A complete list of Working Papers on the last page

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A COMPARATIVE STUDY OF COMPLETE SYSTEMS OF DEMAND FUNCTIONS

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Ten models of consumer demand, including the indirect translog model, the linear ecpenditure system and the Rotterdam model, are applied to Swedish data and compared, using fit and predictive performance as criteria. Data include 8 food commodities. Demand models tend to be superior to naive models, but non-additive models are not clearly better than additive. Each model is estimated on two levels of commodity aggregation. The results show that the estimated structures depend on the level of aggregation.

1. Introduction

Complete systems of demand functions have become a commonly applied tool for economic policy and forecasting. Nevertheless, the application of these models is burdened by both theoretical and practical problems. It is well known that some implications of the theory of demand, like homogeneity and symmetry, do not in general hold good on an aggregate level, and in studies by Barten [1969] and Christensen, Jorgenson and Lau [1975] the classical theory of demand is rejected. In other studies specific assumptions about the utility function, i.e. additivity, have been tested without relinquisihing the tenets of classical demand theory. In, for instance, Deaton [1974 a] the hypotheses of an additive utility function was rejected.

However, in practice data relative to the desired detailed commodity breakdown are scarce and price variation is usually in-

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sufficient to give well determined estimated demand responses to price changes. In order to estimate our models we thus need *a priori* restrictions to reduce the number of unknown parameters, and the only basis we have for choosing such restrictions is the theory of demand. In spite of the implications of previous results, since data are scarce, our problem is to determine whether the classical theory of demand with and without its more restrictive assumptions, like additivity, is good enough to be used in forecasting.

To compensate for a low informational content in data, long time series are needed to obtain sharp tests. In the studies by Barten [1969] and by Christensen, Jorgenson and Lau [1975] the sample period included 31 and 44 years respectively. Deaton's [1974 a] study was also based on long time series covering the period 1900-1970, which, with the war years excluded, gave 48 years. This is more than one can usually expect to find but, what is more important, one may question the validity of these long time series. Commodities included in today's aggregates are vastly different from those included at the beginning of the century. Everyone experienced in the compilation of expenditure and price series is aware of the great difficulties, not only in obtaining consistent series, but also in obtaining reliable series. The consequent "ad hoccery" involved in this kind of statistical work is annoying. With these difficulties to contend with, it is perhaps not so surprising that some of the classical results of demand theory fail to gain statistical support, whereas on the other hand it suggests that the empirical results might be more favourable were higher quality data available.

There are a number of comparative studies, most of which are based on fit criteria (i.e. Parks [1969], Yoshihara [1969], Goldberger and Gamaletsos [1970], Dahlman and Klevmarken [1971], Gamaletsos [1973], Deaton [1974 a] and Theil [1975]). Is is difficult adequately to summarize the results from these studies. Differences in details of model specification, in estimation methods, in definitions of commodities and in compilation of data may at least partly explain the sometimes conflicting results. However, in most studies, models which do not imply an additive utility function show a closer fit to data than those which do. Thus, in a number of studies, variants of the Rotterdam system show themselves superior to, for instance, the linear expenditure system and the direct and indirect addilog models (Parks [1969], Deaton [1974 a] and Theil [1975]). The reported differences in fit between additive models like the linear expenditure system, the addilog models and the Rotterdam system with additivity enforced are smaller and inconsistent. One may also note that the constant elasticity demand system or the double logarithmic model usually obtains a relatively high ranking by fit.

There are only a few studies of relative predictive performance. One example is that of Theil [1975]. His results on British data with four commodities show that his Rotterdam system with block independence has a better performance than the indirect addilog model and the linear expenditure system. The information inaccuracy measure for the linear expenditure system is even higher than for one of Theil's naive models.

It is a general finding that the estimated elasticities are very sensitive to the model specification. Large differences are found between different models estimated on the same data.

In the present study ten models, including two naive models, are applied to Swedish data and compared, using fit, predictive performance and sign and magnitude of estimated elasticities as criteria. In principle, predictions and estimated elasticities depend in a more than trivial sense on the commodity aggregation. In order to explore the aggregation effects each model is estimated twice, once for a four commodity break-down and once for an eight commodity break-down.

Our data include only food commodities. There are several reasons for limiting the study to food. First, the data for food are of a much higher quality than data for other commodity groups. Second, food is non-durable and third, the food market functions relatively freely without the institutional restrictions that limit trade in, for instance, the housing market. Behind this confinement of the study to the demand for food there is an implicit assumption of a weakly separable utility function.

2. Alternative models

There are two naive models to serve as a reference base in evaluating the explanatory ability and predictive performance of the other eight models. They can never, of course, be a substitute for a demand model in simulations of the effects of economic policy. The particular choice of a naive model is somewhat arbitrary. Loosely speaking, expenditure shares are more stable than expenditures, volumes and rates of change in volume. A naive model may thus be more successful in explaining and forecasting expenditure shares than any of the other variables. The first model is thus a trend in expenditure shares and the second a simple autoregressive structure, also in expenditure shares.

Except for the last model below, the remaining seven models have been chosen because they belong to those most commonly applied. Although the constant elasticity of demand model does not satisfy the properties of classical demand theory it has been, since Schultz [1938] and Wold [1952], the best known and most used model of them all, one of its merits being the ease of estimation. To emphasize simplicity, one of the two versions estimated, has no cross-price elasticities. The second version includes all price effects. The possibility of gaining degrees of freedom by local enforcement of the constraints of classical demand theory (cf. Byron [1970]) has not been used. A more recent development with approximately the same advantages and disadvantages is the Rotterdam system (Theil [1965]). Both these models may be looked upon as approximations to an underlying classical demand model. Of the models which do satisfy the constraints of classical demand theory Stone's linear expenditure system (Stone [1954]) is the most widely used (cf. Brown and Deaton [1972]). It is easy to interpret, and although it involves non-linear estimation it is not too difficult to estimate. It has also proved to be a good starting point for useful generalizations, one of which is included in this study, namely the linear expenditure system with habit formation (cf. Pollak and Wales [1969], Pollak [1970] and Dahlman and Klevmarken [1971]). Possible rivals to the linear expenditure system are the addilog models (Houthakker [1960]). There is, however, no conclusive evidence which puts the addilog models before the linear expenditure system and they are, in addition, more difficult to estimate. They have therefore not been used in this study. A possible disadvantage of both the linear expenditure system and the addilog models is that they are derived from an additive utility function. This implies that there are no specific substitution effects and that the own-price elasticity is approximately proportional to the income elasticity, as shown in Deaton [1974 b]. In recent years more general models have been developed, one of which is the translog model (Christensen, Jorgenson and Lau [1975], Christensen and Manser [1977]). The version used here is derived from an indirect utility function, the logarithm of which is of second degree in the logarithm of the price-income ratios. The increased generality, however, is bought at the price of a more difficult estimation and there are as yet but few applications of these models.[1)]

In the comparative studies by Brown and Deaton [1972], Deaton [1974 a] and Theil [1975], the authors argue that the left-hand variables of the competing models should be comparable. They should, for instance, all be expenditure shares or log-changes in expenditure shares. If not, the measures of fit may by definition favour one model before another. In our study, this recommendation is not entirely followed. Each model is preferred in its most commonly applied form, which implies that the stochastic structure will vary from one model to another. However, some of the models which are not usually formulated as expenditure share equations have also been estimated in this form.[2)].

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1) This is not the place to analyse in detail the theoretical properties of the models used. The reader is directed to the references given and to the excellent review article by Brown and Deaton [1972].

2) This transformation is also motivated by heteroscedasticity in consumption expenditures and volumes. In Theil [1975], chap. 5, it is also argued that autocorrelation could be removed by taking first differences.

The following notation is used for all models:

q_{it}	demanded volume per head of commodity i in year t,
p_{it}	price of commodity i in year t,
w_{it}	expenditure share of commodity i in year t,
w^*_{it}	$\frac{1}{2}(w_{it} + w_{it-1})$,
y_t	total food consumption per head in current prices in year t,
p_t	price index for food in year t,
ε_{it}	stochastic disturbance for commodity i in year t,
e_i	income elasticity for commodity i,
E_{ij}	(uncompensated) price-elasticity,
n	number of commodities

α, β, c, Π, κ, σ are parameters. They do not necessarily have the same interpretation in all models. In the Constant Elasticity of Demand Systems, eq. (3) and (4) below, e_i and E_{ij} are also parameters. The operator D is the logarithmic difference operator.

Results are reported for the following ten models:

Trend (Trend-w)

$$w_{it} = \alpha_i + \beta_i t + \varepsilon_{it}; \qquad i = 1, \ldots, n \qquad (1)$$

Autoregressive model (Auto-w)

$$w_{it} = \alpha_i + \beta_i w_{i\,t-1} + \varepsilon_{it}; \qquad i = 1, \ldots, n \qquad (2)$$

Constant Elasticity of Demand System (CEDS-lnq)

$$\ln(q_{it}) = \alpha_i + e_i \ln(y_t/p_t) + E_{ii} \ln(p_{it}/p_t) + \varepsilon_{it}; \qquad i = 1, \ldots, n \qquad (3)$$

Note that all cross-price elasticities are assumed to equal zero.

Constant Elasticity of Demand System (CEDS-lnw)

$$\ln(w_{it}) = \alpha_i + (e_i-1)\ln(y_t/p_t) + (E_{ii}+1)\ln(p_{it}/p_t) +$$

$$\sum_j E_{ij}\ln(p_{jt}/p_t) + \varepsilon_{it}; \qquad i = 1, \ldots, n \tag{4}$$

Symmetry of price responses is not enforced. There are thus $n(n+2)$ parameters to estimate.

Linear Expenditure System with Habit formation (LESH-pq)

$$p_{it}q_{it} = \alpha_i p_{it}q_{i,t-1} + \beta_i(y_t - \sum_k \alpha_k p_{kt}q_{k,t-1}) + \varepsilon_{it}; \quad i = 1, \ldots, n \tag{5a}$$

$$\sum_i \beta_i = 1; \tag{5b}$$

Linear Expenditure System with Habit formation (LESH-w).

$$w_{it} = \alpha_i p_{it}q_{i,t-1}/y_t + \beta_i(1 - \sum_k \alpha_k p_{kt}q_{k,t-1}/y_t)$$

$$+ \varepsilon_{it}; \quad i = 1, \ldots, n. \tag{6a}$$

$$\sum_i \beta_i = 1; \tag{6b}$$

Linear Expenditure System (LES-w).

$$w_{it} = c_i p_{it}/y_t + \beta_i(1 - \sum_k c_k p_{kt}/y_t) + \varepsilon_{it}; \quad i = 1, \ldots, n. \tag{7a}$$

$$\sum_i \beta_i = 1; \tag{7b}$$

Rotterdam System (RD-w^*Dq)

$$w^*_{it}Dq_{it} = \mu_i Dq_t + \sum_j \pi_{ij}Dp_{jt} + \varepsilon_{it}; \qquad i = 1, \ldots, n \tag{8a}$$

$$\sum_j \pi_{ij} = 0 \text{ for all } i; \tag{8b}$$

$$\pi_{ij} = \pi_{ji} \text{ for all } i \text{ and } j; \tag{8c}$$

$$\sum_i \mu_i = 1; \tag{8d}$$

Rotterdam System with Intercept (RDI-w^* Dq)

$$w^*_{it}Dq_{it} = \kappa_i + \mu_i Dq_t + \sum_j \pi_{ij} Dp_{jt} + \varepsilon_{it};\ i = 1, \ldots, n. \tag{9a}$$

$$\sum_j \pi_{ij} = 0 \text{ for all } i; \tag{9b}$$

$$\pi_{ij} = \pi_{ji} \text{ for all } i \text{ and } j; \tag{9c}$$

$$\sum_i \mu_i = 1; \tag{9d}$$

Indirect Translog model (ITRL-w).

$$w_{it} = \frac{\alpha_i + \sum_j \beta_{ij} \ln(p_{jt}/y_t)}{-1 + \sum_j \beta_{Mj} \ln(p_{jt}/y_t)} + \varepsilon_{it}; \qquad i = 1, \ldots, n. \tag{10a}$$

$$\beta_{Mj} = \sum_i \beta_{ij}; \tag{10b}$$

$$\beta_{ij} = \beta_{ji}; \tag{10c}$$

The error terms may be contemperaneously correlated but no autocorrelation is assumed,

$$E(\varepsilon_{it}) = 0 \tag{11a}$$

$$E(\varepsilon_{is}\varepsilon_{jt}) = \begin{cases} \sigma_{ij} & \text{if } s=t \\ 0 & \text{if } s\neq t \end{cases} \tag{11b}$$

Owing to the model specification and the enforcement of the budget constraint, the contemporaneous error moment matrix in models 5 to 10 is singular. In each of these models one equation is redundant and could be left out. With the estimation methods used any equation could be dropped.

It was emphasized above that models (1)-(3) were included in this study because they are simple. This hoolds good a fortiori when it comes to estimation. Each equation has thus been estimated by OLS, although some gain in efficiency might have been obtained in models (2) and (3) by other methods. Model (4), the Constant Elasticity of Demand System with all price effects included, was efficiently estimated by OLS. The three versions of the linear expenditure system were estimated by the quasi- maximum-likelihood method,[3] and the two Rotterdam models were estimated by Zellner's iterative Aitken estimator (IZEF). The TSP program was used to estimate the translog model by a non-linear IZEF procedure. Owing to a restricted supply of computer programs and limited availability of programming facilities the same estimation procedure was not used for the last five models, but the maximum likelihood method and IZEF only differ computationally because, if convergent, IZEF will produce ML estimates (Bradley [1973], Oberhofer and Kmenta [1974], Charnes, Frome and Yu [1976], Christensen and Manser [1977]). No difficulty was experienced in obtaining convergence except with the translog model with eight commodities. This model has therefore been estimated with two alternative assumptions about the error moment matrix. First, the model was estimated as specified above with coarce tolerance limits. In a second estimation round with fine tolerance limits, the moment matrix was fixed at the values otbained at convergence in the first round. In this way convergence was obtained also with fine tolerance limits. The results are reported below under the heading ITRL-w, Ω. The second version of the translog model was obtained by fixing the error moment matrix from the beginning in the following way,

$$\sigma_{ij} = -\sigma^2 \bar{w}_i \bar{w}_j \tag{12a}$$

$$\sigma_{ii} = \sigma^2 (1-\bar{w}_i)\bar{w}_i \tag{12b}$$

where $\bar{w}_i$ is the average expenditure share for commodity i and σ^2 is a constant. The heading used for results with this specification is ITRL-w,Ω_o.

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3) The likelihood function for independently multivariate normally distributed error terms was maximized by using the Harwell Library Sub-routine VAO9AD — a quasi-Newton procedure.

3. Data

The series of expenditures and prices used are revisions of the series published in Dahlman & Klevmarken [1971]. The sample period is 1950-1970. Two years, 1971 and 1972, are reserved for a comparison with ex post forecasts. Data were originally obtained for 16 food commodities, but for this study they were grouped into 8 commodities. Commodities exhibiting similar fluctuations in relative prices were grouped together. In order to study the effects of aggregation on estimated elasticities and predictions the 8 commodities were further aggregated into 4 commodities. The four-commodity breakdown is related to the eight-commodity breakdown as shown below.

8 Commodities	4 Commodities
1. Flour, Bread, Potatoes and derivative products 2. Butter, Egg, Sugar, Spices 3. Milk, Cream	A. Basic Supplies
4. Vegetables 5. Fruit, Berries, Ice-cream, Chocolates and Sweets	B. Vegetables and Fruit
6. Meat and Pork, Cheese 7. Fish	C. Meat and Fish
8. Restaurant Meals	D. Restaurant Meals

Note that the last commodity, "Restaurant Meals", is the same in the two groupings.

In principle each model will determine the proper method of aggregation. Price indices will typically involve the unknown parameters. In this particular study it would be possible to use parameter estimates from the 8-commodity level and estimate price indices for the 4-commodity level. However, in practice this is usually not a feasible procedure, and we have opted in favour of the common practice of aggregating expenditures by simple summation and using price indices of the Edgeworth and Laspeyres type. This makes our analysis of aggregation effects more realistic.

4. Results

There is no obvious standard of comparison for our ten models. They are not nested, they do not have the same dependent variable and their stochastic properties differ. Owing to the fact that they are not nested there is no formal test for discriminating between them. A comparison of sample-period evidence has to be based on descriptive goodness of fit statistics. This comparison could be made for expenditure shares, expenditure levels, volumes or relative changes in volume. The results reported below are based exclusively on comparisons of expenditure shares. The statistics used are information inaccuracies and coefficients of determination. In addition to sample-period fit, the predictive performance for 1971 and 1972 is evaluated by using the same statistics. As a third criterion for discrimination between the ten models the estimated elasticities are evaluated against prior conceptions about their sign and size. For both levels of aggregation parallel results are presented, and this section concludes with some results of the aggregation effects.

Before turning to these results, however, we may report that our results for the translog model permit the same test of the theory of demand as in Christensen, Jorgenson & Lau [1975]. Contrary to the conclusion reached in their study, we connot reject, on a 4 commodity level, the composite hypothesis that the β_{Mj} parameters in (10b) take the same values in each equation and that symmetry according to (10c) is maintained ($\chi_{df=6} = 7.818$ while the 5% critical value is 12.6)[4].

4.1 Goodness of fit

The first column in Tables 1 and 2 give average information inaccuracies for each model for the whole sample period. The second column shows the same statistic, but corrected for degrees of freedom (see Theil [1971] p 649). The third column shows standard deviations for yearly information inaccuracies.

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4) The sample period is too short to give the same test meaning on the eight commodity aggregation level.

For the case of four commodities the linear expenditure system with habit formation yields the closest fit to the data, while the ordinary linear expenditure system shows the worst fit. It fits even more poorly than the "primitive" models[5]. The restrictive assumption of an additive utility function thus does not appear to invalidate the fit of the linear expenditure system, but rather the assumption of a constant subsistence level. The linear expenditure system with habit formation also shows the smallest yearly variation in fit. One may also note that the performance of the constant elasticity of demand model is not much worse than that of the more sophisticated Rotterdam and translog models.

With an eight commodity grouping there are only a few changes in these results. LES still shows the worst fit, but LESH no longer takes a leading position. The translog model now shows a marginally better fit and the unrestricted constant elasticity of demand system fits best of all models. This result is consistent with the preconception that substitution between commodities is more important with a finer than with a coarser commodity break-down. Low frequency variation, however, is still important in the expenditure shares series, as shown by the relatively good fit of the simple autoregressive model. The results in these two tables also indicate that the constant elasticity demand system with a suitable number of non-zero cross-price elasticities is still a good alternative to more recent models.

A comparison of information inaccuracies by commodities reveals approximately the same results, although the ranking of the models is not exactly the same for all commodities. For instance, with four commodities the Rotterdam model explains "Basic Supplies" and "Restaurant Meals" relatively well but comes out worse for "Meat and Fish", which is best explained by the linear expenditure system with habit formation and the translog model.

With the eight commodity grouping, CEDS with no restrictions, owing to its many parameters, comes out best for seven commodities. The translog model with a priori fixed error moment matrix comes out best for one commodity and second best for three commodities.

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5) For those models which do not satisfy the aggregation constraint, all predicted expenditure shares have been normalized to sum to unity. The information inaccuracies for the double-log models, the Rotterdam models and the "primitive" models would otherwise have become larger.

The Rotterdam system with intercepts fits second best for two commodities and the linear expenditure system with habit formation second best for one. The ranking is almost the same when judged by the coefficients of determination, (Tables 5 and 6).

On the aggregate level almost all models fail to explain "Vegetables and Fruit", and at the disaggregate level they similarly show a poor fit to the two commodities "Vegetables" and "Fruit, Berries, Ice-cream, Chocolates and Sweets". Residual plots show that the commodity "Vegetables" is systematically overestimated for the first half of the sample period and underestimated for the second. The reverse is true for "Fruit, Berries, Ice-cream, Chocolates, and Sweets". There is no obvious explanation, however.

4.2 Predictions

Good fit for the sample period does not necessarily imply good predictions. The last three columns of Tables 1 and 2 show the information inaccuracy arising between observed and predicted expenditure shares for 1971 and 1972. These predictions were made with observed values for prices and total food expenditures per head.

The translog model shows the smallest average information inaccuracy followed by the constant elasticity of demand model with no cross-price elasticities. The predictions from the Rotterdam systems as well as those from the naive models are relatively poor, while the linear expenditure system takes an intermediate position. However, there are large differences in predictive ability for the two years, and this makes a comparison between models difficult. One should perhaps pay relatively more attention to the results for 1971, because this was an exceptional year. For the first time since World War II, total private consumption (in constant prices) declined (-1%). Total food consumption also declined by 1 per cent. For this year the simple double-log model with no cross-price effects does better than any of the more recent system models. However, the translog model and the linear expenditure system also predict relatively well.

The same comparison for the eight commodity grouping shows that the autoregressive model has the smallest average information inaccuracy. One is inclined to believe, however, that this good result is exceptional, owing to the very good predictions for 1972. The Rotterdam system (Rd-wDq) and the linear expenditure system with habit formation (LESH-pq) show the second-best predictions. Except for the trend model, the constant elasticity of demand model with no restrictions and the ordinary linear expenditure system, which all give poor predictions, the differences in predictive ability are small.

If we again pay particular attention to the results for 1971 we find that the two linear expenditure systems with habit formation and the constant elasticity of demand model with no cross-price elasticities give the best predictions, closely followed by the translog model (ITRL-w,Ω_o), the Rotterdam model (RD-w*Dq) and the autoregressive model.

It is also interesting to note, for both levels of aggregation that the many parameters in the constant elasticity of demand model with no restrictions and in the Rotterdam model with intercepts do not guarantee better predictions than those obtained by the corresponding more restrictive models.

4.3 Comparison of elasticities

The estimated elasticities can also be used as a basis for comparison. We may, for instance, investigate if the compensated own-price elasticities are negative for all models and commodities, as is suggested by economic theory. We should also expect expenditure elasticities for luxuary food items to be higher than those for non-luxuries. "Meat and Fish" and "Restaurant Meals" should thus be more expenditure-elastic than "Basic Supplies".

Table 7 shows that only two models give an estimated own-price elasticity for "Basic Supplies" with the right sign. These are the linear expenditure system with habit formation and the translog model. However, for the Rotterdam and the CEDS models the estimated standard errors are relatively large.

For the eight commodity grouping, Table 8 exhibits several positive estimates of compensated own-price elasticities. For instance, the constant elasticity of demand model and the two Rotterdam systems have positive elasticities for "Flour, Bread, Potatoes and derivative products". The Rotterdam systems also give positive elasticities for "Fish" and "Restaurant Meals". With all models except the Rotterdam system with intercepts the estimated price elasticity of "Butter, Egg, Sugar and Spices" is positive. The non-negative estimates for the ordinary linear expenditure system and the Constant Elasticity of Demand System with no cross-price effects are not due to random fluctuations, but in most other cases the estimated standard errors are large.

With all models, the estimated income elasticity for "Meat and Fish" is higher than the corresponding elasticity for "Basic Supplies". The same, however, is not true for "Restaurant Meals". While the linear expenditure system gives an estimated income elasticity of 4.2 the translog model suggests that this commodity is inferior. Also, according to the translog model, "Basic Supplies" are income-elastic, while two other models, the constant elasticity of demand model and the linear expenditure system, indicate that they are inferior.

The results for eight commodities show roughly the same pattern. In all models, "Vegetables" and "Fruit, Berries, Ice-cream, Chocolates and Sweets" are expenditure-elastic. With few exceptions this is also true for "Meat, Pork and Cheese" and "Fish". The estimates for the linear expenditure systems with habit formation and the Rotterdam models indicate that "Restaurant Meals" is inelastic while the other models give elasticities well above unity. Almost all models indicate that the three subcommodities within "Basic Supplies" are either inelastic or inferior. The exceptions are the ordinary linear expenditure system ("Flour, Bread, Potatoes and derivative products"), the constant elasticity of demand model with no restrictions ("Milk and Cream") and the translog models ("Milk and Cream"). In conclusion, no model conforms exactly with the _a priori_ expected signs and magnitudes of the elasticities.

The comparison of elasticities offered in Tables 7 and 8 also confirms the finding in previous studies that estimates of elasticities crucially depend on the model used. Our eight models give a vastly different interpretation of data. With these large differences in estimated elasticities the choice of model may become decisive for forecasting and policy.

4.4 On the effects of aggregation

This study offers a possibility of demonstrating numerically a few effects of commodity aggregation. Aggregation has been analysed for linear models – see, for instance, Theil [1954] and Lütjohann [1974] – but it is an almost unexplored field for non-linear models. Consistent aggregation is exceptional in linear models, i.e. consistent aggregation requires unrealistically simple micro relations. However, if aggregation is inconsistent there are in general no macro parameters, i.e. the zeudo macro parameters specified depend on the aggregation process and the variables involved. There is no reason to believe that the situation would generally be more favourable for non-linear models.

The estimated elasticities are thus in general not independent of the level of aggregation. The columns for "Restaurant Meals" in Tables 7 and 8 – note that commodities D and 8 are identical – show that in fact they depend on the grouping. With some models – for instance, the ordinary linear expenditure system and the translog model — the differences in estimated elasticities are large.

There is also an aggregation effect on the goodness of fit as revealed for "Restaurant Meals" by Tables 3-6. Only for those models which satisfy the budget constraint has a comparison of fit between the two levels of aggregation any meaning. All these models, except the ordinary linear expenditure system, show a closer fit to observed expenditure shares for "Restaurant Meals" when the other commodities are aggregated. There are, however, no large differences in fit.

In order to investigate if predictive performance depends on the level of aggregation the eight predicted expenditure shares have, for each model and year, been aggregated to expenditure shares for the four commodities A - D. Using the same information inaccuracy measure as before, they are then compared to observed expenditure shares and to those predicted by the aggregate models. Table 9 shows that of the disaggregate models the translog model, the constant elasticity of demand model with no cross-price elasticities and the linear expenditure system with habit formation give the best predictions. This is the same result as that obtained when the models were estimated on aggregate data (Table 1). For the best models there are no great differences in prediction accuracy due to the level of aggregation, but for some of the inferior models there are. There is however no unique indication that a disaggregate analysis would be superior or vice versa.

5. Conclusions

If we would classify our ten models with goodness of fit as the only criterion, two groups might be distinguished. The naive models, the constant elasticity demand model with no cross-price elasticities and the ordinary linear expenditure system would then be classified as inferior and the other models as superior. If the yearly forecasting ability is also taken into consideration the same grouping is obtained, except that the constant elasticity demand system with no cross-price elasticities would now be classified as superior. One conclusion thus is that demand models tend to be superior to naive models. If we also wish to base our choice on the expected sign and magnitude of estimated elasticities, the linear expenditure system with habit formation and the translog model might be our first preferences, and if, in addition, ease of estimation and applicability are taken into consideration, this study indicates that the linear expenditure system with habit formation is perhaps our best choice.

With these criteria, and contrary to previous studies (cf. Deaton [1974 b]), no severe distortion of measurement is found that is due to the assumption of additivity. The translog model and the Rotterdam system, which are non-additive models, are not found superior to the linear expenditure system with habit formation. Our results, however, also show that there is no obviously best model. Aggregate time series do not give enough information to allow a sharp discrimination between models and, in addition, it has been shown that the estimated structures do depend on the level of aggregation. More work is thus needed to analyse the stability of these models, in both the time and the commodity dimensions. This analysis should, inter alia, be based on predictions for a longer period than two years, and include a comparison not only of expenditure shares but also of consumption volume and relative changes in volume.

Table 1 Average information inaccuracies; 4 commodity grouping

Model	$\bar{\bar{I}}_{50-70}$	$\bar{\bar{I}}^{A}_{50-70}$	$S_{I,50-70}$	I_{71}	I_{72}	$\bar{\bar{I}}_{71-72}$
Trend-w	325	359	270	751	1 552	1 151
Auto-w	267	297	5 167	1 408	190	799
CEDS-lnq	245	283	217	128	306	217
CEDS-lnw	145	203	134	347	767	557
LESH-pq	153	170	128	660	118	389
LESH-w	145	161	107	713	206	459
LES-w	382	422	440	288	743	515
RD-w*Dq	190	224	209	1 011	427	719
RDI-w*Dq	175	219	189	1 170	614	892
ITRL-w	211	250	259	336	12	174

Note: $I_t = \sum_{i=1}^{4} w_{it} \ln(w_{it}/\hat{w}_{it}) \cdot 10^6$

$$\bar{\bar{I}} = \frac{1}{T} \sum_{t=1}^{T} I_t$$

Table 2 Average information inaccuracies; 8 commodity grouping

Model	$\bar{I}_{50-70}$	$\bar{I}^{A}_{50-70}$	$S_{I,50-70}$	I_{71}	I_{72}	$\bar{I}_{71-72}$
Trend-w	818	904	506	2 540	3 490	3 015
Auto-w	448	498	437	1 576	356	966
CEDS-lnq	555	648	500	1 300	1 699	1 500
CEDS-lnw	155	310	112	1 985	4 571	3 278
LESH-pq	444	494	382	1 291	1 354	1 322
LESH-w	381	424	299	1 289	1 929	1 609
LES-w	1 076	1 189	1 059	8 465	1 651	5 058
RD-w^*Dq	372	509	384	1 573	1 027	1 300
RDI-w^*Dq	302	440	260	1 777	1 043	1 410
ITRL-w;Ω	313	420	238	2 306	1 624	1 965
ITRL-w;Ω_0	294	395	193	1 546	1 324	1 435

Table 3 Average information inaccuracies by commodity; 4 commodity grouping

Model	Commodity			
	A	B	C	D
Trend-w	77	124	58	156*
Auto-w	94	140	50	52*
CEDS-lnq	72	113	42	91*
CEDS-lnw	23	84	23	65
LESH-pq	52	67	28	53
LESH-w	46	64	27	51
LES-w	166	142	28	167
RD-w*Dq	24	122	47	44
RDI-w*Dq	21	114	45	41
ITRL-w	63	134	27	50

* See note to Table 4.

Table 4 Average information inaccuracies by commodity; 8 commodity grouping

Model	Commodity							
	1	2	3	4	5	6	7	8
Trend-w	46	169	168	140	151	58	31	161*
Auto-w	49	77	50	67	130	62	29	53*
CEDS-lnq	49	67	80	89	127	45	50	122*
CEDS-lnw	19	10	8	28	65	8	9	27
LESH-pq	20	51	18	162	129	23	22	71
LESH-w	21	46	15	109	140	22	20	57
LES-w	131	98	316	73	369	34	40	162
RD-w*Dq	31	14	25	64	151	51	32	57
RDI-w*Dq	29	13	15	56	127	41	17	47
ITRL-w,Ω	22	30	11	73	113	36	23	48
ITRL-w,Ω_0	17	51	91	52	18	36	16	53

* The inaccuracy measures for Restaurant Meals and the first three models are not the same for both levels of aggregation, because the expenditure shares have not been standardized by the same factor.

Table 5 Coefficients of determination by commodity;
4 commodity grouping

Model	Commodity			
	A	B	C	D
Trend-w	0.9398	0.3800	0.8328	0.6664
Auto-w	0.9202	0.3321	0.8500	0.8518
CEDS-lnq	0.9439	0.4319	0.8803	0.8070
CEDS-lnw	0.9992	0.5739	0.9433	0.8703
LESH-pq	0.9594	0.6650	0.9179	0.8869
LESH-w	0.9638	0.6756	0.9215	0.8882
LES-w	0.8714	0.2903	0.9184	0.6527
RD-w^*Dq	0.9810	0.4050	0.8663	0.9064
RDI-w^*Dq	0.9832	0.4382	0.8733	0.9121
ITRL-w	0.9513	0.3269	0.9213	0.8954

$$R_i^2 = 1 - \sum_{t=1}^{T} (w_{it} - \hat{w}_{it})^2 / \sum_{t=1}^{T} (w_{it} - \bar{w}_{it})^2 \; ; \quad i = A, \ldots, D.$$

Table 6 Coefficients of determination by commodity; 8 commodity grouping

Model	Commodity							
	1	2	3	4	5	6	7	8
Trend-w	0.924	0.966	0.383	0.551	0.023	0.783	0.589	0.656
Auto-w	0.913	0.980	0.796	0.765	0.114	0.765	0.594	0.850
CEDS-lnq	0.922	0.984	0.702	0.716	0.183	0.833	0.349	0.742
CEDS-lnw[1)]	0.971	0.997	0.975	0.904	0.548	0.966	0.876	0.946
LESH-pq	0.965	0.988	0.926	0.410	0.119	0.913	0.691	0.793
LESH-w	0.963	0.989	0.938	0.594	0.025	0.915	0.727	0.829
LES-w	0.790	0.976	-0.145	0.760	-1.425	0.874	0.486	0.668
RD-w^*Dq	0.946	0.997	0.893	0.774	-0.019	0.811	0.556	0.833
RDI-w^*Dq	0.948	0.997	0.936	0.802	0.143	0.847	0.765	0.861
ITRL-w,Ω	0.964	0.993	0.961	0.765	0.274	0.866	0.696	0.895
ITRL-w,Ω_0	0.971	0.834	0.417	0.989	0.934	0.867	0.785	0.887

1) Predictions of expenditure shares do not sum to 1.

Table 7 Income and price elasticities (1960); 4 commodity grouping

Model	Income elasticities commodity				Compensated own-price elasticities commodity			
	A	B	C	D	A	B	C	D
CEDS-lnq	-0.787 (0.154)	2.115 (0.473)	1.273 (0.143)	2.348 (0.585)	0.823 (0.516)	-0.573 (0.225)	0.008 (0.044)	-0.492 (0.079)
CEDS-lnw	0.175 (0.148)	2.094 (0.523)	0.847 (0.202)	2.034 (0.527)	0.542 (0.258)	-0.220 (0.312)	-0.338 (0.128)	-0.528 (0.217)
LESH-pq	0.211 (0.082)	2.355 (0.270)	1.513 (0.164)	0.418 (0.308)	-0.031 (0.016)	-0.341 (0.087)	-0.206 (0.053)	-0.062 (0.072)
LESH-w	0.142 (0.055)	2.536 (0.211)	1.625 (0.147)	0.114 (0.260)	-0.028 (0.018)	-0.493 (0.070)	-0.895 (0.043)	-0.018 (0.090)
LES-w	-0.662 (0.141)	2.224 (0.245)	1.196 (0.131)	4.223 (0.429)	0.263 (0.057)	-0.483 (0.043)	-0.242 (0.039)	-0.633 (0.053)
RD-w*Dq	0.400 (0.143)	2.320 (0.537)	1.181 (0.242)	0.648 (0.420)	0.199 (0.110)	-0.344 (0.222)	-0.173 (0.113)	-0.083 (0.206)
RDI-w*Dq	0.412 (0.140)	2.417 (0.542)	1.156 (0.244)	0.530 (0.428)	0.100 (0.122)	-0.190 (0.290)	-0.231 (0.124)	-0.142 (0.308)
ITRL-w	1.251	0.917	1.138	-0.033	-1.051	-0.828	-1.043	-1.225

Note: Elasticities for the dynamic models LESH-pq and LESH-w are one-period elasticities.

(Assymptotic) standard errors - in parenthesis - are estimated conditional upon observed expenditure shares and volumes. They are not available for the model ITRL-w.

Table 8 Income elasticities using a 8 commodity grouping (1960)

Model	Commodity 1	2	3	4	5	6	7	8
CEDS-lnq	0.039 (0.421)	-1.062 (0.401)	1.018 (0.317)	4.356 (0.660)	1.368 (0.661)	1.289 (0.152)	1.697 (0.280)	2.350 (0.584)
CEDS-lnw	0.274 (0.377)	-0.268 (0.343)	1.242 (0.336)	1.514 (1.034)	1.806 (0.854)	0.521 (0.212)	1.442 (0.549)	1.700 (0.505)
LESH-pq	0.629 (0.104)	0.027 (0.027)	0.447 (0.027)	2.178 (0.205)	1.931 (0.193)	1.491 (0.119)	0.807 (0.211)	0.714 (0.089)
LESH-w	0.463 (0.099)	-0.001 (0.037)	0.461 (0.036)	1.471 (0.162)	2.410 (0.176)	1.561 (0.130)	1.066 (0.216)	0.389 (0.114)
LES-w	1.133 (0.126)	-1.148 (0.144)	-0.568 (0.123)	4.633 (0.364)	2.165 (0.188)	1.198 (0.091)	1.541 (0.199)	1.380 (0.096)
RD-w*Dq	0.197 (0.356)	0.607 (0.236)	0.653 (0.435)	1.603 (1.221)	2.496 (0.833)	1.233 (0.352)	1.109 (0.721)	0.444
RDI-w*Dq	0.211 (0.368)	0.634 (0.286)	0.647 (0.366)	1.603 (1.210)	2.396 (0.825)	1.246 (0.335)	1.191 (0.574)	0.455
ITRL-w,Ω	0.047	0.191	1.565	3.697	1.401	0.933	1.686	1.377
ITRL-w,Ω_0	0.323	3.431	1.245	0.307	1.358	0.853	1.035	1.706

Compensated own-price elasticities (1960)

Model	Commodity 1	2	3	4	5	6	7	8
CEDS-lnq	0.342 (0.114)	0.460 (0.141)	-0.559 (0.107)	-1.046 (0.145)	-0.981 (0.216)	-0.463 (0.064)	-0.087 (0.233)	-0.781 (0.079)
CEDS-lnw	-0.214 (0.126)	0.466 (0.255)	-0.202 (0.157)	-0.586 (0.376)	0.134 (0.586)	-0.830 (0.123)	0.010 (0.514)	-0.703 (0.429)
LESH-pq	-0.198 (0.030)	0.013 (0.012)	-0.163 (0.007)	-0.661 (0.075)	-0.562 (0.064)	-0.359 (0.027)	-0.283 (0.081)	-0.216 (0.035)
LESH-w	-0.157 (0.037)	0.029 (0.015)	-0.176 (0.013)	-0.503 (0.069)	-0.670 (0.055)	-0.390 (0.033)	-0.404 (0.084)	-0.127 (0.046)
LES-w	-0.314 (0.022)	0.308 (0.019)	0.147 (0.012)	-1.134 (0.013)	-0.583 (0.026)	-0.353 (0.022)	-0.382 (0.010)	-0.366 (0.012)
RD-w*Dq	0.054 (0.145)	0.196 (0.127)	-0.097 (0.138)	-0.556 (0.263)	-0.526 (0.276)	-0.302 (0.131)	0.372 (0.354)	0.223
RDI-*Dq	0.013 (0.156)	-0.187 (0.135)	-0.147 (0.145)	-0.636 (0.274)	-0.374 (0.446)	-0.328 (0.131)	0.159 (0.326)	0.311
ITRL-w,Ω	0.169	0.486	-0.089	-0.807	-0.674	-0.343	0.004	-0.097
ITRL-w,Ω_0	-0.138	-0.628	-0.334	-0.823	-0.033	-0.329	-0.129	-0.150

Note: Elasticities for the dynamic models LESH-pq and LESH-w are one-period elasticities.

(Assymptotic) standard errors - in parenthesis - are estimated conditional upon observed expenditure shares and volumes. They are not available for the model ITRL-w.

Table 9 Information inaccuracies for predictions obtained by aggregation from 8 to 4 commodities

Model	I_{71}	I_{72}	$\bar{I}_{71-72}$
Trend-w	735	1 535	1 135
Auto-w	1 008	189	599
CEDS-lnq	105	366	235
CEDS-lnw	611	2 784	1 697
LESH-pq	493	285	389
LESH-w	553	236	395
LES-w	3 038	257	1 645
RD-w^*Dq	1 099	614	856
RDI-w^*Dq	7 045	880	3 963
ITRL-w,Ω	103	109	106
ITRL-w,Ω_0	121	201	161

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