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**RATE OF ECONOMIC DEPRECIATION OF
MACHINES - a study with the Box-Cox
transformation**

by

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RATE OF ECONOMIC DEPRECIATION¹

1.1 Introduction

The objective of this study is to examine if it is possible, using theoretical assumptions, to estimate the rate of economic depreciation on capital. Subsequently a model of vintage asset prices will be designed and used.

Economic depreciation is here defined as the change in asset price due to aging. The rate of economic depreciation is consequently defined as the percentage change in asset price due to aging time.

With capital I here mean physical capital, i e machines and buildings. Human capital is not considered in this study. The reason for this omission is the lack of a used asset market for human capital. Price data from the used asset market for physical capital is essential for this study, since this makes it possible to estimate simultaneously economic depreciation and rate of economic depreciation. Hulten and Wykoff (1980a, b, c, 1981) have made similar studies in the United States which have served as a model for my studies.

In economic applications it is generally assumed that the rate of economic depreciation is a constant, i e the rate of economic depreciation does not change with the assets' age, as in the case when the pattern of the economic depreciation adopts other forms. Hulten and Wykoff (1980b) observe that the pattern for economic depreciation for used american commercial and industrial structures is approximately exponential. Feldstein and Rotschild (1974) argue, on the other hand that a constant rate of economic depreciation is only associated with strong assumptions concerning the capital stock. They also argue that changes in the interest and tax laws influence the optimal economic life time for machinery, which affects the

¹ I would like to express my appreciation and indebtedness to Jan Södersten and Erik Mellander for constructive criticism and invaluable help during the work. The opinions expressed here on, are of course on my own responsibility.

pattern of economic depreciation.

If Feldstein and Rotschilids' argument is correct then current calculations of economic depreciation are wrong. It is therefore important to investigate if this theoretical argument can be confirmed empirically. This study makes no a priori assumptions about an exponential depreciation pattern. The proposed model includes, among other possibilities, the exponential depreciation pattern as a special case which makes it possible to test ex post if the exponential depreciation pattern is a good description of the depreciation pattern.

1.2 The importance of Estimates of Economic Depreciation

Estimates of economic depreciation and the rate of economic depreciation are of great importance when calculating the revenue and valuation of property.

The company incurs an expense in purchasing an asset, such as a machine. This expense is distributed as a cost, over the period for which the asset is expected to yield a positive gross return. In this way, the book value of the asset is depreciated, in accordance with tax regulations. Problems develop, however, to the extent that the book value depreciation differs from the actual economic depreciation. Such a disparity yields a misleading picture of the company's profitability and its development over time. Thus, if this disparity can be shown, it implies that the rules for book value depreciation can be made more accurate and effective.

Incorrect assumptions about economic depreciation also generate serious errors in estimating the net capital stock. In discussing capital stocks, it is essential to distinguish between gross and net capital stock. When the capital stock is estimated, the capital asset valued according to its physical production capacity. An adjustment is consequently made for physical depreciation, i.e. the loss in production capacity over time. For the net capital stock, an additional adjustment is made for economic depreciation.

Calculations of gross capital stock in Sweden are carried out by the Swedish Central Bureau of Statistics, SCB, according to the perpetual inventory (PI-) method. This method is based on the assumption that production capacity is constant over time until the asset is retired, through "sudden death". (An example is a light bulb, which shows this pattern for its production capacity.) However, the time of retirement varies among different capital asset of a vintage. The PI-method means that the gross capital stock at time t , K_t , is estimated as the product of gross investment, I_τ , deflated with investment price index, p_τ , and the part of the capital that still is in use, $\lambda_{t-\tau}$, according to equation (1)

$$K_t = \sum_{\tau = t-\gamma}^{t-1} \frac{I_\tau}{P_\tau} \cdot \lambda_{t-\tau} \quad (1)$$

where γ is the maximum age of the cohort. The assumptions on the share of the capital asset that is still in use are based on studies by Winfrey (1935) and Wallander (1962). Winfrey constructed survivor curves for different types of capital assets and industries on the basis of observations of retirement; Wallander calculated average retirement age for different types of capital assets. With an assumption about the average retirement age, it is possible to calculate from the survivor curves the share of the original cohort that is still in use for given ages.

SCB also calculates net capital stocks where, according to SCB, an adjustment is made for retirement and for the loss in value due to age. (Estimates on the net capital stock are, unfortunately, not published.) Södersten and Lindberg (1983) describe how SCB makes these calculations of the net capital stock. SCB's calculations are based on survivor curves for capital objects. From these one can calculate the average expected remaining life time and the average age of retirement. SCB assumes that the ratio between these two entities constitutes that share of the original value for every separate gross investment that still exists. Using this ratio, the capital stock is adjusted from gross to net.

Economic depreciation is the difference between gross investment and the change in the net capital stock. The economic depreciation rate, implicitly assumed by SCB, δ , is compiled by equation (2) where K_N is the net capital stock and D the economic depreciation at time t .

$$\delta(t) = \frac{D(t)}{K_N(t)} \quad (2)$$

On a high aggregation level, δ is nearly constant for the period 1949–79. Consequently the exponential depreciation pattern in this case is a good approximation to SCB's estimate of economic depreciation. But to what extent does δ in equation (2) correspond to the reality? The assumptions that are used in the calculations of economic depreciation (and therefore also δ) are based on old information. Winfrey's survivor curves are from 1935 and Wallander's average retirement age from 1962.

Since Wallander's study "Verkstadsindustrins maskinkapital", no investigation has been done in Sweden of the lifetime of machines². In the US, however, a number of investigations in this area have been carried out, for example Hulten and Wykoff (1980), Eisner (1972) and Jorgenson(1971).

1.3 Structure

This paper is organized as follows. Economic and physical depreciation are defined in chapter 2 and different depreciation patterns are discussed. Chapter 3 describes the econometric model. Chapter 4 examines the data, focusing on weaknesses such as censored sampling bias, the concepts of lemons and pearls as well as potential adjustment for these. The data material is based on records of used machines sold at ASEA during 1983–1986. Chapter 5 describes the estimation method. Finally, in the last chapter the estimates are reported, comparisons are made with other relevant studies and a summary is presented.

² Wallander's study is the only Swedish study in this area.

2 BASIC TERMS AND DEFINITIONS

2.1 Economic Depreciation

Economic depreciation is defined as the change in asset price due to aging (Hulten and Wykoff, 1980a). This can be illustrated in the following way (compare with Cederblad, 1971).

$I(t)$ = income from capital at time t

t = time

$U(t)$ = operating costs

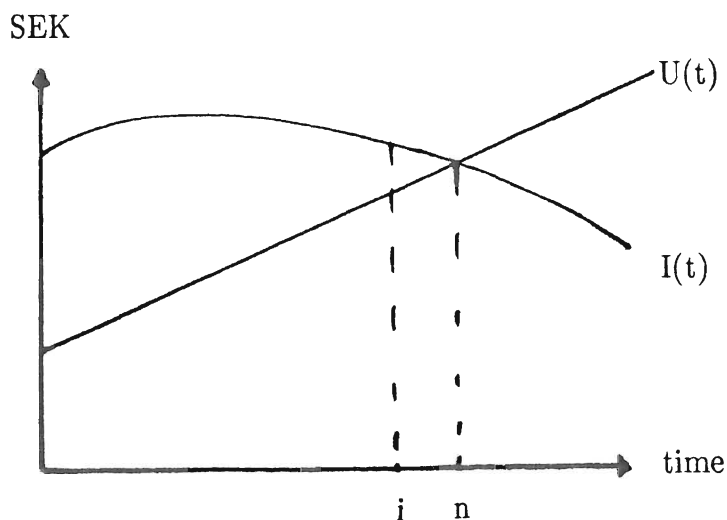
n = economic lifetime

$D(t)$ = discounting factor

$V(i)$ = value at time i

$$V(i) = \int_{t=i}^n (I(t) - U(t)) \cdot D(t) dt$$

Figure 1



The dashed area in figure 1 shows $V(i)$ expressed as the discounted flow of net revenue that the asset is expected to generate during its expected remaining lifetime. The operating costs are expected to rise and the income to fall. This implies that $V(i)$ decreases over time. This a priori discussion is supported by empirical results (Cederblad, 1971).

2.2 Rate of economic depreciation

The rate of economic depreciation is defined as the percentage change in asset price due to aging (Hulten and Wykoff, 1980a). During periods of inflation, the change in price has to be divided into two components, namely economic depreciation and inflation.

If $q(s,t)$ is the value of an s year old asset at time t , the derivative of $q(s,t)$ to time is

$$\begin{aligned} \frac{d q (s , t)}{d t} &= \frac{\partial q (s , t)}{\partial s (t)} \cdot \frac{d s (t)}{d t} + \frac{\partial q (s , t)}{\partial t} \cdot \frac{d t}{d t} \\ &= \frac{-\partial q (s , t)}{\partial s (t)} + \frac{\partial q (s , t)}{\partial t} \end{aligned} \quad (3)$$

because $s=n-t$.

Division with $q(s,t)$ yields

$$\left[\frac{d q (s , t)}{d t} \right] \frac{1}{q (s , t)} = - \left[\frac{\partial q (s , t)}{\partial s (t)} \right] \frac{1}{q (s , t)} + \left[\frac{\partial q (s , t)}{\partial t} \right] \frac{1}{q (s , t)} . \quad (4)$$

The first term on the right side constitutes the rate of economic depreciation and the second term the rate of inflation. When changing-over from continuous to discrete values of s and t equation (4) can be illustrated in a "age-time profile", by the following approach.

s	t	1980	1981	1982
0		$q(0,1980)$	$q(0,1981)$	$q(0,1982) \dots$
1		$q(1,1980)$	$q(1,1981)$	
2		$q(2,1980)$		
.				

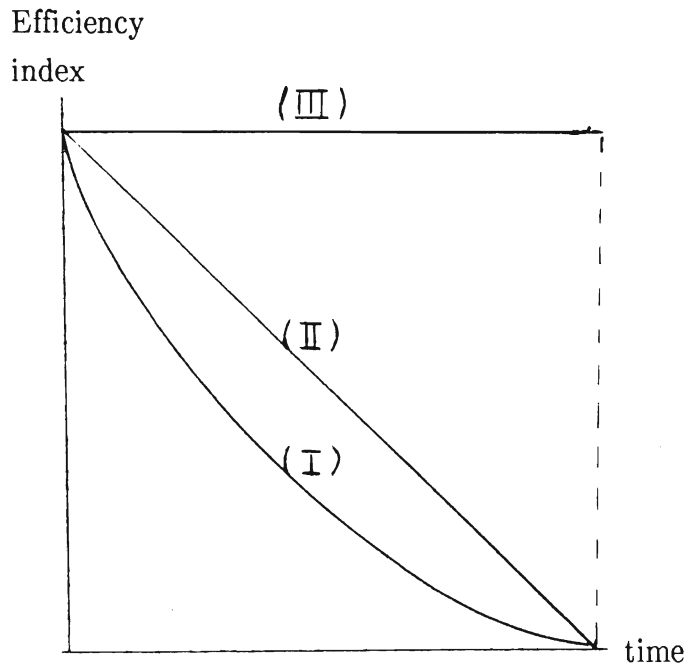
The first term on the right side constitutes the rate of economic depreciation corresponding to the columns of the matrix; for example, the rate of economic depreciation for an asset from 1971 can be calculated by means of column 2. Column t_i is equivalent to the first term on the right side of equation (4) and row s_j is equivalent to the second term. The ideal would be to have a complete "age-time profile". Yet this is a wish that can not be fulfilled. The missing values have to be approximated by some model, which we will return to in chapter 3.

2.3 Physical versus economic depreciation

It is very important to make a clear distinction between physical and economic depreciation. The physical depreciation is the loss in production capacity of a physical asset over time (Hulten and Wykoff, 1980a). A used asset can exhibit its original production capacity and simultaneously show a decline in monetary value if for example, more effective assets have come out on the market. This distinction is essential. Physical depreciation is relevant for the analysis of physical investment and replacement requirements while economic depreciation is relevant for the analysis of taxes and revenue as well as wealth estimations.

Physical depreciation can be illustrated by curves of the production capacity of an asset. Hulten and Wykoff (1980a) assign an efficiency index to every asset. The efficiency index is defined as the marginal rate of substitution in production between a used and a new asset. The efficiency index is equal to one when the asset is new, followed by a decline with time. Figure 2 shows three different efficiency decay profiles.

Figure 2 Example of Physical Depreciation profiles



Source: Hulten and Wykoff (1980a)

The graph (I) illustrates an exponential decrease³ where the asset loses its production capacity with a constant percentage rate (for example when ice melts to water). Graph (II) illustrates a linear decrease, i e the loss in production capacity is constant and graph (III) is the so called one-horse-shay curve. In this case the asset has a constant production capacity over its entire life time until its retirement. At that time the production capacity index assumes the value zero, i e sudden death (for example a light bulb).

³ Almost all literature uses the conception geometric instead of the correct exponential.

A geometric serie

$$s = 1 + (1-\delta) + (1-\delta)^2 + \dots + (1-\delta)^n$$

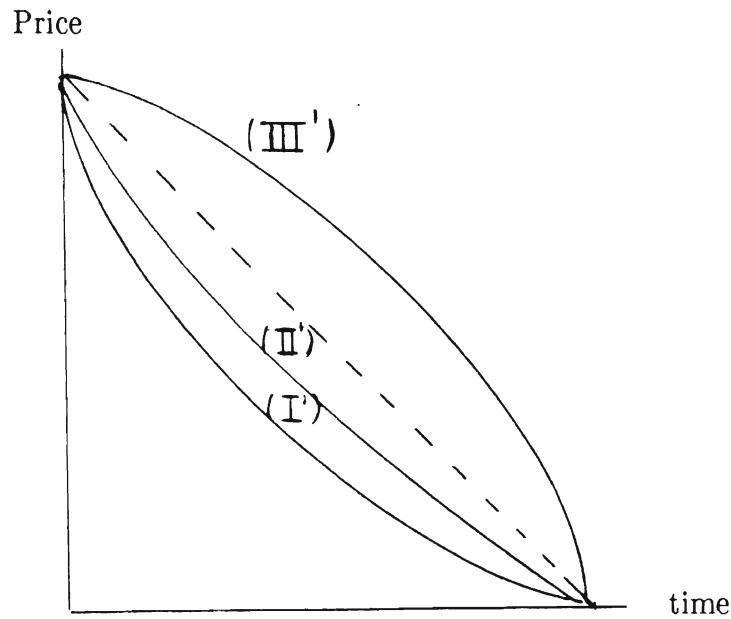
is equivalent to the corresponding exponentiell function

$$\int_{n=0}^{\infty} e^{-\delta n} dn = \frac{1}{\delta}$$

only if $|1-\delta| < 1$ and $n \rightarrow \infty$. The conception exponentiell is therefor preferable.

The corresponding value profiles over time follows a pattern like figure 3 (Hulten and Wykoff, 1980a). In SCB's calculations of the net capital stock an adjustment for this value depreciation is made. Graph (I') corresponds to (I), (II') to (II) and (III') to (III). In most cases the two profiles have different shapes. It is only for one type of asset that the exact pattern is shown for physical depreciation as for economic depreciation, namely, that which follows the exponential form.

Figure 3 Example of Economic Depreciation profiles



Source: Hulten and Wykoff

This can be shown in the following way, in where we, consider for simplicity, an investment of 1 SEK under conditions of zero inflation.

$$q(s) = \text{value of the investment at time } s \quad (5)$$

$$\frac{dq(s)}{ds} = \text{economic depreciation} \quad (6)$$

$$\frac{dq(s)}{ds} / q(s) = \text{rate of economic depreciation} \quad (7)$$

$$-a = \text{rate of physical depreciation} \quad (8)$$

$$i = \text{interest} \quad (9)$$

ρ = gross return immediately after the investment is made (10)

$\rho \cdot e^{-at}$ = gross return at time t (11)

$$q(s) = \int_{t=s}^{\infty} \rho e^{-at} e^{-i(t-s)} dt = \rho e^{is} \int_s^{\infty} e^{-(a+i)t} dt = \rho e^{is} \left[\frac{e^{-t(a+i)}}{-(a+i)} \right]_{t=s}^{\infty} = \frac{\rho}{a+i} \cdot e^{-as}$$
 (12)

The differential of (12) with respect to s is

$$\frac{dq(s)}{ds} = -a \cdot \frac{\rho}{a+i} \cdot e^{-as} = -a q(s)$$
 (13)

which yields

$$\frac{dq(s)}{ds} / q(s) = -a$$

(rate of economic depreciation = rate of physical depreciation). (14)

This special case, when the pattern of physical and economic depreciation coincides and the rate of depreciation is therefore constant, is often assumed in economical calculations.

As noted above, the exponential depreciation profile appears as a special case of the generalized model. This makes it possible to carry out statistical tests to determine, for example, if the exponential depreciation pattern is a good description of the economic depreciation profile.

3 THE ECONOMETRIC MODEL – BOX–COX TRANSFORMATION

A number of models exists with a vintage asset price as the dependent variable when estimating the rate of economical depreciation⁴. In this study a so called Box–Cox transformation model is used⁵.

The model is especially suited for estimating the rate of economic depreciation. As mentioned above the model includes as special cases – exponential, straight–line and one–horse–shay – all the economic depreciation patterns that are mentioned in the economic literature.

The model is

$$q_i^* = \alpha + \beta \cdot s_i^* + u_i \quad (15)$$

where

$$q_i^* = \frac{q_i^{\theta_1} - 1}{\theta_1}, \quad (16a)$$

$$s_i^* = \frac{s_i^{\theta_2} - 1}{\theta_2}, \quad (16b)$$

$$u_i \in N(0, \sigma^2) \forall i \quad (17)$$

$$i = 1, \dots, n$$

with the following definitions of variables

q_i = ratio between market transaction price and purchase price at 1985–prices of asset i

s_i = age of the asset i when the market transaction took place

n = number of observations

⁴ For a discussion of different attempts, see Hulten and Wykoff (1980c).

⁵ This transformation model was developed by Box and Tidwell (1962) and Box and Cox (1964).

The random disturbance term, u_i , is assumed to be normally distributed with a zero mean and a constant variance. The estimate is made on the transformed variables⁶.

The variable q_i can be interpreted as the price for a used asset with purchase price 1 SEK. It therefore takes the value between 0 to 1 if the value does not increase over to time. This model corrects for inflation a priori by deflating the variable q_i . Hulten and Wykoff define the dependent variable as the market transaction price in current prices of a asset i . In addition to the independent variable, s_i , they use an independent variable that is defined as the time when the market transaction took place.

The unknown parameters $\theta = (\theta_1, \theta_2)$ determine the functional form (within the Box-Cox alternative) and the unknown parameters $b = (\alpha, \beta)$ determine the intercept and the slope on the transformed model. As the θ -vector varies, the form of equation (15) changes:

$\theta = (1,1)$ implies a linear pattern for the equation (15)

$$q_i = (1 + \alpha - \beta) + \beta s_i + u_i. \quad (18)$$

An exponential pattern (semi-log) is obtained if $\theta \rightarrow (0,1)$

$$\ln q_i = (\alpha - \beta) + \beta s_i + u_i, \quad (19)$$

⁶ An alternative variable definition defines the dependent variable as market transaction price and expand the right side of the equal-sign in equation (15) with a variable for the cost price, both in current prices. The advantage with (15) is that it is a univocal rate of economic depreciation for a given age.

⁷ $\lim_{\theta \rightarrow 0} \frac{q^\theta - 1}{\theta} = \lim_{\theta \rightarrow 0} \frac{e^{\theta \ln q} - 1}{\theta}$, L'Hospitals rule \rightarrow , $\lim_{\theta \rightarrow 0} \frac{e^{\theta \ln q} - 1}{\theta} =$
 $\lim_{\theta \rightarrow 0} \frac{\ln q e^{\theta \ln q}}{1} = \ln q$

$$q_i = \exp((\alpha - \beta) + \beta s_i + u_i). \quad (19')$$

The vector $\theta = (1,3)$ implies a one-horse-shay pattern

$$q_i = (1 + \alpha - \beta/3) + \beta s_i^3 + u_i. \quad (20)$$

The double-log pattern, finally is given by $\theta \rightarrow (0,0)$

$$\ln q_i = \alpha + \beta \ln s_i + u_i. \quad (21)$$

In terms of equation (15) the rate of economic depreciation can be evaluated. The real price is

$$q_i = \bar{q}_i/d \quad (22)$$

where d is a machine price index and \bar{q}_i is the nominal price on capital asset i .

Derivation of \bar{q}_i on the age, s_i , yields

$$\frac{d \bar{q}_i}{d s_i} = \frac{\partial \bar{q}_i}{\partial q_i} \cdot \frac{d q_i}{d s_i} = d \cdot \frac{d q_i}{d s_i} = d \cdot \beta \cdot q_i^{(1-\theta_1)} \cdot s_i^{(\theta_2-1)} \quad (23)$$

which yields the rate of economic depreciation

$$\delta_i = \frac{\Delta d \bar{q}_i}{d s_i} \cdot \frac{1}{q} = \beta \cdot \frac{s_i^{(\theta_2-1)}}{q_i^{\theta_1}}. \quad (24)$$

where δ_i is the rate of economic depreciation for the observation number i .

The rate of economic depreciation will be negative if β is negative, because s and q are positive. From equation (23) follows that δ_i is normally not constant over its length of life. A constant rate of economic depreciation is obtained only if $\theta_1=0$ and $\theta_2=1$.

The estimated rate of economic depreciation is

$$\hat{\delta}_i = \hat{\beta} \cdot \frac{s_i (\hat{\theta}_2 - 1)}{\hat{q}_i \hat{\theta}_1} . \quad (24')$$

4 THE DATA INFORMATION – VINTAGE ASSET PRICES

4.1 Description of the data material

The data originates from ASEA and gives information about how the secondary market values used assets. Observed variables during the period 1983–86 are purchase price, market transaction price, the year when the market transaction took place and the age of the asset. The material originates from a relatively homogeneous group: milling tools, lathes, drills, grinders e t c. The number of observations is 39. The econometric model was estimated using the method of maximum likelihood techniques. The obtained estimates are consequently only optimal asymptotically. It would be desirable to develop this study using a large time–series material for different types of capital assets.

4.2 WEAKNESS AND POTENTIAL ADJUSTMENTS OF THE DATA

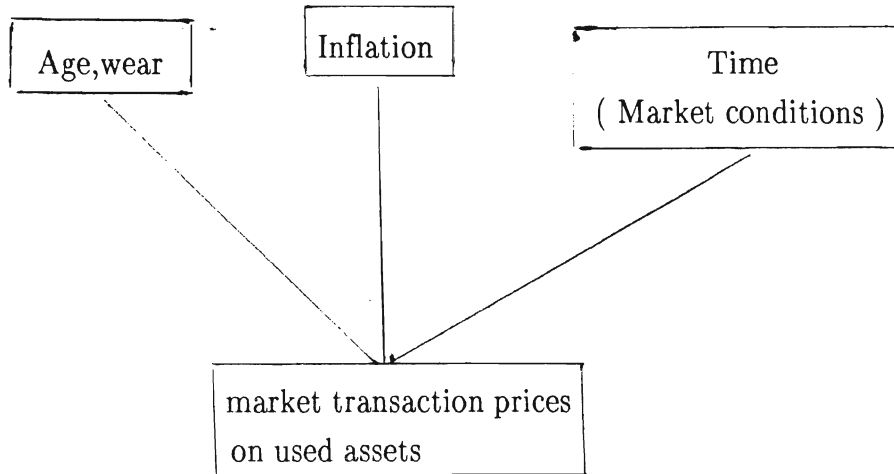
The problems that arise when estimating the rate of economic depreciation based on used asset prices can be divided into three groups:

- a/ potential excluded variables
- b/ censored sample bias
- c/ "lemons and pearls"

4.2.1 Potential excluded variables

The price data we have available are not only affected by age, but also by factors such as inflation and current market conditions. (See figure 4.)

Figure 4 General model structure



The nominal price for an asset is usually influenced negatively by age and positively by inflation⁸. In this study the price is corrected for inflation.

A more precise variable, that adjusts for whether the asset is used during one or more shifts, would be preferable. An asset that has been used in only one shift ought to have a lower rate of economic depreciation compared to the latter. Unfortunately the data material does not give any information about that problem.

Can the effect of time on asset prices be characterized by a smooth, predictable process? Is it possible to predict the future with only past estimates available? Feldstein and Rothschild (1974) note that prices on used assets depend on taxes, interest rates and a lot of other variables which are subject to change over time. Our data material is unfortunately not a

⁸ Exception is capital of investment value, for example veteran car where a value increase happens due to age.

time-series. Thus it is impossible to test if the estimates were stable over the past years. This could be done by dividing the time-series into annual components and then test each year's estimates for stability over time. Hulten and Wykoff (1980b) have done this cross-sectional analysis. They conclude that there is a reasonably good stability of economic depreciation rates over time.

On an underdeveloped secondary market there are objections against using used asset prices when estimating the rate of economic depreciation. The prices on these markets do not reflect the true value of an asset. It is reasonable, however, to assume that the market for used machines is well developed and as a result the true value is reflected in prices.

4.2.2 Censored sample bias

The concept of censored sample bias means that the prices from the secondary market only reflect those assets that survived until the time when the sample was taken. In investment theory, tax policy, capital measurement etc, the economic depreciation pattern of the typical asset from the original cohort is of interest, compared to the assets that have survived the longest. An analysis based on only the survivor implies an overestimate of the net capital stock e t c.

In order to correct for this problem Hulten and Wykoff (1980b) have used Winfrey's (1935) survivor curves⁹. For a given age and an assumption about the average age of retirement, these curves yield probability for surviving. The market transaction price has been corrected by multiplying each price by an estimate of the probability of survival. For example, the price of a seven year old asset is multiplied by the probability of having survived seven years, i e of not being retired during the first seven years.

Even if this approach is an interesting one, it is not completely satisfactory. An independent estimate of the rate of economic depreciation

⁹ See page 3 for a description of Winfrey's survivor curves.

should use as few a priori assumptions as possible. Hulten and Wykoff use, as mentioned above, Winfrey's survivor curves. An alternative approach is to use Mills ratio¹⁰ where retirement is treated as a stochastic process which depends on the value of the asset as well as on its age. The random disturbance is defined as a probability function which describes the probability for surviving to a certain time. I see this approach as a very promising one and hope to develop a method for it in the near future. In this study, no adjustment has been made for censored sample bias.

4.2.3 Lemons and Pearls

The "Lemons and Pearls" (Akerlof, 1970) problem is rooted in the systematic difference between assets which are sold on the open market and those which are not. The assets that come out on the open secondary market are of inferior quality, so called "lemons". The owner keeps all the superior assets, "pearls", in his machinery and sells the "lemons" on the open market. If the owner wants to sell a "pearl", he sells it to a person he knows and trusts. Mutual trust between buyer and seller means that a pearl can be sold for a higher price compared to the open market. As a consequence, the prices on the open market are below the average value of all the used assets that are in service.

This argument assumes asymmetrical information. If buyers and sellers have complete information, this distortion of prices will not occur. An attempt to test this hypothesis has been made. ASEA asked machine dealer to make offers on a number of machines. This apparent sale was motivated in relation to the buyer with a changed product program or that they had double machines and considered to get rid of one of them e t c. An inclosure of these machines when estimating equation (14) does not change the estimates considerably, as is evident from APPENDIX B. The hypothesis regarding Lemons and Pearls does not seem to have relevance for this types of

¹⁰ See for example Fomby, Hill, Johnson (1984).

machines¹¹.

A test of the hypothesis above has been done by Hulten and Wykoff (1980a) in the United States. They compared the prices on the open market with the closed market. No substantial difference was found.

5 ESTIMATION OF THE MODEL

The Box–Cox model was estimated using maximum likelihood techniques. The natural logarithm for the likelihood function is

$$\ln L(\boldsymbol{\theta}, \mathbf{b}, \sigma^2 | \mathbf{q}, \mathbf{Z}) = k + (\theta_1 - 1) \sum_{i=1}^n \ln q_i - \frac{n}{2} \ln \sigma^2 - \frac{1}{2\sigma^2} (\mathbf{q}^* - \mathbf{Zb})' (\mathbf{q}^* - \mathbf{Zb}) \quad (25)$$

where

$$k = -\frac{n}{2} \ln \pi$$

and

$$\boldsymbol{\theta} = \begin{matrix} \Delta \\ (\theta_1, \theta_2) \end{matrix} \quad (26)$$

$$\mathbf{b} = \begin{matrix} \Delta \\ (\alpha, \beta) \end{matrix} \quad (27)$$

$$\mathbf{Z} = \begin{matrix} \Delta \\ (1, \mathbf{s}^*) \end{matrix}. \quad (28)$$

The matrix \mathbf{Z} is consequently a function of the parameter θ_2 via (16b).

¹¹ The credibility of the test depends on how the machine dealers interpret these machines. If the machines are valued with the idea that they are ‘‘pearls’’, the above conclusion can be drawn.

The maximum likelihood function can be concentrated with regard to \mathbf{b} and σ^2 (see APPENDIX C), which yield a function only depending on θ , as seen below.

$$\ln \bar{L}(\theta | \mathbf{q}, \mathbf{Z}) = k' + (\theta_1 - 1) \sum_{i=1}^n \ln q_i - \frac{n}{2} \ln((\mathbf{q}^* - \mathbf{Z}\hat{\mathbf{b}})'(\mathbf{q}^* - \mathbf{Z}\hat{\mathbf{b}})) \quad (29)$$

where

$$k' = k + \frac{n}{2}(\ln n - 1)$$

and $\hat{\mathbf{b}}$ is the OLS-estimate $(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'\mathbf{q}^*$ of \mathbf{b} , for given values of $\hat{\theta}_1$ and $\hat{\theta}_2$, i.e. $\hat{\mathbf{b}} = \mathbf{b}(\hat{\theta})$.

Because of the models non-linearity in the parameters it is not possible to solve the first order conditions to equation (29) analytically (C7, C8' and 13 in APPENDIX C). Equation (29) has consequently been maximized with a numerical iterative method, a so called quasi-Newton method. (See Fletcher, 1970.) For this purpose a Fortran program has been made¹².

6 EMPIRICAL RESULTS AND COMPARISON WITH OTHER STUDIES

6.1 Empirical results

Table 1 contains the maximum likelihood estimate with standard errors in parenthesis.

¹² The algorithm and the Fortran code will be described in a forthcoming IUI Working-paper.

Table 1

$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\alpha}$	$\hat{\beta}$
0.400	-0.184	-0.460	-0.538
(0.162)	(0.582)	(0.394) ¹³	(0.541)

The $\hat{\mathbf{b}}$ -vector is significantly separated from zero on 95 percent level. The estimate of $\hat{\boldsymbol{\theta}}$ and \mathbf{b} imply that the economic pattern is given by equation (30).

$$\hat{q}_i = (-0.351 + 1.167 \cdot s_i^{-0.184})^{2.503} \quad (33)$$

The first-order

$$\frac{\partial \hat{q}}{\partial s} = \hat{\beta} \cdot \hat{q}_i^{1-\hat{\theta}_1} \cdot s_i^{\hat{\theta}_2-1} = -0.538 \cdot \hat{q}_i^{0.600} \cdot s^{-1.184} < 0 \quad (34)$$

and second-order

$$\begin{aligned} \frac{\partial^2 \hat{q}}{\partial s^2} &= \frac{\hat{\theta}_2 - 1}{s_i} \cdot \frac{\partial \hat{q}}{\partial s} + \frac{1 - \hat{\theta}_1}{\hat{q}_i} \cdot \left[\frac{\partial \hat{q}}{\partial s} \right]^2 = \\ &= \frac{-1.184}{s_i} \cdot \frac{\partial \hat{q}}{\partial s} + \frac{0.600}{\hat{q}_i} \cdot \left[\frac{\partial \hat{q}}{\partial s} \right]^2 > 0 \end{aligned} \quad (35)$$

partial derivatives of equation (33) with respect to s show that equation (33) is strictly convex. Equation (33) is plotted in APPENDIX E. Its form resembles the exponential depreciation pattern with a fast economic depreciation for a new asset which declines almost to zero as it reaches the end of the economic life time. However, neither the exponential, linear nor one-horse-shay functional form is significant at a 5 percent level (

¹³ The standard errors for the \mathbf{b} -vector has been derived with Gauss approximations formula – see APPENDIX E.

APPENDIX E).

In accordance with economic theory, $\hat{\beta}$ takes a negative value which implies that the rate of economic depreciation is negative. In accordance with equation (24') the rate of economic depreciation is

$$\hat{\delta}_i = \hat{\beta} \cdot \frac{s_i \hat{\theta}_2 - 1}{q_i \hat{\theta}_1} .$$

Estimated values for delta are reported in table 2. As can be seen from table 2 the rate of economic depreciation is higher for a young asset as compared to an old. Note that the true rate of economic depreciation is somewhat higher (in absolute terms) because of censored sample bias.

As a measure of goodness-of-fit the relation (37) has been used (Haessel, 1978).

$$\cos^2 \phi = \left[\frac{\sum_{i=1}^n (q_i - \bar{q})(\hat{q}_i - \bar{\hat{q}})}{\sum_{i=1}^n (q_i - \bar{q})^2} \right]^2 / \frac{\sum_{i=1}^n (q_i - \bar{q})^2}{\sum_{i=1}^n (\hat{q}_i - \bar{\hat{q}})^2} . \quad (37)$$

$\cos^2 \phi$ takes the value 38.0%. This is considered relatively good, given that the data material is cross-sectional.

Table 2 Rate of Economic depreciation (in percent)

<u>Age</u>	<u>Rate of Economic Depreciation</u>	<u>95% confidence –interval¹⁴</u>
2	–35.0	(–42.1, –27.9)
4	–18.8	(–20.2, –17.5)
5	–15.5	(–16.3, –14.6)
6	–13.2	(–13.5, –12.9)
7	–11.6	(–18.0, – 5.1)
8	–10.3	(–10.4, – 9.8)
10	– 8.5	(– 9.3, – 7.7)
11	– 7.9	(– 8.5, – 7.2)
12	– 7.3	(– 8.4, – 6.2)
13	– 6.8	(– 7.6, – 6.0)
14	– 6.4	(– 8.2, – 4.6)
18	– 5.2	(– 6.2, – 4.3)
19	– 5.0	(– 5.8, – 4.2)

6.2 Comparisons with other studies

Hulten and Wykoff have made similar studies during the 1980s. Their model is of the same type as the model described in chapter three. Some differences should be noted. The dependent variable is defined as the purchase price in current prices, of asset i . In addition to the independent variable, s_i , an independent variable is also included, which is defined as the time when the market transaction took place. The maximum likelihood technique is thereafter applied on a large time serie material. Their estimate is shown in table 3 below.

¹⁴ The standard errors for δ_i has been derived with Gauss approximations formula – see APPENDIX E.

Table 3 **Rate of Economic depreciation for
metal-working machinery**

Age	A	B
	Percent	
1	19.8	13.1
5	11.1	7.0
10	9.3	5.6
15	9.1	5.2
20	9.7	5.1
25	11.0	5.2

Source: Hulten and Wykoff (1980c)

Column A shows estimates for data where an adjustment for censored sample bias has been made with Winfrey's survivor curves¹⁵ and column B shows estimates without any adjustment.

The exponential depreciation pattern is not unambiguously significant for the example described in table 2 or their other types of assets. To examine if the exponential depreciation pattern can be used as a good approximation they have adapted the predicted Box–Cox prices for adjusted data to an exponential relation. They received high R^2 -values throughout (around 97%). The estimated rate of economic depreciation was –14.7% per year for instruments. Their conclusion is that a constant rate of economic depreciation constitutes a good approximation.

Using a similar approach adjusting an exponential functional form, a $\hat{\delta}$ of –9.8% and a R^2 of 92.7% is obtained. But what does this imply? It implies that the exponential functional form is a good approximation for the depreciation pattern with the predicted Box–Cox prices. Of relevance is, how good an exponential adjustment is to the real prices. An OLS estimation of equation (18) results in a $\hat{\delta}$ of –9.0% and a R^2 of only 22%. (Observe though

¹⁵ See page 17 for description.

that the data material is cross sectional data and not time series data. R^2 shows throughout lower values with cross sectional data.) The adaptation of an exponential functional form is consequently not so good.

Södersten and Lindberg (1983) have evaluated the rate of economic depreciation which is implicitly assumed in SCB's calculations¹⁶. The calculations has been made for the period 1949–1979. The estimate is nearly constant over the period. Their results is -7.7% per year for the manufacturing industry. Why is SCB's estimate so different from the estimates in this study? One reason could be that SCB's base assumptions (Winfrey's survivor curves from 1935, Wallander's average age for retirement from 1962 e t c) does not show the reality.

In the Confederation of Swedish Industries planning survey 1978 and 1987 the question was asked " How long economic lifetime (years) do you think that your latest installed most important machines have?". A weighted average value¹⁷ gave a lifetime of 14.3 and 12.7 years respectively. Assuming that the economic depreciation pattern is exponential, the rate of economic depreciation is 7.0 and 7.9% respectively¹⁸. The possibility the companies do not know the economic life time, and most likely underestimate it, is inevitable raised.

7 SUMMARY

A model in which the economic depreciation and the rate of economic depreciation are simultaneously estimated based on prices from the secondary market has been designed. The model makes it possible to statistically test the different depreciation patterns that are discussed in the economic

¹⁶ Se page 3.

¹⁷ The companies machine investment during 1976, 1977 and planned 1978 respectively 1985, 1986 and planned 1987 was used as a weight.

¹⁸ When the depreciation pattern adapts an exponential form the rate of depreciation is the invers of the average life time.

literature. This model has been applied on a Swedish data material.

The results does not support the common idea that the depreciation pattern is exponential. Normal calculations today in Sweden of the rate of economic depreciation are thereby incorrect. For this specific data material a strictly convex depreciation pattern exists with respect to age. A fast economic depreciation for a young asset and a slow economic depreciation for an old asset. A constant rate of economic depreciation, implicitly assumed by SCB, is consequently wrong for this type of asset.

Likewise SCB's calculations of the gross capital stock is incorrect. A strict convex economic depreciation pattern is inconsistent with the sudden death assumption which is made in SCB's gross capital stock calculations. (Sudden death implies a concave economic depreciation pattern, as can be seen from figure 3.)

As has already been emphasized, the estimates only hold for the specific observed machines types in the manufacturing industry and at the specific time. It would be very interesting to develop this study with a larger data material.

APPENDIX A - DATA MATERIAL

Type	Year of purchase	Year of vend	Purchase price (85-years price,tSEK)	Vend price (85-years price,tSEK)
Rörbocknings- maskiner	1975	1985	1 026	116
Klyvsåg	1975	1985	64	8
Svetsläges- tällare	1977	1984	120	28
Mätmaskin BOOM	1980	1984	850	334
Sågaggregat	1978	1983	35,1	2,1
Blästeranlägg- ning SH 1	1976	1983	75	5,3
Blästerskåp	1976	1983	854	8,6
Borrslipmaskin	1973	1983	160	16
Supportsvarv NC	1979	1983	737	288
Sajo	1982	1986	2 001	336
Traub	1982	1986	1 441	504
Sajo	1980	1986	656	280
Max Müller	1980	1986	2 972	336
Schiess	1981	1986	2 906	280
Schiess	1980	1986	3 590	448
Sajo	1979	1986	1 210	336
Sundstrand	1981	1986	6 504	512
Fortuna	1982	1986	434	202
Karusellsvarv	1974	1984	8 904	1 493
Tunnelugn	1982	1984	841	258
Borr o. Fräsv.	1966	1984	13 840	927
Borr o. Fräsv.	1972	1984	3 268	102
Karusellsvarv	1974	1984	5 285	747
Bordhyvel	1972	1984	934	515
Cirkelkalisåg	1965	1984	494	515
Slungbläster med transport- anläggning	1977	1984	1 065	175
Karusellsvarv	1971	1984	2 049	247
Supportsvarv	1976	1984	862	82
Kuggslipmaskin	1965	1984	1 386	114
Kuggslipmaskin	1976	1984	1 748	463
Revolverborr- maskin	1974	1984	1 138	46
Blästerautomat	1980	1985	378	100
Radialborr- maskin	1973	1985	396	65
Karusellsvarv	1972	1985	2 434	210
Formsprutnings- maskin	1974	1985	474	60
Rev. borrar- maskin	1971	1985	2 122	50
Utmattnings- maskin	1983	1985	446	35
Revolverautomat	1975	1985	2 244	350
Lateralfräsverk	1974	1985	124	40

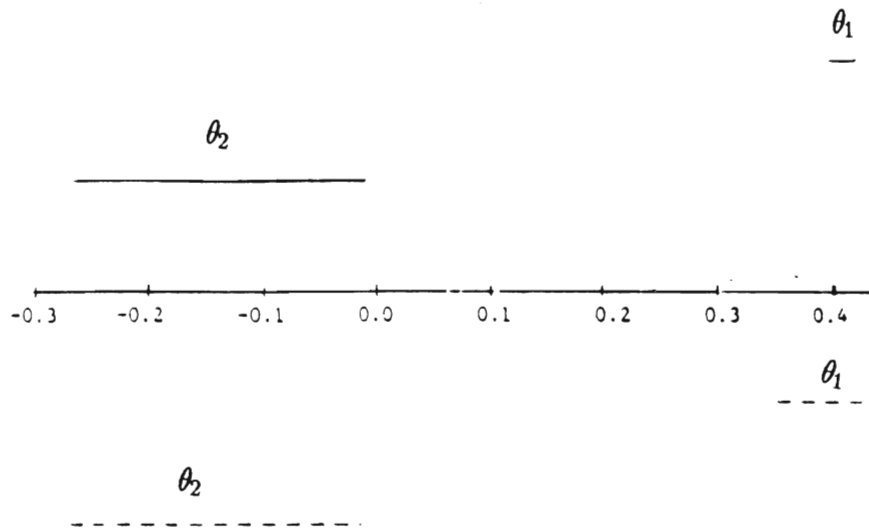
Machineprice index:

1963	100.0
1964	103.6
1965	108.7
1966	112.1
1967	116.2
1968	118.3
1969	115.7
1970	125.9
1971	135.7
1972	142.8
1973	153.2
1974	178.0
1975	202.8
1976	222.9
1977	244.5
1978	273.5
1979	292.7
1980	321.5
1981	350.5
1982	399.8
1983	450.7
1984	466.7
1985	480.5
1986	485.0

Sources: Statistiska Meddelanden 1981–1985 and Analysunderlag till Konjunkturläget dec 1985

APPENDIX B – Lemons and Pearls:

95% – confidence interval for θ :



_____ confidence interval for machines that has been sold,
number of observations are 39

_____ confidence interval based on machines that has been sold
and those that an offer has been made, number of observations are 48.

APPENDIX C

Concentration of the maximum likelihood function with respect to \mathbf{b} and σ^2 are calculated as follows.

$$\ln L(\boldsymbol{\theta}, \mathbf{b}, \sigma^2 | \mathbf{q}, \mathbf{Z}) = k + (\theta_1 - 1) \sum_{i=1}^n \ln q_i - \frac{n}{2} \ln \sigma^2 - \frac{1}{2\sigma^2} (\mathbf{q}^* - \mathbf{Zb})' (\mathbf{q}^* - \mathbf{Zb}) \quad (\text{C1})$$

where

$$k = -\frac{n}{2} \ln 2\pi.$$

The first-order partial derivatives with respect to σ^2 and \mathbf{b} are

$$\frac{\partial \ln L}{\partial \sigma^2} = -\frac{n}{2} \cdot \frac{1}{\sigma^2} + \frac{1}{2\sigma^4} (\mathbf{q}^* - \mathbf{Zb})' (\mathbf{q}^* - \mathbf{Zb}) \quad (\text{C2})$$

respectively

$$\frac{\partial \ln L}{\partial \mathbf{b}} = \frac{1}{2\sigma^2} (2\mathbf{q}^* \mathbf{Z}' - 2\mathbf{b}' \mathbf{Z}' \mathbf{Z}). \quad (\text{C3})$$

The first order condition results in

$$\hat{\mathbf{b}} = (\mathbf{Z}' \mathbf{Z})^{-1} \mathbf{Z}' \mathbf{q}^* \quad (\text{C4})$$

and

$$\hat{\sigma}^2 = \frac{1}{n} (\mathbf{q}^* - \mathbf{Z}\hat{\mathbf{b}})' (\mathbf{q}^* - \mathbf{Z}\hat{\mathbf{b}}). \quad (\text{C5})$$

Insertion of (C4) and (C5) in (C1) gives the concentrated log-likelihood function

$$\ln \bar{L}(\theta | \mathbf{q}, \mathbf{Z}) = k' + (\theta_1 - 1) \sum_{i=1}^n \ln q_i - \frac{n}{2} \ln((\mathbf{q}^* - \mathbf{Z}\hat{\mathbf{b}})'(\mathbf{q}^* - \mathbf{Z}\hat{\mathbf{b}})) \quad (\text{C6})$$

where $k' = k + \frac{n}{2}(\ln n - 1)$.

Differential of $\ln \bar{L}$ with respect to θ_1 gives

$$\frac{\partial \ln \bar{L}}{\partial \theta_1} = \sum_{i=1}^n \ln q_i - \frac{1}{\sigma^2} (\mathbf{q}^* - \mathbf{Z}\hat{\mathbf{b}})' \frac{\partial \mathbf{q}^*}{\partial \theta} \quad (\text{C7})$$

Differential of $\ln \bar{L}$ with respect to θ_2 gives

$$\begin{aligned} \frac{\partial \ln \bar{L}}{\partial \theta_2} &= -\frac{n}{2} \cdot \frac{1}{n\sigma^2} \left[(-2) \frac{\partial \mathbf{q}^* \mathbf{Z}' \hat{\mathbf{b}}}{\partial \mathbf{Z}^C} \cdot \frac{\partial \mathbf{Z}^C}{\partial \theta_2} + \frac{\partial \hat{\mathbf{b}}' \mathbf{Z}' \hat{\mathbf{b}}}{\partial \mathbf{Z}^C} \cdot \frac{\partial \mathbf{Z}^C}{\partial \theta_2} \right] = \\ & \frac{1}{\sigma^2} \left[(\hat{\mathbf{b}}' \otimes \mathbf{q}^*) \frac{\partial \mathbf{Z}^C}{\partial \theta_2} - \frac{1}{2} (\hat{\mathbf{b}}' \otimes \hat{\mathbf{b}}') \cdot \frac{\partial (\mathbf{Z}' \mathbf{Z}^C)}{\partial \mathbf{Z}^C} \cdot \frac{\partial \mathbf{Z}^C}{\partial \theta_2} \right]. \end{aligned} \quad (\text{C8})$$

Because

$$\frac{\partial \mathbf{Z}^C}{\partial \theta_2} = \left[0', \left(\frac{\partial \mathbf{s}^*}{\partial \theta_2} \right)', 0' \right]' \quad (\text{C9})$$

follows that

$$(\hat{\mathbf{b}}' \otimes \mathbf{q}^*) \frac{\partial \mathbf{Z}^C}{\partial \theta_2} = \hat{\beta} \mathbf{q}^* \frac{\partial \mathbf{s}^*}{\partial \theta_2}. \quad (\text{C10})$$

Further more is

$$\frac{\partial (\mathbf{Z}' \mathbf{Z}^C)}{\partial \mathbf{Z}^C} = (\mathbf{I}_3 \otimes \mathbf{Z}') + (\mathbf{Z}' \otimes \mathbf{I}_3) \mathbf{K}_{3n} \quad (\text{C11})$$

and

$$(\hat{\mathbf{b}}' \otimes \hat{\mathbf{b}}') \cdot \frac{\partial(\mathbf{Z}' \mathbf{Z}^c)}{\partial \mathbf{Z}} = 2(\hat{\mathbf{b}}' \otimes \hat{\mathbf{b}}' \mathbf{Z}') \quad (\text{C12})$$

Insertion of (C9), C(10), (C11) and (C12) in (C8) gives

$$\frac{\partial \ln L}{\partial \theta_2} = \frac{1}{\sigma^2} \left[\hat{\beta}_{\mathbf{q}}^* \frac{\partial \mathbf{s}^*}{\partial \theta_2} - \hat{\beta}_{\mathbf{b}}' \mathbf{Z}' \frac{\partial \mathbf{s}^*}{\partial \theta_2} \right] = -\frac{\hat{\beta}}{\sigma^2} (\mathbf{q}^* - \mathbf{Z} \hat{\mathbf{b}})' \frac{\partial \mathbf{s}^*}{\partial \theta} . \quad (\text{C8'})$$

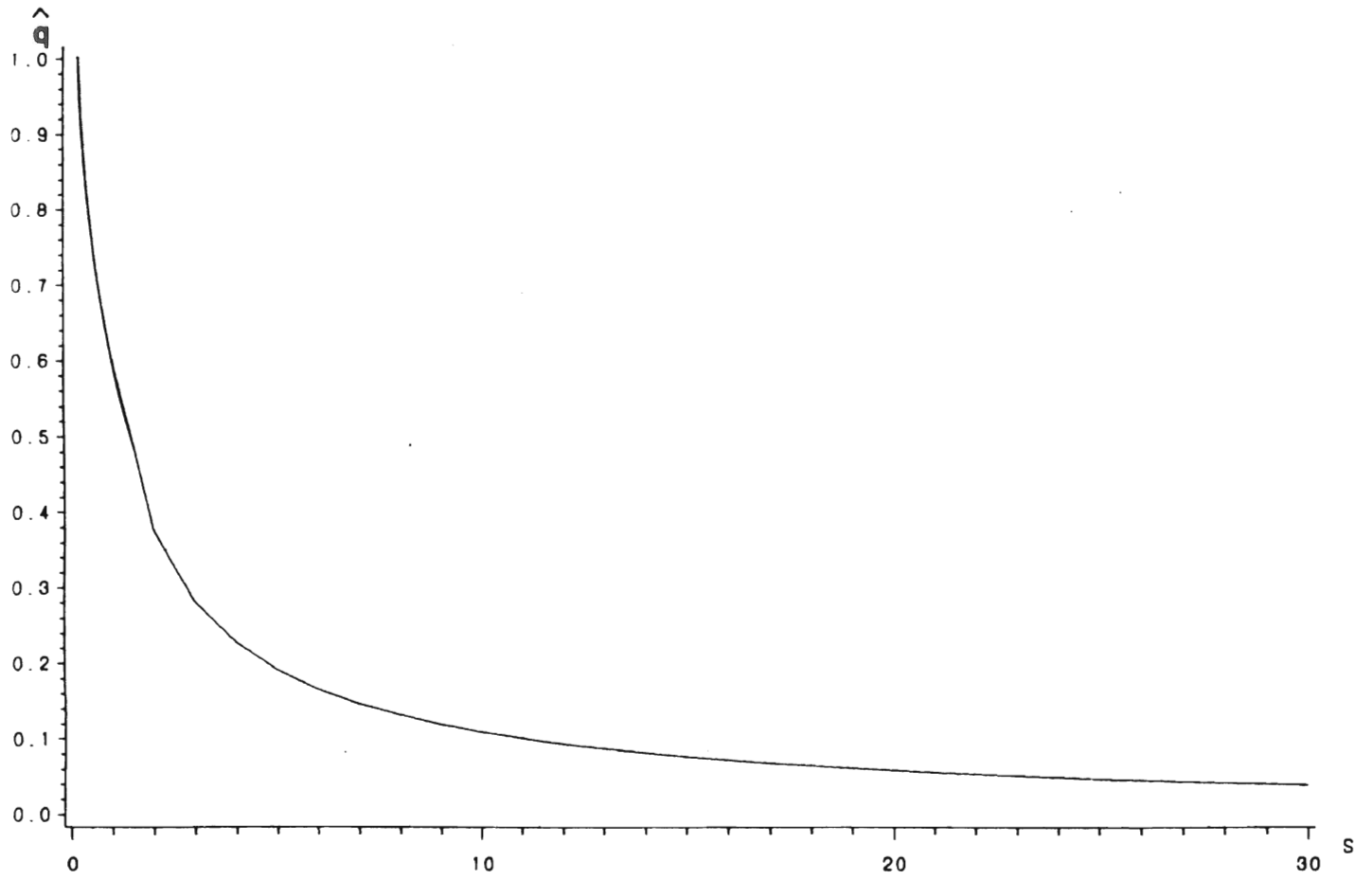
The partial derivatives $\partial \mathbf{q}^* / \partial \theta_1$ in (C7) and $\partial \mathbf{s}^* / \partial \theta_2$ in (C8') have as typical elements

$$\frac{\partial \mathbf{q}_i^*}{\partial \theta_1} = \left[\left(\mathbf{q}_i^* + \frac{1}{\theta_1} \right) \ln \mathbf{q}_i - \frac{\mathbf{q}_i^*}{\theta_1} \right] \quad (\text{C13})$$

and

$$\frac{\partial \mathbf{s}_i^*}{\partial \theta_1} = \left[\left(\mathbf{s}_i^* + \frac{1}{\theta_2} \right) \ln \mathbf{s}_i - \frac{\mathbf{s}_i^*}{\theta_2} \right]. \quad (\text{C14})$$

APPENDIX D



APPENDIX E

Confidence interval for θ :

The estimated standard errors are valued asymptotic which implies

$$\hat{\theta} \in N(\theta, s^2/n) \quad (E1)$$

The confidence interval for θ_1 is given by (E2) at significance level α

$$I_{\hat{\theta}_1} : \left[\hat{\theta}_1 \pm (\lambda_{\alpha/2}) \cdot \frac{s_i}{\sqrt{n}} \right] \quad (E2)$$

where $n = 39$. For $\alpha = 0.05$ the confidence interval for $\hat{\theta}$ are:

$$I_{\hat{\theta}_1} : \left[0.349, 0.450 \right] \quad (E4)$$

$$I_{\hat{\theta}_2} : \left[-0.350, -0.018 \right]. \quad (E5)$$

The linear, exponential and one-horse-shay economic depreciation pattern is thereby not significant at a 95% – level.

Confidence interval for b:

$$\text{Var}(\hat{\alpha}) = \frac{\partial \mathbf{b}_1}{\partial \theta} \cdot \hat{\Sigma}_{\theta} \cdot \left(\frac{\partial \mathbf{b}_1}{\partial \theta} \right)' = 0.1535 \quad (E6)$$

$$\text{Var}(\hat{\beta}) = \frac{\partial \mathbf{b}_2}{\partial \theta} \cdot \hat{\Sigma}_{\theta} \cdot \left(\frac{\partial \mathbf{b}_2}{\partial \theta} \right)' = 0.2929 \quad (E7)$$

where

$\frac{\partial \mathbf{b}_i}{\partial \theta}$ is the i :th row in the $\frac{\partial \mathbf{b}}{\partial \theta}$ – matrice.

For $\alpha = 0.05$ is the confidence interval for \mathbf{b} are:

$$I_{\hat{\alpha}} : \left[-0.58, -0.34 \right] \quad (\text{E8})$$

$$I_{\hat{\beta}} : \left[-0.71, -0.37 \right]. \quad (\text{E9})$$

The $\hat{\mathbf{b}}$ – vector is significant separated from zero at a 95 percent level.

δ_i 's variance:

$\hat{\delta}_i$ is approximated by formula (E10) and an approximation of $\hat{\delta}_i$'s variance is thereafter obtained by Gauss approximation formula.

$$\hat{\delta}_i \approx \hat{\beta} \cdot \frac{s_i^{\hat{\theta}_2-1} \Delta}{q_i^{\hat{\theta}_1}} = g(\hat{\theta}_1, \hat{\theta}_2) \quad (\text{E10})$$

so the variance for $\hat{\delta}_i$, $V(g(\hat{\theta}_1, \hat{\theta}_2))$, is

$$V(g(\hat{\theta}_1, \hat{\theta}_2)) \approx \sum_{i=1}^3 V(\hat{\theta}_i) \left(\frac{\partial g}{\partial \theta_i} \right)^2 + 2 \sum_{i < j} \text{Cov}(\hat{\theta}_i, \hat{\theta}_j) \cdot \left(\frac{\partial g}{\partial \theta_i} \right) \left(\frac{\partial g}{\partial \theta_j} \right). \quad (\text{E11})$$

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