs, in which case $Sys(s^A) \geq Sys(s^I)$. The efficiency ranking (*ii*) versus (*iii*) in Proposition 6 of Tangerås (2023) can therefore be reversed under imperfect competition in the production of electricity.

3 Aggregator competition in the FDR market

The fixed fees and market shares of aggregators in the FDR market are treated as exogenous and symmetric in Tangerås (2023). This section derives such fees as an equilibrium outcome of a multi-stage model. Horizontally differentiated aggregators first compete for market shares in the FDR market in a pre-stage (period 0) before demand (x_1, x_2) is known. Thereafter, they purchase electricity in the real-time market in periods 1 and 2 after demand (x_1, x_2) has been realized. The first subsection considers competition between $A \geq 2$ independent aggregators, the second subsection considers competition between an integrated DSO/aggregator and $A - 1 \geq 1$ independent aggregators.

3.1 Independent aggregators

There are $A \geq 2$ independent aggregators, indexed by $a \in \{1, ..., A\}$.

The demand for household energy services Assume that household i receives random utility $\bar{s} - \rho t_a + \varepsilon_{ia}$ from purchasing the energy service from aggregator a. In the utility expression, $\rho > 0$ is the price response parameter, and ε_{ia} measures household i's idiosyncratic preference for purchasing the energy service from aggregator a. Each household i draws ε_{ia} from an extreme value distribution with location parameter 0 and scale parameter 1. If all aggregators set fees equal to or below some upper threshold $\bar{t}^A > 0$, to be specified, then the market is fully covered.

The market share of aggregator a is then given by

$$L_a = \frac{\exp^{-\rho t_a}}{\sum_{a'} \exp^{-\rho t_{a'}}}, \; \frac{\partial L_a}{\partial t_a} = -\rho L_a(1 - L_a),$$

see e.g., Besanko et al. (1998). The parameter ρ is a measure of the degree of horizontal differentiation. The larger is ρ, the more important is the price for the choice of aggregator. To derive closed-form solutions, I also assume that a quadratic cost function $C(q) = \frac{\phi}{2}q^2$, $\phi > 0$, holds for flexible generation.

Competition in the flexi market The profit of aggregator a equals

$$\Pi^A(t_a, s_a) = L_a t_a - P(x_1 + s)s_a - P(x_2 + \bar{s} - s)(L_a\bar{s} - s_a). \tag{8}$$

in the flexi (real-time) market as a function of the aggregator's market share L_a and the amount $s_a \in [0, L_a\bar{s}]$ of electricity it purchases in the first period to supply the energy service to its customers. The aggregator consumes $L_a\bar{s} - s_a$ in period 2. Under the assumption that aggregators compete in quantities in the flexi market, the marginal profit of this aggregator in the flexi market equals

$$\begin{aligned}\frac{\partial \Pi^A(t_a, s_a)}{\partial s_a} &= P(x_2 + \bar{s} - s) - P(x_1 + s) - P'(x_1 + s)s_a + P'(x_2 + \bar{s} - s)(L_a\bar{s} - s_a) \\ &= \phi(x_2 + \bar{s} - s) - \phi(x_1 + s) - \phi s_a + \phi(L_a\bar{s} - s_a),\end{aligned}$$

where I have applied the functional form assumption for $C(q)$ on the second row.

Assume that all aggregators except a have charged the same fee $t^A \leq \bar{t}^A$ in the first period, whereas a has charged $t_a \leq \bar{t}^A$. All aggregators $a' \neq a$ then have the same market share $\frac{1-L_a}{A-1}$ in the FDR market. Aggregator a withdraws $S_a(\mathbf{x}, L_a)$ from the grid in period 1, whereas each of the other aggregators withdraws $\frac{S_A(\mathbf{x},L_a)}{A-1}$ in the same period, where $\mathbf{x} = (x_1, x_2)$. We can apply the above marginal profit expression to solve for the period 1 electricity withdrawals:

$$S_a(\mathbf{x}, L_a) = \frac{1}{2}[L_a\bar{s} + \frac{x_2 - x_1}{A+1}] \in (0, L_a\bar{s}),\; S_A(\mathbf{x}, L_a) = \frac{1}{2}[(1-L_a)\bar{s} + \frac{A-1}{A+1}(x_2 - x_1)] \in (0, (1-L_a)\bar{s})$$

$$S_a(\mathbf{x}, L_a) = 0,\; S_A(\mathbf{x}, L_a) = \frac{1}{2}[\frac{A - L_a}{A}\bar{s} + \frac{A-1}{A}(x_2 - x_1)] \in (0, (1 - L_a)\bar{s})$$

$$S_a(\mathbf{x}, L_a) = L_a\bar{s},\; S_A(\mathbf{x}, L_a) = \frac{1}{2}[\frac{A-1}{A}(x_2 - x_1) + \frac{A - 2AL_a + L_a}{A}\bar{s}] \in (0, (1 - L_a)\bar{s})$$

Competition for household consumers Assume that aggregators set their fixed fees in period 0 before demand is known in period 1 and 2. Let $\mathbf{x}$ have joint cumulative distribution function $H(\mathbf{x})$ on $\mathbf{X} = [0, \bar{x}]^2$. The cumulative distribution function and the joint density function $h(\mathbf{x}) > 0$ are both continuous. The period 0 expected profit of aggregator a then equals

$$\pi^A(t_a) = L_a t_a - \int_{\mathbf{x}\in\mathbf{X}} [P(x_1 + S(\mathbf{x}, L_a))S_a(\mathbf{x}, L_a) + P(x_2 + \bar{s} - S(\mathbf{x}, L_a))(L_a\bar{s} - S_a(\mathbf{x}, L_a))]dH(\mathbf{x}),$$

where $S(\mathbf{x}, L_a) = S_a(\mathbf{x}, L_a) + S_A(\mathbf{x}, L_a)$ measures the total withdrawal of electricity in period 1 to supply the household energy service. The marginal expected profit of aggregator a equals:

$$\begin{aligned}\pi^{A\prime}(t_a) &= L_a + [t_a - \int_{\mathbf{x}\in\mathbf{X}} P(x_2 + \bar{s} - S(\mathbf{x}, L_a))\bar{s}dH(\mathbf{x})]\frac{\partial L_a}{\partial t_a} - \int_{\mathbf{x}\in\mathbf{X}} [P'(x_1 + S(\mathbf{x}, L_a))S_a(\mathbf{x}, L_a) \\ &\quad -P'(x_2 + \bar{s} - S(\mathbf{x}, L_a))(L_a\bar{s} - S_a(\mathbf{x}, L_a))]\frac{\partial S_A(\mathbf{x}, L_a)}{\partial L_a} dH(\mathbf{x})\frac{\partial L_a}{\partial t_a}.\end{aligned}$$

The sum of the first two expressions on the first row represents the direct effect on aggregator a's profit of a marginal increase in t_a. The second expression in square brackets measures a strategic effect that arises because a marginal reduction in a's market share in the FDR market has a non-negative effect on the amount of electricity that all other aggregators purchase in the flexi market.

Characterization of the equilibrium fees Using the functional form expressions, and $\pi^{A\prime}(t^A) = 0$, it is straightforward to solve for the symmetric equilibrium candidate

$$t^A = \min\{\frac{1}{\rho}\frac{A}{A-1} + \int_{\mathbf{x}\in\mathbf{X}_1} \phi(x_2+\bar{s})\bar{s}dH(\mathbf{x}) + \int_{\mathbf{x}\in\mathbf{X}_2} \phi x_2\bar{s}dH(\mathbf{x}) + \int_{\mathbf{x}\in\mathbf{X}_3} \phi\frac{x_1 + x_2 + \bar{s}}{2}\bar{s}dH(\mathbf{x}); \bar{t}^A\}. \tag{9}$$

In the above expression, $\mathbf{X}_i = \{\mathbf{x} \in \mathbf{X} : x_i - x_{-i} > \frac{A+1}{A}\bar{s}\}$, $i = 1, 2$, whereas $\mathbf{X}_3 = \{\mathbf{x} \in \mathbf{X} : |x_1 - x_2| \leq \frac{A+1}{A}\bar{s}\}$**.** The first [second] period demand is so large for all $\mathbf{x} \in \mathbf{X}_1$ [$\mathbf{x} \in \mathbf{X}_2$] that all electricity is withdrawn in period 2 [1]. The strategic effect vanishes in $\mathbf{X}_1$ and $\mathbf{X}_2$ since in this domain $S_A(\mathbf{x}, L_a) = 0$ [$S_A(\mathbf{x}, L_a) = \bar{s}$] in a neighborhood of L_a around $\frac{1}{A}$. For intermediary $\mathbf{x} \in \mathbf{X}_3$, there is a strategic effect $\frac{\partial S_A(\mathbf{x},L_a)}{\partial L_a} = -\frac{1}{2}\bar{s}$ in the withdrawal of electricity, which yields electricity withdrawal $S_A(\mathbf{x}, \frac{1}{A}) = \frac{1}{2}(\frac{A}{A+1}(x_2 - x_1) + \bar{s})$ from the solution to the first-order condition $\frac{\partial \Pi^A(t^A,s_a)}{\partial s_a} = 0$. The aggregator profit equals

$$\begin{aligned}\pi^A(t^A) &= \frac{1}{A}t^A - \frac{\phi}{A}\int_{\mathbf{x}\in\mathbf{X}_1} (x_2 + \bar{s})\bar{s}dH(\mathbf{x}) - \frac{\phi}{A}\int_{\mathbf{x}\in\mathbf{X}_2} (x_1 + \bar{s})\bar{s}dH(\mathbf{x}) \\ &\quad + \frac{\phi}{2A}\int_{\mathbf{x}\in\mathbf{X}_3} [\frac{A}{(A+1)^2}(x_1 - x_2)^2 - (x_1 + x_2)\bar{s} - \bar{s}^2]dH(\mathbf{x})\end{aligned}$$

in symmetric equilibrium. If $t^A < \bar{t}^A$, then we can plug in the equilibrium from (9) to get

$$\pi^A(t^A) = \frac{1}{\rho}\frac{1}{A-1} + \frac{\phi\bar{s}}{A}\int_{\mathbf{x}\in\mathbf{X}_2} (x_2 - x_1 - \bar{s})dH(\mathbf{x}) + \frac{\phi}{2(A+1)^2}\int_{\mathbf{x}\in\mathbf{X}_3} (x_1 - x_2)^2 dH(\mathbf{x}) > 0$$

after simplification.

If $t^A = \bar{t}^A$, then I need to derive an expression for the outside option $\bar{t}^A$. I assume that this outside option is given by the expected cost of purchasing the energy service at the average flexi price when all other households purchase the energy service from an aggregator, but the deviating consumer uses the same amount of electricity $\frac{1}{2}\bar{s}$ in both periods to produce the energy service. Specifically:

$$\bar{t}^A = v + \frac{1}{2}\int_{\mathbf{x}\in\mathbf{X}}[P(x_1 + S(\mathbf{x}, \frac{1}{A})) + P(x_2 + \bar{s} - S(\mathbf{x}, \frac{1}{A}))]\bar{s}dH(\mathbf{x}) \quad (10)$$

In the above expression, $v \geq 0$ represents an incremental cost above the price of electricity associated with purchasing the good at the average real time price instead of from an aggregator. Households do not trade electricity directly in the market, but use an intermediary, usually a retailer. One can think of v as the incremental marginal cost or profit margin of this retailer. Substituting $\bar{t}^A$ into $\pi^A(t^A)$ yields:

$$\pi^A(\bar{t}^A) = \frac{v}{A} + \frac{\phi\bar{s}}{2A}\int_{\mathbf{x}\in\mathbf{X}_1}(x_1 - x_2 - \bar{s})dH(\mathbf{x}) + \frac{\phi\bar{s}}{2A}\int_{\mathbf{x}\in\mathbf{X}_2}(x_2 - x_1 - \bar{s})dH(\mathbf{x}) + \frac{\phi}{2(A+1)^2}\int_{\mathbf{x}\in\mathbf{X}_3}(x_1 - x_2)^2 dH(\mathbf{x}).$$

This expression is strictly positive for all $v \geq 0$. The above results demonstrate the business case for entering into the FDR market, even if entry is somewhat costly.

Equilibrium existence I now derive a sufficient condition for when $t_a = t^A$ represents a profit-maximizing fee if all other aggregators $a' \neq a$ choose $t_{a'} = t^A$. Specifically, $t_a = t^A$ uniquely maximizes $\pi^A(t_a)$ if $\phi\rho > 0$ is sufficiently close to zero. Deviations by a to $t_a \leq 0$ or $t_a > \bar{t}^A$ are strictly unprofitable because aggregator a earns $\pi^A(t_a) \leq 0$ for all such t_a, whereas $\pi^A(t^A) > 0$, as established above. Hence, all potentially profitable deviations lie in the interval $(0, \bar{t}^A]$. The proof is complicated by the fact that $S_a(\mathbf{x}, L_a)$ and $S_A(\mathbf{x}, L_a)$ are kinked in L_a. The six demand thresholds

$$X_{21}(x_1, L_a) = \max\{x_1 - (A+1)L_a\bar{s}; 0\},\ X_{22}(x_1, L_a) = \min\{x_1 + (A+1)L_a\bar{s}; \bar{x}\},$$

$$X_{23}(x_1, L_a) = \max\{x_1 - \frac{A+1}{A-1}(1-L_a)\bar{s}; 0\},\ X_{24}(x_1, L_a) = \min\{x_1 + \frac{A+1}{A-1}(1-L_a)\bar{s}; \bar{x}\},$$

$$X_{25}(x_1, L_a) = \max\{x_1 - \frac{A-L_a}{A-1}\bar{s}; 0\},\ X_{26}(x_1, L_a) = \min\{x_1 + \frac{A-L_a}{A-1}\bar{s}; \bar{x}\},$$

are relevant for the existence proof. By way of these thresholds

$$S_a(x_1, X_{21}(x_1, L_a), L_a) = 0,\ S_A(x_1, X_{21}(x_1, L_a), L_a) = \frac{1}{2}(1 - AL_a)\bar{s}$$

$$S_a(x_1, X_{22}(x_1, L_a), L_a) = L_a\bar{s},\ S_A(x_1, X_{22}(x_1, L_a), L_a) = \frac{1}{2}[1 + AL_a]\bar{s}$$

$$S_a(x_1, X_{23}(x_1, L_a), L_a) = \frac{1}{2}\frac{AL_a - 1}{A-1}\bar{s},\ S_A(x_1, X_{23}(x_1, L_a), L_a) = 0$$

$$S_a(x_1, X_{24}(x_1, L_a), L_a) = \frac{1}{2}\frac{AL_a + 1 - 2L_a}{A-1}\bar{s},\ S_A(x_1, X_{24}(x_1, L_a), L_a) = (1 - L_a)\bar{s}$$

$$S_a(x_1, X_{25}(x_1, L_a), L_a) = 0,\ S_A(x_1, X_{25}(x_1, L_a), L_a) = 0$$

$$S_a(x_1, X_{26}(x_1, L_a), L_a) = L_a\bar{s},\ S_A(x_1, X_{26}(x_1, L_a), L_a) = (1 - L_a)\bar{s}$$

in interior domain.

Assume first that $t^A < \bar{t}^A$, and consider an upward deviation $t_a \in (t^A, \bar{t}^A]$. In this case, $L_a < \frac{1}{A}$. We can then write

$$\begin{aligned}
\frac{\pi^{A\prime}(t_a)}{L_a} &= 1-\rho(1-L_a)t_a \\
&+\phi\rho(1-L_a)\int_0^{\bar{x}}\left[\int_0^{X_{25}}((x_2+\bar{s}-S(\mathbf{x},L_a))\bar{s}+(2S_a(\mathbf{x},L_a)-L_a\bar{s})\frac{\partial S_A}{\partial L_a})h(x_1,x_2)dx_2\right. \\
&+\int_{X_{25}}^{X_{21}}((x_2+\bar{s}-S(\mathbf{x},L_a))\bar{s}+(2S_a(\mathbf{x},L_a)-L_a\bar{s})\frac{\partial S_A}{\partial L_a})h(x_1,x_2)dx_2 \\
&+\int_{X_{21}}^{X_{22}}((x_2+\bar{s}-S(\mathbf{x},L_a))\bar{s}+(2S_a(\mathbf{x},L_a)-L_a\bar{s})\frac{\partial S_A}{\partial L_a})h(x_1,x_2)dx_2 \\
&+\int_{X_{22}}^{X_{26}}((x_2+\bar{s}-S(\mathbf{x},L_a))\bar{s}+(2S_a(\mathbf{x},L_a)-L_a\bar{s})\frac{\partial S_A}{\partial L_a})h(x_1,x_2)dx_2 \\
&\left.+\int_{X_{26}}^{\bar{x}}((x_2+\bar{s}-S(\mathbf{x},L_a))\bar{s}+(2S_a(\mathbf{x},L_a)-L_a\bar{s})\frac{\partial S_A}{\partial L_a})h(x_1,x_2)dx_2\right]dx_1,
\end{aligned}$$

Taking the second-derivative of the profit function yields

$$\begin{aligned}
\frac{\pi^{A\prime\prime}(t_a)}{\rho L_a} &= -1+\frac{2L_a-1}{L_a}\pi^{A\prime}(t_a) \\
&+\phi\rho L_a(1-L_a)^2\int_{\mathbf{x}\in\mathbf{X}}[\bar{s}\frac{\partial S}{\partial L_a}+(\bar{s}-2\frac{\partial S_a}{\partial L_a})\frac{\partial S_A}{\partial L_a}]dH(\mathbf{x}) \\
&+\frac{1}{2}\phi\rho L_a^2(1-L_a)^2\bar{s}^2\frac{A-1}{A}\int_0^{\bar{x}}\left[\frac{1}{A-1}h(x_1,X_{25}(x_1,L_a))\frac{\partial X_{25}}{\partial L_a}\right. \\
&+h(x_1,X_{21}(x_1,L_a))\frac{\partial X_{21}}{\partial L_a}-h(x_1,X_{22}(x_1,L_a))\frac{\partial X_{22}}{\partial L_a} \\
&\left.+\frac{2A-1}{A-1}h(x_1,X_{26}(x_1,L_a))\frac{\partial X_{26}}{\partial L_a}\right]dx_1
\end{aligned}$$

after simplification. The right-hand side is strictly negative for all $t_a \in (t^A, \bar{t}^A]$ such that $\pi^{A\prime}(t_a) = 0$ if $\phi\rho > 0$ is sufficiently close to zero. If so, then $\pi^A(t_a)$ is strictly quasi-concave in the domain $t_a \in [t^A, \bar{t}]$. It follows that $t_a = t^A$ is the unique-best reply in $t_a \in [t^A, \bar{t}]$ if $\phi\rho > 0$ is sufficiently close to zero.

Consider next a downward deviation $t_a \in (0, t^A)$, so that $L_a > \frac{1}{A}$. In this case, we can write

$$\begin{aligned}
\frac{\pi^{A\prime}(t_a)}{L_a} &= 1 - \rho(1-L_a)t_a \\
&+\phi\rho(1-L_a)\int_0^{\bar{x}}[\int_0^{X_{25}}((x_2+\bar{s}-S(\mathbf{x},L_a))\bar{s}+(2S_a(\mathbf{x},L_a)-L_a\bar{s})\frac{\partial S_A}{\partial L_a})h(x_1,x_2)dx_2 \\
&+\int_{X_{25}}^{X_{23}}((x_2+\bar{s}-S(\mathbf{x},L_a))\bar{s}+(2S_a(\mathbf{x},L_a)-L_a\bar{s})\frac{\partial S_A}{\partial L_a})h(x_1,x_2)dx_2 \\
&+\int_{X_{23}}^{X_{24}}((x_2+\bar{s}-S(\mathbf{x},L_a))\bar{s}+(2S_a(\mathbf{x},L_a)-L_a\bar{s})\frac{\partial S_A}{\partial L_a})h(x_1,x_2)dx_2 \\
&+\int_{X_{24}}^{X_{26}}((x_2+\bar{s}-S(\mathbf{x},L_a))\bar{s}+(2S_a(\mathbf{x},L_a)-L_a\bar{s})\frac{\partial S_A}{\partial L_a})h(x_1,x_2)dx_2 \\
&+\int_{X_{26}}^{\bar{x}}((x_2+\bar{s}-S(\mathbf{x},L_a))\bar{s}+(2S_a(\mathbf{x},L_a)-L_a\bar{s})\frac{\partial S_A}{\partial L_a})h(x_1,x_2)dx_2]dx_1,
\end{aligned}$$

Taking the second-derivative of this profit function yields

$$\begin{aligned}
\frac{\pi^{A\prime\prime}(t_a)}{\rho L_a} &= -1+\frac{2L_a-1}{L_a}\pi^{A\prime}(t_a) \\
&+\phi\rho L_a(1-L_a)^2\int_{\mathbf{x}\in\mathbf{X}}[\bar{s}\frac{\partial S}{\partial L_a}+(\bar{s}-2\frac{\partial S_a}{\partial L_a})\frac{\partial S_A}{\partial L_a}]dH(\mathbf{x}) \\
&+\frac{1}{2}\phi\rho\bar{s}^2L_a(1-L_a)^3\frac{1}{A-1}\int_0^{\bar{x}}[h(x_1,X_{23}(x_1,L_a))\frac{\partial X_{23}}{\partial L_a}+h(x_1,X_{24}(x_1,L_a))\frac{\partial X_{24}}{\partial L_a}]dx_1
\end{aligned}$$

after simplification. By an analogous argument as above, $t_a = t^A$ is the unique-best reply in $t_a \in (0, t^A]$ for $\phi\rho > 0$ sufficiently close to zero. These results complete the proof that t^A is a symmetric equilibrium for $\phi\rho > 0$ sufficiently close to zero. I summarize these results as:

Proposition 5 *Assume that $A \geq 2$ independent, symmetric and horizontally differentiated aggregators compete for flexible distribution resources in period 0. If $\phi\rho > 0$ is sufficiently small, then there exists a symmetric equilibrium in which all aggregators charge the same fee t^A and have the same market share $\frac{1}{A}$. Aggregators earn strictly positive profit in equilibrium.*

3.2 Mixed market structure

Assume that there are $A \geq 2$ aggregators. One is a subsidiary of a parent company that owns also the DSO, and the other $A-1$ aggregators are independent. To simplify the analysis, I assume that the demand for the energy service is not too large relative to other consumption, $\bar{s} < \frac{3}{2}\bar{x}$.

Competition in the flexi market The DSO maximizes

$$\Pi(\tilde{s}^{DSO},\tilde{s}^A,\mathbf{x}) = F+L^{DSO}t^{DSO}-P(x_1+s^{DSO}+s^A)s^{DSO}-P(x_2+\bar{s}-s^{DSO}-s^A)(L^{DSO}\bar{s}-s^{DSO}) \tag{11}$$

over $(\tilde{s}^{DSO}, \tilde{s}^A) \in [0, \bar{s}^{DSO}] \times [0, \bar{s}^A]$ in the flexi market. Proposition 4 in Tangerås (2023) establishes that the DSO chooses $s^A = \bar{s}^A = L^A \bar{s}$ if $x_1 > x_2$ and $s^A = 0$ if $x_1 < x_2$, where $L^A = 1 - L^{DSO}$ is the collective market share of the $A-1$ independent aggregators and L^{DSO} is the market share of the DSO/aggregator.

Optimization over $\Pi(\tilde{s}^{DSO}, s^A, \mathbf{x})$ then yields the first-period electricity withdrawal of the DSO/aggregator:

$$S^{DSO}(\mathbf{x}, L^{DSO}) = \begin{cases} \frac{x_2 - x_1 + (1+L^{DSO})\bar{s}}{4}, & x_1 < x_2 \\ \frac{x_2 - x_1 + (3L^{DSO}-1)\bar{s}}{4}, & x_1 \in [x_2, x_2 + \max\{3L^{DSO} - 1; 0\}\bar{s}] \\ 0 & x_1 > x_2 + \max\{3L^{DSO} - 1; 0\}\bar{s}. \end{cases}$$

The total electricity withdrawal by all aggregators in the first period is given by $S(\mathbf{x}, L^{DSO}) = S^{DSO}(\mathbf{x}, L^{DSO})$ if $x_1 < x_2$ and $S(\mathbf{x}, L^{DSO}) = S^{DSO}(\mathbf{x}, L^{DSO}) + (1 - L^{DSO})\bar{s}$ if $x_1 > x_2$.

Competition for household consumers The DSO/aggregator chooses the fixed fee $\tilde{t}^{DSO}$ to maximize the expected profit

$$\begin{aligned} \pi^{DSO}(\tilde{t}^{DSO}) &= L^{DSO}\tilde{t}^{DSO} - \int_{\mathbf{x}\in\mathbf{X}} [P(x_1 + S(\mathbf{x}, L^{DSO}))S^{DSO}(\mathbf{x}, L^{DSO}) \\ &\quad + P(x_2 + \bar{s} - S(\mathbf{x}, L^{DSO}))(L^{DSO}\bar{s} - S^{DSO}(\mathbf{x}, L^{DSO}))]dH(\mathbf{x}) \end{aligned}$$

By way of the envelope theorem, the marginal expected profit in period 0 equals

$$\begin{aligned} \pi^{DSO\prime}(\tilde{t}^{DSO}) &= L^{DSO} + [\tilde{t}^{DSO} - \int_{\mathbf{x}\in\mathbf{X}} \phi(x_2 + \bar{s} - S(\mathbf{x}, L^{DSO}))\bar{s}dH(\mathbf{x})]\frac{\partial L^{DSO}}{\partial t^{DSO}} \\ &\quad + \int_0^{\bar{x}}\int_{x_2}^{\bar{x}} \phi[2S^{DSO}(\mathbf{x}, L^{DSO}) - L^{DSO}\bar{s}]\bar{s}dH(\mathbf{x})\frac{\partial L^{DSO}}{\partial t^{DSO}}, \end{aligned}$$

where I have used the linear marginal cost assumption $P(q) = C'(q) = \phi q$ to simplify. The expression on the second row is the effect of a decrease in L^{DSO} of a larger $\tilde{t}^{DSO}$, which causes the DSO to modify s^A if $x_1 > x_2$.

The expected profit of aggregator a equals

$$\begin{aligned} \pi^A(t_a) &= L_a[t_a - \int_0^{\bar{x}}\int_0^{x_2} P(x_2 + \bar{s} - S^{DSO}(\mathbf{x}, L^{DSO}))\bar{s}dH(\mathbf{x}) \\ &\quad - \int_0^{\bar{x}}\int_{x_2}^{\bar{x}} P(x_1 + S^{DSO}(\mathbf{x}, L^{DSO}) + (1 - L^{DSO})\bar{s})\bar{s}dH(\mathbf{x})]. \end{aligned}$$

as a function of its fixed fee t_a. In the above expression, I have used $S_a(\mathbf{x}, L_a) = 0$ for $x_1 < x_2$ and $S_a(\mathbf{x}, L_a) = L_a\bar{s}$ for $x_1 > x_2$ by way of the profit-maximizing DSO dispatch. The marginal expected profit of aggregator a equals

$$\begin{aligned}\pi^{A\prime}(t_a) = \quad & L_a + \frac{\partial L_a}{\partial t_a}\frac{\pi^A(t_a)}{L_a} + L_a\int_0^{\bar{x}}\int_0^{x_2} P'(x_2+\bar{s}-S^{DSO}(\mathbf{x},L^{DSO}))\bar{s}\frac{\partial S^{DSO}(\mathbf{x},L^{DSO})}{\partial L^{DSO}}dH(\mathbf{x})\frac{\partial L^{DSO}}{\partial t_a} \\ + \quad & L_a\int_0^{\bar{x}}\int_{x_2}^{\bar{x}} P'(x_1+S^{DSO}(\mathbf{x},L^{DSO})+(1-L^{DSO})\bar{s})\bar{s}(\bar{s}-\frac{\partial S^{DSO}(\mathbf{x},L^{DSO})}{\partial L^{DSO}})dH(\mathbf{x})\frac{\partial L^{DSO}}{\partial t_a}.\end{aligned} \tag{12}$$

Characterization of the equilibrium fees By solving the DSO/aggregator's first order condition, $\pi^{DSO\prime}(\tilde{t}^{DSO})=0$,

$$\begin{aligned}t^{DSO} \quad = \quad & \min\{\frac{1}{\rho}\frac{1}{1-L^{DSO}} + \int_0^{\bar{x}}\int_0^{x_2}\phi(x_2+\bar{s}-S^{DSO}(\mathbf{x},L^{DSO}))\bar{s}dH(\mathbf{x}) \\ & + \int_0^{\bar{x}}\int_{x_2}^{\bar{x}}\phi(x_2+2L^{DSO}\bar{s}-3S^{DSO}(\mathbf{x},L^{DSO}))\bar{s}dH(\mathbf{x});\bar{t}\}\end{aligned}$$

follows directly. If $t^{DSO}<\bar{t}^M$, then one can substitute the equilibrium fee into the profit function to get:

$$\begin{aligned}\pi^{DSO}(t^{DSO}) \quad = \quad & \frac{1}{\rho}\frac{L^{DSO}}{1-L^{DSO}} + \phi\int_0^{\bar{x}}\int_0^{x_2}\frac{x_2-x_1+(1-L^{DSO})\bar{s}}{2}S^{DSO}(\mathbf{x},L^{DSO})dH(\mathbf{x}) \\ & +\phi\int_0^{\bar{x}}\int_{x_2}^{x_2+\max\{3L^{DSO}-1;0\}\bar{s}}\frac{x_1-x_2+(1-L^{DSO})\bar{s}}{2}[L^{DSO}\bar{s}-S^{DSO}(\mathbf{x},L^{DSO})]dH(\mathbf{x}) \\ & +\phi\bar{s}^2(L^{DSO})^2\int_0^{\bar{x}}\int_{x_2+\max\{3L^{DSO}-1;0\}\bar{s}}^{\bar{x}}dH(\mathbf{x})\end{aligned}$$

after simplification. This expression is strictly positive. If

$$t^{DSO}=\bar{t}^M=v+\frac{1}{2}\int_{\mathbf{x}\in\mathbf{X}}[P(x_1+S(\mathbf{x},L^{DSO}))+P(x_2+\bar{s}-S(\mathbf{x},L^{DSO}))]\bar{s}dH(\mathbf{x}),$$

then the DSO/aggregator still earns a positive profit:

$$\begin{aligned}\pi^{DSO}(\bar{t}^M) \quad = \quad & L^{DSO}v + \frac{\phi}{8}\int_0^{\bar{x}}\int_0^{x_2}(x_2-x_1+(1-L^{DSO})\bar{s})^2dH(\mathbf{x}) \\ & +\frac{\phi}{8}\int_0^{\bar{x}}\int_{x_2}^{x_2+\max\{3L^{DSO}-1;0\}\bar{s}}(x_1-x_2+(1-L^{DSO})\bar{s})^2dH(\mathbf{x}) \\ & +\phi\frac{L^{DSO}\bar{s}}{2}\int_0^{\bar{x}}\int_{x_2+\max\{3L^{DSO}-1;0\}\bar{s}}^{\bar{x}}(x_1-x_2+(1-2L^{DSO})\bar{s})dH(\mathbf{x}).\end{aligned}$$

Consider next aggregator a's equilibrium fee. If $t^A<\bar{t}^M$, then I can solve for the *equilibrium profit* directly from the first-order condition $\pi^{A\prime}(t^A)=0$:

$$\pi^A(t^A)=\frac{1}{\rho}\frac{L^A}{A-1-L^A}[1+\frac{\phi\rho\bar{s}^2L^AL^{DSO}}{A-1}(1-\frac{3}{4}\int_0^{\bar{x}}\int_0^{x_2+\max\{3L^{DSO}-1;0\}\bar{s}}dH(\mathbf{x}))]>0.$$

In the above expression, I have substituted in $\frac{\partial L^{DSO}}{\partial t_a} = \rho L_a L^{DSO}$, $P'(q) = \phi$ and

$$\frac{\partial S^{DSO}(\mathbf{x}, L^{DSO})}{\partial L^{DSO}} = \begin{cases} \frac{1}{4}\bar{s}, & x_1 < x_2 \\ \frac{3}{4}\bar{s}, & x_1 \in (x_2, x_2 + \max\{3L^{DSO} - 1; 0\}\bar{s}) \\ 0 & x_1 > x_2 + \max\{3L^{DSO} - 1; 0\}\bar{s}. \end{cases}$$

If $t^A = \bar{t}^M$, then

$$\begin{aligned} \frac{\pi^A(\bar{t}^M)}{L_a} &= v - \frac{\phi\bar{s}}{4}\int_0^{\bar{x}}\int_0^{x_2}(x_2 - x_1 + (1 - L^{DSO})\bar{s})dH(\mathbf{x}) \\ &\quad - \frac{\phi\bar{s}}{4}\int_0^{\bar{x}}\int_{x_2}^{x_2+\max\{3L^{DSO}-1;0\}\bar{s}}(x_1 - x_2 + (1 - L^{DSO})\bar{s})dH(\mathbf{x}) \\ &\quad - \frac{\phi\bar{s}}{2}\int_0^{\bar{x}}\int_{x_2+\max\{3L^{DSO}-1;0\}\bar{s}}^{\bar{x}}(x_1 - x_2 + (1 - 2L^{DSO})\bar{s})dH(\mathbf{x}). \end{aligned}$$

This expression is positive if and only if v is sufficiently large, a property which has interesting implications for the viability of a competitive FDR market. I return to this issue below. Proposition 4 in Tangerås (2023) draws conclusions about the efficiency of the mixed market solution depending on the size of L^{DSO}. If $A \geq 3$ and the aggregators are sufficiently differentiated, then L^{DSO} is close to $\frac{1}{A} < \frac{1}{2}$. In this case, the mixed market equilibrium is less efficient than no FDR market.

Equilibrium existence Assume that $v > 0$ is sufficiently large that $\pi^A(\bar{t}^M) \geq 0$. If so, then unilateral deviations to $\tilde{t}^{DSO} \leq 0$, $\tilde{t}^{DSO} > \bar{t}^M$, $t_a \leq 0$ and $t_a > \bar{t}^M$ are unprofitable because these deviations imply non-positive aggregator profits. Hence, all potentially profitable deviations lie in the interval $(0, \bar{t}^M]$ for all aggregators. The second-derivative of the DSO/aggregator's profit function can be written as

$$\begin{aligned} \frac{\pi^{DSO\prime\prime}(\tilde{t}^{DSO})}{\rho L^{DSO}} &= \frac{2L^{DSO} - 1}{L^{DSO}}\pi^{DSO\prime}(\tilde{t}^{DSO}) - 1 + \phi\rho L^{DSO}(1 - L^{DSO}) \\ &\quad \times[\int_{\mathbf{x}\in\mathbf{X}}\frac{\partial S(\mathbf{x}, L^{DSO})}{\partial L^{DSO}}\bar{s}dH(\mathbf{x}) + \int_0^{\bar{x}}\int_{x_2}^{\bar{x}}(2\frac{\partial S^{DSO}(\mathbf{x}, L^{DSO})}{\partial L^{DSO}} - \bar{s})\bar{s}dH(\mathbf{x})] \end{aligned}$$

after simplification. The second-derivative of the profit function is negative for all $\tilde{t}^{DSO} \in (0, \bar{t}^M]$ that solve the first-order condition $\pi^{DSO\prime}(\tilde{t}^{DSO}) = 0$, if $\phi\rho > 0$ is sufficiently small. Hence, $\pi^{DSO}(\tilde{t}^{DSO})$ is strictly quasi-concave for all $\tilde{t}^{DSO} \in [0, \bar{t}^M]$ if $\phi\rho > 0$ is sufficiently small.

The second-derivative of a's profit function equals

$$\begin{aligned} \frac{\pi^{A\prime\prime}(t_a)}{\rho L_a} &= \frac{2L_a - 1}{L_a}\pi^{A\prime}(t_a) - 1 \\ &\quad - 2\phi\rho\bar{s}^2 L_a(1 - L_a)L^{DSO}[1 - \frac{3}{4}\int_0^{\bar{x}}\int_0^{x_2+\{\max\{3L^{DSO}-1;0\}\bar{s}}dH(\mathbf{x})] \\ &\quad - \frac{9}{4}\phi\rho\bar{s}^3 L_a^2(L^{DSO})^2 Z(L^{DSO})\int_0^{\bar{x}}h(x_2 + (3L^{DSO} - 1)\bar{s}, x_2)dx_2, \end{aligned}$$

where $Z(L^{DSO}) = 0$ if $L^{DSO} < \frac{1}{3}$ and $Z(L^{DSO}) = 1$ if $L^{DSO} > \frac{1}{3}$. Any $t_a \in (0, \bar{t}^M]$ that solves the first-order condition $\pi^{A\prime}(t_a) = 0$ is a strict maximum. Hence, $\pi^A(t_a)$ is strictly quasi-concave in the domain $[0, \bar{t}^M]$. I conclude that (t^{DSO}, t^A) represents an equilibrium of the mixed market structure if $v > 0$ is sufficiently large and $\phi\rho > 0$ is sufficiently small.

Proposition 6 *Assume that an integrated DSO/aggregator competes with $A - 1 \geq 1$ independent, symmetric and horizontally differentiated aggregators in the market for flexible distribution resources in period* 0. *If the disutility $v > 0$ of passive consumption is sufficiently large and $\phi\rho > 0$ is sufficiently small, then there exists an asymmetric equilibrium (t^{DSO}, t^A) in fixed fees. Aggregators earn strictly positive profit in equilibrium.*

Foreclosure An equilibrium with one DSO/aggregator and $A - 1$ structurally independent aggregators exists if $v > 0$ is sufficiently large and $\phi\rho > 0$ is sufficiently small. Assume instead that v is so small that $\pi^A(\bar{t}^M) < 0$. From the marginal profit expression (12), $\pi^{A\prime}(\bar{t}^M) > 0$ in this case. By strict quasi-concavity, $t_a = \bar{t}^M$ then maximizes $\pi^A(t_a)$ over $[0, \bar{t}^M]$. By implication, $\pi^A(t_a) < 0$ for all $t_a \leq \bar{t}^M$. The optimal strategy for a is then to set $t_a > \bar{t}^M$ and earn zero profit. Since this result holds for an aggregator with arbitrary market share $L_a < 0$, it is impossible to sustain $A - 1 \geq 1$ independent aggregators in a market where a DSO/aggregator controls dispatch, if $\pi^A(\bar{t}^M) < 0$. In particular, it is impossible to uphold any market structure with independent aggregators if $v = 0$ because then $\pi^A(\bar{t}^M) < 0$ for all $A \geq 2$. The only viable market structure then is the DSO/aggregator monopoly.

4 Conclusion

This appendix has shown that the findings in Tangerås (2023) do not depend fundamentally on the assumption of competitive supply of generation capacity. Most of the results go through also under the alternative assumption that one large generation owner behaves strategically by withholding production from the real-time market. A main difference between the different modeling assumptions occurs in a market structure where a large generation owner also has a monopoly in the market for flexible distribution resources. Such vertical integration has a pro-competitive effect that does not arise in a model with competitive supply of generation capacity. Vertical integration between generation and aggregation can therefore be more efficient than a market structure in which generators and aggregators behave independently. This result is opposite to the finding in Tangerås (2023). Hence, vertical integration between generation and aggregation can be more or less efficient than independent management of resources depending on the competitiveness of the real-time market.

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Appendix

A.5 Proof of Proposition 3

For the purpose of this proof, define $\tilde{\beta} = \tilde{s}^A - \tilde{y}$ and $\bar{\beta} = \bar{s} - \bar{y}$. The DSO/aggregator then maximizes the objective function

$$\Omega(\tilde{s}^{DSO}, \tilde{\beta}, \mathbf{x}) = -P(x_1 + \tilde{s}^{DSO} + \tilde{\beta})\tilde{s}^{DSO} - P(x_2 + \bar{\beta} - \tilde{s}^{DSO} - \tilde{\beta})(\bar{s}^{DSO} - \tilde{s}^{DSO}). \tag{13}$$

over $\tilde{s}^{DSO} \in [0, \bar{s}^{DSO}]$ and $\tilde{\beta} \in [-\bar{y}, \bar{s}^A]$. Although (13) is strictly concave in each of its separate arguments $\tilde{s}^{DSO}$ and $\tilde{\beta}$, its Hessian matrix has a strictly negative determinant $-[C''(q_1) + C''(q_2)]^2$, so all interior solutions are saddle points. By implication, the profit-maximizing electricity withdrawal (s^{DSO}, β) features corner solutions. Let

$$S^{DSO}(\tilde{\beta}, \mathbf{x}) = \arg \max_{\tilde{s}^{DSO} \in [0, \bar{s}^{DSO}]} \Omega(\tilde{s}^{DSO}, \tilde{\beta}, \mathbf{x})$$

be the profit-maximizing consumption of the aggregator controlled by the DSO as a function of the consumption $\tilde{s}^A$ by all other aggregators and demand $\mathbf{x}$. Let $\omega(\tilde{\beta}, \mathbf{x}) = \Omega(S^{DSO}(\tilde{\beta}, \mathbf{x}), \tilde{\beta}, \mathbf{x})$ be the resulting profit of the integrated DSO/aggregator. Then, $\beta = \bar{s}^A$ for demand configuration $\mathbf{x}$ if $\omega(\bar{s}^A, \mathbf{x}) > \omega(-\bar{y}, \mathbf{x})$, and $s^A = -\bar{y}$ if the strict inequality is reversed. I establish in three claims that $\beta = \bar{s}^A$ for all $x_1 > x_2$. By symmetric arguments, $\beta = -\bar{y}$ for all $x_2 > x_1$. The claims establish the profit-maximizing choice for the different possible signs of $\partial\Omega(\tilde{s}^{DSO}, -\bar{y}, \mathbf{x})/\partial\tilde{s}^{DSO}$ evaluated at $\tilde{s}^{DSO} = \bar{s}^{DSO}$ and $x_1 \geq x_2$.

Claim 1 *If* $\frac{\partial\Omega(\tilde{s}^{DSO}, -\bar{y}, \mathbf{x})}{\partial\tilde{s}^{DSO}}|_{\tilde{s}^{DSO}=\bar{s}^{DSO}} \geq 0$ *for all* $x_1 \geq x_2$*, then* $s^A = \bar{s}^A$ *for all* $x_1 > x_2$*.*

Proof: By this assumption, $S^{DSO}(-\bar{y}, \mathbf{x}) = \bar{s}^{DSO}$ and therefore $\omega(-\bar{y}, \mathbf{x}) = -P(x_1 + \bar{s}^{DSO} - \bar{y})\bar{s}^{DSO}$ for all $x_1 \geq x_2$. Then

$$\omega(\bar{s}^A, \mathbf{x}) - \omega(-\bar{y}, \mathbf{x}) \geq \Omega(0, \bar{s}^A, \mathbf{x}) - \omega(-\bar{y}, \mathbf{x}) = [P(x_1 + \bar{s}^{DSO} - \bar{y}) - P(x_2 + \bar{s}^{DSO} - \bar{y})]\bar{s}^{DSO} > 0$$

for all $x_1 > x_2$ by $P' > 0$.■

Claim 2 *If* $\frac{\partial\Omega(\tilde{s}^{DSO}, -\bar{y}, x_2, x_2)}{\partial\tilde{s}^{DSO}}|_{\tilde{s}^{DSO}=\bar{s}^{DSO}} \leq 0$*, then* $s^A = \bar{s}^A$ *for all* $x_1 > x_2$*.*

Proof: Seeing as $\frac{\partial^2\Omega(\tilde{s}^{DSO},\tilde{\beta},\mathbf{x})}{\partial\tilde{s}^{DSO}\partial\tilde{\beta}}<0$ and $\frac{\partial^2\Omega(\tilde{s}^{DSO},\tilde{\beta},\mathbf{x})}{\partial\tilde{s}^{DSO}\partial x_1}<0$, it follows that

$$\frac{\partial\Omega(\tilde{s}^{DSO},\tilde{\beta},\mathbf{x})}{\partial\tilde{s}^{DSO}}|_{\tilde{s}^{DSO}=\bar{s}^{DSO}}<0$$

under the assumptions of this claim, and therefore $S^{DSO}(\tilde{\beta},\mathbf{x})<\bar{s}^{DSO}$ for all $\tilde{\beta}\in[-\bar{y},\bar{s}^A]$. Moreover,

$$\frac{\partial\Omega(\tilde{s}^{DSO},-\bar{y},x_2,x_2)}{\partial\tilde{s}^{DSO}}|_{\tilde{s}^{DSO}=0}=P(x_2+\bar{s})-P(x_2-\bar{y})+P'(x_2+\bar{s})\bar{s}^{DSO}>0$$

implies $S^{DSO}(-\bar{y},x_2,x_2)>0$. By continuity, $S^{DSO}(\tilde{\beta},\mathbf{x})>0$ for all $(\tilde{\beta},x_1)\in[-\bar{y},\varepsilon]\times[x_1,\delta]$ and some $\varepsilon>-\bar{y}$ and $\delta>x_1$. If $S^{DSO}(\tilde{\beta},\mathbf{x})=0$, then $\omega(\tilde{\beta},\mathbf{x})=-P(x_2+\bar{\beta}-\tilde{\beta})\bar{s}^{DSO}$, in which case $\frac{\partial^2\omega(\tilde{\beta},\mathbf{x})}{\partial x_1\partial\tilde{\beta}}=0$. If $S^{DSO}(\tilde{\beta},\mathbf{x})>0$, then

$$\frac{\partial\omega(\tilde{\beta},\mathbf{x})}{\partial x_1}=-C''(x_1+S^{DSO}(\tilde{\beta},\mathbf{x})+\tilde{\beta})S^{DSO}(\tilde{\beta},\mathbf{x})$$

and

$$\frac{\partial^2\omega(\tilde{\beta},\mathbf{x})}{\partial x_1\partial\tilde{\beta}}=-C''(Q_1^{DSO})\frac{\partial S^{DSO}}{\partial\tilde{\beta}}-C'''(Q_1^{DSO})S^{DSO}(1+\frac{\partial S^{DSO}}{\partial\tilde{\beta}})$$

Observe that $\frac{\partial^2\omega(\tilde{\beta},\mathbf{x})}{\partial x_1\partial\tilde{\beta}}>0$ if

$$\frac{C'''(Q_1^{DSO})S^{DSO}}{C''(Q_1^{DSO})}<\frac{-\frac{\partial S^{DSO}}{\partial\tilde{\beta}}}{1+\frac{\partial S^{DSO}}{\partial\tilde{\beta}}}. \tag{14}$$

Differentiation of the first order condition $\frac{\partial\Omega(S^{DSO},\tilde{\beta},\mathbf{x})}{\partial\tilde{s}^{DSO}}=0$ yields

$$\frac{\partial S^{DSO}}{\partial\tilde{\beta}}=-\frac{C''(Q_1^{DSO})+C'''(Q_1^{DSO})S^{DSO}+C''(Q_2^{DSO})+C'''(Q_2^{DSO})(\bar{s}^{DSO}-D^{DSO})}{2C''(Q_1^{DSO})+C'''(Q_1^{DSO})S^{DSO}+2C''(Q_2^{DSO})+C'''(Q_2^{DSO})(\bar{s}^{DSO}-S^{DSO})},$$

which I can use to obtain

$$\frac{-\frac{\partial S^{DSO}}{\partial\tilde{\beta}}}{1+\frac{\partial S^{DSO}}{\partial\tilde{\beta}}}=1+\frac{C'''(Q_1^{DSO})S^{DSO}+C'''(Q_2^{DSO})(\bar{s}^{DSO}-D^{DSO})}{C''(Q_1^{DSO})+C''(Q_2^{DSO})}$$

Hence, (14) holds by way of assumption (5). Hence, $S^{DSO}(\tilde{\beta},\mathbf{x})>0$ implies $\frac{\partial^2\omega(\tilde{\beta},\mathbf{x})}{\partial x_1\partial\tilde{\beta}}>0$. By implication

$$\omega(\bar{s}^A,\mathbf{x})-\omega(-\bar{y},\mathbf{x})-[\omega(\bar{s}^A,x_2,x_2)-\omega(-\bar{y},x_2,x_2)]=\int_{-\bar{y}}^{\bar{s}^A}\int_{x_2}^{x_1}\frac{\partial^2\omega(\tilde{\beta},y,x_2)}{\partial x_1\partial\tilde{\beta}}dyd\tilde{\beta}>0$$

for all $x_1>x_2$. Finally,

$$\omega(\bar{s}^A,x_2,x_2)-\omega(-\bar{y},x_2,x_2)\geq\Omega(\bar{s}^{DSO}-S^{DSO}(-\bar{y},x_2,x_2),\bar{s}^A,x_2,x_2)-\omega(-\bar{y},x_2,x_2)=0$$

completes the proof that $\omega(\bar{s}^A, \mathbf{x}) > \omega(-\bar{y}, \mathbf{x})$.■

Claim 3 *If* $\frac{\partial\Omega(\tilde{s}^{DSO},-\bar{y},x_2,x_2)}{\partial\tilde{s}^{DSO}}|_{\tilde{s}^{DSO}=\bar{s}^{DSO}} > 0$ *and* $\frac{\partial\Omega(\tilde{s}^{DSO},-\bar{y},\mathbf{x})}{\partial\tilde{s}^{DSO}}|_{\tilde{s}^{DSO}=\bar{s}^{DSO}} < 0$ *for some* $x_1 > x_2$*, then* $s^A = \bar{s}^A$ *for all* $x_1 > x_2$*.*

Proof: By $\frac{\partial\Omega(\tilde{s}^{DSO},0,\mathbf{x})}{\partial\tilde{s}^{DSO}\partial x_1} < 0$ and continuity, there exists a unique $x_1^c > x_2$ such that $\frac{\partial\Omega(\tilde{s}^{DSO},-\bar{y},x_1^c,x_2)}{\partial\tilde{s}^{DSO}}|_{\tilde{s}^{DSO}=\bar{s}^{DSO}} = 0$. Hence $S^{DSO}(-\bar{y}, \mathbf{x}) = \bar{s}^{DSO}$ for all $x_1 \in (x_2, x_1^c]$. A line of argument similar to the one used to prove Claim 1, can then be applied to establish $\omega(\bar{s}^A, \mathbf{x}) > \omega(-\bar{y}, \mathbf{x})$ for all $x_1 \in (x_2, x_1^c]$. If $x_1 > x_1^c$, then

$$\omega(\bar{s}^A, \mathbf{x}) - \omega(-\bar{y}, \mathbf{x}) - [\omega(\bar{s}^A, x_1^c, x_2) - \omega(-\bar{y}, x_1^c, x_2)] = \int_{-\bar{y}}^{\bar{s}^A} \int_{x_1^c}^{x_1} \frac{\partial^2\omega(\tilde{\beta}, y, x_2)}{\partial\tilde{\beta}\partial x_1} dy d\tilde{\beta} \geq 0$$

because $S^{DSO}(\tilde{\beta}, \mathbf{x}) < \bar{s}^{DSO}$ for all $\tilde{\beta} \in [-\bar{y}, \bar{s}^A]$ and $x_1 > x_1^c$. Combining this inequality with $\omega(\bar{s}^A, x_1^c, x_2) > \omega(-\bar{y}, x_1^c, x_2)$ concludes the proof of the claim.■

Combining the above three claims yields $s^A = \bar{s}^A$ for all $x_1 > x_2$. By following qualitatively similar steps as the above, it is straightforward to verify that $s^A = -\bar{y}$ for all $x_1 < x_2$. Summarizing the above three claims yields $s^A = \bar{s}^A$ for all $x_1 > x_2$. If $x_1 > x_2$, then $s^{DSO} + s^A - \frac{1}{2}\bar{s} = s^{DSO} + \frac{1}{2}(\bar{s}^A - \bar{s}^{DSO})$, which is non-negative if $\bar{s}^{DSO} \leq \bar{s}^A$. If $x_1 < x_2$, then $s^{DSO} + s^A - \frac{1}{2}\bar{s} = s^{DSO} - \bar{s}^{DSO} - \frac{1}{2}(\bar{s}^A - \bar{s}^{DSO})$, which is non-positive if $\bar{s}^{DSO} \leq \bar{s}^A$. Hence, $\bar{s}^{DSO} \leq \bar{s}^A$ implies $(s^{DSO} + s^A - \frac{1}{2}\bar{s})(x_1 - x_2) \geq 0$ for all (x_1, x_2). $\bar{s}^{DSO} \leq \bar{s}^A$ is equivalent to $L^{DSO} \leq \frac{1}{2}$.■