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LAY-OFFS**

by  
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This is a preliminary paper.  
Comments are welcome.

May, 1987

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This is very preliminary draft written during my stay at the Industrial Institute for Economic and Social Research, Stockholm.

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## OPTIMAL FIRM SIZE, TAXES, AND LAYOFFS

### Introduction

In his more than useful survey of work on implicit contracts, Rosen (1985) points out that "Contract theory adds no insights into the determination of firm size....". Firm size here being measured by the number of workers attached to the firm (not all need be employed, some may be temporarily laid off). The purpose of this study is to demonstrate that this need not be the case. It will be shown that optimal firm size can be well defined within the context of implicit contract theory, and that when optimal firm size is embedded in a implicit contract model, startlingly different consequences result than those previously obtained in the literature. In particular, it will be demonstrated results obtained by Feldstein (1976) pertaining to layoffs and the unemployment insurance system do not hold when the size of a firm is a choice variable of a firm.

There is now little need to stress the major contribution contract theory has made in the last ten years. In the usual implicit contract model an expected profit maximizing firm, facing an uncertain price for its output, offers a labor contract to a given number of risk averse workers. This contract specifies the wage to be paid to employed workers as well as the numbers to be employed given each possible realization of output price.

Typically, it is demonstrated that such a contract will imply the wage paid to employed workers is independent of the price realization, and it is possible that all attached workers will not be employed given some price realizations. The basic extension of the model presented here is to give the firm the option of choosing for itself the number of workers it wants attached to it, i.e., the firm's size. Although, as will be shown below, this is a relatively straightforward extension, the consequences are important and lead to significantly different results.

In his classic paper, Feldstein (1976) considered the consequences of changes in the unemployment insurance tax system on temporary layoffs. In particular, he argued that an increase in the experience rating factor will reduce temporary layoffs. His argument can be simply explained as follows. Let  $b$  denote the public unemployment insurance payment (per period) to a temporarily laid off worker. Further, let  $eb$  denote the increase in tax paid by a firm when it lays off a worker, where  $e$  is termed the experience rating factor. Typically it is assumed that  $0 \leq e \leq 1$ , although this need not be the case. It comes as no shock the factor  $e$ , taken as a given by firm, can be a powerful tool by which a government can influence employment. If  $e = 1$ , then the firm is said to be fully experience rated, whereas if  $e = 0$ , the firm is not experience rated at all. At first blush, Feldstein's point appears more than reasonable. He argued that when a

government increases the experience rating factor,  $e$ , it increases the marginal cost of laying off a worker to a firm, and thus the number of workers laid off should be reduced, *ceteris paribus*.

In this study Feldstein's argument is reconsidered in some detail. It will be demonstrated that his results rest on the somewhat unreasonable assumption that a firm cannot choose the size of its labor force, as measured by the number of workers attached to the firm. When a firm is allowed to choose the size of its labor force (and this, surely, is the more reasonable case) an increase in the experience rating factor,  $e$ , will reduce a firm's desired size of labor force, and has an indeterminate consequence on the number a firm will lay off. The reasoning behind this result is straightforward. An increase in the experience rating factor does, as Feldstein argues, increase the marginal tax cost of laying off a worker. Nevertheless, such an increase makes a firm more hesitant to hire a worker in the first place as the firm takes it into account that it may want to lay the worker off in the future.

If the alternative to being hired by the firm is unemployment, an increase in  $e$ , will increase the number of unattached unemployed and have an indeterminate consequence on the number of temporary lay-offs. As will be shown, in this case an increase in the  $e$  can increase the total unemployed (unattached unemployed plus temporary layoffs). If, however, the alternative to being hired by

the firm is employment at another firm, an increase in  $e$  will reduce the optimal firm size as well as have an indeterminate consequence on temporary layoffs.

To complete the analysis, changes in other aspects of the unemployment insurance tax system will be considered, as well as changes in unemployment insurance payments. The results obtained are significantly different than those derived when the firm cannot choose its own size.

Although some interesting empirical work has been done on Layoff unemployment and the unemployment insurance system (see, for example, Brechling (1981) and Topel (1983)), the results presented here indicate it may well be worth studying the interaction between layoff unemployment and other types of unemployment.

### The Model

The results will be established in the context of an extended version of a reasonably standard contract model. Suppose a firm offers a contract for a given period of time to a group of homogeneous workers. The firm is a price taker in the market for its output but, unfortunately, does not know the price it will face during the period under consideration. It is, however, known it will face price  $p_1$ , with probability  $\lambda$ , or price  $p_2$ , with probability  $1-\lambda$  ( $p_1 > p_2$ ). As the price to be faced is

revealed before production begins, the contract may be conditioned on the realized price.

The firm selects  $N$  workers and offers each of them a contract which yields an expected utility at least as great as  $W$ . Any contract yielding an expected utility less than  $W$  will be rejected as workers are assumed to be capable of obtaining such an expected utility elsewhere. The factors which determine the precise level of  $W$  will be discussed later.

Any worker who accepts the contract offered by the firm will be termed an attached worker. As will be demonstrated below, not all attached workers need be employed, depending on the price realization. Attached workers not employed will be termed laid off as they are assumed to have no other employment opportunities for the period once they have become attached.

To focus on essentials, a fixed hours model will be used where each employed worker works one unit of time during the period under discussion. Thus,  $f(n)$  indicates the output of the firm when  $n$  of the  $N$  attached workers are employed. The production function  $f(\cdot)$  is assumed to have all the usual properties, i.e.,  $f$  is increasing, differentiable, and strictly concave. The model to be presented can be extended to a variable hours model although this would complicate the model and add little extra to the results. In a recent study, Burdett and Wright (1987) have demonstrated, within the context of a variable hours model, the biases caused when unemployment

insurance (UI) payment is only made to those who are laid off, and not to those who have a reduction in hours worked.

Any attached worker who is laid off receives UI payment  $b$ . Assume the firm contributes to UI payments by paying taxes. In particular, suppose the taxes paid by the firm depend on the number of attached workers it has and the number it lays off. Formally, let

$$(1) \quad eb[N-n] + \delta N + T$$

indicate the taxes paid by the firm when  $n$  of the  $N$  attached workers are employed. The term  $e$  is usually termed the experience rating factor as it specifies the marginal tax cost (in terms of the UI payment) of laying off a worker. Typically, it is assumed  $0 \leq e \leq 1$ , although we only require that  $e \geq 0$ . The second term in (1),  $\delta N$ , denotes a payroll tax,  $\delta > 0$ , whereas the third term,  $T$ , is a pure lump sum tax. The tax system described above is similar to that used by Feldstein (1976), although he does not include the payroll tax.

Any attached worker's utility is assumed to be an increasing function of income received, and a decreasing function of hours worked. In particular, let  $u(y,h)$  denote the utility of a worker when income  $y$  is received for  $h$  units of work. Suppose  $u(\cdot)$  is differentiable such that



$$(2) \quad u > 0: u_2(\cdot) < 0: u_{11}(\cdot) \leq 0: u_{22}(\cdot) \leq 0$$

$$u_{11}(\cdot)u_{22}(\cdot) - u_{12}^2 \geq 0$$

Following convention and because it much simplifies the analysis, assume that if the firm does not employ its attached workers it randomly selects those to be laid off. Thus,  $[N-n]/N$  denotes the probability any of the  $N$  attached workers will be laid off when  $n$  workers are to be employed. The idea of the firm choosing which workers to lay off by random selection may at first appear somewhat unrealistic. Such things as seniority and different skill levels of workers will typically influence a firm when deciding who to lay off. The model specified in this study, however, is too purified to include heterogeneous workers. Nevertheless, it is possible to include such heterogeneity within the implicit contract framework (see, for example, Lowenstein (1983)).

Given the restrictions made above, any contract offered can be represented by a vector of real numbers  $C(N) = (y_1, y_2, n_1, n_2, N)$ , where  $y_i$  is the income received by each of the  $n_i$  employed workers when the realized output price is  $p_i$ ,  $i = 1, 2$ . The expected profit from any particular contract  $C(N)$  can be written as

$$(3) \quad \Pi(C(N)) = \lambda[p_1 f(n_1) - y_1 n_1 - eb[N-n_1] - \delta N - T] \\ + (1-\lambda)[p_2 f(n_2) - y_2 n_2 - [N-n_2] - \delta N - T]$$

Further, the expected utility of a representative worker, given he or she accepts contract  $C(N)$ , can be expressed as

$$(4) \quad U(C(N)) = \lambda[u(y_1, 1)(n_1/N) + u(b, 0)(1-n_1/N)] \\ + (1-\lambda)[u(y_2, 1)(n_2/N) + u(b, 0)(1-n_2/N)]$$

Suppose the firm maximizes its expected profits subject to the constraints it faces. The Lagrangian corresponding to this problem can be written as

$$(5) \quad L = \Pi(C(N)) + \mu[U(C(N))] + \beta_1[N-n_1] + \beta_2[N-n_2]$$

As the concern is with nontrivial contracts in which both income and employment levels will be strictly positive, and because it is straightforward to demonstrate the minimum expected utility constraint will be binding, the necessary conditions for a maximum can be written as

$$(6a) \quad \partial L / \partial y_1 = \lambda n_1 [-1 + \mu u_1(y_1, 1)] = 0$$

$$(6b) \quad \partial L / \partial y_2 = (1-\lambda) n_2 [-1 + \mu u_1(y_2, 1)] = 0$$

$$(6c) \quad \partial L / \partial n_1 = \lambda [p_1 f'(n_1) - y_1 + eb \\ + (\mu/N)(u(y_1, 1) - u(b, 0))] - \beta_1 = 0$$

$$(6d) \quad \partial L / \partial n_2 = (1-\lambda) [p_2 f'(n_2) - y_2 + eb \\ + (\mu/N)(u(y_2, 1) - u(b, 0))] - \beta_2 = 0$$

$$(6e) \quad \partial L / \partial N = -eb - \delta - (\mu / N^2) ([u(y_1, 1) - u(b, 0)] n_1 \lambda + \\ + (u / N^2) [u(y_2, 1) - u(b, 0)] n_2 (1 - \lambda) + \beta_1 + \beta_2 = 0$$

$$(6f) \quad \partial L / \partial \mu = u(y_1, 1) \lambda (n_1 / N) + u(y_2, 1) (1 - \lambda) (n_2 / N) \\ + u(b, 0) [\lambda (1 - n_1 / N) + (1 - \lambda) (1 - n_2 / N)] = 0$$

$$(6g) \quad \partial L / \partial \beta_i = N - n_i \geq 0; \quad \beta_i \geq 0; \quad \beta_i (N - n_i) = 0; \quad i = 1, 2.$$

Any contract which satisfies the above conditions will be termed an optimal contract. Note the above is all reasonably standard with the exception that the firm is assumed to maximize over the number of workers it wishes to have attached to it. An immediate consequence of (6a) and (6b) is stated in the following claim.

Claim 1:

Given  $u_{11}(\cdot) < 0$ , i.e., workers are strictly risk averse in income, any optimal contract will imply  $y_1 = y_2$ . Given  $u_{11}(\cdot) = 0$ , i.e., workers are indifferent to risk in income, there exists an optimal contract such that  $y_1 = y_2$ . Further, an optimal contract will imply

$$(7) \quad N / \mu = u_1(y_i, 1) \quad i = 1, 2.$$

Using the above claim as justification, in what follows it will be assumed that any optimal contract considered will have the "fixed" wage property and let  $y$

indicate the wage paid to employed workers in either state. Indeed, to guarantee such a restriction holds, workers will be assumed to be strictly risk averse in income, i.e.,  $u_{11}(\cdot) < 0$ . This simplification implies (6c), (6d), (6e), and (6f) can be written as

$$(8a) \quad \lambda[p_1 f'(n_1) - y + eb + z(y,b)] - \beta_1 = 0$$

$$(8b) \quad (1-\lambda)[p_2 f'(n_2) - y + eb + z(y,b)] - \beta_2 = 0$$

$$(8c) \quad -eb - \delta - z(y,b)[(\lambda n_1 + (1-\lambda)n_2)/N] + \beta_1 + \beta_2 = 0$$

$$(8d) \quad u(y,1)[(\lambda n_1 + (1-\lambda)n_2)/N] + u(b,0)[1 - (\lambda n_1 + (1-\lambda)n_2)/N] = W$$

where

$$(9) \quad z(y,b) = [u(y,1) - u(b,0)]/u_1(y,1)$$

Note,  $z(y,b)$  indicates how much a worker likes (or dislikes) work relative to being laid off, given a particular optimal contract. It can, of course, be positive or negative, depending on the optimal contract considered, and the particular values of the exogenous parameters. Nevertheless, the sign of  $z(\cdot)$  associated with an optimal contract can be predicted with certainty from only two parameters,  $b$  and  $W$ . This claim follows from (8d) once it is recognized that  $W$  (the minimum expected utility) is a weighted average of the utility of working and the util-

ity from being laid off. Further, the weights are between zero and one. Thus, for example, if  $u(b,0) < W$ , then any optimal contract must imply a wage  $y$  such that  $u(y,1) > W$ , and therefore the optimal contract will be such that  $z(y,b) > 0$ .

Letting the given values of the parameters vary, we may consider the set of optimal contracts. The reasoning presented above allows us to partition this set into two: those optimal contracts which imply  $z(\cdot) \geq 0$  will be termed work contracts, whereas those which imply  $z(\cdot) < 0$  will be termed leisure contracts. The following claim summarizes the above discussion.

Claim 2:

Given a strictly positive employment level in at least one state:

- (a) if  $u(b,0) \geq W$ , the optimal contract will be a work contract;
- (b) if  $u(b,0) < W$ , the optimal contract will be a leisure contract.

The optimal contract will be a work contract in two situations. First, if UI payment  $b$  is also paid to the unattached workers, i.e., those workers who were not offered a contract by any firm. In this case, an unattached worker's expected utility is at least  $u(b,0)$ , and may be greater if there are other, more profitable, job opportunities. Second, even if an unattached worker does not obtain any UI payment, the labor market condi-

tions may be such that if a worker does not obtain an offer from the firm under consideration, there are other job offers elsewhere which yield an expected utility at least as great as  $u(b,0)$ . One of the two scenarios described above may well be satisfied in most real world situations.

Optimal contracts will be leisure contracts if (a) the UI payment to unattached workers is less than that to laid off workers, and (b) the labor market conditions are such that there are no other employment opportunities which yield an expected utility at least as great as  $u(b,0)$ .

It appears likely that  $z(\cdot) > 0$  (working is ex post preferred to being temporarily laid off) is the usual case, although there are reasons for taking seriously the possibility that  $z(\cdot) < 0$ . First, being temporarily laid off can be attractive to some workers in that it may be perceived as a paid vacation without the stigma of a permanent job loss attached to it. Further, for low paid workers the loss of income may not be too great. Second, if the UI payment was set by the government to maximize the expected utility of workers attached to the firm under consideration, it would select a  $b^*$  such that  $u_1(y,1) = u_1(b^*,0)$ . This is, of course, the optimal insurance condition which in this case implies UI payment is set to equate the marginal utilities from both states. If  $u_{12}(\cdot) < 0$ , i.e., the marginal utility of income decreases as leisure increases, then  $z(y,b^*) < 0$ . Thus,

given  $u_{12}(\cdot) < 0$ , we have the somewhat paradoxical result that if UI payments were set optimally, then workers would not only prefer to be laid off but also receive a greater income when this was the case. To my knowledge, this situation is not common. Moral hazard problems may explain why UI payments to temporarily laid off workers are typically less than when they are employed.

Work Contracts:

Throughout this section it will be assumed the value of the parameters  $b$  and  $W$  are such that any optimal contract considered will be a work contract. An immediate consequence of this restriction is presented in the following claim.

Claim 3:

If an optimal contract is a work contract all attached workers will be employed when the realized price is  $p_1$  (i.e.,  $N = n_1$ ). Further, for fixed  $p_1$ , the given price  $p_2$  is such that if  $p_2 < p_2(p_1)$ , then  $n_2 < N$ .

Proof:

Suppose all attached workers are not employed given either price realization. In this case,  $\beta_1 = \beta_2 = 0$  as shown in (6h). This, however, leads to a contradiction, as (8c) cannot now be satisfied. Thus,  $n_1 = N$ , and/or  $n_2 = N$  with an optimal work contract. The second element of the claim is established in Burdett and Hool (1983).

In what follows it will be assumed that  $N = n_1$  and  $n_2 < N$ . Claim 3 has established that this is not unreasonable. Given this restriction, the equations of (9) can be used to perform comparative statics. Before achieving this goal, however, it will be useful to describe the nature of this thought experiment. When considering changes in the parameter,  $e$ ,  $b$ , and  $\delta$ , it will be assumed the minimum expected utility constraint,  $W$ , is held constant. This is in contrast to Feldstein, who holds the expected profit of the firm (assumed equal to zero) constant when considering a change in a parameter. As shown by Burdett and Hool (1983), this can lead to some minor differences in results. Nevertheless, the thought experiment performed here appears the most natural in the situation envisaged. First, consider a change in the experience rating factor,  $e$ . This yields

$$\Delta \begin{vmatrix} dN \\ dy \\ dn_2 \end{vmatrix} = \begin{vmatrix} (1-\lambda)bdz \\ bdz \\ 0 \end{vmatrix}$$

where

$$\Delta = \begin{vmatrix} \frac{\lambda p_1 f''(N) + (1-\lambda)n_2 z(y,b)}{N^2} & \frac{-[\lambda N + (1-\lambda)n_2 z_1(y,b)]}{N} & \frac{-(1-\lambda)z(y,b)}{N} \\ 0 & z_1(y,b) - 1 & p_2 f''(n_2) \\ \frac{-(1-\lambda)n_2 z(y,b)}{N^2} & \frac{\lambda N + (1-\lambda)n_2}{N} & \frac{(1-\lambda)z(y,b)}{N} \end{vmatrix}$$



and

$$(10) \quad |\Delta| = \frac{[z_1(y,b)-1]z(y,b)(1-\lambda)p_1f''(N)\lambda}{N} \\ + \frac{z(y,b)(1-\lambda)^2n_2^2p_2f''(n_2)[z_1(y,b)-1]}{N^3} - \frac{p_1p_2f(n_2)\lambda f''(N)[\lambda N+(1-\lambda)n_2]}{N}$$

As we are considering optimal work contracts, it follows that

$$(11) \quad z_1(y,b) - 1 = 1 - \frac{u_{11}(y,1)}{u_1(y,1)} z(y,b) \geq 0$$

and thus,  $|\Delta| < 0$ . Using Cramer's rule, it can be shown that

$$(12a) \quad \frac{dN}{de} = \frac{(1-\lambda)b}{|\Delta|N} \left\{ [z_1(y,b)-1](1-\lambda)z(y,b)[1-(n_2/N)] \right. \\ \left. - p_2[\lambda N+(1-\lambda)n_2]f''(n_2) \right\} < 0$$

$$(12b) \quad \frac{dy}{de} = - \frac{z(y,b)(1-\lambda)b}{|\Delta|N} \left[ \frac{\lambda p_1 f''(N)N + (1-\lambda)p_2 f''(n_2)n_2}{N} \right] \leq 0$$

$$(12c) \quad \frac{dn_2}{de} = \frac{b}{|\Delta|N} \left\{ \frac{(1-\lambda)^2 n_2 z(y,b)[z_1(y,b)-1]}{N^2} [1 - n_2/N] \right. \\ \left. + [\lambda N+(1-\lambda)n_2]\lambda p_1 f''(N) \right\} = ?$$

Thus, an increase in the experience rating factor,  $e$ , will decrease the number of workers attached to the firm, decrease the wage paid to employed attached workers, and have an uncertain consequence on the number it lays off when  $p_2$  is the realized price. The intuition behind these results is relatively straightforward. If the firm reduces the number of workers attached to it, and  $z(\cdot) > 0$ , then the expected utility of each of the reduced number of workers is increased, holding  $y$  and  $n_2$  constant. The firm can now reduce the expected utility of its attached workers back to  $W$  by lowering the wage paid to employed workers and/or reducing the number it lays off when  $p_2$  is the realized price. Using (8b) it follows that

$$(13) \quad \frac{dn_2}{de} = - [b/p_2 f''(n_2)] - \{[z_1(y,b)-1]/p_2 f''(n_2)\} \frac{dy}{de}$$

This, of course implies  $dn_2/de$  will be positive as long as  $dy/de$  is not too negative. More precisely,

$$(14) \quad dn_2/de > 0 \text{ if and only if } dy/de > - b/[z_1(y,b)-1]$$

Before considering the implications of changes in the experience rating factor in more detail, a brief review of the consequences of changes in the other parameters is presented. An increase in the payroll tax parameter,  $\delta$ , will imply the following changes in the endogenous variables in an optimal work contract

$$(15a) \quad \frac{dN}{d\delta} = \frac{1}{|\Delta|N} [(z_1(y,b)-1)(1-\lambda)z(y,b) - p_2 f''(n_2)(\lambda N + (1-\lambda)n_2)] < 0$$

$$(15b) \quad \frac{dy}{d\delta} = - \frac{1}{|\Delta|N^2} [z(y,b)(1-\lambda)n_2 p_2 f''(n_2)] \leq 0$$

$$(15c) \quad \frac{dn_2}{d\delta} = \frac{1}{|\Delta|N^2} [(z_1(y,b)-1)z(y,b)(1-\lambda)n_2] < 0$$

Thus, an increase in  $\delta$  reduces the number of attached workers the firm desires, reduces the income to the employed attached workers, and increases the number laid off when the realized price is  $p_2$ . Obviously, any change in the lump sum tax will have no consequence on the solution to an optimal contract (given it is assumed that a firm continues to stay in business).

A change in the UI payment to laid off worker,  $b$ , generates the following changes in optimal work contracts:

$$(16a) \quad \frac{dN}{db} = \frac{(1-\lambda)}{|\Delta|N} \{ [e + z_2(y,b)][z(y,b)(z_1(y,b)-1)(1-\lambda)(1-n_2/N) + p_2 f''(n_2)(\lambda N + (1-\lambda)n_2)] - z_2(y,b)p_2 f''(N)(1-n_2/N) \}$$

$$(16b) \quad \frac{dy}{db} = - \frac{(1-\lambda)}{|\Delta|N} \{ [e + z_2(y,b)][(z(y,b)/N)(\lambda p_1 f''(N) + p_2 f''(n_2)n_2)] - z_2(y,b)p_2 f''(n_2)(N - n_2 p_1 f''(N)\lambda) \}$$

$$\begin{aligned}
 (16c) \quad \frac{dn_2}{db} &= \frac{(1-\lambda)}{|\Delta|N} \{ [e+z_2(y,b)] [(z_1(y,b)-1)(1-\lambda)n_2(1-n_2/N) \\
 &+ \lambda p_1 f''(N)(\lambda N+(1-\lambda)n_2)N/(1-\lambda)] \\
 &+ z_2(y,b)(N-n_2)(z_1(y,b)-1)\lambda p_1 f''(N) \}
 \end{aligned}$$

As is seen above, the predictions from an increase in UI payment are somewhat complicated and difficult to sign in general. Indeed, an increase in UI payment can increase, or lower,  $N$ ,  $y$ , and  $n_2$ , depending on the given values of parameters. The reason for this ambiguity is that no assumption has been made so far about the magnitude of  $z_2(y,b) = -u_1(b,0)/u_1(y,1)$ . Of course,  $z_2(y,b)$  is negative, but is it greater, or less, than minus one? The UI payment which maximizes the attached workers' utilities is such that it equates the marginal benefit of working with being laid off, i.e.,  $z_2(y,b) = -1$ . If  $z_2(y,b) < -1$ , attached workers would prefer to transfer income received when employed to income when laid off, whereas if  $z_2(y,b) > -1$ , workers would prefer to transfer income when laid off to income when employed. Of course, they can achieve either of these goals by some suitable savings plan. Such complexities, however, are beyond the scope of the present study.

Utilizing the discussion above, the following conditional predictions can be made:  $dN/db > 0$  and  $dy/db \geq 0$ , when  $|z_2(y,b)| \geq e$ . Thus, a firm will increase its desired size when  $e$  is small enough.

Before finishing this section, it should be noted that an increase in the minimum expected utility constraint will lower the desired size of a firm, i.e.,  $dN/dW < 0$ . The consequence of such a change on income to employed workers and the numbers employed cannot be signed, in general.

In the next section a special case will be considered where  $W = u(b,0)$ . This, of course, will imply that any optimal contract will be such that  $z(y,b) = 0$ . There are two reasons for a detailed analysis of such a situation. First, as Feldstein (1978) has argued this well may be the case in the US. His calculations indicate the net loss of income from unemployment is relatively small such that it may well be balanced by the gain in utility from the extra leisure. Second, as will be shown below, assuming  $z(y,b) = 0$ , much simplifies the analysis and leads to more predictable consequences from policy changes. Such contracts will be termed just work contracts.

#### Just Work Contracts:

This particular case will hold if unattached workers receive the same unemployment compensation as temporarily laid off workers, and workers not offered a contract by the firm face no alternative employment. In this case, any optimal contract will imply  $z(y,b) = 0$ , and  $z_1(y,b) = 1$ . These consequences, of course, will much simplify the analysis. To add some flavor to the situation, suppose for the moment there are  $S$  workers who will

either accept attachment to the firm under consideration, or become (what will be termed) unattached unemployed. Thus, given the firm offers an optimal work contract to  $N$  of the workers  $S \geq N$ , the expected total unemployment,  $EU$ , can be written as

$$(17) \quad EU = S - N + (1-\lambda)[N-n_2]$$

Of course, this is somewhat of a fraud, as no market model has been presented. Nevertheless, any reduction in  $N$  due to a parameter change will lead at least in the very short run to an increase in unattached unemployment. How quickly these workers find jobs is beyond the scope of this paper.

Given  $u(b,0) = W$ , assuming that unattached workers will remain unemployed is not unreasonable. In this case, workers are indifferent between attachment at the firm and unemployment. By convention, however, workers accept any contract which offers an expected utility of  $u(b,0)$ . If all other firms offer the same expected utility as the firm under consideration, then any cost of moving to the other firms, no matter how small, will imply workers prefer to be unattached rather than become attached to another firm.

Given the restrictions made above, the comparative statics results are presented in Table 1.

Table 1

|                 | de   | dδ                         | db<br>( z <sub>2</sub>   ≥ e)                       |
|-----------------|--|----------------------------|---|
| dN              | $\frac{(1-\lambda)b}{p_1 f''(N)\lambda} < 0$ | $\frac{1}{p_1 f''(N)} < 0$ | K > 0,  |
| dy              | 0  | 0                          | $\frac{(e+z_2)}{(\lambda N+(1-\lambda)n_2)} \geq 0$ |
| dn <sub>2</sub> | $\frac{-(1-\lambda)b}{p_2 f''(n_2)} > 0$     | 0                          | $\frac{(e+z_2)}{p_2 f''(n_2)} \geq 0$               |

$$K = -(1-\lambda) \left[ \frac{(e+z_2)e}{p_1 f''(N)} + \frac{z_2(1-n_2/N)}{p_1 f''(N)(\lambda N+(1-\lambda)n_2)} \right]$$

As can be seen in Table 1, the results are relatively straightforward with just work contracts. There is, however, an implication worthy of note. An increase in the experience rating factor can increase, or decrease, total unemployment. In particular,

$$\frac{dEU}{de} = (1-\lambda)b \left\{ \frac{p_1 f''(N) - p_2 f''(n_2)}{p_1 f''(N)p_2 f''(n_2)} \right\}$$

Thus, if  $f''(\cdot)$  is increasing in  $n$ , then an increase in the experience rating factor will increase total unemployment, i.e., the increase in unattached unemployment

dominates the reduction in layoffs. Finally, note that, if  $|z_2| > e$ , an increase in UI payments will reduce the total unemployment as it reduces both unattached and layoff unemployment.

#### Leisure Contracts:

In this final section assume any optimal contract considered implies  $z(.) < 0$ , i.e., a worker would prefer to be laid off rather than work. When this is the case optimal contracts can imply three possible outcomes. First, all attached workers can be employed in either state. Second, all attached workers are employed when  $p_1$  is the realized price, but not all attached workers are employed when  $p_2$  is the realized price. Finally, the parameters may be such that some attached workers are laid off in either state.

Assume first that all attached workers are employed when the high price is realized but not when the low price is realized. In this case it is straightforward to show that all the predictions are the same as with optimal work contracts with the exception that the signs of the predictions about income,  $y$ , are reversed. Thus, the major predictions of this study still hold with optimal leisure contracts when there is layoff unemployment in only one state.

It should be noted that if  $W$ , and  $b$  are such that  $z(.) < 0$ , then workers prefer to be laid off, and thus, for a given amount of employment, firms want many



attached workers as possible. If firms are not experience rated at all, and there is no payroll tax,  $\delta$ , then firms would be indifferent to the number of attached workers they have. This implies there is no sensible optimal firm size in this case as an optimal leisure contract would imply an infinite number of workers attached to each firm. If, however,  $e > 0$ , and/or  $\delta > 0$ , then there is a strictly positive marginal cost to a firm from adding another worker to attachment. This can be such that a firm will choose the number of attached workers such that not all will work in either state. The comparative statics results in this case are exactly the same as with optimal leisure contracts.

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