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A Calibration Algorithm for Micro-simulation Models

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1. Introduction

Thanks to the improvements in the information technologies, large scale micro-simulation models have been increasingly used in the last decades for economic analysis and policy simulations. A major problem of such complex models is the fact that the model specification is very complex and includes a large number of unknown parameters. Therefore even the modellers may have difficult time to understand the "structure" of their models. Moreover, parameter values have to be usually guesstimated since it is usually not possible to get their econometric estimates due to the lack of necessary data. Thus, the "calibration" of large-scale micro simulation models is a problem that needs further research.

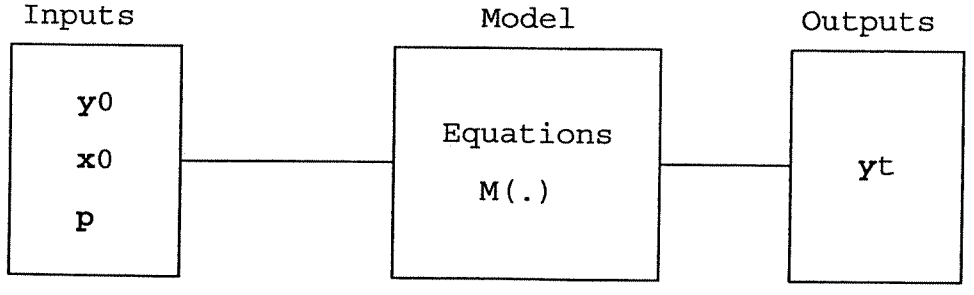
This paper presents an algorithm based on random-search techniques for the use of "calibration" of large-scale micro-simulation models. This algorithm is currently being used for the calibration of the micro-to-macro Model Of the Swedish Economic System, MOSES. The MOSES model focuses on the dynamic coordination of production and the adjustment of output to market price signals. The micro part of the model simulates the behavior of (initially) 225 firms or divisions. 154 of these firms are real firms/divisions that produce almost 30% of the Swedish industrial output. Data about real firms come from the Planning Surveys conducted by IUI and the Federation of Swedish Industries each year since 1976, and from firms' financial reports. The remaining 71 firms are synthetic: they make up the

difference between the micro and macro data.[1] Since the model is rather complex, the reader is referred to Eliasson (1977 and 1985), Bergholm (1989), Albrecht et al. (1989), Taymaz (1991) and Albrecht et al. (1992) for the details. The bibliography in Albrecht et al. (1992) also includes the list of studies based on the MOSES model.

2. Analyzing Model's Structure: Calibration, Sensitivity, Linearity

A micro-simulation model consists of a large number of non-linear equations that, given a set of parameters, convert a set of exogenous variables and the initial values of endogenous variables into series of endogoneous variables (see Figure 1).

Figure 1 Micro-simulation model



Notation: y_0: endogenous variables vector at the base year
x_0: exogenous variables vector
p: parameter vector
y_t: endogenous variables at time t
M(.): a set of model equations

The model, i.e., the set of equations M(.), is so complex that even those who have the complete model code cannot foresee

1) In another MOSES database, there are 504 firms of which 154 are real firms. The size of synthetic firms in this dataset is smaller that those mentioned above.

the impact of changes in initial conditions, $\{y_0, x_0, p\}$, on output, $\{y_t\}$, without simulating the model. The complexity of the model may also obscure understanding of the model, and its representation of economic relations. It is thus necessary to investigate in detail model's "structure" and properties.

There are three major concerns in the analysis of model's structure: calibration, sensitivity, and symmetry/linearity. Calibration is the method used in the "estimation" of parameter values in a predetermined set that "minimize" the difference between simulation results and some choosen control variables observed for the calibration period. In other words, calibration is used to determine the values of a set of parameters[2] that generate the best fit for series of endogenous variables.

Although there are a large number of parameters in a micro-simulation model, many parameters have little effect on model's endogenous variables that are considered as "important". It is important to determine "key" parameters that have significant effects on certain endogenous variables. The calibration algorith could also be used to determine those influential parameters.

Sensitivity analysis refers to the study of the effects of minor changes in initial conditions, $\{x_0, y_0, p\}$, and stochastic factors on simulation results. It is normally preferred to have a "robust" model which is not very sensitive to minor changes in initial conditions. Otherwise, it may be

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2) There is computationally no difference between the parameters of the model and the exogenous variables. Therefore, hereafter, we use the term "parameter" in a broad sense including all variables whose values are constant during a simulation experiment.

difficult to evaluate simulation results because initial variables inevitably have a noise component.[3]

Finally, symmetry and linearity characteristics of the model's endogenous variables may be important. For this purpose, for example, Zellner and Peck's (1973) relative symmetry and linearity tests can be applied (see also Kuh, Neese and Hollinger, 1985: 110-113). Zellner-Peck tests are used

> to determine the extent to which induced changes in the model's endogenous variables are symmetric and/or linear. Symmetry is of interest for its own sake while finding of linearity of near-linearity may be useful in efforts to simplify the model's structure. Also, since the effects of both relatively small and large changes are reported, we gain information on both the local and global properties of the model. (Zellner and Peck, 1973: 152, quoted by Kuh, Neese, and Hollinger, 1985: 110)

Although sensitivity and symmetry/linearity properties are also important, this paper deals only with the calibration problem.

3. A Calibration Algorithm

The model can be thought of as a system of equations containing an r-element parameter vector, $\mathbf{p}$, such that a vector of initial and exogenous variables, $\mathbf{z}$, produces a v-element vector of endogenous control variables $\mathbf{y}^s$ [4].

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3) As mentioned by Kuh, Neese and Hollinger (1985: 18, fn 2), "[i]f a particular model has low parameter sensitivity for endogenous variables of interest, then the Lucas critique -that parameters change in response to stochastic behavior of variables of concern to individual agents- will have less potential practical importance. Conversely, if significant policy parameters induce large responses, the potential importance of Lucas's observations will be all the greater. Even more to the point, parameters though to be endogenous should be treated that way from the outset."

4) Note that M(.) is a stochastic function of p due some stochastic variables in the model. Hence, the y^s is distributed by mean y^{smean} and standard deviation σ_y. If a short time period is used for the calibration procedure, σ_y can be assumed to be very zero since the stochastic variances in the model have significant effects only in the long run.

$$y^s = M(p;z)$$

Our purpose is to estimate the parameter vector, p, that makes the simulated control variables, y^s, as close to their real values, y^r, as possible. If the standard weighted sum-of-squared-errors criterion is used, then the objective function is

$$\min_{p \epsilon S} d(p) = \Sigma_{t \epsilon T} (y_t^s - y_t^r)' W (y_t^s - y_t^r) ,$$

where S is the predetermined parameter space, T the time period for calibration, and W a diagonal weighing matrix. It is assumed that the parameter set S contains the global minimum in its interior.

In addition to the weighted sum-of-squared-errors criterion, absolute and maximum distance criteria may be used. These criteria are defined as follows.

Maximum absolute distance, $d^a(p) = \Sigma_{t \epsilon T} |y_t^s - y_t^r|' w$

Maximum distance, $d^m(p) = \max \{ \max(|y_t^s - y_t^r|) ; t \epsilon T \}$

If the model could mimic exactly the real world, and the actual values of the exogenous variables could be same as those used in the model, than the minimum distance value, d^{min} would be equal to zero. Since this is not the case, $d^{min} > 0$.

Recall that there are many non-linear and non-smooth functions in the model. M(.) and, consequently, d(p) are functions of very complex systems of non-linear functions. It is very difficult, if possible, to search the minimum distance

value by conventional optimization methods (for conventional optimization methods, see Fletcher, 1987). The $d(\mathbf{p})$ function may have many local minima and it is typically a function of large number of parameters.[5] Thus the minimum of the $d(\mathbf{p})$ function can only be searched by random search methods since, as Brooks (1958) stated, the number of experiments in these methods does not depend on the number of parameters and they are not usually restricted by the nature of the distance surface.[6]

Finding the "minimum" of the distance function is only one aspect of the calibration process. The model needs to be robust in the neighborhood of the selected parameter vector $\mathbf{p}^c$, i.e., small variations in parameter values should not have significant effect on system propeties. In other words,

$$| d(\mathbf{p}) - d(\mathbf{p}+\delta) | \leq \epsilon \quad ,$$

where δ is an r-element vector of variations, and ϵ a small positive number.

Finally, the model's long-run properties generated by the parameter vector $\mathbf{p}^c$ should be "reasonable". Hence we seek to establish convergence to a small region surrounding (in some sense) the candidates for the global minimum. The optimality region then is defined by

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5) In the case of MOSES model, more than 60 parameters. But the number of parameters may be much higher than that number.

6) These methods were first proposed by Anderson (1953). For some recent studies, see Solis and Wets (1981), Boender et al. (1982), and references therein.

$S_o = \{ \mathbf{p} \mid \mathbf{p} \in S, d(\mathbf{p}) < d^{min} + \epsilon \}$

where ϵ is a small positive number.

In other words,

1. the parameter vector $\mathbf{p}^c$ does not need to be global minimum of the distance function but $d(\mathbf{p}^c)$ should be a local minimum "close" enough to d^{min};

2. the model should be robust at the neighborhood of $d(\mathbf{p}^c)$; and

3. the long-run properties of $\mathbf{p}^c$ should be satisfactory.

To determine the parameter vector $\mathbf{p}^c$ a simple two-stage calibration procedure can be designed as follows.

Stage 1. Global search

At this stage n number of parameter vectors are randomly drawn from the parameter space, S, and the distance values for each parameter vector are found by simulation. The number of experiments is determined by two criteria.

1. $n \geq \log(1-s)/\log(1-b)$, where b is a predetermined proportion of the parameter space that contains parameter combinations having lower distance values (S_b), and s is the probability of at least one experiment choosen from the parameter sub-space S_b. For example, if we want to get at least one parameter vector drawn from the lowest 5% of the parameter space with 99% probability, then

$n \geq \log(1-.99)/\log(1-.05) \approx 90.$

2. The cumulative distance distribution (CDD) of the sequence of experiments with randomly selected parameter

vectors will converge to the population distribution as $n \to \infty$. Thus n can be choosen so that additional experiments do not change the CDD to a significant extent.

By global search with n random parameter vectors, we can get information about

1. the global minimum,
2. the shape of the distance surface, and
3. influential parameters.

There are a number of techniques, generaly based on de Haan's (1981) analysis of order statistics, for determining the confidence interval for the global minimum. (For an application of this method, see Veall, 1990.) Suppose

$d^{min} = \min \{ d(\mathbf{p}) \mid \mathbf{p} \in S \}$ exists. Using a random sample of $\mathbf{p}^1, \mathbf{p}^2, \ldots \mathbf{p}^n$ from a uniform distribution over S, a confidence interval at the p significance level for d^{min} (under mild conditions on $d(.)$[7]) can be constructed as follows.

$$\{ d^m, Y^1 \} ,$$

where $d^m = [Y^1 - (Y^2 - Y^1)/p^{-1/\alpha} - 1]$, $Y^1, Y^2, \ldots, Y^n$ is the order statistic from $d(\mathbf{p}^1), d(\mathbf{p}^2), \ldots d(\mathbf{p}^n)$, that is, $Y^1 \leq Y^2 \leq \ldots \leq Y^n$, $\alpha = \log(k)/[\log(Y^k - Y^3)/(Y^3 - Y^2)]$, and k is any sequence of integer numbers depending on n such that $k(n) \to \infty$ and $k(n)/n \to 0$ as $n \to \infty$. Note that Y^1 tends to d^{min} almost surely as $n \to \infty$, and d^m is a monotone decreasing function of p such that $d^m \to -\infty$ as $p \to 1$.

This confidence interval can be used for testing the

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7) For the conditions on d(.), see de Haan (1981) and Boenden et al. (1982).

hypothesis that any given minimum is global. Since our purpose is only to find a parameter vector in the optimality region, the confidence interval can be used to assess the "closeness" of the distance value found at Stage 2 to the global minimum.[8]

The CDD obtained in this stage gives an idea about the shape of the distance surface. A relatively smooth curve at low values may show that the model is not sensitive to small variations in parameter values (see Figure 2).

Influential parameters can also be determined at this stage. Recall that parameter values are chosen randomly so that they are independently distributed across experiments. Ignoring joint effects, we can estimate the distance values as an approximate function of each parameter separately. For example, regression analysis may be employed by using linear, quadratic and cubic functions of parameters as explanatory variables. F-statistic of the regression may show the significance of that parameter.

Stage 2. Local search

Local search is based on a method of random "hill-climbing" from an initial point. Direction of the next move is determined randomly. If the distance value decreases at the randomly selected direction, parameter values are changed accordingly. Otherwise, a new random direction is selected. This process is repeated until no further improvement in the distance value is obtained in a predetermined number of trials.

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8) There are some techniques to estimate the value of global minimum. See, for example, Smith (1987).

The algorithm we use is rather simple and can be summarized as follows.

Step 0.

Pick $\mathbf{p}^0 \in S$, set k=nf=ns=0, fix μ_{min}, μ_{-1}, mf, ms, sr, si, con

k : iteration number
$\mathbf{p}^k$: parameter vector for k^{th} iteration
S : parameter space
nf : number of failures
ns : number of successes
μ_k : "diameter" of random search space (step size). $\mathbf{p}^k$ is the center point.
μ_{min} : minimum diameter
mf : number of failures before step size reduction
ms : number of successes before step size increase
sr : step size reduction factor
si : step size increase factor
con : convergence criterion

Step 1.

$$\text{Set } \mu_k = \begin{cases} \mu_{k-1}*sr \text{ if } nf \geq mf \\ \mu_{k-1}*si \text{ if } ns \geq ms \\ \mu_k \end{cases}$$

Stop if $\mu_k \leq \mu_{min}$. Set $\mathbf{p}^c = \mathbf{p}^k$.
Otherwise set $\mathbf{p}^{k+1} = ran(\mathbf{p}^k + \mu_k)$ where ran(.) is a random vector on the "circle" defined by its center $\mathbf{p}^k$ and diameter μ_k.

Step 2.

$$\text{Set } \mathbf{p}^{k+1} = \begin{cases} \mathbf{p}^{k+1} \text{ if } d(\mathbf{p}^{k+1}) < d(\mathbf{p}^k)/1+con \text{ , set ns=ns+1, k=k+1, nf=0} \\ \mathbf{p}^k \text{ otherwise , set ns=0, nf=nf+1, k=k+1} \end{cases}$$

Return to Step 1.

There are two alternatives for the initial parameter vector, $\mathbf{p}^0$: 1) the parameter vector found at Stage 1 that has the lowest distance value, and 2) the parameter vector that is

currently being used in the model. We prefer to use the second alternative because some of the parameters currently used may incorporate our a priori information about their values. Moreover, these parameters determined during the development of the model usually have "reasonable" long-run properties.

Finally the long-run properties of the selected vector, $\mathbf{p}^c$, should be tested. If it generates nonstable, chaotic results after the calibration period, we need to make another search. The sensitivity of the model is easy to check since at least nf number of experiments are done already in the neighborhood of $\mathbf{p}^c$ during the local search.

4. Calibration of the MOSES model

Before beginning the description of the calibration of the MOSES model, let's specify some notations. $\mathbf{p}^{max}$, $\mathbf{p}^{min}$ and $\mathbf{p}^0$ are the vectors of maximum, minimum, and initial parameter values, respectively.[9] $[\mathbf{p}^{max}, \mathbf{p}^{min}]$ defines the parameter space. μ_{-1} is the initial step size vector. There are 75 parameters to be calibrated. Rates of interest, annual growth rates of manufacturing output, prices and employment, and GDP growth were used as control variables. (For details, see Taymaz, 1991.)

To determine the cumulative distance distribution, 92 experiments were run by choosing parameters randomly within the range $[\mathbf{p}^{max}, \mathbf{p}^{min}]$. The CDDs for the first 46 and 92 experiments are shown in Figure 2. There is not any significant change in

9) Strictly speaking, some of the elements of these vectors are not parameters, but exogenous variables (the rate of change of the exogenous technological level, etc.). Since there are computationally no difference between parameters and those exogenous variables, we simply use the term "parameter" to define both.

the frequency distribution from 46 to 92 experiments. Note that at least one parameter vector of those 92 experiments is to be drawn from the lowest 5% of the parameter space with 99% probability. Figure 2 reveals that the landscape defined by the distance values has a relatively flat surface. The distance value is within the range of .00258-.0100 in almost 40% of experiments. By using de Haan's equation with $k=int(n^{1/2})$, we found the confidence interval at the 10% significance level as {.00206-0.00258}.[10]

At this stage, a number of regression analyses were performed to approximate the relations between distance and parameter values. Note that parameters were chosen randomly so that they are independently distributed across experiment. Thus, we can regress the distance values as a function of each parameter separately to get unbiased estimates of the effects of parameters. First, second and third order functions, i.e., linear, quadratic, and cubic functions of parameters were used in regression analysis. The MAXDP (maximum allowable change in prices in one iteration) has the highest coefficient of determination. The MAXDP parameter explains about 20% of the variation in the distance values as shown in the following regression estimation of the cubic function of the MAXDP parameter. The importance of this parameter reflects the relevance of adjustments mechanisms in the model.

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10) The "confidence interval" found from the initial 46 experiments is very close to this one: (.00213-.00258). The interval is also quite insensitive to the formulae of the variable k. The value of d^m is within the range .00200-.00215 for $k \in [9, 92]$.

$$d_i = .015 \underset{(3.10)}{} -3.46*MAXDP_i \underset{(-1.65)}{} +491.23*MAXDP_i^2 \underset{(2.06)}{} -14810.21*MAXDP_i^3 \underset{(-1.95)}{} +\epsilon_i$$

R^2 = 20.65, Number of observations = 92

Having estimated the minimum possible distance value, a local random search was performed. The distance value for $\mathbf{p}^0$ was found .00337. Note that this distance value is close to our confidence interval. It is within the best 15% of the parameter space. The first random search around the $\mathbf{p}^0$ vector had a distance value of .00283. Since it is less than $d(\mathbf{p}^0)/(1+con)$ where convergence criterion, con, is 1%, $\mathbf{p}^1$ was set to that parameter vector.

The local search process proceeded as defined in the preceding section. The minimum distance value by the local search, the $\mathbf{p}^c$ vector, was found to be 0.00207 at the 81^{th} experiment. 20 more experiments were made around the $\mathbf{p}^c$ vector. None of those experiments produced lower distance values. (The step size, μ, was not decreased in those searches around the $\mathbf{p}^c$ vector because it was already decreased twice during the previous searches.)

The long-run properties of of this parameter set was checked by running the model for 20 years. Since results seem to be reasonable, we use $\mathbf{p}^c$ as the calibration vector.

5. Conclusions

In this research paper, the procedure used for the calibration of a large-scale, micro-to-macro simulation model, the MOSES, is summarized. The calibration procedure is rather simple, and seems to be effective. The complete process described here took about 20 hours of the CPU time. Since the model is installed

in a PC computer and the calibration process needs almost no user interaction, it can be done in a day with almost no additional cost.

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Figure 2 Cumulative distance distributions