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SPECULATION, BUBBLES, AND SUNSPOTS UNDER STRUCTURAL UNCERTAINTY

by

Clas Wihlborg

This is a preliminary paper.
Comments are welcome.

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Under Structural Uncertainty

by

Clas Wihlborg
University of Southern California, Los Angeles;
and The Industrial Institute for
Economic and Social Research, Stockholm

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I. Introduction

The efficiency of financial markets has been debated for several decades. In spite of the large number and increasing sophistication of empirical tests, there is little consensus regarding the importance of "destabilizing speculation," "bubbles," "sunspots," and other phenomena which may indicate inefficiencies. In this paper I suggest that the persuasiveness of empirical tests may increase with explicit conceptual and theoretical consideration of structural uncertainty, learning, and consequences thereof for adjustment.

In international finance, the debate beginning with Nurkse (1944) centered on whether speculation is stabilizing or destabilizing with important implications for the welfare effects of floating exchange rates (see e.g., Friedman, 1953; Baumol, 1957; Aliber, 1970, 1973; Hodgson, 1972, and Kohlhagen, 1979). With the rational expectations (RE)-revolution, the debate has shifted to the importance of "bubbles" and "sunspots," the existence of which imply that exchange rate variation is excessive relative to the variation in market fundamentals under the assumption that expectations are formed with knowledge of structural parameters and utilizing all available information (see, e.g., Blanchard and Watson, 1982; Flood and Garber, 1980; Obstfeld and Rogoff, 1983; and Meese, 1986).

The analysis of asset pricing in domestic finance centered after Fama's famous 1970 article on the issue of market efficiency in different forms, but in this area, as well as in international finance, recent analysis centers on price developments relative to a rational expectations path based on fundamentals.

Shiller (1981) has fueled the debate by pointing to the large variance in stock market prices relative to the variance in fundamentals. Shiller (1984) suggests that mass-psychological phenomena may explain the existence of speculative bubbles increasing the variance of prices. A number of authors (e.g., LeRoy and Porter, 1981; Flavin, 1983; LeRoy, 1984; and Flood and Hodrick, 1986) have disputed Shiller's findings and reasoning. For example, LeRoy (1984) points out that non-stationarity, the choice of model for evaluation of expectations, risk-aversion, and the choice of variance measures may explain why there seem to be speculative bubbles though none exist.

One major issue in this debate is the specification of what Shiller calls the "ex post rational price." As Flood and Hodrick (1986) put it: "In bubble research, one particularly important misspecification of the model occurs when the researcher incorrectly specifies agents' beliefs about the time-series properties of market fundamentals." In the terminology of Flood and Hodrick, as well as of Flood and Garber (1980), price changes driven by erroneous expectations about future values of fundamental variables are not "bubbles." Instead, bubbles would be driven by extraneous factors. Such price changes are sometimes called sunspots.

Hamilton and Whiteman (1985) have summarized the state of bubble research by demonstrating that "the proposition that prices are driven by completely extraneous factors is empirically untestable . . ." Therefore, ". . . any claim to have uncovered empirical evidence of sunspots or bubbles rests on an implicit restriction on the dynamics of the variables which are seen by agents but not by the econometrician." For empirical research, they have suggested that if no bubbles exist, then both price and market fundamentals would exhibit stationarity to the same degree. Meese (1986) shows that such tests have low power and employs similar tests of "cointegration"--whether there exists a linear combination of a price and its fundamental determinants, which exhibits time-series characteristics of "rational" forecast errors. Meese's results are consistent with the existence of bubbles, but the validity of the tests depends on the correct choice of fundamentals.

Most papers on bubbles include the assumption that structural parameters and fundamental variables are known by agents. Then, bubbles appear in some sense irrational and are caused, for example, by mechanical trading rules in the marketplace. However, when structural uncertainty is recognized, it is possible that deviations in the price path from the ex post rational path are caused by errors in the perception of structural parameters, including the time series characteristics of fundamentals, as well as by errors in agents' choice of fundamental variables.

The purpose of this paper is to determine explicitly how price paths depend on erroneous perceptions about structural parameters under the assumption that agents do not know with certainty the structural model for the price. The concepts of bubbles and sunspots will be related to errors in perception and learning processes, and the empirical contents of these concepts will be discussed.

Blanchard and Watson (1982) and Obstfeld (1986) have explained bubbles by perception errors about structure, assuming that agents assign probabilities to values of structural parameters. Expected values of these parameters are modeled as if they were equivalent to known parameters. In this paper, there are probability distributions for the time series properties of market fundamentals, and the expected values are not certainty equivalents. Therefore, the variance of these parameters play an explicit role in adjustment.

It will be argued that phenomena that may seem to be bubbles and sunspots could be associated with changes over time in adjustment coefficients to disturbances and that these changes could be the result of learning processes. It will also be demonstrated that seemingly nonsensical regression results from the relationship between actual and forecast changes in price variables can be consistent with reasonable learning processes during the estimation period. Theoretical work on the learning of structure is still in its infancy. Frydman (1982) discusses conditions under which agents learn the correct value of parameters, based on which expectations can be formed. Lewis (1987) contains a test of explicitly modeled learning of money demand parameters in exchange rate determination.

In Section II, I use a simple model of exchange rate determination in order to decompose adjustment into a component corresponding to ex post rational expectations and one component caused by erroneous perceptions about structure. Thereafter, in Section III, learning is discussed and implications are drawn for empirical work on the identification of bubbles, sunspots, and associated learning processes. Section IV contains a summary.

II. A Model of Exchange Rate Determination with Structural Uncertainty

In this section, coefficients of adjustment to market fundamentals and extraneous variables are derived in a simple monetary model of exchange rate determination. Any coefficient (B_j) will be decomposed into two components: adjustment corresponding to ex post rational expectations (RE), and adjustment caused by erroneous expectations about structural parameters. The coefficients will be solved for in terms of "true" structural parameters, errors in expectations about such parameters, and uncertainty about them. In this section, it is assumed that all agents form expectations based on their perceptions about fundamental variables.

The money market is described in the following simple way

$$m_t = p_t^T - b(E_t s_{t+1} - s_t) \qquad (1)$$

where m_t is the log of the money supply in period t, p_t^T is the log of traded goods prices in period t, E_t is the expectations operator, and s_t is the log of the exchange rate in period t.

The log of the price of traded goods is assumed to be equal to the log of the exchange rate

$$p_t^T = s_t \qquad (2)$$

Using (1) and (2), the exchange rate can be solved for:

$$(1+b)s_t = m_t + bE_t s_{t+1} \qquad (3)$$

The true time series properties of m_t are described by the serial correlation coefficient ρ:

$$m_t = \rho_{t-1} m_{t-1} + v_t \qquad (4)$$

where v_t denotes innovations in period t. This variable is independently distributed with mean zero and variance σ_v^2. The serial correlation coefficient may also vary over time and it is uncertain. We return to this issue below.

The exchange rate can be written as:

$$s_t = B_{1,t} m_t + B_{2,t} R_t \qquad (5)$$

where R_t is an extraneous variable which agents may perceive as a fundamental variable. It follows a process

$$R_t = \delta_{t-1}R_{t-1} + w_t \tag{6}$$

In (6), δ_t is a serial correlation coefficient which is uncertain and w_t is the innovation in period t.

Assume that an individual i forms expectations based on perceived fundamentals in the following way:

$$E_t^i[s_{t+1}] = E_t^i[B_{1,t+1}m_{t+1}] + E_t^i[B_{2,t+1}R_{t+1}] \tag{7}$$

For simplicity, it is assumed that m_t and R_t are observed in period t.[1] Therefore:

$$E_t^i[s_{t+1}] = m_tE_t^i[\rho_tB_{1,t+1}] + R_tE_t^i[\delta_tB_{2,t+1}] \tag{8a}$$

and for average expectations

$$E_t[s_{t+1}] = m_tE_t[\rho_tB_{1,t+1}] + R_tE_t[\delta_tB_{2,t+1}] \tag{8b}$$

Inserting (8b) in (3) we obtain that

$$(1+b)s_t = m_t + b\{m_tE_t[\rho_t]E_t[B_{1,t+1}] + m_t \bullet \mathrm{cov}[\rho_t;B_{1,t+1}] + R_tE_t[\delta_t]E_t[B_{2,t+1}] + R_t \bullet \mathrm{cov}[\delta_t;B_{2,t+1}]\} \tag{9}$$

Before solving for adjustment to change in m_t and R_t we distinguish between two components of the adjustment coefficients.

$$B_{j,t} \equiv \bar{B}_{j,t} + \gamma_{j,t} \quad \text{for } j = 1,2 \tag{10}$$

Here $\bar{B}_{j,t}$ is the ex post RE-coefficient in period t under the assumption that individuals know the time-series properties of fundamentals as well as the rule by which all agents form identical expectations in any period t. $\gamma_{j,t}$ is the difference between the actual coefficient and the ex post RE-coefficient. It depends on structural misperceptions.

The expected coefficient for the next period can be written as:

$$E_t[B_{j,t+1}] \equiv \bar{B}_{j,t+1} + \gamma'_{j,t+1} \tag{11}$$

where $\bar{B}_{j,t+1}$ is not known and $\gamma'_{j,t+1}$ is the difference between the expected and the ex post RE-coefficient.

We also define expected time-series properties of disturbances as

$$E_t[\rho_t] = \rho_t + \varepsilon_t^\rho \text{ , and} \tag{12}$$

$$E_t[\delta_t] = \delta_t + \varepsilon_t^\delta \text{ ,} \tag{13}$$

where ε^ρ and ε^δ are differences between actual and expected time-series properties of disturbances. In each period, a certain change in a variable consists of a persistent component, for example $\rho_t \bullet m_t$, and a transitory component, for example v_{t+1}. Expressions (12) and (13) imply that agents are uncertain about the magnitude of each component.

It is assumed in this section that b is a known parameter, but the analysis of implications of uncertainty about time-series properties applies to uncertainty about the parameter b as well.

Deriving ex post RE-coefficients

After inserting (11)-(13) into (9), and (10) into (5), we compare coefficients in (5) and (9). Ex post RE-coefficients ($\bar{B}_1$ and $\bar{B}_2$) are obtained by setting ε_t^ρ, ε_t^δ, $\gamma'_{1,t+1}$ and $\gamma'_{2,t+1}$ equal to zero. Covariances are also set equal to zero for comparisons with RE-models in which parameters are assumed to be known or treated as certainty equivalents.

The following ex post RE-coefficients are derived:

$$\bar{B}_{1,t} = \frac{1 + (\bar{B}_{1,t+1} - \bar{B}_{1,t})b\rho_t}{1 + b(1-\rho_t)} \tag{14}$$

$$\bar{B}_{2,t} = 0 \tag{15}$$

If the true time-series properties of disturbances are constant over time, then $\bar{B}_{1,t+1} - \bar{B}_{1,t} = 0$ and the coefficient in (14) reduces to a standard time-invariant rational expectations coefficient.[2] The coefficient for the extraneous variable in (15) is zero, when the true structure is known.

Deriving Coefficients Due to Erroneous Expectations and Structural Uncertainty

Coefficients describing the difference between actual and ex post RE-coefficients are:

$$\gamma_{1,t} = \frac{b}{1+b}\{\bar{B}_{1,t+1}\varepsilon_t^{\rho} + (\rho_t+\varepsilon_t^{\rho})\gamma'_{1,t+1} + \text{cov}[\rho,B_1]\} \tag{16}$$

$$\gamma_{2,t} = \frac{b}{1+b}\{\delta_t + \varepsilon_t^{\delta})\gamma'_{2,t+1} + \text{cov}[\delta,B_2]\} \tag{17}$$

By using (12) and (13) and by adding and subtracting $\gamma_{j,t}$ in (16) and (17), the following coefficients are derived.

$$\gamma_{1,t} = \frac{b}{1+b-bE_t[\rho_t]}\{\bar{B}_{1,t+1}\varepsilon_t^{\rho} + E_t[\rho_t](\gamma'_{1,t+1} - \gamma_{1,t}) + \text{cov}[\rho,B_1]\} \tag{18}$$

$$\gamma_{2,t} = \frac{b}{1+b-bE[\delta_t]}\{E_t[\delta_t](\gamma'_{2,t+1} - \gamma_{2,t}) + \text{cov}[\delta,B_2]\} \tag{19}$$

Expression (18) shows that exchange rate variation due to variation in fundamental variables may be amplified by erroneous expectations about structural parameters. There are two cases to consider. In one case, average misperceptions are constant over time and the coefficient, γ_1, is constant. If at the same time, the coefficient is uncertain, then we can rewrite $\text{cov}[\rho,B_1]$ as proportional to $\sigma^2_{\varepsilon\rho}$, i.e., to the variance in the policy parameter ρ. Assuming that $b\varepsilon^{\rho}$ is small relative to the denominator in (18), expression (18) can be simplified to:

$$\gamma_1 = K(\varepsilon^{\rho} + \sigma^2_{\varepsilon\rho}), \text{ where } K = \frac{b\bar{B}_1}{1+b-bE[\rho]} \tag{20}$$

(20) shows that exchange rate variation is caused by constant misperceptions about the time series characteristic of the fundamental variables as well as by uncertainty about this parameter. For example, monetary policy regime uncertainty contributes to exchange rate variation in excess of variations in the traditional ex post RE equilibrium. Uncertainty about the parameter b would contribute to adjustment in a similar way.

In the second case, changes occur in the expected value of the parameter ρ. In this case, $\gamma'_{1,t+1} - \gamma_{1,t} \neq 0$[3] in (18) and the coefficient is changing over time. Such changes in adjustment to, say, monetary disturbances occur, for example, when there is an expected shift in monetary policy rules.

Turning to the coefficient for the extraneous variable in (19), it can be seen that the misperception about time-series properties do not enter the expression as an independent term. Therefore, if agents believe that the size of the coefficient for R depends on the magnitude of the serial correlation parameter (δ), then the coefficient γ_2 is equal to the perceived covariance between the parameter δ and the coefficient when the parameter δ is expected to remain constant. Under these circumstances there exists a "sunspot," and if agents learn the true model over time, a bubble could be identified since the exchange rate temporarily adjusted to non-fundamental factors.

Adjustment to an extraneous variable R is inconsistent with equilibrium over the long run, since if δ was expected to remain constant but actually varied, then agents would learn that the true covariance between δ and γ_2 is zero. This reasoning implies that the existence of these bubbles and sunspots would, in general, be associated with learning processes about the true parameters of the model.

III. Empirical Implications and Learning

The coefficients derived in (18) and (20) for fundamental variables express equilibrium exchange rate adjustment as a function of perception errors about time-series properties, under the assumption that all agents form expectations as in expression (8a). For the expressions to remain valid over several periods, agents must be able to learn the parameter b as well. This issue is discussed below.

Under the assumption that the relationship between perceived time-series characteristics of fundamentals and adjustment coefficients is described by the above coefficients, we may observe adjustment to fundamentals that appear to be bubbles and sunspots in ex post analysis. For example, assume that agents have certain average perceptions about the parameter ρ, and believe this parameter to be a constant. If these perceptions include an error (ε^{ρ}), then the adjustment coefficient is equal to $\bar{B}_1 + \gamma_1$ as defined by (14) and (2). If in a later period, agents learn the correct value, then ε^{ρ} goes towards zero and the coefficient becomes smaller. In tests for bubbles based on the assumption that agents know all structural parameters in terms of fundamental variables, this type of learning process would appear as a bubble. However, as long as the perception error remains constant, the adjustment coefficient remains constant (if the true ρ is constant), and there exists no evidence of a bubble.

What appears as a bubble in this analysis would not qualify as such according to the terminology used by, for example, Flood and Garber (1980) and Hamilton and Whiteman (1985), since the adjustment is explained by the information set available to agents about fundamentals. Given structural uncertainty and differences in perception about relevant fundamentals and model parameters, it seems as a near-tautology, however, that the exchange rate at any time depends on information available to agents about fundamentals at the time. Therefore, it could be more fruitful to empirically identify periods of learning of parameters and time-series characteristics of disturbances, as opposed to conducting the perhaps futile search for a proper bubble.

One example of empirical analysis incorporating explicit learning is Lewis (1987). Ideally, learning processes should be specified in such a way that the process is consistent with the information set agents are assumed to have, as Frydman (1982) notes.

Lacking anything but crude specifications of learning processes, it is still possible to use empirical results in order to obtain implicit observations of such processes. For example, as the above expressions suggest, learning would be associated with changes in adjustment coefficients. Econometric methods for the identification of time-varying coefficients exist.

Another method for deriving implicit learning processes is to use econometric results for the relationship between a price and a price forecast, which are seemingly inconsistent with RE. One may derive the implicit learning process that makes the actual results consistent with RE.

Take, for example, Fama's (1983) and Sweeney's (1986) observation that in a regression of the rate of change of the exchange rate on the forward premium, the coefficient for the forward premium is negative, and, in some cases, equal to minus one. What kind of learning process could account for such a result, assuming that the agents are rational and that the forward premium is equal to the expected exchange rate change?

In the Appendix, an expression is derived for the change in the perception error of the time-series properties of monetary policy, which would be consistent with RE, equality between the forward rate and the expected future spot rate, and a coefficient of minus one for the forward premium in a regression. The expression shows that if the serial correlation parameter (ρ) is sufficiently large, and if the parameter is overestimated, then the regression result can be explained by a correction of agents' perception over time. The remaining empirical issue is whether the actual magnitude of the parameter is consistent with the result, and if there were policy rule shifts that could have caused the misperceptions.

For a learning process to explain the negative coefficients for the forward premium over a long period it would be necessary to argue that policy authorities have increased the true coefficients over time, and that changes in agents' perceptions have been lagging. For the '70s, such an argument may be reasonable. Empirical support for this position is provided by estimates of the time pattern of money supply changes in, for example, Kormendi and Meguire (1984). These authors show that through the 70s the percentage change in the money supply in the U.S. is explained by time as well as by lagged changes. The significant coefficient on time indicates growth in the time series parameter ρ.

Expression (20) has implications for variance bound tests as well and helps explain why actual price variance is larger than the variance of the perfect foresight price. The expression says that even if there is no misperception about the time-series properties of disturbances, the variance of this parameter contributes to larger variations in exchange rates and stock prices even if the money supply variance remains constant. Furthermore, changes in the degree of uncertainty about time-series characteristics would appear as bubbles.

Turning to sunspots, i.e., adjustment to an extraneous variable as expressed by γ_2 in (17), it was noted that this coefficient would be non-zero as long as agents have not learned that the "true" covariance between time-series properties and the coefficient γ_2 is zero. Furthermore, over time, agents would be able to learn that the "true" coefficient is zero. Therefore, whether the seeming existence of bubbles depends on perception errors about time-series properties of fundamentals or "true" bubbles in the form of misperceptions about the role of extraneous variables, one would expect that learning processes are occurring over time, rendering adjustment coefficients unstable.

So far, perception errors about time-series properties of disturbances have been discussed. One may ask, however, how the parameter b is learned. Some observations about this issue can be made without going into depth on learning mechanisms and the issue of convergence to ex post RE-equilibrium. It is obviously a fruitful research area in which Frydman (1982, 1987) has taken some initial, important steps. As Frydman has noted, the consideration of expectations of others' expectations are crucial. For illustration, assume that agents have observed in period t that $s_t = B_{1,t}m_t + B_{2,t}R_t$ and that all agents are believed to follow forecast rules as in (8).[4] Each coefficient is here a "signal" regarding the structural knowledge of others. For example, by using (14) and (20), while assuming that all agents perceive coefficients to be constant over time, we know that

$$B_{1,t} = E_t^i \left[\frac{1}{1 + b(1-\rho_t)}\right] + E_t^i\left[\frac{b\bar{B}_{1,t}(\varepsilon^{\rho} + \sigma_{\varepsilon\rho}^2)}{1 + b - bE(\rho_t)}\right] , \qquad (22a)$$

and using (14) for $\bar{B}$, the expression can be rewritten as:

$$B_{1,t} = E_t^i \left\{\frac{1 + b(1-\rho) + b\sigma_{\varepsilon\rho}^2}{[1+b(1-\rho_t)][1+b\{1-E(\rho_t)\}]}\right\} \qquad (22b)$$

This expression implies that the B-coefficient signals to each individual an expression which is a combination of the average expectation about the policy parameter ρ, and the unknown parameter b. With some assumption about either b or $E[\rho]$, and knowing one's own perceptions about ρ, it is possible to make an inference about either others' perceptions or about b.

When individual and average expectations are perceived to be different, the individual may engage in speculative activity provided he or she believes that others will learn his or her own expectations in the future. If, on the average, expectations differ from the information about others' expectations, the coefficient will change.[5]

Observation of coefficients as in (22b) cannot reveal both average perceptions about time-series characteristics and the parameter b. Thus, if there is no other source of information about the former variable, the structural parameter b may never be learned. Frydman (1982) notes that the probability that perceptions about b will not converge to the true value is strictly positive in a similar framework. Adjustment may then always deviate from ex post RE adjustment and "bubbles" would be expected to occur more or less continuously.

In many financial markets, futures and forward markets provide information about average expectations. For example, the forward exchange rate could be interpreted as $E_t[s_{t+1}]$ as expressed in (8b). Therefore, the parameter b in (22) could be identified. In this case, the learning process of relevance for bubbles and sunspots would be the one discussed above for time-series properties of fundamentals and for the choice of fundamentals. Frydman (1982) notes that a condition for b to be learned in this way is that all agents form expectations based on fundamentals as in expression (8a).

IV. Summary

Adjustment coefficients in a financial market depend on structural parameters as well as on perceptions and uncertainty about these parameters. In this paper, a simple model of exchange rate determination was used to show how adjustment can be decomposed into

(1) adjustment with ex post rational expectations as in most models;

(2) adjustment due to erroneous perceptions about the time series characteristics of fundamentals; and

(3) adjustment that is caused by misperceptions about fundamentals in the model.

Both adjustment of type (2) and type (3) may depend on available information about model structure, and therefore, in a sense, be consistent with rational expectations. These types of adjustment may explain the appearance of bubbles and sunspots. In this paper, it has been argued that adjustment of these types should be associated with learning, once it is recognized that structural parameters are uncertain. To the extent that learning occurs, empirical work could identify its existence and duration by analyzing time-variation of adjustment-coefficients.

It was also suggested that results from regressions of actual price changes on forecast price changes can be used to derive implicit learning processes. Such processes may explain results that seem inconsistent with rational expectations. By analyzing policy shifts during the estimation period, it may be possible to evaluate whether the implicit process is reasonable. Frankel and Froot (1987) analyze properties of survey data of expectations in comparison with forward rates, and conclude that expectations may overestimate or underestimate actual price changes for long periods. As the authors suggest, such repeated mistakes may be consistent with rationality if the true model is evolving over time.

More theoretical and empirical research is necessary in order to hypothesize anything specific about the characteristics of such processes, however. Though some exploratory research exists, it is difficult for the researcher to distinguish between "true" learning and changes in expectations caused, for example, by mass-psychology as suggested by Shiller (1984). It may be possible to use an independent source of information about time-series properties in order to analyze whether changes in coefficients are linked to learning of these properties.

Uncertainty about structure as expressed by the variance of the time-series properties of disturbances will also contribute to price variance in excess of the variance in ex post rational expectations equilibrium.

The analysis has important policy implications as well. Government authorities can influence the learning processes about the time-series characteristics of policy variables by announcing their policy rules and by following up on such announcements. Furthermore, since the variance, as well as expected values of policy parameters, influence adjustment coefficients, credibility of policy regimes is important for the variability of prices in financial markets.

Government authorities are obviously unable to do much to improve expectation-formation about non-policy variables except to announce their own expectations. In stock markets, the variance of stock prices would be influenced in a similar manner by the credibility and relevance of information provided by firms.

Appendix: Identifying a Learning Process

Assume that the following result has been obtained when regressing the percentage change in the exchange rate on the forward premium:

$$(s_{t+1} - s_t) = -1(f_{t+1} - s_t) + \varepsilon_{t+1} \tag{A1}$$

where f_{t+1} is the forward rate at time t and ε_{t+1} is a randomly distributed error term.

f_{t+1} is assumed to be equal to the expected exchange rate:

$$E_t[s_{t+1}] = E_t[B_{t+1}\rho_t]m_t \tag{A2}$$

Then (A1) can be rewritten as

$$B_{t+1}m_{t+1} - B_t m_t = -1\{E_t[B_{t+1}\rho_t]m_t - B_t m_t\} + \varepsilon_{t+1} \tag{A3}$$

It is assumed that money is known to be the only fundamental factor in exchange rate determination, i.e., the R_t term in the text is neglected.

In order to find a learning process while retaining the RE-assumption, we deduct the expression $(\bar{B}\rho_t m_t - \bar{B}m_t)$ from both sides of (A3). This procedure implies that if ex post RE-coefficients $(\bar{B})$ had been known, then the exchange rate change had been anticipated and reflected perfectly in the forward rate. We obtain, after deducting this expression, that

$$\bar{B}m_t(\rho_t + E[\rho_t]) - 2m_t\bar{B} + B_{t+1}v_{t+1}$$
$$+ m_t(\gamma_{t+1}\rho_t + E[\rho_t\gamma_{t+1}] - \gamma_t) = \varepsilon_{t+1} \tag{A4}$$

For the coefficient -1 in front of the forward premium to be explained by learning, the left hand side must be randomly distributed with a mean to zero.

(A4) is simplified further by noting that $E[\rho_t] = \rho_t + \varepsilon_t^\rho$, that $\gamma_{t+1} = \gamma_t + (\gamma_{t+1} - \gamma_t)$, and that $\text{cov}[\rho_t;B_{1,t+1}] = \sigma_{\varepsilon\rho}^2$. It is also assumed that $E_t[\varepsilon_{t+1}^\rho] = 0$ and that individuals always expect that the currently perceived policy rule is permanent. Under this condition γ_t can be written as $K(\varepsilon_t^\rho + \sigma_{\varepsilon\rho}^2)$ using (20) in the text. K can be considered a constant if ε^ρ is small relative to $E[\rho]$ in the denominator of (20). After some manipulation, we obtain that:

$$2m_t\bar{B}(\rho-1) + m_t\varepsilon^\rho[K(\rho-1) + \bar{B}] + m_t K\rho\sigma_{\varepsilon\rho}^2 + m_t K\rho(\varepsilon_{t+1}^\rho - \varepsilon_t^\rho) + B_{t+1}v_{t+1} = \varepsilon_{t+1} \qquad \text{(A6)}$$

On the left hand side, $B_{t+1}v_{t+1}$ is randomly distributed with a mean zero (since B_{t+1} may increase or decrease over time, the error process during learning may show heteroskedasticity). The remaining terms on the left hand side must, on the average, sum to zero. Therefore, on the average during the learning process:

$$-K\rho(\varepsilon_{t+1}^\rho - \varepsilon_t^\rho) = 2\bar{B}_t(\rho-1) + \varepsilon^\rho[K(\rho-1) + \bar{B}] + K\rho\sigma_{\varepsilon\rho}^2 \qquad \text{(A7)}$$

(A7) shows that if $\rho > 1$, and if ρ is overestimated ($\varepsilon^\rho > 0$), then the right hand side is clearly positive. For the left hand side to be positive as well, the perception error must be partially corrected. In other words, under the condition that the time-series parameter is sufficiently large and overestimated, the coefficient -1 in the original regression can be explained by a learning process which works in the correct direction. If ρ is growing over time, then the correction may never be sufficient but perceptions lag behind continuously.

Notes

1. By assuming that both m_t and R_t are observed, we disregard the information extraction problem in many macro-models. If these variables were not observed, the model would have to be solved in terms of observed lagged variables (m_{t-1} and R_{t-1}) and current fundamental disturbances (v_t and w_t). In principle the problem caused by parameter uncertainty would still be the same, but it would influence the formation of expectations for a larger number of variables.

2. This coefficient is the infinite sum

$$\frac{1}{1+b} + \frac{b}{(1+b)^2}\rho_t + \frac{b^2 b}{(1+b)^3}\rho_t\rho_{t+1} + \ldots$$

$$+ \frac{b^n}{(1+b)^{n+1}}\rho_t\rho_{t+1}, \ldots, \rho_{t+n-1} = \sum_{j=1}^{\infty}\frac{b^{j-1}}{(1+b)^j}\prod_{t=0}^{j}\rho_t .$$

3. Compare with footnote 2 in which the ex post RE-coefficient is shown to depend on a series of time-series parameters. γ_1 depends in a similar fashion on a series of perception errors.

4. Frydman (1982) argues that subjectively optimal forecasting involves the explicit forecasting of others' expectations. However, as long as a source for extracting such information has not been specified, it seems proper to assume that agents follow rules as in (8).

5. Thus, as Friedman (1953) noted, speculative activity presumes that individuals have different expectations or at least that they believe so. Frydman (1987) analyzes in detail the role of diversity of expectations in a process of learning.

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