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Nonparametric Tests for Perfect Competition: Theory and Application to the Nordic Wholesale Electricity Market

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Nonparametric tests for perfect competition: Theory and application to the Nordic wholesale electricity market[*]

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Abstract

This paper develops a simple nonparametric test for perfect competition in markets for homogenous goods. The method only requires data on prices and some aggregate of output. We then generalize the method to account for variable capacity and intertemporal production decisions. We apply the method to a sample of Swedish data from the Nordic wholesale electricity market. Main results show that the data are approximately rationalizable by perfect competition in bidding zones with low ownership concentration of generation assets, but not in bidding zones characterized by high ownership concentration.

Keywords: Competition, nonparametric methods, Nord Pool power exchange, wholesale electricity markets
JEL Codes: D22, D43, D44

1 Introduction

Exercise of market power creates welfare losses because of underprovision of goods. Analysis of competition is therefore essential to assess market efficiency. This paper contributes to the toolbox of empirical analysis of market performance by introducing simple nonparametric tests for perfect competition in markets for homogenous goods. We illustrate their usefulness by applying them to data from the Nordic wholesale electricity market.

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The most common way of estimating market behavior is through parametric models. These models often build on the assumption of a linear demand function and variable production costs. One commonly used approach is the Bresnahan-Lau (BL) model (Bresnahan, 1982; Lau, 1982). In that model, the price-cost margin is related to a parameter that measures the extent to which companies operate in an imperfectly competitive market. Based on the estimated parameter values, the effects of market power can be measured by comparing the actual market outcome with a theoretical scenario of perfect competition. The quality of such analysis depends fundamentally on how well the specified parametric demand and cost functions reflect true consumer preferences and the actual production technology. If the adopted function forms are poor approximations, then the estimated market parameter is misleading (Kim and Knittel, 2006).

The nonparametric method developed in this paper does not depend on any functional form of demand and cost functions. The fundamental result holds for arbitrary cost functions. Hence, the nonparametric tests for perfect competition proposed in this paper are robust in the sense that they are always consistent with observed data, i.e. how firms and consumers actually act in the market. The method is based on the nonparametric production and consumer approach introduced by Afriat (1972), Hanoch and Rothschild (1972), Diewert and Parkan (1983) and Varian (1984), and later refined by e.g., Chavas and Cox (1990), Cherchye et al. (2014), Chambers and Rehbeck (2022, 2025). This literature derives conditions for testing whether an observed amount of inputs that a firm uses to produce a certain amount of final goods is consistent with the hypothesis that the firm minimizes its costs.

A basic assumption in the literature is that the firm operates either in a fully competitive market or that the prices of the goods produced are regulated. Instead of assuming perfect competition to test for cost efficiency, we start from the assumption that firms' are cost minimizing and proceed to test whether the market is characterized by perfect competition. Perhaps surprisingly, such analysis does not seem to have been carried out before.

In his seminal contribution, Varian (1984) stated analysis of competition as an important application of nonparametric analysis. An early contribution was Ashenfelter and Sullivan (1987). The main purpose was to analyze how a change in a firm's marginal production cost, specifically an increase in the excise tax, affected *imperfect* competition. This work was later generalized by Raper et al. (2007). Carvajal et al. (2013) derived a nonparametric method to asses whether Cournot competition can explain a given sample of observed prices and quantities. Notably, their method relies on the assumption of continuous and convex cost functions at the firm-level. Such regularity assumptions sometimes are restrictive. A prime example is electricity markets where unit startup costs and ramping constraints render firm and industry cost functions discontinuous and non-convex.

The theoretical analysis in Section 2 gives a complete nonparametric characterization of perfect competition. Part of this characterization is an axiom, which we label the *axiom of perfect competition* (APC), that is necessary and sufficient for a data set of price and output quantity data to be consistent with perfect competition. APC is a generalization of the well-known *law of supply*, which states that an increase in price (all other prices constant) results in higher output. APC simply states that the law of supply holds for every subset of firms on the market.

The fundamental assumption underlying our characterization is that firms' cost functions are invariant across the sample period. However, cost functions may display a lot of variation even in the very short-term. For example, in electricity markets short-term variation in the availability of wind power may alter the cost function from one observation to the next in terms of the available production capacity. We adapt our model to such an environment by developing a complete characterization of perfect competition under variable capacity. We derive an axiom, labelled APC-VC, that can be easily implemented to test whether a data set of prices, output quantities and total capacity to produce green output are consistent with perfect competition under variable capacity. APC-VC is very similar to APC, except output is now adjusted to changes in production capacity. We also show that the two axioms incorporate technologies in which production decisions are intertemporal, as is the case with reservoir-based hydro power.

A key application of the methods proposed in this paper is to restructured wholesale electricity markets. These markets offer huge opportunities to exploit market power as: (i) a small number of firms own most of the production capacity; (ii) bottlenecks in the transmission network increase local market concentration; (iii) political and economic barriers prevent large-scale entry into the market; (iv) demand is insensitive to short-term changes in the price of electricity.

Given the properties of the market, the specific (auction-like) market design and relatively high data availability, a large literature has developed which investigates competition in electricity markets. Some studies use firm-specific cost data that allow direct calibration of the industry's cost function that can then be compared with observed market prices (e.g. Wolfram, 1999; Borenstein et al., 2002). Other studies use bid data from individual producers to evaluate market performance (e.g. Wolak, 2003; McRae and Wolak, 2014). Such methods can only be applied in those rare circumstances where individual firm-level data on costs and bidding behavior are made available to outside observers. The procedures developed in this paper have a much broader application as they do not require neither firm nor market level cost data.

Recent studies of the specific Nordic electricity market reject the null hypothesis of perfect competition. Using aggregate bid data from the day-ahead market of the Nord Pool power exchange, price margins have been estimated at approximately 4% (Tangerås and Mauritzen, 2018; Lundin and Tangerås, 2020). However, there is reason to believe that the problems of imperfect competition are more severe than previous studies indicate. During peak demand hours, the transmission network has insufficient capacity to handle the electricity flows necessary to clear the market at an aggregate level. When such bottlenecks arise, Nord Pool divides the market into multiple bidding zones. Market concentration then is much higher in the individual bidding zones than in the Nordic market as a whole. Previous analyses have been carried out at an aggregate level and therefore cannot capture exercise of local market power.

In Section 4, we apply the methods developed in this paper to test for competition at the bidding zone level in the Nordic-Baltic wholesale electricity market. We show that price and aggregated output quantity data from the four Swedish bidding zones in the period 2012-22 generally provide a high goodness-of-fit of the models in the three northernmost bidding zones. These are characterized by excess production, small ownership concentration of dispatchable generation assets and a large degree of market integration. Goodness-of-fit is substantially worse in the southernmost bidding zone where there is excess consumption, ownership concentration is large and transmission bottlenecks are severe, thus indicating a competitive problem. Deeper investigation into this and other local markets appears warranted.

We collect proofs of all theoretical results in the appendix.

2 Theory

In this section, we present our theoretical results. We begin by giving a complete nonparametric characterization of perfect competition. The characterization contains a condition that can be easily implemented to test whether a data set can be rationalized by a model of perfect competition while maintaining minimal assumptions on each firm's cost function. In particular, this allows us to test if a market conforms to perfect competition without having to assume any functional forms for the cost functions. This is particularly convenient from a practical point of view because the exact form or even the properties of the cost functions are typically not observed in empirical applications. We also consider extensions especially relevant for the wholesale electricity market.

2.1 Complete characterization of perfect competition

Assume that an industry consists of $I \geq 1$ firms. Each firm $i \in \mathcal{I} := (1, ..., I)$ produces a homogenous good in amount $q^i \in [0, b^i]$, where $b^i > 0$ measures the total production capacity of firm $i \in \mathcal{I}$. Each firm's production cost is a function $C^i : [0, b^i] \mapsto \mathcal{C} \subset \mathbb{R}_+$. Let $\mathcal{X} \subset \mathcal{I} \cup \emptyset$ denote a (possibly empty) subset of firms and denote by $q^{\mathcal{X}} = \sum_{i \in \mathcal{X}} q^i$ their joint (subset of) production, where $q^{\emptyset} = 0$. Let the price of the good be denoted $p \in \mathcal{P} \subset \mathbb{R}_{++}$. A market is said to be *perfectly competitive* if all firms in the industry act as price takers.

Let an observation t of the market price for a good be denoted p_t and the behavior at observation t of firm $i \in \mathcal{I}$ in the industry be denoted q_t^i. We assume that there exist $T < \infty$ such observations, which are indexed by $t \in \mathcal{T} := (1, ..., T)$. The $I \times T$ "production observations" of (p_t, q_t^i) form the data set $\mathcal{O} = (p_t, q_t^i)_{t \in \mathcal{T}}^{i \in \mathcal{I}}$. We restrict attention to *generic* data sets in the sense that $p_t \neq p_\tau$ for all $t, \tau \in \mathcal{T}$ and $t \neq \tau$.

We say that an observation $t \in \mathcal{T}$ is *perfect-competition (PC)-rationalizable* if the behavior of each firm in the industry is consistent with profit maximization contingent on the assumption that they take output prices as exogenously given.

Definition 1 *A generic data set $\mathcal{O} = (p_t, q_t^i)_{t \in \mathcal{T}}^{i \in \mathcal{I}}$ is PC-rationalizable by I cost functions $(C^i)^{i \in \mathcal{I}}$ if for all firms $i \in \mathcal{I}$ and periods $t \in \mathcal{T}$,*

$$q_t^i \in \arg\max_y \{p_t y - C^i(y)\}$$

subject to $y \in [0, b^i]$.

Our goal is to obtain necessary and sufficient conditions for when $\mathcal{O}$ is PC-rationalizable by I cost functions. Clearly, a necessary condition for PC-rationalizability is that the *law of supply* holds for every individual firm. This means that the quantity of output responds in the same direction as price changes, or in other words, if the price of output increases (all other prices fixed) then the quantity produced increases. Indeed, under the assumption of PC-rationalizability, firm i earns weakly higher profit by producing quantity q_t^i rather than q_τ^i in period t:

$$p_t q_t^i - C^i(q_t^i) \geq p_t q_\tau^i - C^i(q_\tau^i).$$

By the same token, producing q_τ^i in period τ yields weakly higher profit than producing q_t^i in period τ:

$$p_\tau q_\tau^i - C^i(q_\tau^i) \geq p_\tau q_t^i - C^i(q_t^i).$$

Adding the two inequalities and simplifying expressions shows that production costs cancel out and yields the following condition:

$$(p_\tau - p_t)(q_\tau^i - q_t^i) \geq 0,$$

for all $i \in \mathcal{I}$ and all $t, \tau \in \mathcal{T}$. Thus, under PC-rationalizability, the law of supply holds for each individual firm and for all price comparisons. In particular, the law of supply holds for any cost function C^i. The only restriction is that the firm has the *same* cost function for all observations. Summing up the law of supply for every subset of firms delivers the following condition, which we label the *axiom of perfect competition* (APC):

Definition 2 *Consider a generic data set $\mathcal{O} = (p_t, q_t^i)_{t\in\mathcal{T}}^{i\in\mathcal{I}}$. The axiom of perfect competition (APC) holds whenever*

$$\sum_{i\in\mathcal{X}} (p_\tau - p_t)(q_\tau^i - q_t^i) = (p_\tau - p_t)(q_\tau^{\mathcal{X}} - q_t^{\mathcal{X}}) \geq 0,$$

for all $\mathcal{X} \in 2^{\mathcal{I}}$ and all $t, \tau \in \mathcal{T}$.

APC provides for all generic data sets a simple combinatorial test that runs in a finite number of steps. Importantly, APC does not require data on costs, and is implemented by evaluating the sign of the product of price and quantity differentials.[1] Since implementing APC does not require calculating or solving for any unknown parameters, it can be easily implemented using any statistical software. As such, it is a computationally simple task to check if APC holds and it can be applied to large data sets, i.e., $\mathcal{O}$ with I or T large.

Having established that APC is a *necessary* condition for PC-rationalizability, we next state our main result showing that APC also is a *sufficient* condition for a generic data set to be PC-rationalized by I cost functions.

Theorem 1 *Consider a generic data set $\mathcal{O} = (p_t, q_t^i)_{t\in\mathcal{T}}^{i\in\mathcal{I}}$. The following statements are equivalent:*

1. *$\mathcal{O}$ is PC-rationalizable by I cost functions.*
2. *$\mathcal{O}$ satisfies APC.*

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[1] It is obvious that APC is empirically refutable. For example, with $T = 2$ and $I = 2$, it is easy to verify that the following data set: $p_1 = 1, p_2 = 5, q_1^1 = 5, q_2^1 = 1, q_1^2 = 1$ and $q_2^2 = 5$ violates APC. Conversely, the data set: $p_1 = 1, p_2 = 5, q_1^1 = 1, q_2^1 = 5, q_1^2 = 1$ and $q_2^2 = 5$ satisfies APC.

3. *$\mathcal{O}$ is PC-rationalizable by I cost functions that are continuous, strictly increasing and strictly convex.*

There are several interesting features of this result. First, the equivalence of 1 and 3 shows that if some generic data set can be PC-rationalized by I cost functions at all it can, in fact, be PC-rationalized by a set of well-behaved cost functions. Or put another way, violations of continuity, monotonicity and convexity cannot be detected with only a finite number of production observations.[2] The second statement shows that APC gives the sharpest possible test of PC-rationalizability and fully exhausts the empirical content of PC-rationalizability. If the researcher only has access to price and quantity data, it is impossible to test for more than the law of supply without placing additional structure on the underlying model. Third, the proof of Theorem 1 is constructive and shows that APC can be used to construct I continuous, strictly increasing and strictly convex cost functions such that the data $\mathcal{O}$ are PC-rationalized.

In Section 2.3, we give some relevant extensions of Theorem 1 to the wholesale electricity market. But Theorem 1 can be extended in other directions that also should be relevant to other markets. For example, one particular such extension is to consider PC-rationalizations where each firm's cost function is differentiable. The necessary and sufficient conditions for rationalizability guaranteeing differentiability of the cost functions follows from slightly strengthening the APC. We say that a generic data set $\mathcal{O} = (p_t, q_t^i)_{t\in\mathcal{T}}^{i\in\mathcal{I}}$ satisfies the *strong axiom of perfect competition* (SAPC) if: (i) $\mathcal{O}$ satisfies APC, and (ii) $q_t^{\mathcal{X}} \neq q_\tau^{\mathcal{X}}$ for all $\mathcal{X} \in 2^{\mathcal{I}}$ and $t, \tau \in \mathcal{T}$ with $t \neq \tau$.

Proposition 1 *Consider a generic data set $\mathcal{O} = (p_t, q_t^i)_{t\in\mathcal{T}}^{i\in\mathcal{I}}$. The following statements are equivalent:*

1. *$\mathcal{O}$ is PC-rationalizable by I non-decreasing and $\mathcal{C}^2$ cost functions.*
2. *$\mathcal{O}$ satisfies SAPC.*

2.2 Market power and the law of supply

Theorem 1 states that a data set $\mathcal{O} = (p_t, q_t^i)_{t\in\mathcal{T}}^{i\in\mathcal{I}}$ is rationalizable by perfect competition if and only if $\mathcal{O}$ meets the law of supply for every pair of observations and for every subset of firms. A direct follow-up question is whether $\mathcal{O}$ is consistent with other types of rational firm behavior if those same data violates APC. This is indeed the case under imperfect competition, as we illustrate by means of a graphical example.

Figure 1 displays a diagram with quantities on the horizontal axis and prices on the vertical axis. The inverse residual demand function facing the firm equals $P_1(\cdot)$ in period 1 and $P_2(\cdot)$ in period 2. The firm produces output with the same marginal cost function $MC(\cdot)$ in both periods. Under competitive supply, the market-clearing output is found at the point at which price equals marginal cost, marked in the diagram by (q_1, p_1) in period 1 and by (q_2, p_2) in period 2. The pair of observations meet the law of supply because output is larger when the price is higher, $(p_2 - p_1)(q_2 - q_1) > 0$.

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[2] This result is analogous to the well-known Afriat's Theorem, from which it follows that continuity, monotonicity and concavity are nontestable properties in the consumer demand setting with competitive budgets (See Varian (1982) for a discussion).

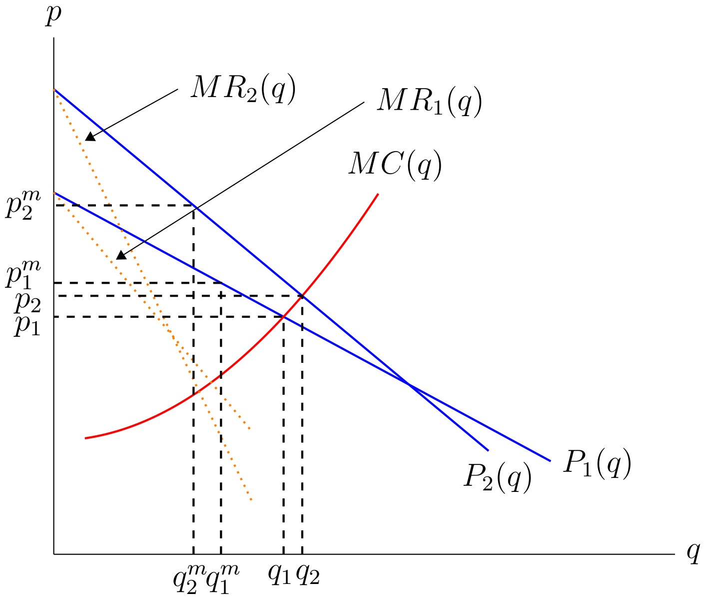

Figure 1: Market power and the law of supply

Suppose instead the firm exercises market power and therefore chooses output to equate marginal revenue and marginal cost in each period. The two marginal revenue functions are indicated by $MR_1(\cdot)$ and $MR_2(\cdot)$ in the figure. Profit-maximization by the firm yields the quantity-price pair (q_1^m, p_1^m) in period 1 and (q_2^m, p_2^m) in period 2. Exercise of market power causes the firm to reduce output in each period compared to the competitive outcome, $q_1^m < q_1$ and $q_2^m < q_2$, resulting in price increases in both periods relative to the competitive solution, $p_1^m > p_1$ and $p_2^m > p_2$.

Period 2 represents the high-price period under imperfect as well as perfect competition. However, the firm now produces less in period 2 than in period 1, thereby causing a violation of the law of supply $(p_2^m - p_1^m)(q_2^m - q_1^m) < 0$. This occurs even if the firm is behaving rationally by optimizing its exploitation of market power. On the one hand, the upward shift in the inverse demand function between periods 1 and 2 makes it more profitable to increase production from one period to the next. On the other hand, the downward rotation in the slope of the inverse demand function makes demand less sensitive to price changes, thereby strengthening the firm's incentive to exercise market power by withholding more output from the market in period 2 than period 1.

In Figure 1, the market power effect dominates the direct price effect causing the firm to produce less output when the price is high compared to when the price is low. Hence, price and quantity combinations that are inconsistent with perfect competition (and therefore violate APC) are consistent with exercise of market power. This illustration shows that APC has power against a wide range of different types of rational firm behavior (other than perfect competition).

2.3 Extensions to the wholesale electricity market

In this section, we consider extensions of the model that are especially relevant for the wholesale electricity market, but may also be applicable to other markets. In particular, we pay special attention to short-term variability of production capacity of firms and intertemporal production decisions.

2.3.1 Variable capacity

The characterization of perfect competition in Theorem 1 relies on the assumption that cost functions are constant across the sample period and that all sources of price variation stem from exogenous changes in demand. As such, firms move up and down their fixed marginal cost curves when choosing output. One possible source of error would be to mistake exogenous changes of the cost functions for non-competitive market behavior. Holding demand constant, changes in the marginal cost curve can yield price and output changes as firms move up and down the demand curve. This would generate a negative relationship between prices and output in a competitive market and thus cause a violation of APC even if the market was indeed competitive. A standard solution is to partition the data set into sufficiently narrow subsamples such that the cost functions are likely to remain constant within each subsample, and then apply APC to every subsample. However, this approach is likely to fail in important applications of the theory, since there is no empirically consistent and simple way of determining the range of the subsamples.

Let us consider wholesale electricity markets. Over the last two decades many jurisdictions have implemented support schemes to increase the amount of renewable electricity production. These support schemes have mainly lead to an increase in *variable renewable energy* (VRE) such as solar and wind power. These technologies have two fundamental properties. First, they produce electricity with zero marginal cost. This means that cost-minimizing firms will dispatch such units for any positive price. Second, available VRE capacity is likely to vary from one dispatch period to the next depending on changes in predicted weather conditions. These two factors jointly imply that the cost of incremental production also is likely to be subject to short-term variation.

To address variable capacity in the context of wholesale electricity markets, we extend the characterization in Theorem 1 by relaxing the assumption that cost functions are constant across all periods. In particular, we assume that there are two types of technologies, black (b) and green (g) that differ in terms of their production cost and available capacity within the sample period. In electricity markets, we can think of black technologies as dispatchable thermal generation such as nuclear power or fossil-fueled generation such as coal and gas power. Wind and solar power represent green technologies.

Suppose that firm $i \in \mathcal{I}$ produces black output in quantity $q_t^i(b) \in [0, b^i]$ in period $t \in \mathcal{T}$, where $b^i \geq 0$ measures the firm's total capacity to produce black output. The associated production cost for a firm with $b^i > 0$ is a *strictly increasing function* $C^i : [0, b^i] \mapsto \mathcal{C} \subset \mathbb{R}_+$. This cost function is invariant across the sample period. Assume that firm $i \in \mathcal{I}$ produces green output in quantity $q_t^i(g) \in [0, g_t^i]$ in period $t \in \mathcal{T}$, where $g_t^i \in [0, \bar{g}^i]$ measures the total capacity to produce green output in period $t \in \mathcal{T}$, and $\bar{g}^i \geq 0$ is firm i's installed capacity of the green technology. The production cost of the green technology is zero. We assume that $b^i + \bar{g}^i > 0$ for all $i \in \mathcal{I}$, so that each firm has the capacity to produce some output, although not all firms necessarily produce both types of output. We denote firm i's total output in period t by $q_t^i = q_t^i(b) + q_t^i(g)$. Black and green output are sold at the same price $p_t > 0$ per unit of output. In addition, green output may receive a subsidy that amounts to $a_t \geq 0$ per unit of output in period t.

We define an extended generic data set by $\widetilde{\mathcal{O}}_{\mathcal{G}} = (p_t, a_t, q_t^i\,(b)\,, q_t^i\,(g)\,, g_t^i)_{t\in\mathcal{T}}^{i\in\mathcal{I}}$ such that $p_t \neq p_\tau$ for all $t, \tau \in \mathcal{T}$ with $t \neq \tau$. For every period $t \in \mathcal{T}$, $\widetilde{\mathcal{O}}_{\mathcal{G}}$ contains information about the market price p_t, the subsidy a_t per unit of green output, and in addition, for every firm $i \in \mathcal{I}$, the quantity $q_t^i\,(b)$ of black and $q_t^i(g)$ of green output as well as green production capacity g_t^i. Recall the definition of $q_t^{\mathcal{X}}$ as the joint production in period $t \in \mathcal{T}$ of the subset $\mathcal{X} \subset \mathcal{I} \cup \emptyset$ of firms. Define $g_t^{\mathcal{X}} = \sum_{i\in\mathcal{X}} g_t^i$ as those same firms' aggregate capacity to produce green output in period $t \in \mathcal{T}$, where we define $g_t^{\emptyset} = 0$.

We now turn to the issue of rationalizability. We first show that any firm $i \in \mathcal{I}$ that maximizes profit and treats (p_t, a_t) as exogenously given, will utilize the green technology to its full extent, i.e. set $q_t^i\,(g) = g_t^i$. Let $(q_t^i\,(b)\,, q_t^i\,(g))$ be firm i's profit-maximizing production vector, and suppose $q_t^i\,(g) < \min\{q_t^i; g_t^i\}$. This output vector results in the profit

$$p_t q_t^i + a_t q_t^i\,(g) - C^i(q_t^i - q_t^i\,(g))$$

of firm $i \in \mathcal{I}$.

Consider an alternative production plan $(\tilde{q}_t^i\,(b)\,, \tilde{q}_t^i\,(g))$ in which green production is set to $\tilde{q}_t^i\,(g) = \min\{q_t^i; g_t^i\}$ and black production to $\tilde{q}_t^i\,(b) = \max\{q_t^i - g_t^i; 0\}$. Total production is still given by $\tilde{q}_t^i\,(g) + \tilde{q}_t^i\,(b) = q_t^i$, so firm $i \in \mathcal{I}$ earns

$$p_t q_t^i + a_t \min\{q_t^i; g_t^i\} - C^i(\max\{q_t^i - g_t^i; 0\})$$

under the alternative plan. Subtracting the first profit expression from the second yields

$$a_t(\min\{q_t^i; g_t^i\} - q_t^i\,(g)) + C^i(q_t^i - q_t^i\,(g)) - C^i\left(\max\{q_t^i - g_t^i; 0\}\right) > 0.$$

This expression is strictly positive because $a_t \geq 0$, and the cost of producing the black output is strictly increasing. This result contradicts the assumption that $q_t^i\,(g) < \min\{q_t^i; g_t^i\}$ maximizes firm i's profit. For any q_t^i, it is therefore optimal for firm $i \in \mathcal{I}$ to produce green output in quantity $q_t^i\,(g) = \min\{q_t^i; g_t^i\}$ and black output in quantity $q_t^i\,(b) = \max\{q_t^i - g_t^i; 0\}$. The firm maximizes green output because it has zero production cost and is potentially associated with a production subsidy.

If $q_t^i < g_t^i$, then the firm has operating profit $(p_t + a_t)q_t^i - C^i(0)$, which is strictly increasing in output. Hence, the profit-maximizing output choice of firm $i \in \mathcal{I}$ satisfies $q_t^i \geq g_t^i$, in which case the firm produces green output at full capacity, $q_t^i\,(g) = g_t^i$, and black output in quantity $q_t^i\,(b) = q_t^i - g_t^i$ at resulting profit

$$p_t q_t^i + a_t g_t^i - C^i(q_t^i - g_t^i).$$

We can treat $a_t g_t^i$ as a negligible constant. Thus, PC-rationalizability under variable capacity is defined as follows:

Definition 3 *A generic data set* $\widetilde{\mathcal{O}}_{\mathcal{G}} = (p_t, a_t, q_t^i\,(b)\,, q_t^i\,(g)\,, g_t^i)_{t\in\mathcal{T}}^{i\in\mathcal{I}}$ *is PC-rationalizable under variable capacity by I strictly increasing cost functions* $(C^i)^{i\in\mathcal{I}}$ *if for all firms* $i \in \mathcal{I}$ *and periods* $t \in \mathcal{T}$,

$$q_t^i \in \arg\max_y \{p_t y - C^i(y - g_t^i)\}$$

subject to $y \in [g_t^i, g_t^i + b^i]$.

The cost function of firm $i \in \mathcal{I}$ varies across observations if the available capacity g_t^i of green output varies across observations. However, PC-rationalizability under variable capacity relies only on observability of each firm's total output $(q_t^i)_{t \in \mathcal{T}}^{i \in \mathcal{I}}$ and green capacity $(g_t^i)_{t \in \mathcal{T}}^{i \in \mathcal{I}}$, instead of amounts of output produced under black and green production, $(q_t^i(b), q_t^i(g))_{t \in \mathcal{T}}^{i \in \mathcal{I}}$, respectively. Green capacity and green production will be identical under profit maximization since the production cost of green output is zero. The subsidy $(a_t)_{t \in \mathcal{T}}$ does not matter for allocations because the revenue from the subsidy is independent of total output subject to firms producing green electricity a full capacity. Under perfect competition, observation of $\widetilde{\mathcal{O}}_{\mathcal{G}}$ does not provide any useful information beyond what can be deduced from the more restricted data set $\mathcal{O}_{\mathcal{G}} = (p_t, q_t^i, g_t^i)_{t \in \mathcal{T}}^{i \in \mathcal{I}}$. This result is useful because plant level output data often are unobservable to outsiders.

Our next result shows that PC-rationalizability *does not* imply the law of supply if the firm has variable production capacity. Under PC-rationalizability, firm i earns weakly higher profit by producing green output in quantity g_t^i and black output in quantity $q_t^i - g_t^i$ in period t rather than green output in quantity g_t^i and black output in quantity $q_\tau^i - g_\tau^i$:

$$p_t q_t^i - C^i(q_t^i - g_t^i) \geq p_t(g_t^i + q_\tau^i - g_\tau^i) - C^i(q_\tau^i - g_\tau^i).$$

Likewise, the profit of producing green output in quantity g_τ^i and black output in quantity $q_\tau^i - g_\tau^i$ in period τ is at least as high as producing green output in quantity g_τ^i and black output in quantity $q_t^i - g_t^i$ in period τ:

$$p_\tau q_\tau^i - C^i(q_\tau^i - g_\tau^i) \geq p_\tau(g_\tau^i + q_t^i - g_t^i) - C^i(q_t^i - g_t^i).$$

Adding the two inequalities and simplifying the expression delivers a necessary condition

$$(p_\tau - p_t)(q_\tau^i - q_t^i - g_\tau^i + g_t^i) \geq 0 \ \forall i \in \mathcal{I} \text{ and } \forall t, \tau \in \mathcal{T} \tag{1}$$

for PC-rationalizability under variable capacity. If $p_\tau > p_t$, then firm i violates the law of supply under perfect competition if $g_t^i - g_\tau^i > q_t^i - q_\tau^i > 0$. Under these conditions, firm i's capacity to deliver output at zero marginal cost is so large in period t compared to period τ that it produces a larger total quantity in period t than τ, although the price is actually lower in t than τ. A failure to account for variable capacity may thus cause a false rejection of APC. Aggregating (1) for every subset of firms instead delivers the following axiom, which we label the axiom of perfect competition under variable capacity:

Definition 4 *Consider a generic data set* $\mathcal{O}_{\mathcal{G}} = (p_t, q_t^i, g_t^i)_{t \in \mathcal{T}}^{i \in \mathcal{I}}$. *The axiom of perfect competition under variable capacity (APC-VC) holds whenever* $(p_\tau - p_t)\left(q_\tau^{\mathcal{X}} - q_t^{\mathcal{X}} - g_\tau^{\mathcal{X}} + g_t^{\mathcal{X}}\right) \geq 0$ *for all* $\mathcal{X} \in 2^{\mathcal{I}}$ *and all* $t, \tau \in \mathcal{T}$.

Observe that APC-VC reduces to APC in the special case when $g_t^i = g_\tau^i$ for all $(i, t, \tau) \in \mathcal{I} \times \mathcal{T} \times \mathcal{T}$. Our next result gives a complete characterization of PC-rationalizability under variable capacity.

Theorem 2 *Consider a generic data set* $\mathcal{O}_{\mathcal{G}} = (p_t, q_t^i, g_t^i)_{t \in \mathcal{T}}^{i \in \mathcal{I}}$. *The following statements are equivalent:*

1. $\mathcal{O}_{\mathcal{G}}$ *is PC-rationalizable under variable capacity by* I *strictly increasing cost functions.*

2. *$\mathcal{O}_{\mathcal{G}}$ satisfies APC-VC.*

3. *$\mathcal{O}_{\mathcal{G}}$ is PC-rationalizable under variable capacity by I cost functions that are continuous, strictly increasing and strictly convex.*

The additional informational requirement of Theorem 2 compared to Theorem 1 is that one needs access to data on (some aggregate) of the green production capacity $(g_t^i)_{t\in\mathcal{T}}^{i\in\mathcal{I}}$ to test for perfect competition under variable capacity. Theorems 2 and 1 share several similarities. Most importantly, violations of continuity, monotonicity and convexity in both theorems cannot be detected with only a finite number of production observations. Furthermore, as APC gives the sharpest possible test of PC-rationalizability in Theorem 1, APC-VC gives the sharpest possible test of PC-rationalizability under variable capacity.

2.3.2 Intertemporal production decisions

This section explores consequences of resource extraction problems when the opportunity cost of deferring production to a future date represents a substantial part of a firm's optimization problem. This extension is relevant for several markets, for instance the crude oil market, but we illustrate the model within the context of hydro power production in the wholesale electricity market. The model and setup should be general enough to fit other markets with few alterations.

Many countries rely fundamentally on hydro power for electricity supply. Hydro power has zero marginal production cost, but is largely a predictable source of energy in the short-run, contrary to solar and wind power. The profit-maximizing dispatch of hydro power represents an intertemporal problem because reservoir capacity utilized for electricity production in one dispatch period cannot be used for production in a futúre dispatch period. Hence, the fundamental economic factor that limits hydro power extraction is an opportunity cost measured by the expected value of storing hydro power for future use. This *water value* depends on expectations about the future electricity wholesale prices, and is likely to be subject to short-term variation because of the variability of electricity prices on the spot market.

Add hydro production capacity $h^i \geq 0$ to the generation portfolio of firm i. To incorporate the dynamic component, let firm $i \in \mathcal{I}$ have a planning horizon of $T \geq 2$ periods, indexed by $\mathcal{T} := \{1, ..., T\}$. The firm enters period 1 with a reservoir measured in terms of $H_0^i > 0$ MWh output, and has decided to maintain reservoir capacity $H_T^i \in [0, H_0^i)$ at the end of period T. We assume no uncertainty, no discounting and that there are no direct production costs associated with hydro power production. We also ignore inflow and evaporation, which would affect reservoirs from one period to the next.

Firm i's additional decision problem is how to allocate the planned production $H_0^i - H_T^i$ across the T periods. Let $q_t^i(h)$ be the quantity of hydro-electric power produced by firm i in period $t \in \mathcal{T}$. Denote by $q_t^{\mathcal{X}}(h) = \sum_{i\in\mathcal{X}} q_t^i(h)$ the joint hydro production of the subset $\mathcal{X}$ of firms in period t, where we set $q_t^{\emptyset}(h) = 0$. Define the extended generic data set $\widetilde{\mathcal{O}}_{\mathcal{G}} = (p_t, a_t, q_t^i(b), q_t^i(g), q_t^i(h), g_t^i, H_0^i, H_T^i)_{t\in\mathcal{T}}^{i\in\mathcal{I}}$.

Under perfect competition, firm i's problem is how to choose $(q_t^i(b), q_t^i(g), q_t^i(h))_{t\in\mathcal{T}}$

to maximize total profit

$$\sum_{t\in\mathcal{T}}[p_t q_t^i + a_t q_t^i(g) - C^i(q_t^i - q_t^i(g) - q_t^i(h))]$$

subject to the capacity constraint $q_t^i(b) \in [0, b^i]$ on black output, $q_t^i(g) \in [0, g_t^i]$ on green production, and $q_t^i(h) \in [0, h^i]$ on hydro power for each period $t \in \mathcal{T}$, as well as the resource constraint $\sum_{t=1}^{T} q_t^i(h) \leq H_0^i - H_T^i$. In this maximization program, the firm produces a total output of $q_t^i = q_t^i(b) + q_t^i(g) + q_t^i(h)$ in period t.

By similar logic as above, any firm $i \in \mathcal{I}$ that maximizes profit and treats (p_t, a_t) as exogenously given, maximizes green output by setting $q_t^i(g) = g_t^i$ for all $t \in \mathcal{T}$. Firm i can reduce production cost or increase revenue by increasing green output and reducing black output for all $q_t^i(g) < g_t^i$, while keeping everything else constant. Firm i's profit then equals

$$\sum_{t\in\mathcal{T}}[p_t q_t^i - C^i(q_t^i - g_t^i - q_t^i(h))] + \sum_{t\in\mathcal{T}} a_t g_t^i,$$

where $\sum_{t\in\mathcal{T}} a_t g_t^i$ is a constant that can be disregarded. Thus, we define PC-rationalizability under variable capacity and dynamic production as follows:

Definition 5 *A generic data set* $\widetilde{\mathcal{O}}_{\mathcal{G}} = (p_t, a_t, q_t^i(b), q_t^i(g), q_t^i(h), g_t^i, H_0^i, H_T^i)_{t\in\mathcal{T}}^{i\in\mathcal{I}}$ *is PC-rationalizable under variable capacity and dynamic production by I strictly increasing cost functions* $(C^i)^{i\in\mathcal{I}}$ *if for all firms* $i \in \mathcal{I}$,

$$\left(q_t^i, q_t^i(h)\right)_{t\in\mathcal{T}} \in \arg\max_{(y_t, y_t(h))_{t\in\mathcal{T}}} \sum_{t\in\mathcal{T}}[p_t y_t - C^i(y_t - g_t^i - y_t(h))]$$

subject to $y_t - y_t(h) \in [g_t^i, g_t^i + b^i]$ *and* $y_t(h) \in [0, h^i]$ *for all* $t \in \mathcal{T}$, *as well as* $\sum_{t=1}^{T} y_t(h) \leq H_0^i - H_T^i$.

Under PC-rationalizability, firm i earns weakly higher profit in period t by producing black output in quantity $q_t^i - g_t^i - q_t^i(h)$ relative to black output in quantity $q_\tau^i - g_\tau^i - q_\tau^i(h)$, all else equal

$$p_t q_t^i - C^i(q_t^i - g_t^i - q_t^i(h)) \geq p_t(g_t^i + q_t^i(h) + q_\tau^i - g_\tau^i - q_\tau^i(h)) - C^i(q_\tau^i - g_\tau^i - q_\tau^i(h)).$$

The profit of producing black output in quantity $q_\tau^i - g_\tau^i - q_\tau^i(h)$ in period τ is at least as high as producing black output in quantity $q_t^i - g_t^i - q_t^i(h)$ in period τ, all else equal:

$$p_\tau q_\tau^i - C^i(q_\tau^i - g_\tau^i - q_\tau^i(h)) \geq p_\tau(g_\tau^i + q_\tau^i(h) + q_t^i - g_t^i - q_t^i(h)) - C^i(q_t^i - g_t^i - q_t^i(h)).$$

By combining these inequalities, we obtain a necessary condition

$$(p_\tau - p_t)(q_\tau^i - q_t^i - g_\tau^i + g_t^i - q_\tau^i(h) + q_t^i(h)) \geq 0 \qquad \forall i \in \mathcal{I} \qquad \text{and} \qquad \forall t, \tau \in \mathcal{T} \qquad (2)$$

for PC-rationalizability under variable capacity and dynamic production. This condition does not depend on separate observability of black and green production nor of the production subsidy.

The allocation problem of reservoir capacity is linear under perfect competition so that firm i produces as much as possible in the periods with the highest price. Partition

$\mathcal{T}$ into a subset $\mathcal{T}^i(h)$ containing the peak price periods and a subset $\underline{\mathcal{T}}^i(h)$ containing the off-peak price periods from the viewpoint of firm i. Let $t^i(h) \in \mathcal{T}^i(h)$ be such that $p_{t^i(h)}$ is the smallest peak price contained in $\mathcal{T}^i(h)$ and allocate the entire production capacity to the peak periods, $\sum_{t\in\mathcal{T}^i(h)} q_t^i(h) = \min\{H_0^i - H_T^i; Th^i\}$. Based on (2) and the definitions of $\mathcal{T}^i(h)$ and $\underline{\mathcal{T}}^i(h)$, we obtain the following condition:

Definition 6 *Consider a generic data set* $\mathcal{O}_{\mathcal{G}} = (p_t, q_t^i, g_t^i, H_0^i, H_T^i)_{t\in\mathcal{T}}^{i\in\mathcal{I}}$. *The axiom of perfect competition under variable capacity and dynamic production (APC-VCDP) holds whenever*

$$(p_\tau - p_t)\left(q_\tau^{\mathcal{X}} - q_t^{\mathcal{X}} - g_\tau^{\mathcal{X}} + g_t^{\mathcal{X}} - q_\tau^{\mathcal{X}}(h) + q_t^{\mathcal{X}}(h)\right) \geq 0,$$

for all $\mathcal{X} \in 2^{\mathcal{I}}$ *and all* $t, \tau \in \mathcal{T}$, *and for each firm* $i \in \mathcal{I}$, $(q_t^i(h))_{t\in\mathcal{T}}$ *is characterized by:*

$$\begin{aligned} q_t^i(h) = &\begin{cases} 0 & \forall t \in \underline{\mathcal{T}}^i(h) \\ h^i & \forall t \in \mathcal{T}^i(h),\ t \neq t^i(h) \end{cases} \\ q_{t^i(h)}^i(h) = &\ \min\{H_0^i - H_T^i; Th^i\} - (|\mathcal{T}^i(h)| - 1)h^i. \end{aligned} \tag{3}$$

Firm i produces at full capacity h^i in the $|\mathcal{T}^i(h)|-1$ periods with the highest price and allocates the rest of the reservoir capacity to the remaining high price period. Our next result gives a complete characterization of PC-rationalizability under variable capacity and dynamic production.

Theorem 3 *Consider a generic data set* $\mathcal{O}_{\mathcal{G}} = (p_t, q_t^i, g_t^i, H_0^i, H_T^i)_{t\in\mathcal{T}}^{i\in\mathcal{I}}$. *The following statements are equivalent:*

1. $\mathcal{O}_{\mathcal{G}}$ *is PC-rationalizable under variable capacity and dynamic production by I strictly increasing cost functions.*
2. $\mathcal{O}_{\mathcal{G}}$ *satisfies APC-VCDP.*
3. $\mathcal{O}_{\mathcal{G}}$ *is PC-rationalizable under variable capacity and dynamic production by I cost functions that are continuous, strictly increasing and strictly convex.*

Theorem 3 shows how to adapt the model with variable capacity to include dynamic production into firms' generation portfolio. In deriving this result, we have ignored several relevant aspects of actual hydro power production that merit comment.

Production possibilities are affected by inflow into and possibly evaporation from the reservoir. However, such external flow variables do not qualitatively affect the profit maximizing program, which is to allocate as much production to peak price periods as possible. Accounting for reservoir inflow would expand the subset of peak price periods $\mathcal{T}^i(h)$, whereas evaporation would yield a contraction of $\mathcal{T}^i(h)$.

Hydro production may also be subject to minimal flow restrictions, which yields minimal production of $\underline{h}^i \geq 0$ in every period, but without any consequence for the firm's desire to allocate as much as possible of the residual production $H_0^i - H_T^i - T\underline{h}^i$ to the peak price periods. Environmental constraints may restrict a hydro power plant's ability to increase or decrease production from one period to the next. Such *ramping constraints* $\underline{\varepsilon}^i$ and $\bar{\varepsilon}^i$ imply production constraints defined as $-\underline{\varepsilon}^i \leq q_{t+1}^i(h) - q_t^i(h) \leq \bar{\varepsilon}^i$. Serial correlation of electricity demand implies that peak price periods tend to cluster. By implication, ramping constraints will be non-binding in the interior of $\mathcal{T}^i(h)$ and $\underline{\mathcal{T}}^i(h)$, and will affect production only for intermediary prices where the firm ramps up production from 0 to h^i or vice versa.

Finally, uncertainty implies that prices seldom are fully known at the outset of the planning period. However, linearity of the maximization problem under perfect competition implies that the hydro producer does not need to form beliefs about price levels. It is sufficient to know which periods belong to $\mathcal{T}^i(h)$ and $\underline{\mathcal{T}}^i(h)$, respectively. Whether prices will be higher from one period to the next, is much easier to predict than the level of those prices. Hence, Theorem 3 is likely to hold also in more uncertain and constrained environments than the one considered above.

Applying Theorem 3 to test APC-VCDP requires substantially more information than testing APC-VC. In particular, the initial reservoir capacity H_0^i and the end capacity H_T^i for each individual firm is required in order to derive $(q_t^i(h))_{t\in\mathcal{T}}$ over the planning period. Data at such granularity can be difficult to obtain. However, a necessary condition for APC-VCDP requires much less information. Note that PC-rationalizability implies:

$$\begin{aligned}
&\sum_{s\in\mathcal{T}}[p_s q_s^i - C^i(q_s^i - g_s^i - q_s^i(h))] \\
&\geq \sum_{s\neq t,\tau}[p_s q_s^i - C^i(q_s^i - g_s^i - q_s^i(h))] + p_t(g_t^i + q_\tau^i - g_\tau^i) - C^i(q_\tau^i - g_\tau^i - q_\tau^i(h)) \\
&+ p_\tau(g_\tau^i + q_t^i - g_t^i) - C^i(q_t^i - g_t^i - q_t^i(h))
\end{aligned}$$

for all $i \in \mathcal{I}$ and $\forall t, \tau \in \mathcal{T}$, where the expressions in the second and third rows return firm i's profit of producing hydro output $q_\tau^i(h)$ and black output $q_\tau^i - g_\tau^i - q_\tau^i(h)$ in period t and hydro output $q_t^i(h)$ and black output $q_t^i - g_t^i - q_t^i(h)$ in period τ. Simplifying the inequality produces (1). We therefore conclude:

Corollary 1 *A generic data set $\mathcal{O}_\mathcal{G} = (p_t, q_t^i, g_t^i)_{t\in\mathcal{T}}^{i\in\mathcal{I}}$ is PC-rationalizable under variable capacity and dynamic production by I strictly increasing cost functions only if:*

$$(p_\tau - p_t)\left(q_\tau^\mathcal{X} - q_t^\mathcal{X} - g_\tau^\mathcal{X} + g_t^\mathcal{X}\right) \geq 0$$

for all $\mathcal{X} \in 2^\mathcal{I}$ and all $t, \tau \in \mathcal{T}$.

Thus, a violation of APC-VC implies a violation of APC-VCDP, although the contrary is not true.

3 Measuring and testing departures

APC (and its extensions) gives a binary response to whether observed market behavior is consistent with perfect competition. However, even if the data violate APC, it may well be that industry behavior is sufficiently close to satisfying perfect competition such that the deviation from APC is negligible in practice. This may arise because of measurement errors, small optimization error or other types of randomness. In this section, we propose methods to measure and test departures from perfect competition.

3.1 Goodness-of-fit

We begin by presenting a measure of goodness-of-fit that provides information about how close observed market behavior is to satisfying perfect competition. This measure is related to the nonparametric goodness-of-fit measures proposed by Afriat (1972) and Varian (1990) for single firms. However, there is one important difference. The models in Afriat (1972) and Varian (1990) are linear in cost, making it natural to let efficiency of production be measured in terms of cost in those models. On the other hand, our model of perfect competition is nonlinear in cost but linear in revenue. Therefore, measuring (market) production efficiency in terms of revenue makes more sense within our framework.

The APC can be written in the equivalent form:

$$\sum_{i\in\mathcal{X}} p_\tau q_\tau^i + \sum_{i\in\mathcal{X}} p_t q_t^i \geq \sum_{i\in\mathcal{X}} p_\tau q_t^i + \sum_{i\in\mathcal{X}} p_t q_\tau^i, \tag{4}$$

for all $\mathcal{X} \in 2^{\mathcal{I}}$ and all $t, \tau \in \mathcal{T}$. The left-hand side of this inequality captures the actual sum of revenues for any subset of firms in any pair of observations $t, \tau \in \mathcal{T}$, while the right-hand side are revenues evaluated at the counterfactual prices for the same subset of firms. Define the vector $\mathbf{e}^i = (e_1^i, ..., e_T^i)$ for every firm $i \in \mathcal{I}$, where $e_t^i \geq 1$, and multiply each element with the corresponding revenue in the APC:

$$\sum_{i\in\mathcal{X}} e_\tau^i p_\tau q_\tau^i + \sum_{i\in\mathcal{X}} e_t^i p_t q_t^i \geq \sum_{i\in\mathcal{X}} p_\tau q_t^i + \sum_{i\in\mathcal{X}} p_t q_\tau^i. \tag{5}$$

When $e_t^i = 1$ for every $t \in \mathcal{T}$ and $i \in \mathcal{I}$, we obtain the standard form of the APC. If $e_t^i > 1$ then $100 \times (e_t^i - 1)$ is a percentage measure of the required increase in revenue of firm $i \in \mathcal{I}$ at observation $t \in \mathcal{T}$ for APC to hold. Given this, we can interpret e_t^i as an (in)efficiency index for firm $i \in \mathcal{I}$ at observation $t \in \mathcal{T}$ and the vector $\mathbf{e}^i = (e_1^i, ..., e_T^i)$ as an (in)efficiency vector for firm $i \in \mathcal{I}$. Let $\mathbf{e} = (\mathbf{e}^1, ..., \mathbf{e}^I)$ be the concatenated $T \times I-$dimensional vector of the efficiency vectors for all firms in the market. As a measure of goodness-of-fit, we propose the vector $\mathbf{e}$ closest to the unit vector in some norm, i.e., $\Omega(\mathbf{e}) = \inf_{\mathbf{e}\geq\mathbf{1}} \|\mathbf{e} - \mathbf{1}\|$, subject to the inequalities (5). Since the data $\mathcal{O} = (p_t, q_t^i)_{t\in\mathcal{T}}^{i\in\mathcal{I}}$ satisfies the APC whenever $\mathbf{e} = \mathbf{1}$, it holds that $\mathcal{O}$ satisfies the APC if $\Omega = 0$.

Although $\|\cdot\|$ may be chosen in many different ways, a natural choice is the Minkowski norm:

$$\Omega_\rho^M(\mathbf{e}) = \left(\sum_{i\in\mathcal{I}} \sum_{t\in\mathcal{T}} \frac{(e_t^i - 1)^\rho}{T \times I}\right)^{1/\rho},$$

where $\rho \geq 1$. By the properties of the Minkowski norm, we have $\Omega_a^M \leq \Omega_b^M$ for any $a \leq b$. We define the APC deviation index of order ρ (APCDI$_\rho$) as the set of efficiency values closest to the unit vector in the Minkowski norm such that condition (5) holds.

Definition 7 *The APC deviation index of order ρ (APCDI$_\rho$) is defined as the set of efficiency values* $\mathbf{e}$ *solving:*

$$\inf_{\mathbf{e}\geq\mathbf{1}} \left\{ \Omega_\rho^M(\mathbf{e}) \mid \sum_{i\in\mathcal{X}} e_\tau^i p_\tau q_\tau^i + \sum_{i\in\mathcal{X}} e_t^i p_t q_t^i \geq \sum_{i\in\mathcal{X}} p_\tau q_t^i + \sum_{i\in\mathcal{X}} p_t q_\tau^i \right\}.$$

In our empirical analysis, we report results for APCDI$_1$ and APCDI$_\infty$,

$$\inf_{\mathbf{e}\geq\mathbf{1}} \Omega_1^M(\mathbf{e}) = \inf_{\mathbf{e}\geq\mathbf{1}} \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}} \frac{(e_t^i - 1)}{T\times I},$$
$$\inf_{\mathbf{e}\geq\mathbf{1}} \Omega_\infty^M(\mathbf{e}) = \inf_{\mathbf{e}\geq\mathbf{1}} \max_{i\in\mathcal{I}}\max_{t\in\mathcal{T}} \left\{e_t^i - 1\right\},$$

for two reasons. First, together they bound the APCDI. Second, calculating APCDI$_1$ and APCDI$_\infty$ is an easy task in practice. In particular, calculating APCDI$_1$ is a linear problem which can be achieved using standard linear programming techniques. APCDI$_\infty$ can also be calculated using linear programming techniques by noticing that APCDI$_\infty$ can be equivalently defined as:

$$\inf_{e\geq 1}\left\{e \mid \sum_{i\in\mathcal{X}} e p_\tau q_\tau^i + \sum_{i\in\mathcal{X}} e p_t q_t^i \geq \sum_{i\in\mathcal{X}} p_\tau q_t^i + \sum_{i\in\mathcal{X}} p_t q_\tau^i\right\}. \tag{6}$$

Thus, APCDI$_\infty$ can be calculated by solving the following linear program (since $\sum_{i\in\mathcal{X}} p_\tau q_\tau^i + \sum_{i\in\mathcal{X}} p_t q_t^i > 0$):

$$\min_{e\geq 1}\left\{e \mid \frac{\sum_{i\in\mathcal{X}}\left(p_\tau q_t^i + p_t q_\tau^i\right)}{\sum_{i\in\mathcal{X}}\left(p_\tau q_\tau^i + p_t q_t^i\right)} \leq e\right\}. \tag{7}$$

Hence, since both APCDI$_1$ and APCDI$_\infty$ can be calculated using linear programming techniques, they are both efficiently solvable in polynomial execution time.

APCDI$_\rho$ can be easily generalized to give analogous measures of goodness-of-fit for APC-VC and APC-VCDP. For example, in the case of APC-VC, the analogous expression of (5) is given by:

$$\sum_{i\in\mathcal{X}} e_\tau^i p_\tau q_\tau^i + \sum_{i\in\mathcal{X}} e_\tau^i p_t q_t^i \geq \sum_{i\in\mathcal{X}} p_\tau q_t^i + \sum_{i\in\mathcal{X}} p_t q_\tau^i + \sum_{i\in\mathcal{X}} \left(p_\tau - p_t\right)\left(g_\tau^i - g_t^i\right),$$

for all $\mathcal{X} \in 2^{\mathcal{I}}$ and all $t, \tau \in \mathcal{T}$. The only difference between this condition and (5) is the right-hand side which now contains the additive term $\sum_{i\in\mathcal{X}} \left(p_\tau - p_t\right)\left(g_\tau^i - g_t^i\right)$. The APC-VC deviation index of order ρ is then defined as the set of efficiency values $\mathbf{e}$ solving:

$$\inf_{\mathbf{e}\geq\mathbf{1}}\left\{\Omega_\rho^M(\mathbf{e}) \mid \sum_{i\in\mathcal{X}} e_\tau^i p_\tau q_\tau^i + \sum_{i\in\mathcal{X}} e_\tau^i p_t q_t^i \geq \sum_{i\in\mathcal{X}} p_\tau q_t^i + \sum_{i\in\mathcal{X}} p_t q_\tau^i + \sum_{i\in\mathcal{X}} \left(p_\tau - p_t\right)\left(g_\tau^i - g_t^i\right)\right\}.$$

3.2 Measurement errors

In this section we consider the situation when measurement errors in the observed quantity data are causing violations of the APC. Following Varian (1985), we assume that the observed firm behavior $(q_t^i)_{t\in\mathcal{T}}^{i\in\mathcal{I}}$ and the "true" (unobserved) firm behavior $\left(\overline{q}_t^i\right)_{t\in\mathcal{T}}^{i\in\mathcal{I}}$ is connected through the Berkson multiplicative measurement error model:

$$\overline{q}_t^i = q_t^i \varepsilon_t^i, \tag{8}$$

where $(\varepsilon_t^i)_{t\in\mathcal{T}}^{i\in\mathcal{I}}$ are unobserved and random measurement errors.

The purpose is to test the null hypothesis that the "true data" $\overline{\mathcal{O}} = \left(p_t, \overline{q}_t^i\right)_{t\in\mathcal{T}}^{i\in\mathcal{I}}$ is PC-rationalizable against the alternative that $\overline{\mathcal{O}}$ is not PC-rationalizable. Thus, if the observed data $\mathcal{O} = (p_t, q_t^i)_{t\in\mathcal{T}}^{i\in\mathcal{I}}$ violates the APC but the null cannot be rejected then violations are spurious due to measurement errors, and in this case, the "true data" $\overline{\mathcal{O}}$ satisfies APC. In contrast, if the null is rejected then there are systematic violations of APC, in which case $\overline{\mathcal{O}}$ violates APC. Suppose that the errors $(\varepsilon_t^i)_{t\in\mathcal{T}}^{i\in\mathcal{I}}$ are independent normal random variables with unit mean and variance σ^2. Hence, $(\varepsilon_t^i - 1)/\sigma$ is standard normally distributed, implying that:

$$\frac{1}{\sigma^2}M_\varepsilon = \frac{1}{\sigma^2}\sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}}\left(\varepsilon_t^i - 1\right)^2 = \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}}\left(\frac{\varepsilon_t^i - 1}{\sigma}\right)^2,$$

is distributed chi-square with $T \times I$ degrees of freedom where $M_\varepsilon = \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}}\left(\varepsilon_t^i - 1\right)^2$. Under the null, $M_\varepsilon/\sigma^2 \leq c_\alpha$, where c_α is the chi-square critical value at nominal significance level α, meaning that the null is rejected if $M_\varepsilon/\sigma^2 > c_\alpha$.

In practice, such inference is infeasible since the errors $(\varepsilon_t^i)_{t\in\mathcal{T}}^{i\in\mathcal{I}}$ are unobserved. To obtain a feasible test-procedure, consider the following quadratic problem to calculate the minimal perturbation of the observed quantity data $(q_t^i)_{t\in\mathcal{T}}^{i\in\mathcal{I}}$ such that these data satisfy the APC:

$$\min_{\epsilon\geq\mathbf{0}}\left\{\sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}}\left(\epsilon_t^i - 1\right)^2 \mid \sum_{i\in\mathcal{X}}\left(p_\tau - p_t\right)\left(q_\tau^i\epsilon_\tau^i - q_t^i\epsilon_t^i\right) \geq 0\right\}.$$

Let $\widehat{M_\epsilon} = \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}}\left(\widehat{\epsilon}_t^i - 1\right)^2$ denote the optimal solution to this program. Since this problem solves for the minimal errors (in the quadratic norm) such that the data satisfies APC, we have $\widehat{M_\epsilon} \leq M_\varepsilon$, Thus, under the null, $\widehat{M_\epsilon}/\sigma^2 \leq M_\varepsilon/\sigma^2 \leq c_\alpha$. For a given variance σ^2 the null is therefore rejected whenever $\widehat{M_\epsilon}/\sigma^2 > c_\alpha$. This procedure can be used to test the null assuming that the variance of the "true errors" $(\varepsilon_t^i)_{t\in\mathcal{T}}^{i\in\mathcal{I}}$ is known, and in such case, the procedure will have at least the desired nominal size.

If we don't have any prior knowledge of σ^2 then we can rephrase the statistical decision rule as: for what values of σ^2 can the null be rejected? Notice that the decision rule to reject the null can be equivalently stated as as $\sigma^2 < \widehat{M_\epsilon}/c_\alpha$. Define the "bound statistic": $\overline{\sigma}^2 = \widehat{M_\epsilon}/c_\alpha$, which serves as a measure of the smallest σ^2 for which the null would be rejected. In other words, if $\overline{\sigma}^2$ is smaller then ones prior belief concerning the largest possible (allowable) σ^2, we may well want to accept that the "true data" $\overline{\mathcal{O}} = \left(p_t, \overline{q}_t^i\right)_{t\in\mathcal{T}}^{i\in\mathcal{I}}$ is PC-rationalizable.

4 Empirical application

4.1 The Nordic wholesale electricity market

The Nordic wholesale electricity market covers Denmark, Finland, Norway, Sweden, and all the three Baltic countries, with a total population of approximately 33.5 million people. It is partitioned into 16 bidding zones, constituting five zones in Norway, four in Sweden, two in Denmark, one in Finland, and one for each of the three Baltic countries. Figure

Figure 2: Bidding zones in the Nordic-Baltic wholesale electricity market

2 illustrates this bidding zone partition. Most of the electricity produced in the region is sold on the day-ahead market of the Nord Pool power exchange.[3] Nord Pool is coupled with the other power exchanges in continental Europe, and all are cleared simultaneously using the *Euphemia* pricing algorithm.

Every day before noon, generation owners submit price-dependent offers of how much electricity they want to sell in each of the 24 hours the following day. Specifically, they submit individual offers for each of the bidding zones in which they own generation capacity. Simultaneously, electricity retailers and large industrial consumers place price-dependent bids for how much electricity they are willing to purchase every hour the following day in each zone in which they own consumption capacity. Producers (consumers) are not allowed to participate in bidding zones in which they do not own generation (consumption) capacity. Network owners submit the available transmission capacities between the different bidding zones. These capacity bids determine the maximal amount of trade between zones.

All hourly sales bids across all bidding zones are aggregated to create one separate supply curve for the entire Nordic market for every single hour the following day. A corresponding demand curve is constructed on the basis of aggregating all individual

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[3] Nord Pool was the single power exchange in the Nordic-Baltic market until June 2021, when EPEX-Spot started operations, but EPEX is still a fringe power exchange in the market.

demand bids. The hourly *system price* is found at the level at which aggregate supply equals aggregate demand. This is also the equilibrium price of the day-ahead market that hour if there are no transmission bottlenecks in the system. This clearing procedure generates 24 system prices every day, one for every hour.

Bottlenecks sometimes arise when the transmission network does not have sufficient available capacity to manage all the electricity flows needed to balance aggregate supply and demand at the system price. The bidding zones are defined to reflect the possibility of such transmission constraints. In the event of binding transmission constraints, the electricity price is decreased in export-constrained zones and increased in import-constrained zones until the point at which local electricity prices are such that total demand equals total supply, but the associated flow of electricity across zones has been adjusted to the available capacity of the transmission network. By way of this clearing procedure, the Nordic day-ahead market can have as many as 15 equilibrium prices for every hour. Importantly, all production (consumption) cleared in the day-ahead market receives (pays) the zonal price in the bidding zone in which the production (consumption) is located. Under this market design, consumer expenditures are larger than producer revenue if there are bottlenecks in the system. The difference between consumer expenditures and producer revenue goes to the owners of the transmission network as a *congestion rent*.

4.2 Data and setup

Nord Pool supplies data on the hourly market-clearing prices for each bidding zone as well as the system price. Nord Pool also reports the hourly import and export capacities for each bidding zone and the equilibrium volume of electricity purchased and sold within each bidding zone for every hour. Thus, Nord Pool provides quantity data aggregated to the bidding zone level, but not firm level data. Nord Pool also publishes data on expected hourly wind power production for Denmark, Sweden and the Baltic countries aggregated to the bidding zone level. Wind power data from Sweden are available as of 2016.

We evaluate competition in the four Swedish bidding zones that were introduced in 2012, indicated by SE1 to SE4 in Figure 2. The full data sample contains hourly price and aggregated quantity data from January 1, 2012 until December 31, 2022 (24 price-quantity observations for each day). Omitting days with incomplete data leaves us with data for 3282 days (In total 78768 ($= 24 \times 3282$) production observations). Using the wind power data leaves us with 2779 days in the sample (In total 66696 ($= 24 \times 2779$) production observations).

For each of the four bidding zones and each trading day we calculate the APC and APC-VC deviation indices for $\rho = 1$ and $\rho = \infty$. Thus, we calculate the APC deviation index separately for each of the 3282 days, and the APC-VC deviation index separately for each of the 2779 days. Note that since we are using aggregated quantity data at the bidding zone level, a violation of APC(-VC) at the aggregated level is sufficient to reject APC(-VC) since it must hold for all subsets of firms for the data to be PC-rationalizable. In other words, the APC and APC-VC deviation indices calculated using aggregated quantity data are lower bounds on the "true" APC and APC-VC deviation indices.

The fundamental assumption underlying the different axioms of perfect competition is that the cost function for every firm and bidding zone is invariant across all periods within a given sample. In the Swedish electricity market, the portfolio of non-intermittent

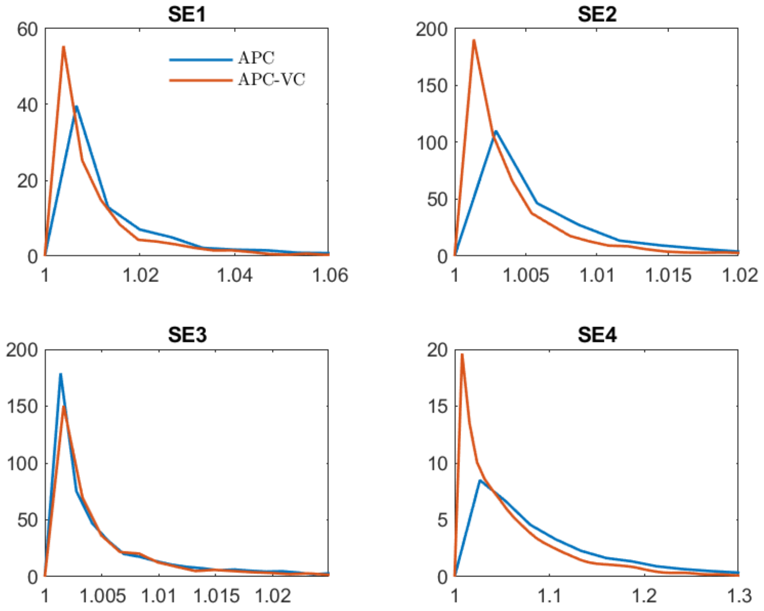

Figure 3: Distribution of goodness-of-fit for APC and APC-VC with $\rho = 1$

production capacity may comprise hydro power, nuclear power, combined heat and power or fossil-fuel thermal power, depending on the specific firm, time of year or bidding zone. Fuel input prices and hydro reservoir inflows change at most on a daily basis. Furthermore, all electricity bids or offers for each of the 24 delivery hours the following day are submitted simultaneously into the day-ahead market at noon the previous day. By implication, each firm places all its 24 bid or offer curves for the following day based on the same information set. These properties imply that the cost function for each individual firm and bidding zone remains invariant over the sample period since we set every sample period equal to the 24 delivery hours within the same day.

4.3 Results

Figure 3 plot the distributions of the APC and APC-VC deviation indices for each bidding zone using $\rho = 1$, while Figure 4 plot the same distributions for $\rho = \infty$. As seen from both figures, the goodness-of-fit is almost uniformly better for APC-VC than APC. Thus, a model that also accounts for hourly differences in the wind power prognosis provides a better fit to the data than a model that only accounts for hourly differences in production. This is clear for all biding zones except possibly for SE3 where the distributions of the APC and APC-VC deviation indices are almost tangent.

Table 1 gives summary statistics of the distributions in Figures 3 and 4. The table reports the mean, minimum (min), 25th percentile (P25), median, 75th percentile (P75) and maximum (max) of every distribution. Recall that numbers closer to 1 indicate better goodness-of-fit, and that this measure is monotonic so that numbers further away from 1 indicate a worse fit. The goodness-of-fit are, in our opinion, generally low except possibly for SE4. Looking, for instance, at the mean fit using $\rho = 1$ for APC-VC in bidding zone

Table 1: Summary statistics over the distribution of goodness-of-fit for APC and APC-VC evaluated at $\rho = 1$ and $\rho = \infty$. P25 and P75 denotes the 25th and 75th percentiles, respectively.

	Bidding	mean		min		P25		median		P75		max	
Axiom	zone	$\rho = 1$	$\rho = \infty$	$\rho = 1$	$\rho = \infty$	$\rho = 1$	$\rho = \infty$	$\rho = 1$	$\rho = \infty$	$\rho = 1$	$\rho = \infty$	$\rho = 1$	$\rho = \infty$
APC	SE1	1.002	1.013	1.000	1.000	1.000	1.001	1.001	1.004	1.002	1.010	1.244	1.657
APC-VC	SE1	1.002	1.010	1.000	1.000	1.000	1.001	1.000	1.003	1.001	1.009	1.084	1.388
APC	SE2	1.002	1.008	1.000	1.000	1.000	1.001	1.001	1.003	1.002	1.007	1.073	1.284
APC-VC	SE2	1.001	1.004	1.000	1.000	1.000	1.001	1.000	1.002	1.001	1.005	1.033	1.132
APC	SE3	1.002	1.006	1.000	1.000	1.000	1.000	1.000	1.002	1.001	1.005	1.076	1.136
APC-VC	SE3	1.000	1.005	1.000	1.000	1.000	1.001	1.001	1.002	1.001	1.005	1.063	1.162
APC	SE4	1.035	1.109	1.000	1.001	1.008	1.028	1.018	1.063	1.038	1.127	1.940	3.596
APC-VC	SE4	1.021	1.066	1.000	1.000	1.005	1.020	1.012	1.045	1.026	1.088	1.377	1.750

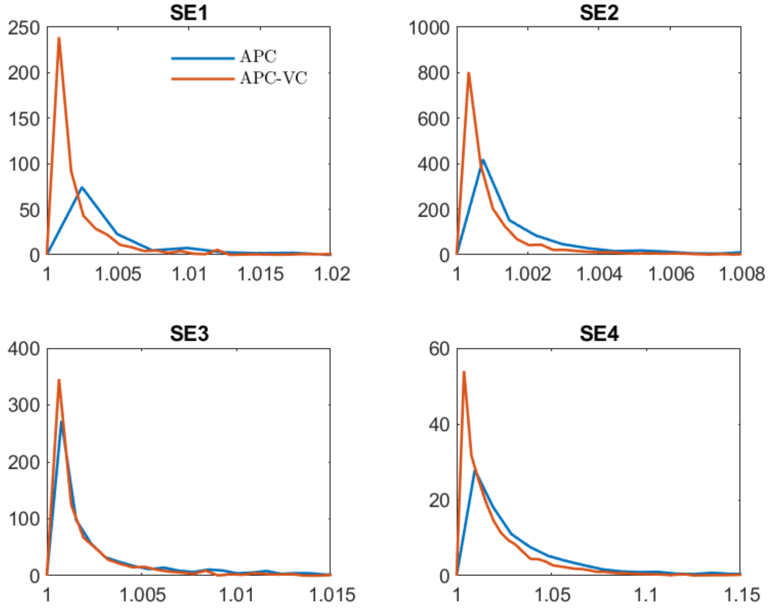

Figure 4: Distribution of goodness-of-fit for APC and APC-VC with $\rho = \infty$

SE2 shows that the aggregated revenue only need to increase by 0.1 percentage points for the market to be perfectly competitive (Using $\rho = \infty$ shows that the revenue need to increase by 0.4 percentage points). This generally holds across all summary statistics.

Hence, although we are effectively only reporting a lower bound of the "true" goodness-of-fit, we believe that the data are close enough to being rationalized by perfect competition for a large number of days in the sample period 2012-22. As such, violations of APC-VC (and to a large extent also APC) could plausibly be the result of noise. These results are reassuring in the sense that SE1-SE3 have structural properties that characterize markets with competitive pressure, such as excess production, a small degree of local market concentration and a large degree of market integration.

The summary statistics of goodness-of-fit for the southernmost bidding zone SE4 are substantially larger in order of magnitude compared to those for SE1-SE3. But in contrast to the other bidding zones, SE4 has characteristics associated with weak competitive pressure, such as excess demand, large degree of local market concentration and small degree of market integration. Of course, the violations of APC-VC could be attributed to noise also in SE4, but it is unclear a priory why such noise should be systematically larger in magnitude in SE4 than in the other three Swedish bidding zones. Thus, the violations of APC-VC in SE4 could well result from imperfect competition. Additional analysis into market performance in SE4 seems warranted.

5 Conclusion

This paper has developed nonparametric tests for perfect competition in markets for homogeneous goods. The tests are based only on price and aggregate quantity data and are easy to implement (No data on firm costs are required). The tests rely on minimal assumptions about demand and cost functions. Such flexibility is particularly important in industries where cost functions are highly non-convex, for instance because of unit startup costs and ramping constraints. The proposed tests can be easily implemented to evaluate competition in specific industries. Failure of industry data to meet the tests should warrant a deeper investigation, for instance by competition authorities or other agencies with legal authority to collect data at such detailed level that is required to conduct more sophisticated analyses of market performance in concentrated industries.

Finally, we briefly mention a few extensions of the models and methods. First, it would be interesting to develop a stochastic framework to implement and test the models. Such a framework could potentially be based on the moment inequality based tests in econometrics. Another possible way to incorporate stochastic elements in the analysis would be to combine parametric production analysis with the methods proposed in this paper to increase the power of the tests.

The second extension concerns recoverability. Given that a data set satisfies APC, can we recover or bound the cost functions of the firms? Can we recover other types of underlying technological constraints from observed market behavior? The third issue concerns extrapolation. Given observed market behavior in some economic environments how can we forecast behavior in other environments? To which extent is counterfactual analysis feasible within our framework?

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A Proofs

A.1 Proof of Theorem 1

The proof 3⇒1 is trivial. We established 1⇒2 in Subsection 2.1.

2⇒3. Our first step is to construct a piecewise linear marginal cost function that is positive, strictly increasing, and continuous almost everywhere. Rank $\mathcal{T}$ in increasing price order, so that $p_t < p_{t+1}$ for all $t \in \{1, ...T-1\}$. By (ii) and the rank-order property of $\mathcal{T}$, $q_t^i \leq q_{t+1}^i$ for all $t \in \{1, ...T-1\}$. The subset $\mathcal{T}^i = \{t \in \mathcal{T} : q_t^i > 0\}$ represents all periods in which firm i produces positive output. Note that $\mathcal{T}^i = \emptyset$ if $q_T^i = 0$. If $q_T^i > 0$, then we let p_{t^i} be the smallest price in the sample for which firm i produces positive output: $q_{t^i}^i > 0$, and $q_t^i = 0$ for all $t \in \{1, ..., t^i - 1\}$ if $t^i \geq 2$. Hence, $\mathcal{T}^i = \{t^i, ..., T\}$ if $\mathcal{T}^i \neq \emptyset$. Let $MC^i : [0, b^i] \to \mathbb{R}_+$ be the marginal cost function of firm i. If $\mathcal{T}^i = \emptyset$, then we set $b^i > 0$ and let $MC^i(y) = p_T + y$. If $\mathcal{T}^i \neq \emptyset$, then we set firm i's capacity to $b^i > q_T^i$.

The marginal cost function is defined as follows. First, $MC^i(y) = p_1 \frac{y}{q^i_1}$ for all $y \in [0, q^i_1]$ if $q^i_1 > 0$, whereas $MC^i(y) = p_{t^i-1} + (p_{t^i} - p_{t^i-1}) \frac{y}{q^i_{t^i}}$ for all $y \in [0, q^i_{t^i}]$ if $q^i_1 = 0$. For large quantities, $MC^i(y) = p_T \frac{y}{q^i_T}$ for all $y \in (q^i_T, b^i]$. For $t \geq t^i$, $MC^i(y) = p_t \frac{q^i_{t+1}-y}{q^i_{t+1}-q^i_t} + p_{t+1} \frac{y-q^i_t}{q^i_{t+1}-q^i_t}$ for all $y \in (q^i_t, q^i_{t+1}]$ such that $q^i_t < q^i_{t+1}$. This marginal cost function is left-continuous, whereas right-discontinuity possibly occurs for all quantities for which $q^i_t = q^i_{t+1}$. For all such observations, $\lim_{y \to q^{i+}_{t+1}} MC^i(y) \geq p_{t+1} > p_t = MC^i(q^i_t)$ by construction. Hence, $MC^i(y)$ is positive for all $y > 0$, strictly increasing in the domain $[0, k^i]$ and continuous almost everywhere. Every non-decreasing function defined on an interval is integrable. For each firm i, we can then define a cost function

$$C^i(y) = \int_0^y MC^i(\tilde{y}) d\tilde{y} \tag{9}$$

that has the properties specified in (iii) of the theorem. We finally need to verify that the chosen function implements the observed quantities given the observed prices. Firm i maximizes $\Pi^i_t(y) = p_t y - C^i(y)$ over $y \in [0, b^i]$. Let $\mathcal{T}_i \neq \emptyset$, and assume that $t \geq t_i$. In this case,

$$\begin{aligned}
\Pi^i_t(q^i_t) - \Pi^i_t(y) &= p_t q^i_t - C^i(q^i_t) - p_t y + C^i(y) \\
&= \int_{q^i_t}^{y} (MC^i(\tilde{y}) - p_t) d\tilde{y} = \int_{q^i_t}^{y} (MC^i(\tilde{y}) - MC^i(q^i_t)) d\tilde{y} > 0
\end{aligned}$$

for all $y \neq q^i_t$. Therefore, firm i's optimal choice is to set $y = q^i_t$ for all $t \in \{t_i, ..., T\}$. The proof for $\mathcal{T}_i \neq \emptyset$ is done if $t_i = 1$. Assume next that $t_i \geq 2$. In this case,

$$\Pi^i_t(0) - \Pi^i_t(y) = \int_0^y (MC^i(\tilde{y}) - p_{t_i-1} + p_{t_i-1} - p_t) d\tilde{y} > 0$$

for all $t \in \{1, ..., t_i - 1\}$ and $y > 0$. Therefore firm i's optimal choice is to set $y = q^i_t = 0$ for all $t \in \{1, ..., t_i - 1\}$, $t_i \geq 2$. If $\mathcal{T}_i = \emptyset$, then

$$\Pi^i_t(0) - \Pi^i_t(y) = \int_0^y (MC^i(\tilde{y}) - p_t) d\tilde{y} = \int_0^y (\tilde{y} + p_T - p_t) d\tilde{y} > 0$$

for all $t \in \mathcal{T}$ and $y > 0$. Therefore, firm i's optimal choice is to set $y = q^i_t = 0$ for all $t \in \{1, ..., T\}$ if $\mathcal{T}_i = \emptyset$. To summarize, q^i_t is the unique optimum for all $i \in \mathcal{I}$ and for all $t \in \mathcal{T}$ under the specified cost function for firm i.

A.2 Proof of Proposition 1

1⇒2. Suppose that the generic data set $\mathcal{O} = (p_t, q^i_t)^{i \in \mathcal{I}}_{t \in \mathcal{T}}$ is PC-rationalizable. By Theorem 1, we know that this implies that APC holds. Since the cost function for firm $i \in I$ is C^1, the marginal cost is unique (at every point), so clearly since $\mathcal{O}$ is generic we have $p_t \neq p_\tau$ for all $t, \tau \in \mathcal{T}$ with $t \neq \tau$, which then implies $q_t(\mathcal{X}) \neq q_\tau(\mathcal{X})$ for all $\mathcal{X} \in 2^{\mathcal{I}}$ and $t, \tau \in \mathcal{T}$ with $t \neq \tau$, which is SAPC.

2⇒1. Since APC holds we can construct the function MC as in the proof of Theorem 1. This function can be regularized by convolution to obtain a $\mathcal{C}^2$ function (See e.g., Chiappori and Rochet 1987). In particular, define the function:

$$\begin{aligned}
k(A) &= \frac{\exp\left(-\frac{1}{|A|-1}\right)}{\int_{\mathbb{R}} \exp\left(-\frac{1}{|A|-1}\right)} \text{ if } |A| < 1, \\
k(A) &= 0 \text{ if } |A| \geq 1.
\end{aligned}$$

Let $k_\eta(A) = k\left(\frac{A}{\eta}\right)/\eta$ for some $\eta > 0$. Then k_η is symmetric differentiable and zero for any value $|A| \geq \eta$. Let:

$$\overline{MC}^i(y) = \int_{\mathbb{R}} MC_i(y - \varepsilon)\, g_\eta(\varepsilon)\, d\varepsilon.$$

This function is positive for all $y > 0$, non-decreasing in the domain $[0, b^i]$, and $\mathcal{C}^1$. As in the proof of Theorem 1, for every firm i, we define the cost function:

$$C^i(y) = \int_0^y \overline{MC}^i(\tilde{y})d\tilde{y}.$$

Thus, the function $C^i(y)$ is non-decreasing and $\mathcal{C}^2$. PC-rationalizability then follows as in the proof of Theorem 1.

A.3 Proof of Theorem 2

The proof 3⇒1 is trivial. We established 1⇒2 in Subsection 2.3.

2⇒3. We can replicate the same steps as in the proof of Theorem 1 by which we first rank $\mathcal{T}$ in increasing price order and then construct for each firm i a positive, strictly increasing, piecewise linear and left-continuous marginal cost function $MC^i(\cdot)$ of black output that is right-continuous almost everywhere. This marginal cost function has the property $MC^i(q_t^i - g_t^i) = p_t$ for all $t \in \mathcal{T}^i = \{t^i, ..., T\}$, i.e. p_t for which $q_t^i > g_t^i$, and $MC^i(0) = p_{t^i-1}$ for all $t \in \underline{\mathcal{T}}^i = \{1, ..., t^i - 1\}$, i.e. p_t for which $q_t^i = g_t^i$. From $a_t \geq 0$ and the zero-cost assumption of green production, it follows that optimal green production equals g_t^i in every period because the firm's cost function is strictly increasing. It remains to check that q_t^i maximizes $\Pi_t^i(y) = p_t y - C_i(y - g_t^i)$ for all $y \in [g_t^i, g_t^i + b^i]$ and all $(i, t) \in \mathcal{I} \times \mathcal{T}$. If $t \in \mathcal{T}^i$ so that $q_t^i > g_t^i$, then

$$\begin{aligned}
\Pi_t^i(q_t^i) - \Pi_t^i(y) &= \int_{q_t^i}^y (MC^i(\tilde{y} - g_t^i) - p_t)d\tilde{y} \\
&= \int_{q_t^i}^y (MC^i(\tilde{y} - g_t^i) - MC^i(q_t^i - g_t^i))d\tilde{y} > 0
\end{aligned}$$

for all $y \neq q_t^i$. Hence, q_t^i is the unique optimum in this case. If $t \in \underline{\mathcal{T}}^i$ so that $q_t^i = g_t^i$, then

$$\begin{aligned}\Pi_t^i(g_t^i) - \Pi_t^i(y) &= \int_{g_t^i}^{y} (MC^i(\tilde{y} - g_t^i) - p_t) d\tilde{y} \\ &= \int_{g_t^i}^{y} (MC^i(\tilde{y} - g_t^i) - MC^i(0) + p_{t^i-1} - p_t) d\tilde{y} > 0\end{aligned}$$

for all $y > g_t^i$.

A.4 Proof of Theorem 3

The proof 3⇒1 is trivial. We established 1⇒2 in Subsection 2.3.

2⇒3. We can replicate the same steps as in the proof of Theorem 2 by which we first rank $\mathcal{T}$ in increasing price order and then construct for each firm i a positive, strictly increasing, piecewise linear and left-continuous marginal cost function $MC^i(\cdot)$ of black output that is right-continuous almost everywhere. This marginal cost function has the property $MC^i(q_t^i - q_t^i(h) - g_t^i) = p_t$ for all $t \in \mathcal{T}^i$, i.e. p_t for which $q_t^i > g_t^i + q_t^i(h)$, and $MC^i(0) = p_{t^i-1}$ for all $t \in \underline{\mathcal{T}}^i$, i.e. p_t for which $q_t^i = g_t^i + q_t^i(h)$. From $a_t \geq 0$ and the zero-cost assumption of green production, it follows that optimal green production equals g_t^i in every period because the firm's cost function is strictly increasing. It remains to verify that $(q_t^i, q_t^i(h))_{t\in\mathcal{T}}$ maximizes

$$\Pi^i((y_t, y_t(h))_{t\in\mathcal{T}}) = \sum_{t=1}^{T} [p_t y_t - C^i(y_t - y_t(h) - g_t^i)]$$

for all $i \in \mathcal{I}$ subject to the production constraints of the respective technologies. After some manipulation of expressions we can write

$$\begin{aligned}\Pi^i(\left(q_t^i, q_t^i(h)\right)_{t\in\mathcal{T}}) - \Pi^i((y_t, y_t(h))_{t\in\mathcal{T}}) = &\sum_{t\in\mathcal{T}^i} \int_{q_t^i - q_t^i(h)}^{y_t - y_t(h)} [MC^i(\tilde{y} - g_t^i) - MC^i(q_t^i - q_t^i(h) - g_t^i)] d\tilde{y} \\ &+ \sum_{t\in\underline{\mathcal{T}}^i} \int_{g_t^i}^{y_t - y_t(h)} [MC^i(\tilde{y} - g_t^i) - MC^i(0)] d\tilde{y} \\ &+ \sum_{t\in\underline{\mathcal{T}}^i} [p_{t^i-1} - p_t][y_t - y_t(h) - g_t^i]] \\ &+ \sum_{t\in\mathcal{T}} p_t(q_t^i(h) - y_t(h))\end{aligned}$$

The expressions on the first two rows are non-negative because the marginal cost function is increasing. The expression on the third row is non-negative because $p_t \leq p_{t^i-1}$ for all $t \in \underline{\mathcal{T}}^i$. This leaves the expression on the last row, which we can write as

$$\begin{aligned}\sum_{t\in\mathcal{T}} p_t(q_t^i(h) - y_t(h)) &= \sum_{t\in\mathcal{T}} (p_t - p_{t^i(h)})(q_t^i(h) - y_t(h)) + p_{t^i(h)} \sum_{t\in\mathcal{T}} (q_t^i(h) - y_t(h)) \\ &\geq \sum_{t\in T^i(h)\setminus t^i(h)} (p_t - p_{t^i(h)})(h^i - y_t(h)) + \sum_{t\in\underline{T}^i(h)} (p_{t^i(h)} - p_t) y_t(h) \geq 0\end{aligned}$$

for arbitrary $s \in \mathcal{T}$. The first inequality on the second row follows from the capacity and resource constraint $\sum_{t=1}^{T} q_t^i(h) = \min\{Tk_i; H_0^i - H_T^i\} \geq \sum_{t=1}^{T} y_t(h)$. This completes the proof.