The Research Institute of Industrial Economics

WORKING PAPER No. 484, 1997

MULTI-MARKET COMPETITION AND COORDINATION IN GAMES

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LICENTIATE DISSERTATION IN ECONOMICS, MAY 1997

FOREWORD

The Research Institute of Industrial Economics (IUI) has a long tradition in studying strategic interaction and international competition. This ambition is currently pursued within the project "Industrial Organization and International Specialization". In the three essays contained in this volume Mattias Ganslandt applies game theoretic methods to such diverse problems as the role of capacity in multi-market entry deterrence, the effects of strategic uncertainty on equilibrium selection in coordination games and, finally, the role of simplicity and communication in teams.

This study was defended on May 30, 1997, as a licentiate thesis at the Department of Economics, University of Lund. It is the 54th doctoral or licentiate dissertation completed at IUI since its founding in 1939.

IUI would like to thank the thesis advisor, Hans Carlsson at the University of Lund, for his encouragement and support and also the discussant Johan Stennek at the Institute for International Economics Studies (IIES) for valuable comments.

Financial support from Marianne and Marcus Wallenberg foundation is gratefully acknowledged.

Stockholm in June 1997

Ulf Jakobsson
Director of IUI

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Acknowledgements

First and foremost I wish to thank my advisor Hans Carlsson for his support, criticism, comments and advice. Fredrik Andersson, Pontus Braunerhjelm, Karolina Ekholm and Jerker Holm gave indispensable suggestions and critique on various parts of this thesis. Tony Venables and Dan Anderberg provided me with valuable comments on early drafts of the first and second essay. I am also grateful to comments from seminar participants at Lund University, Stockholm School of Economics, Umeå University, IUI, The Workshop on The Geography of Multinational Firms in Stockholm, The International Conference on Simplicity in Tilburg, The Workshop on Dynamic Games in Arhus, EARIE'96, EEA'96 and ESEM'96. I am grateful to IUI (The Research Institute of Industrial Economics) for financial support and for providing an excellent environment for research and discussion. Finally, to Helena and Emilia, thank you for being such great sources of inspiration, support and love.

Introduction

This thesis consists of one essay in industrial organization and two essays in non-cooperative game theory. The first essay (Chapter I) concerns credibility in multi-market competition. The analysis focuses on the incumbent's possibilities to exploit first-mover advantages with investments in capacity as it competes with potential entrants in several markets. The other two essays (Chapter II and III) both examine coordination problems. The former studies the effects of uncertainty, while the latter investigates the effects of communication. The analysis in Chapter II indicates that uncertainty in a coordination game influences the problem of equilibrium selection. It is shown that a unique equilibrium is robust to perturbations of the players' strategies. Moreover, the noise-proof equilibrium is inefficient. The results in Chapter III suggest that structured and costly pre-play communication allow players to select among multiple strict Nash equilibria. The expected outcome has a short description in the language constructed by players. Moreover, the equilibrium is likely to be inefficient with respect to Pareto optimality in the underlying game. In the remaining parts of this introduction we shall give a brief outline of the problems analysed and the main results in each paper.

1. The role of investment in multi-market deterrence

Investment in capacity allows first-movers to make credible commitments. More precisely, the incumbent can install capacity to restrict competition from potential entrants. Investment in capacity shifts the incumbent's incentives and makes the firm more aggressive. Thus, as Dixit [2] and Spence [3] have argued, investment is a potential entry deterrent.

Intertemporal commitments with investments are possible because the capacity cost is sunk. The firm restricts its own future freedom of action by making irreversible decisions. However, if companies compete in several markets the effect of an investment is less clear.

A general issue in the literature on multi-market competition is whether presence in many markets facilitate or obstruct the incumbents possibilities to prevent competition. In the former case, when multi-market competition facilitate entry deterrence, we can expect integrated markets to be more concentrated than segmented markets. In the latter case the opposite holds. Therefore the question of market linkages should be at the heart of any theory of market concentration.

The framework of multi-market competition opens up perspectives which are relevant for decision making by multinational, diversified or integrated companies. An international firm can choose either concentrated production in a single plant, and supply foreign markets through exports, or to establish local plants in each market. In the first case the firm is free to redistribute its total capacity between different markets as the local market conditions change. In the latter case, a reasonable assumption is that production of goods will be adopted to local preferences and the firm can only use its local capacity in other markets if it incurs additional costs. Chapter I deals with the issue of how much capacity is needed to successfully deter entry from potential competitors in the two cases. We analyse whether an international firm that competes in several markets needs more total capacity to deter entry if capacity can be redistributed between markets without cost. Furthermore, we examine if there are less costly strategies to deter potential competitors in local markets.

The results in Chapter I suggest that a multi-market incumbent has to install more capacity to deter entry from potential competitors when capacity can be distributed globally and divided on local markets without additional costs. The result highlights the difficulties associated with credible strategies. A threat is only credible to the extent that the incumbent has incentives to carry it out if he faces local competition. If capacity can be redistributed without cost and the incumbent meets local competition, an emergency exit remains open and his original threat is less credible.

If the firm chooses dispersed production less total capacity is needed to deter entry. A condition for the local capacity to be an efficient market commitment is that the additional cost the firm has to incur in order to use the capacity in other local markets is sufficiently high. If this is true the future freedom of action for the incumbent is restricted and the commitment is credible.

Thus, the results in Chapter I indicate that strategic motives may induce a multi-market firm to choose dispersed production in several local plants rather than concentrated production, even if the production technology exhibits increasing returns to scale at the plant level.

2. Strategic uncertainty and equilibrium selection

The second essay in this thesis deals with the problem of equilibrium selection in coordination games.

The Stag Hunt Game in the figure below is a variant of such a game:

	H	L
H	2, 2	0, 1
L	1, 0	1, 1

In the case of the two-player Stag Hunt, player 1 selects a row (H or L) and player 2 a column (H or L). Payoffs are defined as the intersection of a row and a column, with player 1's payoff specified first. This game is interesting because it has two strict equilibria, (H,H) and (L,L), where the former is Pareto efficient. Both players would prefer the outcome in (H,H). However, the H strategy involves a risk since coordination failure would result in 0, while the L strategy guarantees the player a payoff of 1, even if coordination failure occurs.

In the second essay of this thesis we are concerned with the effects of uncertainty in a more general version of this coordination game. The game studied in Chapter II is due to Bryant [1]. In Bryant's game a group of players choose efforts from a compact interval and each player's payoff is determined by the minimum effort in the group minus the cost of his own effort. This game exhibits a continuum of Nash equilibria that can be ranked with respect to efficiency. All equilibria in the original game are strict. Therefore traditional refinements fail to select a unique equilibrium.

In a tacit version of the game players face a hard prediction problem if they wish to correctly forecast which equilibrium will occur. This intuition is supported by experimental findings which suggest that players beliefs are widely dispersed in Bryant's coordination game. In chapter II we investigate how this uncertainty influences the problem of equilibrium selection. What strategies can we expect rational players to choose in the coordination game if there is uncertainty about how strategies are translated into efforts?

The results in Chapter II indicate that if the utility functions are strictly concave, there is a unique equilibrium which is robust to perturbations of players' strategies. This point is called a noise-proof equilibrium. This is an interesting result because we are able to select a unique equilibrium in a model based on a non-cooperative game while traditional refinements fail to solve the equilibrium selection problem. Moreover, we can make precise predictions about how the uncertainty influences the outcome of the game.

If the utility function is sufficiently concave the noise-proof equilibrium is an interior point or the lowest effort in the continuum of equilibria in the original game. Hence, the Pareto efficient point cannot be obtained as an equilibrium in the uncertain version of the game. The inefficiency in the unique noise-proof equilibrium increases as the number of players increases. This result agrees with the findings in experiments based on variants of Bryant's game.

3. Structured communication and equilibrium selection

In a coordination game there are many equilibrium outcomes in which all players choose a mutual best response. While intuition might suggest that players should always be able to reach the Pareto efficient equilibrium, formal analysis does not support this conclusion. Even the strongest refinements often fail to select a unique equilibrium.

The problem of equilibrium selection has stimulated some recent research efforts among game theorists. It is argued that communication will never persuade rational agents to act contrary to their own interests, but, if agents have mutual interests, it may help them to coordinate their efforts.

One research approach assumes that an equilibrium in a coordination game is the result of pre-play discussion. It is argued that if equilibrium arises as the result of costless negotiations between the players, they would never settle down in a Nash equilibrium which is Pareto dominated by another equilibrium outcome. Dominated equilibria are never *renegotiation proof.*

A consequence of this argument would be that if players are allowed to meet and costlessly discuss their strategies without restrictions they will reach an efficient outcome in any coordination game, even if the environment and the most efficient behaviour is very complex. Why then, are most rules of behaviour simple?

A fact which may influence the outcome in a coordination game, and motivate players to choose simple rather than efficient strategies, is that pre-play communication is usually neither costless nor unrestricted. In chapter III we study how the structural conditions for costly pre-play communication influences the equilibrium selection problem in a game with multiple strict Nash equilibria.

We consider a set-up with asymmetric information. Both players know that any pure strategy is a best reply to itself, but only one player knows the revenue-maximizing equilibrium. The informed player can send a costly message to his uninformed counterpart through a channel that admits binary code only. The same code is used in a variety of situations, with few as well as many strict Nash equilibria. Does this mean that pre-play communication favours efficient Nash equilibria? Could an outside observer who knows the structural conditions for communication, i.e. the cost of communication and the qualities of the channel for transmission, make a prediction of the outcome?

Our results show that players would choose a code with short code-strings for strategies that occurs in problems with few equilibrium strategies, i.e. choice problems that appear more simple to the players. Moreover, when the cost of communication is high and the efficiency gains are small, the informed player has a motive to choose a short code-string as a message. The corresponding strategy is highly regular, and the equilibrium is most likely inefficient with respect to Pareto optimality in the underlying game.

However, when the cost of communication is lower, or the potential efficiency gains are larger, it is no longer true that players will always choose the shortest code string and a uniform sequence of actions. Nevertheless, players choose sequences of actions with relatively short descriptions if they are approximately as good as the revenue-maximizing strategy. In this case the equilibrium selection is a trade-off between efficiency and ease of describability.

References

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[2] Dixit (1980), The Role of investment in entry-deterrence, Economic Journal 90:95-106

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I. Multi-Market Competition and Market Commitments

1. Introduction

First-mover advantages allow established firms to restrict or prevent competition. Established firms can invest in capacity to the extent that entry by other firms is deterred. The crucial condition is that the incumbent can make early decisions in order to restrict its future freedom of action. However, when firms compete in many markets, e.g. multinational enterprises, this condition may change. Even if the cost of capacity is sunk, the multi-market incumbent can redistribute some of its capacity from markets with tough, to markets without competition. Thus, the firm maintains some degrees of freedom when it is present in more than one market. Consequently, the multi-market firm's threat to fight potential entrants is less credible.

This paper is therefore motivated by two sets of questions. How is the incumbent's possibility to exploit first mover advantages influenced by the fact that it competes in many markets? Can a foreign direct investment be regarded as a commitment to a specific market in order to restrict or prevent competition in that market?

If multi-market competition facilitates entry-deterrence we should expect integrated markets to be more concentrated than segmented markets. On the other hand, if the opposite holds and multi-market competition obstructs the incumbent's possibilities to restrict competition we should expect integrated markets to be less concentrated. Hence, the issue of market-linkages is important for any theory of market concentration.

It is often assumed that firms are allowed to take decisions exclusively at the multi-market level, referred to as the integrated market hypothesis, or at the local level, referred to as the segmented market hypothesis. One exception is Venables [12]. In his model firms take capacity decisions on an integrated basis and other decisions, e.g. price and sales decisions, on a national basis. Our model is close to Venables' in the attempt to analyse the importance of investment on entry-deterrence when capacity can be used on a multi-market level, while sales decisions are taken on a local basis. Unlike Venables, we focus on the role of local as well as global capacity in entry-deterrence.

This paper consider a market situation described as a stage game in which the incumbent first selects local and global capacities, then meets potential competition from local entrants in n markets.

In our model, the possibility to redistribute global capacity from a market with entry to a market without entry makes entry deterrence more difficult and more costly. Therefore, the multi-market incumbent has an incentive to induce market segmentation. If it is possible, the incumbent may choose to assign parts of its multi-market capacity to specific markets in order to facilitate entry-deterrence.

In particular the incumbent may induce market segmentation through bundling of products and services. Firms can bundle their tradable products with locally produced and consumed nontradables. If the product cannot be used without local services then the capacity is assigned to the local market when the marginal cost of expanding the local capacity of services in other markets is sufficiently high. In this respect our results relate to Horn and Shy [8] where market segmentation is endogenously determined through bundling of tradables with nontradables.

The paper is also related to many previous contributions in the literature. Spence [10] and Dixit [4] made the original Stackelberg story consistent by interpreting Stackelberg's output quantities as irreversible capacities.[1] In a seminal article Dixit [4] shows that a generalized Stackelberg outcome can be derived in a two-firm, two-stage model in which one firm makes an irreversible investment in the first stage and compete with another firm in quantities in the second stage.[2] In Dixit's model entry can be deterred if the setup cost is sufficiently high. With a simple Leontief technology no idle capacity will be installed to deter entry. [3]

We extend the Spence-Dixit analysis to the multi-market setting. Like Spence and Dixit, we consider a model with constant marginal cost of capital. Furthermore, demand is considered to be independent between markets. However, the firm is free to redistribute its global capacity between different markets. Hence, there is a

[1] In Spence [10] it is assumed that the threat by the established firm of producing at a level equal to established capacity is believed by the prospective entrant. Dixit [4] basically shows that if the post-entry game is restricted in such a way that only Nash-equilibria may be considered the subgame perfect equilibrium never exhibit idle capacity (with linear demand and Leontief technology).

[2] Eaton and Lipsey [5], Eaton and Ware [6], Gilbert [7] and Spulber [11] specify similar models with related results.

[3] It should be noted that these results does not hold in general. In a similar two-firm, two-stage game with iso-elastic demand the incumbent will hold excess capacity which is idle and would be utilized only in the event of entry.. This result is easily shown in a simple model originally due to Bulow, Geanakopolos and Klemperer [2]. Similar results are shown with multiple incumbent firms by Barham and Ware [1].

strategic link between different markets. [4]

Bulow, Geanakopolos and Klemperer (henceforth BGK) [3] present a multi-market model which relates to our analysis. They study a multi-market game in which two firms compete in one market but one of the firms is a monopoly in a second market. Technology exhibits decreasing returns to scale. A positive shock in the firm's home market turns out to be a strategic disadvantage since increasing production in one market increases marginal cost in both markets. More generally, BGK show that strategic substitutes and decreasing returns to scale makes the incumbent appear less aggressive in the multi-market setting. Strategic complements and increasing returns make the incumbent firm more aggressive. If the two markets exhibit joint economies then the positive effect in one market is positive on entry deterrence in the other market if products are strategic substitutes or strategic complements.

In our model the technology exhibit a constant marginal cost. Hence, BGK's analysis does not apply. We show that if firms compete in strategic substitutes and the incumbent is free to redistribute capacity between local markets, the firm is less aggressive.

The rest of this paper is organised as follows. Section 2 introduce four versions of the multi-market game. Section 3 is devoted to the first version of multi-market game, which is similar to Selten's [9] chain store game. In this version a multinational firm competes sequentially with n potential entrants in n markets. In the second version of the multi-market game, in section 4, the incumbent competes with n firms simultaneously after the capacity choice has been made. In the third version of the multi-market game, in section 5, the multinational competes with a second large player that is potential entrant in all n markets. Section 6 introduces market commitments and analyse what circumstances lead the incumbent to serve markets from a single multi-market plant versus many local plants. Finally, section 7 concludes the paper.

2. Multi-market entry deterrence

We consider four versions of a multi-market game. In the first three versions it is assumed that production capacity is used at the multi-market level. The incumbent is not allowed to assign parts of its total capacity to local markets. Instead,

[4] Other contributions to the literature have considered other market-linkages. Production can exhibit increasing or decreasing returns to scale which makes production in different markets dependent. A second possibility is that demand in different markets is related. This interdependence can change the marginal incentives to produce. For an extensive summary on multi-market competition models in which potential entrants are existing firms, see Witteloostuijn and Wegberg [13].

capacity can be redistributed between different markets without additional costs as the market conditions change. The first three cases differ with respect to potential competition and timing.

In the first version of the multi-market game an incumbent meets sequential competition from local entrants. The sequential structure is plausible when firms independently try to specify the product. They consider entry as soon as the product specification is correct and they have raised enough money for local production. This happens first to firm one, then to firm two etc. In this first version of the game we assume that potential entrants only consider local entry. One rational for this assumption is that before it can raise money for international expansion the firm has to succeed in its domestic market.

In the second version of the multi-market game the incumbent faces simultaneous competition from local entrants. The simultaneous structure arises when the incumbent owns a global patent which expires simultaneously in all local markets. In this case local competitors already have a correct specification of the product. As soon as the patent expires they immediately consider entry in the domestic market. In the second version of the game we maintain the assumption that potential competitors only consider local entry.

In the third version of the multi-market game the incumbent meets simultaneous competition in all markets from a single multi-market competitor. This market structure is plausible if the first competitor that finishes the process of product specification immediately consider a global strategy, or a global patent expires in all markets simultaneously and the potential entrant is able to raise enough money for multi-market entry.

After the analysis of the first three versions of the multi-market game we change the assumptions about the incumbent's possibilities to restrict competition. In the fourth version of the game we allow the incumbent to assign parts of its multi-market capacity to local markets. In order to assign local capacity to the domestic market the incumbent must install a local plant. The choice of a production organization is a trade-off between ease of entry-deterrence and scale economies in production at the plant level.

Generally, there is some asymmetry present when the incumbent is able to choose between large scale production and market commitments. The asymmetry can be geographical, due to product properties or consumers' preferences. A necessary condition is that goods produced in a single plant can be accepted by all consumers but local products can only be accepted by consumers in the domestic market.

As an extreme case, with very high transportation costs, local production may be necessary in order to sell the product with positive profit in local markets. Under such circumstances the local markets are segmented with respect to production and local capacity is assigned to distinct markets. In this case there is no alternative for the incumbent but to choose dispersed production. However, under less strict conditions the incumbent may choose to produce in a single plant, i.e. having a multi-market capacity, or produce in many local plants, i.e. assigning parts of the total capacity to local markets. These circumstances can arise in various situations:

First, firms can bundle their tradable products with locally produced and consumed nontradables, e.g. services. If the product cannot be used without local services the capacity is assigned to the local market if the marginal cost to expand the service capacity is sufficiently high.

Secondly, market commitments can be possible due to a horizontally differentiated product space. If local markets exhibit unique preferences, such as cultural habits or a unique language, capacities can be assigned to the domestic market as consumers in other countries are unable to use locally customized products. Correspondingly, market commitment can be possible due to network lock-ins. If the producer induces local network lock-ins by introducing local standards it can assign capacities to a specific market.

Thirdly, market commitment can be induced by asymmetric trade regulations. Products from one market can be sold to a second and third market, respectively. However, products from the second market cannot be sold without additional tariffs and quotas to the third market and vice versa. In much the same way transportation costs can allow market commitments as intermediate locations enable positive sales in both markets but local production in one market exhibit transportation costs that prevents consumption in the other market.

Finally, licensing can be regarded as market commitment. If market commitments are not possible in any other way, licensing production to independent local producers may commit capacities to local markets. Whether this strategy is more profitable than single-plant production depends on the relative bargaining positions of the multi-market firm and the licence-taking firms.

Thus, our model of endogenously determined multi-market production potentially applies in many different cases of international competition. After this presentation of the general features of the model we analyse each version in detail.

3. Sequential competition from local entrants

A multinational firm, also called player 0, has advertised and meet demand for its product in n markets, numbered 1 to n. In each market there is a potential entrant, player t, who might raise enough funding from creditors to establish a firm in market t selling the same product as the multinational enterprise.

Entry in market t is associated with a fixed, sunk cost A, which can be thought of as an advertising cost that makes consumers in market t aware of the entrant. Advertising makes all consumers in market t aware of the firm and its products but does not affect aggregate demand for the goods. There is no personal arbitrage since consumers are only aware of firms that make advertising in their home market. Therefore prices need not be internationally equalized. In the first version of the multi-market game we focus on a situation in which each potential competitor considers advertising in a single market only and, consequently, remaining local.

At the start of the game none of the potential entrants has specified the product correctly in order to start production. But as time goes on one after the other finish the process of specification and raise enough credit to (possibly) enter the local market. This will happen first to player 1, then to player 2, etc. As soon as player t has specified the product correctly he must decide to enter or stay out of the market. If he decides to stay out, he stops being a potential competitor. If a second firm enters market t, player 0 and player t choose outputs simultaneously and the market clears as a duopoly. If the potential entrant stays out monopoly will prevail.

After this description of the market situation in the first version of the multi-market game we turn to a formal specification of the model:

The game, Γ_n^1, has $n+1$ players, player 0 and player $1, ..., n$ $(n \geq 1)$. The game is played over a sequence of periods $t = 0, ..., n$. In period 0 the incumbent, player 0, must choose a pre-entry capacity k which is immediately made known to all players. At the beginning of period $t = 1, ..., n$, player t decides to enter or stay out. Player t's decision is made known to all players. If player t decides to enter, player 0 and player t will choose x_t^0 and x^t simultaneously. If player t decides to stay out monopoly prevails. The output decision is immediately made known to all players, too. At the end of period t, market t clears and payoffs are distributed to player 0 and player t. Next, for $t = 1, ..., n-1$ period $t+1$ begins and is played according to the same rules. The game ends after period n.

Player 0's payoff is the sum of n partial payoffs for $t = 1, ..., n$. Player 0's revenue in market t is $\pi\left(x_t^0, x^t\right)$. Cost of capital is additive and marginal capital cost is $r > 0$. The objective of player 0 is to maximize his total payoff:

$$v^0\left(k, x_1^0, .., x_n^0, x^1, .., x^n\right) = \sum_{t=1,..,n} \pi\left(x_t^0, x^t\right) - rk \text{ s.t. } \sum_{t=1,..,n} x_t^0 \leq k \tag{3.1}$$

Setting up a firm, i.e. enter market t, is associated with a fixed cost $A > 0$ for player t. Player t's revenue is $\pi\left(x^t, x_t^0\right)$. Marginal capital cost is $r > 0$ and additive. The objective of player $t = 1, ..., n$ is to maximize her payoff:

$$v^t\left(x^t, x_t^0\right) = \begin{cases} \pi\left(x^t, x_t^0\right) - rx^t - A & \text{if enter} \\ 0 & \text{if stay out} \end{cases} \tag{3.2}$$

Next, we introduce some notation before we proceed with our analysis. We define strategic substitutes, introduce a necessary and sufficient condition on entry-deterrence and define the deterrence level.

We shall call x^i a *strategic substitute* for x^j if $\frac{\partial^2 \pi}{\partial x^i \partial x^j} < 0$. Strategic substitutes imply that when a firm plays a more aggressive strategy, the optimal response by the other firm is *less* aggressive play. The condition that x^i is a strategic substitute for x^j is referred to as S:

$$\text{(S)} \qquad \frac{\partial^2 \pi\left(x^i, x^j\right)}{\partial x^i \partial x^j} < 0$$

Secondly, we introduce a best reply function with a non-binding capacity restriction of player 0, denoted $\beta^0\left(x^1\right)$, in the one-period game, Γ_1^1, implicitly defined by $\frac{\partial \pi}{\partial x_1^0}\left(\beta^0\left(x^1\right), x^1\right) = 0$. In the same fashion let the best reply function of player 1, $\beta^1\left(x_1^0\right)$, be defined $\frac{\partial \pi}{\partial x^1}\left(x_1^0, \beta^1\left(x_1^0\right)\right) - r = 0$. If potential competitor decides to enter in period 1 we have the following Nash equilibrium when the capacity constraint is non-binding for the incumbent: $\{\overline{x}_1^0, \overline{x}^1\}$, where $\overline{x}_1^0 = \beta^0\left(\overline{x}^1\right)$, $\overline{x}^1 = \beta^1\left(\overline{x}_1^0\right)$. If $k \leq \overline{x}_1^0$ the incumbent will use all capacity, but with $k > \overline{x}_1^0$ some capacity will be left idle. The unique Nash equilibrium in the subgame with entry is $\left\{\widehat{x}_1^0(k), \widehat{x}^1\right\}$ where $\widehat{x}_1^0(k) = \min\left\{k, \overline{x}_1^0\right\}$ and $\widehat{x}^1 = \beta^1\left(\widehat{x}_1^0(k)\right)$.

In the second subgame in the second stage, with no entry, we obtain the following Nash equilibrium when the capacity constraint is non-binding for the incumbent: $\left\{\overline{\overline{x}}_1^0, 0\right\}$, where $\overline{\overline{x}}_1^0 = \beta^0(0)$. Thus, $\overline{\overline{x}}_1^0$ is the monopoly level the incumbent would choose if the capacity does not restrict output.

When the firms compete in strategic substitutes the potential entrants profit is decreasing in the incumbent's output. However, the incumbent does not choose an output above the limit $\overline{x}_1^0$ if the potential competitor enters the local market. Thus, under condition (S) a necessary condition on entry deterrence is that profit of the potential entrant is non-positive in a Nash equilibrium with entry and a non-binding capacity restriction for the incumbent. This condition will be denoted D:

$$\text{(D)} \qquad \pi\left(\overline{x}^t, \overline{x}_t^0\right) - r\overline{x}^t - A \leq 0$$

If the necessary deterrence condition D is satisfied, condition S is a sufficient condition on entry deterrence. However, it can easily be shown that S is not a necessary condition for the result. In particular, the result can hold even if the strategic variables are strategic complements.

If D is satisfied and player t would earn a positive profit as monopoly it follows from the Theorem of Intermediate Values that at some positive level of output by the incumbent the profit of the entrant must be equal to zero. This *deterrence level* will be denoted $\widetilde{x}$ and defined

$$\pi\left(\beta^{t}\left(\widetilde{x}\right),\widetilde{x}\right)-r\beta^{t}\left(\widetilde{x}\right)-A\equiv 0$$

Thus, if the established firm successfully commit to an output $\widetilde{x}$ he would deter entry.

Now, we will show three results from the first version of the multi-market game, Γ_n^1. First, D is a sufficient condition on entry deterrence in the multi-market game. Second, if firms compete in strategic substitutes then D is not only a sufficient, but a necessary condition on entry deterrence. Third, if both conditions S and D are satisfied the incumbent installs strictly more than $n \cdot \widetilde{x}$ to deter entry in Γ_n^1.

If D is satisfied the local entrant would not earn a positive profit in $\{\overline{x}_t^0, \overline{x}^t\}$, and thus stay out of the local market. To see that D is a sufficient condition on entry deterrence assume that the incumbent has installed more capacity in period 0 than he would ever use. Thus, every market can be treated independently and the unique Nash equilibrium in every market t is $\{\overline{x}_t^0, \overline{x}^t\}$ and therefore entry deterrence is possible.

Proposition 1. *Entry deterrence is possible in Γ_n^1 if D is satisfied.*

Proof. $(D \Rightarrow \text{possible entry-deterrence})$. Let the pre-commitment capacity installed in period 0, k, be very large. The capacity constraint is not binding in any subgame. The objective of the incumbent is to maximize $\sum_{t=1,\ldots,n}\pi\left(x_t^0, x^t\right)$ with respect to x_t^0 for all t, $\frac{\partial v^0}{\partial x_t^0}=\frac{\partial \pi\left(x_t^0, x^t\right)}{\partial x_t^0}=0\ \forall t$. Hence, this problem is additively independent and each market can be considered as a separate one-market game Γ_1^1. If the capacity constraint is not binding the unique Nash equilibrium with entry is $\{\overline{x}_t^0, \overline{x}^t\}$ where $\overline{x}_t^0=\beta^0\left(\overline{x}^t\right)$, $\overline{x}^t=\beta^t\left(\overline{x}_t^0\right)$. Since $\pi\left(\overline{x}^t, \overline{x}_t^0\right)-r\overline{x}^t-A\leq 0$ player t would choose to stay out and monopoly prevails. ■

If firms compete in strategic substitutes we can say even more. In that case the deterrence condition (D) is not only sufficient but also a necessary condition on entry deterrence. Strategic substitutes (S) imply that the profit of a potential entrant is monotonically decreasing in the incumbent output. If k does not restrict output, then $\{\overline{x}_t^0, \overline{x}^t\}$ is the unique Nash equilibrium with entry in market t. Furthermore, $\overline{x}_t^0$ is the highest output the incumbent selects with any capacity k. Hence, if the potential competitor earns a positive profit in $\{\overline{x}_t^0, \overline{x}^t\}$ the same would be true in any Nash equilibrium. Thus he enters market t and entry deterrence is not possible.

Proposition 2. *If condition S is satisfied and condition D is violated then entry deterrence is not possible in* Γ_n^1.

Proof. (S and $\neg D \Rightarrow \neg$ entry-deterrence). First note that $\overline{x}_t^0$ is the highest output level of player 0 in a subgame with entry in market t. From (S) $\pi\left(x^t, x_t^0\right)$ is monotonically decreasing in x_t^0 and hence $\pi\left(x^t, x_t^0\right)$ reach its minimum at $\overline{x}_t^0$. If $\neg D$, i.e. $\pi\left(\overline{x}^t, \overline{x}_t^0\right) - r\overline{x}^t - A > 0$, player t could ensure himself a positive profit if he enters market t. ■

After these two qualitative results we can state a more precise result that characterize the disadvantage of multi-market competition on entry-deterrence. If firms compete in strategic substitutes and the necessary deterrence condition is satisfied, the incumbent must install $k > 2\widetilde{x}$ to deter entry in the two–market game Γ_2^1.

We illustrate the basic intuition by means of an example. Why is not twice the deterrence level, $\widetilde{x}$, enough to deter entry in two markets? The main reason is that if one potential competitor enters and the other stays out, the incumbent has an incentive to redistribute capacity to the monopoly market.

In the last period the remaining capacity is $k - x_1^0$. If condition D is satisfied $k - x_1^0 \geq \widetilde{x}$ would deter entry. Working backward to period 1 there are two subgames. If player 1 stays out the incumbent will split the capacity equally in both markets. If the potential competitor enters, the marginal incentive to use capacity in market 1 and 2 must be equal:

$$\frac{\partial \pi}{\partial x_1^0}\left(x_1^0, \beta^1\left(x_1^0\right)\right) = \frac{\partial \pi}{\partial x_2^0}\left(k - x_1^0, 0\right)$$

It follows from strategic substitutes that $k - x_1^0 > x_1^0$. Thus, if $k = 2\widetilde{x}$ then $x_1^0 < \widetilde{x}$ and entry is not deterred in the first market.

Proposition 3. *If D and S are satisfied in the first version of the two-market game,* Γ_2^1, *then the multi-market incumbent installs capacity* $\overline{k}^1 \in \left(2\widetilde{x}, \widetilde{x} + \overline{\overline{x}}^0\right]$ *to deter entry.*

Proof. Appendix A ■

4. Simultaneous competition from local entrants

Consider a market situation similar to the first version of the multi-market game. Again a multinational firm, called player 0, has advertised and meet demand for its product in n markets, numbered 1 to n. In each market there is a potential entrant, player t, who might raise enough funding from creditors to establish a firm in market t selling the same product as the multinational enterprise.

In this version the incumbent owns a global patent that expires at the same time in all markets and potential competitors can enter the local markets simultaneously. If a potential competitor challenges the established firm in a local market, the incumbent and the entrant choose outputs simultaneously and the market will clear as duopoly. If the potential entrant stays out monopoly will prevail.

The rules of the second version of the multi-market game are defined as follows. The game,Γ_n^2, has $n+1$ players, player 0 and player $1,...,n$ $(n \geq 1)$. The game is played over a sequence of two periods $m = 0, 1$. In period 0 the incumbent must choose a pre-entry capacity k. At the beginning of period 1, player $t = 1, ..., n$ must decide to enter or stay out of market t simultaneously. Player t's decision is immediately made known to all other players. If player t decides to enter market t then the incumbent and the entrant choose x_t^0 and x^t simultaneously. At the end of period 1, all markets clear and payoffs are distributed to the incumbent and players $1, ..., n$. Player 0's payoff is given by eq. (3.1) and player t's payoff by eq. (3.2).

The analysis in the second version of the multi-market game is similar to the analysis in the first version. If players compete in strategic substitutes and the necessary deterrence condition is satisfied the entry can be deterred also in the second version of the game. To deter entry the established firm must install $k > 2\widetilde{x}$ in the two–market game, Γ_2^2. In the second version of the two-market game there are four subgames in the last stage. In two of the four subgames one potential competitor enters, while the other stays out. To see why twice the single market deterrence capacity does not suffice, consider the profit maximizing conditions when $k = 2\widetilde{x}$:

$$\frac{\partial \pi}{\partial x_1^0}\left(x_1^0, \beta^1\left(x_1^0\right)\right) = \frac{\partial \pi}{\partial x_2^0}\left(2\widetilde{x} - x_1^0, 0\right)$$

Strategic substitutes imply that the output in the duopoly market is strictly lower than the deterrence level, i.e. $x_1^0 < \widetilde{x}$. Thus, entry would not be deterred.

Proposition 4. *If D and S are satisfied in the second version of the two-market game, Γ_2^2, then the incumbent installs capacity $\overline{k}^2 \in \left(2\widetilde{x}, \widetilde{x} + \overline{\overline{x}}^0\right]$ to deter entry.*

Proof. Appendix B ■

The first and second version of the multi-market game differ in one important respect. If the incumbent installed enough capacity to deter simultaneous entry by all potential competitors but not enough to deter unilateral entry by one potential competitor, e.g. $k = 2 \cdot \widetilde{x}$, then the potential entrants face a coordination problem in the second version of the game. This coordination problem does not occur in the first version of the game. In the first version player 1 enters and player 2 stays out.

In the second version both potential competitors wish to enter if they are the only entrant, but not otherwise. [5]

The coordination problem in the second version of the game remains unsolved since both Nash equilibria are strict. However, we will not deal with this problem further since we are mainly interested in the conditions on entry deterrence but in a real market situation the coordination problem may affect the decisions by the entrants and possibly facilitate entry-deterrence.

5. Simultaneous competition from a multi-market entrant

Again a multinational firm has advertised and meet demand for its product in n markets. Now, in the third version of the multi-market game a single potential competitor, another multinational company, considers entry in all local markets selling the same product as the established firm. The incumbent's global patent expires at the same time in all markets and the potential competitor may enter all local markets at the same time. Entry in each market is associated with a fixed sunk cost, which can be thought of as an advertising cost making consumers in the domestic market aware of the entrant. The entrant remains unknown in all markets where it is not advertising. If the second multinational firm enters the local market, the incumbent and the entrant choose outputs simultaneously and the market will clear as a duopoly.

Now, the rules of the third version of the game are defined as follows. The game,Γ_n^3, has 2 players, called player 0 and player 1. The game is played over a sequence of two periods $m = 0, 1$. In period 0 the established firm must choose a pre-entry capacity k. At the beginning of period 1, the potential competitor must decide to enter or stay out in n separate markets called $t = 1, ..., n$. Player 1's decision is immediately made known to player 0. If player 1 decides to enter market t, players will choose x_t^0 and x^t simultaneously. If player 1 decides to stay out, monopoly prevails in that market. At the end of period 1, all markets clear and payoffs are distributed to player 0 and player 1.

The incumbent's payoff is given by eq. (3.1). Entry in market t, is associated with a firm-specific fixed cost $A > 0$ for player 1. Let E be the set of all markets that player 1 enters and O where it stays out. Player 1's partial revenue, in a market where it enters, is $\pi\left(x^t, x_t^0\right)$. Marginal capital cost is $r > 0$ and additive. The objective of player 1 is to maximize her total payoff:

$$v\left(x^1, ..., x^n, x_1^0, ..., x_n^0\right) = \sum_{t \in E}\left(\pi\left(x^t, x_t^0\right) - rx^t - A\right) \tag{5.1}$$

[5] This is a version of the "chicken" game.

Next we will show that inequality D is a sufficient condition on entry deterrence in the third version of the multi-market game, too. If the incumbent invests in a sufficiently large capacity which makes the capacity constraint non-binding in every subgame, the optimal output in every market can be determined independently. The potential competitor chooses her optimal strategy in each market separately and the best reply functions in all market are identical. The unique Nash-equilibrium output in every market is $\{\overline{x}_t^0, \overline{x}^t\}$. Thus player 1's partial revenue does not cover the fixed and variable costs in any market and the total payoff is negative.

Proposition 5. *Entry deterrence is possible in Γ_n^3 if D is satisfied.*

Proof. ($D \Rightarrow$ entry-deterrence). Let the pre-commitment capacity installed in period 0, k, be very large. The capacity constraint is not binding in any subgame. The objective of player 0 is to maximize $\sum_{t=1,\dots,n} \pi\left(x_t^0, x^t\right)$ with respect to x_t^0 for all t, $\frac{\partial v^0}{\partial x_t^0} = \frac{\partial \pi\left(x_t^0, x^t\right)}{\partial x_t^0} = 0\ \forall t$. If the capacity constraint is not binding the unique Nash equilibrium with entry is $\{\overline{x}_t^0, \overline{x}^t\}$ where $\overline{x}_t^0 = \beta^0\left(\overline{x}^t\right)$, $\overline{x}^t = \beta^t\left(\overline{x}_t^0\right)$. Without loss of generality assume that player 1 enters $t \in E = \{1, .., m\}$. Since $\sum_{t \in E}\left(\pi\left(x_t^1, x_t^0\right) - r x_t^1 - A\right) = m \cdot \left(\pi\left(\beta^t\left(x_1^0\right), x_1^0\right) - r\beta^t\left(x_1^0\right) - A\right) < 0$ for $m = 1, .., n$ player t would choose to stay out and monopoly prevails. ■

In fact the strategic interaction in the second and third versions of the multi-market game is identical, except for the coordination problem present in the second version of the game. Two factors make the strategic decisions in the two games identical with respect to entry deterrence. First, to the entrant in the third version of the multi-market game the strategic variables $x^1, \dots, x^n$ are independent and she will choose her optimal strategy in each market separately. Thus, the best reply function of player 1 in market t is identical to player t's best reply function in the second version of the multi-market game.

Secondly, since the fixed cost A is the same in all markets, the revenue in each market that the potential competitor enters must cover the variable and fixed costs. Player 1 would only enter a market in which the expected payoff is positive. This is exactly the condition on entry to a local competitor in Γ_n^2. The analysis from the second version of the game therefore applies to the third version, too. Player 0 must install $k > 2\widetilde{x}$ to deter entry in the two-market game Γ_2^3.

Proposition 6. *If D and S are satisfied in the third version of the two-market game, Γ_2^3, then the incumbent installs capacity $\overline{k}^3 \in \left(2\widetilde{x}, \widetilde{x} + \overline{\overline{x}}^0\right]$ to deter entry.*

Proof. Appendix B ■

In the previous sections we have characterized the difficulties of entry deterrence in the first, second and third versions of the multi-market game. To deter entry of many potential competitors in a sequential or simultaneous market structure in the post-investment phase, or a single multi-market competitor, takes more capacity than n times the deterrence level. In fact we can say something even more precise. The established firm install exactly the same capacity to deter entry in Γ_2^1, Γ_2^2 and Γ_2^3. Thus, the unique optimal deterring capacity is independent of the market situation as described in the first, second and third versions of the multi-market game.

Proposition 7. *If conditions D and S are satisfied the global capacity needed to deter entry in the two-market game is independent of the timing of the game, i.e. sequential or simultaneous entry of the potential competitors, and independent of the number of markets considered by the potential entrant(s). More precisely,* $\overline{k}^1 = \overline{k}^2 = \overline{k}^3$ *in* Γ_2^1, Γ_2^2 *and* Γ_2^3.

Proof. $\overline{k}^1, \overline{k}^2, \overline{k}^3$ are implicitly defined by $\frac{\partial \pi}{\partial x_1^0}\left(\widetilde{x}, \beta^1\left(\widetilde{x}\right)\right) - \frac{\partial \pi}{\partial x_2^0}\left(\overline{k}^i - \widetilde{x}, 0\right) = 0$, for $i = 1, 2, 3$. ■

6. Market commitments

In this section we extend the analysis and let the incumbent first decide on an organization of its production, either in a single plant at the multi-market level or in many plants at the local level. We will show that if the multinational enterprise chooses a multi-plant strategy with local production then plant-capacities will be assigned to local markets.

We study a three stage game similar to the two-stage game in the previous section. Now, in the first stage the multi-market firm can decide to set-up local plants in both markets or produce in a single multi-market plant. Setting up two plants incurs an extra fixed cost compared to the single plant strategy. Stage two and three correspond to stage one and two in the two-stage game in the previous section.

We can now describe the rules of the fourth version of the game. The game, Γ_n^4, has two players, player 0 and player 1. The game is played over a sequence of two periods. In the first stage the incumbent must first choose to build $d \in \{1, n\}$ plants (where $D = (1, ..., d)$ is the set of plants), and secondly k which is a multi-market capacity and k_t, for all $t \in D$, which are plant-specific capacities for every plant installed. All decisions of the established firm is immediately made known to the potential competitor. At the beginning of the third stage, player 1 must decide to enter or stay out in n separate markets called $t = 1, ..., n$. Player 1's decision is made known to the incumbent. If player 1 decides to enter market t player 0 and player 1 will choose x_t^0 and x^t simultaneously. Finally, all markets clear and payoffs are distributed to player 0 and player 1.

The the cost of local capacity is $r_1 > 0$, the cost of multi-market capacity is $r_2 > 0$ and $\sum_{t=1,..,n} x_t^0 \leq k$. For simplicity we assume that $r = r_1 + r_2$. Each plant is associated with a fixed cost $G > 0$.

The incumbent's payoff is given by:

$$v^0\left(k, x_t^0, x^t\right) = \begin{cases} \sum_{t=1,..,n} \pi\left(x_t^0, x^t\right) - rk - G & \text{if d=1} \\ \sum_{t=1,..,n} \left(\pi\left(x_t^0, x^t\right) - r_1\left(\max\left\{x_t^0, k_t\right\}\right)\right) + r_2 k - nG & \text{if d=n} \end{cases}$$

To the potential competitor entry in market t, is associated with a firm-specific fixed cost $A > 0$. Let E be the set of all markets that player 1 enters and O the set of markets where he stays out. Player 1's revenue is $\pi\left(x^t, x_t^0\right)$. Marginal capital cost is $r > 0$ and additive. The objective of player 1 is to maximize her payoff given by eq. (5.1).

We call k_t a *market commitment* if this part of the total capacity k in a multi-market firm is assigned to market t and cannot profitably be used for production of goods sold in other local markets. A sufficient condition on market commitments is that the marginal cost to increase local capacity is larger than the marginal incentive to increase the output in a monopoly market at the deterring level $\overline{x}^0$. We refer to this condition as (C).

$$\text{(C)} \quad r_1 > \frac{\partial \pi}{\partial x_t^0}\left(\overline{x}^0, 0\right)$$

Condition C simply guarantees that it is not profitable for player 0 to redistribute capacity to a monopoly market if entry occurs in other markets. To deter entry in market t if condition C is satisfied and condition D is satisfied with equality it is sufficient for player 0 to install a local capacity equal to the deterrence level $k_t = \overline{x}^0$ and a multi-market capacity $k = n\overline{x}^0$.

Proposition 8. *If conditions C, D and S are satisfied in the fourth version of the two-market game, Γ_2^4, capacity $\overline{k} = \sum_t \overline{k}_t = 2\widetilde{x}$ is sufficient to deter entry.*

Proof. Entry deterrence is possible in Γ_2^4 due to (D). Player 0 will choose a two-plant strategy and installs capacity $\overline{k} = 2\widetilde{x}$ and $\overline{k}_t = \widetilde{x}$ for $t = 1, 2$. If player 1 enters both markets, symmetric incentives imply that $x_1^0 = x_2^0 = \widetilde{x}$ and D that the profit of player 1 is not positive. If player 1 enters market 1 and stays out of market 2 then to deter entry the following inequality must hold $\frac{\partial \pi}{\partial x_1^0}\left(\overline{x}^0, \beta^1\left(\overline{x}^0\right)\right) + \left(r_1 - \frac{\partial \pi}{\partial x_2^0}\left(2\widetilde{x} - \overline{x}^0, 0\right)\right) \geq 0$. The first part of the LHS is equal to zero and from (C) the second part is positive and thus the inequality holds. Equal parts of the total capacity should be assigned to both markets, i.e. $\overline{k}/2$. $\widetilde{x}$ deters entry in market t, hence $2\overline{x}^0$ is enough to deter entry in both markets. ■

The incumbent installs strictly less capacity with market commitments compared to the capacity needed to deter entry if the capacity is not assigned to specific markets, i.e. $2\overline{x}^0 < \overline{k}^3 = \overline{x}^0 + \overline{\overline{x}}^0$. The difference in the established firm's profit, if C is satisfied in Γ_n^4, between the two-plant and one-plant strategy is called the commitment premium. We define the *commitment premium* as

$$\Delta v \equiv v\left(2\overline{x}^0, \overline{x}^0, 0\right) - v\left(\overline{k}^3, \overline{k}^3/2, 0\right)$$

Working backward the multi-market firm will choose a multi-plant strategy if the commitment premium is positive.

Proposition 9. *If C is satisfied in Γ_2^4 then the multi-market firm will choose a two-plant strategy to deter entry i.f.f.* $(2 \cdot \pi(\overline{x}^0, 0) - 2\overline{x}^0 r) - \left(2 \cdot \pi\left(\overline{k}^3/2, 0\right) - r\overline{k}^3\right) > G$.

Proof. The profit of a one-plant strategy is less than a two-plant strategy if $2 \cdot \pi\left(\overline{k}^3/2, 0\right) - r\overline{k}^3 - G < 2 \cdot \pi(\overline{x}^0, 0) - 2r\overline{x}^0 - 2G$. Rewrite $(2 \cdot \pi(\overline{x}^0, 0) - r2\overline{x}^0) - \left(2 \cdot \pi\left(\overline{k}^3/2, 0\right) - r\overline{k}^3\right) > G$. ■

It follows from this proposition that a two-plant strategy is more likely the lower the plant-specific setup cost. The organization of production within the multinational firm is primarily determined by the relationship between economies of scale at plant level and the commitment premium and not the fixed cost A. If market commitments are not possible, multi-plant production never takes place in this model.

7. Conclusion

Multi-market competition without market commitment makes the incumbent's possibilities to exploit first mover advantages more difficult. A firm's opportunity in one market influences its possibility to successfully commit to its optimal strategy in a second market. The incumbent has to install a higher level of global capacity to successfully deter entry in all markets. If exogenous or endogenous factors allow the incumbent to assign parts of its capacity to local markets, multi-plant production can be profitable even under increasing returns to scale at the plant level. The results suggest that local investments can be regarded as market commitments in order to restrict or prevent competition in distinct markets.

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Appendix A

Proof. *Step 1.* Start in period 2. Let the remaining capacity be $k_2 = k - x_1^0$. There are two subgames; either player 2 has entered or stayed out of market 2. In the subgame with entry the unique Nash equilibrium is $\{\widehat{x}_2^0(k_2), \widehat{x}^2\}$ where $\widehat{x}_2^0(k_2) = \min\{k_2, \overline{x}^0\}$ and $\widehat{x}^2 = \beta^2(\widehat{x}_2^0(k_2))$. If player 2 decides to stay out in period 2 we have the following limit Nash equilibrium $\left\{\overline{\overline{x}}^0, 0\right\}$ where $\frac{\partial\pi}{\partial x_2^0}\left(\overline{\overline{x}}^0, 0\right) = 0$. The unique Nash equilibrium in the subgame with no entry is $\{\widehat{x}_2^0(k), 0\}$ where $\widehat{x}_2^0(k_2) = \min\left\{k_2, \overline{\overline{x}}^0\right\}$. From S it follows that $\overline{\overline{x}}^0 > \overline{x}^0$.

Step 2. Player 2 would enter if $k_2 < \widetilde{x}$ and stay out if $k_2 \geq \widetilde{x}$. To deter entry player 0 would need $k_2 \geq \widetilde{x}$. Now, assume that enough unused capacity remains to deter entry. Now, rewrite the equilibrium output by player 0 in period 2 as a function of k and x_1^0, i.e. $x_2^0(k, x_1^0) = \min\left\{k - x_1^0, \overline{\overline{x}}^0\right\}$.

Step 3. Working backwards to period 1 we have two subgames; either player 1 enters or stays out of market 1. First, capacity k would ensure a successful commitment by player 0 in market 1 to an output $\widetilde{x}_1^0$ if and only if: $\frac{\partial\pi}{\partial x_1^0}\left(\widetilde{x}_1^0, \beta^1(\widetilde{x}_1^0)\right) + \frac{\partial x_2^0}{\partial x_1^0}(k, \widetilde{x}_1^0)\frac{\partial\pi}{\partial x_2^0}(x_2^0(k, \widetilde{x}_1^0), 0) \geq 0$. Now, $\frac{\partial x_2^0}{\partial x_1^0}(k, \widetilde{x}_1^0) = -1$ if $k \leq \overline{\overline{x}}^0 + \widetilde{x}_1^0$ and $\frac{\partial x_2^0}{\partial x_1^0}(k, \widetilde{x}_1^0) = 0$ if $k > \overline{\overline{x}}^0 + \widetilde{x}_1^0$. To deter entry player 0 has to commit to $\widetilde{x}$ in the subgame with entry. The following inequality must be satisfied:

$$\frac{\partial\pi}{\partial x_1^0}\left(\widetilde{x}, \beta^1(\widetilde{x})\right) - \frac{\partial\pi}{\partial x_2^0}(k - \widetilde{x}, 0) \geq 0 \tag{7.1}$$

If $x^1 > 0$ it follows from (S) that $k - \widetilde{x} > \widetilde{x} \Rightarrow k > 2\widetilde{x}$. If (D) holds with equality, i.e. $\widetilde{x} = \overline{x}^0$, then the first part of the LHS of inequality [7.1] is equal to zero and the inequality is satisfied if and only if $k - \widetilde{x} = \overline{\overline{x}}^0 \Rightarrow k = \widetilde{x} + \overline{\overline{x}}^0$. Let $\overline{k}^1$ be implicitly defined by $\frac{\partial\pi}{\partial x_1^0}\left(\widetilde{x}, \beta^1(\widetilde{x})\right) - \frac{\partial\pi}{\partial x_2^0}\left(\overline{k} - \widetilde{x}, 0\right) = 0$. Secondly, in the subgame without entry $\frac{\partial\pi}{\partial x_1^0}(x_1^0, 0) + \frac{\partial x_2^0}{\partial x_1^0}(k, x_1^0)\frac{\partial\pi}{\partial x_2^0}(k - x_1^0, 0) = 0 \Rightarrow x_1^0 = k/2$.

Step 4. Player 1 would stay out if $k \geq \overline{k}^1$ and enter if $k < \overline{k}^1$.

Step 5. Working backward to period 0, the incumbent would install $\overline{k}^1$ to deter entry in both markets. ■

Appendix B

This proof is valid for the main result in the second and third versions of the multi-market game.

Proof. *Step 1.* Start in stage two. The objective of player 0 in the second stage is to solve the following program:

$$\begin{aligned} &\max \ \pi\left(x_1^0, x^1\right) + \pi\left(x_2^0, x^2\right) \\ &\text{s.t. } x_1^0 + x_2^0 \leq k \end{aligned}$$

If $x_1^0 + x_2^0 < k$ then $\partial\pi\left(x_1^0, x^1\right)/\partial x_1^0 = 0$ and $\partial\pi\left(x_2^0, x^2\right)/\partial x_2^0 = 0$. If $x_1^0 + x_2^0 = k$ then $\partial\pi\left(x_1^0, x^1\right)/\partial x_1^0 = \partial\pi\left(k - x_1^0, x^2\right)/\partial x_2^0$.

Step 2. In the last stage there are four subgames. First, player 1 and player 2 enter market 1 and 2, respectively. If $k > 2\overline{x}^0$ then $\partial\pi\left(x_1^0, x^1\right)/\partial x_1^0 = 0$ and $\partial\pi\left(x_2^0, x^2\right)/\partial x_2^0 = 0 \Rightarrow x_1^0 = x_2^0 = \overline{x}^0$. If $k \leq 2\overline{x}^0$ then $\partial\pi\left(x_1^0, x^1\right)/\partial x_1^0 = \partial\pi\left(k - x_1^0, x^2\right)/\partial x_2^0 \Rightarrow x_1^0 = x_2^0 = \frac{k}{2}$.

Step 3. Second, player 1 enters market 1 and player 2 stays out of market 2. If $k > \overline{x}^0 + \overline{\overline{x}}^0$ then $\partial\pi\left(x_1^0, x^1\right)/\partial x_1^0 = 0$ and $\partial\pi\left(x_2^0, 0\right)/\partial x_2^0 = 0 \Rightarrow x_1^0 = \overline{x}^0$ and $x_2^0 = \overline{\overline{x}}^0$. If $k \leq \overline{x}^0 + \overline{\overline{x}}^0$ then from (S) $\pi\left(x_1^0, x^1\right)/\partial x_1^0 = \partial\pi\left(k - x_1^0, 0\right)/\partial x_2^0 \Rightarrow x_1^0 < \frac{k}{2}$ and $x_2^0 > \frac{k}{2}$. The symmetric solution applies if player 2 enters market 2 and player 1 stays out of market 1.

Step 4. Finally, both players stay out. If $k > 2\overline{\overline{x}}^0$ then $\partial\pi\left(x_1^0, 0\right)/\partial x_1^0 = 0$, $\partial\pi\left(x_2^0, 0\right)/\partial x_2^0 = 0$ and $x_1^0 = x_2^0 = \overline{\overline{x}}^0$. If $k \leq 2\overline{\overline{x}}^0$ then $\partial\pi\left(x_1^0, 0\right)/\partial x_1^0 = \partial\pi\left(k - x_1^0, 0\right)/\partial x_2^0$ and $x_1^0 = x_2^0 = \frac{k}{2}$.

Step 5. Player 0 chooses an output $\min\left\{\frac{k}{2}, \overline{x}^0\right\}$ in a market with entry except in a subgame with entry in one market but not the other and a binding capacity restriction. In that case the optimal output will be determined by $\partial\pi\left(x_1^0, x^1\right)/\partial x_1^0 = \partial\pi\left(k - x_1^0, 0\right)/\partial x_2^0$. Player 0 wants to commit to $\widetilde{x}$ to deter entry: $\partial\pi\left(\widetilde{x}, x^1\right)/\partial x_1^0 = \partial\pi\left(k - \widetilde{x}, 0\right)/\partial x_2^0$ and $k > 2\widetilde{x}$. If (D) holds with equality, i.e. $\widetilde{x} = \overline{x}^0$, then $\partial\pi\left(\widetilde{x}, x^1\right)/\partial x_1^0 = \partial\pi\left(k - \widetilde{x}, 0\right)/\partial x_2^0$ and $k = \overline{x}^0 + \overline{\overline{x}}^0$.

Step 6. Working backward to the first stage. Denote the monopoly level x^M; implicitly given by $\partial\pi\left(x^M, 0\right)/\partial x_1^0 - r = 0$. Now, the incumbent capacity per market is larger than the monopoly level (the proposition refer to entry deterrence, not blocked entry). Hence $\frac{\partial v^0}{\partial k} = \frac{1}{2}\partial\pi\left(k/2, 0\right)/\partial x_1^0 + \frac{1}{2}\partial\pi\left(k/2, 0\right)/\partial x_1^0 - r < 0 \Rightarrow \overline{k}^2 \in \left(2\widetilde{x}, \overline{x}^0 + \overline{\overline{x}}^0\right]$ where $\overline{k}^2$ is determined by $\partial\pi\left(\widetilde{x}, x^1\right)/\partial x_1^0 = \partial\pi\left(\overline{k}^2 - \widetilde{x}, 0\right)/\partial x_2^0$. ■

II. Noisy Equilibrium Selection in Coordination Games[1]

1. Introduction

We study a Keynesian model, originally due to Bryant [1], with a continuum strict Nash equilibria. It has been suggested that this model capture some important features of the provision of public goods, tacit coordination in teams or the problem of coordination among input suppliers to a shared production process. Bryant's model, and variants, have been studied frequently in the literature (see for instance Cooper and John [2], Van Huyck, Battalio and Beil [10], Crawford [3], [4], Monderer and Shapley [9]).

In Bryant's game the players face two coordination problems. First, players may fail to correctly forecast which equilibrium will occur and thus regret their individual choice. Such coordination failure results in disequilibrium. Secondly, a common coordination problem arises when equilibria can be ranked with respect to Pareto-optimality. Players may give a mutual best response but nevertheless implement a Pareto-dominated equilibrium. In that case each player is satisfied with his individual choice but he regrets the level implemented by their joint actions.

The multiplicity of equilibria causes a great deal of uncertainty. Unfortunately, there is no consensus among game theorists how to characterize the uncertainty in Bryant's game. We propose a model in which players cannot choose efforts directly. Instead players choose strategies which are translated into efforts by the addition of noise terms. Introducing noise in the game has important implications for the equilibrium selection problem. We show that in the noisy variant of the original game the continuum of equilibria shrinks to a unique point. Moreover, the unique equilibrium is inefficient and inefficiency increases as the number of players becomes large. We also show that the outcome in the noisy game accurately predicts the experimental results obtained in Van Huyck, Battalio and Beil [10] (henceforth VHBB).

Our results differ from the results obtained in the traditional refinement literature. All strict equilibria survive even the strongest refinements. One reason for this difference is that traditional equilibrium analysis does not address the uncertainty

[1] co-authored with Hans Carlsson

in Bryant's coordination game.

Recently, other approaches to the problem of equilibrium selection have been suggested. The effects of strategic uncertainty is embodied in a modified version of the "general theory of equilibrium selection" proposed by Harsanyi and Selten [5]. According to Harsanyi and Selten's theory, payoff-dominance should have absolute precedence over risk-dominance in games of mutual interest. However, as we disregard payoff-dominance and solve for the risk-dominant equilibrium the theory would capture some effects of strategic uncertainty. Indeed, Bryant's game has a unique risk dominant equilibrium.

The risk dominant equilibrium is different from the noise-proof equilibrium. Unfortunately, VHBB's experiment cannot discriminate between the different theories. However, a slightly modified experimental design may give further insights to the benefits of the different concepts.

Yet another approach to the equilibrium selection problem is suggested in Crawford [4]. He combines the structure of evolutionary games with a model in which players learn from experience. A learning model is estimated (treating the belief variables as exogenous) and hence it is possible to explain the patterns observed in the Van Huyck, Battalio and Beil's experiments. Although Crawford is able to characterize the dynamics of beliefs formation, we show that a prediction of the outcome can be obtained without invoking exogenous variables.

The remainder of this paper is organized as follows. Section 2 describes a coordination game with continuous strategy spaces and introduces a noisy variant that we will study. This section contains the paper's main result (Proposition 1). In section 3 the predicted outcome is evaluated in the context of Bryant's [1] discussion on coordination failure, while section 4 is devoted to the experimental findings in VHBB [10]. The approaches to equilibrium selection that have been proposed in Harsanyi and Selten [5], Crawford [4] and Monderer and Shapley [9] are outlined in section 5. Finally, section 6 concludes the paper with a general discussion on equilibrium selection in coordination games.

2. A model with noisy coordination

Consider the following tacit coordination game Γ, which is a variant of Bryant's [1] Keynesian coordination game.

n players simultaneously choose strategies $e_1, ..., e_n$, which may be interpreted as efforts, from an interval $[0, M]$. The lowest effort $\min\{e_1, ..., e_n\}$ determines the output of a public good (or equivalently, a private good which is divided equally among the players). For a vector of real numbers $r = (r_1, ..., r_m)$ we let $\underline{r}$ denote $\min\{r_1, ..., r_n\}$.

The payoff to player i under strategy profile $e = (e_1, ..., e_n)$ may be written

$$u_i(e) = g(\underline{e}) - e_i \tag{2.1}$$

where $g : [0, M] \to \mathbb{R}$ expresses the value of the public good when utility is measured in effort units. We assume that g is concave and that there is a unique point e^1, $0 < e^1 \leq M$, such that $g'(e^1) = 1$. It is clear that all pure strategy equilibria are symmetric. Conversely, for each t in $[0, e^1]$, there exists a Nash equilibrium with $e_i = t$ for all i. Hence there is a continuum of Nash equilibria. The equilibria are ranked by the common effort with respect to Pareto efficiency so the model has the double coordination problem. The first problem is individual. Each player must choose a strategy from a continuum of equilibrium strategies. If he fails to forecast the other players' actions disequilibrium results. Each player will regret his individual choice. The second coordination problem is common. Even if all players have identical beliefs that a strategy profile will be played, and therefore able to coordinate in a Nash equilibrium, the players' joint efforts may implement an outcome which is inefficient.

The first problem will influence each agent's decision. Even the most rational agent is uncertain which equilibrium strategy other players will use. One possible way to treat this uncertainty is to assume that individuals can make errors in their actions. This may reflect the fact that a subject may not have settled down on any particular approach about how to play the game. Alternatively, individuals can be uncertain about the common production process and thus unable to translate their strategies into efforts without additional noise.

More precisely, we consider a noisy variant Γ^ε of the above model. In this variant, players cannot choose efforts directly. Instead each player i chooses a strategy s_i which is translated into effort by the addition of a noise term. Thus

$$e_i = s_i + \varepsilon \cdot X_i \tag{2.2}$$

where ε is a scale parameter and X_i is a random variable. We assume that the random variables have independent and continuous distributions with zero mean and take values on $[-1, 1]$. They need not be identical.

We let A_i^ε denote the set of pure strategies for player i which survive iterated elimination of strictly dominated strategies in Γ^ε. Moreover $A^\varepsilon = A_1^\varepsilon \times ... \times A_n^\varepsilon$ and

$$\begin{aligned} a_i^\varepsilon &\equiv \inf A_i^\varepsilon \qquad a^\varepsilon \equiv (a_1^\varepsilon, ..., a_n^\varepsilon) \\ b_i^\varepsilon &\equiv \sup A_i^\varepsilon \qquad b^\varepsilon \equiv (b_1^\varepsilon, ..., b_n^\varepsilon) \end{aligned}$$

We assume there is at most one point e^n, $0 \leq e^1 \leq M$, such that $g'(e^n) = n$. If no such point exists, we set $e^n = 0$. We can now state our main result that the rationalizable sets shrink to the point e^n for each player as the noise vanishes:

Proposition 1. *For all* i $\lim_{\varepsilon \to 0} a_i^\varepsilon = \lim_{\varepsilon \to 0} b_i^\varepsilon = e^n$.

The Proposition is proved by means of the following four Lemmas. Define $P_i^\varepsilon(s)$ as the probability that player i's realized effort will be strictly lower than any other player's effort when the strategy profile s is used in Γ^ε. Let $\underline{e}^\varepsilon(s)$ denote the expected lowest effort under s. The result of the first Lemma are more or less immediate from our assumptions about the noise. Note, in particular, that the probability that the efforts of two different players will be exactly the same is zero under any strategy profile.

Lemma 1. *For all* i *and* s *(a)* $P_i^\varepsilon(s)$ *is continuous in* s_i, *(b)* $\sum_i P_i^\varepsilon(s) = 1$ *and (c)* $\frac{\partial \underline{e}^\varepsilon(s)}{\partial s_i} = P_i^\varepsilon(s)$.

Lemma 2 follows immediately from the fact that $\partial P_i^\varepsilon(s)/\partial s_j \geq 0$ for all s and $j \neq i$.

Lemma 2. *If* $s \in A^\varepsilon$, $P_i^\varepsilon(a^\varepsilon) \leq P_i^\varepsilon(s \setminus a_i^\varepsilon)$ *and* $P_i^\varepsilon(b^\varepsilon) \geq P_i^\varepsilon(s \setminus b_i^\varepsilon)$.

Lemma 3 provides useful upper and lower bounds on $P_i^\varepsilon(a^\varepsilon)$ and $P_i^\varepsilon(b^\varepsilon)$, respectively.

Lemma 3. *For all* i *(a)* $P_i^\varepsilon(a^\varepsilon) g'(a_i^\varepsilon + \varepsilon) \leq 1$ *or* $a_i^\varepsilon = M$ *and (b)* $P_i^\varepsilon(b^\varepsilon) g'(b_i^\varepsilon - \varepsilon) \geq 1$ *or* $b_i^\varepsilon = 0$.

Proof. (a) Obviously, $a_i^\varepsilon \in A_i^\varepsilon$ so there does not exist $s_i \in S_i$ which is better against all $s_{-i} \in A_{-i}^\varepsilon$ for player i. In particular, unless $a_i^\varepsilon = M$, the net gain from an increase in a_i^ε should be nonpositive for some $s_{-i} \in A_{-i}^\varepsilon$, i.e.

$$\frac{\partial Eu(a_i^\varepsilon, s_{-i})}{\partial s_i} \leq 0 \tag{2.3}$$

where E denotes expectation. Noting that $g'(\underline{e})$ cannot be smaller than $g'(a_i^\varepsilon + \varepsilon)$ when i plays a_i^ε it is easily seen that

$$\frac{\partial \underline{e}^\varepsilon(a_i^\varepsilon, s_{-i})}{\partial s_i} g'(a_i^\varepsilon + \varepsilon) - 1 \leq \frac{\partial Eu(a_i^\varepsilon, s_{-i})}{\partial s_i} \tag{2.4}$$

Hence, combining the above inequalities with Lemma 1 (c), we get that

$$P_i^\varepsilon(a_i^\varepsilon, s_{-i}) g'(a_i^\varepsilon + \varepsilon) \leq 1 \tag{2.5}$$

for some $s_{-i} \in A_{-i}^\varepsilon$ and the result follows from Lemma 2. (b) is shown in an analogous way. ■

Lemma 4. *For all i, $\lim_{\varepsilon\to 0} a_i^\varepsilon = \alpha$ and $\lim_{\varepsilon\to 0} b_i^\varepsilon = \beta$ where α and β are independent of i.*

Proof. It suffices to show that $a_i^\varepsilon - a_j^\varepsilon \leq 2\varepsilon$ and $b_i^\varepsilon - b_j^\varepsilon \leq 2\varepsilon$ for any i and j. Assume $a_i^\varepsilon - a_j^\varepsilon > 2\varepsilon$. Then $P_i^\varepsilon(s) = 0$ for any strategy profile where $s_i = a_i^\varepsilon - \varepsilon$ and $s_{-i} = a_{-i}^\varepsilon$. Hence, $s_i = a_i^\varepsilon - \varepsilon$ is a strictly better response than any $s_i \in A_i^\varepsilon$ to some $s_{-i} \in A_{-i}^\varepsilon$, a contradiction. The second inequality is shown in a similar way. ■

We can now state the proof of our main result.

Proof. (*of the Proposition*) For any ε, let m denote the player i which is associated with the largest $P_i^\varepsilon(a^\varepsilon)$. As, by Lemma 1 (b), $P_m^\varepsilon(a^\varepsilon) \geq \frac{1}{n}$, we get $g'(a_m^\varepsilon + \varepsilon) \leq n$ or $a_m^\varepsilon = M$ from Lemma 3. Thus using Lemma 2, $\lim_{\varepsilon\to 0} g'(a_i^\varepsilon) = g'(\alpha) \leq n$ or $\alpha = M$. In a similar way one can show that $g'(\beta) \geq n$ or $\beta = 0$. Combining these two results with the obvious fact that $\alpha \leq \beta$ we obtain the Proposition. ■

3. Coordination failure

The previous section was devoted to that in a noisy model, based on Bryant's game, the players' actions converge to a profile of efforts which corresponds to a vector of mutual best responses in the original game. However, the common coordination problem is not solved. On the contrary, the common problem is emphasized. Players fail to coordinate in the original Pareto-optimal equilibrium. The unique equilibrium in $\{\Gamma^\varepsilon\}_{\varepsilon\downarrow 0}$ is determined by $g'(e^n) = n$ with $e_i = e^n$ for all i. In this equilibrium the effort is decreasing in the number of players and whenever g is strictly concave the inefficiency increases as the number of players becomes large. Thus, Bryant's [1] argument that a rational expectations model may exhibit underemployment equilibria is strengthened. If the efficient outcome can be obtained as a Nash equilibrium in Γ, one may ask why this is not an equilibrium in the noisy variant of the game?

Cooper and John [2] show that if a game exhibit (strictly) positive spillovers all equilibria are inefficient. Positive spillovers arise if an increase in one player's effort increases the payoffs of the other players. The original game, Γ, does not exhibit (strictly) positive spillovers and the most efficient outcome $e_i = e^1$ for all i can be obtained as an equilibrium. However, players may fail to coordinate in the most efficient equilibrium due to imperfect information.

In a noisy game the situation is quite different. In Γ^ε uncertainty is a source of positive spillovers. At the limit $\{\Gamma^\varepsilon\}_{\varepsilon\downarrow 0}$ the unique equilibrium is $e_i = e^n$ for all i. In a symmetric strategy profile we define $P_j^\varepsilon(s)$ as the probability that player j's realized effort will be strictly lower than any other player's effort in Γ^ε. The partial derivative of player i's payoff function with respect to an increase in player

j's strategy is $\frac{\partial u_i}{\partial e_j}(s) = P_j^{\varepsilon}(s) \cdot g'(\underline{e}) > 0$ for some player j. Thus Γ^{ε} exhibit positive spillovers and the implemented equilibrium is inefficient.

Our result has an interesting connection to the literature on moral hazard in teams. Holmström [6] studies a model in which agents' action determines a joint outcome which must be allocated among the agents. It is shown that if the joint outcome is *differentiable* in players actions the efficient outcome will not be a Nash equilibrium. Holmström argues that as long as there are externalities present (budget-balancing) one cannot achieve an efficient outcome. Holmström shows that the principal can introduce penalties that are sufficient to police all agents' behaviour. In technical terms the incentive scheme will make the joint outcome non-differentiable in players' actions.

In Bryant's game there is no need for a residual claimant as long as strategies are translated into efforts without noise. Deviations from the Pareto-efficient strategy profile will be penalized. The joint outcome is non-differentiable in players actions and therefore the Pareto efficient outcome is sustainable as a Nash equilibrium. However, if there is uncertainty who was at fault, some player has incentive to free-ride on the other players' efforts. In technical terms the uncertainty in the noisy model makes the joint outcome differentiable in players actions and, therefore, Holmström's result applies.

4. Experimental evidence

In this section we summarize and discuss VHBB's [10] experiment. In the experiments, VHBB study coordination games with discrete strategy sets $\{1, ..., 7\}$ and linear payoff functions

$$u_i(e) = a\underline{e} - be_i \tag{4.1}$$

where $a > b \geq 0$. Groups of 2 to 16 subjects played series of one-stage simultaneous move games. No communication was allowed before or during play. VHBB's experiments minimum experiments comprise essentially three different games, called A, B and C. Games A and B used groups of 14-16 subjects. Game C used small groups of 2 subjects, randomly selected from the entire set of subjects. In game A and C payoff parameters were set at $a = 0.20$ and $b = 0.10$. In game B parameter $b = 0$, thus making $e_i = 7$ a weakly dominant strategy. After each period game, the minimum action was publicly announced. The treatments in VHBB [10] are summarized in table 1.

It is easy to verify that any symmetric profile is a Nash equilibrium and, if $b > 0$, a strict one. Using a simple renormalization and applying Proposition 1 to

Treatment	Payoff function	No. of subjects
A	$0.20 \cdot \underline{e} - 0.10 \cdot e_i$	14-16
B	$0.20 \cdot \underline{e}$	14-16
C	$0.20 \cdot \underline{e} - 0.10 \cdot e_i$	2

Table 1: VHBB (1990) experimental treatments

continuous versions with strategy sets $[1, 7]$, of these games, one sees that $(1, ..., 1)$ is the unique noise-proof equilibrium if $a < bn$ while $(7, ..., 7)$ is the unique noise-proof equilibrium if $a > bn$. If $a = bn$, any symmetric pure strategy profile is a noise-proof equilibrium.

Hence, $(1, ..., 1)$ is the unique noise-proof equilibrium in (the continuous version of) game A, $(7, ..., 7)$ is the unique noise-proof equilibrium in game B, while any symmetric pure strategy profile is a noise-proof equilibrium in Game C.[2]

In game A subjects' initial efforts were widely dispersed and then approached the lowest effort $e_i = 1$. By period ten 72 percent adopt the minimum effort, and the minimum in all seven experiments was 1. VHBB also conducted a slightly modified version of game A. In treatment A' game A was played, and after each period the distribution of actions was revealed. In treatment A' the convergence of efforts to the lowest effort $e_i = 1$ was even more rapid than in treatment A. By period five 84 percent adopt the minimum effort. In game B the initial efforts were not as widely dispersed as in game A. In game B the payoff dominant effort $e_i = 7$ was chosen by 84 percent of the subjects and 96 percent reached the maximum effort by the 5th period. Finally, in game C subjects' efforts varied substantially in all periods. 23 percent of the initial efforts were $e_i = 1$ and 37 percent were $e_i = 7$. Subjects' choices drifted but with no discernible trend.[3]

In the large group experiments subjects' efforts mainly converged to a level predicted by the unique noise proof equilibrium. In game A uncertainty vanishes slowly, and mutual best-response outcomes are rare. The results from the small

[2]Harsanyi and Selten's (1988) theory of equilibrium selection gives precedence to payoff dominance, and therefore predicts $e_i = 7$ in all treatments. This predicted play is far from the observed behaviour in VHBB's experiments. The theory without payoff dominance would predict a common effort $e_i = 1$ in game A, $e_i = 7$ in game B (elimination of weakly dominated strategies), and $e_i = 4$ in game C (applying a uniform prior over undominated strategies).

[3]Further experimental findings can be found in Isaac *et al* (1989). In one experiment, Isaac *et al* study a minimum game as in VHBB with $n = 4$, payoff parameters set at $a = 1.2$ and $b = 1.0$, and discrete strategy sets $\{0, 62\}$. The game has two strict Nash equilibria. The unique noise proof equilibrium (in a continuous version) is $(0, ..., 0)$. Initial efforts were widely dispersed and subjects' efforts converged rapidly to the level predicted by the noise proof equilibrium. By period ten 79 percent of the efforts were $e_i = 0$.

group experiment are very different from the large group treatments. In game C the experimental behaviour do not discriminate between equilibria. Thus, VHBB's experimental results from game C does not contradict the prediction by the noise proof equilibrium. In VHBB the wide dispersion of initial efforts and different paths of convergence suggest that uncertainty has important implications for equilibrium selection. Moreover, the level of efforts is decreasing in the number of players as predicted by the noise proof equilibrium.

The large strategy space in the VHBB's experiments allows for rich dynamics. However, with linear payoff functions as in VHBB the marginal incentives and the effects of strategic uncertainty at the margin are identical in every equilibrium point in the interior of the continuum of equilibria. Accordingly, the suggested equilibrium selection concepts all predict boundary solutions and the dynamics tend to converge to a point at the boundary of the set of strategies. It would also be interesting to study the behaviour of subjects when the marginal incentive varies in the interior of the strategy set. With strictly concave payoff functions it would be possible to test the interior solution predicted by the noise proof equilibrium. Moreover, VHBB's experiments cannot discriminate between the noisy model proposed in section 2 and risk dominance. In the next section we propose an experimental design which allows to compare the predictions by the different theories.

5. Alternative approaches to equilibrium selection

In section 2 it is shown that adding noise to the minimum game shrinks the continuum of equilibria to a single point. The result is surprising because all equilibria in the continuum are strict and immune against traditional refinements. In this section we discuss some principles of equilibrium selection that are based on comparisons of the relative riskiness of equilibria.

Harsanyi and Selten's [5] "general theory of equilibrium" selection discriminate between strict equilibria. In HS's theory payoff-dominance should have absolute precedence and players should have no trouble coordinating their expectations at the commonly preferred equilibrium point. In the minimum game this is a unique point with highest effort by all players, i.e. $e_i = e^1$ for all i. It turns out that this prediction is far from the play observed in the experiments conducted by VHBB [10]. More promising is a variant of the theory that eliminates the precedence they give to payoff-dominance.

The risk dominance concept selects a unique equilibrium in the minimum game.[4]

[4] Harsanyi and Selten's theory is based on finite choice sets. However, in the minimum game we can take the limit as the distance between two compared equilibrium points go to zero to obtain an approximation with continuous strategy spaces.

The definition is based on comparison between equilibrium points two by two. In a game with more than two equilibrium points the risk dominant equilibrium is a point that is not risk dominated by any other point. A hypothetical process starts from an initial situation where it is common knowledge that either of two points will be the solution.

According to Harsanyi and Selten, players reason in the following way. Player i attach subjective probability z_i to the event that all opponents choose the first equilibrium and $(1 - z_i)$ to the second. Beliefs are independent and uniformly distributed. Player i choose a best reply to his beliefs. Adaption is achieved by using the tracing procedure. In this particular game the tracing procedure is simple. The iteration comes to an end at the first iteration since the situation is symmetric and all players have a unique best reply to his prior. We assume there is at most one point $v(n) \in [0, M]$ with $g'\left(e^{v(n)}\right) = v(n)$, for $n \geq 2$. In appendix A we show how to derive the risk dominant equilibrium. Thus, we obtain a characterization of the risk dominant equilibrium in the minimum game:

$$\left(g'(e) - 1\right)^{n-1} - g'(e)^{n-2} = 0 \tag{5.1}$$

where $e^{v(n)}$ solves the equation. The risk dominant equilibrium is $e_i = e^{v(n)}$ for all i. Risk dominance and the approach proposed in section 2 yield the same result if the number of players is equal to two. However, with more than two players the selected equilibrium is different with risk dominance and our approach. More precisely, the efforts in the risk-dominant equilibrium is higher than the efforts in the noise-proof equilibrium, i.e. $e^{v(n)} > e^n$ for all $n > 2$.

The different outcomes result because the approaches differ in their assumptions on what players believe about the amount of correlation in beliefs and strategies of their opponents. Risk dominance rely on more or less ad hoc thought processes to model the players' reasoning about the game, while the approach presented in section 2 is based on a fully specified noncooperative game. The relative advantages of the different theories cannot be settled a priori.

However, the risk dominant equilibrium and the limit equilibrium selected in the noisy game are both well defined and thus further experiments can test which approach gives better predictions in Bryant's coordination game. Using a simple renormalization ($b = 1$) in a linear version of the game with efforts in $[0, M]$ the unique equilibrium is determined by the payoff parameter a and the number of players n. The strategy profile $(M, ...M)$ is the unique noise proof equilibrium if $a > n$, and $(0, .., 0)$ if the opposite holds. Correspondingly, $(M, ...M)$ is the risk dominant equilibrium if $\ln\left(a^{-1}\right) / \ln\left(1 - a^{-1}\right) + 1 > n$, and $(0, .., 0)$ if the opposite holds.

In figure 1 we illustrate the different concepts. $(M, ...M)$ is the unique noise proof equilibrium below the solid line and $(0, ..., 0)$ above. $(M, ...M)$ is the risk dominant equilibrium below the dotted line and $(0, ..., 0)$ above. At any point D between the solid and the dotted lines risk dominance and noise proofness select different equilibria. Points A and C illustrate games A and C in VHBB [10]. Thus, a modified experiment can possibly test the different theories.

Recently, Monderer and Shapley [9], pointed out that the game in VHBB [10] is a potential game.[5] It can easily be verified that the same is true for the game presented in section 2. To see this, define a potential function P:

$$P(e) = g(\underline{e}) - \sum_{j=1}^{n} e_j \tag{5.2}$$

It has been suggested that the potential maximizing equilibrium can be used to predict the outcome in potential games. This argument is partly based on the experimental findings in VHBB. Interestingly, the limit solution in our noisy model always coincide with the potential maximizing equilibrium. P is maximized at the strategy profile e, such that $g'(e) = n$, which is exactly the condition for an interior limit equilibrium in the sequence $\{\Gamma^{\varepsilon}\}_{\varepsilon\downarrow 0}$ of noisy games with vanishing noise. This may explain the success of the potential maximizing equilibrium to predict behaviour in Bryant's model. It is open to future research to investigate the relation between the potential maximizing equilibrium and noiseproof equilibrium more thoroughly.

Yet other approaches to the equilibrium selection problem departure from rational criteria to evolutionary or adaptive models. Like risk-dominance evolutionary stability responds to differences in sizes of basins of attraction. Interestingly, VHBB's [10] experimental environment satisfies the structural assumptions of evolutionary game theory. Crawford [4] combines the structure of evolutionary games with a model in which players learn from experience. He shows how the strategic uncertainty in the minimum game interacts with the learning process to determine the probability distribution of the dynamics in VHBB's experiment. By treating the belief variables as exogenous it is possible to estimate a learning model and Crawford is able to give a unified explanation to the patterns observed in VHBB. Thus, he can characterize the process of learning in coordination games. Our model however shows that the limiting outcome can be explained without invoking any exogenous variables.

[5] Γ is a potential game if it admits a potential, i.e. a function $P : Y \to \mathbb{R}$ such that for every $i \in N$ and for every e_{-i} we have $u_i(e_i, e_{-i}) - u_i(\widetilde{e}_i, e_{-i}) = P(e_i, e_{-i}) - P(\widetilde{e}_i, e_{-i})$ for every $e_i, \widetilde{e}_i \in [0, M]$.

6. Conclusions

The main lesson to be drawn from the present paper is that the problem of coordination in games with many equilibria need not be as difficult as it appears at first sight. Starting from a game with a continuum of strict equilibria we obtained a unique solution by a slight perturbation of the original model. This approach has several attractive features: In contrast to other equilibrium selection approaches it has a strictly noncooperative basis and we manage to derive our solution without invoking factors exogenous to the strategic game. Moreover the solution is derived by means of a relatively weak equilibrium concept, viz. repeated elimination of strictly dominated strategies. Thanks to this our result is likely to be applicable to rationalistic contexts as well as to equilibration mechanisms based on evolution or learning (with regard to learning, see in particular Milgrom and Roberts [8]).

The solution we derived fits the experimental evidence. The existing experiments, however, do not allow to discriminate between several competing explanation. Hence more experiments are needed for an accurate assessment of the solution's predictive power. Another interesting task for future research is to see whether the above approach can be applied to a larger class of games.

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Appendix A

In this appendix we derive the unique risk dominant equilibrium in Γ. Compare two equilibrium point $\mathbf{e} = (e, ..., e)$ and $\mathbf{e}' = (e + \Delta e, ..., e + \Delta e)$, where $\Delta e > 0$. The prior of player i is $p_i(e) = \Delta e \cdot (g(e + \Delta e) - g(e))^{-1}$ and $p_i(e') = 1 - p_i(e)$. Equilibrium $\mathbf{e}$ riskdominates $\mathbf{e}'$ if

$$\left(1 - \Delta e \cdot (g(e + \Delta e) - g(e))^{-1}\right)^{n-1} (g(e + \Delta e) - g(e)) - \Delta e \leq 0 \tag{6.1}$$

Now, compare two equilibrium points $\mathbf{e} = (e, ..., e)$ and $\mathbf{e}'' = (e - \Delta e, ..., e - \Delta e)$. The prior of player i is $p_i(e) = 1 - p_i(e'')$ and $p_i(e'') = \Delta e \cdot (g(e) - g(e - \Delta e))^{-1}$. Equilibrium $\mathbf{e}$ riskdominates $\mathbf{e}''$ if

$$\left(1 - \Delta e \cdot (g(e) - g(e - \Delta e))^{-1}\right)^{n-1} (g(e) - g(e - \Delta e)) - \Delta e \geq 0 \tag{6.2}$$

A unique risk dominant equilibrium must satisfy both conditions. We can find a solution at the limit. If both conditions are satisfied at the limit then they are also satisfied for every $\Delta e > 0$. Hence, we can use the definition of the derivative ($\Delta e \downarrow 0$) to obtain a characterization of the risk dominant equilibrium in the minimum game:

$$(g'(e) - 1)^{n-1} - g'(e)^{n-2} = 0 \tag{6.3}$$

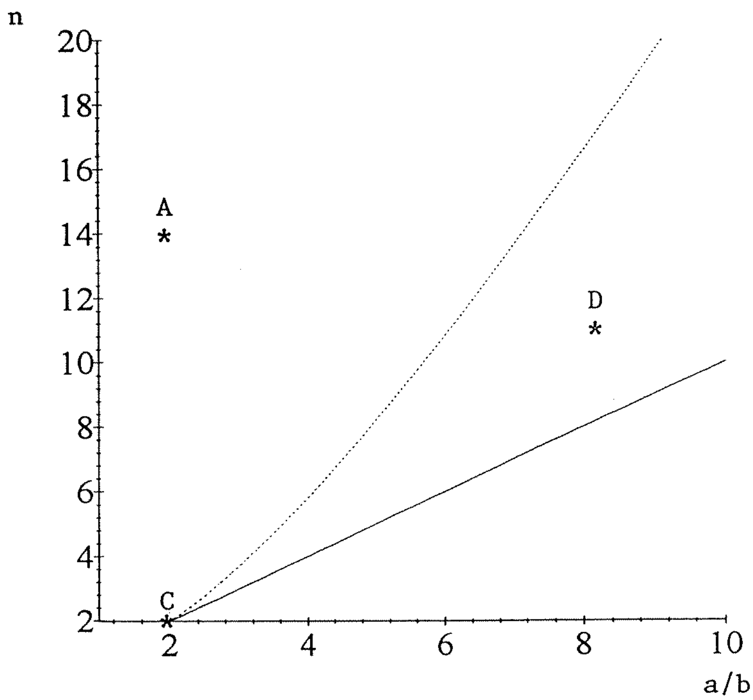

Figure 1.

III. Simplicity and Communication in Coordination Games

1. Introduction

Communication is one avenue through which players might achieve coordination of their expectations in games of mutual interest. Pre-play communication should help players to avoid coordination failures. Furthermore, transmission of information should help players to optimize their collective behaviour. We expect agents to take better decisions if they know much about the situation than if they know little. Does this mean that pre-play communication guarantees successful coordination? Moreover, does pre-play communication favour Pareto-optimal Nash equilibria in the underlying game?

Most coordination games that are studied in the game theoretic literature exhibit multiple *strict* Nash equilibria. While intuition might suggest that players should be able to coordinate on a Pareto-optimal equilibrium the traditional refinements in game theory fail to select an efficient outcome. Still worse, they even fail to select a unique outcome. All strict equilibria survive even the strongest refinements. This conflict between intuition and formal analysis has given birth to several efforts among game theorists.

The first approach allows agents to send costless pre-play signals before they choose actions. This costless pre-play communication is called *cheap talk*. Unfortunately, in terms of Nash equilibria, cheap talk does not help players to coordinate in the efficient outcome. More precisely, there exist equilibria in which players have decisions rules that are constant and therefore unaffected by the message received from the other players (cf. Weibull [21], p. 61). Hence, both problems of coordination, i.e. the problem of equilibrium selection and the problem of social inefficiency, remain unsolved.

The second approach suggests that if equilibrium arises as the result of costless negotiations between the players, then team members should be able to coordinate in a Pareto-optimal outcome. It is argued that it must not be profitable for any player to propose that a strategy combination be abandoned for another equilibrium in which everybody is better off. Only Pareto-optimal equilibria are *renegotiation-*

proof (Fudenberg and Tirole [9], p. 174f).

This criterion is controversial when applied to one-shot games. At least two critical points can be made. First, messages need not be credible in coordination games. For instance, in some games each player is better off if he can convince the other player to choose a high effort, regardless of his own intended play. As Aumann [1] argues, it is not clear that the players should expect that their opponents to believe their announcements.

A second objection is that we often observe inefficient behaviour in coordination problems (e.g. Van Huyck *et al* [15]), while renegotiation proofness suggest that players should always settle down in a Pareto-optimal equilibrium. Renegotiation proofness predict complex behaviour in situations when complex behaviour is optimal.[1] Why then are most rules of behaviour in real life simple, when the external environment, and consequently the potentially most efficient behaviour, is complex? After all, most rules of behaviour are easy to describe. In this paper we will investigate the relationship between complexity, defined as the length of the description of the behaviour, and Pareto-optimality in coordination games.[2]

It could be argued that players do neither as poorly as some cheap talk models suggest in terms of equilibrium selection, nor as well as renegotiation proofness predicts in terms of efficiency. Therefore, we suggest that some underlying assumptions must be changed. In this paper we assume that information must be transmitted in a structured form through a given channel, and that the transmission is costly. We expect the structural conditions for communication to influence the equilibrium selection problem. In order to simplify the analysis we assume that the coordination game does neither involve conflicts of interest, such as the Battle of Sexes, nor problems of trustworthiness, as in the Stag Hunt Game.

This paper focuses on a variant of the Dodo game (Binmore [2]). In the Dodo game all players have identical interests and there are no incentives to send insincere messages. In our model the game has many strict Nash equilibria, and players have asymmetric information. The informed player will know the relative Pareto ranking of all Nash equilibria before players choose actions, while the uninformed player expects all symmetric pure strategy combinations to be strict Nash equilibria with the same payoff *ex ante*. Following the literature on renegotiation proofness we assume that the uninformed player believes credible messages.

We model the pre-play communication in two steps. Before one player is informed

[1] Harsanyi and Selten's [10] "general theory of equilibrium" selection discriminate between strict equilibria. In HS's theory payoff-dominance should have absolute precedence and players should have no trouble coordinating their expectations at the commonly preferred equilibrium point. Thus, HS's theory predicts the same outcome as renegotiation proofness in games of mutual interests.

[2] Complexity is defined in a quite different way in other contributions to the literature, see for instance Holm [11].

about the ranking of equilibria both players negotiate without cost. Once the informed player has learned the Pareto-ranking of equilibria every message is costly. In this game players can use communication both to coordinate their expectations in a specific equilibrium and to optimize their collective behaviour.

We can illustrate our basic results in a simple version of the game:

G_1:

	H	L
H	2, 2	0, 0
L	0, 0	1, 1

G_2:

	H	L
H	1, 1	0, 0
L	0, 0	2, 2

In this Dual-Dodo game two players choose H or L. Player 1 selects a row and player 2 selects a column and Nature determines the state of the world (G_1 or G_2). The payoffs are given as the intersection of a row and a column, where player 1's payoff is specified first. Before either player is informed about Nature's choice they can meet and decide how to communicate after Nature has told player 1 about its choice. Assume that they decide to play (H, H) if no information is transmitted. In other words they choose the H strategy as a convention. Next, they can decide that if player 1 transmits a signal to player 2 they should both change their strategies to L, i.e. (L, L).

If Nature selects G_1 they are both happy with the convention and no information is transmitted, but what if the other state of the world occurs? It immediately follows that players are ready to give up one unit of utility each to transmit a message which would trigger L-play. If the cost is higher they will remain in the (H, H) equilibrium.

Thus, if the cost of communication is sufficiently high, players would choose the strategy described by the empty string. This equilibrium is the most simple in two ways. First of all, the empty string is the shortest description available in the language chosen by players. In that sense the equilibrium is the most easy to describe. Secondly, the equilibrium is simple because the behaviour is not conditional on the state of the world: players would choose the same action independent of Nature choice.

It is worth noting that, if communication is very costly, players do not transmit any information in equilibrium. Accordingly, they do not incur any cost of communication. In this sense players would behave as in the models of renegotiation proofness. Nevertheless, we get an equilibrium which is not Pareto-optimal in the underlying game.

2. Main features of the model

This paper studies a simple coordination game with multiple strict Nash equilibria. The multiplicity of equilibria causes strategic uncertainty.[3] Findings in experiments with similar games suggest that players' beliefs are initially widely dispersed and players get low expected payoffs in the initial periods of the treatments (cf. Van Huyck, Battalio and Beil [15]). Thus, players have strong incentives for pre-play communication. We analyse how the structure of a common language influences the equilibrium selection problem in coordination games when players are allowed to transmit messages to coordinate their behaviour in a language, i.e. a labelling procedure and a code, which is optimal for a class of coordination games.[4]

In order to model this problem we consider a set of payoff structures that vary with respect to the number of strict equilibrium points. We refer to this as the *meta-game*. While players still remain behind the veil of ignorance, i.e. none of the players know the Pareto-ranking of the Nash equilibria, they will construct an optimal language for the meta-game. The language consists of a labelling procedure and a code which is optimal with respect to the joint distribution over possible payoff structures and the channel for communication. Specifically, the alphabet contains few letters in relation to the large number of strategies (equilibria) that the players potentially wish to describe. It is assumed that the channel admits transmissions in binary code only. Consequently, only few strategies can be described with short code-strings, so the descriptions of equilibrium strategies vary in complexity.[5]

After the first stage Nature selects a specific payoff structure. One of the two players are informed about Nature's choice. We refer to the resulting choice problem as the *coordination game*.

Next, the informed player can send a message to his less informed counterpart. The informed player can use a message to convey some information about the optimal strategy. He will choose to transmit a message which results in maximized payoff for both players given the cost of transmitting the message. More precisely, the informed player will make a suggestion about what players should do in the game. This suggestion is a list specifying a strategy for each player. If the message is credible we want to assume that the uninformed player believes it. It is assumed

[3]Much recent discussion in game theory has focused on simple coordination experiments. Coordination problems have been used by game theorists to test various hypotheses on learning, equilibrium selection and strategic uncertainty (for numerous examples and references see van Huyck, Battalio and Beil [15], [16], [17]; van Huyck et al [18]; van Huyck, Battalio and Rankin [19] and van Huyck, Cook and Battalio [20]).

[4]Our approach to equilibrium selection is rationalistic. This is different from other approaches to equilibrium selection in coordination games, such as models of learning (Crawford [4], Fudenberg and Levin [8]), models of stochastic dynamics (Young [22], Kandori, Mailath and Rob [12]) or evolutionary models (Weibull [21]).

[5]This is a parallel to the fact that most binary strings are complex (Chaitin [6]).

that the message is transmitted at a unit cost per bit.

The coordination problem is modelled as a repeated binary choice problem. Both players make a binary choice in each of T periods, without observing previous choices. Any symmetric profile of actions is a strict Nash equilibrium. It is assumed that T is drawn from some probability distribution.

The set of payoff structures has two features. First, the payoff structures can be ranked with respect to the number of strict Nash equilibria. As the number of periods increases the number of strict equilibria increases. In this sense the coordination problem is harder for large number of time periods and therefore the decision problem becomes more complex. The most simple coordination problem is when players have to choose between two strict equilibria only, i.e. the one-period game. Secondly, in every game of a given length all equilibria have exactly the same expected value before uncertainty is resolved, and exactly one equilibrium which is strictly better than all other *ex post*. Both players would prefer to coordinate in the Pareto-optimal equilibrium, but they cannot resort to payoff dominance as an equilibrium selection rule because one player remains uninformed about the ranking of equilibria. More exactly, he can never be sure which equilibrium is the Pareto-optimal one until the game is over.

3. The binary choice game

Now, consider a simple coordination problem in which two players are required to choose one or the other of two actions, called a_1 and a_2. Before players choose actions Nature has decided which of the two equilibrium profiles is dominant, i.e. which strategy profile is associated with a "superior" and "inferior" outcome respectively. When Nature selects A_1, strategy profile (a_1, a_1) dominates (a_2, a_2) and when it chooses A_2 the dominance relation is reversed. In a superior equilibrium each player gets x and in an inferior equilibrium both get 1 each. If players fail to coordinate they both get 0. The two payoff matrices A_1 and A_2 are defined as follows:

$$\begin{array}{c|c|c|} & a_1 & a_2 \\ \hline a_1 & x,x & 0,0 \\ \hline a_2 & 0,0 & 1,1 \\ \hline \multicolumn{3}{c}{A_1} \end{array} \qquad \begin{array}{c|c|c|} & a_1 & a_2 \\ \hline a_1 & 1,1 & 0,0 \\ \hline a_2 & 0,0 & x,x \\ \hline \multicolumn{3}{c}{A_2} \end{array} \tag{3.1}$$

where $x > 1$. In this game both symmetric strategy profiles are strict Nash equilibria.

The binary choice game is a meta-game with asymmetric information. The players face the coordination problem described above T times. The rules of the meta-game are defined as follows.

First, players meet to construct a code in an alphabet $\mathcal{A}$ before the number of periods in the game is determined and a specific payoff structure is chosen. Before Nature's choice both players know the joint distribution over time periods and payoff structures. It is assumed that players will choose an optimal code for the whole class of payoff structures.

Second, the number of periods in the game is drawn, i.e. $T \in \Omega$, where $\Omega = \left\{1, 2, ..., \widehat{T}\right\}$. It is assumed that there is a probability function $\pi : \Omega \to (0,1)$, such that $\sum_{T \in \Omega} \pi(T) = 1$ and $\pi(T) > 0$ for all $T \in \Omega$. Next, Nature selects a sequence of matrices which determines payoffs in every period. Let $A^t \in \{A_1, A_2\}$ be the payoff matrix in period t and $A = (A^1, ..., A^T)$ be the sequence of matrices which defines the payoff structure. There is a probability function $p_T : \times_{t \in T} \{A_1, A_2\} \to (0,1)$, such that $p_T(A) = 2^{-T}$ for all $A \in \times_{t \in T} \{A_1, A_2\}$. At the end of the second stage player 1 is informed about A and T without noise.

Third, the informed agent, i.e. player 1, can send a message m coded in alphabet $\mathcal{A}$ which is received by player 2. We assume that player 1 can transmit a message to player 2 through a channel which admits transmissions in binary code only. For this purpose fix an alphabet $\mathcal{A} = \{0, 1\}$. Let $\mathcal{A}^*(c)$ denote the set of all strings $z = z_1 z_2 z_3 ... z_c$ of length c with elements $z_k \in \mathcal{A}$. Define the union of all strings $\mathcal{A}^* = \bigcup_{c \geq 1} \mathcal{A}^*(c)$. The complexity of a message m coded in $\mathcal{A}$ is defined as the length, c, of the string.[6] The cost of transmission is w per bit.

The message which is transmitted from the informed to the uninformed player is a suggestion about what players should do in the coordination game. A suggestion is a list specifying a strategy for each player. The suggestion is *consistent* if the strategy profile is a mutual best response (see Farrell [7]). We assume that player 1 will only make consistent suggestions. As any strict equilibrium is a symmetric pure strategy profile, this assumption implies that a consistent suggestion can be reduced to a description of a single pure strategy.

Fourth, players choose strategies. A strategy of player i is an ordered string of actions, written $s_i \in S_i$, where $S_i = \times_{t \in \widehat{T}} \{a_1, a_2\}$. We assume that in a T period game the sequence of actions is truncated after the T:th element. Finally, the game is played T periods and players receive payoffs. The payoff is the average period revenue minus the cost of communication. There is no observation of actions or payoffs until the game ends.

[6]The definition of complexity as the length of the description is inspired by algorithmic information theory (cf. Calude [3]).

4. Labels

In contrast to classical game theory, this paper contributes to a strand in the literature which was initiated by Gauthier [5]. A player chooses an option under a description. A choice problem exists only if the player conceives a set of distinct alternative options. Thus, we can consider the coordination problem to be defined by the agent's description of the game. Each player i knows which player she is. It is also assumed that all players make a mutual distinction between the one-period actions a_1 and a_2 before the game starts.

We shall use the term label for the description by which players recognise pure strategies.[7] A labelling is a function L_i which assigns a label $L_i(s_{ig})$ to each strategy $s_{ig} \in S_i$ of each player i, such that for each player, each of her pure strategies has a distinct label. We assume that the players construct a *common language*, i.e. $L(s) \equiv L_1(s) = L_2(s)$ for all pure strategies $s \in S_i$.

In order to model the language selection problem it is assumed that players both players are allowed to meet and construct a labelling and subsequently a code in the binary alphabet $\mathcal{A}$ before uncertainty is resolved. The idea is to set up a procedure so that players will choose an appropriate language for the entire class of payoff structures. To do this we will assume that players are situated behind a veil of ignorance. It is assumed that players know distributions, but not the particular choice by Nature.

Now, players can proceed in the following manner. They attach one label to each action in the games in one period, call them y_1 and y_2. Moreover, the players will associate the two labels with two pure strategies in every game in $\widehat{T}$ periods. Next players will choose labels for the sequences of actions in the two-period games which remain to be named, call them y_3 and y_4. These labels are also associated with two pure strategies in every game in more than two periods. Next players will choose four labels in the games in three periods, for sequences of actions which remain unnamed, call them $y_5, ..., y_8$. Continue in this way to name 2^{k-1} sequences of actions in the k-period games and let these labels be associated with strategies in every game in $\widehat{T}$ periods. Denote the set of all labels with $Y = \{y_1, y_2, ...\}$.

The procedure described above leaves many questions unresolved. The procedure only implies that when k is small a label y_k is used in a wider range of games. For instance y_1 and y_2 must be attached to the actions in the one-period game but it is arbitrary which strategies these labels are used for in games in more than one period. However, we can apply one more assumption to provide more structure to the code.

[7] cf. Sugden [14]

To both players a T-period game can be decomposed in $\lceil T/k \rceil$ games in k periods, where the last game is possibly truncated.[8] Consider a strategy called $\widetilde{y}$ in a k-period game. This strategy is a sequence of actions. Repeat this sequence $\lceil T/k \rceil$ times. This results in a sequence of actions which is a strategy in the T-period game. The resulting sequence is labelled $\widetilde{y}$ in the T-period game. We refer to this as *invariance with respect to decomposition.*

This assumption implies that our labelling procedure is highly structured. Indeed there is a unique labelling, up to symmetric transformations, which satisfies this condition. For instance, the one-period game labels y_1 and y_2 would describe uniform sequences of actions in any T-period game. Any T-period game can be decomposed in T one-period games with a uniform action labelled y_1 or y_2. Denote repetition with $*$. Labels y_1 or y_2 refer to $(a_1)*$ and $(a_2)*$. Correspondingly, labels y_3 or y_4 refer to $(a_1, a_2)*$ and $(a_2, a_1)*$. We can proceed to construct this labelling in the same manner for y_5, y_6 etc.

5. Optimal coding

Now, we can proceed to the problem of coding. A *code* is a function $\varphi : Y \rightarrow \mathcal{A}^*$ and the elements of $\varphi(Y)$ are called code-strings.

The players' goal is to find a code which maximizes the expected payoff . Now, introduce the function $bin : \mathbb{N} \rightarrow \mathcal{A}^*$, where bin is a binary expansion of $n \geq 0$, such that $(n)_2 = 1bin(n)$. By definition $bin(1) = \lambda$. To simplify notation let $\log k \equiv \lfloor \log_2(k) \rfloor$, where $\lfloor \cdot \rfloor$ denotes the "floor" of the real (rounding downwards). Define $\log 0 \equiv 0$.

We consider two situations. In the first case the empty string can be used as a message, in the second case it can not be used as a message. We define the following condition:

(C) λ is a code-string.

where λ is defined as the empty string. Condition C is satisfied in the first case and violated in the second. For the first case we obtain the following result:

Proposition 1. *If condition C applies then $\varphi(y_k) = bin(k)$ for $k = 1, 2, ...$ is an optimal code.*

The probability distribution over sequences of payoff matrices is uniform. Therefore, all pure equilibrium strategies are identical with respect to expected revenue. We will make use of the following simplifying lemma.

[8] $\lceil \alpha \rceil$ denotes the "ceiling" of the real α, (i.e. rounding upwards).

Lemma 1. *The expected revenue in any equilibrium in pure strategies is* $\frac{1}{2}(x+1)$ *before Nature has selected* A.

Proof. The expected revenue in any equilibrium before Nature has selected A is $E[u] = \frac{1}{T}2^{-T}\sum_{k=0}^{T}\binom{T}{k}((T-k)x+k) = \frac{1}{2}(x+1)$. ■

We can now provide the proof of the main result.

Proof. *Step 1.* There are 2^n unique code-strings of length n in φ and in $\mathcal{A}^* \bigcup \{\lambda\}$ for all $n \geq 0$. Thus, φ uses all strings in $\mathcal{A}^* \bigcup \{\lambda\}$. *Step 2.* The length of a code-string $\varphi(y_k)$ is $|\varphi(y_k)| = |bin(k)| = \log k$, which is increasing in k. *Step 3.* Using the lemma the expected value of the sequence of equilibria generated by the labels (y_k, y_k) for $T \geq 1 + \log(k-1)$ is $\sum_{t=1+\log k,\ldots,\widehat{T}}\left[\pi(t)\frac{1}{2}(x+1) - \pi(t)\cdot w\cdot|\varphi(y_k)|\right]$. The expected revenue (the first part in the squared brackets) is independent of the code. Therefore we can reduce the problem of finding an optimal code to a minimization problem of the expected cost. The optimal code must solve:

$$\min_{\varphi} w \sum_{k=1}^{2^{\widehat{T}}} \left(\sum_{t=1+\log k}^{\widehat{T}} [\pi(t)\cdot|\varphi(y_k)|] \right)$$

From *step 1* it follows that all code-strings in $\mathcal{A}^* \bigcup \{\lambda\}$ are used. Therefore, the assumption that $\pi(t) > 0$ for all t implies that short code-strings must be used for small k, i.e. $|\varphi(y_k)|$ must increase monotonically in k, which is shown in step 2. ■

As condition C is met players decide to associate the empty string with a strategy in the one-period game and therefore in any game in more periods. If the players wish to play this strategy they do not need to transmit any information through the channel. It is as if they had chosen a convention in the game. Under the assumption of invariance with respect to decomposition that means that the players had decided to define a uniform sequence of actions, $(a_1)*$ or $(a_2)*$, as the convention.

For the second case when condition C is not applied we obtain a similar result:

Proposition 2. *If condition C does not apply then* $\varphi'(y_k) = bin(k+1)$ *for* $k = 1, 2, \ldots$ *is an optimal code.*

Proof. *Step 1.* There are 2^n unique code-strings of length n in φ and in $\mathcal{A}^*$. Thus, φ uses all strings in $\mathcal{A}^*$. *Step 2.* The length of a code-string $\varphi'(y_k)$ is $|\varphi'(y_k)| = |bin(k+1)| = \log(k+1)$, which is increasing in k. *Step 3.* Using the lemma we can see that the expected value of the sequence of equilibria generated by (y_k, y_k) for $T \geq 1 + \log(k-1)$ is $\sum_{t=1+\log k,\ldots,\widehat{T}}\left[\pi(t)\frac{1}{2}(x+1) - \pi(t)\cdot w\cdot|\varphi'(y_k)|\right]$. The expected revenue (the first part in the squared brackets) is independent of the code.

Therefore we can reduce the problem of finding an optimal code to a minimization problem of the expected cost. The optimal code must solve:

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From step 1 it follows that all code-strings in $\mathcal{A}^*$ are used. Therefore, the assumption that $\pi(t) > 0$ for all t implies that short code-strings must be used for small k, i.e. $|\varphi'(y_k)|$ must increase monotonically in k, which is shown in step 2. ■

In the second case we obtain a symmetric code. Both strategies in the one-period game are associated with one-bit code strings. This situation is reasonable if player 2 is genuinely uninformed. For instance, we can think of a situation when both players know the rules of the game but the uninformed player does not know at which point in time the game will occur. In that case the first bit of the message has a very high coordination value.

The results in proposition 1 and 2 are not surprising. Players will use all strings of length zero before they use code-strings of length one, and strings of length one before they use code-strings of length two, and all strings of length two before strings of length three etc. Put in other words: they will attach a label to each node in a binary tree. Thus, the problem of finding a code is reduced to the problem of associating code-strings of a given length with some particular labels. To minimize the expected average length it suffices to attach the most likely labels with the shortest strings. The two codes φ and φ' are two such examples.

6. Simplicity and efficiency

Player 1 can send a message m coded in an alphabet $\mathcal{A}$ to player 2. After communication, player 1 and 2 each chooses a strategy s_1 and s_2, respectively. Players choose strategies simultaneously and for all periods. No observations are done until the game ends. Finally, players receive payoffs determined by A and strategies s_1 and s_2. The cost of transmission is w per bit.

If the efficiency gains are small and communication is costly it is always profitable to coordinate in a Nash equilibrium with the shortest description. Now, we can present the following result:

Proposition 3. *Assume condition C applies. If $x - 1 < w$ then player 1 would choose to transmit λ as a message to the uninformed player. None of the players would incur any cost of communication.*

Proof. (i) The minimum payoff of the least complex message is $\underline{u} = 1$. It can easily be seen that the maximum payoff transmitting further steps of a more complex message is $\overline{u} = x - w$. Now, $x - w < 1$ if $x - 1 < w$ ∎

Secondly, we can proceed to the case in which the empty string cannot be used as a message. A similar result holds if condition C does not apply:

Proposition 4. *Assume condition C does not apply. If $x - 1 < 2w$ then player 1 would choose to transmit a 1 bit code-string to the uninformed player.*

Proof. (i) The minimum payoff of the least complex message is $\underline{u} = \frac{1}{2}(x+1) - w$. It can easily be seen that the maximum payoff transmitting further steps of a more complex message is $\overline{u} = x - 2w$. Now, $\frac{1}{2}(x+1) - w > x - 2w$ if $x - 1 < 2w$. ∎

It is worth noting that the value of communication is high in both cases. If players were choosing an equilibrium strategy at random the expected payoff would be $\frac{1}{4}(x+1)$, which is clearly lower than the expected payoff in the first case and lower than the expected payoff in the second case if $w < \frac{1}{4}(x+1)$. Second, as we restrict our attention to a labelling that satisfy the principle of invariance with respect to decomposition the Nash equilibrium with the shortest description is a strategy profile with a sequence of actions with the most regular pattern. In that case we can expect players to choose the same action in every period if the cost of transmitting information is high.

Corollary 1. *As the labelling satisfy invariance with respect to decomposition the expected equilibrium strategy is a sequence of actions which is uniform, i.e. $(a_1)*$ or $(a_2)*$, if (i) $x - 1 < w$ and condition C applies, or, (ii) $x - 1 < 2w$ and condition C does not apply.*

The results in proposition 3 and 4 suggest that high costs of communication and small differences between the revenues in different equilibria gives both players incentives to keep the transmission of information at a minimum level. Both results are rather extreme in the following sense: players would not transmit more than the minimum number of bits even if that would result in successful coordination in the equilibrium with the highest revenue. If communication costs are high they prefer to transmit the shortest string available even if they only succeed to coordinate in the least efficient equilibrium. The reason is that the efficiency gains are outweighed by the additional cost of transmitting extra bits. In the game studied in this paper this is equivalent to choose the most regular pattern of behaviour if the labelling is invariant with respect to decomposition.

However, short descriptions and simple strategies do not exist merely at high communication costs. At lower levels of the communication cost the problem of choosing an optimal equilibrium is a trade-off between ease of describability and efficiency. This is possible to illustrate with two simple examples.

EXAMPLE 1. Consider a labelling which is invariant with respect to decomposition. Then $(a_1, a_2)*$ or $(a_2, a_1)*$ must be labelled y_3. For instance, let y_3 denote $(a_1, a_2)*$. As condition C applies the code-string for this label is one bit. Of course, for this strategy there exists exactly one state of the world for which the sequence of actions is optimal with respect to revenues. However, for every T there exist T sequences of payoff matrices in which $(a_1, a_2)*$ is almost optimal in terms of revenue, i.e. it is optimal in every period except one. For each of these sequences approximately half of the matrices are A_1 and A_2, respectively.[9] Naturally, that means a uniform sequence of actions, $(a_1)*$ or $(a_2)*$, is far from optimal with respect to revenues. If the number of periods is large, $T > 7$, then the sequences of actions with the shortest descriptions are never optimal in more than $T - 3$ periods.

Now, if the state is one of the sequences close to $(a_1, a_2)*$ the first bit transmitted from the informed to the uninformed player would increase each agents revenue with at least two times the difference between the revenues in the inferior and superior subperiod outcome at the cost of w. The second bit transmitted would only increase the revenues with half as much, but at the same cost. More precisely, an intermediate communication cost,

$$w \in (0.125x - 0.125, 0.25x - 0.25) \tag{6.1}$$

is a sufficient condition to ensure that it is optimal for the informed player to choose the equilibrium strategy with the 1-bit code-string, $\varphi(y_3)$, rather than try to coordinate in the equilibrium with the highest revenue or the equilibrium with shortest description (the empty string λ). Indeed, the optimal choice of both players is a trade-off between ease of describability and efficiency. Players approximate the perfect fit with some sequence of actions that is easy to describe in order to save communication costs.

EXAMPLE 2. The previous example was devoted to show how the players can approximate some specific strategy with a sequence of actions with short description. Now, this example will show how players choose messages at some given number of time periods as the cost of communication varies. We are interested in the expected average length of the message.

[9] There is at least $0.5(T-3)$ matrices of each kind in every sequence.

To this aim we define the average length of the code-string with respect to p_T to be the number:

$$L_{\varphi}(w) = \sum_{A \in \times \{A_1, A_2\}} \left[p_T(A) \left| \varphi(y(A, w)) \right| \right] \tag{6.2}$$

where $\varphi(y(A,w))$ is the optimal code-string in state A at cost w.

Again, consider a labelling which is invariant with respect to decomposition. Assume that condition C applies and let $T = 8$ and $x = 1.5$. We can easily solve for the two extreme cases. As the communication cost is zero, $w = 0$, the players would naturally choose to coordinate in the revenue-maximizing outcome in any state of the world. The average length of the message transmitted would be $L_{\varphi}(0) \approx 6.01$. At the other extreme, when the cost of communication is high, $w > 0.5$, the informed player would choose the empty string as a message in every state of the world. In this case the average length of the message transmitted would be $L_{\varphi}(0.5) = 0$.

To see what the average code length would be at intermediate levels of the communication cost we have conducted some numerical simulations. Let $\widetilde{n}$ denote the number of states in which players choose a different strategy than the most efficient in terms of revenues and, correspondingly, let $\widetilde{n}_1$ denote the number of states in which the players choose λ rather than the most efficient in the underlying game. The number of deviations from the revenue-maximizing strategy, i.e. $\widetilde{n}$ and $\widetilde{n}_1$, should be related to the total number of states which is 256.

The results of our simulations are reported in Table 1. The average length of the code-string transmitted decreases monotonically as the cost of communication increases. It is worth noting that players do not alter to the shortest description at some threshold, but rather change their behaviour gradually. At relatively low levels of w, players would start to play more easily described strategies. For instance, when $w = 0.04$ the probability that players would play a more easily described strategy is 0.86. Hence, players are very likely to alter from the revenue-maximizing strategy to some more easily described sequence of actions. However, the probability that they would play the strategy with the shortest description is only 0.23. In 159 of 256 states the informed player would transmit some, but not all, information about the payoff structure to the uninformed player.

In terms of communication, players would start from a situation with zero communication cost when the uninformed player learns the Pareto-optimal behaviour perfectly and then change gradually to a situation when the uninformed player remains without any knowledge about the state of the world at very high cost of transmission of information. In terms of communication costs, players would not incur any cost of transmission at zero and very high costs. At intermediate lev-

Table 1: Numerical simulation.

w	L_φ	$\widetilde{n}$	$\widetilde{n}_1$
0.00	6.012	0	0
0.01	5.902	4	4
0.02	4.484	77	16
0.03	2.492	185	31
0.04	1.629	219	60
0.05	1.105	235	86
0.06	1.105	235	86
0.07	0.637	247	111
0.08	0.637	247	111
0.09	0.613	249	113
0.10	0.559	250	120
0.20	0.184	252	208
0.30	0.035	254	246
0.40	0.004	254	254
0.50	0.000	255	255

els, however, they would use the channel for transmission of information and the expected cost of communication is strictly positive.

7. Conclusions

It is shown that simplicity can select among multiple strict Nash equilibria. Not surprisingly, choosing a message (and an equilibrium) is a trade-off between efficiency and ease of describability. Simple patterns of behaviour occur if talk is costly.

As communication is costly players will coordinate in a Nash equilibrium in which the sequences of actions have descriptions that occur in games in few as well as many periods. The equilibrium appears to be simple to the players since it is obtained with a description that occurs in a wide range of games, including the least complex coordination problems (with few strict Nash equilibria). In this way the observed equilibrium behaviour is: (i) easy to describe because the code-string attached to the strategy is short, and (ii) simple because it is a replication of a behaviour from a much less complex decision problem. We expect team-behaviour to be highly regular. In Herbert Simon's [13] words, man is not only a concept forming, but also a patternfinding animal.

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