Welfare Effects of Changes in Income Tax Progression in Sweden

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1. INTRODUCTION

In this paper we will investigate the implications of the theory of optimal income taxation for the graduation of the income tax schedule in Sweden.

The optimal income tax problem deals with the trade-off between equality and efficiency, when deciding on the progressiveness of the income tax schedule. The trade-off problems considered in the literature are of two kinds:

(i) between equity and efficiency losses due to distortions of labor-leisure choice. (See, e.g., Diamond, 1968; Mirrlees, 1971, and Phelps, 1973.)

(ii) between equity and distortions of the incentives to invest in human capital. (See, e.g., Atkinson, 1973, Phelps, 1973, and Sheshinski, 1976).

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* The authors wish to express their gratitude to Ragnar Bentzel, Michael Bruno, Martin Feldstein and Schlomo Yitzaki for helpful comments and conversations on an earlier draft of this paper, and to Mikael Jern for his programming assistance.

So far there are few works where these trade-offs have been studied in connection with an actual tax system.[1]

We will investigate the first mentioned trade-off problem in connection with the Swedish system of personal income taxation. Even though we cast the problem into an optimal taxation mould, we do not intend to find the optimal tax schedule. Instead we will look for welfare improving tax reforms.

The instrument used in this analysis is an extended version of the model for simulating the Swedish system of personal income taxation first presented in Jakobsson and Normann (1972). The original simulation model belongs to a class of models with explicit public parameters that by now is quite common.[2] This article might be seen as an attempt to indicate how these models can be extended to include behavioral relations, which opens up the possibility of using them for a broader range of problems than today.

The next section of the article is a description of the model used. We start by presenting the original simulation model by which tax revenues at the individual and aggregate levels can be computed. The original model provides us with one of the essential features of the optimal tax problem, namely a tax function defined on individual

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[1] The only examples we know of are Bruno and Habib (1976) and Rosen (1976). None of these works however did primarily investigate the rate structure of the tax system.

[2] For early examples or models of this type see Pechman (1970) and Rechtenwald (1972)

income. This model is then extended to encompass the other main ingredients of the optimal income tax problem as posed by Mirrlees (1971). These are individual utility functions defined on consumption and leisure, a skill distribution, a social welfare function defined on individual utilities, and a production relation. We give a fairly detailed description of how this extension is made in the last part of Section 2.

To find the optimal tax system, the social welfare function is maximized subject to two constraints. The first is that the individual maximizes his utility subject to his income constraint. The second is that the total labor supplied can produce the total quantity of goods demanded. Welfare improving tax reforms will analogously be tax changes that improve social welfare subject to these two constraints. Sections 3 and 4 of the article are devoted to finding that kind of tax changes, where the present Swedish tax system is the initial state. This is done by simulation in the extended tax model.

We find that under the assumptions usually made in the literature on optimal income taxation progression in the Swedish income tax should be decreased. The most striking result is that all statutory marginal tax rates should be diminished in brackets above Skr 30,000 (ca $7,500) which was well below the median income in 1975, the year covered in the study.[1] The main explanation for this turns out to be a "perverse revenue" effect. Revenues will actually be increased when marginal tax rates are diminished. The extra revenues could

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[1] Skr 30,000 in 1975 correspond to Skr 43,000 or $10,000 in 1979 prices.

be used for introducing a lump sum transfer. This combination of parameter changes will obviously increase the utility for everybody. Therefore the specification of the social welfare function is not important for the result mentioned, as long as we restrict ourselves to Paretian functions.

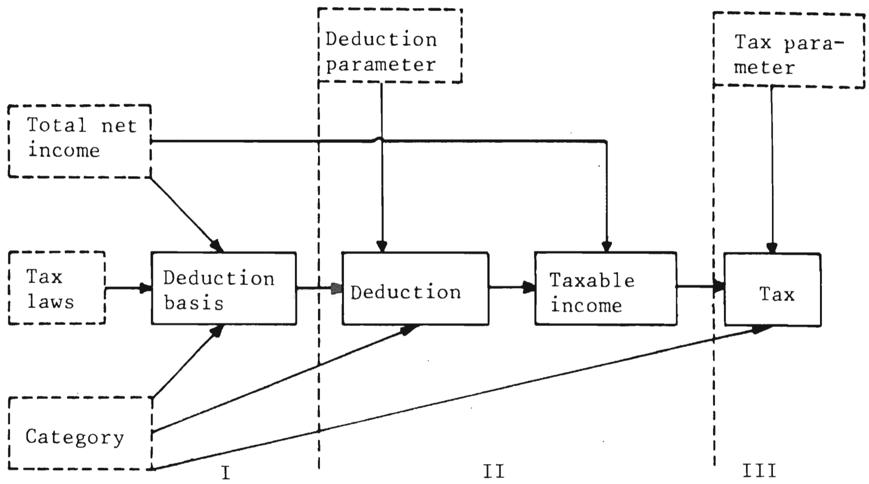
What is important, however, is the labor supply response to a change in marginal tax rates, since this response obviously is crucial for the "perverse revenue effect." In Section 5 we investigate how sensitive this effect is to different assumptions on the elasticity of substitution (σ) between consumption and leisure in the individual utility function. It is found that this effect appears in most rate brackets for $\sigma > 0.4$.

In the last section we briefly discuss what kind of conclusions can be drawn from our results.

2. MODEL DESCRIPTION

The original model consists of two parts namely a micro part and an aggregative part. The former part is constructed to compute the tax for a random individual. The individuals were partitioned in ten categories such that all individuals in a category are treated at least approximately equal by the tax laws. The categories are of the type single persons (age 17-66) without children, married men (age 17-66) and so on. An individual is characterized in the model not only by the category he belongs to but also by the level of his income before tax. Thus the micro model is an algorithm that for a given set of public parameters computes the tax for an individual on the

Figure 1. Chart of the micro-model



basis of two pieces of information of him, namely: (1) the individual's level of income before tax; (2) the category the individual belongs to.

As can be seen from Figure 1 the micro model is the place where the public parameters are introduced. Jakobsson and Normann (1972) give a short description[1] of how the tax laws were formalized and to some extent simplified so that they could be integrated in the model.

If we consider a specific category a condensed description of the micro model is given by:

$$t = F(y;\ P) \tag{1}$$

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[1] For a full description, see Jakobsson and Normann (1974).

where t = individual tax payments
y = individual income before tax[1]
P = set of deduction and tax parameters.

To get from (1) to a macro-relation between income and taxes an aggregation procedure is introduced. The one we have used relies on knowledge of the income distributions in different categories. Still considering a specific category the total tax (T), paid by the category is given by

$$T = N \cdot \int_{y_{min}}^{y_{max}} F(y;P) \cdot \psi'(y)dy \qquad (2)$$

where N = number of persons in the category,
$\psi'(y)$ = density function of incomes in the category.

In this simulation model it is possible to distinguish and compare the effects on, e.g., revenues and income distribution after tax of different specified changes in the parameter set. The level and distribution of income before taxes also appear explicitly so the built-in-flexibility of the tax system can be investigated. An important limitation of the model, however, is that income before tax is exogenous. By introducing, in the micro-model, utility maximizing choice between labor and leisure on part of indviduals, this assumption is relaxed in the present version of the model.

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[1] The income before tax concept used here is total net income (sammanräknad nettoinkomst). Our choice of this concept that is defined by the tax law has been dictated by the existing data on income distribution.

2.1. Individual Behavior

The assumptions on individual behavior made here are those of standard labor leisure analysis. We will thus assume that individuals have identical preferences and try to maximize their utility. It is also assumed that consumptions of goods (C) and consumption of leisure (L) enter a utility function U(C;L). Each individual makes his (C;L) choice under his budget constraint;

$$C = f(wH;P) \equiv wH - F(wH;P) \tag{3}$$

where H = hours worked (H=Q-L; Q=hours available)
w = wage rate
f represents the function from income before tax to income after tax.

The formulation of the budget constraint implies two assumptions, both common in the optimal tax literature:

(i) Savings are ignored.

(ii) Other income than wage income is ignored, i.e., y = wH.

In order to make a quantitative analysis it is necessary to be more specific on the form of the individual utility function. We have here chosen the standard assumption that the utility function is of the Cobb-Douglas type. In a special section we will discuss how sensitive our basic results are to this assumption.

On the assumption that the individual tries to maximize his utility, he will face the following optimum problem:

$$\text{Max } U = C^{\alpha}(Q-H)^{1-\alpha} \quad \text{subject to } C=f(wH;P) \tag{4}$$

The optimal labor supply of the individual will be

$$H = \frac{Q \cdot e \cdot \alpha}{1 - \alpha + e \cdot \alpha} \qquad (5)^{1}$$

where

$$e = \frac{f_1(wH;P) \cdot wH}{f(wH;P)} = \text{residual progression.}^{2}$$

If we suppose that the wage rate (w) for each individual is given exogenously then (5) in principle can be solved for H, provided that f is completely specified. Furthermore it is clear that to each specific set of public parameters (P) we get a related solution for H. So (5) defines a function from (w;P) to H or

$$H = g^{1}(w;P) \tag{6}$$

By (6), the budget-restriction (3), and the utility function we get

$$U = g^{2}(w;P) \tag{7}$$

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[1] Q stands for maximal labor supply. Supposing that there is a limit at 16 hours per day every day, we get for a full year Q=5,840. To get realistic values on labor supply we have chosen α=0.33. Experimentation with different values on α indicates that our results are not sensitive to changes in α.

[2] For a discussion of this concept see Jakobsson (1976).

Since we are assuming that $y = wH$, we also get by (6) and (1) individual tax payments

$$t = g^3(w;P) \tag{8}$$

2.2 Aggregation over Wage Rates

A basic difference between the micro-model defined by (1) and that defined by the preceding equations is that the wage rate is exogenous in the latter while income is exogenous in the original model. From the empirical point of view this represents a difficulty since the only information we have got on individuals is the distribution of income. In order to aggregate the model (6)-(8) it is therefore necessary to relate individual income in the initial position to wage rates. This is done by (5). At the existing tax system we can observe the income distribution before tax. Formula (5) then relates each income to a specific value of H. Since $y = wH$, we also get a specific wage rate associated with each income level in the initial stage. From the observed income distribution we can then derive a distribution of wage rates that is exogenously given in the model and constant throughout the experiments carried out here. For a specific category of income earners aggregate tax payments can be obtained as

$$T = N \cdot \int_{w_{min}}^{w_{max}} g^3(w;P)\ \phi'(w)dw \tag{9}$$

where $\phi'(w)$ is the "derived" distribution of wages. We will assume that this distribution is equivalent to the skill distribution in the optimal income tax problem. Concerning production we adopt the assumption that the production of each worker equals his wage.

2.3 The Social Welfare Function

A central element for the whole concept of an optimal tax schedule is an interpersonal comparison of utilities. The valuation of utilities for different persons is made by a social welfare function. The proper specification of this function is of course a very difficult problem. We have, however, chosen the form most commonly used in the literature on optimal taxation, namely addition of individual utilities raised to the power of $1-\varepsilon$, where ε could be interpreted as social inequality aversion (Atkinson, 1970) $(U^{1-\varepsilon}/(1-\varepsilon);$ $\varepsilon > 0;\ \varepsilon \neq 1)$. By this function we have social welfare

$$W = \frac{1}{1-\varepsilon} N \cdot \int_{w_{min}}^{w_{max}} [g^2(w;P)]^{(1-\varepsilon)} \phi'(w)dw \qquad (10)$$

Restricted as this form might seem it still allows for a wide range of social preference orderings. Included are the strictly utilitarian approach ($\varepsilon=0$) and the Rawlsian welfare function, max-min, ($\varepsilon \to \infty$). This illustrates the well-known fact that the sensitivity of the function W to changes in different parts of the distribution is affected by the valaue of the parameter ε. The higher the value of ε the larger is the weight given to changes in the lower part of the distribution. A higher value does also increase the general sensitivity for inequality.

By (10) our extended simulation model is complete and it will now be used to investigate what effects we get when public parameters are changed. By simulations with the model we compute partial derivatives of H, U, t (individual level), W and T (aggregate level) with respect to specific public parameters P_j.

3. SIMULATION RESULTS

All simulations are restricted to the category married men in active ages with wives having no income. Important for our analysis is that in this category a very high fraction of total income is wage income. Table 1 gives for this category average pre-tax income in each income class (1975) and corresponding average and marginal effective tax rates in the 1975 tax system.

The policy instruments we are going to consider are the statutory marginal tax rates at national taxation, the local tax rates and the basic tax deduction. In addition to these existing parameters we consider the effects of the introduction of a lump-sum transfer equal to all persons in the distributions.

3.1. Effects on The Individual

On the individual level we can, according to (6)-(8), compute $\partial H/\partial P_i$; $\partial U/\partial P_i$; $\partial t/\partial P_i$, etc., for each specific wage rate. Before we report on the results of these computations we shall indicate

Table 1. Tax rates and income distribution for married men (wife not assessed) in 1975

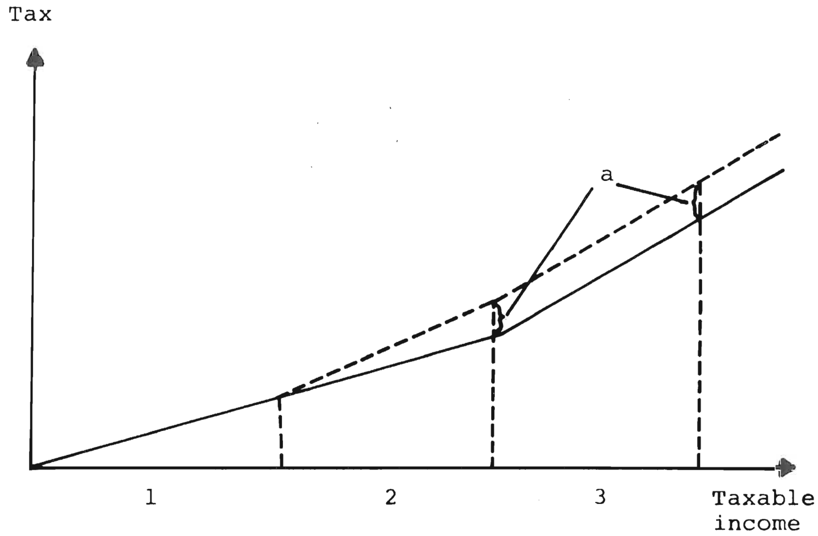
Income class	Relative frequency of tax-payers %	Pre-tax mean income Skr	Average tax rate %	Marginal tax rate %	Residual pro-gression[a]
1	2.2	118	0	0	1.00
2	0.1	2 801	0	0	1.00
3	2.0	9 076	0	31	0.69
4	4.1	14 411	9	31	0.76
5	5.7	20 259	16	36	0.76
6	8.2	25 598	20	41	0.74
7	13.6	31 416	24	46	0.71
8	17.6	36 634	28	52	0.66
9	14.6	42 373	31	52	0.69
10	15.1	49 323	35	57	0.65
11	6.6	61 274	40	62	0.63
12	5.7	74 882	44	72	0.50
13	2.1	98 865	51	72	0.57
14	2.3	161 158	61	80	0.51

[a] Elasticity of income after tax with respect to income before tax.

the nature of the different parameter changes and the kind of individual response we might expect under the assumptions made.

The effect on individual labor supply from a tax change can be divided in an income effect and a substitution effect. The income effect is positive, which in this context means that an isolated increase in the average tax rate will increase labor supply. The negative substitution effect implies that an isolated increase in the marginal tax rate will lead to a diminished labor supply. For a given tax schedule a specific revenue is collected from the individual.

Figure 2. Increase of the statutory marginal tax rate within a specific bracket



The tax schedule in the Swedish tax system can be described as an increasing step-wise linear function from income to tax payments. The general shape of the function is determined by the statutory marginal tax rates at national taxation and the so called basic tax deduction. Figure 2 illustrates an increase of the statutory marginal tax rate within a specific bracket (bracket 2 in the figure). Obviously, people below this bracket will not be affected by the change. Everybody in bracket 2 and above will have their utility levels diminished. An individual within the bracket gets his marginal tax rate as well as his average tax rate increased, so the effect on labor supply is in principle undetermined and so is the revenue effect. If the effect on labor supply is positive, the revenue effect will of course also be positive. A negative supply effect might, however, diminish the tax-base enough to offset the effect on revenue from the upward shift in the tax schedule.

As the tax increase in bracket 3 and above is of the same nature as an additional lump-sum tax labor supply in these brackets will be greater than before and so will revenues collected. Utility levels, however, will of course be diminished.

If we now go to the local tax it could mainly be seen as a linear tax with constant marginal tax rate which is equal to the local tax rate. It is clear that for the whole range of income an increase in this tax rate will give rise to exactly the same effects as we met within bracket 2 in the preceding paragraph.

The qualitative effects of changes in the other two instruments (basic tax deduction, lump-sum transfer) are obvious since they do not affect marginal tax rates and therefore only give rise to income effects.

Results on the micro level for changes in the statutory marginal tax rates in brackets Skr 0-10,000 and Skr 30,000-40,000, can be seen in Table 2. Each of these parameters has been increased by one percentage unit. In the table the resulting changes in percent of initial values are given for tax payments, hours worked and individual utilities at different income levels. To pick an example we can in row 8, column 9, read the value of $(\partial t/\partial P_j)/(t)\cdot 100$ at income level ≈36,600, where P_j stands for the marginal tax rate in the bracket Skr 30,000-40,000.

Table 2. Effects of parameter changes on the individual at different income levels

Income class	Pre-tax mean income before tax change (1)	Increase of statutory marginal tax rate in taxable income bracket 0-10,000 Skr: Marginal tax rate (2)[a]	Work effort (3)[b]	Utility (4)[b]	Tax payment (5)[b]	30,000-40,000 Skr: Marginal tax rate (6)[a]	Work effort (7)[b]	Utility (8)[b]	Tax payment (9)[b]
1	118	0	0	0	0	0	0	0	0
2	2 801	0	0	0	0	0	0	0	0
3	9 076	+1	-1.1	-0.1	0.0	0	0	0	0
4	14 411	+1	-0.6	-0.2	4.8	0	0	0	0
5	20 259	0	0.5	-0.2	5.5	0	0	0	0
6	25 598	0	0.4	-0.2	2.3	0	0	0	0
7	31 416	0	0.4	-0.1	2.1	0	0	0	0
8	36 634	0	0.4	-0.1	1.6	+1	-2.1	-0.0	-3.7
9	42 373	0	0.3	-0.1	1.3	+1	-1.8	-0.1	-2.5
10	49 323	0	0.3	-0.1	1.1	0	0.3	-0.1	1.1
11	61 274	0	0.3	-0.1	0.8	0	0.3	-0.1	0.8
12	74 882	0	0.3	-0.1	0.8	0	0.3	-0.1	0.8
13	98 865	0	0.3	-0.1	0.4	0	0.2	-0.1	0.4
14	161 158	0	0.2	-0.1	0.3	0	0.2	-0.1	0.3

[a] Change given in percentage units.

[b] Change given in percent of initial value.

As could be expected, utilities are decreased for all individuals affected by the tax increase. Furthermore, those individuals that get their tax rates increased with unchanged marginal tax rates will increase their hours worked. The amount of tax collected from these people will, of course, also increase. These results do not depend on our specific choice of utility function for the individual. The Cobb-Douglas assumption is, however, important in the brackets where marginal tax rates are increased. Here we get a decrease in labor supply. For individuals with taxable income in the bracket Skr 30,000-40,000 this effect is strong enough to produce a negative overall effect on their tax payments.

This negative effect is essential for the results we will give later on. Some readers might find it so extreme that it would rule out any form of the individual utility function producing this effect. However, as soon as any incentive effects at all are admitted, a perverse revenue effect does not seem to be too far fetched which should be clear from the following example.

Consider a full time worker supplying 2,000 hours/ year at a wage rate of 22.5 Skr/hour. This gives a yearly wage of Skr 45,000 and a taxable income of approximately Skr 40,000. Tax payments are roughly Skr 12,000. Now let the marginal tax rate in the brackets above the taxable income Skr 30,000 be increased by one percentage unit. At a taxable income of 40,000 this gives an initial tax increase on Skr 100 or 0.8 percent of taxes paid. By how much must hours worked be diminished in order to offset this positive revenue effect? Since the elasticity of tax payments with respect to income in this bracket is roughly equal to 2, an adjustment in hours worked by 0.4%, or 8 hours per year, would be sufficient to give a zero revenue effect. Higher adjustments than 8 hours per year will consequently give negative revenue effects.

3.2 Aggregate Effects

From the aggregative part of he model (e.g., (9)-(10)) we can investigate the effects of specific parameter changes on tax revenues and the social welfare function. Table 3 gives computed values of $\partial T/\partial P_i$ and $\partial W/\partial P_i$ for different parameters.

The most striking result of the table is that the perverse revenue effects we could observe at the micro-level in certain cases give rise to similar effects at the macro-level. Take, e.g., the bracket Skr 30-40,000. From the table we can see that a rise of the marginal tax-rate in this bracket by 1 percentage unit will decrease the aggregate tax revenues by Skr 19 million. From the micro-simulations (Table 2) it is clear that this figure is the net effect of diminished revenues from people within the bracket getting their marginal tax rate increased and revenue increases from people above the bracket, where the average tax rate is increased while the marginal tax rate is unchanged.

The interpretation of the perverse revenue effects for certain brackets is that the tax schedule in these brackets is not Pareto-optimal under the assumptions on individual behavior made here. Lowered marginal tax rates would increase utilities for the persons affected at the same time as total revenues would be increased.

Table 3. Aggregate effects of parameter changes on social welfare and tax revenue

	Parameters: National income tax schedule — Taxable income bracket. Thousands of Skr	Initial statutory marginal tax rate, %	Change of parameter	Effect on: Tax revenue mill. Skr $\partial T/\partial P_i$	Social welfare[a] $\partial W/\partial P_i$, ε=0.8	ε=3.0	ε=6.0
P1	0-15	7	+1 p.u[b]	99	$-4.21\ 10^{-1}$	$-8.9\ 10^{-3}$	$3.3\ 10^{-5}$
P2	15-20	12	"	27	-1.41 "	-2.7 "	-.8 "
P3	20-25	17	"	22	-1.27 "	-2.3 "	-.6 "
P4	25-30	22	"	10	-1.03 "	-1.7 "	-.4 "
P5	30-40	28	"	-19	-.89 "	-1.3 "	-.3 "
P6	40-45	33	"	-30	-.25 "	-.3 "	-.1 "
P7	45-65	38	"	-0	-.44 "	-.4 "	-.0 "
P8	65-100[c]	43	"	-14	-.46 "	-.39"	-.0 "
P9	100-[d]	52	"	-33	-.68 "	-.49"	-.0 "
P10	Lump sum transfer[e]		+100 Skr	-71	1.0	1.0	1.0
P11	Basic tax deduction[f]		+100 Skr	-35	$1.73\ 10^{-1}$	$3.4\ 10^{-3}$	$1.2\ 10^{-5}$
P12	Local income tax[g]		+1 p.u	33	-6.40 "	-12.9 "	-4.5 "

[a] These effects are normalized so that the effect of the introduction of a lump-sum transfer by 100 Skr is equal to one.

[b] Percentage unit.

[c] Two brackets put together. The statutory marginal tax rate is 48% in the subbracket 70,000-100,000 Skr.

[d] Cf. c). The statutory marginal tax rate is 56% in the subbracket 150,000-.

[e] This parameter does not exist in the actual tax system.

[f] Presently 4,500 Skr allowed to all income earners subject to the restriction that taxable income should not become negative.

[g] Flat rate of approximately 26% applied to taxable income.

We can also observe that the effect on social welfare of introducing a lump-sum transfer, with one exception is much greater than any other welfare effect. The exception is the rate of the regressive local tax. For $\varepsilon=0.8$ it would not increase social welfare to finance an increased lump-sum transfer with an increase in the local tax rate.

For higher values of ε the welfare effect of other parameter changes become almost negligible compared to the welfare effect of a change in the lump-sum transfer.

4. WELFARE IMPROVING POLICIES UNDER A FIXED BUDGET-CONSTRAINT

We are now equipped to answer the question of which parameter changes to choose in order to increase social welfare. As we do not consider other branches of public policy than personal income taxation it is natural to restrict the changes in the tax schedule to leave total net revenues constant. Under the assumptions made here this restriction is equivalent to the restriction that changes in consumption shall be equal to changes in production (see Stern, 1976). By the help of Table 3 it is easy to design policies, i.e., combinations of parameter changes that improve social welfare keeping total revenues constant.

In terms of our previous notation our task is to find combinations of parameter changes dP_k; dP_c such that

$$dW = \frac{\partial W}{\partial P_k} \cdot dP_k + \frac{\partial W}{\partial P_c} \cdot dP_c > 0$$

$$dT = \frac{\partial T}{\partial P_k} \cdot dP_k + \frac{\partial T}{\partial P_c} \cdot dP_c = 0 \qquad (11)$$

In Table 4 we give a selection of combined parameter changes that fulfills (11). The results are in accordance with those reached by Mirrlees (1971) and Phelps (1973). Both authors present results indicating that the optimal marginal tax rates should be falling at higher income levels. Here it is clear that marginal tax rates in brackets above 30,000 should be lowered. In Table 4, II and III are examples of such policies. It should also be mentioned that these two policies are of special interest since they as well as policy VI represent Pareto improvements.

We have introduced the possibility of a lump-sum transfer in the tax system. Our results strongly indicate that such an element should be included in the actual tax system. This is of course also in accordance with the results reached in theoretical literature.

In our analysis this result can be explained by the heavy weight attached to income in the lowest part of the distribution, already by the utilitarian sum of utilities. This tendency is reinforced by the social welfare function. It should also be pointed out that the financing of such policies is comparatively easy in the category married men since it has few persons in the lower end of the income spectrum (see Table 1).

Table 4. Combination of parameter changes improving social welfare under a fixed revenue constraint

	I		II		III	
Parameters involved	P1[a] marginal tax rate bracket 0'-15' Skr	P10[b] lump-sum transfer	P6[a] marginal tax rate bracket 40'-45' Skr	P10[b] lump-sum transfer	P6[a] marginal tax rate bracket 40'-45' Skr	P1[a] marginal tax rate bracket 0'-15' Skr
Parameter changes	+0.71	+1	-2.33	+1	-3.3	-1

	IV		V*		VI	
Parameters involved	P1[a] marginal tax rate bracket 0'-15' Skr	P3[a] marginal tax rate bracket 20'25,	P12[a] local tax rate	P10[b] lump-sum transfer	P12[a] local racket	P1[a] marginal tax rate bracket 0'-15' Skr
Parameter changes	-1	+4.5	+2.2	+1	-3	+1

[a] Change given in percentage units.

[b] Change given in hundreds of Skr.

* For ε=0.8 the indicated combination of changes in local tax rate (P12) and lump-sum transfer (P10) leads to a decreased value of the social welfare function.

Another general conclusion from the results is that the valuation of different policies do not change much with the value of ε. For the piecemeal policy analysis done here, it is in most cases indifferent if ε is equal to zero (the strictly utilitarian approach) or if we let ε tend to infinity (the Rawlsian criterion). A related point is that utility changes in the higher income classes mostly could be neglected. What is important here is the revenue effect. Therefore the assumptions made on disincentives in these classes are important for the results we will get.

From Table 3 it is seen that an increase in the local tax rate combines a low revenue effect with a high welfare loss. Policies V and VI in Table 4 are both encompassing a change in the local tax rate (P12). When it is used to finance an increased lump-sum transfer we get a welfare increase only when ε is greater than 0.8. This increase is much less than the one we get when the local tax rate is lowered in combination with an increase in the marginal tax rate in the lowest bracket (policy VI).

5. DISINCENTIVES AND THE REVENUE EFFECT

A clear-cut result of our previous analysis is that, under the assumptions made, marginal tax rates should be decreased in all brackets above Skr 30,000. This result depends crucially on the fact that in these brackets a decreased marginal tax rate leads to an increase in aggregate tax revenues (T).

It is important to check how sensitive this result is to changes in the elasticity of substitution between consumption of goods and consumption of leisure. We have done this by letting the individual's labor supply be governed by a utility function of the CES-type.[1] By simulating the response of hours worked and revenues for different values of σ for a change in the marginal tax rates in each one of the brackets above 30,000 Skr we get an indication of the range of σ where the disincentive effect is strong enough to create a perverse revenue effect.

From Table 5 it is seen that in the two highest brackets there is quite a wide range of values on σ that will give a perverse aggregate revenue effect. For the lower brackets, however, we get a picture that is a bit more mixed. Still, the Cobb-Douglas assumption does not seem to be essential for our results. An interesting result in this connection is provided by Stern (1976) who calculated implied elasticities of substitution from supply curves estimated by Ashenfelter and Heckman. This calculation gives $\sigma = .4$ which indicates that the range of σ in Table 5 for most brackets contains realistic values.

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[1] $U = \left[\alpha C^{-\mu}(1-\alpha)(T-H)^{-\mu}\right]^{1/\mu}$ $\left(\sigma = \frac{1}{1+\mu}\right)$. If U is maximized subject to the budget constraint the number of hours worked will be determined implicitly by the following equation

$$\frac{C}{T-H}^{-(\mu+1)} = \frac{1-\alpha}{\alpha w f_1'(wH;P)}$$

In lack of data on hourly wage rates we have computed values on w from yearly incomes on the assumption that everybody <u>initially</u> is working 2,000 hours/year.

Table 5. Least value on σ in the CES-function where an increased tax rate produces diminished aggregate tax revenues

	P 5	P 6	P 7	P 8	P 9
Bracket of the tax schedule (thousands of Skr)	30-40	40-45	45-65	65-100	100-
Revenues will be diminished for $\sigma \geq$	0.8	0.4	1.0	0.4	0.3

6. CONCLUDING REMARKS

A clear-cut conclusion of our analysis is that the graduation of the Swedish income tax schedule differs greatly from what would be prescribed by the theory of optimum income taxation with its usual assumptions. One may then take either the position that the tax system should be changed or the position that the assumptions in the theory of optimal income taxation need re-examination.

Certainly one would like to have more empirical evidence on individual behavior before using our results for policy prescriptions. The analysis made has highlighted the crucial importance of the labor supply response to tax changes. Therefore one objection against the results reached might be that the assumptions on disincentives have little empirical support. Econometric work in this area indicates that labor force participation and average hours of adult men are affected relatively little by changes in tax rates. As we could see in Section 5, calculations made by Stern (1976) indicate that the elasticity of substitution between labor and leisure among adult men still is high enough to produce the "perverse revenue effect" in a wide range of tax brackets. A more important fact, however, is that there is a downward bias in the estimates of these studies since they only are concerned with one dimension of labor supply, namely hours of work, while more important dimensions are left out, like work effort, choice of job, demand for education.

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A Procedure for Testing the Signalling Hypothesis

James W. Albrecht

1. Introduction*

The signalling model of the returns to education as developed by Spence (1974), Arrow (1973) and Stiglitz (1975) represents an important theoretical contribution to the economics of information, but whether this contribution is of significant empirical consequence is an open question. This paper develops and applies a general method for addressing this question.

The signalling interpretation of the returns to education depends upon employers' lack of information about job applicants. Workers (applicants) are assumed to have a good idea about their productivities, but, *a priori*, employers are not. If the less productive cannot be induced to admit to that fact, then the employer considering job applicants will be forced to "estimate" applicants' productivities.

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* This paper has gone through several versions and two data sets. The earliest version was presented at the 1974 Econometric Society meetings in San Francisco. The guidance and encouragement of Roy Radner on the early versions is gratefully acknowledged. This version has also been published in Journal of Public Economics, February 1981.

It is suggested that educational background may serve as an ideal observable trait for the employer to use to infer other, unobservable traits related to productivity. That education can be so used depends upon the assumption that the cost of education varies inversely with productivity. Under this assumption only the inherently more productive will find extra education worthwhile. Employers' initial beliefs that the educated are more productive will be self-fulfilling.

This basic objection to the signalling hypothesis is that the educational screen is a costly one. Ought not there exist less expensive alternative mechanisms to elicit information about productivities from applicants? It is sometimes asserted, for example, that any signalling component to the rewards to education would be eroded by the establishment of "testing firms". Alternatively, firms may be able to structure their promotion policies in such a way as to deter applicants from misstating their qualifications; that is, applicants may be induced to self-select into the proper job slots. These arguments, however, lack any empirical basis.

My approach to the signalling hypothesis will be to examine directly the question of whether employers reward education for purely informational purposes in the hiring decision. The role of education in the hiring decision will be decomposed into a pure "productivity component" and a pure "information component". This is most naturally done within a 2-way analysis of covariance framework with interactions between education and "information".

In the next section I develop the statistical procedure for testing the signalling hypothesis. Then, in the third section, I present an application of this method to a recruitment by the Swedish auto manufacturer Volvo. The results of this application provide both an illustration of the procedure and some substantive evidence about the signalling hypothesis. Finally, in a concluding section, I summarize the method and relate my procedure to another approach presented in Riley (1979).

2. A GENERAL PROCEDURE

Suppose an employer is considering applicants for a position who can be characterized by their educational background and by their "information level", i.e., the amount of *a priori* information the employer has about them. According to the signalling hypothesis, employers need to use education as a source of information about applicant productivities, i.e., applicants cannot be induced to properly selfselect by some cheaper means. Therefore, if the signalling hypothesis is valid, employers will be forced to rely more heavily on education when considering those applicants about whom they have the least information. The test procedure presented below is an exploitation of this simple idea.

Typically "information level" will be a qualitative variable, and often educational attainment will be as well. Let $i = 1,\ldots,I$ index educational categories, and let $j = 1,\ldots,J$ index informational categories. The k^{th} individual in the $(i,j)^{th}$

cell has observable characteristics X_{ijk}. Assume that the (lifetime, discounted, etc.) marginal product (= Z_{ijk}) of this applicant as perceived by the prospective employer can be expressed as a linear combination of these characteristics plus a $N(0,\sigma^2)$ error term. That is,

$$Z_{ijk} = X_{ijk}\eta + U_{ijk}, \qquad (1)$$

where U_{ijk} is $N(0,\sigma^2)$.

The employer's decision problem can be modelled as one of accepting only those applicants whose perceived marginal product exceeds a critical value w. Then, the probability that the k^{th} applicant in the $(i,j)^{th}$ cell will be accepted can be written as

$$P_{ijk} = Pr(Z_{ijk} > w) = Pr(X_{ijk}\eta + U_{ijk} > w) =$$

$$= Pr(U_{ijk} < X_{ijk}\eta - w)$$

$$= \int_{-\infty}^{(X_{ijk}\eta - w)/\sigma} (2\pi)^{-1/2} e^{-z^2/2} dz$$

$$= \Phi(X_{ijk}\eta^*), \qquad (2)$$

where η^* is the standardized parameter vector and $\Phi(\cdot)$ is the distribution function of the standardized normal random variable.

To pursue the 2-way analysis of covariance approach, assume

$$X_{ijk}\eta^* = \mu + \alpha_i + \beta_j + \lambda_{ij} + \sum_{h=1}^{H} \delta_h Y_{hk}, \qquad (3)$$

where

$$\sum_i \alpha_i = \sum_j \beta_j = \sum_i \lambda_{ij} = \sum_j \lambda_{ij} = 0.$$

The interpretation of the parameters is as follows:

μ = mean (standardized, perceived) productivity

α_i = main effect on productivity of being in educational category i

β_j = main effect on productivity of being in informational category j

λ_{ij} = interaction effect on productivity of being jointly in educational category i and informational category j

δ_h = effect of the h^{th} concommitant variable on productivity.

The main effects of education are the effects of educational categories averaged across all informational categories, and likewise for the main effects of information. The interaction effect in the $(i,j)^{th}$ cell is the effect of the i^{th} level of education on the employer's perception of applicant productivity specific to the j^{th} informational category; that is, it is the effect of the i^{th} level of education above and beyond the main effect, α_i.[1]

It is the interaction effects which are of principal interest. To see this it is useful to consider a simple "2×2" example. Imagine an applicant pool

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[1] The concepts of main effects and interaction effects in 2-way analysis of variance models are lucidly discussed in Scheffé (1959).

differentiated according to high versus low education level and high versus low information level.

If employers are forced to use education for information, then the interaction effects can be expected to take on the sign pattern indicated below:

		Information	
		High	Low
Education	High	-	+
	Low	+	-

We expect education to receive a positive overall weight in the employer's assessment procedure. If part of this positive overall weight can be ascribed to an informational component, then the positive effect of education ought to be decreased in the presence of alternative information; i.e., we expect the interaction effect for high education together with high information to be negative. Analogous arguments can be made to sign the other interaction terms, but these are redundant since there is only one independent interaction parameter in this 2x2 case. Alternatively, if the employer is not forced to use education as a source of information, then the effect of education should be constant across all information levels. Thus, a test of the hypothesis that the employer does not use education for informational purposes may be expressed as

$$H: \lambda_{ij} = 0; \quad i=1,\ldots,I \quad j=1,\ldots,J.$$

It is to be emphasized that the hypothesis of zero interaction effects is <u>not</u> the hypothesis that the employer is indifferent about the educational attainment of applicants, nor is it the hypothesis that the employer is indifferent about the amount of _a priori_ information available about prospective employees. These hypotheses instead translate into hypotheses about the main effects.

Nor does the hypothesis of zero interaction effects imply that an employer's preference for applicants about whom more information is available need solely reflect a preference for more information. There may be differences in average productivity across information classes, but these differences ought to be reflected in the main effects of information, rather than in differential rewards to education. However, one must be on guard for other mechanisms that might introduce an interaction between education and information, and such alternative mechanisms are easier to imagine when information is not "neutral". The point, of course, is that one must be careful in specifying "information classes".

3. AN APPLICATION

The data used in this application come from records of applicants for entry-level blue collar positions at Volvo's Torslanda auto works for the month of June 1978.[1] Excluding those applying for

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[1] These data were kindly made available to me by Göte Bernhardsson and Anne-Marie Qvarfort of the Employment Commission in the Swedish Ministry of Labor (sysselsättningsutredningen). Their report on Volvo's recruitment practices is available in mimeo as "Personalrekryteringen till Volvo-Torslandaverken, Juni 1978", _Sysselsättningsutredningen_, October 1978.

part-time work, a total of 515 applicants were considered and of these 291, or 56.5%, were hired. Data on the educational attainment and on the recruitment source of each applicant are available from these records. Educational attainment is a dichotomous variable with "low education" identified with attainment of less than the gymnasium level. The gymnasium is normally attended for 3 years in Sweden between the ages of 16-19 and roughly corresponds to the last years of senior high school plus parts of junior college in the U.S. Today the completion rate in the gymnasium is quite high, but this is a very recent phenommenom, and in this sample 42% of the applicants have not completed the gymnasium.

The information class of the applicant is identified with the source of his or her recruitment. The first recruitment source - and this is the source to be identified with greater prior information - is recommendation by a current Volvo employee; that is, the applicant has given the name of a Volvo employee who has informed him of the job opening and from whom the personnel department can solicit an evaluation. Of course, such an evaluation may not be unbiased, but it seems reasonable that the company can take the caliber of the reference into account. The other two recruitment sources are identified with less prior information. The first of these relatively low information sources is the Swedish Labor Market Board (AF). This refers to job seekers who have searched AF's position announcements and have then come to Volvo with a notification from that Board. No active placement on AF's part is implied. Secondly, there are those who have simply applied in response to newspaper advertisements (plus a small group from "miscellaneous" sources). In principle, those who come via AF and those who come via advertisement are in an equally low information category. However, there is the possibility of more active placement on the part of the AF for some candidates. This potentially has both the implication of more information and the implication of a decrease in the probability of hire for those candidates since AF is more likely to make an active effort on behalf of those who are "difficult to employ". These two low information categories have been combined in the empirical results presented below.[1]

Besides the information about education and recruitment source, data are available on the age, the nationality, the residence and the sex of each applicant. These data are presented in Table 1. Ignoring any covariation between these variables for the moment, Table 1 indicates a preference for (1) more highly educated applicants, (2) applicants in the high information category, (3) younger applicants, (4) Swedish and Finnish nationals, (5) non-Gothenburg residents and (6) males. The only surprise in the data is the preference given to those living out of the greater Gothenburg region where the plant is located. However, the relatively low number of non-Gothenburg residents

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[1] In fact, Bernhardsson and Qvarfort conjecture that some applicants recorded as recruited via advertisement may also have searched the AF position announcements. There are 2 bases for this suspicion: (i) some applicants may feel that any identification with AF hurts their chances and (ii) the fraction of applicants coming from AF seems "abnormally low".

Table 1 The basic data

		Appli-cants	Hired	Relative frequency
	Total	515	291	.565
Education:	Low	215	101	.470
	High	300	190	.633
Information:	Rec	180	110	.611
	AF	115	58	.504
	Ad	220	123	.559
Age:	⩽20	202	129	.639
	21-27	186	104	.559
	⩾28	127	58	.457
Nationality:	Swedish	298	182	.611
	Finnish	122	76	.623
	Other	95	33	.347
Residence:	Gothenburg	415	228	.549
	Other	100	63	.630
Sex:	Male	455	270	.593
	Female	60	21	.350

Source: Unpublished data from the Employment Commission in the Swedish Ministry of Labor.

(and the even lower number of females) among the applicants should be noted.

The model that has been estimated inverts equation (2) to express $\Phi^{-1}(p)$ as a constant plus a sum of main effects for education, information, age, nationality, residence and sex plus an education-information interaction. The parameters have been estimated using maximum likelihood (probit), and test statistics for assessing the significance of the main and interaction effects have been computed as -2 times the logarithm of the appropriate likelihood ratio. The test statistics are asymptotically χ^2 with degrees of freedom equal to the number of independent restrictions implied by the null hypothesis. These parameter estimates and test statistics are presented in Table 2.

The parameter estimates may be interpreted with the aid of a simple example. An applicant who (1) has a low level of education, (2) falls in the high information category, (3) is between the ages of 21-27, (4) is Swedish, (5) is a Gothenburg resident and (6) is male would be hired with an estimated probability of $\Phi(0.305) = 0.620$. An applicant with a high level of education but otherwise identical attributes would be hired with an estimatted probability of $\Phi(0.569) = 0.715$ with the change ascribable to the increase via the main effect of education (from -0.191 to +0.191) and to the decrease via the education-information interaction (from +0.059 to -0.059).

The pattern of main effects in Table 2 is in basic accord with that suggested by the raw data in Table 1. Completion of the gymnasium, Swedish or Finnish nationality and being male strongly increase the chance of getting hired, and these main effects are significant at the 1% level. Having a Volvo employee to use as a reference also increases the hire probability, but not as strongly; and the factors of age and residence, while retaining the same pattern as in the raw data, become much less important. In fact, the anomalous apparent preference for non-Gothenburg residents essentially becomes zero when the covariation between residence and other variables is taken into account. The significance probabilities for the main effects of information, age and residence (0.15,

Table 2 Probit estimates and significance tests

Estimate	Maximum likelihood (χ^2)	Test statistic
Mean	-0.145	-
Main effects		
Education		9.48*
High	0.191	
Low	-0.191	
Information		2.18
High	0.115	
Low	-0.115	
Age		1.98
≤20	0.095	
21-27	-0.022	
≥28	-0.073	
Interaction effect		0.54
High Ed.× High Inf.	-0.059	
High Ed.× Low Inf.	0.059	
Low Ed. × High Inf.	0.059	
Low Ed. × Low Inf.	-0.059	
Nationality		14.78*
Swedish	0.185	
Finnish	0.240	
Other	-0.425	
Residence		0.34
Gothenburg	-0.016	
Other	0.016	
Sex		10.98*
Male	0.320	
Female	-0.320	

* Significant at 1 percent level.

0.35 and 0.65, respectively) are above conventionally accepted levels.

The interaction effects take on the sign pattern suggested by the signalling hypothesis, i.e., the positive effects of extra education are decreased in the presence of extra information, but these effects are quite small in magnitude. The significance probability for the education-information interactions is only slightly less than 0.5. The hypothesis of zero interaction effects clearly cannot be rejected; that is, the hypothesis that there is no purely informational component to the preference exhibited for the more educated applicants cannot be rejected. Volvo's hiring behavior gives no support to the signalling hypothesis in this instance.

Finally, it should be noted that the results are insensitive to re-parameterization of the basic model. Alternative models have been estimated with (1) age as a continuous variable, (2) 3 information categories instead of 2, (3) interactions between education and nationality and information and nationality and (4) residence and sex supressed as separate variables. In addition the model with residence and sex supressed has been re-estimated by the alternative technique of "minimum normit chi-square", i.e., weighted least squares based on the cell relative frequencies, as developed by Berkson (1955). The basic conclusions remain the same.

4. DISCUSSION

This paper has presented a procedure for testing the signalling hypothesis based on a decomposition of the role of education in the hiring decision into a pure "productivity" component and a pure "information" component. The procedure was applied to a recruitment of auto workers by Volvo, and in this instance Volvo's hiring behavior indicates no support for the signalling hypothesis. Volvo prefers applicants with more education and (weakly) prefers applicants about whom more information is available, but in the absence of that extra information no significantly different premium is attached to extra education. That is to say, Volvo does not appear to rely on education for purely informational purposes in the hiring process.

Of course, this same procedure could be applied to different sets of data, and one aim of this paper is to motivate the collection of richer data sets for replication. As explained above, and as illustrated in the Volvo application, the trick is to define the concept of "information level" in a suitable way.

The procedure developed in this paper is very "micro" in the sense that it focuses on the significance of signalling at the level of the individual job and at the level of the individual employer. More "macro" approaches are also possible, and such approaches can be considered complementary to the method advocated here. In my opinion, the best of these macro approaches is presented in Riley (1979).[1] Riley's method is based on an idea

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[1] Some other empirical papers on signalling are Layard and Psacharopoulos (1974), Taubman and Wales (1973), and Wolpin (1977). Riley gives a good discussion and critique of these papers.

similar to that of information levels. However, instead of differentiating among applicants for a particular job according to the amount of available prior information, he divides occupations into those for which productivity may be easily ascertained versus those for which signalling might conceivably be important. A test of the signalling hypothesis is then based on a comparison of lifetime earnings functions at each level of education for those in the "screened" sector versus those in the "unscreened" sector. Using this test, Riley concludes that signalling is a significant phenomenon.

However, as Riley points out, there is no obvious best method for classifying occupations as screened or unscreened. In fact, he is forced to use *ex post* data analysis to perform the classification. Nor is there any way to ensure the differences in earning profiles between the screened and unscreened sectors for a given education level can be solely ascribed to the screening function of education. But these practical problems are analogous to those which make the application of this paper's procedure difficult; namely, suitably defining what one means by "information" and ensuring that specious interactions between education and information are controlled.

To summarize, empirical analysis of the significance of signalling appears to have reached the point where well-founded techniques are becoming available. However, the data requirements imposed by these techniques have proved to be rather stringent. One advantage of the procedure and application presented in this paper is that these data requirements have been clarified, and one can hope that further applications will be possible.

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