A study of the Swedish system for personal income taxation

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1. Introduction

This paper is a presentation of an econometric study of the Swedish System for personal income taxation during the period 1952-67.

The study is part of a larger project, aiming at an empirical investigation of the effects of fiscal policy. Our method will be to cover different parts of the Swedish Economy by building Econometric Models where the fiscal policy parameters as far as possible appear explicitly. In every sub-project we should be able to study the direct effects of fiscal policy, and therefore we hope that every sub-project will be of interest in its own right, but we will at the same time look at the models as macro relations that could be integrated in a total model.

The core of the present study, on personal income taxation, is the construction of an algorithm, that from a given income-distribution before taxes and a given set-up of tax, deduction and duty parameters computes revenues from federal and local government taxes, and old-age pension fees. It also computes how the direct tax-burden is distributed between different income classes and family categories. We can consequently also compute the income distribution after taxes but, before other transfers.

Since, according to our general aim, the decision variables of the Public Authorities appear explicitly in the model, we can distinguish and compare the effects of different specified changes in the parameter set. The level and distribution of income before taxes also appear explicitly, why e.g. the effects of a uniform change in income, with given tax parameters, or in other words the built-in flexibility of the tax system, can be investigated.

By treating national taxes, local taxes and old-age pension fees, we cover 97 % of the revenues from direct taxation of the households. The most important direct taxes that we don't treat are tax on property, tax on gifts, and inheritance tax.

Table 1, below, serves to illustrate the fact that a large and increasing share of the households' total net income goes to the payment of those direct taxes that are treated by our model.

Table 1. Average level of different taxes, in percent of the households' total net income

Year	National income tax	Local income tax	Old age pension fees	Total
1952	11	9.6	0.7	21.3
1960	10	9.9	2.6	22.5
1967	12	15	2.3	29.3

Source: SOS: Taxeringsutfallet

In terms of revenues about 85 % of all national direct taxes come from personal taxation, while the rest comes from corporate taxation. On the local side the analogous figure is 90 %.

If we look at the financing of the activities of the Local and National Authorities, about half of the revenues has come from direct taxes. For the National Authorities there has, though, been a sharp decline in the revenue share of the direct taxes. This can be seen from table 2.

Table 2. The revenue-share of direct taxes for National and Local Authorities

Year	National Authorities	Local Authorities
1952	59 %	49 %
1960	44 %	48 %
1967	40 %	54 %

Source: NR (National Accounting data)

From what has been said, we can conclude that the direct taxes treated are playing an important role in the budgets of the individual households as well as in the budgets of the Authorities. This lends a special interest to a careful investigation of the relation between changes in tax parameters and changes in the revenues, and the distribution of the burden of direct taxation. If this is done with a model approach, it will be seen below that a whole range of empirical questions concerning the tax system can be answered.

The plan of this paper is to present our tax model and to some length go into one field of applications. The presentation of the model starts in section 2, by a general discussion of our type of fiscal policy models. In section 3 we give an outline of the model. In section 4 we present some tests and the predictionary power of the model. In section 5 we give a detailed account of how the model was used to investigate direct effects of automatic and discretionary tax policy. In section 6 we give a review of further applications of the model.

2. Fiscal policy models

It should be clear from the introduction, that the plan of our investigation of the effects of fiscal policy is to build, for different sectors, econometric models where the direct effects of measures of fiscal policy and different variables in the economy can be studied. But, citing Bent Hansen,[1] "To be able to say something about the effects of the measures of fiscal policy ... one has primarily to make sure which measures those really are. In a more technical vocabulary this means that one has to know which 'parameters' the Authorities can control."

From this point of view the study of the effects of fiscal policy should be a study of how the Authorities' parameters affect the economy. I.e. the fiscal policy models must explicitly include the parameters or the decision, as exogeneous variables. This was a main point raised by Hansen[1] and Tinbergen.[2]

Their works are both mainly theoretical. But Hansen's arguments for an explicit treatment of the Authorities' parameters are of course relevant also for applied econometric models. It seems rather self-explanatory that a study of the quantitative effects of a measure of fiscal policy should be

1) B. Hansen, The Economic Theory of Fiscal Policy, London 1958.

2) J. Tinbergen, On the Theory of Economic Policy, Amsterdam 1952.

a study of the effects of the parameter changes the measure comprised. To make such a study with an econometric model it seems mandatory that the changed parameters actually appear in the model.

Furthermore we will also point out that econometric models, not directly intended for an analysis of fiscal policy, could be improved by an explicit treatment of, at least, the most central public parameters. The relations that are considered as structural, and that are often used for the regression estimations[1] may namely in most cases be directly affected by a change in the public parameters. So there is an error of specification for a time-series regression if important parameters, that has been changed during the period considered, do not appear in the structural relations.

Now when the case seems to be strong for letting the public parameters appear explicitly in econometric models, it is natural to ask why models of this type are so rare. To our knowledge there are very few works of this type. Balopoulos: "Fiscal Policy Models of the British Economy"[2] is one of them that has been a point of departure for our study.

We believe that the main reason, that such an approach as a rule is not taken, even in large-scale project like i.e. the Wharton model,[3] is the very high degree of disaggregation that is needed for the suggested approach to be meaningful. The best example, here, is probably provided by the system for personal income taxation. The public parameters are here plentiful and differentiated on one hand with respect to family categories and on the other hand with respect to income levels. Consequently an introduction of the tax parameters in an econometric model calls for an introduction of the income distribution for those family categories that the tax law discriminates between.

To stretch the example further, the use of the tax parameters in regional policies would give a model that is disaggregated with respect to regions. The less general the fiscal policy is, the more difficult will it be to comprise it in an econometric model.

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1) See e.g. E. Malinvaud, Statistical Methods of Econometrics, chap. 16, Amsterdam 1966.

2) E. Balopoulos, Fiscal Policy Models of the British Economy, Amsterdam 1967

3) See e.g. Evans, Macroeconomic activity, Philadelphia 1969.

3. Outline of the model

3.1. In this section we will describe the principles of the tax model. The model consists of two parts, namely a micro part and an aggregation part. When they are described in the sequel, we have for the sake of simplicity assumed that we are interested only in the national tax. The other tax types are namely computed analogously.

3.2. The micro part is constructed to compute the tax for an individual picked at random. For an ideal model of this kind, information about the economic conditions of all individuals would be necessary. It would not then be sufficient with information about the individuals' net income. One would also for all individuals have to know all the deduction bases, i.e. all the income concepts, costs and expenditures, that constitute the bases for the different expenditures. To get such information for a range of years and then work with it is of course an impossibility. Our method, here, has been to partition the individuals into categories,[1)] such that every individual in a category is treated at least approximately equal by the tax laws. An ideal partition, i.e. one that would give exactly equal treatment within each category, would call for about one hundred different categories. By ruling out those categories that, of different reasons, contain very few individuals and by putting together those categories, between which the tax law discrimination is very slight, we ended up with only ten categories.

An individual is characterized not only by the category he belongs to, but also by his level of the total net income.[2)] This income concept is used because the data on income distribution are given in terms of total net

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1) The categories are of the type; single persons (age 17-66) without children, married men (age 17-66) and so on.

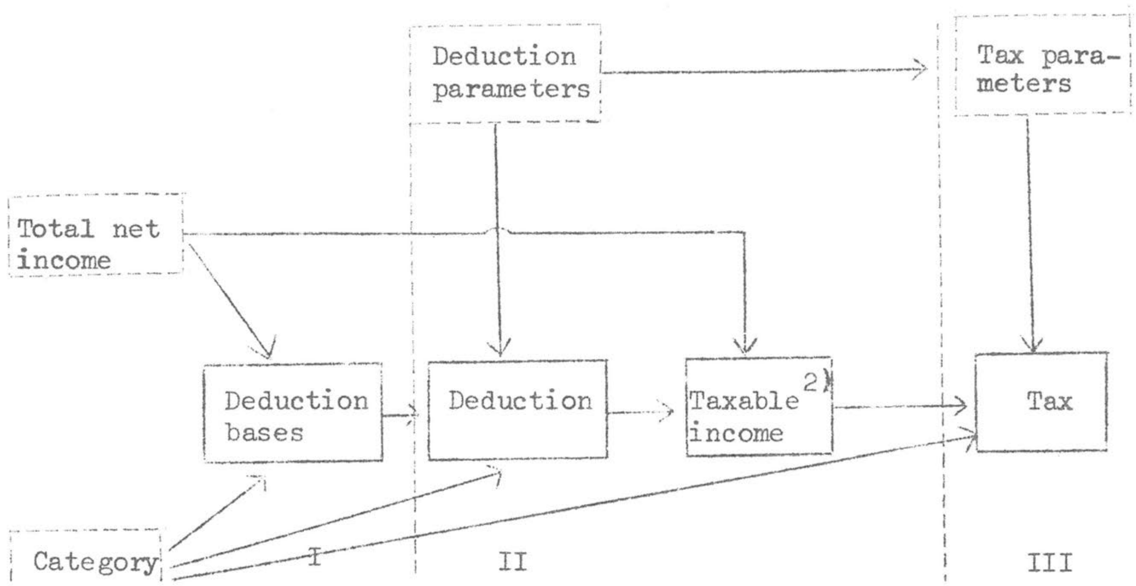
2) In Sweden the taxpayers' income and property are assessed yearly in order to determine taxable income and taxable property. The assessments are based on returns of income and property made by the taxpayers to local assessment boards. From the point of view of taxation, seven different sources of income are distinguished, viz. agricultural real estate, other real estate, trades and professions, partnership in trading company, or shipping company, wages and salaries, occasional earnings, and capital.
Total net income is the sum of income from the seven sources mentioned.
Assessed income consists of total net income less certain general deductions. Different deductions are applied to national tax and local tax, and income assessed for national tax differs in consequence from income assessed for local tax. Where the taxpayer is a private individual, taxable income is arrived at after deduction of tax-free amount from the assessed income.

income. Thus our micro model is an algorithm that for a given set-up of public parameters computes the tax for an individual and the ground of two pieces of information of him, namely:

1) the individual's level of total net income
2) the category the individual belongs to.

The construction of the micro model is illustrated by fig. 1.

Fig. 1. Chart of the micro model



3.3 As can be seen from fig. 1, the micro model is the place where the public parameters are introduced. In this sub-section we go into some details to explain how we formalized and simplified the tax laws so they could be integrated in the model as public parameters. For a start, we pick an individual at random out of category ℓ.

Let the individual's total net income[2] be represented by the stochastic variable $\xi 1_\ell$, defined over all incomes in the category. At present we are not interested in the distribution of the variable. In the same way the individual's federal tax is represented by the stochastic variable ζ_ℓ.

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1) See page 5, note 2.

2) N.b. this is a before tax concept.

Our task is to formalize the tax structure in a function F, with incomes in the category as domain and tax levels in the category as co-domain. That is an F should be specified such that

$$\xi 1_{\ell} \xrightarrow{F} \zeta_{\ell} \tag{1}$$

where the specification of F shall contain the governmental parameters in an explicit form.

The deductions for allowances and personal expense are also stochastic variables, here denoted $\eta_{k\ell}$; $k = 1, 2, \ldots, n$.

An investigation of the deduction rules that has been in force during the period 1951-67 will show that each deduction could be described as a function of a deduction base (e.g. amount of earnings, amount of insurance premiums paid), and, at most, four government parameters.

That is, for our individual, the level of the k:th deduction will be

$$\eta_{k\ell} = g(\gamma_{k\ell};\ M1_{k\ell},\ldots,M4_{k\ell}) \tag{2}$$

where the stochastic variable $\gamma_{k\ell}$ is the individual's k:th deduction base, and $M1_{k\ell},\ldots,M4_{k\ell}$ are the government parameters of the k:th deduction working on category ℓ. The form of the function g is given by the following formulae (with subscripts k, ℓ left out).

$$\begin{aligned} &M2\gamma + M3 \leq M1 && \Longrightarrow \eta = M1 \\ &M1 < M2\gamma + M3 < M4 && \Longrightarrow \eta = M2\gamma + M3 \\ &M4 \leq M2\gamma + M3 && \Longrightarrow \eta = M4 \end{aligned} \tag{3}$$

This form is general enough to cover all deductions, and it reflects the fact that most deductions are computed as some percentage (M2) of the deduction base often with an added constant (M3), mostly there also is a minimum (M1) and/or a maximum (M4) of the deduction. It should be noted that the government parameters appear explicitly in this formulation of the g function.

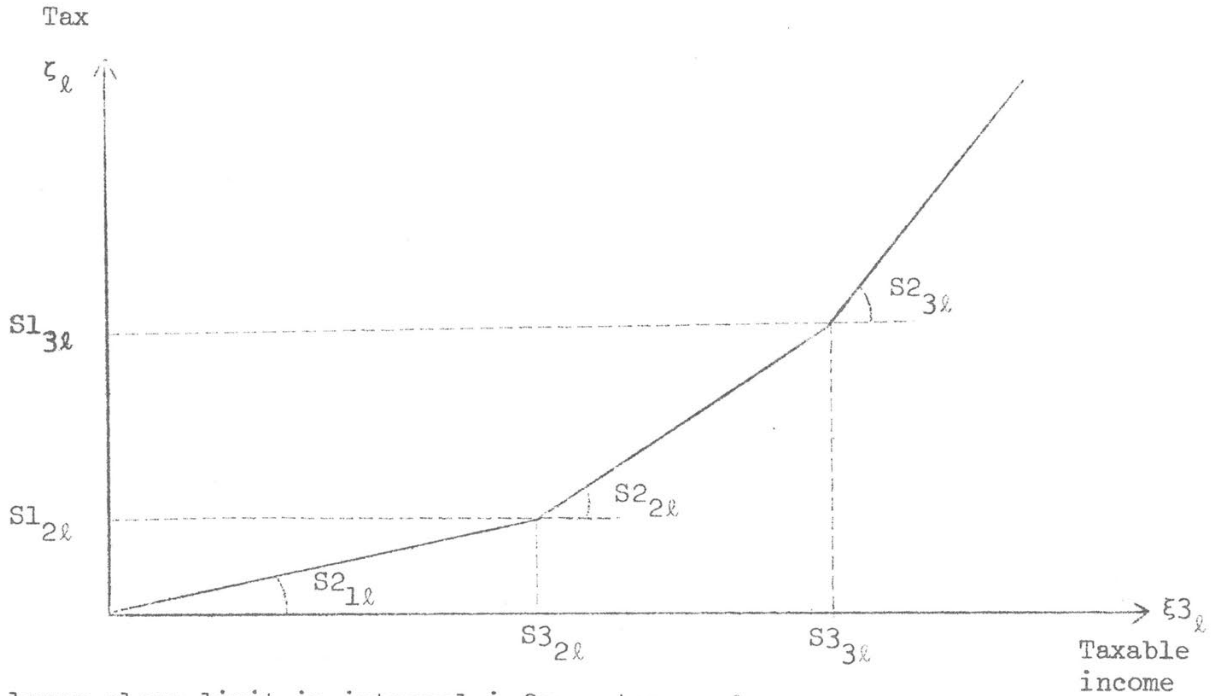
A major difficulty in connection with the deductions is that the income of an individual is not in any exact way related to the deduction bases. As can be seen from (1) and from the scheme in fig. 1, the idea of the micro model is to give the tax level as function of total net income. But there are no systematic observation on joint distributions of income before tax and exemptions bases. That implies that we on the ground of fragmentary observations and a priori assumptions have had to specify relations between the deduction bases and the income before tax, i.e.

$$\gamma_{k\ell} = H_{k\ell}(\xi 1_{\ell}) \tag{4}$$

The form of the $H_{k\ell}$ function can be very complicated as, for example, in the case of the deductions for local government taxes, where the tax base is the tax that was paid to local government last year. That is the tax laws of the last few years must be taken account of in the specification of $H_{k\ell}$.

The individuals National tax is given, in the tax laws, as a function of the taxable income, which is what is left when all deductions are made from the total net income. The tax function as it is stated in the tax laws can for an individual in a given category be illustrated by fig. 2.

Fig. 2



$S3_{i\ell}$ = lower class limit in interval i for category ℓ
$S2_{i\ell}$ = marginal tax rate in interval i for category ℓ
$S1_{i\ell}$ = tax level at lower class limit in interval i for category ℓ

The parameter $S3_{i\ell}$, $S2_{i\ell}$ and $S1_{i\ell}$, that are different for different categories are explicitly stated in the tax laws.

If the taxable income for an individual in a certain category ℓ is denoted by $\xi 3_\ell$, it should be clear from the figure that the tax (ζ) for the individual is given by the function

$$\zeta_\ell = S1_{i\ell} + S2_{i\ell}(\xi 3_\ell - S3_{i\ell}); \quad S3_{i\ell} \leq \xi 3_\ell < S3_{(i+1)\ell}; \; i=1,\ldots,m \qquad (5)$$

$$\zeta_\ell = h(\xi 3_\ell ; S_\ell) \qquad (5)'$$

where S_ℓ denotes the vector of tax parameters working on category ℓ. Now the relation between $\xi 1_\ell$ and ζ_ℓ could easily be established. Firstly $\xi 3_\ell = \zeta 1_\ell - \Sigma\eta_{k\ell}$ which relation by (2) and (4) can be written as

$$\xi 3_\ell = \xi 1_\ell - \sum_k g(H_{k\ell}; M_{k\ell}) \qquad (6)$$

If we substitute for $\xi 3_\ell$ in (5)' we get

$$\zeta_\ell = h\{\xi 1_\ell - \sum_k g[H_{k\ell}(\xi 1_\ell); M_{k\ell}]; S_\ell\} \qquad (7)$$

or

$$\zeta_\ell = F(\xi 1_\ell ; M_\ell ; S_\ell) \qquad (7)'$$

where M_ℓ is the vector of all deduction parameters and S_ℓ is the vector of all tax parameters, which is the relation we wanted. It should be clear from this short description that our concept of a public parameter is rather narrow: Exemption parameters are those entities by which the authorities specify the function from deduction base to deduction and tax parameters are the entities by which the authorities specify the functional relationship between taxable income and tax.

3.3. To get from (7)' to a macro relation between incomes and taxes we have to introduce an aggregation procedure. The one we have used relies on knowledge of the income-distributions in the different categories. The distribution function for the stochastic variable $\xi 1_\ell$ is by definition

$$\psi_{\xi 1_\ell}(x1_\ell) = P(\xi 1_\ell \leq x1_\ell).$$

The density function is the derivative of the distribution function and we represent it $\psi'_{\xi 1_\ell}$. The expected value of a function of a stochastic variable is given by

$$E[f(\xi)] = \int_{-\infty}^{+\infty} \psi'(x)f(x)dx.$$

Now the tax paid by an individual in category ℓ is a stochastic variable, ζ_ℓ, that by (7)' is a function of the same individual's total net income, $\xi 1_\ell$ a stochastic variable whose distribution is supposed to be known. The expected value of the tax paid by an individual in category ℓ is consequently given by the formula

$$E(\zeta_\ell) = E[F(\xi 1_\ell;\ M_\ell;\ S_\ell)] = \int_{I_{min}}^{I_{max}} \psi'_{\xi 1_\ell}(x1_\ell)F(x1_\ell,\ M_\ell,\ S_\ell)dx1_\ell \qquad (8)$$

As the total number of persons (N_ℓ) in category ℓ is known, the expected value of total tax payments from category ℓ is $Tot_\ell = E[\zeta_\ell]N_\ell$. The expected value of total tax payments is then the sum of expected tax payments from all categories or

$$Tot = \sum_\ell Tot_\ell;$$

The expected values have been estimated from formula (8) where the function F is known from our micro model and the density functions $\psi'_{\xi 1_\ell}$ are specified and then estimated from data on income distributions.[1)]

Our primary objective when specifying the form of the density functions to be estimated has been to get as close to the observed category distributions as possible.

A simple and from many points of view convenient method would be to apply some standard distribution as e.g. the log-normal distribution, estimate the parameters in the distribution and then let the obtained density func-

1) The main source is the series SOS: Skattetaxeringarna samt fördelningen av inkomst och förmögenhet 1952-68. Statistiska Centralbyrån, Stockholm.

tion serve as $\psi'_{\xi 1_\ell}$ in formula (8). Already a superficial investigation of our data makes it evident that the structure of the income distributions are too complicated to lend itself to a careful description by some standard distribution over the whole interval. The description we have chosen is close to the data in the respect that the relative frequences of the income brackets used in the presentation of the primary data are retained in our representation of the distribution function. We have for each income bracket applied a special two-parametric density function[1] where the parameters have been estimated under the restriction that:

$$\int_{I3_i}^{I1_i} \psi'_{\xi 1_\ell}(x1_\ell)dx1_\ell = f_{i\ell};$$

where

$f_{i\ell}$ = the observed relative frequency in income bracket i

$I1_i$ = upper limit of income bracket i

$I3_i$ = lower " " " " i

The actual estimation methods are very close to those used in a study by Kaitz & Leibenberg.[2] When $\psi'_{\xi 1_\ell}$ is exactly specified everywhere it is possible to solve the integral in formula (8). It is though too complicated to be solved analytically. This is therefore done by numerical methods in a computer.

By the aggregation procedure the model for determination of the revenues from National tax is complete. The principles for the models over the other tax types are the same as those of the national tax model. So when we know the income distribution before tax and the relevant fiscal policy parameters the sum of all revenues from direct taxes can be determined. It is intrinsic in the construction of the model that it also can be used to determine the distribution of the direct tax burden and the distribution of income after taxes.

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1) We have used a parabolic specification in the lowest bracket, a linear specification in the middle brackets and a Pareto distribution in the upper brackets.

2) Kaitz & Leibenberg, "An Income Size Distribution" in <u>Studies in Income and Wealth</u>, Vol. 13, p. 143, NBER.

4. Predictive power of the model

The construction of the model is done without any observations whatsoever of the endogeneous variables in the model. This gives us good opportunities to make tests of the model's ability to make predictions.

Now there exist observations for the years 1952-67 on the following variables that are endogeneous in the model.

G1 National tax
G4 Old age pension fees
E3 Taxable income for national tax
E4 Taxable income for local tax
E5 Assessed income for national tax
E6 Assessed income for local tax

We shall compare observations on these variables with the predictions on them that are supplied by the tax model. The comparison is done in terms of percentage deviations of predictions from observed values. These deviations are tabulated below (table 3). The table indicates that the model predictions are reliable. This impression is strengthened by the fact that on no occasion the direction of change is wrongly predicted.

5. Direct effects of budget policy on public revenue

When discussing the effects of the government budget on the performance of the economic system it seems to be of analytical value to make a distinction between effects of changes in parameters under public control and so called automatic effects. By automatic effects on the budget are usually meant such changes in the components of the budget that are generated by changes in tax duty and expenditure bases at constant parameters.

It should be clear that the model presented in this paper cannot be used for a total analysis of budget policy. This is partly because our analysis restricts to an examination of only one of the components of the budget and partly because we have not used a "complete" econometric model.

But the model can be used for a partial analysis of fiscal policy worked out through the system of individual income taxation. It is thus in the first place possible to estimate the direct effect on public revenue of changes in single parameters or in groups of parameters. In the second place we can draw some conclusions concerning the system of individual income taxation as a built-in stabilizer.

Table 3. Percentage deviations of predictions from observed values on variables G1, G4, E2, E4, E5 and E6

Year	G1	G4	E3	E4	E5	E6
1952[1)]	5.01	-3.53	6.58	2.68	-2.44	2.15
53	-2.26	-2.13	-2.42	1.76	-1.08	1.28
54	0.13	-2.55	0.18	3.24	0.16	2.33
55	-2.19	-6.07	-2.51	-3.95	-1.94	-3.99
56	1.25	-6.38	0.87	-1.72	0.48	-1.97
57	0.09	-8.73	0.40	-1.32	0.06	-1.72
58	-3.45	-15.09	-1.99	-2.12	-1.21	-1.15
59	0.51	3.96	0.67	-1.81	0.75	-0.66
1960	-2.36	2.70	-1.42	-3.46	-0.68	-1.77
61	-2.23	2.96	-1.33	4.01	-0.58	-1.78
62	-2.52	4.43	-0.87	-2.85	-0.34	-1.85
63	-1.08	4.93	0.44	-1.15	0.69	-0.55
64	-2.22	3.40	-1.63	-2.61	-1.13	-1.29
65	-0.69	4.24	-0.22	-1.59	-0.02	-1.10
66	-2.35	0.99	-1.89	-2.88	-1.07	-2.30
67	1.13	2.71	-0.15	-2.60	-0.17	-3.34
Mean	-0.83	-0.89	-0.33	-1.02	-0.53	-1.14
S.E.	2.13	5.86	2.16	2.60	0.91	1.76

1) Due to incompleteness in the data, predictions "should" be worse this year than other years.

5.1. Built-in flexibility contra built-in stability

In this section we want to state how we use some important terms in this study and in this connection we make some clarifications important to have in mind when interpreting some of the results presented later.

Let us start by regarding the following simple model

$$T = f(Y_h)$$

where T stands for personal income taxes and Y_h for total personal income.

We assume this aggregate tax function holds for a given set of tax parameters (in a wide sense) and for a given income distribution. The built-in flexibility of the tax system is then defined either as the effective marginal tax rate $\frac{dT}{dY_h}$ or as the elasticity $\frac{dT}{dY_h} \cdot \frac{Y_h}{T}$.

The great interest that has been shown for the concept of built-in flexibility[1] is due to the connection with the built-in stabilizing effect of the tax system. A well-known measure in a static context[2] of the built-in stabilizing effect of a tax system i.e. of its degree of built-in stability is the following[3]

$$\alpha = 1 - \frac{\Delta Y}{\Delta Y_a}$$

where ΔY is the actual change in national income and ΔY_a is the corresponding change under the assumption of a built-in flexibility $(\frac{dT}{dY_h})$ equal to zero. Thus α will tell us by how many percentage units the change in Y is reduced due to the existence of built-in flexibility in the system.

It is easily shown,[4] within the framework of a simple macro model that an increase in the effective marginal tax rate will, under certain realistic assumptions, lead to an increase in the value of α. Recognizing that the

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1) See for example Joseph A. Pechman, "Yield of the Individual Income Tax During a Recession", in Policies to Combat Depression (Princeton Univ.Press) 1956 , E.J. Mishan and L.A. Dicks-Mireaux, "Progressive Taxation in an Inflationary Economy", American Ec. Review, Sept. 1958, P.H. Pearse, "Automatic Stabilization of the British Taxes on Income", Review of Ec. Studies, Febr. 1962, J.O. Blackburn, "Implicit Tax Reductions", American Ec. Review, March 1967.

2) For an approach in a dynamic setting, see D.J. Smyth, "Built-in Flexibility of Taxation and Automatic Stabilization", Journal of Political Economy, Aug. 1966.

3) Introduced by R.A. Musgrave and M.H. Miller, "Built-in flexibility", American Ec. Review, March 1948.

4) See for example Pearse, op.cit.

marginal rate can be written as a product of elasticity and average effective tax rate it is also clear that built-in stability will, ceteris paribus, increase with increases in any of these concepts.

So by built-in flexibility we mean the direct effect of changes in income on tax revenue i.e. multiplier effects are not included. The concept of built-in stability on the other hand can be said to measure the total stabilizing effect on the economy of a certain degree of built-in flexibility.

5.2. Estimating the built-in flexibility: General approach and method of calculation

In our investigation of the magnitude of built-in flexibility of the individual income tax system in Sweden we have chosen to work with the following model for a specific type of income tax.[1)]

$$T_i = t_i \cdot B_i, \quad t_i = t_i(B_i), \quad B_i = B_i(Y_h)$$

where T_i stands for revenue from tax source i, B_i the corresponding tax base and t_i is defined as $t_i = \frac{T_i}{B_i}$.

The built-in flexibility of this tax can then, formulated as a derivative, be written

$$\frac{dT_i}{dY_h} = t_i \cdot B_i'(Y) + B_i \cdot t_i'(B_i) \cdot B_i'(Y_h) \tag{9}$$

From this we can see, given the tax and exemption rates and given the distribution of income, how the magnitude of built-in flexibility depends on the initial values of t_i and B_i and on changes in these.

If we write the elasticity of the tax base with respect to household income as $E_{B_iY_h} = \frac{dB_i}{dY_h} \cdot \frac{Y_h}{B_i}$ and the elasticity of t_i with respect to B_i as

$E_{t_iB_i} = \frac{dt_i}{dB_i} \cdot \frac{B_i}{t_i}$ we can derive the following interesting relation

1) C.f. R.A. Musgrave, The Theory of Public Finance, New York 1959, pp 505-510.

$$E_{t_iY_h} = E_{B_iY_h}(1 + E_{t_iB_i}) \tag{10}$$

where $E_{T_iY_h} = \frac{dT_i}{dY_h} \cdot \frac{Y_h}{T_i}$.

This formulation makes it possible to compare the effects on $E_{T_iY_h}$ of $E_{B_iY_h}$ and $E_{t_iB_i}$. Tax bases (B_i) in this paper are taxable income and assessed income.

As to the national income tax (T_1) the statutory marginal tax rate is increasing with taxable income (B_i) which will result in a $\frac{dt_1}{dB_1}$ greater than zero. It will therefore be interesting to compare the contributions to built-in flexibility that are due to deductions on the one hand and due to rising marginal rates on the other. As the local income tax (T_2) is proportional with respect to taxable income $\frac{dt_2}{dB_2}$ 1) will equal zero and thus $E_{t_2B_2}$ will equal zero too. The progressivity of this tax system is therefore entirely due to the deductions that constitute the difference between household income and taxable income.

The old age pension fee (T_3) is proportional to assessed income at national taxation (B_3)2) which in this case is taken as the base. There exists, however, an upper absolute limit to the amount that should be payed, which implies a regressive element in the old age pension fee and thus we have in this case $\frac{dt_3}{dB_3} < 0$.3)

The elasticity in total revenue from these three sources can be written as

$$E_{TY_h} = \frac{dT}{dY_h} \cdot \frac{Y_h}{T}\ , \text{ where } T = \sum_{i=1}^{3} T_i .$$

1) Taxable income (and assessed income) at local and national taxation are not identical concepts in Sweden. For the difference of magnitude, see section 4.

2) 1965 and later however the base is taxable income (at national taxation).

3) 1965 and later $\frac{dt_3}{dB_1} < 0$.

This overall concept can be formulated as a weighted average of the single elasticities. The weights will then be the share of each component of the total revenue from these sources. We thus have

$$E_{TY_h} = \sum_{i=1}^{3} \frac{T_i}{T} E_{T_iY_h}$$

from which we can isolate the contributions of the single sources to the overall elasticity.

The numerical computation of the built-in flexibility measures is accomplished by simulating small changes in the total sum of household income generated in such a way as to give each income earner a percentage change in income equal to the percentage change in total income. The implication of this is of course that we keep the structure of income distribution unchanged although the level (or scale) of the distribution has changed (the income distribution is kept "stable").

The method can be stated more precisely in the following way, using the notation of section 3.3.

$$Tot(xl_\ell) = \sum_\ell N_\ell \int_{I_{min}}^{I_{max}} \psi'_{\xi l_\ell}(xl_\ell)F(Lxl_\ell, M_\ell, S_\ell)dxl_\ell \tag{11}$$

This expression refers to total national income tax revenue when the income of all taxpayers is changed with (L-1)100 percent.

Differences in outcome in relation to the initial values are then calculated for taxes, bases, pre-tax income and the tax/base quotas (t_i). By varying L in a small interval around L = 1[1] we get a number of observations from which derivatives can be estimated by linear regression

$$\frac{dT_i}{dY_h} = \frac{cov(\Delta T_i, \Delta Y_h)}{var(\Delta Y_h)}, \quad \frac{dB_i}{dY_h} = \frac{cov(\Delta B_i, \Delta Y_h)}{var(\Delta Y_h)} \quad \text{and so on.}$$

When the derivatives are known the elasticities are easily calculated.

---

1) In these calculations L have taken 11 values, inclusive the value 1, in the interval $0.95 \leq L \leq 1.05$.

5.3. Built-in flexibility: Results

Before presenting and discussing our numerical results we give some comments on their interpretation. It is of great importance to point out that our calculations of taxes and duties are estimates of the tax liability each year.

With respect to the collection of taxes on individual income the Swedish tax law makes a distinction between A-tax and B-tax. A-tax being payed by wage and salary earners and B-tax by other taxpayers. As to the A-tax a "pay-as-you-earn" system is used which guarantees a close relationship between tax payments and tax liability. At 1967 the part of total income liable to A-tax was approximately 90 % and at 1957 the corresponding figure was approximately 85 %.

The relation between tax liability and tax payments a given year could be expected to be less close when B-tax is regarded. This tax is payed by selfemployed people and here the tax payments year t is in principle made dependent on the income earned in year t-2. If, however, the divergence between income in years t and t-2 happens to exceed a certain degree the law states an adjustment in payments which makes for a somewhat closer relationship with liability.

This institutional framework is important to have in mind when interpreting our estimates of built-in flexibility. When yearly data are used this measure is of course meant to describe how actual tax payments during a year, as distinguished from tax liabilities, react to changes in income this same year.

The following table illustrates for some years the difference between tax-repayments of the Authorities and complementary payments of the taxpayers, the difference taken in relation to tax liability separating, however, A-tax from the sum of B- and C-tax. C-tax represents income taxes payed by corporate business and is included because of the lack of data for B-tax taken for itself.

Table 4. Percentage divergence of tax payments from tax liability

Income year	A-tax	B- and C-tax
1961	-1.0	0.64
1962	2.6	3.8
1963	0.41	2.9
1964	-3.9	-0.60
1965	-4.3	0.61
1966	-1.7	2.6
1967	-0.07	6.2

The figures derived from data published in Appendix 1 of the Budget proposition 1968. (Finansplanen Bil.2, Riksrevisionsverkets inkomstberäkning.)

In our opinion the conclusion that may be drawn from this table is that the magnitude of the difference between tax payments and tax liability is only of secondary importance to our estimates of built-in flexibility.[1)]

In tables 5-8 below we present some results of our computations but first we give a list of symbols used in these tables.

T_1 = National income tax

T_2 = Local income tax

T_3 = Old age pension fee

$T = \sum_{i=1}^{3} T_i$ = Total amount of personal income tax

Y_h = Total net income (c.f. page 5 note 2)

B_1 = Taxable income at national assessment

B_2 = " " " local "

B_3 = Assessed " " national "

$t_i = \frac{T_i}{B_i}$

E_{AB} = Elasticity of A with respect to B.

1)For another recent discussion of this problem see Bent Hansen, Fiscal Policy in Seven Countries 1955-1965, OECD 1969 where the same conclusion is drawn concerning the Swedish economy. The interested reader will there find an excellent survey of the Swedish tax system.

Table 5. Aggregate average tax rates and ratios of revenues from single tax sources to the total revenue of personal income taxation

Income year	Aggregate average tax rates				Shares of total income tax		
	T_1/Y_h	T_2/Y_h	T_3/Y_h	T/Y_h	T_1/T	T_2/T	T_3/T
1953	0.088	0.098	0.0071	0.19	0.46	0.50	0.037
1957	0.096	0.10	0.016	0.21	0.45	0.47	0.074
1958	0.094	0.094	0.014	0.20	0.47	0.46	0.070
1961	0.11	0.11	0.025	0.24	0.45	0.44	0.10
1962	0.097	0.11	0.024	0.23	0.42	0.47	0.11
1965	0.12	0.13	0.021	0.27	0.45	0.48	0.077
1966	0.11	0.14	0.021	0.27	0.41	0.52	0.076
1967	0.12	0.15	0.023	0.29	0.42	0.50	0.080

Table 6. Measures of built-in flexibility

Income year	Derivatives				Elasticities			
	$\frac{dT_1}{dY_h}$	$\frac{dT_2}{dY_h}$	$\frac{dT_3}{dY_h}$	$\frac{dT}{dY_h}$	$E_{T_1Y_h}$	$E_{T_2Y_h}$	$E_{T_3Y_h}$	E_{TY_h}
1953	0.18	0.12	0.0056	0.31	2.09	1.28	0.79	1.63
1957	0.21	0.12	0.0092	0.34	2.15	1.24	0.59	1.59
1958	0.21	0.13	0.0079	0.35	2.21	1.39	0.55	1.72
1961	0.25	0.14	0.018	0.41	2.29	1.37	0.71	1.70
1962	0.23	0.15	0.015	0.39	2.37	1.41	0.64	1.73
1965	0.29	0.17	0.0089	0.47	2.35	1.30	0.42	1.71
1966	0.26	0.18	0.026	0.47	2.39	1.28	1.27	1.74
1967	0.28	0.18	0.029	0.49	2.34	1.25	1.22	1.70

Table 5 gives average aggregate tax rates for some years and expresses also the time-path of the relative importance to the public authorities of the three taxes. It can be observed that the average aggregate rate of personal income taxation (taken with respect to total net income) has increased from 19 to 29 percent over the 15 years investigated. It can also be seen that at the end of the period the local income tax represents the largest component of personal income taxation.

From table 6 we see that the built-in flexibility of the personal income tax system has increased, measured as a derivative, from 0.31 in 1953 to 0.49 in 1967. The largest contribution has during the whole period come from the national income tax. Looking at the overall elasticity we observe a striking constancy at a level of approximately 1.7 from 1958 and on. We also note the significantly higher level of the national income tax elasticity. If we take unitary elasticity as a benchmark we can make the observation that the old age pension fee was inflexible before 1966 and flexible thereafter.[1)]

The increases in the aggregate average and marginal national income tax rates (T_1/Y_h and dT_1/dY_h) that can be observed from the tables are entirely due to the automatic rate-increasing effect of increases in the income level. The adjustments of the national income tax system that have occurred during the investigation period have all been of the nature to reduce effective rates.

In table 7 are some derivatives that express the sensitivity of tax bases to changes in total net income and the sensitivity of tax/base quotas (t_i) to changes in bases. The progressivity of the national income tax, proportionality of the local income tax and regressivity of the old age pension fee with reference to their bases respectively is clearly illuminated in this table.

The results presented in table 8 make it possible, using equation (10), to compare the contributions to the built-in flexibilities of exemptions on one hand and of the statutory tax rates on the other. Regarding for example the national income tax it can be concluded that the relative importance

1) The main reason for this is the change in the base of the fee, from B_3 to B_1, that took place in 1966.

of the base-elasticity has decreased during the period. In 1967 the contributions of the base and rate elasticities seem to be of approximately equal magnitude while in 1953 the base elasticity was the dominating factor.[1)]

Table 7. The sensitivity of bases to changes in total net income and of tax/base quotas (t_i) to changes in bases.

Income year	Bases			Tax/base quotas		
	$\frac{dB_1}{dY_h}$	$\frac{dB_2}{dY_h}$	$\frac{dB_3}{dY_h}$	$\frac{dt_1}{dB_1}$	$\frac{dt_2}{dB_2}$	$\frac{dt_3}{dB_3}$
1953	0.95	0.98	0.98	$1.94 \cdot 10^{-12}$	0	$-1.04 \cdot 10^{-13}$
1961	0.97	0.96	0.98	$2.65 \cdot 10^{-12}$	0	$-2.78 \cdot 10^{-13}$
1965	0.99	0.99	0.99	$2.27 \cdot 10^{-12}$	0	$-2.70 \cdot 10^{-13}$

Table 8. Elasticities of bases with respect to total net income and of tax/base quotas (t_1) with respect to bases

Income year	Bases			Tax/base quotas		
	$E_{B_1Y_h}$	$E_{B_2Y_h}$	$E_{B_3Y_h}$	$E_{t_1B_1}$	$E_{t_2B_2}$	$E_{t_3B_3}$
1953	1.79	1.28	1.13	0.17	0	-0.30
1957	1.64	1.24	1.15	0.31	0	-0.49
1958	1.68	1.39	1.17	0.31	0	-0.53
1961	1.58	1.37	1.16	0.45	0	-0.39
1962	1.63	1.41	1.17	0.46	0	-0.45
1965	1.53	1.30	1.17	0.54	0	-0.64
1966	1.56	1.28	–*	0.53	0	-0.18**
1967	1.54	1.25	–*	0.52	0	-0.21**

* Not computed

** $E_{t_3B_1}$ c.f. page 21 note 1.

1) Deductions completely proportional to income, i.e. $E_{B_1Y_h} = 1$, would have implied a value of $E_{T_1Y_h}$ equal to 1.17 in 1953 and 1.52 in 1967, ceteris paribus. Proportionality in the statutory rates on the other hand, i.e. $E_{t_1B_1} = 0$, would have implied $E_{T_1Y_h}$ equal to 1.79 in 1953 and 1.54 in 1967, ceteris paribus.

5.4. A comparison with a crude method for estimation of the tax function

In section 2 we touched upon some weaknesses of statistical nature that often seem to be present in empirically estimated tax functions of the following type:

$$T = a + bY_h + u.$$

Here we give an idea of the consequenses of the specification error, that will be present in the above relation, when estimated from uncorrected time-series data. The difficulty with a time-series regression of this type is that the parameters a and b cannot be regarded as autonomous because changes in the structure of tax, duty and deduction rates as well as in the distribution of income will change the relation i.e. change the values of a and b.

In fig. 3 we have drawn a regression line estimated in this crude way and also illustrated the results for some years of simulation experiments[1) with our model. We have in the figure national income tax on the vertical and total net income on the horizontal axis. The broken line represents the regression equation computed from observations during 1951 to 1967. The simulated relations that are depicted refer to years immediately after important adjustments in the national income tax system i.e. 1952, 1953, 1957, 1960, 1962 and 1966.

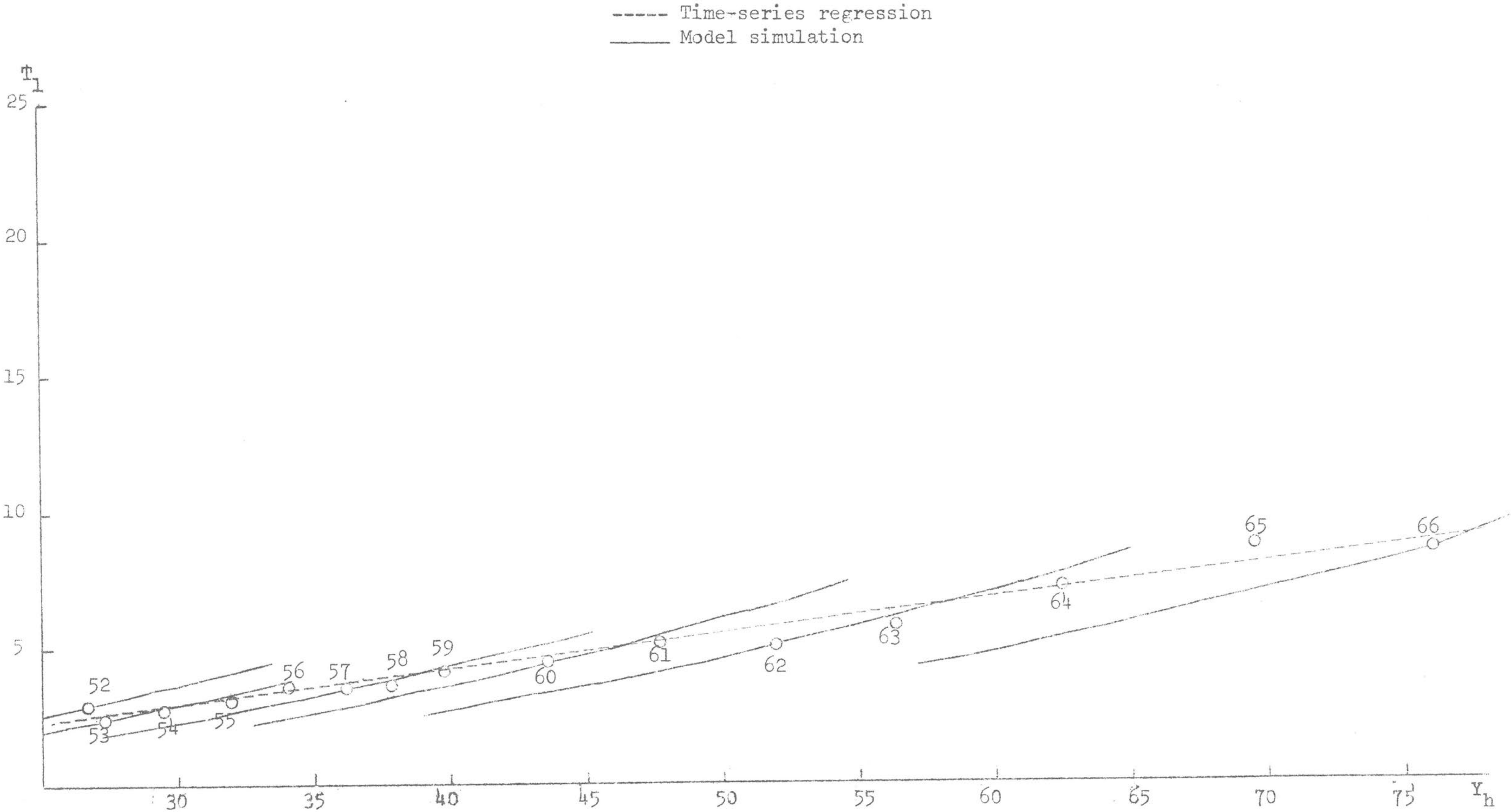
In the first place we observe from the figure that simple regression from time-series data underestimates the income effect on national income tax for all investigated years.

In the second place we see that the simulated relations are nonlinear which of course is a result of the progressivity of the system. Finally we note that the tax-income observation for a year between two revisions in the national income tax system is found in the neighbourhood of the curve representing the tax system ruling at that point of time. The most important reason why they do not lie on the curve is that the local income tax rate has risen every year, which will give the observed tendency since local income taxes are deductable at the national assessment.

---

1) In working out these simulations we varied the total income (Y_h) between the limits 0.75 Y_h and 1.25 Y_h in such a way as to give an equal relative change in income to all individuals.

Fig. 3. Results of estimation of the income effect of national income tax



Regression line: $T_1 = -1.018 + \underset{(0.004)}{0.132}\, Y_h$ $\quad R^2 = 0.9844$

5.5. Effects of discretionary changes

As the pure parameters of the public authorities appear explicitly in the model it is possible to compute the effect on tax revenue of changes in these parameters. We here describe the method used in estimating partial derivatives of revenue with respect to changes in single parameters. Our approach to this problem has been to use a method of simulation similar to the one used in the calculation of built-in flexibility. In the equation below we have varied the value of L in the interval $0.95 \leqq L \leqq 1.05$ and computed the tax revenue for each value.

$$\mathrm{Tot}(Mi_{k\ell}) = \sum_{\ell} N_{\ell} \int_{I_{min}}^{I_{max}} \psi'_{\xi 1_{\ell}}(x1_{\ell})F(x1_{\ell}, M1_{1\ell}, \ldots, LMi_{k\ell}, \ldots, M4_{n\ell}, S_{\ell})dx1_{\ell} \quad (12)$$

This gave us a set of observations of public revenue at different values of the parameter under study, ceteris paribus. Assuming a linear relation we then used ordinary least squares to estimate the value of the partial derivative in question.

In table 9 a collection of derivatives is given. Our intention has been to present derivatives which potentially and from historical experiences must be judged as particularly interesting.

It can be seen from the table that an increase in the rates of regional tax deduction for married and unmarried taxpayers of S.kr 100 will reduce the national tax liability by S.kr 111 million. On the other hand an increase in the "levy-percentage" by one percentage unit will increase national tax liability by S.kr 98.3 million.

As has been stated before the national income tax is the only component of the Swedish personal income tax system that is progressive in the sense of having a statutory marginal tax rate increasing with income. In table 10 below we reproduce for some brackets the statutory marginal tax rates of the national income tax system in 1967. The brackets are chosen so as to include pre-tax income within the range of S.kr 20,000-30,000. Between these limits we had in year 1967 853,974 tax units (288,244 taxed as unmarried) or 25 % of all units.

Table 9. The effect of selected instruments of fiscal policy (P_i) upon the aggregate tax liability in year 1967. Millions of Swedish kronor

Instrument of fiscal policy (P_i)	Unit of measurement of the parameter	$\frac{\partial T_1}{\partial P_i}$	$\frac{\partial T_2}{\partial P_i}$	$\frac{\partial T_3}{\partial P_i}$
1) Regional tax deduction[1]				
Unmarried taxpayers	S.kr	0.30	-0.26	-0.05
Married taxpayers	S.kr	-0.40	-0.33	-0.06
2) Deduction effect year t of change in local tax rate year t-1[2]	%	-129.6	-	-6.6
3) The "levy-percentage"[3]	%	98.3	-	-
4) Local income tax rate	%	-	636.6	-
5) The rate of the old age pension fee	%	-	-	323.4
6) The maximum limit of the old age pension fee	S.kr	-	-	0.33

1) These regional tax deductions were during most of the investigated years the most important deductions at national assessment. Only in 1966 and 1967 the deduction of local taxes was larger at national assessment. At the local level the regional tax deductions were always the most important. In 1967 the rates were S.kr 4,500 for married taxpayers together and S.kr 2,250 for others at national as well as local assessment. The regional differentiation of the deductions was annulled in 1962 and since 1958 the rates have been equal at national and local assessment.

2) The deduction at the national assessment for income year t of local taxes liable in income year t-1 was during 1951 to 1965 the next largest deduction at that assessment. In 1966 and 1967 certain minimum levels were statuted for this deduction which put it up as the most "expensive" deduction for the central government these years.

3) The levy-percentage (uttagsprocenten) is a scale factor in the national tax system which most of the years has been fixed at 100 %. The parliament has every year to take a decision on the level of this parameter which at its introduction was meant as a means of stabilization policies.

Within the interval we have split up taxable income in all rate brackets that have appeared during our investigation period. This approach will increase our opportunities of choice when simulating simulate the effect on revenues of different sets of tax parameters in this interval by using the partial derivatives reported in the same table. Given a specific division of the taxable income interval in rate brackets we have in each bracket only one independent statutory tax parameter to manipulate: the marginal tax rate or the average tax rate at the upper limit of the bracket. We have preferred to manipulate marginal tax rates.[1)]

Table 10. Effects on national income tax revenue of changes in statutory marginal tax rates expressed as partial derivatives. Millions of S.kr. Year 1967

Rate bracket	Taxable income Thousands of S.kr	Unmarried taxpayers		Married taxpayers*	
		Marginal tax rate %	Partial derivative	Marginal tax rate %	Partial derivative
1	10-12	27	13.4	10	25.9
2	12-14	27	8.6	15	22.2
3	14-15	27	3.7	15	10.4
4	15-16	31	3.7	15	10.1
5	16-18	31	4.4	22	18.0
6	18-20	31	3.5	22	15.3
7	20-24	36	4.0	27	24.6

* Single persons with children are taxed as married people.

This table tells us for example, that if the statutory marginal tax rate for unmarried people in bracket 5 is raised by one percentage unit, ceteris paribus, the revenue of the national income tax system will increase by S.kr 4.4 millions. If at the same time the rate for married couples in this bracket is raised by one percentage unit the taxes will increase by S.kr 22,2 millions.

Knowledge of the values of these derivatives for all brackets makes it very simple to compute the effects on public revenue of specific alternative tax schedules the year in question.

1) It should be noted that because of the continuity of the tax function a change of the marginal rate in one bracket will affect all higher brackets through changes in their average rates.

5.6. The past development of yield from personal income taxation

Until now we have been discussing partial derivatives of tax revenue with respect to certain action parameters and the income level. In order, however, to explain the actual development of income tax yield between successive years it will of course be necessary to know what parameters have actually changed and to what extent as well as how the income distribution has changed.

In this section we want to demonstrate how the model makes it possible to split up the total yearly change in the revenue of the system of personal income taxation in parts explained by certain factors. In a first round, on which we shall concentrate here, we separate the effects of the following sets of explanatory variables:

1. Change in statutory tax rates
2. " " " deduction rates
3. " " pattern of income distribution
4. " " average income[1) a) Inflation effect
   b) Change in real terms

The effects on national income tax, local income tax and old age pension fee are all computed. In a second round we have looked more in detail at the explanatory factor marked 2 above. Some results of this investigation are mentioned in passing.

In formal terms the method used can be described as follows. Once again we take as our point of departure equation (8) which refers to the national income tax. Index t indicates the year in question.

$$Tot_t = \sum_{\ell} N_{\ell t} \int_{I_{min}}^{I_{max}} \psi'_{\xi 1_{\ell t}}(x1_{\ell t}) F(x1_{\ell t}, M_{\ell t}, S_{\ell t}) dx1_{\ell t} \tag{13}$$

Suppose that our aim is to explain the actual change in national income tax revenue between years t and t+1. To get the effect of the change in statutory tax rates we simply substitute the vector $S_{\ell(t+1)}$ for $S_{\ell t}$, ceteris paribus. The effect of the change in deduction rates is given, taking again equation (8) as a starting point, by substituting instead $M_{\ell(t+1)}$ for $M_{\ell t}$, ceteris paribus.

---

1) The different development of average income for different categories of taxpayers have been taken care of.

The effect of the change in the pattern of income distribution is given by

$$\widehat{Tot}_t = \sum_{\ell} N_{\ell(t+1)} \int_{I_{min}}^{I_{max}} \psi' \xi 1_{\ell(t+1)} (x1_{\ell(t+1)}) F(\frac{x1_{\ell(t+1)}}{V_{\ell t}}, M_{\ell t}, S_{\ell t}) dx1_{\ell(t+1)} \quad (14)$$

where $V_{\ell t} = \frac{\overline{x1}_{\ell(t+1)}}{\overline{x1}_{\ell t}}$; $\overline{x1}_{\ell t}$ expresses the mean income in category ℓ, year t.

The effect of the inflatory increase in the income level can be written

$$\widetilde{Tot}_t = \sum_{\ell} N_{\ell t} \int_{I_{min}}^{I_{max}} \psi' \xi 1_{\ell t} (x1_{\ell t}) F(\frac{P_{t+1}}{P_t} x1_{\ell t}, M_{\ell t}, S_{\ell t}) dx1_{\ell t} \quad (15)$$

where P_t is an index of the absolute price level in year t. Finally we have the level effect in real terms as

$$\Sigma N_{\ell t} \int_{I_{min}}^{I_{max}} \psi' \xi 1_{\ell t} (x1_{\ell t}) F(\frac{P_t}{P_{t+1}} V_{\ell t} x1_{\ell t}, M_{\ell t}, S_{\ell t}) dx1_{\ell t} \quad (16)$$

In tables 11 and 12 we present for a couple of years the results of our explanation of the actual change in tax revenue.

Tables 11 and 12. Allocation of the changes in the yield of personal income taxation among its determinents, millions of S.kr

Table 11. 1961-62

Cause of change	Amount: National tax	Local tax	Old age pension fee	Total
1. Change in statutory tax rates	-551	82	0	-469
2. " " deduction rates	-428	-270	-4	-702
3. " " pattern of income distribution	-106	-51	-11	-168
4. " " average income				
a) inflation effect	416	277	29	722
b) change in real terms	581	390	51	1022
5. Estimated total change (1+2+3+4)	-88	428	65	405
6. Total change according to the model	-165	438	62	335
7. Residual (6-5)	-77	10	-3	-70
8. Actual change	-78	Unknown	43	Unknown

Table 12 1965-66

Cause of change	Amount			
	National tax	Local tax	Old age pension fee	Total
1. Change in statutory tax rates	-811	545	78	-188
2. " " deduction rates	-765	-89	-90	-944
3. " " pattern of income distribution	52	86	8	146
4. " " average income				
a) inflation effect	1159	796	32	1987
b) change in real terms	368	260	15	643
5. Estimated total change (1+2+3+4)	3	1598	43	1644
6. Total change according to the model	-41	1657	112	1728
7. Residual (6-5)	-44	59	69	84
8. Actual change	-33	Unknown	156	Unknown

As can be seen from the tables we had in 1962 and 1966 important revisions of the statutory tax rates in the national income tax system. These revisions were both years complemented by heavy increases in some deduction rates. The most important measures in 1962 were changes in the structure and increases in the rates of regional tax deductions (c.f. table 9, note 1)[1]. As to 1965/66 the large effect of changes in deduction rates at national taxation was almost entirely due to the introduction of a minimum rate at the deduction of local income taxes[2], c.f. the relatively modest effect at local taxation where this deduction is not allowed.

The aim with these adjustments was partly, for reasons of equity and incentive to compensate for the strong automatic tax-increasing effects that are built into the system and that had given rise to sharp increases in the effective national income tax rates. The year 1962 can be characterized as pressed by a tendency to recession and in 1966 the slack was quite obvious,[3] so the timing of the measures fit well into a pattern of countercyclical policy.[4]

1) These measures counted for S.kr 346 million (S.kr 220 million) of the total deduction effect at national (local) taxation.

2) We have estimated the effect of this single measure to S.kr -654 million.

3) See W. Heller et al,, Fiscal Policy for a Balanced Economy, OECD 1968.

4) The loss of revenue to the public was, however, in both cases to a certain degree compensated by increases in the rates of indirect taxation.

For every year in the sixties we have had increases in the average[1] statutory rate of local taxation. In 1966 this increase was particularly high (from 17.25 % in 1965 to 18.29 % in 1966) the effect of which can be seen from table 12 (c.f. for a comparison of the order of magnitude the relevant derivative in table 9).

It should be mentioned there that row 2 of the tables are not only including genuine effects of changes in deduction parameters since, according to our method, if the statutory local tax rate is changed between years t and t-1 this will show up as an effect of a change in deduction rules for national taxation as well as for the old age pension fee.

Between 1966 and 1967 the effects of changes in the deduction rules were according to our computations S.kr 183 million. Inspection of table 9 indicates that something like S.kr 100 million of that change should be referred to as a deduction effect of the change in the local income tax rate of approximately one percentage unit between 1965 and 1966.

Finally, it is clear from the tables that the most important single determinant of changes in revenue is variations in the income level aspect of income distribution. This factor counted in 1965/66 for no less than 67 % of the gross change in total revenue and in 1961/62 for 56 %.

6. Further applications

6.1. In this section we will give a very brief description of how we have used the model in some other fields of application.

6.2. By using only the micro part of the model we can study how the tax burden varies with income in the different categories. We have 1952-67 for each category computed the tax levy, in percent, as a function of income before tax. Fig. 4 can serve as an illustration. We can there see how the tax burden for a certain category has varied with income, different year.

To illuminate the connection between inflation and real tax burden a corresponding diagram can be made with deflated income on the horizontal axis. This is made in fig. 5.

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1) Computed as a weighted mean over regions.

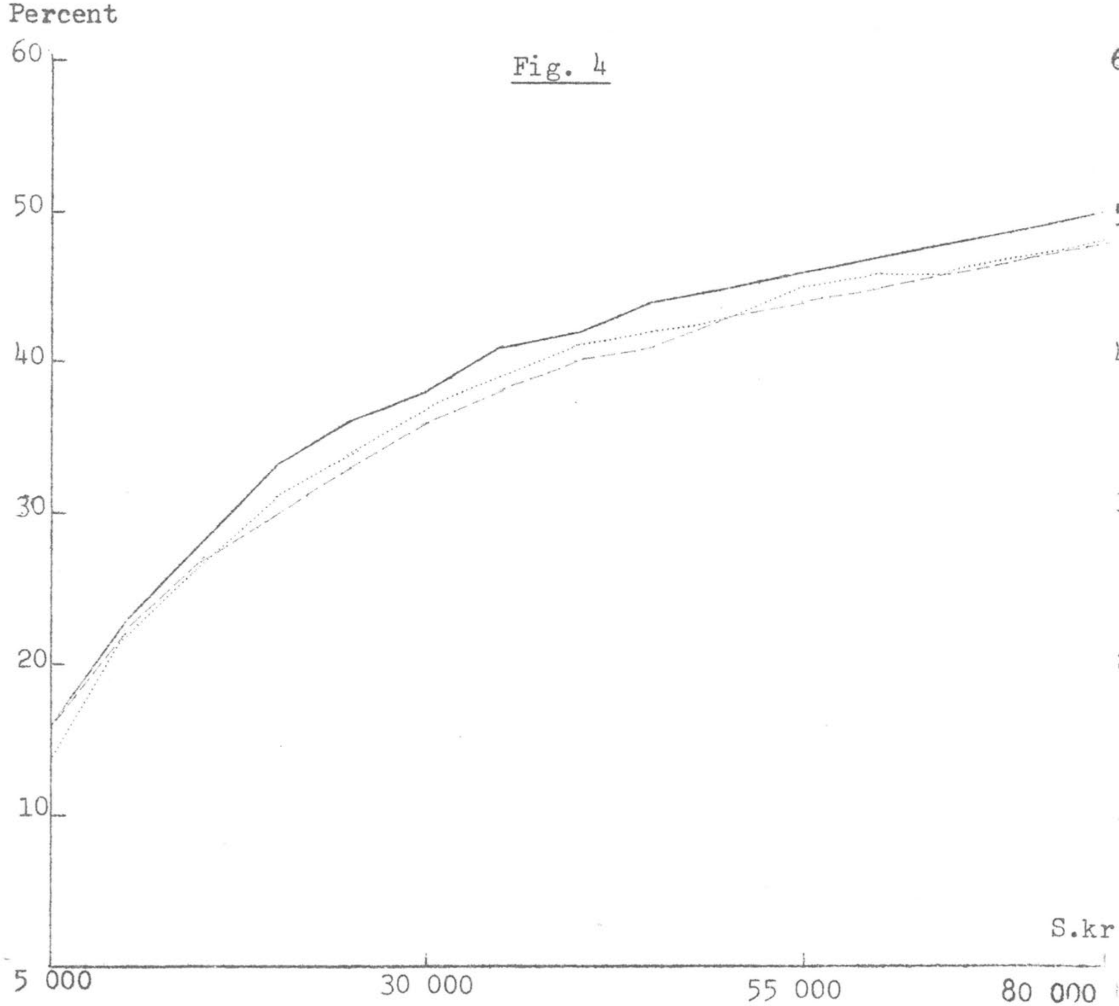

Fig. 4

Tax-levy in percent of total net income for unmarried persons without children.

1953
1960
1966

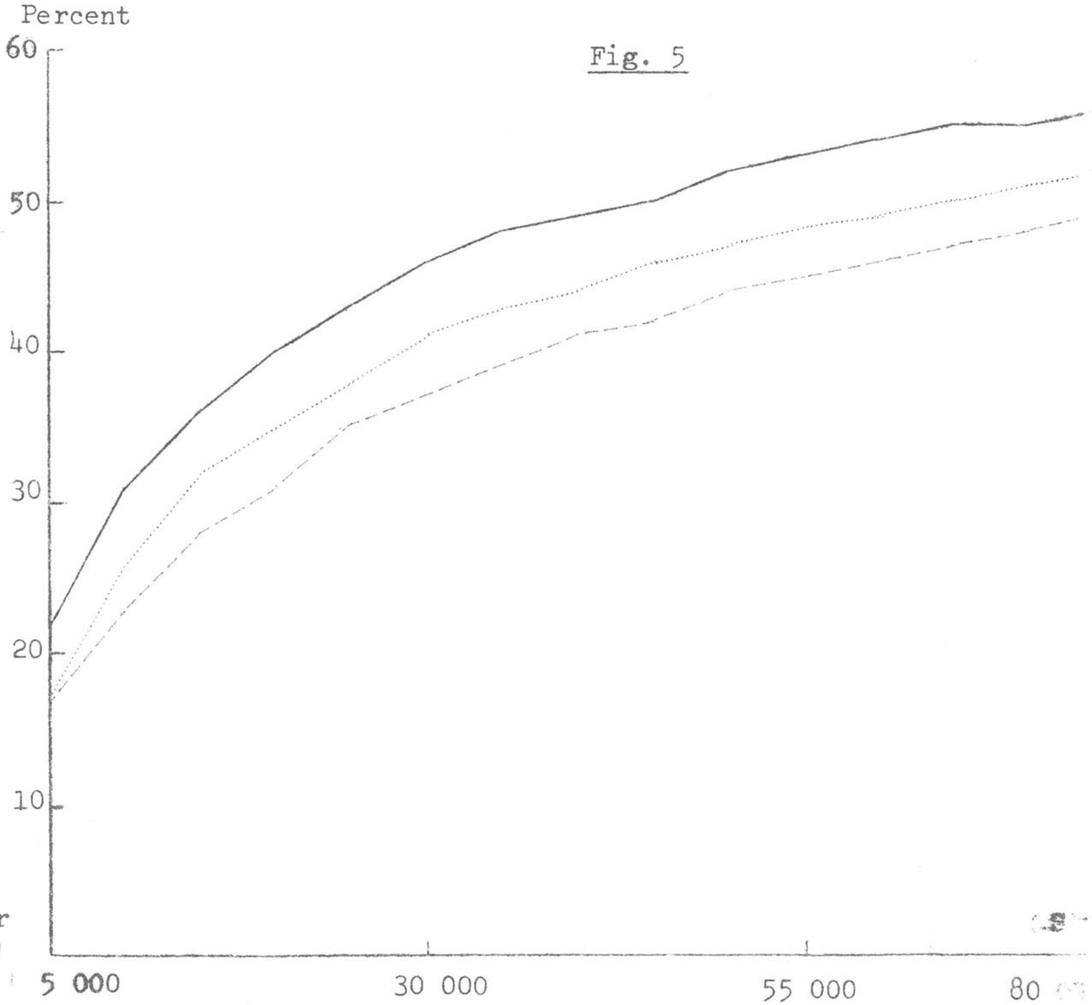

Fig. 5

Tax-levy in percent of deflated total net income for unmarried persons without children.

1953
1960
1966

6.3. One way of investigating a tax system is to compare it with a perfectly proportional tax, giving the same revenue. We have, different years, made that comparison for the Swedish tax system. In each category we have applied a proportional tax with a tax rate equal to the category's average tax, by the ruling tax system. Now we can calculate the number of individuals in each category that get a lower tax by the ruling progressive tax system than by the hypothetical proportional system.

In table 13 we give the percentage of "gainers" in two important categories.

Table 13. Percentage of tax payers that were favoured by the ruling tax system compared with a proportional one

Category	Year		
	1952	1960	1966
	percent		
Single person, age 17-66 without children	68	71	66
Married men, age 17-66 (wife not assessed)	82	83	79

6.4. Furthermore the sums of money gained by each individual gainer in a category can be summed up to the total amount of money accruing to the gainers (T) by the ruling tax system compared with the proportional one. In table 14 we give total gains in a category as a percentage of the total amount of income after tax (F 7) in the category.

Table 14. T/F7

Category	Year		
	1952	1960	1966
	percent		
Single persons, age 17-66 without children	2.1	3.0	3.3
Married men, age 17-66 (wife not assessed)	3.4	3.7	5.1

6.5. For a given tax system we can for each category compute the tax liability at a given income level. We can therefore corpute the income distribution after tax, that can be represented and compared with income distribution before tax, either in tables, diagrams (Lorenz-curves) or special measures of income inequality. We have constructed Lorenz-curves for income before and after tax for different categories. A measure of inequality suggested by Atkinson[1)] has also been used. To present results from this investigation would, however, carry us beyond the limits of this paper.

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1) A.B. Atkinson , On the Measurement of Inequality, Journal of Economic Theory 2, 1970.

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