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OPTIMAL PRICING IN THE TELECOMMUNICATIONS MARKET

by

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Summary

The paper addresses the question whether current tariffs for telecommunications services at the Swedish telecommunications company, Televerket (TVT), are optimal as second-best "Ramsey" prices. Focusing on two tariffs for telecommunications services – those on subscription and calling time – the paper estimates whether there is room for an increase of consumers' surplus with current net revenue for TVT unchanged. To this end, a micro model is constructed in which consumers differ as to "taste" for using a telephone and as to income. In order to make or receive calls, the consumer has to buy a subscription. The utility maximising consumer chooses if to subscribe or not and, if subscribing, how much to call. The resulting individual demands are aggregated over a density of taste and income. This simple model gives the following two relations between elasticities, where S is the number of subscriptions, X the number of calling-minutes, p_s and p_x the corresponding prices:

$$\frac{\epsilon X}{\epsilon p_s} = 2\,\frac{\epsilon S}{\epsilon p_s}\,\frac{x_m}{x}$$

$$\frac{\epsilon S}{\epsilon p_x} = \frac{\epsilon S}{\epsilon p_s}\,\frac{p_x x_m}{p_s}$$

Here x is the average calling time for all subscribers, x_m is a weighed average of calling time over marginal subscribers. These formulas give us the cross-elasticities $\epsilon X/\epsilon p_s$ and $\epsilon S/\epsilon p_x$ once the own-price elasticity $\epsilon S/\epsilon p_s$ and the levels of x and x_m are known. The value of $\epsilon S/\epsilon p_s$ is taken from the literature; the value of x from a data set that has been collected jointly by IUI and TVT: about 5000 subscribers' calling times has been measured for three (non-adjacent) weeks.

This data set has also been used to estimate the own-price elasticity $\epsilon x/\epsilon p_x$ when the number of subscribers is held fixed. The demand $x(p_x)$ is assumed be exponential, i.e., $\epsilon x/\epsilon p_x$ is proportional to p_x. The tariff for calling time is differentiated in two dimensions: as to distance zone and as to point in time of the week (day-night, holiday-weekday). Demand is assumed to show substitution effects across points in time, but not across different distance zones. This makes it possible to estimate $\epsilon x/\epsilon p_x$(and in principle also the cross

elasticity across points in time, although these are not accurate). The result is that $\epsilon x/\epsilon p_x = 0.012 \cdot p_x$, where p is öre/minute. However, total demand X is also influenced via S so, from the micro model, (we assume that $x=x_m$) the total effect is

$$\frac{\epsilon X}{\epsilon p_x} = \frac{\epsilon x}{\epsilon p_x} + 2\,\frac{\epsilon S}{\epsilon p_s}\,\frac{x p_x}{p_s}$$

The consumers' surplus is affected also via external effects: the network externalities of subscriptions. We argue that the value for other consumers of a subscription at the margin is equal to p_s.

Equipped with these numbers, we calculate the optimal <u>marginal</u> price adjustment that keep TVT's net revenue constant. The result suggests that price on subscription and very long distance calls are too high, whereas price on local calls is too low. The results are presented in appendix C.

APPENDIX A:
The Model of Demand

A consumer has the utility $u(x,y;\theta,S)$, where x is his consumption of telephone call time, y consumption of other goods, θ is a "taste" parameter and S is the total number of subscribers. $x=\{x_{ji}(t)\}$, where $x_{ji}(t)$ is a phone call in distance zone j, at time period i of duration t. With p_y≡price on good y, p_S≡price on subscription, I≡income and $p_{ji}(t)$ the telephone call tariff, the consumer chooses to subscribe iff $u^S \geq u^n$, where

$$u^S(p,p_S,\theta,I,S) \equiv \max_{x,y} u(x,y;\theta,S) \text{ s.t. } \sum_{i,j} \int_0^\infty p_{ji}(t)x(t)\, dt + p_y y + p_S = I$$

$$u^n(\theta,I,S) \equiv \max_{y} u(0,y;\theta,S) \text{ s.t. } p_y y = I$$

We define $v \equiv u^S - u^n$ which is assumed be increasing in θ. This means that the consumer subscribes iff $\theta \geq \theta_0(I)$ where $\theta_0(I)$ defined by

$$v(p,p_S,\theta_0,I,S) = 0. \tag{1}$$

From now on we simplify notation so x ≡ demand for telephone call time, and p_x=price on calling time. A fuller account of the model is given in the Swedish version (the rationale for the more complicated version is that tariffs are not proportional).

The consumers are supposed to be distributed in θ and I according to the density function $g(\theta,I)$. Hence aggregated demand for calling time is

$$X = \int_0^\infty \int_{\theta_0(I)}^\infty x(\theta,I)g(\theta,I)\, d\theta\, dI$$

and the total number of subscribers is

$$S= \int_0^\infty \int_{\theta_0(I)}^\infty g(\theta,I)\, d\theta\, dI \tag{2}$$

It is important to note that S enters the utility function. In fact, we show in the fuller Swedish version that under the assumption that subscriptions are neither substitutes nor complements for each other, the derivative (CS= consumer's surplus) $\partial CS/\partial S = p_S$.

As an example, let us derive the second elasticity formula given in the Summary. The full derivations of all formulas are given in the Swedish version.

Differentiating (2) w.r.t. p_S gives

$$\frac{\partial S}{\partial p_S} = - \int_0^\infty g(\theta_0(I),I)\frac{\partial \theta_o}{\partial p_S}\, dI$$

and from (1)

$$\frac{\partial \theta_o}{\partial p_S} = - \frac{v_{p_S} + v_S \frac{\partial S}{\partial p_S}}{v_\theta}$$

hence

$$\frac{\partial S}{\partial p_S} = \int_0^\infty g(\theta_o(I),I)\ \frac{v_{p_S} + v_S \frac{\partial S}{\partial p_S}}{v_\theta}\, dI$$

We define

$$H = \int_0^\infty g(\theta_o(I),I)\frac{v_S}{v_\theta}\, dI$$

and get

$$(1-H)\frac{\partial S}{\partial p_S} = \int_0^\infty g(\theta_o(I),I)\frac{v_{p_S}}{v_\theta}\, dI \tag{3}$$

We also define h(I) by

$$h(I) = \frac{g(\theta_o(I),I)v_{p_S}(\theta_o(I),I)/v_\theta(\theta_o(I),I)}{(1-H)\frac{\partial S}{\partial p_S}} \tag{4}$$

which is thus a density function: it integrates to 1 and is positive since both numerator and denominator are negative.

The following formula is derived in the same way as (3):

$$(1-H)\frac{\partial S}{\partial p_x} = \int_0^\infty g(\theta_o(I),I)\frac{v_{p_x}}{v_\theta}\, dI \tag{5}$$

By Roy's identity,

$$v_{p_x} = -u_I^S x, \qquad v_{p_S} = -u_I^S \quad \text{, i.e.,}$$

$$v_{p_x} = v_{p_S} x$$

Inserting this into (5), we get, using (4),

$$(1-H)\frac{\partial S}{\partial p_x} = \int_0^\infty g(\theta_o(I),I)\frac{v_{p_S} x}{v_\theta}\, dI$$

$$= (1-H)\frac{\partial S}{\partial p_S} \int_0^\infty h(I)x(\theta_o(I),I)\, dI \tag{6}$$

Here the last integral is a weighed average of marginal consumers' demand for calling time. We introduce the notation

$$x_m \equiv \int_0^{\infty} h(I)x(\theta_0(I),I)\ dI$$

and by (3) and (6) we get

$$\frac{\partial S}{\partial p_x} = \frac{\partial S}{\partial p_s} x_m \quad \text{i.e.,} \quad \frac{\epsilon S}{\epsilon p_x} = \frac{\epsilon S}{\epsilon p_s} \frac{p x_m}{p_s}$$

APPENDIX B:
Demand Elasticity of Telephone Call Time

The model

It is often observed that the demand elasticity for calling time is increasing in price. Partly leaning against this, we assume that the elasticity E is proportional to price p: $E = \theta p$. This corresponds to the demand function $x_{d,t} = A \cdot e^{-\theta p}$, where $x_{d,t}$ is expected demand at time t for calling time for distance zone d type calls under, say, one hour. The constant A is assumed to vary with point in time, t, with d and with other prices. A subscriber who contemplates to make a zone d call must decide at which time t to make the call, and he is then assumed to take prices for type d calls at various times into account, but not explicitly the distance zone d. This leads to a specification $A=f(d)h(t,q)$ where q is the vector of prices for type d calls at time periods other than t.

We assume that the income effect is negligible, and this causes a (Slutsky) symmetry restriction on demand. The simplest demand function we can figure out consistent with the above restrictions is

$$x_{d,t} = f(d)e^{-\theta p}(g(t)-c\cdot e^{-\theta p'})$$

where p' is the sum of all prices for type d calls at time periods different from t. Actually, c could depend on d, but we restrict c to be a constant to avoid an over-parameterisation problem.

The Data

The data set consists of information on total calling time divided into 24 categories for about 4000 individuals during 3 weeks: one week in March 1988, one in June and one in September. The 24 categories are defined by 4 periods of time during the week, and 6 distance zones. The time periods are weekdays 8-12, 12-18, 18-22, and other time; the distance zones are national trunk calls <45 km, long distance calls <45 km, 45-90 km, 90-180 km, 180-270 km and >270 km. In each of these 24 categories, the price on calling time is constant during each measured week, but there is a substantial price change between the first measuring period and the second. The equation has been estimated separately for the three weeks. It would seem natural to exploit the price shift between the measuring weeks, but we haven't done that for the reasons that there is a lot of seasonal variation of demand and also a strong time trend. In order to correct for these (as well other exogenous shifts), we need a much longer time series than we have access to.

Estimation

Since $x_{d,t}$ is _expected_ demand, the regression equation is

$$x_{d,t} = f(d)e^{-\theta p}(g(t)-c\cdot e^{-\theta p'}) + \epsilon_{d,t}$$

where $E[\epsilon_{d,t}] = 0$ and $E[\epsilon^2_{d,t}] = \sigma^2_{d,t}$, and $E[\epsilon_{d,t}\epsilon_{\delta,\tau}] = 0$ if $d \neq \delta$ or $t \neq \tau$. The estimation technique is non-linear least squares, with the standard errors computed according to the formula referenced in White [1980] but adjusted for degrees of freedom. The results are presented in table 1. As we can see, the March equation suffers from multicolinearity; the lower fit compared to the June equation explains about 80% higher standard deviations, but most of the actual ones are well over 500% larger. However, the June and September equations yield very similar results, and θ is reasonably well defined in these equations. We have no good explanation to offer why the March equation performs so badly. If the results for θ are weighed together to minimise the variance, we get (p is in öre/min)

June+September		March+June+September	
$\hat{\theta} = -0.0127$	SD=0.00329	$\hat{\theta} = -0.0122$	SD=0.00316

In principle it is also possible to compute the cross elasticities for calls between different time periods. However, we refrain from doing this for two reasons. First, these cross effects are rather crudely specified (the parameter c is not allowed to vary with d, for instance); secondly, the estimates of the coefficients determining them are not accurate enough to merit much interest in the result. Thus, we confine the analysis to the own-price elasticity θp.

Table 1

The estimated equation is $s = e^{\Sigma\alpha_i D_i}(e^{\Sigma\beta_j T_j} - \delta e^{\theta p'})e^{\theta p}$ where D_i are dummies for the distance categories, T_j dummies for the time periods (β_1=1); s is total calling time in seconds per subscriber during one full week; p and p' are in öre/min.

coefficient	March	June	September
α_1	1.39 (2.76)	2.31 (0.646)	2.63 (0.957)
α_2	3.37 (2.75)	4.24 (0.405)	4.37 (0.670)
α_3	3.89 (2.75)	4.60 (0.375)	4.80 (0.657)
α_4	3.89 (2.79)	4.84 (0.451)	5.22 (0.838)
α_5	3.57 (2.86)	4.34 (0.467)	4.93 (0.846)
α_6	4.22 (2.81)	5.04 (0.443)	3.83 (0.817)
β_2	0.475 (1.07)	0.107 (0.120)	0.170 (0.208)
β_3	0.780 (1.47)	0.385 (0.179)	0.405 (0.303)
β_4	0.774 (1.61)	0.220 (0.192)	0.126 (0.305)
δ	-0.689 (4.63)	0.657 (0.730)	0.527 (1.14)
θ	-0.00597 (0.0114)	-0.0127 (0.00371)	-0.0129 (0.00713)
$\tilde{R}^2$	0.950	0.985	0.973

Note: asymptotic standard deviations in parenthesis.

Reference

White, H. "A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity." Econometrica 48:721-746

APPENDIX C:

Simulation Results

EA is the marginal external utility of a subscription. We can show that (under reasonable assumptions) this is (close to) Pa, the price on subscription. However, we also use EA = Pa/2 in the simulations.

Q is a parameter in the aggregate demand for calling time. If $\bar{x}$ = average calls made by a subscriber, then $Q \cdot \bar{x}$ new calls are generated by a marginal subscriber. One can argue that Q=2 (the new subscriber makes $\bar{x}$ calls and receives $\bar{x}$ calls), but we also use Q=1 and Q=2 in the simulations.

"Lambda" is a Lagrange multiplier. If dr is a marginal change in revenues for TVT due to price changes, then the resulting loss of consumers' surplus is Lambda·dr.

The marginal price adjustments are in percentage points and normalised so that their squared sum equals 100.

Q=2, EA=Pa

subscription	-8.526
local calls weekd 08-18	1.743
local calls other time	2.016
class 103 weekd 08-12	0.006
class 103 weekd 12-18	0.007
class 103 weekd 18-22	0.007
class 103 other time	0.007
class 104 weekd 08-12	-0.042
class 104 weekd 12-18	-0.005
class 104 weekd 18-22	0.008
class 104 other time	0.031
class 105 weekd 08-12	-0.438
class 105 weekd 12-18	-0.316
class 105 weekd 18-22	-0.324
class 105 other time	-0.176
class 106-108 weekd 08-12	-2.460
class 106-108 weekd 12-18	-2.320
class 106-108 weekd 18-22	-2.477
class 106-108 other time	-1.484
lambda =	2.115

Q=1, EA=Pa

subscription	-3.522
local calls weekd 08-18	4.601
local calls other time	4.317
class 103 weekd 08-12	0.026
class 103 weekd 12-18	0.036
class 103 weekd 18-22	0.035
class 103 other time	0.029
class 104 weekd 08-12	0.091
class 104 weekd 12-18	0.193
class 104 weekd 18-22	0.239
class 104 other time	0.209
class 105 weekd 08-12	-0.587
class 105 weekd 12-18	-0.273
class 105 weekd 18-22	-0.181
class 105 other time	0.008
class 106-108 weekd 08-12	-4.266
class 106-108 weekd 12-18	-3.622
class 106-108 weekd 18-22	-3.528
class 106-108 other time	-1.852
lambda =	1.830

Q=0, Ea=Pa

subscription	-0.514
local calls weekd 08-18	3.204
local calls other time	2.673
class 103 weekd 08-12	0.026
class 103 weekd 12-18	0.034
class 103 weekd 18-22	0.034
class 103 other time	0.029
class 104 weekd 08-12	0.042
class 104 weekd 12-18	0.154
class 104 weekd 18-22	0.200
class 104 other time	0.194
class 105 weekd 08-12	-0.816
class 105 weekd 12-18	-0.466
class 105 weekd 18-22	-0.397
class 105 other time	-0.130
class 106-108 weekd 08-12	-5.369
class 106-108 weekd 12-18	-4.737
class 106-108 weekd 18-22	-4.781
class 106-108 other time	-2.652
lambda =	1.169

Q=2, Ea=Pa/2

subscription	-8.526
local calls weekd 08-18	1.743
local calls other time	2.016
class 103 weekd 08-12	0.006
class 103 weekd 12-18	0.007
class 103 weekd 18-22	0.007
class 103 other time	0.007
class 104 weekd 08-12	-0.042
class 104 weekd 12-18	-0.005
class 104 weekd 18-22	0.008
class 104 other time	0.031
class 105 weekd 08-12	-0.438
class 105 weekd 12-18	-0.316
class 105 weekd 18-22	-0.324
class 105 other time	-0.176
class 106-108 weekd 08-12	-2.460
class 106-108 weekd 12-18	-2.320
class 106-108 weekd 18-22	-2.477
class 106-108 other time	-1.484
lambda =	1.812

Q=1, Ea=Pa/2

subscription	-3.522
local calls weekd 08-18	4.601
local calls other time	4.317
class 103 weekd 08-12	0.026
class 103 weekd 12-18	0.036
class 103 weekd 18-22	0.035
class 103 other time	0.029
class 104 weekd 08-12	0.091
class 104 weekd 12-18	0.193
class 104 weekd 18-22	0.239
class 104 other time	0.209
class 105 weekd 08-12	-0.587
class 105 weekd 12-18	-0.273
class 105 weekd 18-22	-0.181
class 105 other time	0.008
class 106-108 weekd 08-12	-4.266
class 106-108 weekd 12-18	-3.622
class 106-108 weekd 18-22	-3.528
class 106-108 other time	-1.852
lambda =	1.569

<u>Q=0, Ea=Pa/2</u>

subscription	-0.514
local calls weekd 08-18	3.204
local calls other time	2.673
class 103 weekd 08-12	0.026
class 103 weekd 12-18	0.034
class 103 weekd 18-22	0.034
class 103 other time	0.029
class 104 weekd 08-12	0.042
class 104 weekd 12-18	0.154
class 104 weekd 18-22	0.200
class 104 other time	0.194
class 105 weekd 08-12	-0.816
class 105 weekd 12-18	-0.466
class 105 weekd 18-22	-0.397
class 105 other time	-0.130
class 106-108 weekd 08-12	-5.369
class 106-108 weekd 12-18	-4.737
class 106-108 weekd 18-22	-4.781
class 106-108 other time	-2.652
lambda =	1.002

APPENDIX D:

Computer program

```
dim a(18), b(18), c(36), d(18,36), pi(18), p(18), dLp(18)
dim klass$(18)
klass$(0)= "             abonnemang"
klass$(1)= "  lokalsamtal vard 08-18"
klass$(2)= "  lokalsamtal övrig tid"
klass$(3)= "   klass 103 vard 08-12"
klass$(4)= "   klass 103 vard 12-18"
klass$(5)= "   klass 103 vard 18-22"
klass$(6)= "    klass 103 övrig tid"
klass$(7)= "   klass 104 vard 08-12"
klass$(8)= "   klass 104 vard 12-18"
klass$(9)= "   klass 104 vard 18-22"
klass$(10)="    klass 104 övrig tid"
klass$(11)="   klass 105 vard 08-12"
klass$(12)="   klass 105 vard 12-18"
klass$(13)="   klass 105 vard 18-22"
klass$(14)="    klass 105 övrig tid"
klass$(15)="klass 106-108 vard 08-12"
klass$(16)="klass 106-108 vard 12-18"
klass$(17)="klass 106-108 vard 18-22"
klass$(18)=" klass 106-108 övrig tid"
' '
'INITIERING AV MATRISERNA a, b, c, d'
dim Tr(18), Nr(18), xderT(3), xderN(3)
' '
'priser september 1988'
'---------------------'
PA=171.*4/52*100        'pris abonnemang öre/vecka'
c0=3                    '1 period i min lokalsamtal vard 08-18'
c1=6                    'd:o övrig tid'
p(1)=7.67               'öre/min lokalsamtal vard 08-18'
p(2)=3.83               'd:o övrig tid'
p(3)=22                 'öre/min klass 103 vard 08-12'
p(4)=15                 'öre/min klass 103 vard 12-18'
p(5)=13                 'öre/min klass 103 vard 18-22'
p(6)=12                 'öre/min klass 103 vard 22-08, lör, sön'
p(7)=45                 'öre/min klass 104 vard 08-12'
p(8)=31                 'öre/min klass 104 vard 12-18'
p(9)=26                 'öre/min klass 104 vard 18-22'
p(10)=23                'öre/min klass 104 vard 22-08, lör, sön'
p(11)=81                'öre/min klass 105 vard 08-12'
p(12)=58                'öre/min klass 105 vard 12-18'
p(13)=49                'öre/min klass 105 vard 18-22'
p(14)=43                'öre/min klass 105 vard 22-08, lör, sön'
p(15)=125               'öre/min klass 106-108 vard 08-12'
p(16)=92                'öre/min klass 106-108 vard 12-18'
p(17)=77                'öre/min klass 106-108 vard 18-22'
p(18)=66                'öre/min klass 106-108 vard 22-08,      '
                        'lör, sön'
'nivåer i september 1988'
'-----------------------'
Tl0=1466965./60/1000    'antal lok.samtalsmin, samtal >1 mark
                        'vard 08-18'
Tl1=1528541./60/1000    'd:o övrig tid'
Tr(1)=1927735./60/1000  'antal lokalsamtalsmin vard 08-18'
```

```
Tr(2)=2292367./60/1000    'd:o övrig tid'
Tr(3)=1121.*5/60/1000     'antal samtalsmin, samma ordning som p
Tr(4)=2480.*5/60/1000     'ovan'
Tr(5)=2650.*5/60/1000
Tr(6)=10511./60/1000
Tr(7)=7124.*5/60/1000
Tr(8)=11627.*5/60/1000
Tr(9)=15192.*5/60/1000
Tr(10)=54905./60/1000
Tr(11)=7661.*5/60/1000
Tr(12)=12317.*5/60/1000
Tr(13)=18499.*5/60/1000
Tr(14)=79535./60/1000
Tr(15)=70672./60/1000
Tr(16)=129269./60/1000
Tr(17)=203881./60/1000
Tr(18)=177684./60/1000
' '
Nk0=5351./1000            'antal lokalsamtal =1 mark vard 08-18'
Nk1=5827./1000            'd:o övrig tid'
Nr(1)=7310./1000          'antal lokalsamtal vard 08-18'
Nr(2)=7106./1000          'd:o övrig tid'
Nr(3)=3.17*5/1000         'antal samtal, samma ordning som p ovan'
Nr(4)=6.33*5/1000
Nr(5)=4.86*5/1000
Nr(6)=20./1000
Nr(7)=19.47*5/1000
Nr(8)=36.43*5/1000
Nr(9)=28.05*5/1000
Nr(10)=153./1000
Nr(11)=26.99*5/1000
Nr(12)=40.90*5/1000
Nr(13)=29.33*5/1000
Nr(14)=227./1000
Nr(15)=61.57*5/1000
Nr(16)=96.*5/1000
Nr(17)=79.*5/1000
Nr(18)=391./1000
' '
'elasticiteter och identiteter'
'-----------------------------'
Aelast=-0.4               'priselasticitet på abonnemang'
A=1                       'allt räknas per 1 abonnent'
EA=PA                     'marg extern nytta av abonnemang=pris'
APA=A*Aelast/PA
Q=2                       'nytt abb. genererar Q*(genomsnittligt'
                          'antal ringda samtal) nya samtal'
                          ' '
def fnTelast(k)=-0.013*p(k)
                          'priselast. på samtalstid som'
                          'funktion klass för fix abonnent'
def fnNelast(k)=0
                          'pris/min-elast. på samtal som'
                          'funktion av klass för fix abonnent'
xderTlok=0                'xderivata tid inom lokalsamtal'
xderNlok=0                'xderivata antal inom lokalsamtal'
xderT(0)=0.042/60         'xderivata tid inom klass 103'
xderN(0)=0                'xderivata antal inom klass 103'
xderT(1)=0.11/60          'xderivata tid inom klass 104'
```

```
115 xderN(1)=0              'xderivata antal inom klass 104'
116 xderT(2)=0.042/60       'xderivata tid inom klass 105'
117 xderN(2)=0              'xderivata antal inom klass 105'
118 xderT(3)=0.024/60       'xderivata tid inom klass 106-108'
119 xderN(3)=0              'xderivata antal inom klass 106-108'
120 ' '
121 'marginalkostnader'
122 '-----------------'
123 c(0)=1000./52*100       'marg kostnad för abonnemang öre/vecka'
124 ' '
125 c(1)=0                  'marg kostnad för lokalsamtal, vard 08-18'
126 c(3)=0                  'd:o övrig tid'
127 c(5)=0                  'marg kostnad för samtal, samma ordning'
128 c(7)=0                  'som för p ovan'
129 c(9)=0
130 c(11)=0
131 c(13)=0
132 c(15)=0
133 c(17)=0
134 c(19)=0
135 c(21)=0
136 c(23)=0
137 c(25)=0
138 c(27)=0
139 c(29)=0
140 c(31)=0
141 c(33)=0
142 c(35)=0
143 ' '
144 c(2)=0                  'marg kostnad för lok.samtalsmin,'
145                         'vard 08-18'
146 c(4)=0                  'd:o övrig tid'
147 c(6)=0                  'marg kostnad för samtalsminuter, samma'
148 c(8)=0                  'ordning som p ovan'
149 c(10)=0
150 c(12)=0
151 c(14)=0
152 c(16)=0
153 c(18)=0
154 c(20)=0
155 c(22)=0
156 c(24)=0
157 c(26)=0
158 c(28)=0
159 c(30)=0
160 c(32)=0
161 c(34)=0
162 c(36)=0
163 ' '
164 'initiering'
165 '----------------'
166 p(0)=PA
167 ' '
168 a(0)=-A+EA*APA
169 a(1)=(-1+EA*APA/A)*(c0*Nk0+Tl0)
170 a(2)=(-1+EA*APA/A)*(c1*Nk1+Tl1)
171 for j=3 to 18
172  a(j)=(-1+EA*APA/A)*Tr(j)
173 next
```

```
174 ' '
175 d(0,0)=APA
176 for j=1 to 18
177  d(0,2*j-1)=Q*APA*Nr(j)/A
178  d(0,2*j)=Q*APA*Tr(j)/A
179 next
180 for j=1 to 2
181  for k=1 to 2
182   d(j,2*k)=xderTlok
183   d(j,2*k-1)=xderNlok
184  next
185 next
186 d(1,0)=APA*(c0*Nk0+Tl0)/A
187 d(1,1)=fnNelast(1)/p(1)*Nr(1)
188 d(1,2)=fnTelast(1)/p(1)*Tr(1)
189 d(1,3)=0
190 d(1,4)=0
191 d(2,0)=APA*(c1*Nk1+Tl1)/A
192 d(2,1)=0
193 d(2,2)=0
194 d(2,3)=fnNelast(2)/p(2)*Nr(2)
195 d(2,4)=fnTelast(2)/p(2)*Tr(2)
196 for n=0 to 3
197  m1=3+4*n
198  m2=6+4*n
199  for j=m1 to m2
200   for k=m1 to m2
201    d(j,2*k)=xderT(n)
202    d(j,2*k-1)=xderN(n)
203   next
204  next
205 next
206 for j=3 to 18
207  d(j,0)=APA*Tr(j)/A
208  d(j,2*j-1)=fnNelast(j)/p(j)*Nr(j)
209  d(j,2*j)=fnTelast(j)/p(j)*Tr(j)
210 next
211 for j=1 to 18
212  for k=1 to 18
213   d(j,2*k)=d(j,2*k)+Q*Tr(k)*APA*Tr(j)/A/A
214   d(j,2*k-1)=d(j,2*k-1)+Q*Nr(k)*APA*Tr(j)/A/A
215  next
216 next
217 ' '
218 b(0)=PA*APA+A+Q*APA*(c0*p(1)*Nk0+p(1)*Tl0)/A
219 b(0)=b(0)+Q*APA*(c1*p(2)*Nk1+p(2)*Tl1)/A
220 for j=3 to 18
221  b(0)=b(0)+p(j)*Q*APA*Tr(j)/A
222 next
223 b(1)=c0*fnNelast(1)*Nk0+fnTelast(1)*Tl0          'xderTlok och'
224 b(1)=b(1)+c0*Nk0+Tl0                             'xderNlok    '
225 for j=1 to 18
226  b(1)=b(1)+Q*Nk0*APA*Tr(j)*c0*p(1)/A/A
227  b(1)=b(1)+Q*Tl0*APA*Tr(j)*p(1)/A/A
228 next
229 b(2)=c1*fnNelast(2)*Nk1+fnTelast(2)*Tl1          'sätts till 0'
230 b(2)=b(2)+c1*Nk1+Tl1
231 for j=1 to 18
232  b(2)=b(2)+Q*Nk1*APA*Tr(j)*c1*p(2)/A/A
```

```
233  b(2)=b(2)+Q*T11*APA*Tr(j)*p(2)/A/A
234 next
235 for j=3 to 18
236  b(j)=Tr(j)
237  for k=3 to 18
238   b(j)=b(j)+d(j,2*k)*p(k)
239  next
240 next
241 ' '
242 'SLUT PÅ INITIERINGEN'
243 'UTRÄKNING AV OPTIMAL MARGINELL PRISJUSTERING'
244 ' '
245 for j=0 to 18
246  pi(j)=b(j)
247  for k=0 to 36
248   pi(j)=pi(j)-d(j,k)*c(k)
249  next
250 next
251 for j=0 to 18
252  numerator=numerator+p(j)*p(j)*pi(j)*a(j)
253  denominator=denominator+p(j)*p(j)*pi(j)*pi(j)
254 next
255 lambda=-numerator/denominator
256 for k=0 to 18
257  dLp(k)=p(k)*a(k)+lambda*p(k)*pi(k)
258  delta2=delta2+dLp(k)*dLp(k)
259 next
260 delta=sqr(delta2)
261 for k=0 to 18
262  dLp(k)=10*dLp(k)/delta
263  print klass$(k) tab(27);
264  print using"###.###"; dLp(k)
265 next
266 print "                    lambda =" tab(27);
267 print using"###.###"; lambda
268 system
```