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CORPORATE AND PERSONAL TAXATION AND THE GROWING FIRM

by

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Abstract

The elements of corporate and personal taxation are integrated into a corporate growth model describing a value maximizing firm. The choice parameters of the firm are (1) the growth rate (2) the debt ratio (3) the capital-labour ratio. Dividends are determined residually. The corporate tax considered is a flat-rate tax on profits as defined by the tax laws. The personal tax is a linear tax schedule. The main results of the paper are:

1. When the tax laws allow for free depreciation of all internally financed investments the corporate tax will be neutral or non-distortionar

2. A scheme of true (free) economic depreciation will be distortionary. An increased tax rate will in this case give a lower (higher) growth rate, a (higher) lower debt ratio and a more (less) labour intensive technique of production.

3. Within the framework of the straight-line depreciation and declining-balance depreciation rules, a change towards faster depreciation will always give a higher growth rate, a higher debt ratio and a less labour intensive technique.

4. For normal rates of tax depreciation and relatively modest debt ratios an increased corporate tax rate will lead the firm to increase its growth rate, its capital-labour ratio and its debt ratio.

5. An increase of the marginal tax rate of the personal income tax or a decrease of the tax rate on capital gains will lead the firm to increase its growth, its debt ratio and its capital labour ratio.

Corporate and personal taxation and the growing firm*

1. Introduction

There are numerous studies of the effect of changing tax laws on the investments, financial policy and dividend policy of the firm. More often than not, however, the different areas are treated separately, i.e. it is common to study the effects of a tax change on investments without taking account of the simultaneous effects on leverage and dividend policy. This might be explained by the fact that the analysis is carried out without the use of a complete model of the firm.

In this paper, however, the elements of corporate and personal taxation are integrated into a closed formal corporate growth model, which is an extension and modification of a model presented by Solow [1971] . The firm's objective is to find a growth path that maximizes the value of its shares. In doing so the firm has to choose three parameters, namely (1) the growth rate (or the rate of net investment), (2) the debt ratio, and (3) the capital-labour ratio. When these parameters are determined, dividends are also determined. The corporate taxation considered is a flat rate tax on profits, where profits are computed after deductions for capital depreciation according to rules specified in the tax laws. The personal tax discussed here is a linear tax schedule with a constant marginal tax rate.

The first question to which we address ourselves is which depreciation scheme makes corporate taxation neutral or non-distortionary, in the sense that a change in the tax rate will not affect the choice parameters of the firm. In the literature there are two main positions on this question. According to one, free depreciation, that is immediate writing off of all investments, makes corporate taxation non-distortionary.[1)]

The other position is that true economic depreciation is neutral.[2)]

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1) See Brown [1948] , Musgrave [1959,p.343] , Shoup [1969, pp 301,302] and Smith 1963 .

2) See e.g. Samuelson [1964] and in recent articles Stiglitz [1973] and King [1974].

In his article Solow [1971, p.338] pointed out very clearly that the latter position is compatible with a profit-maximizing firm in a static environment. For a growing value-maximizing firm, however, the true depreciation scheme would be distortionary according to Solow. Here Stiglitz and King differs with Solow since they get their result with a dynamic model of a value-maximizing firm as their framework of analysis.

The result of the analysis in this paper is that free depreciation for tax purposes of all internally financed investments makes the corporate tax neutral. In this case the corporate tax is equivalent to a special tax on dividends. Both free depreciation of investments and true economic depreciation are distortionary. It can be shown, however, that true economic depreciation for the borrowing value maximizing firm is neutral with respect to the firms partial condition for optimal borrowing. Thus Stiglitz' and King's result might be explained by their considering borrowing the only relevant source of finance.

Our second question is how the firm responds to tax changes under the rules of depreciation that are actually used. The rules considered are linear depreciation and declining balance depreciation.

As could be expected, we find that a change in the tax laws towards faster depreciation will always induce firms to choose a higher growth rate. The firm will also increase its debt ratio and choose a more capital intensive technique of production.

When we analyse the effects of a change in the tax rate it is found that the debt position is of crucial importance for the direction of induced change in the choice parameters of the firm. This result is in accordance with the findings from the section on neutral depreciation schemes and it might seem evident that it has to do with the deductability of interest payments. Many authors, including Baumol & Malkiel [1967] , Lintner [1962] and Modigliani & Miller [1963] have pointed out the distorting effects of dividends being taxed while interest payments are not. However, our analysis shows that the main factor explaining the importance of the debt position for the growing firm is that a levered firm gets a completely untaxed contribution of borrowed money to its cash flow.

Moreover, it is shown that for normal rates of tax depreciation and relatively modest debt ratios an increased tax rate will lead the firm to increase its growth rate, its capital labour ratio and its debt ratio.

The third question is how changes in the personal income tax affect the policy of the firm. As could be expected[1)] a change in the marginal tax rate will affect the policy of the firm only when there is a differential treatment of capital gains on one hand and dividends and other income on the other hand. If we consider a system where tax rates on dividends and capital gains are determined independently we find that an increase of the marginal tax rate on income or a decrease of the tax rate on capital gains both will lead the firm to increase its growth, its debt ratio and its capital labour ratio. Before we continue some of the shortcomings of the analysis should be pointed out. Thus the firm we are analyzing acts under complete certainty. This is of course unrealistic. The defense for the assumption is that other students of the questions discussed in this paper make the same assumption. And hopefully it will give some useful insights to clarify what happens in the simple world of certainty.

Another important limitation is that, like the main body of literature on corporate growth, the analysis is restricted to the micro-level. Therefore, the task still remains to reconcile our results on corporate taxation and growth with the results within the static general equilibrium framework of the Harberger model.

The plan of the paper is as follows. After a list of notation and definitions given in section 2, the basic model used in the analysis is presented in section 3. The first-order conditions for optimum of the taxed firm are developed and interpreted in section 4. Section 5 considers the question of neutral depreciation schemes. The prerequisites for a comparative dynamics analysis of the firm are given in section 6 and the appendix. These are used in section 7 where the firm behaviour under different depreciation rules is analyzed. Taxation of personal income and capital gains is considered in section 8. The main conclusions of the paper are listed in section 9.

2. List of symbols

In the sequel we shall use the following notation, where index t indicates point of time. Variables without index are not time dependent. Concerning prices, output is the numeraire.

1) See e.g. King [1973] .

point of time. Variables without index are determined in period zero, and are then constant ever after.

K_t = physical capital; L_t = labour employed; $\ell = L_t/K_t$;

Q_t = volume of output (sales); = labour employed: $\ell = L_t/K_t$;

w = wage rate relative to price of output;

K_t^I = equity capital; $K_t^y = K_t - K_t^I$; $h = K_t^y/K_t$ (debt ratio);

f = rate of physical depreciation; A_t = tax depreciation;

a = tax depreciation parameter;

r = rate of return on K; r^I = rate of returns on K^I;

C_t = corporate tax payments; c = corporate tax rate;

x' = personal income tax rate; x = 1-x';

g = rate of growth; i = borrowing rate of interest; k = rate of discount;

P_t = value of shares; D_t = dividends

Π_t^I = profits on K^I; $d = D_t/\Pi_t^I$.

3. The model

The firm is on a steady state growth path where the parameters h, g and ℓ are chosen at t = 0. Once the values are chosen they are expected to persist forever. So are the values of the exogenous parameters m,w,f,p,a and x. Also given is the amount of internal funds at t = 0. Or, in other words, mK_0^I is fixed. When the firm has decided on the value of g the growth of capital is given by

$$K_t = K_0 e^{gt}. \qquad (1)$$

The production function is homogeneous of the first degree. We will furthermore assume that there are certain costs associated with expansion. As the firm grows faster a higher fraction of scarce management has to be devoted to the organization of expansion *per se*.[1] Another example of growth costs is training of new personell.[2]

The growth costs are represented by the function T(g) $[T(0) = 1; T'(g) < 0]$ in a way that is given by the production function

$$Q_t = T(g) \cdot f(\ell) \cdot K_t. \qquad f'(\ell) > 0;\ f''(\ell) < 0 \qquad (2)$$

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1) Penrose [1951]
2) Rotschild [1971]

The debt ratio is determined by the choice of the parameter h, which in turn affects the borrowing costs according to the relation

$$i = i(h) \quad [i'(h) > 0]. \tag{3}$$

It is now clear that apart from taxes the firm has the following receipts and expenditures at time t.

$Q_t = T(g)f(\ell).K_0e^{gt}$	= Income from total sales	(4-a)
$mgK_t^y = mg \cdot hK_0e^{gt}$	= Increase in external funds	(4-b)
$mgK_t = mgK_0e^{gt}$	= Costs of net investments	(4-c)
$wL_t = w \cdot \ell K_0e^{gt}$	= Wage costs	(4-d)
$mfK_t = mfK_0e^{gt}$	= Replacement costs	(4-e)

So far, tax payments are not considered. We shall include these by assuming that the firm is taxed according to a flat rate p working on a base determined by

$$Q_t - wL_t - mhiK_t - mA_t \tag{5}$$

where A_t stands for depreciation deductions allowed by tax laws. We shall later investigate different depreciation formulas. At present we only state A_t as a general function of the growth rate and the public parameter a

$$mA_t = mA(a,g) \cdot K_0e^{gt} \tag{6}$$

where $\frac{\partial A}{\partial a} > 0$; $\frac{\partial A}{\partial g} > 0$.

We can now express total tax payments at time t as

$$C_t = K_0e^{gt}\ [T(g)f(\ell) - w\ell - mhi - mA(a,g)]\,\varrho. \tag{7}$$

It is assumed that $C_t > 0$.

The dividends (D_t) are the difference between receipts and expenditures including taxes at time t. Collecting all terms in the cash flow and observing that $K_0 = K_0^I/(1-h)$ we get.

$$D_t = \frac{K_0^I e^{gt}}{(1-h)} [T(g)f(\ell)(1-p) - gm(1-h) - w\ell(1-c) - mf - mih(1-c) + mcA]. \quad (8)$$

To see the relation between this model and other work on corporate growth, as well as for interpretation of some of our results, the following identities are useful

$$\Pi_t^I = D_t + mgK_t^I; \quad r^I = \frac{\Pi_t^I}{mK_t^I}; \quad d = \frac{D_t}{\Pi_t^I} \quad (9\text{-a})$$

whereby

$$r^I d = r^I - g. \quad (9\text{-b})$$

Although the corporate growth model presented here is new[1)] from a technical point of view, the assumptions made are fairly standard in the literature on corporate growth[2)]. Notably lacking in most corporate growth models, however, is an explicit production function that permits substitution between capital and labour. Such an element is included in this model. Most studies on the relation between taxation and corporate investments fail to distinguish between investments for capital deepening and investments for expansion. The inclusion of a production function in the model permits us to make that distinction here.

1) Although it owes a lot to the Solow [1971] model.
2) See e.g. Marris [1964], [1971] and Gordon [1962].

4. The optimum position of the firm

We will suppose that the firm acts as a maximizer of the value of its shares[1)] The equilibrium value of the shares is the stream of future dividends discounted to present value by the discount rate k. So when t = 0 we have

$$P_0 = \frac{[K_0^I \;\; T(g)f(\ell)(1-c) - gm(1-h) - mih(1-c) - w\ell\,(1-c) - mf + cmA]}{(1-h)(k-g)} \qquad (10)$$

The problem for the firm is to find values of ℓ, h and g that maximize P_0. The necessary conditions for a maximum are:

$$\frac{\partial P_0}{\partial g} = \frac{K_0^I[T'(g)f(\ell)(1-c) + mp\,\frac{\partial A}{\partial g}]}{(1-h)(k-g)} - \frac{K_0^I}{(k-g)} + \frac{P_0}{(k-g)} = 0 \qquad (11\text{-a})$$

$$\frac{\partial P_0}{\partial h} = \frac{K_0^I\, m\Big[g - (1-c)[hi'(h) + i]\Big]}{(1-h)(k-g)} + \frac{P_0}{(1-h)} = 0 \qquad (11\text{-b})$$

$$\frac{\partial P_0}{\partial \ell} = \frac{K_0^I\, m(1-c)\Big[T(g)f'(\ell) - w\Big]}{(1-h)(k-g)} = 0\;. \qquad (11\text{-c})$$

The interpretation of (11-a) and (11-b) requires an extra comment. Beginning with (11-a), it is helpful to explain the meaning of each specific term in the expression for $\frac{\partial P_0}{\partial g}$. The first term is the discounted present valu of all future growth costs, as represented by the T(g) function, and growth gains, from tax depreciation, created by one extra unit of growth. We call this term MC_g The investment of equity capital in the initial period is equal to gK_0^I. Since the firm is restricted to steady states a change in growth rate by one unit affects all future investments in equity capital. The second term, which we might call MI_g, is the discounted present value of these investments.

While the two first terms represent growth effects on P_0 via dividend changes in the initial period and ever after, the third term represent the pure <u>dividend growth</u> effect on P_0 from a changed growth rate $\left(\frac{\partial P_0}{\partial g}\Big| D_0=\text{const.}\right)$ To sum up, we can rewrite (11-a) in the following way:

$$MEI_g = \left(\frac{\partial P_0}{\partial g}\Big| D_0=\text{const.}\right) + MC_g\;. \qquad (11\text{-a}')$$

Condition (11-b) is more easily interpreted when it is rewritten[2)]

$$(1-c)[i'(h)h + i] = r^I \qquad (11\text{-b}')$$

1) Several possible objectives of the firm have been put forward in the literature. The work of Solow 1971 , however, suggests that at least for the comparative dynamics of the firm, the choice of objective might not be all that important.

2)The expression for $(\partial P_0)/(\partial h)$ is multiplied through by $(1-h)(k-g)$ to give: $K_0^I\, mg - K_0^I(1-c)[hi'(h)+i]+U_0 = 0$, which by (9-a) gives (11-b).

Thus, it is seen that the marginal cost of debt net of taxes is equal to the net return on equity (r^I).

Concerning (11-c) it is immediately seen that it implies the traditional marginal condition on labour.

It is interesting to note that in the absence of taxes ($p=0$), (11-a)' implies $mK_0^I < P_0$, which in turn gives the well known inequality[1)] of Lintner [1964].

$$r^I > k > \frac{mK_0^I r^I}{P_0}. \qquad (12)$$

When the firm is subject to taxation the validity of inequality (12) depends on the relative magnitudes of the terms $T'(g)f(\ell)(1-c)$ and $cm(\partial A)/(\partial g)$.

When the marginal tax gains from growth are greater than the marginal costs of the firm[2)] i.e. when $cm(\partial A)/(\partial g) > (1-c)\cdot T'(g)f(\ell)$ we get a reversal of the inequality signs in (12). This also means that the relation between equity capital and the value of shares is reversed. Even if the inequalities are not reversed it is clear that the relation between r^I and k generally is distorted by the corporate tax.

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1) i) $mK^I < P_0 \iff mK^I < \frac{r^I K^I u}{k-g} \iff k-g < r^I u = r^I - g$; i.e. $k < r^I$.

ii) $k = \frac{ur^I mK^I}{P_0} + g > \frac{r^I mK^I}{P_0} \iff g > \frac{(1-u)r^I mK^I}{P_0} \iff mK^I < P_0.$

2) This situation is one of net marginal growth gains. One might wonder whether this is compatible with an internal optimum solution. All that is needed, however, is that $T''(g)f(\ell)(1-c)+cm\,(\partial^2 A)/(\partial g^2) < 0$, when $(\partial P_0)/(\partial g) = 0$

We have namely from (12-a)

$$\frac{\partial^2 P_0}{\partial g^2} = \frac{K_0^I[T''(g)f(\ell)(1-c) + mc\frac{\partial^2 A}{\partial g^2}] + K_0^I[T'(g)(\ell)(1-c) - m(1-h) + mc\frac{\partial A}{\partial g}](1-h)}{(1-h)^2(k-g)^2} +$$

$$+ \frac{(\frac{\partial P_0}{\partial g} + P_0)}{(k-g)^2}.$$

By using (12-a) again we can conclude that

$$\frac{\partial P_0}{\partial g} = 0 \Rightarrow \frac{\partial^2 P_0}{\partial g^2} = \frac{K_0^I\,[T''(g)f(\ell)(1-c) + mc\frac{\partial^2 A}{\partial g^2}]}{(1-h)^2(k-g)^2}.$$

5. Neutral depreciation schemes

A depreciation scheme is non-distortionary for the value maximizing firm if it makes the tax-base proportional to the value of the firm. This is so because then the value of the firm after tax is a fraction of its value before tax and the firm policy that maximizes the present value also maximizes any fraction of the present value. From formula (10) we see that all terms in the expression for P_0 will be included in the tax base if we put

$$A = f + g - gh \tag{13}$$

which gives

$$P_0 = \frac{(1-c)K_0^I \; [T(g)f(\ell) - gm(1-h) - mih - w\ell - mf]}{(1-h)(k-g)} \; . \tag{14}$$

Here the tax is proportional to the maximand of the firm and is consequently non-distortionary. For the interpretation of (13) we recall that $A_t = mK_t \cdot A$ and get

$$A_t = mfK_t + mgK_t(1-h). \tag{15}$$

Therefore, the neutral depreciation deductions are equal to all internally financed investments (including replacements). As the method of deriving (15) does not allow us to rule out the possibility that there are other non-distortionary depreciation schemes, we will show in section 6 that free depreciation as well as true economic depreciation are in fact distortionary for the growing levered firm.

We can, however, already here give an intuitive argument for this proposition. Let us examine what happens to P_0 in the case of <u>free depreciation</u>. Then $A = f+g$ and from (10) we get

$$P_0 = \frac{K_0^I\{(1-c) \; [T(g)f(\ell)-gm-mih-w\ell-mf] + gh\}}{(1-h)(k-g)} \; . \tag{10)'$$

Obviously the term $g \cdot h$ which gives a positive contribution to P_0 is completely unaffected by the corporate tax. When the tax rate c is increased it clearly pays off to increase the term $g \cdot h$. This can be done by speeding up growth as well as by increasing leverage. So both these variables are bound to be affected positively by a change in the tax rate (c) in the case of free depreciation.

Let us now turn to true economic depreciation. We substitute for A = f in (10) and get

$$P_0 = \frac{K_0^I(1-c)\ [T(g)f(\ell)-mih-w\ell-mf]}{(1-h)(k-g)} - \frac{K_0^I\ gm}{(k-g)}\ . \qquad (10)''$$

Now there is a term that gives a negative contribution to P_0 that is unaffected by the tax. This is so because internally financed investments ($K_0^I \cdot gm$) reduce cash flow without being tax deductible. When the tax rate increases so does, ceteris paribus, the relative weight of the last term in expression (10)". The firm could to some extent redress the balance by reducing g and h. Thus, in this case we can expect growth and leverage to be reduced by an increased tax rate.

In view of the recent articles by Stiglitz [1973] and King [1973] it is of interest to point out that the true economic depreciation scheme is in fact neutral with respect to the marginal condition (11-b) on borrowing.

By inserting the expression for P_0 in (11-b) we get after some manipulations the following condition that is equivalent to (11-b)

$$(1-c)[i'(h)h+i] = T(g)f(\ell)(1-c) - mih(1-c) - wc(1-c) - mf + cmA. \qquad (11-b)'$$

When the tax laws permit true economic depreciation, i.e. when A = f we get

$$(1-c)[i'(h)\ h+i] = (1-c)[T(g)f(\ell)-mih - w\ell - mf]. \qquad (16)$$

Clearly the marginal condition on borrowing will not be affected by the tax rate here.

6. Second-order conditions and comparative dynamics

In this section we begin the analysis of firm behaviour in response to changes in the tax-laws. We do this by a traditional comparative dynamics analysis. Thus the effects on g, ℓ and h of a change in e.g. the parameter a is determined by total differentiation of system (11) or more specifically by the solution of the following system of equations written in matrix form

$$\left[\frac{dg}{da}; \frac{dh}{da}; \frac{d\ell}{da}\right] = \left[-\frac{\partial^2 P_0}{\partial g \partial a}; -\frac{\partial^2 P_0}{\partial h \partial a}; -\frac{\partial^2 P_0}{\partial \ell \partial a}\right] H^{-1}, \qquad (17)$$

where H is the Hessian matrix of second-order derivatives of g, h and ℓ with respect to P_0. The second-order conditions for a maximum of P_0 imply that the diagonal elements of H are negative when P_0 attains its maximum. In the appendix it is shown that the signs of all the off-diagonal elements can be derivedfrom the condition (11). It is then possible to determine the signs of the inverse H^{-1}.

Remembering that $\frac{\partial A}{\partial a} > 0$ the signs of the row-vector on the right hand side of equation (17) can be determined from equation (11). We then get the following sign pattern on the right hand side of equation (17)

$$[-;\ -;\ 0] \cdot \begin{bmatrix} - & - & + \\ - & - & + \\ + & + & - \end{bmatrix} \tag{18}$$

Now (17) and (18) give

$$\frac{dg}{da} > 0;\ \frac{dh}{da} > 0;\ \frac{d\ell}{da} < 0.$$

The primary effects of the increase in a is (1) to increase the possibilities for lower tax costs by more rapid growth, (2) to give a positive effect on r^I which by (11-b) leads to an increase in leverage which affects growth positively. The change in ℓ is an adjustment to the changes in growth and leverage.

The question of the effect of a change in the corporate tax rate is analyzed in the same way. We now have:

$$\left[\frac{dg}{dc};\ \frac{dh}{dc};\ \frac{d\ell}{dc}\right] = \left[-\frac{\partial^2 P_0}{\partial g \partial c};\ -\frac{\partial^2 P_0}{\partial h \partial c};\ -\frac{\partial^2 P_0}{\partial \ell \partial c}\right] H^{-1} \tag{19}$$

$$\qquad\qquad ? \qquad\qquad - \qquad\qquad 0$$

Under the row-vector of cross derivatives the signs of the element as they can be derived from system (11)[1)] are indicated. We observe that the

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1) i) By (11-a) $\frac{\partial^2 P_0}{\partial g \partial c} = \frac{K_0^I\left(-T'(g)f(\ell) + m\frac{\partial A}{\partial g}\right)}{(1-h)(k-g)} + \frac{1}{(k-g)} \cdot \frac{\partial P_0}{\partial c}$.

The first term is positive, the second is negative and the sign of $(\partial^2 P_0)/(\partial g \partial c)$ could not be determined.

ii) By (11-b) $\frac{\partial^2 P_0}{\partial h \partial c} = \frac{K_0^I[m(hi'(h)+i)(1-h) - T(g)f(\ell) + mih + w\ell + mA]}{(1-h)^2(k-g)}$,

but in optimum $(\partial P_0)/(\partial h) = 0$ so again by (11-b) we get:

$-(1-h)(1-c)\left(hi'(h)+i\right) = (1-c)\left(mih + w\ell - T(g)f(\ell)\right) + mf - cmA$, whereby

$\frac{\partial^2 P_0}{\partial h \partial c} = \frac{K_0^I(mA-mf)}{(1-h)^2(k-g)} > 0$ if and only if $A > f$. I.e. if and only if tax depreciation is faster than physical depreciation.

iii) By (11-c) $\frac{\partial^2 P_0}{\partial h \partial \ell} = 0$.

sign of the cross derivative $\frac{\partial^2 P_0}{\partial g \partial c}$ is undetermined in the general case. Remembering that the cross derivative only stands for a partial effect, the interpretation of the undetermined sign is that an increase in the tax rate affects the possibilities of increasing the value of shares in two ways:

1) positively: the presence of deductible depreciation makes growth relatively cheaper than before,

2) negatively: the cash flow resulting from the increase in growth is taxed harder than before which makes it less worth-while for share-holders to spend their money on growth.

Another partial effect comes from the leverage side. As long as depreciation for tax purposes is faster than physical depreciation, the sign of $(\partial^2 P_0)/(\partial h \partial c)$ is unambigously positive which reflects the fact that a tax increase, in this case, reduces the marginal cost for leverage more than it reduces the marginal gains of increased leverage. A positive sign of $(\partial^2 P_0)/(\partial h \partial c)$ in the model is bound to affect growth positively. However, from (19) and from knowledge of the sign pattern in H^{-1} we can conclude that the overall effect on ℓ, g and h from the increase in c remains undetermined. This ambiguity, however, disappears for certain specifications of A. To see this we need to examine the cross derivative $\frac{\partial^2 P_0}{\partial g \partial c}$ more closely.

From (11-a) we get:

$$\frac{\partial^2 P_0}{\partial g \partial c} = \frac{K_0}{(k-g)}\left[-T'(g)f(\ell) + \frac{m\partial A}{\partial g} - \overbrace{\frac{T(g)f(\ell) - w\ell - mhi - mA}{k-g}}^{C}\right] \tag{20}$$

Again by (11-a) we have in optimum:

$$(T'(g)f(\ell))(1-c) - m(1-h) + mf\frac{\partial A}{\partial g} + (1-c)\underbrace{\frac{(T(g)f(\ell) - ih - w\ell - mA)}{k-g}}_{C} +$$

$$+ \frac{mA - gm(1-h) - mf}{(k-g)} = 0 \tag{21}$$

Substituting for C in (21) we get:

$$\frac{\partial^2 P_0}{\partial g \partial c} = \frac{K_0}{(k-g)} \left[\frac{m}{1-c} \cdot \frac{\partial A}{\partial g} - \frac{m(1-h)}{1-c} + \frac{m}{1-c} \cdot \frac{A - f - g(1-h)}{k-g} \right] \qquad (22)$$

so $\frac{\partial^2 P_0}{\partial g \partial c} \gtreqless 0$ when

$$\frac{\partial A}{\partial g} - (1-h) + \frac{A - f - g(1-h)}{(k-g)} \gtreqless 0. \qquad (23)$$

Let us now in the light of (23) examine the case of free depreciation of all investments, that is A = f + g. Then $\frac{\partial A}{\partial g} = 1$ and the expression i (23) reduces to the single term h · k which is positive for the borrowing firm. Then $\frac{\partial^2 P_0}{\partial g \partial c}$ is also positive and we get by (19)

$$\frac{dg}{dc} > 0; \frac{dh}{dc} > 0; \frac{d\ell}{dc} < 0.$$

We now turn to the case of true economic depreciation (A = f). Then $\frac{\partial A}{\partial g} = 0$ and the expression in (23) collapses into $-(1-h)(1+\frac{g}{k-g})$ which is negative since h is always less than one. So now $\frac{\partial^2 P_0}{\partial g \partial c}$ will be negative. We also recall from the analysis in the preceding section that A = f implies

$\frac{\partial^2 P_0}{\partial g \partial h} = 0$ (See also note 1, page 11).

Therefore we now get

$$\frac{dg}{dc} < 0; \frac{dh}{dc} < 0; \frac{d\ell}{dc} > 0;$$

7. Declining balance and straight line depreciation

We shall first find the form of (23) for the declining balance depreciation scheme. The declining balance scheme or the "a percent rule" permits the corporation to write off each year a percent of : (the present year's investment) + (last year's book value). The development of book value is thus described by:

$$\frac{dB}{dt} = - a(B(t) - \frac{dB}{dt}) + (1-a)\ I\ (t) \qquad (24)$$

or

$$\frac{dB}{dt} = - \frac{R}{1-a} \cdot B(t) + I\ (t) \qquad (24)'$$

We also have that the depreciation at time t is

$$A(t) = I(t) - B'(t) \qquad (25)$$

In our steady-state model $I(t) = (g+f)K_0e^{gt}$. So the steady-state development of the book-value is obtained by solving

$$\frac{dB}{dt}(\frac{a}{1-a})B(t) = (1-a)(g+f)K_0e^{gt}. \qquad (26)$$

If we impose the condition that for $a = 1$ $A(t) = I(t)$ (that is $B'(t) = 0$). The solution of (26) is

$$B(t) = \frac{(1-a)(g+f)}{g(1-a)+a} K_0e^{gt}, \qquad (27)$$

and by (26)

$$A(t) = \frac{a(g+f)}{g(1-a)+a} K_0e^{gt}, \qquad (28)$$

so in this case

$$A = \frac{a(g+f)}{g(1-a)+a} \quad \text{and} \quad \frac{\partial A}{\partial g} = \frac{a[g^2+(a+f)(1-g)]}{[g(1-a)+a]^2}.$$

Substituting these expressions into (23) we get the following condition for $(\partial^2 P_0)/(\partial g \partial p)$ to be greater than zero in the declining balance case:

$$\frac{a[g^2+(a+f)(1-g)]}{[g(1-a)+a]^2} - (1-h) + \frac{A - f - g(1-h)}{k-g} > 0 \qquad (29)$$

The inequality has been investigated for parameter values in the follow range:

$$\begin{cases} 0 \leq f \leq 0.05; \quad 0 \leq g \leq 0.1; \quad 0 \leq k \leq 0.15; \quad g < k. \\ 0.2 \leq a \leq 0.5; \quad 0 \leq h \leq 1. \end{cases}$$

It turns out that the inequality holds for all values of f, g and k in this range if:

(a = 0.3-0.4 and h > 0.3) or (a = 0.5 and h > 0.2). I.e. <u>for reasonable values of rates of physical depreciation, growth and discount an increase in corporate taxation will give a higher optimal growth rate for firms with a leverage of 30 % or more if firms are permitted to deduct 30 % of the book value</u>. Under these circumstanses we also have $(dh/d\rho) > 0$ and $(\partial h/d\rho < 0$.

Now we turn to the case of straight line depreciation. In this case an investment [$I(\tau)$ made at time τ] is written off fully during the period $\{\tau; \tau + \frac{1}{a}\}$ where a is a public parameter. In our model, we have $I(\tau) =(g+f)K_0$ $I(\tau) = (g+f)K_0e^{g\tau}$. Therefore depreciation for tax purposes at point t should be

$$H(t) = \int_{t-(1/a)}^{t} a(g+f)K_0e^{g\tau}d\tau = a\frac{(f+g)}{g}(1-e^{-\frac{g}{a}})K_0e^{gt} . \qquad (30)$$

As we want to interprete the value of the parameter a as a time period measured in unit intervals we have to interprete $H(t)$ in the following way:

At time t the tax laws permit the firm to make the deduction $H(t)$ against gross profits earned during the last unit interval (i.e. not against the profits earned at the specific moment t). Since in our model the firm must have continuous book keeping with continuous tax payments, the rule "deduct $H(t)$ from profits earned during time interval $\{(t-1);t\}$" has to be transformed into a rule saying: "deduct $A(t)$ from profits earned at moment t". This transformation is given by the requirement that the stream of depreciations $A(t)$ made during the interval $\{t-1;t\}$ should sum up to $H(t)$. Therefore we have

$$H(t) = \int_{t-1}^{t} A(\tau)d\tau = A^*(t) - A^*(t-1), \qquad (31)$$

where $A^*(t)$ is the primitive function to $A(t)$. $Q(t)$ is determined in (30) so that we have:

$$\frac{a(f+g)}{g}(1-e^{-\frac{g}{a}})K_0e^{gt} = A^*(t) - A^*(t-1), \qquad (32)$$

whose general solution is:

$$A^*(t) = C + \frac{a(f+g)(1-e^{-\frac{g}{a}})}{g(1-e^{-g})}K_0e^{gt}. \qquad (33)$$

We are actually interested in:

$$A(t) = \frac{dA^*(t)}{dt} = \frac{a(f+g)(1-e^{-\frac{g}{a}})}{(1-e^{-g})}K_0e^{gt}, \qquad (34)$$

which is the rate of depreciation we are looking for. (As a check we observe that for a = 1 we get $A(t) = (g+f) K_0 e^{gt}$, i.e. immediate full depreciation.

With our previous notation

$$A = \frac{a(f+g)(1-e^{-\frac{g}{a}})}{(1-e^{-g})} \quad \text{and}$$

$$\frac{\partial A}{\partial g} = \frac{[(f+g)(a-1)+a]\, e^{-g(1+\frac{1}{a})} + (f+g-a)e^{-\frac{g}{a}} - a(1+f+g)e^{-g} + a}{(1-e^{-g})^2} .$$

If these expressions are substituted into (23) we get

$$(k-g)[\{(f+g)(a-1)+a\}e^{-g(1+\frac{1}{a})} + (f+g-a)e^{-\frac{g}{a}} - a(1+f+g)e^{-g} + a](1+c) +$$

$$+ (1-e^{-g})\, a\, (f+g)(1-e^{-\frac{g}{a}}) - (1-e^{-g})^2 \{(1-h)k+f\} > 0. \tag{35}$$

The inequality has been investigated for the same range of parameter values as inequality (29). One finds that inequality (35) holds for all values of f, g and k in this range if (a = 0.2 and h $\geq$ 0.3) or (a = 0.3 - 0.4 and h $\geq$ 0.2) or (a = 0.5 and h $\geq$ 0.1). I.e. a tax increase will have a positive growth effect on firms with a leverage of 30 % or more if 5 years straight line depreciation is permitted. For these firms the tax increase will also have a positive leverage effect and a negative effect on the labour/capital ratio.

8. Taxation of Personal Income

In this section personal income taxation is introduced. We make the assumption that all personal income from profits and income from interest payments except capital gains is taxed at the rate x' and x = 1-x'. Furthermore we assume that there is no tax whatsoever on capital gains.[1] The development of the total value of shares P(t) is then given by the differential equation:

$$xkP(t) = xU(t) + \frac{dP}{dt} \tag{36}$$

1) The argument is the same if we suppose that the capital gains tax is positive but completely independent of the personal income tax.

In our model the steady-state path of dividends is given by

$$U(t) = U_0 e^{gt} \tag{37}$$

The general solution of (36) and (37) is $P(t)=\frac{xU_0e^{gt}}{xk-g}$, especially $P_0 = \frac{xU_0}{xk-g}$. If we suppose that the objective of the firm still is to maximize P_0, we get optimum conditions completely analogous to (11):

$$\frac{\partial P_0}{\partial g} = \frac{xK_0^I\left[T'(g)f(\ell)(1-c) - m(1-h) + cm\frac{\partial A}{\partial g}\right]}{(1-h)(xk-g)} + \frac{P_0}{xk-g} = 0 \qquad (11\text{-a})''$$

$$\frac{\partial P_0}{\partial h} = \frac{xK_0^I\left[g - (1-c)\left(hi'(h) + i\right)\right]}{(1-h)(xk-g)} + \frac{P_0}{(1-h)} = 0 \qquad (11\text{-b})''$$

$$\frac{\partial P_0}{\partial \ell} = \frac{xK_0^I m(1-c)\left[\alpha T(g)f''(\ell)^{-1} - w\right]}{(1-h)(xk-g)} = 0. \qquad (11\text{-c})''$$

It is clear that the new element does not change the signs in the H matrix and its inverse. A comparative dynamics analysis of changes in g, h and ℓ in response to a change in x is given by:

$$\left[\frac{dg}{dx}; \frac{dh}{dx}; \frac{d\ell}{dx}\right] = \left[-\frac{\partial^2 P_0}{\partial g \partial x}; -\frac{\partial^2 P_0}{\partial h \partial x}; -\frac{\partial^2 P_0}{\partial \ell \partial x}\right] H^{-1} \tag{38}$$

The signs of the cross derivatives in the row-vector on the right hand side ar readily determined from system (11)".

We get $\frac{\delta^2 P_o}{\delta g \delta x} < 0$; $\frac{\delta^2 P_o}{\delta h \delta x} = 0$; $\frac{\delta^2 P_o}{\delta \ell \delta x} = 0$

whereby from (38) and the sign pattern of H^{-1} we obtain

$$\frac{dg}{dx} < 0; \quad \frac{dh}{dx} < 0 \quad \text{and} \quad \frac{d\ell}{dx} > 0 \tag{39}$$

As $x = 1-x'$ where x' is the tax-rate, the interpretation of the derivatives are that an <u>increase</u> in the personal income tax will increase growth and leverage while labour intensity will be reduced. This can be explained by the fact that the growth of stock-value is given by the growth of dividends. When the tax rate is increased (i.e. x reduced) the relative value of growth <u>per se</u> is increased. This implies that the value maximizing firm is induced by a tax increase to lower the dividend rate somewhat and instead raise its growth rate.

This argument holds for a capital gains tax that is completely independent of the personal income tax on dividends. The analysis will be similar for a capital gains tax that is proportional to the personal income taxation. In this case however it is not possible to establish an unambiguous effect of a tax increase on the growth rate unless we impose further restrictions on the parameters involved.[1)]

9. Concluding remarks

The elements of corporate and personal taxation have been integrated in a balanc growth model of the firm. As the public parameters as well as the choice parameters of the firm appear explicitly, the model lends itself very readily to an analysis of the effects of different specifications of the tax laws. Parameter changes within the framework of a given structure can be dealt with in a straightforward manner by means of comparative dynamics analysis. The main results of the paper can be listed as follows:

1. When the tax laws allow for free depreciation of all internally financed investments the corporate tax will be neutral or non-distortionary.
2. A scheme of true economic depreciation will be distortionary. An increased tax rate will in this case give a lower growth rate, a lower debt ratio and a more labour intensive technique of production.

3. Free depreciation of all investments is also distortionary. Now an increased tax rate will induce the firm to choose a higher growth rate, a higher debt ratio and a less labour intensive technique.

4. Within the framework of the straight-line depreciation and declining balance depreciation rules, a change towards faster depreciation will always give a higher growth rate, a higher debt ratio and a less labour intensive technique.

5a. The effect of an increase in the tax rate under declining balance depreciation and straight line depreciation will depend on the debt ratio of the firm and the specific rate of depreciation permitted.

5b. For normal rates of tax depreciation and relatively modest debt ratios an increased corporate tax rate will lead the firm to increase its growth rate, its capital-labour ratio and its debt ratio.

6. An increase of the marginal tax rate on income or a decrease of the tax rate on capital gains will lead the firm to increase its growth, its debt ratio and its capital labour ratio.

1) The relation between dividends and growth costs in the optimum is crucial. High growth costs and small dividends make the firms less inclined to speed up growth in response to a tax increase.

Appendix

1. Determination of the signs of the elements in the matrix H.

Second order conditions for maximiting P_0 are that the principal components in the Hessian matrix H have alternating signs - + -. H is given by

$$H = \begin{bmatrix} \frac{\partial^2 P_0}{\partial g^2} & \frac{\partial^2 P_0}{\partial g \partial h} & \frac{\partial^2 P_0}{\partial g \partial \ell} \\ \frac{\partial^2 P_0}{\partial h \partial g} & \frac{\partial^2 P_0}{\partial h^2} & \frac{\partial^2 P_0}{\partial h \partial \ell} \\ \frac{\partial^2 P_0}{\partial \ell \partial g} & \frac{\partial^2 P_0}{\partial \ell \partial h} & \frac{\partial^2 P_0}{\partial \ell^2} \end{bmatrix} \quad \text{(A-1)}$$

The second-order conditions imply that the diagonal elements in H are negative. The signs of the off diagonal elements remain to be determined. As the matrix H is symmetrical it is sufficient to study e.g. the elements on the left hand side. From (11-b) we get:

$$\frac{\partial^2 P_0}{\partial h \partial g} = \frac{(1-h)^2(k-g)K_0^I m(1-h) + (1-h)\frac{\partial P_0}{\partial g} - (1-h)(k-g)\frac{\partial P_0}{\partial h}}{(1-h)^4(k-g)^2}. \quad \text{(A-2)}$$

The first term in the numerator is obviously positive while, according to (11-a) $\frac{\partial P_0}{\partial g} = 0$ and according to (11-b) $\frac{\partial P_0}{\partial h} = 0$ so $\frac{\partial^2 P_0}{\partial h \partial g} = \frac{\partial^2 P_0}{\partial g \partial h} > 0$.

From (11-c)

$$\frac{\partial^2 P_0}{\partial \ell \partial g} = \frac{K_0^I m\left[(1-h)(k-g)\,\alpha T'(g) f'(\ell)(1-g) + (1-h)(k-g)\frac{\partial P_0}{\partial \ell}\right]}{(1-h)^2(k-g)^2}. \quad \text{(A-3)}$$

$T'(g) < 0$ by assumption and $\frac{\partial P_0}{\partial \ell} = 0$ by (11-c) so $\frac{\partial^2 P_0}{\partial \ell \partial g} = \frac{\partial^2 P}{\partial g} < 0$; Again from (11-c) $\frac{\partial^2 P_0}{\partial \ell \partial h} = \frac{\partial^2 P_0}{\partial h \partial \ell} = 0$. Consequently the signs of H are given by

$$(H) = \begin{bmatrix} - & + & - \\ + & - & 0 \\ - & 0 & - \end{bmatrix} \qquad \text{(A-4)}$$

2. <u>Determination of signs in H^{-1}.</u>

As H and therefore also H^{-1} are negatively semi-definite the diagonal elements of H^{-1} are all negative. Since H^{-1} is symmetric, it is again sufficient to determine the signs of the elements to the left of the diagonal.

A typical element h^{-1}_{ij} in H^{-1} is given by $\frac{\Delta h_{ij}}{\det H}$ where Δh_{ij} is the cofactor of the element h_{ij} in the matrix H. By "multiplying signs" from (A-4) we get:

i) $\Delta h_{21} = [(+)(-) - (0)(-)](-1)^3 > 0$ implying

that $h^{-1}_{21} = \frac{\Delta h_{12}}{\det H} < 0$.

ii) $\Delta h_{31} = [(+)(0) - (-)(-)]\ (-1)^4 < 0$ implying

that $h^{-1}_{31} = \frac{\Delta h_{31}}{\det H} > 0$.

iii) $\Delta h_{32} = [(-)(0) - (+)(-)](-1)^5 < 0$ implying

that $h^{-1}_{23} = \frac{\Delta h_{23}}{\det H} > 0$.

Consequently

$$H^{-1} = \begin{bmatrix} - & - & + \\ - & - & + \\ + & + & - \end{bmatrix} \qquad \text{(A-5)}$$

Litterature

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