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UNEMPLOYMENT, VACANCY DURATIONS AND WAGE INCREASES:

Applications of Markov Processes to Labour Market Dynamics

by
Nils Henrik Schager

THE INDUSTRIAL INSTITUTE FOR
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ABSTRACT

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This study investigates the optimal wage and hiring decisions of a firm, which faces a search labour market. It also considers the aggregate hiring rate as the outcome of the interaction of unemployment and vacancies in such a market.

The method applied is that of stochastic processes and their control. The solution to the firm's stochastic decision problem is a Markov decision process, which is amenable to detailed analysis. Such an analysis is carried out with respect to two wage policy regimes, one of complete wage flexibility and one of downward wage rigidity. In the latter case it is shown that a stopping control in the form of explicit job vacancies is an indispensable part of the optimal policy.

The classical issue of the effect of unemployment on the firm's wage decision is considered. It is shown that if an increase in unemployment raises the expected rate of hiring considerably more than it raises the wage sensitivity of this rate, the optimal wage will decline. As a consequence it is argued that a causal Phillips relation running from unemployment to wages may well exist at the micro level.

The expected rate of hiring is the reciprocal of the expected duration of a vacancy. In the context of an aggregate labour market analysis it is demonstrated that a relation between the duration of vacancies and the stock of unemployment is stable as long as the efficiency of vacancy/unemployment interaction in bringing about new hires is constant. Such a relation should consequently be given the interpretation that has traditionally been given to the UV-curve.

In summary the study presents a case for the existence of a causal effect of labour market conditions, measured by the durations of vacancies, on the wage decision of a firm and hence on the wage dynamics of the economy as a whole.

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FOREWORD

Wage inflation, high unemployment and inflexible labor markets have afflicted Western economies for almost two decades. And modern economic theory, though technically advanced, has not performed well in explaining these experiences. When those responsible for economic policy ask for advice from economists they often get conflicting answers, which fail to convince.

The way out of this unhappy state of affairs is not to abandon technically advanced methods. The economy is a complex system of interacting dynamic processes and its investigation requires advanced analytical tools. Rather we must require that relevance and technique go together. To be specific, if the economy is not in short-run equilibrium, it is of little use to elaborate short-run equilibrium models for an explanation.

In this research report, which is also a doctoral dissertation at the University of Uppsala, the author, Nils Henrik Schager, addresses the important issues of wage increases, unemployment and labour market efficiency. He reconsiders the foundations of search theory models of the labour market by applying the tools and concepts of stochastic processes. As a result, Schager is able to indicate interesting features of the wage inflation mechanism, which have been largely ignored in earlier economic research. Although the report is largely theoretical, it also contains new empirical results on the development of labour market efficiency in Sweden.

Stockholm, April 1987

Gunnar Eliasson

AUTHOR'S PREFACE

This book is a study in the dynamics of the labour market and wage formation. Its origins can be traced as far back as 1976, when I was asked by a Government Committee to investigate "the relation between wages and profits" in Swedish industry. At that time I was working as an economist at the Swedish Employers' Confederation, and that position gave me access to data that would otherwise have been unobtainable. On the other hand, my job at the Confederation was not easily combined with theoretical research.

Some empirical results based on cross-section data were produced in two reports to the Government Committee around 1980. Perhaps lack of results is a better phrase, as the reported regression estimates were inconclusive. Partly because of this, partly because of the general breakdown of established explanations of aggregate wage increases in the face of widespread stagflation, I became increasingly dissatisfied with existing theories of wage dynamics. In 1981 I used the opportunity to move to the Industrial Institute for Economic and Social Research in order to think more thoroughly about the matter.

Already in 1978, however, I developed the idea that was to be decisive for the methods chosen in my later research. The idea was simply that the time it takes for a firm to recruit an employee should be of crucial importance for its decision on recruitment means, not least its wage decision. In fact, this waiting time should be the quantity, through which the labour market conditions reveal themselves to the firm.

This notion led me to make a successful attempt to relate the amount of aggregate wage drift to the expected durations of reported vacancies in an empirical study on Swedish data [Schager (1981) in the list of references]. Nevertheless, the role played by vacancy durations in a theoretical setting was only intuitively indicated in that study.

My ambition to incorporate the concept of vacancy durations in a rigorous theory of wage dynamics soon led into the area of stochastic processes and their control. This is not an easy subject and it is largely unfamiliar to economists. It has required much time and effort on my part to get acquainted with this area of research and to acquire a workable competence in it. I think it is worth pointing out, however, that the technique used in the present study is really a consequence of elaborating rigorously the implications of an originally simple idea.

It is up to the reader to judge the merits of my contribution. Nevertheless, I myself feel satisfied with the return on my investment in understanding of Markov and Markov decision processes. They are forceful tools, which, once mastered, are able to yield forceful results. I think that their value will become increasingly appreciated in future economic research.

Although empirical work was the starting point of my investigations into the wage formation process, this study contains little empirical application. This absence is solely due to restrictions with respect to time and volume in producing a dissertation. I hope to be able to demonstrate soon the potential of the theoretical models of this book in empirical work. A respecification of the regression equations of my 1981 study on aggregate wage increases is the most immediate task. Further I intend to use the model of optimal firm behaviour on my cross-section data, for which it is particularly well suited.

During the long formation process of this study I have benefited from the support of many. The management of the Swedish Employers' Confederation has granted me not only access to valuable data sources but also generous financial support, which has facilitated greatly my theoretical work. For the opportunity to carry out my research at the Industrial Institute for Economic and Social Research, my thanks go to the director of the Institute, Gunnar Eliasson.

As far as individual support is concerned, I would like to mention three persons in particular. The former head of the Research Department at the Employers' Confederation, Karl-Olof Faxén, has constantly encouraged and helped me through the various phases of my research. He has never failed to show confidence in my ability to carry through my ambitions. His ingenious suggestions have improved my thinking on many points; e.g. the analysis and results of chapter I are largely due to a seminal remark made by Karl-Olof Faxén. Kurt Brännäs at the University of Umeå was engaged in my research in connection with the development of econometric methods, suitable for testing models of wage formation on cross-section data. He has had to await the yield of his efforts, while I have struggled with theory. Nevertheless, Kurt Brännäs has always shown patience and willingly taken part in my theoretical writings by constructive comments. Frank van der Duyn Schouten, Free University of Amsterdam, came in touch with my work at a conference in 1984. I am happy to say that he found my research to be an interesting and challenging opportunity for a specialist in Markov decision theory. Frank v.d. Duyn Schouten has scrutinized the technical parts of my analysis with an expert's eye. Without his assistance, I would have felt much less confident in the reliability of my results. I feel greatly indebted to these persons for their keen and disinterested engagement in my research.

At an early stage of my research I benefited from discussions with the secretary of the Government Committee, Berndt Öhman. Since 1981 my former and present colleagues at the Industrial Institute have assisted me considerably with valuable advice. My thanks go especially to Harald Lang, Bertil Holmlund, Anders Björklund, Bo Axell, Stephen Turner and Erik Mellander. At the final seminar at the Department of Economics, Uppsala University, useful suggestions, based on careful scrutiny of chapter I and II, respectively, were given by Ante Farm, Institute for Social Research, Stockholm University, and Jörgen Weibull, Institute for International Economic Studies, Stockholm University.

I also have every reason to be grateful to Bengt-Christer Ysander, my professor of economics at Uppsala University. Without his clear discernment and good judgement I would hardly have succeeded in putting together my present research results in a reasonably well organized and limited form.

Finally, I would like to thank Birgitta Burman and her co-workers at the Industrial Institute for their good work and friendly assistance in putting my thoughts into readable book-pages.

My reference to persons or institutions above does not imply any responsibility on their part for the contents of this book. For remaining deficiencies I am solely liable.

Stockholm, April 1987

Nils Henrik Schager

CONTENTS

INTRODUCTION

The present book is a search theoretical study on the dynamics of the labour market. In substance the analyses can be said to be founded on some of the contributions in the now classic book 'Microeconomic Foundations of Macroeconomics', edited by Edmund Phelps in 1970, notably Holt (1970), Mortensen (1970) and - to a somewhat lesser extent - Phelps (1970). The first chapter of this book builds on the analysis in Holt (1970) of the interaction between unemployment and vacancies in a search labour market. The second chapter analyzes the optimal wage behaviour of a firm which faces such a labour market in the spirit of Mortensen (1970) and his successors.

Two famous 'curves' of economics are at the focal point of our investigations. The first chapter aims at demolishing the usual interpretation of the UV-curve. The second one considers and tries to reestablish the microeconomic basis for a modified Phillips curve. How these results are brought about is briefly indicated later in this introduction.

A search labour market is characterized by uncertainty. The timing and the contents of a job and wage offer to the job searcher are uncertain and so are the corresponding features of a job applicant contact to the firm. The flows of offers and of contacts are consequently stochastic and form stochastic processes in time.

The fact that a search labour market is made up of simultaneous stochastic processes has of course been implicitly recognized in the search theory literature. Holt used the phrase 'stochastic interaction' to characterize the matching between vacancies and unemployment in a search labour market. For the most part, however, the vast body of literature on stochastic processes and their applications has not been used in economic search theory. The only exception is some recent contributions to the theory of individual job search [cf. Zuckerman (1983); Burdett and Mortensen (1978); Albrecht, Holmlund and Lang (1986)].

The established theory of stochastic processes divides into the descriptive theory of the processes and the theory of their control. There exists a substantial literature on the first subject, much of which is of a highly technical and abstract character. A moderately advanced exposition, which devotes more space to possible applications than to rigorous proofs is given in the classic book by Cox and Miller (1965).

A stochastic process may be defined in discrete or in continuous time. An example of the first type of process is the Markov chain, in which the system moves from one state to another at discrete intervals of time according to some transition probability law. This approach has computational advantages, but phenomena do not usually follow such simple time paths. As far as theoretical model building is concerned, a representation of the process in continuous time is preferable. The process is now to be specified not only with regard to the probability of moving from one state to another but also with regard to the probability that a transition takes place at any point of time.

We referred to the Markov chain above; its name indicates that it possesses the Markov property, which is an essential concept in the theory of stochastic processes. A process is Markov, if its future probability law of transition is completely determined by its present state. As a consequence the sojourn time in any state of a continuous time Markov process is exponentially distributed, because the instantaneous probability of moving to any other state is constant. This property is essential for making the problem of controlling a Markov process tractable.

On a general level the class of continuous time processes (whether Markov or not) can be partitioned into

a) diffusion processes
b) jump processes
c) drift processes

The diffusion processes are continuous not only in time but in the state space as well. They require the advanced theory of stochastic calculus to be handled. Diffusion processes and their control have been applied especially in finance but also in economics as a way to model price uncertainty. Malliaris and Brock (1982) give an extensive account of such applications.

Jump processes are discrete in the state space. Such processes have long been applied in various branches of operations research, such as queuing and inventory theory. Typically, the state space is given by a variable, that can only take on a natural number, such as the number of served customers or the number of stored items. It is also typical that the use of jump processes has been more problem-oriented than theory-oriented, so that the theory of jump-processes can at present be said to be less developed but potentially more complicated than that of diffusion processes [cf. Davies (1984)].

The drift process, finally, is simply the deterministic process, the time path of which is known with certainty. This is, of course, the usual way to represent the dynamics of an economic model. Moreover, it has also been the way in which the process of job applicants getting in touch with a recruiting firm has been modelled in search theory applications to the labour market. To describe the control of this process the behaviour of the firm has consequently been modelled in accordance with any of the techniques available for deterministic control [for an account of such techniques, see e.g. Kamien and Schwartz (1981)].

From a methodological point of view, the aim of this book is to replace the deterministic process approach with an appropriate stochastic one. It should be clear from the preceding comments that the flow of job applicants and of job offers is most suitably represented by a jump process. In the analyses to follow we will work with the special type of jump processes, that are called birth-and-death processes. They possess the Markov property and any jump (transition) from a state can only occur to an adjacent state (i.e. one unit is either added or subtracted).

It might appear somewhat surprising that the application of jump processes in economics have been sparse compared to the application of diffusion processes [cf. Malliaris and Brock (1982)].

The reason is that stochastic processes have been introduced in economic theory via econometrics. The basic process in diffusion theory is the Wiener process, in which the increments are normally distributed; the corresponding role in jump process theory is played by the Poisson process, in which the increments are Poisson distributed. Taking the view that the stochastic element in the process considered is introduced through a stochastic "disturbance", amended to a basically deterministic economic structure, the properties of the Wiener process offer themselves as a natural starting point. In this sense the stochastics of the jump processes that emerge from search theory is more firmly based on the economic model itself. Although there certainly exist expected values of the realization of the processes, which can on occasion be usefully referred to, this is not to say that those values reflect the economic structure of the model in any basic sense. The structure is given by the instantaneous transition probabilities ("infinitesimal generators"), which define the processes.

A useful and accessible description of birth-and-death processes can be found in Chapter 4 of Cox and Miller (1965). The control of such processes, which subject falls under the heading Markov decision processes, has been analyzed for some time at different levels of generality and applicability. A standard reference of a general nature is Ross (1970). Howard (1971) gives a very comprehensive textbook introduction to the subject, oriented towards operations research applications. As birth-and-death processes figure extensively in queuing theory, Markov decision processes have been studied under the heading queue control theory. A useful review of such more problem-oriented work is found in Stidham and Prabhu (1974). A more recent contribution, addressing specifically birth-and-death Markov decision processes, is Serfozo (1981). At a somewhat higher level of abstraction van der Duyn Schouten (1983) and Vermes (1985) treat the control of mixed jump and drift processes (Markov decision drift processes).

After this short review of the concepts and methods of stochastic processes we return to the subject matter of this book. Chapter I is devoted to a treatment of the interacting processes of vacancies and unemployment. The presentation is by no means technically difficult; we concentrate attention on the instantaneous transition probabilities, which fully characterize the processes, including the causal structure that is embedded in them. Only in the Appendix to the chapter do we touch upon the transient and equilibrium behaviour of the processes.

By concentrating on the basic properties of the vacancy and unemployment processes and their interaction, we are able to show that the efficiency of vacancies and unemployment in bringing about new hires cannot be indicated by the relation between stocks of vacancies and of unemployment (the UV-curve). Making explicit the implications of the process interaction, that was proposed already in Holt and David (1966), we show that hiring efficiency is indicated by a stock-duration relation. This interesting new result is produced just because of a careful utilization of the concepts and tools of stochastic process analysis. As the chapter (including the Appendix) makes clear, confusion is likely to arise, when concepts belonging to static and dynamic probability models and process properties of instantaneous, transient, and equilibrium nature are mixed together without sufficient care.

In Chapter II, the theory of Markov decision processes is applied. We consider the decision problem of a firm which faces a Poisson process of contacts with job applicants. The firm can control the intensity of the hiring process through its wage level; it can also stop the hiring process by direct intervention at any time. The formal analysis of the decision process is technical and is presented in a Technical Supplement to Chapter II.

The analysis establishes the optimal wage policy as a stationary policy in the employment state of the firm (a closed-loop control). When wages are completely flexible, the optimal wage policy forms a non-increasing trajectory in the employment state. When wages are downward rigid, the optimal wage trajectory is constant; moreover, it is shown that the optimal policy involves stopping the recruitment process by direct intervention.

In the main text of Chapter II the properties of the solution to the recruitment decision model are further analyzed and discussed. We demonstrate that the possibility of multiple solutions depends on the shape of the distribution of reservation wages among the job applicants. We draw special attention to the possibility that the reservation wage distributions of the unemployed and of the employed job searchers, respectively, may differ. A possible outcome is that the firm may apply a wage at which the wage sensitivity is low with respect to recruitment of the unemployed and high with respect to recruitment of the employed job searchers.

The solution to the decision model is made subject to sensitivity analysis. Besides some intuitively reasonable results, the effect of a change in the unemployment stock on the optimal wage turns out to be particularly interesting. Unemployment affects the optimal wage through its impact on the expected speed of the hiring process, and on the wage sensitivity of this speed. If both of them are (roughly) equally affected, the optimal wage is non-decreasing in the unemployment stock, as Mortensen (1970) and others have found. If, however, the wage sensitivity in the neighbourhood of the optimal wage is little affected by a change in unemployment, the optimal wage is non-increasing in this change. Such a situation occurs when the conditions described at the end of the preceding paragraph hold at the optimal wage. Consequently, under such conditions a "Phillips curve" response in the wage of the firm obtains.

The remainder of Chapter II focuses on the consequences of downward wage rigidity. We argue that the existence of such rigidity makes it more likely that firms are in such positions with respect to their optimal wages that unemployment has a nonincreasing impact on the wage. Otherwise, the most conspicuous implication of downward wage rigidity is the likely prevalence of "wage disequilibria" among firms. As the firm cannot respond to wage-decreasing parameter changes by actually decreasing wages, the effect of such changes is to bring about a wage disequilibrium at the firm. This feature produces a nice explanation of the observed pattern of quantity and wage adjustments during a business cycle. The timing of parameter changes during the cycle has important consequences for their visible effect on e.g. wage increases; the observed relationship between tight labour markets and rapid wage increases may be explained partly by this mechanism.

The discussion on business cycle phenomena in Chapter II is tentative, as we do not develop rigorously a market interdependence model. The same is true for the concluding discussion on the Phillips curve. As indicated above, there is a case for reestablishing a causal chain running from unemployment via the expected speed of the hiring process (or, equivalently, the expected duration of vacancies) to the wage level. One source of instability in such a relation is a lack of stability in the unemployment-vacancy duration relation. This is a subject of investigation in Chapter I, and empirical evidence for Sweden indicates very clearly a shift in this latter relation in the years 1967-69. It is remarkable that wage increases in Sweden have continued to be closely related to the duration of vacancies (as opposed to the unemployment rate) over these years, as Schager (1981) and Bosworth and Lawrence (1987) show. These findings constitute strong evidence in favour of the empirical relevance of the theoretical argument in this book.

Nevertheless, the wage dynamics of a search labour market is a very complex theoretical issue, that is as yet largely unexplored. Presumably the turnover of employees between firms plays a more crucial role than e.g. the behaviour of the unemployed in overheated states of the labour market, in which wages tend to accelerate. Whether such a process of wage increases can be sustained depends on the mechanism, by which new demand is fed to the product market. An endogenous inflationary process of a type similar to that proposed by Bent Hansen (1951) is clearly not ruled out.

The economic applications of Markov and Markov decision processes in this book are not particularly advanced. With respect to the analysis of the unemployment and vacancy processes in Chapter I, an extension to apply more elaborate queuing models should yield further useful insights in the functioning of a search labour market. The Markov decision model of a recruiting firm in Chapter II can be extended in several directions, as we point out in the course of its presentation. As stochastic jump processes have only occasionally been used in economic theory (as opposed to operations research), their value as an analytical tool has not been subject to any serious test. Their potential may be considerable and we hope that the results of the present applications will stimulate further research in this area.

CHAPTER I

HIRES AS THE OUTCOME OF STOCHASTIC INTERACTION BETWEEN UNEMPLOYMENT AND VACANCIES: A REINTERPRETATION OF THE UV-CURVE

1 Introduction

Since the article by Dicks-Mireaux and Dow (1958) the relationship between the stock of unemployed U and the stock of unfilled vacancies V - or the UV-curve - has been used as an indicator of the degree of flexibility in the labour market. These authors claimed that at a given degree of flexibility changes in labour demand correspond to movements along a rectangular hyperbola in the UV-plane, while an outward shift in this curve indicates increased 'maladjustment' in the labour market.

The UV-curve was introduced in the same years as the Phillips curve and appeared to be a natural complement to it. As dynamic price theory at that time predicted that the rate of change of money wages is a function of the excess demand in the labour market and excess demand ought to be measured as the difference between the stock of vacancies and the stock of unemployed, a stable UV-relation ought to be the basis of a stable relation between money wage changes and unemployment.

The interpretation of shifts in the UV-curve as changes in 'ability of adjustment' was almost self-evident within a neoclassical conceptual framework. The only real embarrassment was caused by the neoclassical implication that unfilled vacancies and unemployment cannot coexist in one market. To reconcile theory with facts it was thus necessary to assume that the labour market consisted of several imperfectly connected submarkets, where either unfilled vacancies or unemployment could persist. Hansen (1970) showed that several restrictive assumptions, unlikely to be met in practice, are needed to yield a stable aggregate UV-curve under such conditions.

By the time Hansen wrote his article the theory of labour market dynamics was undergoing a drastic change, as models of search under imperfect information in a few years completely replaced the neoclassical models. With special regard to the analysis of vacancies and unemployment in a search theoretical framework the pioneering work was done by Charles Holt in a series of contributions [notably Holt and David (1966), Holt (1970)].

It is the aim of this chapter to demonstrate that the position of the UV-curve cannot be given the traditional interpretation as an indicator of 'adjustment ability', when it is regarded as the outcome of search under uncertainty in the labour market. It is remarkable that this conclusion can be reached without adding anything substantially new to the analysis by Holt. As a matter of fact it seems as if Holt overlooked those elements in his own analysis which pointed to a reinterpretation of the UV-relation. It also appears reasonably clear that this happened because of the strong influence of the Phillips curve (see Section 4 below).

The interest in the UV-curve reached its peak in the mid-seventies, but the research was almost exclusively empirically oriented. Since then the apparently capricious behaviour of the UV-curves in different countries has produced a diminishing confidence in its usefulness. Still, the following quotation is representative of the current state of opinion: 'The question whether the functioning of the labour market has deteriorated is usually illuminated with the help of so called UV-curves ... For every given structure of the labour market ... it is assumed that there exists a given UV-curve, i.e. a given negative relation between vacancies and unemployment ... Changes in aggregate demand will give rise to movements along a given UV-curve, while changes in the functioning of the labour market at given aggregate demand will reflect itself in a shift in the UV-curve ...' [Persson-Tanimura and Johannesson (1987), translated from Swedish].

As an authoritative example of the wavering belief in the UV-curve analysis (or the 'Beveridge curve' as it is nowadays often called) we may quote a recent OECD Working Paper: 'Another possible, though indirect, indicator of labour mobility, which must, however, be used with considerable caution, is the relationship between vacancy and unemployment rates. On ... strong assumptions ... increased labour market mismatches are indicated if vacancies and unemployment rise simultaneously. Inspecting the 'Beveridge curves' for twelve OECD countries points to outward shifts since the late sixties or early seventies in several countries ... However, ... there does not remain much, if any, support for the notion that labour market mismatches have universally increased.' [Klau and Mittelstädt (1985)]

Quite recently, however, Jackman, Layard and Pissarides (1984) have looked into the foundations of the UV-curve and found that a relation between the stock concepts is not necessarily the most appropriate one. The findings of these authors are in important respects related to ours, but the focal point of our attention is rather obscured in their presentation as they hasten to consider many other aspects of job vacancy creation.

We will make frequent references to Holt's contributions as well as to the paper of Jackman et al. as our discussion proceeds. It will be organized as follows: First we will present a treatment of the labour market search in terms of the established theory of stochastic processes, leading to a reinterpretation of the UV-relation. Then there follows a critique of the traditional use of the UV-curve and we propose an alternative indicator. The framework is then widened to include on-the-job search. The concept of efficiency in the matching of vacancies and unemployment is further discussed in one section and the paper concludes with an empirical demonstration, making use of Swedish data.

2 Unemployment and vacancies as realizations of stochastic processes

As Holt pointed out already in the late sixties, when there is job search under uncertainty the labour market is to be conceived as a 'dynamic stochastic system'. Surprisingly enough, there has since then been little attempt to develop the stochastic approach beyond some basic concepts. Although the processes in the labour market may in reality be very complex, this is no reason why one should not try to apply some more elaborate, but still tractable probabilistic tools such as simple Markov processes in representing them.

This is precisely what we are going to do. More specifically we will make use of the established theory of time-homogeneous birth-and-death processes, the application of which to labour market flows offers itself quite readily (see Cox and Miller (1965)).

We will retain all the standard assumptions of the basic models used in Holt and David (1966), Holt (1970) and in Jackman et al. (1984). Vague points and diverging opinions will be currently commented on. We will adhere to the assumption of homogeneity throughout, so the unemployed job searchers and the vacant jobs (and later on the on-the-job searchers) will have all relevant characteristics in common. Homogeneity prevails also with respect to time, i.e. we do not leave the area of Markov processes in our analysis. Furthermore, we will be content just to consider expected values, i.e. we make use of the deterministic approximation of the processes.

In modeling the unemployment process the most immediate alternative to choose should be the immigrant-death-process (in the terminology of Cox and Miller (1965)). This means that there is an inflow intensity of individuals into the unemployment stock at a rate u, at the same time as every individual in that state has a constant instantaneous probability, μ, of leaving unemployment. So with U denoting the number of individuals in unemployment, i.e. the stock of unemployed, the outflow intensity from unemployment is $U * \mu$. This is in turn equal to an inflow intensity into employment, as we follow the basic models in assuming that the only way to leave unemployment is by entering employment.

For the immigrant-death-process to be applicable it must hold that both u and μ are independent of U. As we shall see, however, the exact nature of the inflow process is of no relevance for our main results. We just note that the possible effect on u is likely to derive from μ rather than from U. What is of crucial importance, though, is whether the outflow intensity is a linear-death process, i.e. whether μ is independent of U.

With the possible exception of Holt and David (1966), this question is hardly addressed in an explicit way by earlier writers. It should be answered affirmatively, however, if the stocks of vacancies and of the unemployed are not strongly imbalanced. Otherwise, congestion phenomena in the labour market may arise. If, for example, the number of the unemployed by far exceeds the number of vacant jobs, the search of an unemployed individual is not only an attempt to locate a vacancy for immediate comparison of claims and offers. It is rather an attempt to locate a queue to a vacant job. In such cases a more elaborate queuing model should be applied, which allows for the waiting time in the queue [cf. also footnote 15 in Holt (1970)].

The relevance of applying such queuing models is apparent, as depressed labour markets have been the rule rather than the exception in many Western European countries in recent years. This contribution does not have such an ambitious aim, however. Nevertheless, it is worth pointing out that there exists a limiting case in the form of the immigrant-emigrant process that can serve as simple model of the unemployment process, when the stock of unemployment has been persistently much larger than that of vacancies.

In the emigrant process the outflow intensity is independent of the volume of the stock of unemployment. Denoting this intensity β, the individual probability of leaving unemployment varies inversely to the stock volume, as

$$\mu = \frac{\beta}{U}$$

It is very important to understand the difference between the causal structure in the two types of processes. In the immigrant-death process, the independent parameters are u and μ, in the immigrant-emigrant process they are u and β. This distinction is of crucial importance as our further analysis will show. (For convenience the processes will from now on be referred to as I-D-processes and I-E-processes, respectively.)

Regardless of how the outflow intensity of the unemployed depends on U, it should depend on V, the stock of vacancies. Otherwise, the notion of an interaction between unemployment and vacancies in the labour market, giving rise to new employment, is deprived of any causal content; we will illustrate this point more in detail later on.

Fortunately, there exists empirical evidence that bears upon this matter. Several studies of the determinants of the duration of unemployment have investigated the relation between this variable and the stock of vacancies quite thoroughly [Barron (1975), Axelsson and Löfgren (1977), Björklund and Holmlund (1981)]. Given the homogeneity assumptions of the present analysis, μ^{-1} is nothing but the expected duration of unemployment. Hence we are able to use the findings of these studies in ascertaining that there exists a strongly significant influence from V on μ.

Here we must add a qualification. As pointed out in the studies referred to, the measured effect of V on μ can be expected to be the resultant of two forces working in opposite directions. One is the pure 'probability' or 'availability' effect, which reflects the impact on the employment probability of more employment opportunities being available. This is the effect of V on the outflow intensity that we have focused on. But there is possibly also a 'supply' effect, reflecting the fact that the unemployed may become more demanding in their choice as more opportunities present themselves. We will have more to say on this matter in Section 7 below. For the time being we note that the findings reported in Holmlund (1976) and in Björklund and Holmlund (1981), based on the comparatively reliable Swedish vacancy data, are consistent with the hypothesis that the 'availability' effect of vacancies on μ is linear in V.

To sum up, we may conceive the flow intensity out of unemployment into employment as

$$U * \mu = U * k * V \qquad \text{for an I-D-process}$$
$$U * \mu = k * V \qquad \text{for an I-E-process}$$

It now turns out that we have in fact also deduced the type of process that applies to vacancies. This is a consequence of the fact that the flow out of unemployment and the flow out of vacancies are both equal to the flow of hires. Denoting the hiring intensity by e, we have

$$e = U * \mu = V * \lambda$$

where λ is the instantaneous probability of a vacancy being filled.

If unemployment follows an I-D-process as specified, we get

$$e = U * \mu = U * k * V = \lambda * V \Rightarrow \lambda = k * U$$

If k is independent of V we get the symmetric result that the vacancy process is also an I-D-process. Consequently, the truly symmetric interaction system obtains:

$$\mu = k * V$$
$$\lambda = k * U$$
$$e = k * U * V$$

The reader may recognize the basic Holt model in these relations, especially as it is presented in Holt and David (1966).

If unemployment follows the specified I-E-process, we get

$$e = U * \mu = k * V = \lambda * V$$

Again if k is independent of V, vacancies follow an I-D-process, but now it is independent of the unemployment process, which is completely passive as regards the employment intensity. Hiring is determined by vacancies only. On the margin there is really no interaction in the system:

$$\mu = \frac{k * V}{U}$$
$$\lambda = k$$
$$e = k * V$$

It is now due time to pay attention to the important parameter k in our system of processes. In our

I-D/I-D-system the availability of vacancies has a linear impact on the probability of an unemployed person being employed; the availability of the unemployed has the same linear impact on the probability of a vacancy being filled. k is this probability adjusted for the availability effect. Consequently, k reflects those characteristics of labour market search which are regarded as constituents of search efficiency or adjustment ability: the intensity of search among the unemployed and firms with vacant job positions, the willingness of job-applicants to accept offered jobs and wages and the willingness of employers to accept the qualities of job-applicants; a more detailed discussion on some of these matters is carried out in Section 7. An indicator of search efficiency should be able to identify changes in k in an unambiguous way. This conclusion also holds when the I-E/I-D-system is applicable. The only difference is that the marginal availability effect of U on the vacancy filling probability is non-existent so the indicator must be chosen accordingly.

We end this section by calling the reader's attention to the fact that nowhere in our presentation has it been necessary to assume that the processes of unemployment and vacancies are in equilibrium. In fact, for an I-E-process an equilibrium may not even exist! [See Cox and Miller (1965), p. 170.]

3 The UV-relation and the 'hiring production function'

In the preceding section we demonstrated how the applications of two simple stochastic systems lead to mappings of the stocks U and V into the hiring intensity e. With a more general (or rather more heuristically oriented) formulation, we write

$$e = k(U,V) * U * V \qquad (1)$$

where k is made dependent on U and V in an unspecified, but probably non-increasing way.

Our I-D/I-D-system implies that k is independent of U and V, while in the I-E/I-D-system it holds that $k(U,V) = k * U^{-1}$. To complete the taxonomy, we note that $k(U,V) = k * V^{-1}$ is consistent with an I-D/I-E-system and $k(U,V) = k * (U * V)^{-1}$ with an I-E/I-E-system.

What is really the expression (1)? It is a 'hiring production function', transforming the stocks of unemployment and vacancies into a hiring intensity. Provided that the absolute value of the elasticity of k with respect to both U and V are not too large (and well below one) or varies irregularly, (1) will behave as an ordinary production function at the same time as the I-D/I-D-system should serve as a good approximation of the process interaction. Likewise it is clear that (1) defines a stable, convex relation between U and V at any fixed hiring intensity. Thus the UV-curve is nothing but an isoquant of the 'hiring production function'. There are as many UV-curves as there are levels of hiring flows.

The situation is particularly transparent when the pure I-D/I-D-system or the basic Holt model applies. The linear availability effect on the transition probabilities μ and λ corresponds unambiguously to the scale effect in the production function (1). The 'adjustment ability' or search efficiency factor k is the factor of 'technical efficiency' in (1). The UV-curve can move outwards for two reasons: hiring is produced on a larger scale or the efficiency in hiring production is reduced.

In principle the same dichotomy holds when e is not strictly linear in both U and V. However, the identification of scale shifts vs. efficiency shifts becomes less easy, unless one is prepared to maintain parts of the Holt assumptions. For example, by postulating that the availability or scale effect is linear in U and V, the remaining effect of these variables on e is interpreted as endogenous changes in hiring efficiency; alternatively, the scale effect is defined as the influence of U and V on e, and hence changes in hiring efficiency is by assumption exogenous to these variables (see also Section 7 below). Again, a queuing model approach should be the most satisfactory way to handle this problem theoretically.

If the labour market is so depressed that an I-E/I-D-system applies, (1) degenerates into

$$e = k * V$$

As pointed out in our discussion in the preceding section, hiring is now a function of vacancies only. Unemployment has no influence and the UV-curve has no meaningful interpretation.

Let us end this section by pointing to the consequence of assuming that the vacancy and unemployment processes form an I-E/I-E-system. This is equivalent to rejecting the concept of a hiring function in terms of vacancies and/or unemployment.

The hiring intensity is independent of both vacancy and unemployment stocks. Vacancies are filled and the unemployed are hired at a rate that is determined by factors that are not causally linked to these stocks. The neoclassical model is really to be interpreted as such a case, in which any UV-relation is only a statistical regularity, which happens to show up because of correlations in cyclical variables.

4 A review of earlier research

Equipped with a framework of reference, we will now scrutinize earlier research on the UV-curve, that are based on search theories of the labour market.

As we indicated in our introduction, the seminal contribution in this field is Holt and David (1966). They introduced the idea of job matching as a result of stochastic interaction between stocks of unemployment and vacancies. Substantiating their argument with reference to empirical studies they proposed a model that is equivalent to a pure I-D/I-D-system and they explicitly wrote down our Equation (1), k being independent of U and V.

Holt and David did not probe too deeply into the structure of their model, but noted that at a constant hiring rate it would generate a stable hyperbolic UV-curve. Such stability was in accordance with empirical evidence, they thought. On the other hand, they seem to have been quite aware of the meaning of the parameter k: 'Estimates of the value of k ... will give a quantitative notion of the effectiveness of present information channels in bringing together workers and vacancies in compatible combinations.' (ibid. p. 105).

Holt developed these ideas in his later contribution Holt (1970). Some of the sharpness of the analysis is lost, however, in a multitude of interesting, but formally loosely connected observations and comments. Moreover, Holt chose to conduct his discussion exclusively in an equilibrium setting, which made possible an error in his analysis.

The crucial sentences can be found in Holt (1970), p. 235; we quote,

> 'The near constancy of U·V occurs because F, T_s and P_{oa} all tend to have stable values.....The quit and lay-off components, whose sum in equilibrium determines F, fluctuates cyclically and countercyclically, respectively, so their fluctuations largely cancel out.'

In our notation F=e and P_{oa}/T_s=k; the quit and lay-off rates are the inflow into unemployment, in our notation u.

Rephrazing Holt's statement, it says that 'u in equilibrium determines e'. A process or a system of processes may be in equilibrium or not, but that condition does not change its causal structure. Taken literally, Holt's statement implies a novel causal structure of the hiring process, so that any addition to unemployment is automatically offset by an increase in hires.

Such a structure of the hiring process is of course not consistent with the idea of hires resulting out of the interaction between the stocks of vacancies and of unemployment. Closer reading of Holt (1970) reveals that he has in fact not intended to propose a new model; he just observes that in equilibrium u and e are equal (and, we can add, equal to the inflow of vacancies). But the unfortunate word 'determines' creeps somehow into the text and, which is worse, distorts Holt's further analysis.

By stating that e is cyclically stable (because u is so!) Holt was able to conclude that the UV-curve is cyclically stable. Otherwise, it would certainly be a more sound procedure to look at the stability of the hiring rate directly; clearly, one would not expect it to be stable over the cycle. Consequently, the conclusion is that the UV-curve must be subject to cyclical shifts to the corresponding extent (and as a by-product that the unemployment-vacancy processes are <u>not</u> in a stable equilibrium during a cycle).

Holt's strange error can only be explained by the influence of the Phillips curve hypothesis, which had at that time reached its peak of authority. But it was really at the expense of violating his own model that Holt was able to conclude:

> 'Because the expression on the right of the equation (15) (i.e. UV = e/k in the present notation, our note) tends to change slowly, we see that cyclical fluctuations in unemployment are highly correlated with those of the vacancy rate, so ... we can obtain a fairly stable Phillips relation that suppresses the role of vacancies.' [Holt (1970), p. 241.]

Having made his basic analytical mistake, Holt's further arguments were consequently confused. e (in our notation) was not just the hiring rate; keeping to his process equilibrium framework, for Holt it was the general turnover rate of the unemployed and of vacancies. Clearly Holt was aware that an increase in the 'turnover rate' must shift the UV-curve outwards. He took great pains in finding arguments why such a shift ought to be interpreted as increased mismatches in the labour market, so that any shift outwards of the UV-curve could be seen as a sign of increased maladjustment. With the correct interpretation of e such an effort is of course in vain.

In the same celebrated volume in which Holt (1970) was published, there is an impressive contribution by its editor Edmund Phelps [Phelps (1970)]. Focusing on money wage dynamics, he also discussed the relation between hires, vacancies and unemployment. Without using the notion of interacting stochastic processes, he arrived at the conclusion that the volume of hires is a function of the stock of vacancies and unemployment: 'My theory denies a strict and simple short-run relation between the unemployment rate level and the vacancy rate level. ... unemployment and vacancy levels together determine the rate of change of employment ...' (ibid. p. 149). In other words, Phelps argued that the UV-curve is not cyclically stable and stated the reason for it: unemployment and vacancy stocks 'produce' new employment. His conclusion is in complete accordance with our analysis.

Phelps' contribution did not go unnoticed by other researchers; his UV-analysis is described e.g. in Holmlund (1976) together with Holt's. However, Phelps presented his assumptions in a way that looked unnecessarily ad hoc and he hastened to transform the rate of change of employment into one of unemployment, a procedure which required additional assumptions. In that way Phelps was able to come closer to the ubiquitous Phillips relation, and he claimed quite consistently that it should be amended with the rate of change of unemployment as an additional explanatory variable. Thus the notion of a basic causal relation between the hiring rate and the stocks of vacancies and unemployment fell into the background and was obscured. The contradiction between Holt's and Phelps' simultaneously published contributions on the UV-relation was not conceived as so fundamental as it really was.

Those researchers who used the UV-relation during the seventies did not question its traditional interpretation, at least not on its weakest point. Apart from the empirical problem, whether registered unemployment and vacancies corresponded to the 'true' variables, most concern centered around the possible effects of various heterogeneities in the labour market [see e.g. Foster (1974) with references, Holmlund (1976)].

The contributions by Bertil Holmlund seem to merit special attention. His dissertation [Holmlund (1976)] and still more a revised and condensed version of it [Holmlund (1978)] reveal how very close Holmlund was to a reinterpretation of the UV-relation: '... the position of the curve in the UV-plane depends i.a. on how efficiently the search processes in the labour market work, i.e. how rapidly a job applicant can find a job and a vacancy can be filled. Besides, the position of the curve is determined by the volume of unemployment inflow' [Holmlund (1978), p. 438, translated from Swedish].

Apart from the confusion about which flow to focus on [an heritage from Holt (1970)], the conclusion seemed to be just at hand that the UV-curve should be replaced by some relation that eliminates the effect of the flow component and isolates the effect of the degree of efficiency of the search processes. However, Holmlund withdrew at the last step and repeated the usual interpretation of shifts in the UV-curve.

At the same time we want to emphasize that Holmlund's contributions [later in cooperation with Anders Björklund in Björklund and Holmlund (1981)] contain penetrating studies of the duration of vacancies and of unemployment. As was made clear in the preceding section, the empirical evidence on the determinants of the durations is essential for understanding the character of the interacting processes of vacancies and unemployment.

The presentation given above should give an accurate picture of the state of the UV-analysis up to the publication of Jackman et al. (1984). Before we consider their contribution it is convenient to summarize the results which follow from our analysis in the preceding sections.

5 The alternatives to the UV-curve

Let us return to Equation (1)

$$e = k * U * V$$

where $k = k(U,V)$ may depend on U and V.

Now it is clear that for any homogeneous birth-and-death process e/U and e/V are to be interpreted as the instantaneous probability of an unemployed person being employed, μ, and of a vacancy being filled, λ, respectively. So we have in general

$$\mu = k * V; \qquad \lambda = k * U \tag{2}$$

as we found for the I-D/I-D-system. The only difference is that k may not be a constant. It is easy to check that the other systems of processes which we explicitly considered earlier are covered by (2).

Expressing (2) in terms of the expected duration of a vacancy and an unemployment spell, which we denote by T_v and T_u, respectively, we have

$$T_u = \frac{1}{k * V}; \qquad T_v = \frac{1}{k * U} \tag{2'}$$

So if the pure I-D/I-D-system (or the basic Holt model) is applicable, <u>it is a change in the relation between the duration of vacancies and the stock of unemployment, or equivalently, between the duration of unemployment and the stock of vacancies that indicates changes in the hiring efficiency of the labour market</u>.

In the I-E/I-D-system, which we claimed might be a suitable model for a depressed labour market, we have

$$T_u = \frac{U}{k * V}; \qquad T_v = \frac{1}{k}$$

It is immediately clear that cyclical changes are solely reflected in the unemployment duration, while changes in the vacancy duration correspond unequivocally to changes in hiring efficiency.

It is now convenient to compare our results with those of Jackman et al. (1984). First of all, their study must be given credit for having reestablished the notion of a hiring function in vacancies and unemployment and for conducting their UV-analysis in these terms. It should also be acknowledged that the approach of Jackman et al. has inspired the presentation in this chapter as regards the systematic comparison between the notion of a hiring production function and that of interacting stochastic processes.

Nevertheless, the specification of the stochastic mechanisms in Jackman et al. (1984) is not particularly successful. With reference to a certain example of search behaviour they argue that the hiring function is homogeneous of the first degree in the vacancy and unemployment stocks. A scrutiny of their example (in footnote on p. 6) shows that if we transform their fixed period chain into the corresponding continuous time process, their search model implies an I-D/I-E-system, i.e. hiring is determined solely by the unemployment stock, while vacancies are completely passive. To demonstrate this requires some technical exercises. We have relegated these to an Appendix, which can also be used as a guide to how the instantaneous probabilities defining the processes are used to derive their transient and equilibrium behaviour.

Consequently, the corresponding hiring function has the suggested homogeneity property only in a degenerate sense:

$$e = k * U; \qquad \mu = k; \qquad \lambda = \frac{k * U}{V}$$

While one might conceive of an over-heated labour market, where such a model can apply, it is not very appropriate for the British labour market in recent years, with which Jackman et al. are primarily concerned. In fact, their data, showing small variations in vacancy durations and steadily increasing unemployment durations since 1975, point more to an I-E/I-D-system at work, where the unemployment I-E-process is clearly out of equilibrium.

If one accepts the homogeneity assumption of Jackman et al. for the sake of the argument, however, one immediately finds that their reasoning follows the same line as ours. Let $f(U,V) = e$ have the proposed homogeneity property: then it clearly holds that

$$f(\frac{U}{e}, \frac{V}{e}) = 1$$

For example, it may be that

$$f(U,V) = k * U^{\alpha} * V^{1-\alpha}$$

and, consequently,

$$\frac{1}{k} = (\frac{U}{e})^{\alpha} * (\frac{V}{e})^{1-\alpha}$$

In this case the elimination of the scale effect in the production of hires yields a relation in T_u and T_v, which changes when there are changes in hiring efficiency. Consequently, Jackman et al. argue that this relation should replace the UV-relation. Thus their principle aim is the same as ours: to eliminate the effect of the volume of hires on a vacancy-unemployment relation in order to establish a measure that can be used as an indicator of hiring efficiency.

As we showed the search theoretical argument in favour of a hiring function that is homogeneous of the first degree in vacancies and unemployment is weak. However, this assumption does take care of a complication that may otherwise arise. One would like to think that changes in size between labour markets (as between different countries) should be reflected in equiproportional changes in flows of hires and in stocks of unemployment and vacancies, ceteris paribus. If U and V are interpreted as absolute numbers, this result does not hold in the basic Holt model or our I-D/I-D-system; it exhibits strong 'economies of scale' in hiring. Incidentally, Holt (1970) addressed this question and proposed increased 'compartmentalization' in large labour markets as a counteracting force.

However, if U and V are interpreted as the ratio of the stocks to the total labour force (or total employment) and e as the corresponding relative hiring intensity, the intuitive result with respect to changes in the size of the labour market obtains. To see this let us denote the absolute number of the unemployed and of vacancies by U_a and V_a, respectively, and the labour force size by L, so that $V = V_a/L$ and $U = U_a/L$.

If the outflow intensities depend on V and U we get

$$\lambda = k * \frac{U_a}{L} ; \qquad \mu = k * \frac{V_a}{L}$$

and

$$e_a = V_a * \lambda = k * \frac{V_a * U_a}{L} = U_a * \mu ,$$

where e_a is the absolute hiring intensity.

Hence

$$e = \frac{e_a}{L} = k * \frac{V_a}{L} * \frac{U_a}{L} = k * U * V$$

So with the suggested interpretation of the variables U, V and e as ratios, our Equation (1) obtains in a form that is neutral with respect to equiproportional changes in the volume of hiring flows, unemployment and vacancy stocks and the labour force size.

It should be noted that the procedure adopted above is often used in the application of birth-and-death processes in chemistry, where the transition probabilities are made dependent on the concentration ratio rather than the volume of an active substance in a large system [cf. Gardiner (1983), Section 7.2.3, especially example b)].

6 An extension to on-the-job search

An extension of the model to include on-the-job search offers few additional conceptual difficulties. The supply side of the labour market is now made up by two independent processes, one referring to the unemployed and the other referring to the employed job searchers.

Let the stock of employed job searchers be denoted by J and the expected search duration until a new job is found by μ_J. We might expect that the stock of vacancies has the same kind of impact on μ_J as on μ (which we will in this section denote by μ_u) so that

$\mu_J = k_J * V; \qquad \mu_u = k_u * V$

As vacancies will be filled by both types of job searchers the hiring function now reads

$$e = U * k_u * V + J * k_J * V \tag{3}$$

and, repeating the above argument,

$$\lambda = U * k_u + J * k_J$$

So we get the relations

$$\begin{aligned} \mu_u &= k_u * V = T_u^{-1} \\ \lambda &= U * k_u + J * k_J = T_v^{-1} \end{aligned} \tag{4}$$

The relation between V and T_u reflects as before the ability of the labour market to turn the unemployed into employees, whereas the relation between U and T_v is now in addition also dependent on the stock of on-the-job searchers and their 'availability effect' adjusted reemployment probability. The straightforward conclusion is that changes in hiring efficiency are reflected in changes in a relation between T_v, U <u>and</u> J.

When there is no reliable information available on the stock of employed job searchers, one may take advantage of the fact that the inflow into the stock seems to be highly labour market induced. The Swedish data on the volume of this inflow show a strong procyclical pattern and Holmlund (1984) explains this observation by the impact of the changing probability of finding a job offer, i.e. the availability effect of vacancies, over the cycle. This conclusion can be formalized by stating that the propensity of employees to enter job search is an increasing function of $\mu_J = k_J * V$. Provided now that the function is not strongly non-linear in μ_J, we should expect the equilibrium value of J to be approximately the same in the face of changes in labour market conditions and hence J should tend to be stable at the same value. In that case the UT_V-relation should be only marginally disturbed by the existence of on-the-job search; its changes can be interpreted as formerly stated. [Note that the inflow into unemployment - at least in Sweden according to Björklund (1981) - is <u>not</u> labour market induced, so short run equilibrium arguments cannot apply to U.]

7 Further remarks on the concept of hiring efficiency

It should by now be clear that our analysis focuses on the efficiency of the labour market in a well-defined sense, i.e. the efficiency of the interaction between vacancies and unemployment in bringing about new hires. In this section we broaden the discussion somewhat in order to clarify some concepts. However, a comprehensive treatment of the "efficiency" of a search labour market requires a more general approach that extends beyond the border of unemployment-vacancies analysis.

In this context it is worthwhile to reflect on the conspicuous absence of an explicit hiring production function interpretation of the UV-relation in the literature between Phelps (1970) and Jackman et. al. (1984). As we have argued, this causal interpretation is really one of the important results that emerges from the application of search theory to the labour market. The hiring efficiency concept is an immediate consequence of that result. How can it be that it has been overlooked?

The answer to that question seems to be that there exists a vague 'efficiency' notion that has veiled the one inherent in the search theoretical approach to vacancies and unemployment. Regardless of which mechanism it is that governs the flow of hires, the stock of vacancies and of unemployment can always be interpreted as temporarily unsatisfied demand for hires (i.e. for new employees and for new jobs, respectively). So in a sense these stocks can be seen as indicators of how 'efficiently' the labour market satisfies the demand for hires. However, even in this sense they remain vague, as one may clearly assign the same intuitive meaning to e.g. the duration of vacancies and of unemployment.

The essential point, however, is that search theory assigns an important complementary meaning to vacancies and unemployment. They do not only constitute demand for hires, they are the 'resources' by which hires are 'produced' through stochastic interaction. The vague notion of 'efficiency' in demand satisfaction can be replaced by the strict concept of production efficiency.

It is here instructive to compare the search market model with the neo-classical one. In the ideal case the Walrasian auctioneer does all what is needed to match vacant jobs and the unemployed. The efficiency in producing hires is infinitely high by definition, so the question of varying degrees of hiring efficiency cannot arise. Stocks and durations are kept at zero. When 'frictions' have been imposed on the functioning of a neoclassical labour market, as in the first theories of the Phillips curve, the deficiency in the competence of the auctioneer has not been in establishing contacts between vacancies and job searchers. Rather it has been in his ability to adjust wages so that the stocks of vacancies and of unemployment balance exactly, before they are eliminated. With such an interpretation, the stocks indicate (lack of) efficiency in the wage equilibrating ability of the market (auctioneer).

The connection with 'hiring efficiency' is at most indirect; hiring may reach its maximum at the equilibrium wage, but such an outcome depends on the slopes of the demand and supply curves of labour. Given the traditional assumptions on this matter, hiring will be a decreasing function of the absolute difference between the stocks of vacancies and of unemployment (one of them ought to be zero). Clearly the conceptual differences between a search market and a neo-classical market model are so great, that one cannot heedlessly make reference to both of them.

This does not imply, however, that wage adjustments play no role in the observed hiring efficiency of a search labour market. On the contrary, as the analysis in Chapter II demonstrates, the probability of a hire at a recruiting firm is the product of the probability of a contact with a job applicant and the probability that the offered wage exceeds his reservation wage. Generally speaking, the hiring efficiency should increase if the wage offer distribution over vacancies is shifted to the right in comparison to the reservation wage distribution over job applicants. Conditional on a contact having occurred, the probability of a hire has increased. As we claimed in Section 2, the efficiency factor k should reflect such effects.

When there are more vacancies available, so that the probability of a wage offer to any job applicant increases, he should respond by increasing his reservation wage [cf. the studies referred to in Section 2 as well as the theoretical analysis in Albrecht, Holmlund and Lang (1986)]. This is true even if the wage offer distribution is unchanged. Hiring efficiency may decrease because of this 'supply' effect, as was noted in Section 2. This special type of relative shift in wage distributions is of cyclical character, so k will not be cyclically stable, if the basic Holt model is imposed a priori. As the correlation between the stock of vacancies and the position of the reservation wage distribution is very high, the supply effect of vacancies shows up just as a non-linearity in the estimated relation between e and V.

Some kind of a stock effect on the wage distributions is always to be expected. Consider, for example, a change in the stock of the unemployed. As the analysis in Chapter II demonstrates, an effect on the wage offer of a firm exists, although it is not clear-cut. Following the traditional assumption, according to which there is no change in the composite reservation wage distribution facing the firm, the wage offer should increase, so the wage offer distribution shifts to the right as a response to an increased unemployment stock. As a consequence hiring efficiency should increase by this 'supply' effect, induced by the increased availability of unemployed job searchers. On the other hand, it is conceivable or even likely that the 'supply' effect of unemployment works in the opposite direction, according to the results of Chapter II. If so, the supply effects of vacancies and of unemployment may tend to offset each other, as the stocks are in general inversely correlated.

When one of the wage distributions begins to move the other one should in general respond to this shift. How such a sequence of interdependent shifts evolves over time is the kernel of the yet unexplored wage dynamics of a search labour market. Some aspects of this issue should be noted, though, especially as it has been given a rather one-sided treatment in other research contributions.

A change in the wage offer distribution should lead to a change in reservation wage distribution in the same direction, but there is in general no reason to think that the latter change should completely offset the first one, so that any recruiting firm will find itself in an unchanged position as to its hiring intensity (but see below!). As the unemployed make use of more favourable conditions in the form of more vacancies as an opportunity to decrease their expected sojourn time in unemployment, more favourable wage offer conditions should in part have the same effect.

This observation justifies a comment on the empirical studies on the determinants of unemployment durations, referred to in Section 2. Besides the 'availability' and 'supply' effects of vacancies, the data should also be influenced by a 'wage offer' effect. This wage offer effect is presumably correlated with the vacancy effects, because the wage offer distribution should shift at such phases of the business cycle, when the stock of vacancies is large. The 'wage offer' effect being unspecified in an econometric model, the result is of course to increase the reported 'availability' effect and decrease the 'supply' effect at high levels of vacancies.

A more general remark concerns the fashionable interpretation of job search models as to the unemployed being completely satisfied with their present expected unemployment duration, irrespective of the position of the wage offer distribution. If the wage offer distribution shifts to the right, so will the reservation wage distribution to a completely offsetting extent. There exists no genuine 'wage offer' effect. Hence only unperceived wage offer distribution changes can affect the unemployment duration. If this account of labour market phenomena were true to any significant extent, it would be detrimental to the hiring efficiency concept. Hiring efficiency is registered as temporarily increasing just because the unemployed are fooled into employment by false expectations; eventually, as the job searchers realize their mistake, they quit into unemployment and hiring efficiency is restored to its lower level.

Although the above version of unemployment dynamics happened to be associated with the search theory of the labour market at an early stage, it is neither a part of search theory per se, nor has empirical evidence validated it. We are prepared to judge the effect of misperceived wage offer distribution changes as unimportant in practice.

Nevertheless, the idea that job-searchers accept job offers without knowing the true characteristics of the offer is clearly not unrealistic. In Holmlund and Lang (1985) a model is presented in which there exist job attributes, that are unobservable, unless the job offer is accepted and tried out. This mechanism produces a nice explanation of the high quitting rate of employees with short job tenures. Clearly, such a kind of uncertainty introduces the possibility of 'hiring failures' in the sense that hiring should not have occurred, if the job attributes were perfectly known beforehand. When the truth is revealed, the hire results in a quit.

The conclusion to be drawn from these observations is twofold: the hiring efficiency concept is just one aspect of the efficiency of job matching and it is a reliable indicator to the extent that the frequency of 'hiring failures' because of false or uncertain perceptions does not change significantly. It is a maintained hypothesis of the present analysis that such changes can be disregarded. Of course, this hypothesis is testable by observing the on-the-job search behaviour of employees; casual inspection of the figures for Sweden, presented in Holmlund (1984), supports it, as no conspicuous change in this behaviour can be observed.

Having discussed the 'quality' of realized hires as the output of the hiring production function, it is quite natural to address the question of the 'quality' of the inputs. Leaving the vacancies and the employed job searchers aside, we could rephrase the question: are the unemployed job searchers the only input in the production of hires? By definition the unemployed are those in the labour force without a job at present but actively searching for one. This definition seems to be exhaustive, but there are pitfalls. The category of the unemployed borders in a fuzzy way both to those outside the labour force - the discouraged workers - and to those engaged in labour market programmes. One may argue that hiring efficiency should reflect the ability of the labour market to turn these latter categories into regular employees. After a period of long unemployment durations both the group of discouraged workers and of individuals in labour market programmes are likely to increase. As they are not reported as unemployed, hiring efficiency does not appear as low as it is. When they return as unemployed, because of better labour market prospects, observed hiring efficiency gets an artificial downward bias.

One has to be careful here, though. To include the marginal groups as input in the production of hires, we must assume them to be prepared to search for a job at short notice. The fact that their search is at present negligible must reflect a well-founded belief that search is practically useless. Consequently, we should expect such a situation to arise primarily when the unemployment process has become approximately an I-E-process. As we noted above, hiring is then determined by vacancies. In such a case an assessment of the 'true' unemployment figure is justified rather as an estimate of the amount of idle resources that could be used in hiring production under more favourable circumstances, not as an estimate of the correct hiring efficiency measure.

When the labour market is fairly balanced, so that unemployment forms an I-D-process and the Holt model applies, the effect of an (over)ambitious labour market policy is to take unemployed job searchers out of the market, at least temporarily. The result on the production of hires can be looked upon in two ways. The first view focuses on the market process and disregards the individuals in labour market programmes completely. The effect of the programmes is consequently a reduction in unemployment and an increase in the duration of vacancies at an unchanged hiring efficiency. The other view regards the individuals as being 'unemployed' but with very small search intensity/high reservation wages. The effect is consequently an unchanged unemployment stock but increased vacancy durations because of decreased hiring efficiency. We adhere to the former view as being in the spirit of the present analysis; the formally unemployed (according to Labour Force Survey definitions) are those engaged in the market interaction with vacancies and any extension to include other groups is bound to be fraught with arbitrary judgements.

Let us at last make clear what should by now be obvious; we do not venture into the area of analyzing the social efficiency of search labour markets, in which the realized allocation of search costs and job positions is compared to the socially optimal one. The interested reader should consult Pissarides (1984) with references for that type of analysis.

8 Empirical evidence for Sweden

We are now going to apply our analysis to Swedish labour market data, the reliability of which we are able to assess.

As has been made clear we need figures on the stocks and durations of vacancies and of unemployment. As regards the employed job searchers there are some figures available since 1976 for those who search through the public employment offices. We do not make use of them except as a complement at the end of this section, however, as we need series for longer periods of time.

Data on vacancies are found in the monthly statistical reports of the Labour Market Board. They are made up by all unfilled job positions that are notified by employers at the public local employment offices. As employment exchange is a public monopoly in Sweden, all notified vacancies are included in the statistical reports.

The vacancies are reported as being notified for the first time during the month as well as remaining unfilled at the end of the month. They are divided into different categories e.g. by occupation. We have used the figures for manufacturing occupations besides the figures for the total.

Denoting the number of unfilled vacancies at the beginning and the end of a month by V_0 and V_1, respectively, and the number of new notified vacancies by $v * t$ (where v is an intensity and t is the length of the month), we have, assuming constant inflow and outflow intensities during the month,

$$V_0 * e^{-\lambda t} + \frac{v * t}{\lambda * t} (1 - e^{-\lambda t}) = V_1$$

This equation determines λt and, as t is given, λ. Hence we obtain an estimate of the expected value of T_v. [The derivation of the equation is presented in detail in Schager (1981), pp. 410-411.]

Apart from the homogeneity of the vacancies, which condition is always violated to some extent in empirical work, the most pertinent question concerns the effect of some vacancies not being notified at the employment offices. In 1977 compulsory notification was introduced in Sweden, taking effect gradually in subsequent years as the new rules were extended into more regions. A study, carried out at the first stage of the introduction, indicated that the flow of new vacancies was increased by forty percent and the stock of unfilled vacancies by fifty percent as a consequence of the compulsory notification [see Labour Market Board (1983)].

The reliability of this result may be questioned as to the impact on vacancy figures in recent years. As the inflow of notified vacancies during this time has reached unprecedented lows, an upward correction of earlier vacancy figures with as much as forty percent would produce an implausibly sharp decline in 'real' vacancies for the last years. Presumably the effect is of less magnitude than was reported at first.

We are left with the disturbing conclusion that the stock figures on vacancies are not consistent over the two last decades and that a correction is not easily accomplished. There is, however, some encouraging evidence, too. With regard to vacancies in manufacturing occupations, we know that the notification rate has traditionally been much higher than for salaried occupations or for occupations in the public sector (ibid.). Hence we are entitled to assume that the stock of notified vacancies in manufacturing occupations has been far less influenced by the new legislation than the total vacancy stock, so that the inconsistency over time for this vacancy category is not too disturbing. Another reason for paying special attention to data on manufacturing occupations is of course the higher degree of homogeneity within this group.

With regard to the calculated duration of vacancies, it is clear that this measure is invariant at equiproportional changes in the stock and inflow variables. If we accept the reported magnitudes of the overall effect on these variables of the introduction of compulsory vacancy notification, the result should be an increase of the duration measure by around seven percent as the compulsory notification rules took full effect (calculated on an equilibrium basis). A disturbance of such a magnitude is not likely to affect our results. For the calculated vacancy duration in manufacturing occupations the disturbance can be expected to be even smaller.

Turning to the quality of the unemployment figures, the situation is reversed compared to that of vacancies, insofar as the stock figures are more reliable than the calculated duration measures. Data on unemployment are obtained from the Labour Force Surveys, which are carried out according to international standards. Although the stock figures are subject to sampling errors, these can be safely ignored for yearly averages of both total unemployment and unemployment in manufacturing occupations, especially from 1970 and onwards when the surveys have been made on a monthly basis. Earlier the survey figures were based on only one month for each quarter. Nevertheless, one can conclude that the stock figures are consistently calculated since 1963.

There is, however, no immediate information on the inflow into unemployment between two consecutive surveys, so we cannot apply the same method of calculating the outflow intensity as we did for vacancies. The problem of estimating the expected duration of unemployment from Labour Force Surveys is notorious, as is made clear in Björklund (1981). One approach, which is applied in Axelsson and Löfgren (1977), is to make use of the figures on the number of unemployed persons with different durations of experienced unemployment spells, published in the Survey reports. We will use this approach, too, but in a slightly different way than what these authors did.

From the Survey reports we have the number of persons who have been unemployed for at most four weeks. Assuming that four weeks is also the period between the consecutive monthly surveys, which should hold approximately, we conclude that the remaining part of the stock of the unemployed (with unemployment spells of at least five weeks) consists only of those individuals who were reported unemployed in the preceding survey. Hence we have

$$U_0 * e^{-\mu t} = U_1 - U_{1,T_u} < 4$$

and can calculate μt where t is equal to four weeks and consequently the expected duration of unemployment T_u.

The measure of T_u, calculated in this manner is as good as the available data permit, but its precision is presumably not very accurate. The elimination of the inflow component during the period between consecutive surveys is only approximate and the sampling error in the monthly estimates of the stock components may not be negligible. It is not surprising then, that the calculated monthly outflow intensities show a more unstable pattern for unemployment than for vacancies.

To make a check of the calculated outflow intensities we have applied the equation above to the yearly averages of the respective stocks. It turns out that such a calculation produces almost the same figures as a yearly average of calculated monthly intensities. In fact, the more simple calculation procedure must be applied to the years before 1970, where data are not available for every month. So for the pre-1970 years we calculate a yearly average directly by solving

$$U * e^{-\mu t} = U - U_{T_u < 4}$$

for μt, U and $U_{T_u < 4}$ being yearly averages of the stocks.

Unfortunately, the data presented in the Survey reports are not so detailed that we can calculate the duration of unemployment in manufacturing occupations. This deficiency ought to be remedied in a more detailed study, but here we must be content to consider the total labour market only.

The figures on reported stocks and calculated durations of vacancies and unemployment for the manufacturing occupations and the total labour market are reproduced in Table I.1 and Table I.2, respectively.

In Figures I.1-5 the observations on stocks and durations are depicted as to illustrate the relations between these variables.

Figure I.1 shows that there is no stable relation between the rates of unemployment and of vacancies in manufacturing occupations. The same is true for the total labour market; the observations are not graphed here, as they are also depicted in e.g. Klau and Mittelstädt (1985), Chart 7. As has been argued in this paper, this lack of stability in the UV-relation should not surprise us.

Figures I.2-4 all depict stock-duration relations of type (2'), changes in which should reflect changes in hiring efficiency. They convey very interesting information in that they all point to <u>a marked deterioration in efficiency around the years 1967-69</u>. There are fairly few observations available for the sixties. Still the evidence is that the years 1963-67 produce observations generated by one hyperbolic relation, the years 1969-86 by another; 1968 seems to be a year of transition.

As it happens the quality of the data is best for the stock of unemployment and the duration of vacancies as we just demonstrated. Hence the $T_V U$-relation should be the most reliable one, provided that the job-search process of the employed does not change independently of that of the unemployed. Utilizing the immediately observable shift in the curves 1967-69, we introduce a dummy variable to take care of this shift, exclude the 1968 observation and obtain as OLS estimations for the period up to 1984 (t-values in parenthesis):

<u>Manufacturing occupations (Figure I.2)</u>

$$\ln T_V = \underset{(10.77)}{1.34} + \underset{(9.21)}{1.02}\, D - \underset{(-7.41)}{1.01} \ln U, \quad \bar{R}^2 = 0.83$$

D = 0 for 1964-67; D = 1 for 1969-84

<u>Total labour market (Figure I.4)</u>

$$\ln T_V = \underset{(22.15)}{1.04} + \underset{(12.79)}{0.59}\, D - \underset{(-9.64)}{0.72} \ln U, \quad \bar{R}^2 = 0.89$$

D = 0 for 1963-67; D = 1 for 1969-84

We want to stress that these estimates as well as those which follow should not be interpreted as a test of an hypothesis of a shift in hiring efficiency: the "hypothesis" is generated by the data themselves, as the comments in the preceding paragraph made clear. The estimates are a quantification of the parameters of the hiring production function and their shifts. The intercepts of the estimated equations are estimates of -ln k.

These equations confirm the strong impression given by visual inspection of the figures. Something obviously happened to Swedish hiring efficiency in the late sixties. It is striking that this deterioration took effect so rapidly and that the lower level of efficiency has prevailed up to now. No obvious reason for this sudden change is easily produced. It may be noted that as the traditional UV-curve also moved at first around 1968, the seeming shift in the functioning of the labour market was in issue considered by Swedish economists in the early seventies; no real conclusions emerged from those discussions [cf. Holmlund (1976)].

There is an interesting piece of evidence, however, in the figures on interregional mobility. In 1969 there occurred a sharp change in the migration pattern to the effect that people stopped moving from the rural areas to the big city areas and began to move in the opposite direction [cf. e.g. Jonsson and Siven (1986), p. 30]. If the structure of labour demand did not change correspondingly, a decrease in hiring efficiency is to be expected (cf. the discussion on aggregation effects below). It appears that a new investigation, taking a regional perspective in the spirit of Holmlund's studies, is warranted.

The same estimation procedure applied to the T_uV-relation yields

<u>Total labour market (Figure I.3)</u>

$$\ln T_u = \underset{(41.57)}{1.82} + \underset{(7.75)}{0.40}\, D - \underset{(-8.97)}{0.59} \ln V, \quad \bar{R}^2 = 0.91$$

D = 0 for 1963-67; D = 1 for 1969-84

This result confirms the conclusion from the preceding estimations, although unfortunately, the calculation procedure to obtain a measure of the duration of unemployment has to be changed just in the critical period between the years 1969 and 1970.

As regards the effect of the introduction of compulsory vacancy notification on the estimated T_uV-relation, a look at the residuals reveals that an increase of the reported vacancy stock of a magnitude of up to twenty percent is compatible with a roughly unchanged relation before and after the introduction. A reported stock increase of fifty percent, as was initially indicated, would of course imply higher hiring efficiency in recent years. However, as we argued, such a strong change in the notification rate seems implausible; the estimated T_vU-relation, which is just marginally disturbed by the change in notification rules, indicates no substantial change in hiring efficiency in these years.

As the estimated equations stand, they seem to indicate a linear effect of the unemployment stock on the vacancy filling probability in manufacturing occupations, while the corresponding effect for the total labour market shows an absolute elasticity of less than one. However, to investigate the properties of the hiring function we must take into account the strong (although not perfect!) correlation between U and V. Hence we estimate an equation

$$T_v = \frac{1}{k} * U^b * V^c$$

from which we can deduce the elasticities of the outflow intensity e with respect to U and V as

$$\frac{V}{T_v} = e = k * U^{-b} * V^{1-c}$$

OLS-estimation yields

Manufacturing occupations

$$\underset{}{\ln T_v} = \underset{(7.96)}{1.01} + \underset{(10.43)}{0.93\ D} - \underset{(-3.84)}{0.60\ \ln U} + \underset{(3.61)}{0.26\ \ln V}, \quad \bar{R}^2 = 0.90$$

D = 0 for 1964-67; D = 1 for 1969-84

Total labour market

$$\ln T_v = \underset{(12.75)}{0.80} + \underset{(13.87)}{0.51\ D} - \underset{(-2.69)}{0.30\ \ln U} + \underset{(4.38)}{0.35\ \ln V}, \quad \bar{R}^2 = 0.94$$

D = 0 for 1963-67; D = 1 for 1969-84

The presence of multicollinearity prevents us from assigning much weight to the separate coefficients of ln U and ln V, but taken together they imply elasticities of hiring with respect to U and V which amount to 1.34 for manufacturing and 0.95 for the total. (The corresponding calculation on T_u yields a value of 1.31 for the total but should be less reliable on data quality grounds.)

The estimated sum of the availability effects (elasticities) of the stocks of vacancies and unemployment is consequently less than two. This could be seen as an indication of the existence of fairly strong supply effects. Nevertheless, with reference to the discussion in Section 7 we do not favour this explanation to any larger extent. In general, the stocks are correlated in such a way that the supply effect of vacancies on the reservation wages of the unemployed is counteracted by a supply effect of unemployment on the wage offers of firms. In addition there might be an offsetting independent 'wage offer' effect. We like to draw attention, however, to the observation for 1977 (see Figures I.1 and I.2). In that year the stock of vacancies was unusually small compared to the (small) stock of unemployment. The supply effect of vacancies was consequently small compared with e.g. the situation in 1970, when unemployment was of the same magnitude as in 1977. Obviously, the seeming tendency to increased hiring efficiency in 1977 may reflect an unusual combination of small supply effects of vacancies on the reservation wage distribution and strong supply effects of unemployment on the wage offer distribution.

In general, though, we find the suggestion in Holt (1970) more appealing that lower measured elasticities of the outflow into employment than those predicted by the basic Holt model originate from heterogeneities in the labour market. This is in accordance with our findings, where the elasticities are larger for manufacturing occupations than for the total labour market. It would be an interesting research project to study the relations between the relevant variables for a labour market which is more narrowly defined with respect to occupations and geographical proximity.

We should add that aggregation of homogeneous (and equally efficient) submarkets, where the basic Holt model applies, does not automatically lead to estimated elasticities of the aggregate outflow intensity with respect to aggregate stocks that are less than one, when the intensities in the submarkets depend on the ratio of the stocks to labour force size rather than on the sheer volume of the stocks. As a consequence and in contrast to the results presented in Holt (1970), compartmentalization is not a sufficient condition for reducing the outflow intensity at given aggregate stocks; if either the unemployment or the vacancy ratios are the same in all submarkets, compartmentalization will have no effect. To get a reduction in the aggregate outflow intensity, it must hold that vacancies and unemployment are distributed in an imbalanced way, so that vacancies are concentrated to some submarkets, unemployment to others. This is likely to be the usual case, but if some submarkets have more of both vacancies and unemployment, the estimated aggregate elasticities may even exceed one. Formally, the aggregate outflow intensity is linearly dependent on

$\sum_i \frac{V_{ai} * U_{ai}}{L_i}$, i being the submarket index,

but we estimate its dependence on

$$\frac{\sum_i V_{ai} * \sum_i U_{ai}}{\sum_i L_i} = \frac{V_a * U_a}{L} ;$$

from this difference the above conclusions follow.

As it happens that the outflow of vacancies for the total labour market seems to be nearly homogeneous of the first degree in the stocks of unemployment and of vacancies according to our estimates - by chance presumably - we should be prudent and check whether the stock-duration relation is seriously disturbed by cyclical factors. Consequently, we form the duration-duration relation, proposed by Jackman et al. Estimation of T_u and T_v yields

<u>Total labour market (Figure I.5)</u>

$$\ln T_u = \underset{(21.73)}{2.46} + \underset{(11.56)}{0.84}\, D - \underset{(-7.17)}{0.98} \ln T_v, \quad \bar{R}^2 = 0.87$$

This result confirms the general conclusion from earlier estimations. The shift in hiring efficiency around 1967-69 and its subsequent stability are robust results.

Finally, we should consider the available data on the stock of on-the-job searchers. As we showed above, changes in this stock may influence the relation between the duration of vacancies and the unemployment rate.

The only source for this variable is the data on job searchers registered at the local employment offices. This is a fairly recently established statistical series, and figures are available since 1976 only. In Table I.3 the figures on the stock of job searchers, who are not registered as unemployed (and prepared to take a job immediately) are reproduced.

As we see, the stock does not vary very much during the years 1976-81, especially not in comparison with the unemployment figures. This is in accordance with our suggestion that entrance into on-the-job search is largely labour market induced. However, in the following three years there has been a sharp increase in the stock of job searchers who are not unemployed. This should produce an inward shift in the $T_V U$-curve for the total labour market, ceteris paribus, but no such shift can be traced for these years. On the surface there seems to have occurred an additional deterioration in hiring efficiency since 1982.

The series on registered job searchers has hardly been used for analytical purposes yet, however, and its properties are not well known. There is some evidence that in recent years the registered job search of the unemployed has increased more than corresponds to the Labour Force Survey figures, because more people have had reason to 'indicate' unemployment status without really searching for a new job. Whether there is some recent systematic trend also in the remaining part of the registered stock of job searchers we do not know; the published figures are too crude to allow any investigation of the issue. We prefer to be prudent and just conclude that by also taking the on-the-job search process into account, we do not find any evidence in favour of a better development of hiring efficiency in Sweden in recent years than what is indicated by the $T_V U$-relation.

9 Summary and concluding comments

In this chapter we have demonstrated that the relation between the stocks of unemployment and of vacancies is not stable at unchanged degree of hiring efficiency as has traditionally been perceived. By exploiting the notion of unemployment and vacancies forming interacting stochastic processes - which need not be in equilibrium - we have shown that hiring is a function of vacancy and unemployment stocks in a production function sense. A stable relation between the stocks is but an isoquant of this hiring production function.

If the interacting processes have such properties as to produce a model of the type that was proposed by Charles Holt in his seminal contributions, it is a relation between vacancy stocks and unemployment durations or between unemployment stocks and vacancy durations that is stable in the face of unchanged hiring efficiency. If the processes are not as efficient as in the Holt model in producing new hires (presumably because of heterogeneities that have not been controlled for) a relation between the durations of unemployment and of vacancies may serve as a complementary indicator of changes in efficiency.

If, finally, the labour market has been constantly depressed or overheated for some time, hiring may be a function of only the vacancy or the unemployment stock, respectively. There is no interaction on the margin between the processes, and hiring efficiency is reflected in either the vacancy duration (the depressed case) or the unemployment duration (the overheated case).

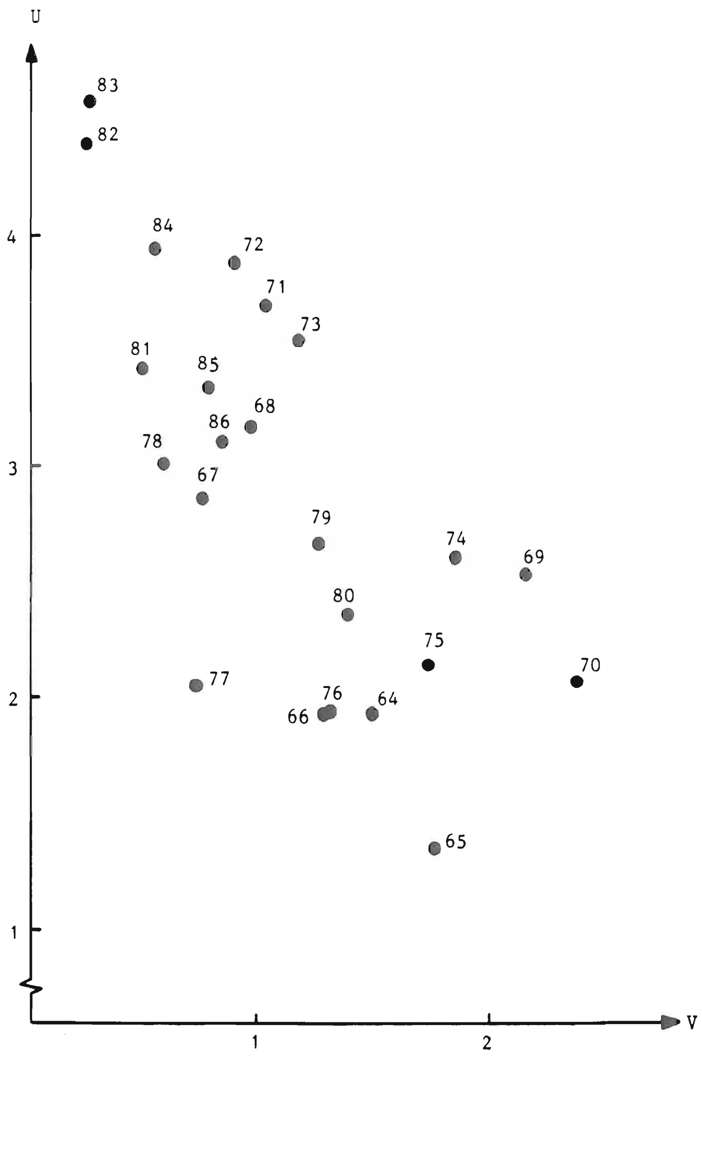
The results of the theoretical analysis have been applied to Swedish labour market data. The investigation reveals very clearly that there occurred a marked deterioration in Swedish hiring efficiency in the late sixties, which has not yet been compensated. This is a new discovery, which is not possible to trace with the help of traditional UV-curve analysis.

It is indeed justified to suggest that the labour markets of other countries, the hiring efficiency of which has for a long time been evaluated by an erroneous UV-relation, should also be made subject to investigations of the same kind as far as the availability and quality of data permit. Let us point to just one interesting piece of evidence, given in Klau and Mittelstädt (1985). In our introduction we quote these authors as they describe the outward shifts of the UV-curve in several countries since the late sixties; we omitted the end of their sentence, which reads '... most visibly in the case of the United States'. Of course, it is counterintuitive to think of the U.S. as having the worst development of hiring efficiency among the OECD countries! The answer to this seeming paradox is found in the following lines: 'In selected European countries there has been a trend decline in hirings ... In contrast, in the United States a rising trend in layoffs was outweighed by a rising trend in hirings ...' (ibid.). On the basis of the analytical results of the present chapter, the conjecture is of course that the outward shifting UV-curve in the U.S. reflects increased hiring flows and not decreased hiring efficiency.

Let us finally point to a more far-reaching possible consequence of our reinterpretation of the UV-curve. The stability of this curve became a maintained hypothesis as it was perceived as a prerequisite for a stable Phillips curve. Now we have seen that under stable efficiency conditions there is instead a stable relation between the unemployment stock and the duration of vacancies. A few years ago I argued on fairly intuitive grounds that the duration of vacancies should be a good indicator of excess demand in the labour market and consequently a good explanatory variable in a nominal wage increase equation; I managed to show that wage drift in Sweden was indeed best explained by this measure [Schager (1981); cf. also Bosworth and Lawrence (1987)]. Most important with respect to the present analysis is the fact that a shift in wage drift behaviour in the late sixties, left unexplained by the unemployment variable, is taken care of by the vacancy duration variable.

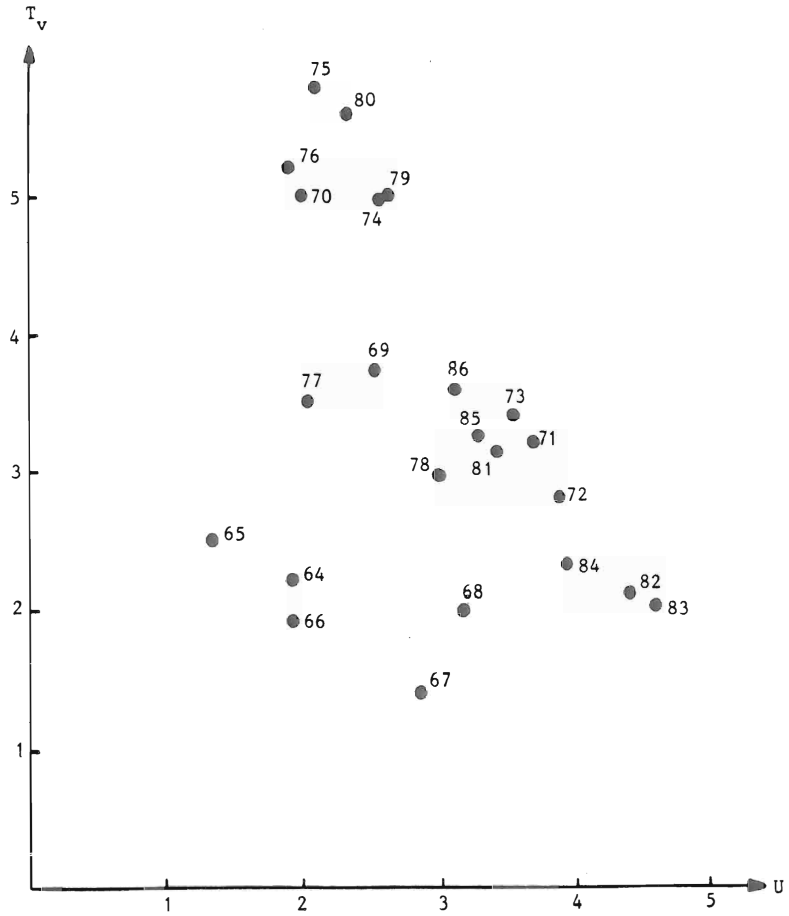
In Chapter II of this book we show theoretically that the effect of unemployment on the wage behaviour of an individual firm does work via its impact on the expected duration of the vacancies at the firm. We also show that decreases in the unemployment stock, which affect the expected duration of vacancies but leave the wage sensitivity of the duration (largely) unchanged, make the firm increase its wage. Consequently, the analysis of this chapter does not only give rise to a reinterpretation of the UV-curve; it also constitutes a part of restoring a Phillips curve relation, establishing a causal chain running from the labour market to aggregate wage increases, but substituting durations of vacancies for unemployment.

Figure I.1 Vacancies (V) and unemployment (U) in manufacturing occupations, percent, 1964-86



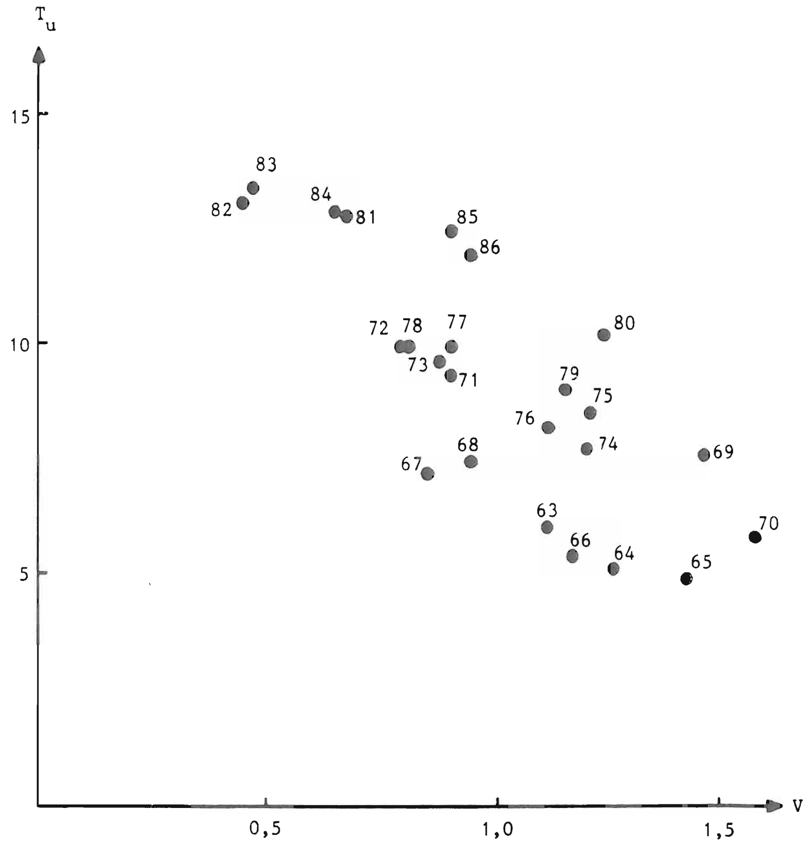
Source: See Table I.1.

Figure I.2 Duration of vacancies (T_v), weeks, and rate of unemployment (U), percent, manufacturing occupations 1964-86



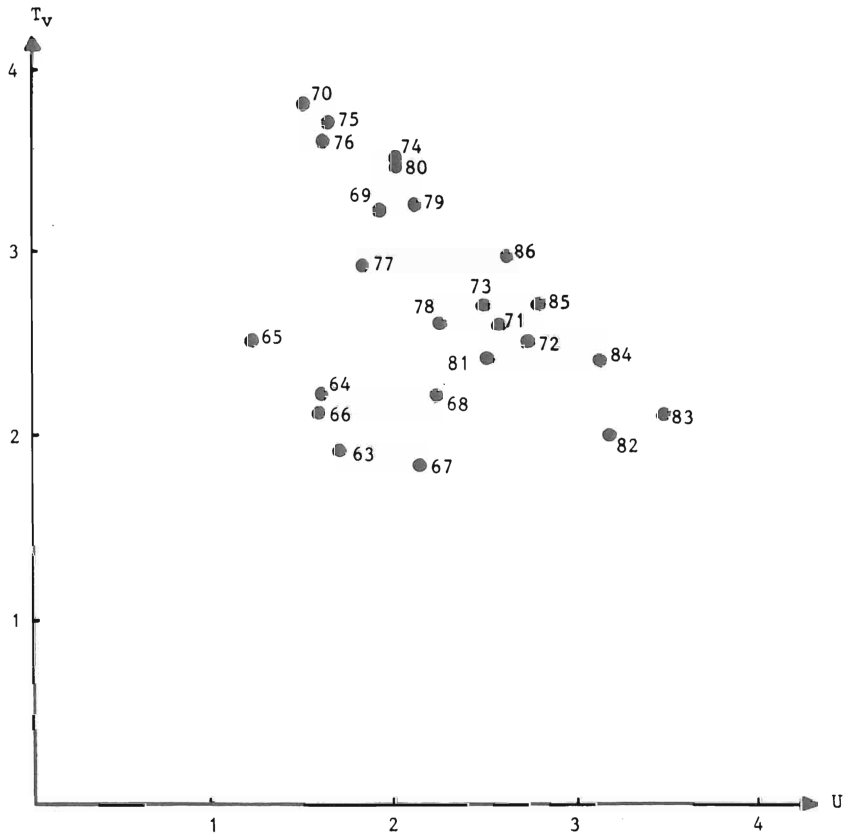
Source: See Table I.1.

Figure I.3 Duration of unemployment (T_u), weeks, and rate of vacancies (V), percent, total labour market 1963-86



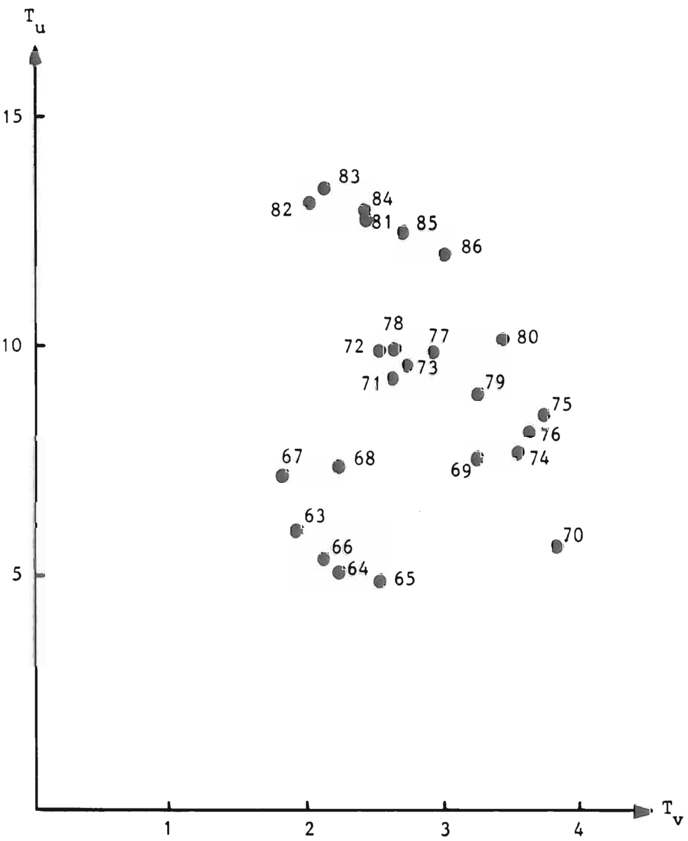
Source: See Table I.2.

Figure I.4 Duration of vacancies (T_v), weeks, and rate of unemployment (U), percent, total labour market 1963-86



Source: See Table I.2.

Figure I.5 Duration of unemployment (T_u), weeks and duration of vacancies (T_v), weeks total labour market 1963-86



Source: See Table I.2.

Table I.1 Manufacturing occupations

	T_V, weeks	V, %	U, %
1964	2.2	1.49	1.92
1965	2.5	1.76	1.34
1966	1.9	1.28	1.92
1967	1.4	0.76	2.86
1968	2.0	0.98	3.17
1969	3.7	2.16	2.53
1970	5.0	2.39	2.07
1971	3.2	1.04	3.70
1972	2.8	0.90	3.88
1973	3.4	1.19	3.54
1974	5.0	1.85	2.59
1975	5.8	1.74	2.13
1976	5.2	1.31	1.93
1977	3.5	0.73	2.05
1978	3.0	0.60	3.01
1979	5.0	1.26	2.66
1980	5.6	1.39	2.35
1981	3.1	0.50	3.42
1982	2.0	0.25	4.41
1983	2.0	0.27	4.59
1984	2.3	0.55	3.94
1985	3.2	0.76	3.32
1986	3.6	0.83	3.10

T_V = Duration of vacancies, based on monthly calculations of the outflow probability.

V = Reported stocks of vacancies as a ratio to labour force.
Source: Labour Market Statistics, Labour Market Board.

U = Reported stocks of unemployment as a ratio to labour force.
Source: Labour Force Surveys, Central Bureau of Statistics.

Table I.2 Total labour market

	T_v, weeks	T_u^M, weeks	T_u^Y, weeks	V, %	U, %
1963	1.9		6.0	1.12	1.66
1964	2.2		5.1	1.27	1.56
1965	2.5		4.9	1.44	1.18
1966	2.1		5.4	1.18	1.56
1967	1.8		7.2	0.86	2.11
1968	2.2		7.4	0.95	2.22
1969	3.2		7.6	1.48	1.89
1970	3.8	5.7	(5.8)	1.59	1.51
1971	2.6	9.3	(8.5)	0.99	2.54
1972	2.5	9.9	(10.0)	0.80	2.70
1973	2.7	9.6	(10.1)	0.89	2.46
1974	3.5	7.7	(8.4)	1.21	1.99
1975	3.7	8.5	(8.6)	1.22	1.63
1976	3.6	8.2	(8.4)	1.12	1.60
1977	2.9	9.9	(9.4)	0.91	1.80
1978	2.6	9.9	(9.6)	0.82	2.23
1979	3.2	9.0	(9.6)	1.16	2.07
1980	3.4	10.2	(9.7)	1.25	1.98
1981	2.4	12.8	(11.6)	0.69	2.48
1982	2.0	13.1	(13.2)	0.46	3.15
1983	2.1	13.4	(13.1)	0.48	3.46
1984	2.4	12.9	(13.3)	0.66	3.09
1985	2.7	12.4		0.83	2.85
1986	3.0	12.1		0.89	2.67

Definitions of T_v, V and U: see Table I.1.

T_u^M = Duration of unemployment, based on monthly calculations of the outflow probability.

T_u^Y = Duration of unemployment, based on a yearly average calculation of the outflow probability.

Table I.3 Registered job searchers

	Unemployed, thousands	Not unemployed, thousands
1976	72.9	61.0
1977	81.7	64.2
1978	102.6	76.0
1979	98.7	76.1
1980	94.6	66.2
1981	123.0	69.2
1982	161.3	80.3
1983	178.3	95.4
1984	159.3	111.1

Unemployed = registered job searchers in 'category 1' according to Labour Market Board definitions.

Not employed = all registered job searchers except those in 'category 1'

Source: Labour Market Statistics, Labour Market Board.

APPENDIX TO CHAPTER I

A COMMENT ON THE VACANCY - UNEMPLOYMENT MATCHING PROCESS PRESENTED IN JACKMAN, LAYARD AND PISSARIDES (1984), 'ON VACANCIES'

In the paper Jackman et al. (1984) the interaction between vacancies and unemployment as to bring about new hires (or engagements) is very briefly specified on page 6, especially in the first footnote.

The model makes no distinction between realized contacts between unemployed job applicants and unfilled vacancies, on the one hand, and realized engagements, on the other. This feature is not essential for our discussion, however.

The model specifies that every unemployed person makes one and only one application per period. The number of unfilled vacancies at the beginning of the period is denoted by v. The paper states that the probability that a vacancy remains unfilled during the period is $(1-\frac{1}{v})^u$, where u is the number of unemployed at the beginning of the period. Furthermore, it is claimed that $(1-\frac{1}{v})^u$ can be approximated by $e^{-u/v}$. This is obviously done by letting u and v go to infinity in such a way that the ratio u/v approaches some bounded value.

It is clear from this statement that the authors apply a special static allocation model, which can be generally phrased as 'put u unnumbered tokens in v boxes; each box can contain any number of tokens' [see e.g. Chung (1979), section 3.3]. This last assumption is important because it means that an unemployed individual searches over all initially unfilled vacancies v regardless of the possibility that it has already been filled by another applicant. If it has been, the unemployed individual who encounters it is 'absorbed' into a non-search state by the assumption that he searches only once during the period.

It seems clear from this specification of the search mechanism that the model must be interpreted as a typical shortrun one. This is also confirmed by a look at the original application of the model as it appears in Hall (1977). The proper way to analyze both the transient behaviour and the equilibrium of the hiring process is consequently to extend the model to a sequence of periods.

The main deficiency of the analysis given both by Jackman et al. and by Hall is the neglect to make this multi-period extension. They argue as if the situation at the end of the first period were the relevant state to focus on. As a consequence their model is made to produce curious results. Before discussing that point, however, I want to propose a better technique for analysing the search and contact processes.

As is often the case, the application of a discrete time period model leads to a fairly unwieldy analysis; the continuous time model, which focuses on the instantaneous transition probabilities of the process is much more attractive and eliminates automatically some features that are really unnecessary technical artefacts in the discrete model.

Instead of looking at the time period [0,1] we consider the very short time period [0,dt]. It is natural to assume that the total number of contacts between job applicants and vacancies during [0,dt] has a probability distribution such that there is a positive probability that one contact occurs and a zero probability that more than one contact occurs. In the model of Jackman et al. the generation of a contact between an unemployed individual and an initially unfilled vacancy is deterministic, but here we assume that every unemployed individual gets in touch with a vacancy with a constant probability μ during [0,dt]. Hence the probability that one vacancy will be contacted during [0,dt] is $u \cdot \mu$ (and the probability that more than one vacancy is contacted is zero).

This description of the contact process corresponds to the standard assumptions of birth-and-death processes [see e.g. Cox and Miller (1965) for a general discussion of such processes and Bartholomew (1973) for their application within the social sciences]. In an infinitesimally short period of time the system moves from the state (u, v, e) to the state (u-1, v-1, e+1) with probability $u \cdot \mu$ and remains in the original state with probability $1 - u \cdot \mu$ (u is the number of unemployed, v the number of vacancies, e the number of hires). Given that $e = 0$ at $t = 0$, the expected state of the system at the end of the time period [0,t] is $[\max\{u-v; ue^{-\mu t}\}, \max\{0; v-u(1-e^{-\mu t})\}, \min\{v; u(1-e^{-\mu t})\}]$. The number of hires up to time t is determined by u and μ, provided that $v - u(1-e^{-\mu t}) \geq 0$; otherwise the system has at time t reached an equilibrium state [u-v, 0, v]. With $u < v$ the equilibrium state is [0, v-u, u] and it is reached when t goes to infinity.

Jackman et al. claim that their system is in the state $[u - v(1-e^{-u/v}), v \cdot e^{-u/v}, v(1-e^{-u/v})]$ at the end of the first "period". This is not the expected state at $t = 1$ in the continuous time process, because the assumption that the unemployed search over vacancies that might have been filled and then stop search until the end of the period is of no consequence when the period is very short. The unemployed search all the time and only over unfilled vacancies.

As we remarked Jackman et al. pay as much attention to their one-period end state as if it were an equilibrium state and not merely a transitory phenomenon, the properties of which arise mainly because of the fixed period assumption at that. Let us, however, reconstruct the birth-and-death process in continuous time which has the basic properties and an equilibrium state equal to those given by Jackman et al.

Consider the process which moves the state (u_s, u_p, v_r, e) to $(u_s-1, u_p, v_r-1, e+1)$ with probability $u_s \cdot \mu \cdot \frac{v_r}{v_r+e}$ and to (u_s-1, u_p+1, v_r, e) with probability $u_s \cdot \mu \cdot \frac{e}{v_r+e}$ in an infinitesimally short period of time; u_s denotes the number of unemployed searchers, u_p the number of unemployed non-searchers, and v_r the number of real vacancies. This is the process implicit in the model of Hall and of Jackman et al., if their assumptions hold in a long-term perspective.

The search goes on in the same way as in the former process: the instantaneous probability that $u_s \to u_s-1$ is $u_s \cdot \mu \cdot [\frac{v_r}{v_r+e} + \frac{e}{v_r+e}] = u_s \cdot \mu$. The probability of a hire, $e \to e+1$, is not any longer $u_s \cdot \mu$, however; it is $u_s \cdot \mu \cdot \frac{v_r}{v_r+e}$. Given that the process starts at $v_r = v$ and $e = 0$ it always holds at any point of time that $v = v_r + e$ so the instantaneous hiring probability is $u_s \cdot \mu \cdot \frac{v_r}{v}$. Consequently the probability that a real vacancy is filled is $\lambda(t) = \frac{\mu}{v} \cdot u_s(t)$ at any point of time t.

The expected value of $\lambda(t)$ is of course $E[\lambda(t)] = \frac{\mu}{v} \cdot E[u_s(t)] = u_s(0) \cdot \frac{\mu \cdot e^{-\mu t}}{v}$. Keeping to the simplifying assumption that search contacts are realized as expected (compare the deterministic structure of contacts in the Jackman et al. model), the probability that a vacancy is filled during [0,t] is

$$p(t) = 1 - e^{-\int_0^t \lambda(\tau)d\tau} = 1 - e^{-\frac{u_s(0)}{v}\int_0^t \mu \cdot e^{-\mu\tau}d\tau} =$$

$$= 1 - e^{-\frac{u_s(0)}{v}[1-e^{-\mu t}]} .$$

The equilibrium value of p(t) is $p = 1 - e^{-\frac{u_s(0)}{v}}$ as stated by Jackman et al. and the equilibrium state of the system is their one-period end state;
$u_s = 0$; $u_p = u_s(0) - v \cdot p$; $v_r = v(1-p)$; $e = v \cdot p$.

This second hiring process is very inefficient compared to the first one. It is not only less efficient in the rather innocent sense that the unemployed search over vacancies that are already filled; this feature can only delay the hiring process but does not alter its equilibrium state. The crucial (and in my opinion very dubious) feature is the assumption that the unemployed stop search after their first unsuccessful trial!

As a matter of fact the highly inefficient character of this search model was acknowledged already by Hall (1977), but his only way to modify it was to allow for a second 'period' of search and contacts to take place. In his immediate comment Stiglitz (1977) explicitly voiced criticism on this point and our investigation shows his critical remarks to be warranted.

To sum up:

1. The discrete time period model of Jackman et al. and of Hall is a fairly unwieldy device for analyzing short-run phenomena; a continuous time process approach that works with instantaneous transition probabilities is much more tractable.

2. In a long-run context the assumptions implicit in the model of Jackman et al. and of Hall are not convincing; its equilibrium properties should not be used to describe the properties of a hiring function in vacancies and unemployment.

3. Taking the short-run interpretation of the model of Jackman et al. and transforming it to the corresponding continuous time process with instantaneous transition probabilities we obtain a hiring intensity $e = \mu \cdot u$ where μ is independent of both u and v. Consequently $v(> 0)$ does not influence the hiring intensity (but $v = 0$ implies $e = 0$). This is the basis for our claim in the main text of the chapter.

CHAPTER II

THE OPTIMAL WAGE AND HIRING POLICY OF A FIRM AS A CONTROLLED MARKOV PROCESS

1 Introduction

The theories of search in the labour market are by now a firmly established part of economic theory. At first concentrating its attention to the behaviour of the (unemployed) job searcher, the theory was later extended to treat the behaviour of the firm [see Mortensen (1970), Salop (1973), Pissarides (1976), Siven (1979), Virén (1979)]. Several interesting results emerged from the analyses of a firm's optimal response to the special kind of uncertainty that characterizes a search labour market.

The Phillips curve relation was one of the first issues addressed with the help of models of optimal firm behaviour in a search labour market. It turned out that such a relation cannot be derived from the optimizing behaviour of an individual firm in the sense that a decrease in the unemployment rate increases the wage paid by the firm. On the contrary, Mortensen (1970) found that the outcome, although somewhat ambiguous, is likely to be a decrease in the wage. This conclusion is strengthened in Siven (1979), which ascertains that the wage will decrease as a response to decreased unemployment.

Pissarides (1976), however, suggests that decreased unemployment causes the firm to increase its wage offer. As his recruitment model is characterized by some rather special properties, partly as a result of assumptions introduced for technical reasons, his results have not modified the common view that search theory does not imply a causal relation of the Phillips type, running from changes in unemployment to changes in wages.

As a consequence the Phillips curve relation of received search theory is not a theory of wage dynamics. It is rather a theory of unemployment dynamics. The typical presentation of the inflationary mechanism and its repercussions on unemployment, repeated on innumerable occasions, states that prices increase as a result of an increase in nominal demand, e.g. as a result of monetary expansion. As a response firms increase their wages. The crucial assumption is then introduced that the unemployed misperceive this shift in the wage offer distribution and do not adjust their reservation wages accordingly. Consequently they find a larger proportion of the wage offers acceptable and leave the state of unemployment more rapidly, by which the unemployment stock decreases.

This is in epitome the purported causal chain behind the search theoretical Phillips curve. It runs from unexpected wage increases to decreases in unemployment. A celebrated implication of the argument is of course that with correct expectations (or with expectations that are not correlated with actual wage increases) there is no observable Phillips relation at all. It is notable that the wage dynamics of the model has very little to do with its search theoretical foundations.

The standard search model of optimal firm behaviour of the Mortensen type seems to suggest, however, that the wage offer of a firm is not invariant to but increasing in the unemployment stock. In such a case the usual description of the wage and unemployment dynamics does not fully picture the implications of search theory. When wages increase more rapidly than expected by the unemployed, the unemployment stock decreases. As an optimal response to this decrease, any firm should lower its wage! The firms act in such a way as to make the incorrect expectations of the unemployed less incorrect. This phenomenon introduces a stabilizing influence of a rather peculiar nature on the wage dynamics of the model and disturbs its nice dichotomy between the expected inflation rate and the labour market situation.

More important is to realize that a neglected result of search theory seems to be the existence of a Phillips relation, orthodox in its causal structure but perverse in its qualitative properties. To the extent that the unemployment flows are not dominated by the effect of false expectations - and it would be a bold proposition to suggest that they are - this kind of Phillips relation should be observable according to standard search theory. If this conclusion is disturbing, one should regard it as one reason for a closer look at the models of optimal wage behaviour of a firm, which faces a search labour market.

The aim of this chapter is precisely to specify a rigorous model of the optimal wage and hiring policy of a firm, recruiting from a search labour market, and to investigate its properties, e.g. with respect to changes in the unemployment stock.

The model and its properties are presented in the comprehensive Technical Supplement, which gives a selfcontained account of the basic assumptions and a complete and rigorous analysis of the dynamic optimization problem. The method used is an application of the special type of stochastic dynamic programming which is called Markov decision processes.

The Technical Supplement is of a highly mathematical nature. The basic assumptions of the model and an outline of the optimization procedure are also presented in Section 2 of this chapter and Section 3 gives an account of the results of the Supplement on the structure of the optimal policy in graphical and verbal form.

In Section 4 we discuss further the properties of the optimal wage policy with regard to the shape of the reservation wage distribution of job applicants. We consider in particular the possible effects of different distributions of reservation wages for unemployed and employed job applicants, respectively. Section 5 of the chapter is devoted to sensitivity or 'comparative static' analysis. There we consider e.g. the effect of a change in the stock of the unemployed and show that if such a change is modelled as a uniform change in the search intensity, leaving the reservation wage distribution intact, the results by Mortensen and Siven are confirmed. We also show, however, that if a change in the unemployment stock results in a change in the search intensity exclusively at the lower part of the range of this distribution, a traditional Phillips curve response by the firm is a likely outcome.

In Section 6 we give some conjectures on the properties of a model, that is extended to include quits. Such a model seems to amend to but not to alter our previous results.

In the last two sections we discuss some connections between our results on optimal firm behaviour and the cyclical behaviour of wages and labour market variables. We consider especially the implications of downward wage rigidity. The Phillips curve issue is again addressed in this setting. The discussion must necessarily be tentative as it is outside the scope of the present contribution to analyze the full interdependence between the optimal actions of all agents in a search labour market.

To the main text of the chapter there is appended an excursus on the formal similarities between our stochastic recruitment model and a deterministic model in which employees quit as well as become hired.

To facilitate the reading a list on the notation used is given at the end of the chapter.

2 The recruitment model: basic assumptions and an outline of the optimization procedure

The basic model of the analysis is that of search in the labour market under incomplete information. The job applicants know the distribution of wage offers over vacant jobs and the firms know the distribution of reservation wages over job applicants, but no one can tell where a specific wage offer or job applicant is to be found.

Mortensen (1970) is the first contribution to analyze strictly the optimal wage policy of a firm in such a labour market environment. Its crucial feature is the ability of the firm to control the (expected) speed of its hiring process by its offered wage level. Hence the firm faces the dynamic optimization problem of balancing higher wage costs against reaching more rapidly more profitable employment states. Mortensen studied this problem by using deterministic control methods, assuming that expected expansion is always realized, and his analysis has been extended by subsequent authors [Salop (1973), Pissarides (1976), Siven (1979), Virén (1979), Leban (1982a,b)].

No extension of the model of firm behaviour under labour market search has explicitly taken into consideration the stochastic nature of the hiring flow to the firm [at least not as far as analytical models are concerned; Eaton and Watts (1977) simulated numerical results on the basis of a stochastic model]. From a methodological point of view the novel feature of the present contribution is its treatment of the firm's dynamic optimization problem as an explicitly stochastic one. The stochastic flow of job applicants to the firm is modelled in analogy to the way in which the flow of customers to a service station is modelled in operations research. Such models belong to the area of queuing theory, so technically speaking we use methods of optimal control of queues, although applied in a somewhat unconventional setting. Markov decision processes is a more theoretically disposed notion, which largely overlaps that of queue control. To use yet another concept our method is an example of stochastic dynamic programming.

Before proceeding with the optimal control aspects of our analysis we present the basic assumptions of the model. They are partly standard in search theory applications to the labour market.

The firm is modelled as a one-product production unit. Labour is the only variable factor of production. The production technology is characterized by fixed coefficients of production over separate employment intervals, so that the i:th employee produces a_ℓ units of the product if i belongs to $[N_{\ell-1}+1, N_\ell]$; $\ell=1,2...L$; $N_0=0$. We denote by $[N_{\ell-1}+1, N_\ell]$ the ℓ:th productivity or employment interval. N_L is the physical capacity level (in terms of the number of employees).

We assume that the marginal productivity of labour is non-increasing in the productivity intervals, so that $a_\ell \geq a_{\ell+1}$, $a_L > 0$. This assumption is a generalization of the traditional notion in economics of a strictly concave production function in the volume of employment; that case is obtained by letting strict inequality hold and by narrowing the productivity intervals as to contain one employee only. Our assumption can also be justified by reference to the so-called vintage approach to technical progress, where the production possibilities of a firm are the result of technical advances embodied in equipment of different age and hence of different productivity.

If the firm has i employees, i belonging to the ℓ:th productivity interval, the rate of production q(i) is

$$q(i) = \sum_{j=1}^{\ell-1} a_j(N_j-N_{j-1}) + a_\ell(i-N_{\ell-1}), \qquad \ell=1,...L$$

By defining $\Delta a_k = a_k - a_{k+1} \geqq 0$, we can write

$$q(i) = \sum_{j=1}^{\ell-1} \Delta a_j . N_j + a_\ell . i, \qquad \ell = 1, \ldots L$$

This expression is easily interpreted; all i employees will in the first place produce a_ℓ units each; in addition $N_{\ell-1}$ (<i) employees will produce another $\Delta a_{\ell-1}$ units each and so on.

We further assume that the firm can sell all it produces at a fixed price p. If the firm pays the wage w_i in employment state i, it will in that state earn a profit rate per unit of time equal to

$$r(i,w_i) = p \cdot q(i) - i \cdot w_i = p \cdot \sum_{j=1}^{\ell-1} \Delta a_j \cdot N_j + i(p \cdot a_\ell - w_i)$$

$$i \in [N_{\ell-1}+1, N_\ell], \qquad \ell = 1, \ldots L$$

The stochastic feature of the model emanates from the firm's interaction with the labour market. When the firm announces vacant jobs, it will be contacted by job applicants, who arrive according to a stochastic process. We will assume that the process of contacts is Poisson with an intensity denoted γ. It should be a natural assumption, if the firm is small compared to the size of the labour market and the stock of job applicants. As a consequence the time between successive contacts is exponentially distributed with expectation γ^{-1}.

According to results from labour market search literature, each job applicant should under fairly general conditions conduct his search by calculating a reservation wage such that a job offer should be accepted if and only if the corresponding wage offer exceeds the reservation wage [see Zuckerman (1983) with references]. We denote by $F(\cdot)$ the distribution function of reservation wages over all job applicants. The corresponding density function $f(\cdot)$ is supposed to be continuous and differentiable. It has a range $[\underline{v}, \bar{v}]$, $\underline{v} \geq 0$; $\bar{v} \leq \infty$. Any randomly selected job-applicant would accept a job offer from the firm with probability $F(v)$, if the firm offers the wage v.

Let the firm offer the wage w_i in employment state i. Then the recruitment or hiring process is also Poisson and has the intensity $\lambda(w_i)=\gamma \cdot F(w_i)$. The time until the (i+1):th employee is hired - or the sojourn time in employment state i - is exponentially distributed with expectation $[\lambda(w_i)]^{-1}$. In this way the firm is able to control the intensity of its hiring process by its wage offer.

Besides the possibility of controlling the expected speed of the hiring process the firm has at its disposal a stopping control, $\delta_i = \{0,1\}$. When $\delta_i = 1$, vacancies are announced and the hiring process goes on at the intensity $\lambda(w_i)$ as indicated above. When $\delta_i = 0$, no vacancies are announced in employment state i and the process is stopped. The firm will remain in state i forever after.

We assume that the firm pursues the traditional aim of maximizing expected total profits. It faces an infinite horizon and discounts profits continuously at an intensity α. To reach its objective it has to select optimally a sequence of wages $\{w_i\}_n^S$ and a stopping state S, such that $\delta_S = 0$, $\delta_i = 1$, i = n...S-1, n being the initial employment state. In technical terms, the firm faces a Markov decision process with discounting. According to results from basic theory of Markov decision processes the solution to the firm's decision problem corresponds to solving the following functional equation of dynamic programming

$$L(i) = \max_{w_i,\delta_i} \left| \frac{r(i,w_i)}{\delta_i \cdot \lambda(w_i)+\alpha} + \frac{\delta_i \cdot \lambda(w_i)}{\lambda(w_i)+\alpha} \cdot L(i+1) \right| ,$$

where L(i) is the maximum expected discounted total profits in employment state i and i and w_i are subject to appropriate restrictions [cf. e.g. Stidham and Prabhu (1974)].

It is a property of Markov decision processes - or, equivalently, of controlled exponential systems - that the optimal policy is dependent only on the state of the process. In principle the value of the control variables w(t) and $\delta(t)$ can vary continuously in t. We need only to consider stationary policies, i.e. values w_i and δ_i, however, in our search for the optimal policy, if the state is, as we have it, completely characterized by the employment volume i.

As a consequence the optimal policy is time-independent. At any given point of time after the start of the recruitment process the number of employees will obey a probability distribution, so the optimal policy, deterministic in the state variables, is stochastic in the time variable. This poses no problem within this framework, however; the firm simply observes its present state and chooses the optimal values of its policy variables accordingly, irrespective of time. As is typical for dynamic programming models, the optimal policy is a closed loop control.

It is easy to see how the exponentially distributed sojourn times in the Markov process give rise to the functional equation above. Let the firm be in employment state i and let the value of applying the optimal policy in state i+1 be L(i+1). The profit rate in state i is $r(i,w_i)$, when the firms pays the wage w_i; the present value at time zero of the profit rate at time τ is $r(i,w_i)e^{-\alpha\tau}$. If the sojourn time in state i is t, the total discounted profit in that state is

$$\int_0^t r(i,w_i)e^{-\alpha\tau}d\tau.$$

As we saw, t is exponentially distributed and has the probability density function $\lambda(w_i) \cdot e^{-\lambda(w_i)t}$ on $[0,\infty)$, so the expected discounted profit in state i is

$$\int_{t=0}^{\infty}\int_{\tau=0}^{t} r(i,w_i)e^{-\alpha\tau}\lambda(w_i)e^{-\lambda(w_i)t}d\tau dt = \frac{r(i,w_i)}{\lambda(w_i)+\alpha}$$

After the time t spent in state i, the firm enters state i+1, from which state and onwards it will earn total expected discounted profits, evaluated at time t, equal to L(i+1); evaluation at time zero yields

$$e^{-\alpha t} \cdot L(i+1).$$

Taking expectation over the pdf of t gives

$$\int_0^{\infty} L(i+1)e^{-\alpha t} \cdot \lambda(w_i)e^{-\lambda(w_i)t} dt = \frac{\lambda(w_i)}{\lambda(w_i)+\alpha} \cdot L(i+1)$$

Adding the two contributions to total expected discounted profits, evaluated at time zero, in state i when the wage w_i is paid, and maximizing over w_i yields the functional equation above for

$\delta_i = 1$, i.e. when the firm chooses to expand employment from i to i+1.

As it is not necessarily optimal for the firm to expand, it may choose to stay in state i (i.e. it may choose S=i) and earn the profit rate $r(i,w_i)$ forever, so that total discounted profits are

$$\int_0^\infty r(i,w_i)e^{-\alpha\tau}d\tau = \frac{r(i,w_i)}{\alpha} ;$$

putting $\delta_i = 0$ and maximizing over w_i reduces the functional equation to this form.

L(i) is the overall maximum which emerges from a comparison of the two solutions.

A rigorous analysis of the functional equations corresponding to the firm's decision problem is carried out in the Technical Supplement to this chapter. The sketch of the optimization procedure given above corresponds to that version of the firm's decision problem, in which wages are assumed to be flexible. We also treat the case, when wages are downward rigid. The difference in wage policy feasibility has formal consequences. When wages are downward rigid, the state must be characterized not only by the employment volume but also by the wage level itself. The control variable is not the wage but the wage increase (besides, as before, the stopping control).

In the Technical Supplement the two cases as to the feasible wage policy are treated on equal terms and the result of the analysis is comprehensively, but non-technically described in the following section.

In the sequel we will be more brief in treating the case of flexible wages, however. The reason for this difference in emphasis is that we find the assumption of downward rigid wages more relevant in a model of a firm's wage behaviour.

The existence of downward wage rigidity is as acknowledged by economists as they find it hard to derive from the optimizing behaviour of job applicants and firms. The theoretical issue is far from settled, though some recent approaches may yield more convincing results in the future [see Yellen (1984), Lindbeck and Snower (1986)]. Our position is simply that as firms seem to be constrained in their choice on feasible wage policies, an analysis of the optimal wage policy should take such a constraint into account. As will be shown, the existence of downward wage rigidity has important consequences for the behaviour of a firm.

The case of flexible wages deserves proper attention for at least two reasons, however. There are analytical insights to be gained from a comparison between the two cases. Besides, wage flexibility is an outstanding feature of the traditional deterministic models of firm behaviour in search theory; Mortensen (1970), Siven (1979) and Pissarides (1976) all build their models on that assumption. [It is true that Pissarides (1976) also treats a 'fix-wage' case, but then he assumes that wages are indeed fixed in all directions and dimensions, so the wage is no decision variable to the firm.] Consequently we need to evaluate to what extent any difference in results between our analysis and the traditional ones depends on different assumptions about the feasible wage policy.

One theoretical weakness of the wage flexibility assumption should be particularly noted. Typically this assumption leads to an optimal policy such that wages decrease when employment expands. This result has been criticized by Leban (1982a,b) and Virén (1979) as running counter to observed facts and they have analyzed (deterministic) models of the firm's dynamic optimization problem under the assumption of downward money wage rigidity. In addition Virén points out that the assumption of wage flexibility is not only a bad empirical generalization; it is also inconsistent with the standard models of job search behaviour:

> 'If we allow wages to be downward flexible, an inconsistency arises between this and the (job) search model, in which it is assumed that the firm's wage offer also implies the same wage rate for the participant in the future. This is the same as assuming that an employee ignores the possibility that an eventually high offer implies only a temporary high wage in the firm. ... There seems to be no reason why an employee would behave in such a nonoptimal way.
>
> There are two obvious ways out of this dilemma: one is to postulate downward rigid wages with regard to each firm ...; the other is to consider explicitly the employee's expectations formation with respect to the future wages in a firm, when the search model is specified.'
> [Virén (1979), p. 78)]

The first approach is the one chosen by Virén as well as in this chapter. The second one does not appear to have attracted any research efforts; it is understandable, because the technical difficulties presented by such a model seem to stand in inverse proportion to its relevance.

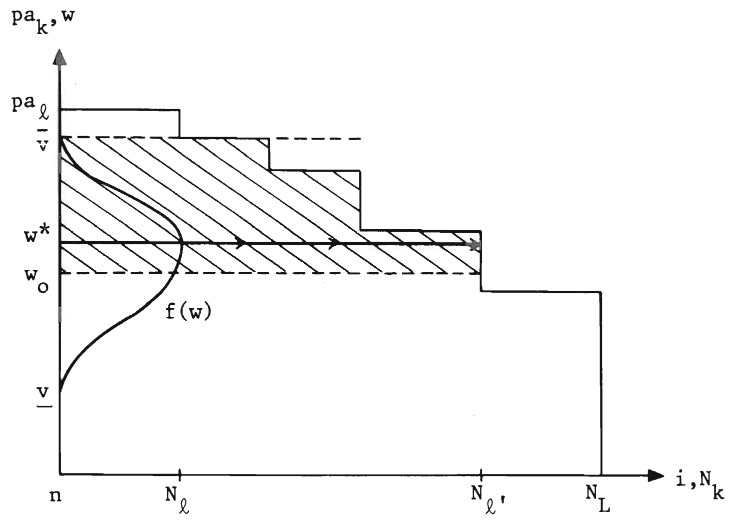
3 The recruitment model: a summary of the results on the optimal wage and vacancy policy

The structures of the optimal wage and vacancy policies, as they have been derived in the Technical Supplement, are graphically depicted in Figure II.1. Graph a) illustrates the case of downward wage rigidity, graph b) the case of flexible wages. In both graphs the marginal productivities pa_k, $k=\ell...L$, and the wage level w or w_i are measured along the vertical axis and employment states i above the initial state n are measured along the horizontal axis; the N_k's, $k=\ell...L$ are the end-points of productivity intervals. The probability density curve f(w) along the vertical axis with range $[\underline{v},\bar{v}]$ is that of the reservation wage distribution. These parts of the graphs represent the parameters of the control problem common to both wage regimes. In the case of downward rigidity an initial wage level w_o must be specified; it has a counterpart in the flexible case in the form of an externally given minimum wage $\underline{w}$.

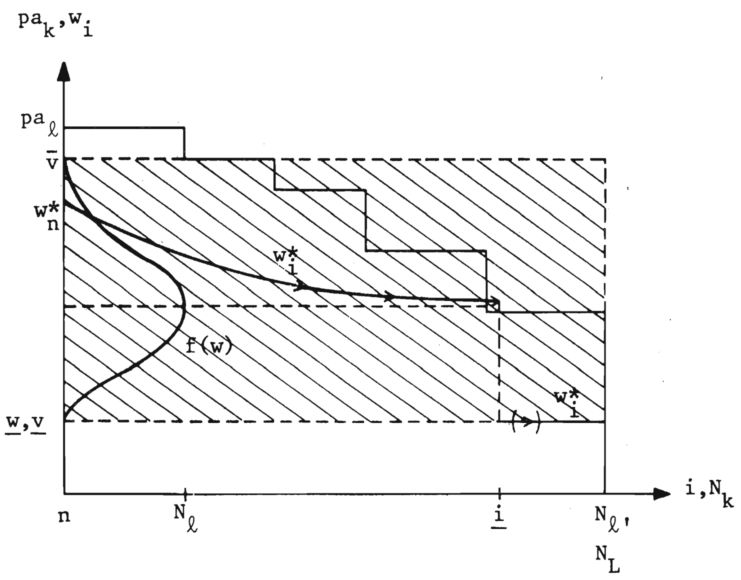
The feasible combinations of optimal wages and optimal expansion are shown by the shaded areas in the two graphs. In both cases it is easy to see that the optimal wage during expansion will never exceed $\bar{v}$; the recruitment process has at $w=\bar{v}$ reached its highest speed and further wage increases mean just higher costs. In addition downward wage rigidity implies that the optimal wage during optimal expansion into any employment state cannot exceed the marginal productivity in that state; because wages cannot be lowered, a higher wage must imply perpetual negative marginal profits with respect to expansion into that state.

Figure II.1 The optimal wage and vacancy policy of a recruiting firm

a) Downward wage rigidity



b) Wage flexibility



Such a condition on the combination of optimal wage levels and optimal expansion does not hold when wages are flexible; wage decreases can always restore the necessary optimality requirement that marginal profits with respect to expansion are positive.

In both wage regimes there exists a lower bound on the set of feasible wage levels, and they have the common feature that expansion is inoptimal beyond the employment state $N_{\ell'}$, above which the marginal productivity becomes less than this lower bound. Otherwise the nature of the lower bound is quite different in the two cases: under downward wage rigidity it is the wage w_o which was chosen by the firm before employment state n was entered; under wage flexibility it is an externally given minimum wage $\underline{w}$. This is also the reason why we have chosen $w_o > \underline{w}=\underline{v}$ in the graphs of Figure II.1.

Consider now the dynamics of a point (i,w_i) in the shaded areas, i.e. of a stationary policy process that is feasible and obeys the necessary optimality conditions above. Such a point moves through time to the right in discrete steps by one, corresponding to employment expansion with a velocity, the expected value of which is proportional to the area under the reservation wage probability density curve f(w); denoting the reservation wage distribution $F(\cdot)$, the expected velocity is $\gamma \cdot F(w_i)$, where γ is the expected rate at which job applicants contact the firm. The higher the point is situated in the shaded area, the more rapidly it moves to the right (in expectation) and as long as $w_i > \underline{v}$ the movement will go on, unless it is stopped by the vacancy control.

While the stationary policy point can move freely upwards or downwards at the discretion of the firm, when wages are flexible, it can move at most upwards, when wages are downward rigid. Consequently, in the latter case the point must move (in expectation) at a non-decelerating velocity to the right and eventually hit the right-hand boundary of the optimality region. Unless stopped by the vacancy control it would enter the inoptimal region, so vacancies must be put to zero at the boundary. It is intuitively clear and also true that stopping should take place only at an employment state N_k, where the marginal productivity is strictly decreasing.

It is an important result of the analysis of the case of downward wage rigidity that a vacancy control is an indispensable part of the optimal policy. The firm should not accept anyone job applicant who is prepared to accept to work for its wage. [For the sake of completeness we may take into account the unlikely possibility that $w_o \leq \underline{v}$ so that a point (n, w_o) with no expansion at all is included in the set of feasible and possibly optimal policies. Whenever expansion occurs, however, a vacancy control needs to be applied.] The relation between the optimal wage policy and the optimal vacancy control is also an important result that we will shortly consider.

When wages are fully flexible the vacancy control plays a far less prominent role. If it plays any role at all, depends on the relation between $\underline{w}$ and $\underline{v}$. The vacancy control is needed to prevent the firm from expanding beyond the right-hand boundary of the optimality region. But with flexible wages this can always be accomplished more profitably by a wage decrease, so that the stationary policy point is moved downwards, until eventually $\underline{w}$ is reached. As we have drawn graph b) in Figure II.1, $\underline{w} = \underline{v}$ and $F(\underline{v}) = 0$. In such a case the vacancy control is obsolete; when the firm wants to stop expansion, which must happen at least when it reaches employment state $N_{\ell'}$, it just chooses $\underline{w}$ as the optimal wage and expansion stops automatically at the same time as current profits are maximized. If it should happen that $\underline{w}>v$ or - perhaps more realistically - that $F(\underline{v}) > 0$, because of a clustering of job-applicants at a reservation wage equal to the minimum wage, the firm will eventually reach $(N_{\ell'},\underline{w})$ as the endpoint of the optimal expansion process and use the vacancy control to stop further expansion into the inoptimal region.

Under a flexible wage regime the firm has either no use for a vacancy control or otherwise its optimal use is immediately given by the relation between the externally imposed minimum wage and the firm's value productivity structure. When downward wage rigidity prevails, however, the optimal vacancy control is an integral part of the total optimal policy of the firm.

It is one of the main results of the Technical Supplement that under downward wage rigidity the optimal wage policy is to choose a constant wage level to hold in all future employment states during expansion. In graph a) of Figure II.1 the optimal policy is the trajectory indicated by the straight line with arrows and the constant value of the optimal wage level is denoted w^*. In the initial state (n,w_0), the firm should choose a wage level $w^* \geq w_0$ and expand at a constant (expected) speed $\gamma \cdot F(w^*)$ up to an employment state N_k^* such that the marginal productivity at N_k^* is greater than w* and at N_k^*+1 lower than w*. At N_k^* the vacancy control is used to stop the process.

Of course, all wage levels w between w_o and $\bar{v}$ (or pa_ℓ, if $pa_\ell < \bar{v}$) are possible candidates for being the optimal one. By virtue of the concavity of the production function and the condition for optimal stopping the values of the optimal stopping states $N_k(w)$ corresponding to a chosen wage level w form a monotone (non-increasing) function of w as indicated by the stepwise falling right-hand boundary of the optimality region in graph a) of Figure II.1. Consequently, we just have to maximize total expected discounted profits over all constant-wage trajectories w to find w*. The computational procedure depends on how regularly the function $N_k(w)$ behaves, i.e. how regular the marginal productivity structure is.

Provided that the production function is strictly concave on at least some employment intervals within the region of optimal expansion, as in Figure II.1, so that $N_k(w)$ is strictly decreasing for some w, we get the interesting result that under downward wage rigidity a higher wage level means more rapid expansion but a lower level of announced vacancies and of ultimate employment. The fact that wages cannot be lowered constitutes a cost to the firm, which is reflected in the employment state, where the recruitment process is stopped, by not only a higher wage but also a lower value added compared to a situation where wages are flexible. The optimal values of wages and of vacancies are jointly determined by all the parameters of the decision problem and the firm must in the initial state balance the effect of its wage decision on the (expected) recruitment speed against the effect on the profit margin in all future employment states, including the possibility that the profit margin changes sign and induces changes in the stopping state.

When wages are flexible, the firm needs to consider the cost effects of its wage decision on the profit margin in the current employment state only. As a consequence it can pay more attention to the effect of its wage decision on the (expected) speed of expansion to higher and more profitable employment states. It is another main result of the Technical Supplement that when wages are flexible the optimal wage trajectory is non-increasing in the employment states during expansion. As a matter of fact it is strictly decreasing up to an employment state $\underline{i}$, from which state and onwards the wage is constantly held at the minimum wage $\underline{w}$. The optimal trajectory is depicted in graph b) of Figure II.1 as the curve with arrows, denoted w_i^*. If $F(\underline{w})=0$, as Figure II.1 indicates, expansion is automatically stopped at $\underline{i} \leq N_{\ell'}$, which then becomes the implicit optimal stopping state of the decision process. In that case all employment expansion takes place at a strictly falling wage level.

The decreasing optimal wage level in terms of employment states means that (expected) expansion is most rapid at the beginning of the recruitment process and slows down gradually to a minimum as the exogenously determined capacity limit $N_{\ell'}$ is approached. This property is a reflection of the fact that the increases in employment states become less valuable to the firm as employment expands; technically speaking, maximized expected discounted total profits are concave in the employment state.

To clarify matters, we should point out that a similar feature exists in the case of downward wage rigidity. In the graphs of Figure II.1 and the concomitant comments above, n is the initial employment state and i>n are the employment states subsequent to the initial one. In the case of wage flexibility any i can also be regarded as an initial state; there is formally no difference between initial and subsequent states. When wages are downward rigid, however, there is a difference between the two concepts. In the initial state $[n,w_o]$ w_o is by definition arbitrary, so an initial state at higher employment levels is defined as $[i,w_o]$, w_o arbitrary. In a subsequent state $[i,w_i]$, however, w_i is not arbitrary; on the contrary, w_i is equal $w^*(n)$, if an optimal policy is followed.

Let us now consider a sequence of initial states $\{n,w_o\}_0^{N_L}$, where the arbitrary w_o's are put equal to $\underline{w}$, say. It is a result of the analysis in the Technical Supplement, intimately coupled to its main result on the structure of the optimal policy in the case of downward wage rigidity, that $w^*(n)$ is non-increasing in n. In analogy to the optimal wage trajectory w_i^* in the case of flexible wages the sequence $w^*(n)$ will decrease monotonically in n until $w^*(n)=\underline{w}$ for some n, which occurs at least at $n=N_L$. The sequence $w^*(n)$ is not equal to the trajectory w_i^*, of course. For n=i, a relation $w^*(n) \leq w_i^*$ clearly holds, such that $w_i^*=\underline{w}$ implies $w^*(n)=\underline{w}$ and $w^*(n)>\underline{w}$ implies $w^*(n)<w_i^*$. These relations follow from the fact that under downward wage rigidity the optimal wage level at any n must necessarily imply an 'inoptimally' high wage level in the future compared to the case of flexible wages and the firm counteracts this effect by choosing a lower wage.

Returning to the case of flexible wages we have drawn graph b) of Figure II.1 in such a way as to illustrate two features of the optimal solution. The first one is the possibility that the profit rate might be negative in some states; the firm might choose to forgo profits in lower, transient employment states in order to reach the most profitable ultimate state more rapidly. For reasons already discussed this cannot happen, when wages are downward rigid. The second feature concerns a relation between the optimal solution and the reservation wage distribution. As shown in the Technical Supplement an interior optimal solution at any employment state must be situated at a wage level where this distribution function is concave (the probability density function f(w) is falling). With an unimodal pdf of reservation wages, as depicted in Figure II.1, a discontinuous drop in the optimal wage is consequently bound to occur at the end of the recruitment process. In the case of downward rigidity the (second-order) condition for an interior optimal wage level does not necessarily imply concavity of the reservation wage distribution at that wage level.

4 The optimal wage policy and the reservation wage distribution

With the exception of the explicit comment in the last paragraph of the preceding section, none of our results presented up to now - i.e. the results of the analysis in the Technical Supplement - depends on any assumptions concerning the shape of the reservation wage distribution (other than the condition that its pdf is continuous and differentiable). The properties of this distribution become important, when the multiplicity of optimal solutions, or rather the multiplicity of interior local maxima of expected discounted total profits (the value function), is considered.

In the case of downward wage rigidity a multiplicity of interior maxima may arise, when changes in the constant-wage trajectory w are considered, because of sudden changes in the optimal stopping state. In the Technical Supplement we demonstrate that the derivative with respect to w of the value function increases discontinuously at such values of w where the optimal stopping state is reduced. With the exception of a short comment in connection with Figure II.2, graph b) we will not consider this complication in the sequel, however, but limit our attention to the special case, in which the production function is linear (up to a given capacity limit). As a consequence there will be no further discussion on the vacancy control issue, which is trivial in such a case. The loss in this respect is compensated by a clearer presentation of other important aspects of the model.

Having eliminated any possible influence from a discontinuously changing optimal stopping state, the number of possible candidates for the optimal wage policy is crucially dependent on the shape of the reservation wage distribution. Let us consider the case of downward wage rigidity. The value function then reads

$$\alpha V(n,w) = (pa-w)\{n + \frac{\lambda}{\alpha}[1 - (\frac{\lambda}{\lambda+\alpha})^N]\}$$

where α is the discount intensity, pa is the value productivity, now constant for all employment intervals, $\lambda = \gamma \cdot F(w)$ and N is the now unique optimal stopping state.

The essential properties of V(n,w) are most easily observed if we begin by looking at the situation in which N is infinite (strictly speaking this case is not treated in the Technical Supplement, but uniform convergence of V ensures that a solution to the decision problem exists and has the same properties as shown for the case with N bounded). Clearly it holds

$$\lim_{N\to\infty} \alpha \cdot V(n,w) = \alpha W(n,w) = (pa-w)(n+\frac{\lambda}{\alpha});$$

$$\frac{d\alpha W(n,w)}{dw} = \alpha W'(n,w) = -n - \frac{\lambda}{\alpha} + (pa-w)\frac{\lambda'}{\alpha} =$$

$$= -n + \frac{\gamma}{\alpha}[(pa-w)f(w) - F(w)]$$

$$\alpha \cdot W''(n,w) = \frac{\gamma}{\alpha}[(pa-w)\ f'(w) - 2f(w)].$$

Consider the expression

$$\alpha \cdot W'(n,w) + n = \frac{\gamma}{\alpha}[(pa-w)f(w) - F(w)];$$

it holds that $F(\underline{v}) = 0$; $F(w|w>\underline{v}) > 0$;
$(pa-w)f(w) = 0$ for $w = \min[pa;\bar{v}]$.

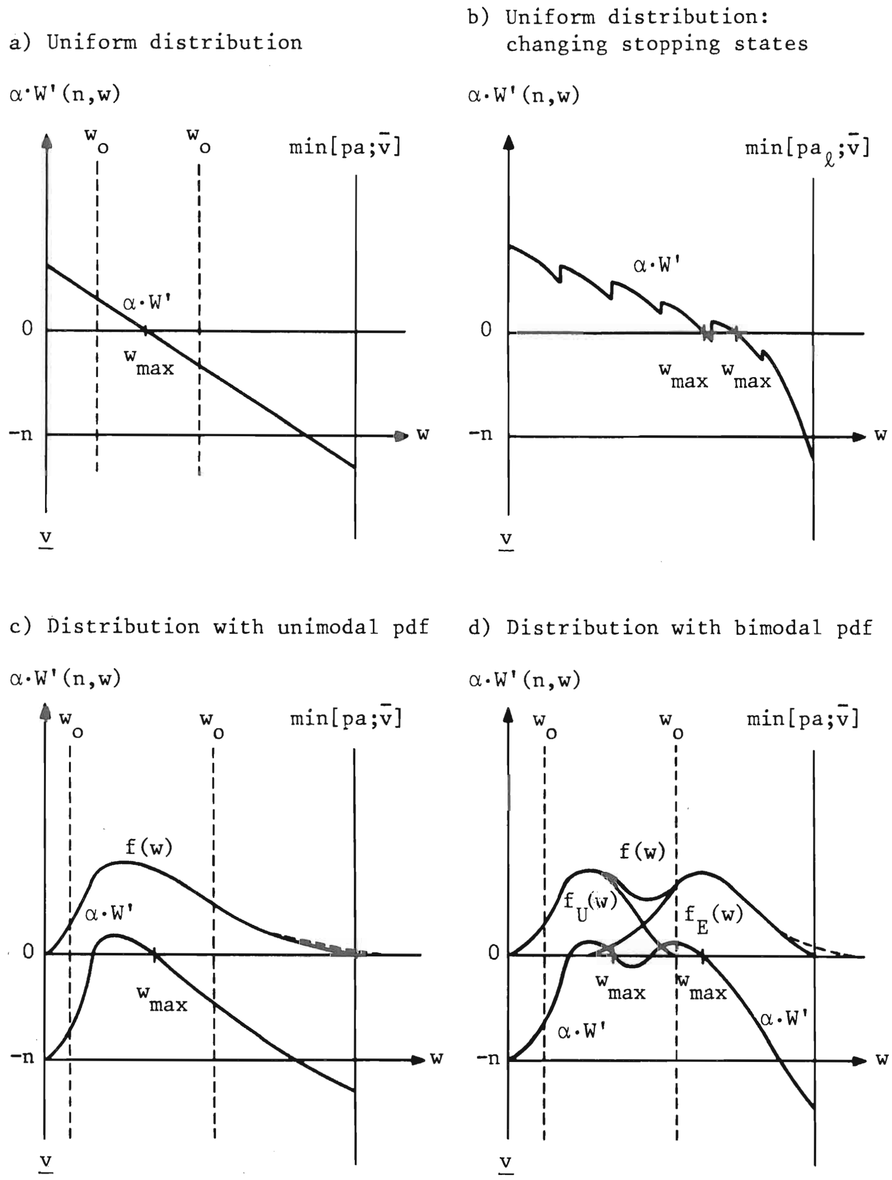
Consequently, $\alpha W'(n,w) + n$ (and a forteriori $W'(n,w)$) is negative at the upper boundary of the optimality region.

It is obvious that uniqueness of an interior solution is guaranteed if f(w) is non-increasing, i.e. if the probability density of job applicants is never higher at a higher reservation wage than at a lower. In such a case $f(\underline{v})$ is positive and $\alpha W'(n,w) + n$ decreases monotonically from a positive to a negative value as w increases in the optimality region. In Figure II.2, graph a), we have used the uniform distribution to illustrate the case of a non-increasing pdf of reservation wages; we have also added a graph b) to depict the effect of a changing optimal stopping state in this simple example (the legitimacy of going from an unbounded to a bounded N while preserving the general properties of W'(n,w) will be vindicated shortly).

In Figure II.2 the interior maxima are denoted by w_{max}. When the reservation wage density is non-increasing, a unique $w_{max}(n)$ exists for all sufficiently small n. Because $\alpha \cdot W'(n,w)$ decreases in n, $w_{max}(n)$ decreases "continuously" in n until $w_{max} = \underline{v}$, from which employment state and onwards it ceases to exist. If $w_{max}(n)$ exists and is feasible, i.e. greater than w_o, it is also the optimal wage level w*(n). Note that even with this uncomplicated distribution function, the existence of changing optimal stopping states introduces multiple interior maxima as graph b) in Figure II.2 illustrates, so that the optimal wage jumps downwards from one w_{max} to another when n increases and the optimal stopping state is changed.

To clear up any remaining confusion we should point out that the optimal wage level w*(n) is, as before, the optimal constant wage trajectory as a function of the initial employment state n, w_o arbitrary; in terms of subsequent states i the optimal wage is constant.

Figure II.2 The optimal wage level at different types of reservation wage distributions



It is hardly a satisfactory empirical generalization, though, to postulate a non-increasing probability density function of reservation wages. Little is known of such density functions in practice, but available evidence supports the conjecture that the reservation wages of the unemployed job applicants are distributed according to an unimodal pdf, presumably reaching its peak quite rapidly and tapering off at higher wage levels [see Lancaster and Chesher (1983)].

If the reservation wage distribution has an unimodal pdf of any standard distribution type (the beta distribution is the obvious candidate, if $\bar{v}$ is to be bounded), the expression for $\alpha \cdot W'(n,w)$ behaves as follows when w increases from $\underline{v}$ to $\min[pa;\bar{v}]$: it is first increasing from zero up to a maximum, then decreasing to zero and finally increasingly negative as $\min[pa;\bar{v}]$ is approached. This plausible result is demonstrated in some detail in Schager (1986b). The qualitative behaviour of $\alpha \cdot W'(n,w)$ is depicted in Figure II.2, graph c), in which the pdf f(w) is also reproduced. The peak of $\alpha \cdot W'(n,w)$ must lie to the left of the mode of f(w), but we cannot say a priori whether f(w) is increasing or decreasing at a w_{max}.

As in the former case the $w_{max}(n)$ is unique if it exists, and the conclusions as to its existence are the same as in the case of a non-increasing pdf of reservation wages. The condition for $w_{max}(n)$ to be the optimal wage $w^*(n)$ is not the same, however. As is illustrated by the w_o-value to the left in graph c), it may hold that $W(n,w_o) \geq W[n,w_{max}(n)]$ even if $w_{max}(n) > w_o$; i.e. even if $w_{max}(n)$ exists and is feasible, it is not necessarily equal to $w^*(n)$. As a consequence $w^*(n)$ does not necessarily approach the lower bound on w "continuously" as n increases, but can make a downward jump to it from $w_{max}(n)$ at some critical level of n (as illustrated by a lowering of the $\alpha W'$-curve in graph c) of Figure II.2). We also see that the position of the lower bound w_o on feasible wage levels has a somewhat complicated impact on $w^*(n)$: let $w_{max}(n)$ exist and w_o increase from $\underline{v}$ to $\min[pa;\bar{v}]$; then $w^*(n)$ is at first equal to w_o, later on equal to $w_{max}(n)$ and at last again equal to w_o. Another implication, which is important when sensitivity analysis results are considered, is the possibility that small variations in parameter values can cause the firm to make sudden changes between a low-wage, minimum-expansion strategy and a high-wage, rapid-expansion one.

Even if the reservation wage distribution of the unemployed can be represented by a distribution with a unimodal pdf, we must take into account that the firm recruits employed as well as unemployed job applicants. Clearly one should not expect the employed job searchers to have reservation wages distributed in the same way as the unemployed. The empirical evidence on the matter seems to be non-existent, but on the other hand one is entitled to assume a priori that the reservation wage of an employed job searcher is equal to the wage he earns at his present job plus a premium to make up for his transition costs. Consequently the reservation wage distribution of the employed can by stylized as the distribution of paid wages over all employees with some shift to the right. According to empirical evidence such a wage distribution has a fairly symmetrical, unimodal pdf (see e.g. Jonsson and Siven [1986] for Swedish data).

Given an optimal search behaviour of the unemployed, we should expect their reservation wage distribution to be situated to the left of the distribution of paid wages [cf. Axell (1976), Chapter 5, where a corresponding result is shown for price distributions in a search goods market]. The total distribution of reservation wages is thus the weighted convolution of two separate distributions, each with an unimodal pdf, such that

$$F(w) = \frac{\gamma_U \cdot F_U(w) + \gamma_E \cdot F_E(w)}{\gamma_U + \gamma_E} ;$$

$$\gamma_U + \gamma_E = \gamma ; \; F_U(w) > F_E(w)$$

F_U and F_E are the reservation wage distributions of the unemployed and the employed job searchers, respectively. γ_U and γ_E are the corresponding contact intensities.

We immediately note that the recruitment intensity

$$\lambda(w) = \gamma \cdot F(w) = \gamma_U \cdot F_U(w) + \gamma_E \cdot F_E(w) =$$
$$= \lambda_U(w) + \lambda_E(w)$$

is now the sum of two independent intensities, one representing recruitment of unemployed job applicants, the other recruitment of employed ones.

Given that $f_U(w)$ and $f_E(w)$ are unimodal, $f(w)$ can clearly take on different shapes. Bimodality is one perfectly possible outcome. Although such a property of $f(w)$ is not necessary for establishing some important results of ours in the sequel, the bimodal case illustrates nicely the consequences of a dichotomized recruitment process. (In order to get a bimodal pdf $f(w)$ out of the constituent unimodal pdf's $f_U(w)$ and $f_E(w)$, they must be sufficiently separated from each other in the sense that the distance between their modes is significant and their variances are not too large. In addition γ_U and γ_E must be of the same order of magnitude; otherwise the shape of $f(w)$ will be determined essentially by the pdf associated with the dominating contact intensity.)

Simple substitution establishes that if recruitment of job applicants is generated in the way described in the preceding paragraphs the expression for $\alpha \cdot W'(n,w) + n$ becomes

$$\alpha \cdot W'(n,w) + n = \frac{\gamma_U}{\alpha} [(pa-w)\ f_U(w) - F_U(w)] + \\ + \frac{\gamma_E}{\alpha} [(pa-w)\ f_E(w) - F_E(w)].$$

The additive property of the constituent recruitment processes carry through to establish that $\alpha W(n,w)$ is the sum of the value to the firm of each process taken separately; hence $\alpha \cdot W'(n,w) + n$ is the sum of two $\alpha \cdot W'(n,w)$-curves as depicted in Figure II.2, graph c), although differently peaked. Bimodality of $\alpha \cdot W'(n,w) + n$ does not necessarily follow from the bimodality of $f(w)$ but it is a highly plausible outcome. In graph d) of Figure II.2 such a situation is depicted.

Following the same argument as in the earlier cases, it is easily seen how $w^*(n)$ may decrease from the higher $w_{max}(n)$ to the lower one and further to the lower bound on feasible wage levels, as n increases. It is worth observing that the lower w_{max} may never be optimal, even if it is feasible. An increase in w_o from $\underline{v}$ to $\min[pa;\bar{v}]$ may now produce a sequence of $w^*(n)$ such as $\{w_o$, low $w_{max}(n)$, w_o, high $w_{max}(n)$, $w_o\}$. We can identify the low-w_{max}-solution as a decision to recruit primarily unemployed applicants, while the high-w_{max}-solution relies to a large extent also on the employees of other firms as a source for recruitment.

The possibility of a potentially bimodal shape of $\alpha \cdot W'(n,w)$, generated by recruitment of both employed and unemployed job applicants, introduces interesting effects, which bear upon the Phillips curve issue. We will consider them further in Section 5 on sensitivity analysis results.

Before we briefly consider the case of flexible wages, we must take into account that $V(n,w)$ rather than $W(n,w)$ is the relevant value function to analyze. The qualitative properties of the derivative of the value function with respect to w as described above are preserved, however. We have

$$\alpha \cdot V'(n,w) + n = \frac{\gamma}{\alpha}\{(pa-w)f(w)[1 - (\frac{\lambda}{\lambda+\alpha})^{N-n} - \frac{\alpha}{\lambda+\alpha} \cdot (N-n)(\frac{\lambda}{\lambda+\alpha})^{N-n}] - F(w)[1 - (\frac{\lambda}{\lambda+\alpha})^{N-n}]\}.$$

Compared to the expression for $\alpha \cdot W'(n,w) + n$, the terms $(pa-w)f(w)$ and $F(w)$ are multiplied by factors smaller than one and greater than zero; the factor applied to $(pa-w)f(w)$ is the smaller one but its positivity is ensured by considering its serial form

$$\frac{\alpha}{\lambda+\alpha} \sum_{i=1}^{N-n} [(\frac{\lambda}{\lambda+\alpha})^{i-1} - (\frac{\lambda}{\lambda+\alpha})^{N-n}].$$

Furthermore, both factors are decreasing in λ and hence in w. Consequently, the effect of introducing a bounded capacity limit is to contract $\alpha \cdot W'(n,w)$ and to decrease further its positive term. The general shape of $\alpha \cdot V'(n,w)$ remains the same as that of $\alpha \cdot W'(n,w)$ but any w_{max} must necessarily be lower; its existence and its optimality may also disappear when N becomes bounded. The reason is of course that the value to the firm of a more rapid expansion through a higher wage diminishes, when expansion must be stopped.

Let us now consider the case of flexible wages. We recall that the decision variable is now the wage level w_n to hold at the present employment state n only, so there is no conceptual distinction between initial and subsequent states. It turns out that an analysis of the impact of the shape of the reservation wage distribution on the optimal solution is much simpler in this case. With flexible wages we can consider the current decision separately, taking for granted that optimal decisions are taken at the next stage of the process, independently of the current decision. Differently expressed, with flexible wages the firm can take any decision now, optimal or not, but nevertheless choose the optimal one at the next instant; any mistake is reversible. With downward rigid wages, an inoptimal wage increase may establish too high a wage level and that mistake is irreversible and has perpetual consequences to the firm.

In the Appendix to the Technical Supplement it is established that the properties of the optimal solution w_n^* is determined by the implicit expressions

$$\hat{L}'(n,w_n) = \frac{1}{\gamma+\alpha}\,\{-n + \gamma \cdot f(w_n)[L(n+1) - L(n)]\}$$

$$\hat{L}''(n,w_n) = \frac{\gamma}{\gamma+\alpha} \cdot f'(w_n) \cdot [L(n+1) - L(n)]$$

so that a $w_{n,max}$ is found where $-n + \gamma \cdot f(w_n)[L(n+1) - L(n)] = 0$ and $f'(w_n)[L(n+1) - L(n)] < 0$; $L(n+1)$ and $L(n)$ are the maximized expected total profits in employment states n+1 and n, respectively, and $L(n+1) - L(n)$ is positive (and decreasing in n) according to the results of the Technical Supplement.

Let us at first compare the expression $-n + \gamma \cdot f(w_n)\ [L(n+1) - L(n)]$ with the corresponding expression when wages are downward rigid (for simplicity with N unbounded)

$$-n - \frac{\gamma \cdot F(w)}{\alpha} + \gamma \cdot f(w) \cdot \frac{pa - w}{\alpha}\ .$$

The term $-\frac{\gamma \cdot F(w)}{\alpha}$ is the (expected) cost effect with respect to all future employees of a wage increase, that must be accounted for when the increase is irreversible. This term has no counterpart when wages are flexible.

The factor $(pa-w)/\alpha$ is immediately seen to be equal to $W(n+1,w) - W(n,w)$ and corresponds to $L(n+1) - L(n)$. The important difference between them is that $W(n+1,w) - W(n,w)$ depends on w, the decision on the current <u>and</u> future wage level, while $L(n+1) - L(n)$ does not depend on w_n, the decision on the current wage level.

Consequently, when wages are flexible and w_n is allowed to vary, we consider variations in the simple expression $-n + \gamma \cdot f(w_n)[L(n+1) - L(n)]$,

$L(n+1) - L(n)$ being a positive constant. Of course, $-n + \gamma \cdot f(w_n)[L(n+1) - L(n)]$ has a many maxima as $f(\cdot)$ has modes on the interval $[\underline{v},\bar{v}]$. Consequently, the maximal number of $w_{n,max}$'s is equal to the number of non-adjacent decreasing intervals of the pdf of the reservation wage distribution. They must be situated at values where the pdf is falling according to the second-order condition $f'(w_n)[L(n+1) - L(n)] < 0$. Whether all potential $w_{n,max}$'s are realized depends as before on the value of n; the main result of the Technical Supplement on the case of flexible wages is a corollary to the finding that $-n + \gamma \cdot f(w_n)[L(n+1) - - L(n)]$ is strictly falling in n.

The comments as to the situation when the pdf of the reservation wage distribution has different shapes are by large the same as in the downward rigidity case. The graphs of Figure II.2 can also be used to depict the case, when wages are flexible, if the following changes are observed: $\alpha \cdot W'(n,w)$ is replaced by $(\gamma+\alpha)\ \hat{L}'(n,w_n)$; $\min\{pa;\bar{v}\}$ is replaced by $\bar{v}$ and the $(\gamma+\alpha)\ \hat{L}'$-curve never enters below the $(-n)$-line; w_o is replaced by $\underline{w}$ (which should be situated not too much above $\underline{v}$) and w is replaced by w_n.

Note also that discontinuities because of sudden changes in the optimal stopping state, as depicted in graph b) of Figure II.2, do not occur when wages are flexible. To produce a downward-sloping $\hat{L}'$-curve as in graph a) $f(w_n)$ must be strictly decreasing; a uniform distribution gives rise to a horizontal $\hat{L}'$-line. As we just established, $\hat{L}'(n,w_n)$ culminates at exactly those w_n-values at which the pdf has its peaks (and not to the left of them, as when wages are downward rigid). Contrary to the downward rigidity case a peak of the pdf is not only necessary but sufficient to produce a maximum of $\hat{L}'(n,w_n)$.

We should observe that wages like $w_{n,max}$ and $\underline{w}$ are indeed realized along the optimal trajectory as n increases, because they are optimal in subsequent as well as in initial states when wages are flexible. Let us assume that there exists a bimodal reservation wage distribution, made up by the reservation wages of unemployed and employed job applicants in the way we described earlier; then the recruiting firm may at first decide on a high $w_{n,max}$, which attracts a majority of the employed job searchers, change abruptly to a low $w_{n,max}$ at a higher employment state and rely primarily on unemployed job applicants and eventually settle down at $\underline{w}$ with a minimum speed of recruitment. A firm under a downward wage rigidity regime must on the contrary decide on the composition of its recruitment sources from the outset and stick to it during its expansion.

5 Sensitivity analysis

In this section we investigate the impact on the optimal solution of changes in the parameter values of the decision problem. Traditionally such an investigation is called 'comparative statics analysis' in the economics literature, but that term is better replaced by 'sensitivity analysis' in the present (and perhaps also in a traditional) context.

We are not going to give a comprehensive analysis of all possible sensitivity analysis results that offer themselves in our recruitment model. For the case of downward wage rigidity (with a constant capacity limit N) a more detailed investigation is carried out in Schager (1986b); as the value function is then explicit in the decision variable w, the procedure is quite straightforward. For the case of wage flexibility the value function is implicit in the decision trajectory, i.e. it is explicit in w_n conditional on $\{w_i^*\}_{n+1}^N$ being chosen. Applying, as we have already done in the preceding section, without explicitly referring to it, the method of virtual decision epochs, described in the Appendix to the Technical Supplement, the principal difficulty lies in establishing how L(n+1) - L(n) is affected by a change in parameter values. This problem can usually be handled by using induction from the ultimate state L(N).

It is useful to consider separately those parameters which affect directly the profit rate structure and those which affect the process structure.

The parameters with a direct impact on the profit rate structure $\{pa-w_i\}_n^N$ are the value productivity pa and the capacity limit N. Under both wage regimes a change in either pa or in N changes the optimal wage in the same direction, as one would expect. The value to the firm of the decision process also changes in the same direction. We summarize:

<u>The optimal wage and the maximized expected total profits are non-decreasing in the value productivity and in the capacity limit</u>.

Of course, non-decreasing can be replaced by strictly increasing if $w^*(n)$ and w_n^* are interior solutions at the initial value of the parameter.

The situation becomes more involved when changes in the process structure are considered. The process structure is characterized by $\lambda(\cdot)$, i.e. by the contact intensity γ and the reservation wage distribution $F(\cdot)$. We should also add the discount intensity α to the set of process structure parameters; as we have seen, α appears in a way that is formally similar to $\lambda(\cdot)$ and it can be interpreted as the instantaneous probability that the horizon is reached. Although we have treated the horizon as infinite, the presence of discounting enables us to think of the horizon as being a stochastic variable, exponentially distributed with finite expectation α^{-1}. The process of contacts and of hires competes in a sense with a process of "horizon arrivals".

To see this, consider the expression, applicable when wages are downward rigid and the capacity is unbounded,

$$\alpha \cdot W'(n,w) + n = \frac{\gamma}{\alpha} [(pa-w)\ f'(w) - F(w)].$$

From this expression it is clear that, besides experiencing a windfall gain or loss, when α is changed, the firm is indifferent to simultaneous proportional changes in α and γ. Differently stated, a simultaneous change in the expected horizon and in the expected time between contacts, such that the expected number of contacts up to the expected horizon, i.e. γ/α, is unaltered, leaves the firm in an unchanged position as to its decision.

Some reflection on the behavior of

$$W'(n,w) = \frac{1}{\alpha}\{-n + \frac{\gamma}{\alpha}[(pa-w)\ f'(w) - F(w)]\}$$

when α or γ changes makes it clear that the optimal wage must be increasing in γ and decreasing in α. As the same result is proved for the case of flexible wages in the Appendix to the Technical Supplement, we can summarize:

<u>When wages are flexible or when they are downward rigid and the capacity is unbounded, the optimal wage as well as the maximized expected discounted total profits is non-decreasing in the contact intensity and non-increasing in the discount intensity</u>.

As is shown in Schager (1986b) this result does not necessarily hold when wages are downward rigid and the capacity is bounded. If, for example, n=0, implying that w*(0) is an interior solution, unaffected by γ when capacity is unbounded, the existence of a capacity bound makes w*(0) decreasing in γ. Otherwise it seems to be no easy matter to establish useful conditions for the optimal wage to be non-decreasing or non-increasing in γ or α. Loosely speaking, to get w* non-decreasing in γ there must remain much to be gained in the future recruitment process and that is not guaranteed when the process is stopped at some capacity limit.

It is now time to address the issue, announced in our introduction, of the firm's response to a change in the unemployment rate. The stock of unemployment, U, affects the firm through the contact intensity, as we have $\gamma_U = U \cdot \theta_U$, where θ_U

is the search intensity to the firm of any unemployed job applicant. Ignoring the recruitment of employed job applicants and simply putting $\gamma = \gamma_U$, we find that U (and θ_U) affects the optimal wage of the firm in the same way as γ does.

Consequently, with the exception of the case of downward wage rigidity with capacity bounded, an increase in the unemployment stock must have a non-decreasing impact on the wage level. It would be hazardous to suggest that a "representative" firm faces such a combination of parameter values (i.a. a small number of vacancies and a small number of employees) that the existence of downward wage rigidity with bounded capacity should reverse this relation in empirical findings, although this possibility should deserve further attention. We are rather inclined to say that the result of Mortensen and of Siven is confirmed by our model when γ is identified with γ_U.

How is this result brought about? Consider again the expression

$$W'(n,w) = \frac{1}{\alpha}\{-n + \frac{\gamma}{\alpha}[(pa-w)\ f(w) - F(w)]\} =$$

$$= \frac{1}{\alpha}\{-n + \frac{1}{\alpha}[(pa-w)\ \lambda'(w) - \lambda(w)]\}$$

A change in γ changes $\lambda'(w)$ and $\lambda(w)$ proportionately. Let us, however, for a moment forget about the relation $\gamma \cdot F(w) = \lambda(w)$ and just think of $\lambda(w)$ as the intensity of hires and $\lambda'(w)$ as the wage sensitivity of this intensity at the wage offer w. Then it is clear that a change such that $\lambda(w)$ is increased but $\lambda'(w)$ remains constant decreases $W'(n,w)$. The optimal wage is non-increasing in such a change.

This is in fact the way in which Pissarides (1976) arrives at his deviating results, which we referred to in the introduction. He identifies an increase in the unemployment stock with an increase in the supply of labour to the firm, a statement that is clearly in accordance with our relation $\lambda_U(w) = U \cdot \theta_U \cdot F_U(w)$. But Pissarides does not consider any explicit reservation wage distribution, so he feels unhampered to postulate that $\lambda'(w)$ (in our notation) is unchanged when the supply of labour to the firm, i.e. $\lambda(w)$, increases. It is in complete accordance with the results of our model that with such a characterization of the effect of a change in the unemployment stock, an increase in this stock should lead to a decreasing (non-increasing) optimal wage.

It is difficult, though, to accept Pissarides' approach in the context of a search theoretical model of the labour market. What he implicitly assumes is that every additional unemployed individual has a reservation wage equal to $\underline{v}$, or rather, to get the model consistent, that all the unemployed have that reservation wage. So $F_U(w) = 1$ for all $w \geq \underline{v}$ and $f_U(w) = 0$ for all $w \neq \underline{v}$, i.e. the reservation wage distribution of the unemployed is degenerate. On the other hand, such an assumption does not imply that the firm's decision problem degenerates, as long as there exists a reservation wage distribution $F_E(w) \neq F_U(w)$ over the employed job applicants.

What Pissarides' assumptions amount to, when amended with a reservation wage distribution of employed job applicants, is a limiting case of a bimodal total reservation wage distribution. The important thing is that his results can be obtained with such a distribution without imposing any implausible restrictions on it. Consider again our decomposition of $\lambda(w)$ according to the preceding section.

$$\lambda(w) = \gamma \cdot F(w) = \gamma_U \cdot F_U(w) + \gamma_E \cdot F_E(w) =$$
$$= \lambda_U(w) + \lambda_E(w)$$

An increase in the stock of unemployment increases γ_U as before. As a result γ is increased but $F(w)$ is also shifted to the left. We consider consequently a simultaneous change in γ and in $F(w)$. In complete analogy an increase in γ_E increases γ but shifts $F(w)$ to the right. (This will be the only type of change in the process parameter $F(\cdot)$ considered in this section on sensitivity analysis.)

Although $F(w)$ is shifted, the constituent distributions $F_U(w)$ and $F_E(w)$ are unchanged, so we can fruitfully exploit the additive properties of $\lambda(w)$. Recall the expression

$$\alpha \cdot W'(n,w) + n = \frac{\gamma_U}{\alpha} [(pa-w)\ f_U(w) - F_U(w)] +$$
$$+ \frac{\gamma_E}{\alpha} [(pa-w)\ f_E(w) - F_E(w)]$$

For notational convenience, we put

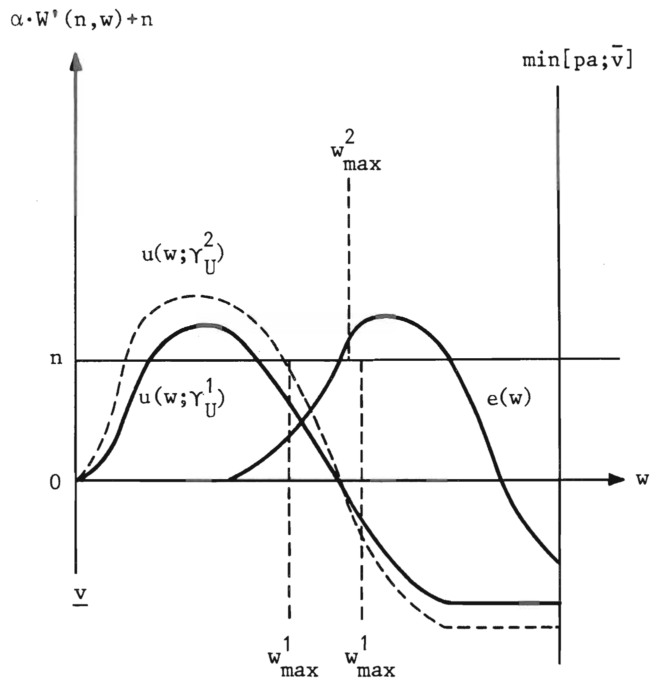
$$u(w) = \frac{\gamma_U}{\alpha} [(pa-w)\ f_U(w) - F_U(w)];$$

$$e(w) = \frac{\gamma_E}{\alpha} [(pa-w)\ f_E(w) - F_E(w)]$$

$u(w)$ and $e(w)$ are depicted in Figure II.3. $u(w;\gamma_U^1)$ is the position of the curve at a lower unemployment stock, $u(w;\gamma_U^2)$ its position at a higher one (the dashed curve); $e(w)$ is unaffected by γ_U. We have drawn the curves so that two w_{max}-solutions appear at γ_U^1; at those values denoted w_{max}^1 it holds that $u(w;\gamma_U^1) + e(w) = n$. The second order condition requires that $u'(w) + e'(w) < 0$ at a w_{max}.

Figure II.3 The effect of a change in the supply of unemployed job applicants on the optimal wage

Downward rigid wages



The important distinction between the w^1_{max}'s is that $u(w;\gamma^1_U) > 0$ at the lower w^1_{max}, while $u(w;\gamma^1_U) <$ < 0 at the higher one. An increase in γ_U to γ^2_U does not change the sign of $u(w)$ but pivots proportionally the curve clockwise around the w-value where $u(w) = 0$, as indicated by the dashed curve. Consequently, the conditions for a w_{max}-solution require that the lower w_{max} is increased and that the higher one is decreased, when γ_U increases. As the curves are drawn in Figure II.3 the lower w_{max} actually disappears when γ_U is increased to γ^2_U, so the two w^1_{max}'s collapse into the intermediate w^2_{max} at $\gamma_U = \gamma^2_U$.

What is produced is really the possibility of either a "Mortensen" or a "Pissarides" outcome of a change in unemployment. The decisive factor as to which of them is realized is whether a change in the firm's wage decision at its optimal level affects primarily the speed of recruitment of unemployed or of employed job applicants. To get the optimal wage decreasing in γ_U, $u(w)$ must be negative at the optimal wage. For this to happen the pdf's $f_U(w)$ and $f_E(w)$ must lie well apart (so that $u(w)$ and $e(w)$ lie well apart) and the employed job applicants must constitute a significant source of recruitment to the firm; $u(w) < 0$ implies $e(w) > n$ in the neighbourhood of the optimal wage, so $e(w)$ must be able to produce a w_{max} 'on its own'.

Again we should consider the effect of a bounded capacity N on our results in the case of downward wage rigidity. As we recall that an increase in γ, $F(w)$ unchanged, will not necessarily lead to a non-decreasing optimal wage when capacity is bounded, the possibility that the optimal wage is decreasing in γ_U is strengthened. We have

$$\alpha \cdot V'(n,w) + n = \frac{\gamma_U \cdot f_U(w) + \gamma_E \cdot f_E(w)}{\alpha} (pa - w) \cdot$$

$$\cdot \{1 - [\frac{\lambda(w)}{\lambda(w)+\alpha}]^{N-n} - \frac{\alpha}{\lambda(w)+\alpha} (N-n) [\frac{\lambda(w)}{\lambda(w)+\alpha}]^{N-n} \} -$$

$$- \frac{\gamma_U \cdot F_U(w) + \gamma_E \cdot F_E(w)}{\alpha} \{1 - [\frac{\lambda(w)}{\lambda(w)+\alpha}]^{N-n} \}$$

The decision process is not as simply dichotomized as with N unbounded but the same general structure emerges: the second term is negative and increasing in absolute value with respect to γ_U as it is equal to $\sum_{i=1}^{N-n} [\frac{\gamma_U \cdot F_U(w) + \gamma_E \cdot F_{E(w)}}{\gamma_U \cdot F_U(w) + \gamma_E \cdot F_E(w) + \alpha}]^i$; the factor within curly brackets in the first term is, as earlier established, positive but decreasing in λ and hence in γ_U. So, in accordance with our earlier findings, at a w_{max} where $f_U(w)$ is low, an increase in γ_U makes $\alpha \cdot V'(n,w)$ turn negative and calls for a reduction in the optimal wage, if originally equal to that w_{max}.

When wages are flexible the situation can be described in terms similar to those of the preceding discussion. Also in this case increases in unemployment can have a non-increasing effect on the optimal wage because of the same mechanism. We do not carry through a formal parallel demonstration, though.

To sum up, we have demonstrated that an increase in the unemployment stock (or in the search intensity of the unemployed) has an increasing impact on the optimal wage if it also increases the probability density of the job applicants considerably at the optimal wage level; it has a decreasing impact on the optimal wage, if the reverse is true.

Our analysis of the effect of a change in γ_U can easily be adapted to handle a change in γ_E. It is clear that the optimal wage must in general be non-decreasing in γ_E, i.e. non-decreasing in the supply of employed job applicants.

It is worth noting that under wage flexibility, the firm applies, as we observed earlier, different strategies as to the composition of its recruitment sources at different stages of expansion, corresponding to the changing optimality of high and low wages at different employment levels. Under downward wage rigidity the firm has decided once and for all on the composition of its recruitment sources. Consequently, it should respond qualitatively uniformly to a change in the unemployment stock at different stages of expansion.

On the other hand, when wages are downward rigid, the firm cannot lower its wage along the optimal trajectory. The wage level in subsequent states during expansion is in a kind of 'disequilibrium' as we note in Section 3 of the Technical Supplement. This introduces some interesting features with respect to sensitivity analysis results in a dynamic context. We will consider them in Section 7, where we discuss tentatively some possible interdependency effects in a labour market, a part of which is the present model of optimal firm behaviour.

It is at this stage of the analysis obvious, that the effect of unemployment on the wage decision of a firm is an involved question, which cannot be answered by theory alone but requires empirical information. Especially much more evidence is needed on the composition of the supply of labour to the firms and its impact on the composite reservation wage distribution facing them. According to the hypothesis put forward in this section these conditions play a crucial role. Here search theory needs the support of empirical investigations and data collection on a large scale.

6 Conjectural remarks on an extended recruitment-cum-quit model

The results of the present chapter build on the analysis in the Technical Supplement and are consequently established with a considerable degree of rigour. We have deliberately chosen to confine our contribution to provide a rigorous, yet tractable analytical model, that can also be used as a part of and a step to further extensions in future research. Nevertheless, it is worthwhile to venture somewhat into the structure of a wider search theoretical framework, consistent with our stochastic dynamic programming model. This we will do in the present and the following sections.

As noted in the introduction to the Technical Supplement, a drawback of the present recruitment model is the absence of quits. This deficiency is acknowledged and the reason for neglecting quits is purely technical: a stochastic model, in which the firm moves to both higher and lower states according to a probabilistic law of motion, is much less tractable than a model, in which the movement is one-way. This is especially true when wages are assumed to be downward rigid.

It is at this point to be noted, that from a formal point of view a stochastic recruitment control and a deterministic control of both recruitment and quits have a similar structure. With a slight reformulation of the contact process, such that the contact intensity is increasing in the number of vacancies announced by the firm, these similarities become especially apparent, as the Excursus to this chapter demonstrates. This is not surprising as the effect of a deterministic approximation of the recruitment and quit processes is to make the composite net process one-way at any wage level.

The first extension of the present stochastic recruitment model to aim at should consequently be the inclusion of a stochastic quit process, which is influenced by the wage decision of the firm. In Section 5 of the Technical Supplement we give some conjectures as to the structure of the optimal policy in such a stochastic recruitment-cum-quit model. The reader is referred to that section, which is non-technical, for a presentation of our conjectures.

It is obviously of interest to see whether our results from the sensitivity analysis of the preceding section, for example the effect of a change in unemployment, can be expected to hold in such an extended model.

The policy-determining expression can easily be formulated for the flexible wage case in its virtual decision epoch form. An investigation of its structure reveals, that we have no reason to doubt that a change in the unemployment stock has the same effects on the optimal wage as those discussed in the preceding section. As the wage sensitivity of the quit rate is not directly influenced by γ_U, any non-increasing effect of γ_U on the optimal wage would in fact be strengthened.

The conjectured structure of the optimal policy is complex in the case of downward rigid wages. Nevertheless, we may gain some insights by considering the value function, corresponding to a constant wage trajectory w in all subsequent states and an unbounded capacity limit; the value to the firm of such a policy in the recruitment-cum-quit model is

$$W_T(n,w) = \frac{pa-w}{\alpha+\mu(w)}\left[n + \frac{\lambda(w)}{\alpha}\right],$$

where $\mu(\cdot)$ is the quit intensity of an employee at the firm.

$\max\limits_{w \geq w_o} \{W_T(n,w)\}$ is <u>not</u> the optimal value, unless n=0, according to the conjecture. It might be optimal to increase strictly the wage, when a lower employment state is entered, although it is optimal to keep it fixed, when a higher one is entered, if the conjecture is true.

W_T is a straightforward generalization of $W(n,w) = \frac{pa-w}{\alpha}\left[n + \frac{\lambda(w)}{\alpha}\right]$; the existence of quits diminishes the value of the decision process to the firm.

Differentiation of W_T with respect to w yields

$$W_T'(n,w) = \frac{n + \frac{\lambda(w)}{\alpha}}{\alpha+\mu(w)}\left\{-1 + (pa-w)\left[\frac{\lambda'(w)}{\alpha\cdot n+\lambda(w)} - \frac{\mu'(w)}{\mu(w)+\alpha}\right]\right\}$$

Rearranging the expression for $W'(n,w)$ yields

$$W'(n,w) = \frac{n + \frac{\lambda(w)}{\alpha}}{\alpha} \{-1 + (pa-w) \frac{\lambda'(w)}{\alpha \cdot n+\lambda(w)}\}$$

The properties of $W'(n,w)$ have been thoroughly investigated in earlier sections. A comparison between W_T' and W' shows that the inclusion of a quit function $\mu(w)$ does not simplify the structure of possible interior maxima. Depending on the shape of $\mu'(w)$, i.e., on the shape of the pdf of the wage offer distribution, new interior maxima may come into existence. We also see that the factor within curly brackets increases when quits are introduced; in addition, W_T' is contracted compared to W' and it is contracted more at low values of w than at high ones. At $w = \min[pa;\bar{v}]$ W_T' is negative. Consequently, if a 'high w_{max}' exists in the recruitment model, a corresponding w_{max} exists in the extended model, too, and its value is higher; furthermore, if this w_{max} is optimal in the recruitment model, its counterpart is 'optimal' (conjecturally, of course) in the extended model.

If an increase in the stock of unemployment decreases the optimal wage in the recruitment model in the way we described in Section 5, the factor within curly brackets in the expression for $W'(n,w)$ must decrease at the optimal wage and at higher wage levels. Consequently, the factor within curly brackets in the expression for $W_T'(n,w)$ must decrease in the neighbourhood of its 'optimal' wage level, when unemployment increases [$\mu(w)$ and $\mu'(w)$ being unaffected by this change]. We conclude that the 'optimal' wage (again in a conjectural sense) is decreasing in the unemployment stock in the extended model, when the corresponding effect holds in the recruitment model.

Summing up our discussion on the extended model, we see no reason why our qualitative results on the recruitment model would drastically change, if quits are included. This is of course no excuse for abstaining from efforts to establish rigorous results as to the stochastic recruitment-cum-quit decision process; as we stated earlier, this should be an important task for future research.

7 A discussion on business cycle dynamics under downward wage rigidity

In this section we comment tentatively on some dynamic aspects of our model of optimal firm behaviour in a market interdependence setting. In particular we investigate some features of the decision process under downward wage rigidity, taking into consideration the fact that the firm will usually not be in an initial but in a subsequent employment state after some time has elapsed.

As is pointed out in some detail in Section 3 of the Technical Supplement, under downward wage rigidity the firm will be increasingly in 'disequilibrium' with respect to its wage level as employment expansion goes on. This is easily visualized with the help of Figure II.2; an increase in the employment state causes the $\alpha \cdot W'(n,w)$-curve to shift downwards. In a subsequent state the firm can not respond to this downward shift by a reduction in the wage level. Consequently, $\alpha \cdot W'(i,w^*)$ will be negative and increasingly so, as employment expands. Succinctly stated, $0 \geq W'(n,w^*) > W'(i,w^*) > W'(i+1,w^*)$, $i > n$.

This property of the recruitment process has important consequences when sensitivity analysis is applied in a dynamic context. In the first place it is immediately obvious that any non-increasing parameter change (with respect to the optimal wage) will leave the optimal constant wage trajectory unchanged. At the same time it will establish or increase a wage disequilibrium in the sense just defined along the optimal trajectory.

Secondly, any non-decreasing parameter change will be strictly increasing only to the extent that it operates on a wage along the optimal trajectory, at which $W'(i,w^*) = 0$. Otherwise it will just decrease the disequilibrium at state (i,w^*), leaving the wage unchanged. Consequently, if the firm has been successful in expanding employment, a non-decreasing parameter change will partly or wholly be "exhausted" in reducing the disequilibrium at the higher employment state and the more employment has expanded the more the parameter must change in order just to restore the equilibrium condition $W'(i,w^*) = 0$. Expressed in other terms, a considerable parameter change of non-decreasing character may be required to free the firm from the consequences of its earlier wage increase decision and make a new wage increase warranted.

Let us think of the firm, having initially expanded employment, facing an (unexpected) series of parameter changes of at first a non-increasing and later on a non-decreasing character. Employment expansion itself produces a wage disequilibrium. This disequilibrium is increased by the first non-increasing parameter change. If the parameter affects the profit rate structure directly (and especially if the production function is strictly concave in the relevant range) the recruitment process might also be stopped at a lower employment state. At a later stage a non-decreasing parameter change occurs. Under the same conditions as above it may increase the optimal stopping state, but the change has to be considerable to make up for the established wage disequilibrium. If we think of two consecutive non-decreasing parameter changes occurring at the end of the series of changes, the first one is likely to have its main effect in eliminating the wage disequilibrium, while the second one has a greater chance of affecting the wage level.

The important aspect of this generally phrased example is that it can be seen as an illustration of the series of events that faces the firm during a business cycle. While still expanding at the peak of the cycle, the firm experiences worsening business conditions, e.g., a fall in the price of its products. The wage level is unaffected by this, but the number of vacancies may be reduced (perhaps to zero). If other firms are also hit, so that some of them stop recruiting, things obviously happen in the labour market, but we ignore this important aspect for a moment. The firm is now experiencing a period of marked wage disequilibrium, at an early stage of which it is likely to have reached its (reduced) stopping state and ceased recruiting. When business conditions improve, e.g. in the form of an increase in the product price, we may expect the firm to open up vacancies again but the price increase must be considerable in order to produce a wage increase; it is not enough that the pre-slump price level is restored.

We should also note that the firm may find itself at too high an employment level when the slump has occurred, i.e. its actual employment exceeds the (reduced) optimal stopping state. If the firm does not resort to lay-offs, this introduces a second kind of disequilibrium in the form of labour hoarding in the slump period. As a consequence vacancies will not immediately or fully respond to an improvement in the product market, part of which is used to eliminate the disequilibrium in the employment level. It is a striking feature of the process that the first recovery phase may leave the decision variables of the firm, the wage level and the creation of vacancies, unchanged. What happens to the firm in the form of reduced disequilibria remains largely hidden under the surface.

We have now described what can be called a firm-specific business cycle process. As a first step towards a macroeconomic interpretation we must take into account what happens in the labour market, when most firms face the same type of parameter changes simultaneously. Clearly the effect to the firm must be a change in the parameters of the process structure, i.e. the reservation wage distribution and the contact intensity.

When the slump sets in, many firms will cancel their vacancies and stop recruiting. Such a change must necessarily imply that job applicants face a decreased intensity of wage offers. It is by no means certain, however, that the wage offer distribution itself changes in any systematic fashion at this stage of the cycle; anyone firm has an unchanged wage level, and we cannot say a priori that high-wage firms stop recruiting to a larger extent than low-wage firms. As an optimal response to a decreased intensity of wage offers the unemployed should decrease their reservation wages [cf. Albrecht, Holmlund and Lang (1986)]. So the reservation wage distribution of the unemployed is shifted to the left when recruitment is reduced.

Although the total effect of a decreased wage offer intensity, a concomitant optimal decrease in the reservation wage and, possibly, an optimal change in the individual search effort on the probability of an unemployed individual of becoming recruited is not explored in Albrecht, Holmlund and Lang (1986), it is likely to be a decrease. If the arrival rate into unemployment is at least constant (it might be increasing at this stage of the cycle because of a higher frequency of lay-offs), the unemployment stock will increase. Consequently, after the economy has been hit by a slump, the firm will experience a successive increase in the contact intensity of the unemployed job applicants, γ_U, as well as a shift to the left of the pdf of the reservation wage distribution, $f_U(w)$.

It is an important consequence of downward wage rigidity that if the firm has chosen to recruit at such a high wage level as to attract a significant part of the employed job applicants at the previous business cycle peak (and we can add with reference to the extended model, as to prevent its own employees from quitting on a large scale), it has to stick to that high level in the slump period. In such a case the change in the labour market during the recession, manifesting itself to the firm as a rise in $\gamma_U \cdot F_U(w) = \lambda_U(w)$, will have a non-increasing impact on the optimal wage, as we demonstrated in Section 5. Consequently, the disequilibrium at the unchanged wage level will be further increased.

We have not taken into account the possibility that the total number of vacancies announced in the labour market may influence directly the contact intensity to the firm, given the stock of unemployment. As we discuss in Chapter I on the interpretation of the UV-curve, different search "technologies" can be envisaged; the aggregate data presented there do not point to the need of including a separate effect of vacancies on the contact intensity, given the stock of unemployment. One reason can be that the individual job searcher adjusts his search intensity, when the offer intensity changes in a way consistent with the model of Albrecht, Holmlund and Lang (1986). As it were, any direct influence from the stock of vacancies implies that both γ_U and γ_E increase in the slump. At a 'high' wage level such an additional change in γ_U increases the disequilibrium, the change in γ_E decreases it. The composite effect is uncertain and presumably of little importance. Obviously any change in $F_E(w)$ can be safely ignored, as it is closely linked to the distribution of paid wages; it takes massive lay-offs of systematic character to change it in a recession, when wages are downward rigid.

To sum up, we conclude that a firm, which has been relying on recruitment of both employed and unemployed job applicants on a comparable scale will find itself in wage disequilibrium at the end of the slump period for three additive reasons: former employment expansion, worsened conditions in the product market and increased number of unemployed with lowered reservation wages in the labour market. It might also find itself in employment disequilibrium with hoarded labour.

If a majority of the firms is in a 'high wage' position, our conclusions as to the effect on a single firm of the parameter changes in the recovery phase of the business cycle hold in the aggregate; they are in fact strengthened by the recession effects on the labour market parameters. Consequently, the improved conditions in the product market in the form of higher prices and increased stopping states are at first 'absorbed' by higher profit margins (and possibly decreased labour hoarding) without affecting wages (or vacancies).

For the sake of the argument we can assume that the parameters of the profit rate structure are restored to the level of the previous business cycle peak. This also restores the optimal stopping states, eliminates labour hoarding and gives rise to vacancies and recruitment. The wage disequilibrium is not eliminated completely by this parameter change, however. First of all, the expanding firms were in disequilibrium exactly because of expansion at the previous peak and this condition must still hold. Secondly, the labour market has changed during the recession and the higher unemployment stock gives rise to an additional wage disequilibrium. The reservation wage distribution of the unemployed may be fairly quickly restored to its pre-slump position (although that is not certain) but the unemployment stock can only diminish through the time-consuming recruitment processes of the firms. In fact, it might require a prolonged period of time until the labour market is restored to its pre-slump situation.

Two characteristics of the outlined aggregate dynamic process deserve to be explicitly noted. One is that the combination of parameter values at a business cycle peak must constitute a strictly increasing, non-infinitesimal change (with respect to the wage level) compared to the parameter value combination at the previous peak in order to produce a strictly positive wage increase. Increasing changes from slump period values, which fail to meet this requirement, just reduce existing wage disequilibria. The other feature is the importance of the ordering in time of the sequence of increasing parameter changes during the business cycle. If, as we have described it, increasing changes in the recovery phase emanate from the product market, they will have little visible effect on wages; they show up in increasing profits, recruitment and production. In the mature phase of the boom labour market parameters change in an increasing manner, as indicated, and as now former wage disequilibria have largely been eliminated, firms respond by wage increases. Obviously, the first kind of parameter changes is per se no less 'wage-inflationary' than the second kind; it is just that the timing becomes very significant, when wages are downward rigid.

This implication of downward wage rigidity should be important for understanding the observed pattern of wage increases during a business cycle. Theoretical work on wage dynamics has as a rule disregarded this possibility, which is not surprising, as the case of downward wage rigidity has not been attended to in the more rigorous contributions on the subject. The paper by Leban (1982b), which is an exception in this respect, has apparently not attracted enough attention to change this state of affairs; although that paper is strictly confined to an investigation of the decision problem of a single firm, Leban's model produces similar disequilibrium effects as we have discussed and tried to apply in a context of labour market interdependence. [It should be mentioned that Leban (1982b) is primarily focusing on disequilibria in the employment state of the firm, leaving the concept of wage disequilibrium largely implicit.]

Let us now take into consideration the possible effect on the wage and employment dynamics of including quits. When the slump occurs, the existence of quits alleviates to some extent the situation for a firm which is brought into an employment disequilibrium; for the firm, which still recruits, realized quits will diminish the wage disequilibrium. As a general slump decreases the intensity of wage offers, both 'equilibrating' effects are working slowly and in the aggregate they should partly offset each other; the recruiting firm, which still aims at expansion, although possibly to a lower stopping state, is hardly likely to experience a net loss of employees in the slump because of quits. Even so, in order to produce a wage increase, its employment volume must decrease below its historical minimum and beyond (as the parameter values were changing decreasingly at the occurrence of the slump) according to the conjecture on the optimal policy in the recruitment-cum-quit model.

The existence of on-the-job search and resulting quits accounts for the fact that there exist vacancies and labour turnover even in a prolonged slump period. Given the parameter values characterizing this phase of the business cycle and the fact that most firms, which aim at expansion at the beginning of the slump, are likely to have been at least partly successful in their attempts, the turnover of employees implies mainly a 're-shuffling' of wage disequilibria between firms. Wage increases would seldom occur because of this.

The turnover of employees will play a much more significant role in the boom period. When the recovery sets in, because of a favourable change in the profit rate structure parameters, vacancies are opened up and recruitment restarts on a broad scale. Because of the accumulation of a large stock of unemployed job applicants during the recession, a larger proportion of the new employees comes from that recruitment source. As a consequence most firms will experience an increase in employment volume, although the quit intensities of their employees also increase as a response to the increased wage offer intensity.

As time goes on after the start of the recovery phase, the unemployment stock decreases and a larger proportion of any expansion must take place at the expense of a net reduction in employment of other firms. At this phase of the cycle, however, many firms are in wage equilibrium. Consequently, labour turnover may have a strong impact on wage increases on the average, because the firms with net losses in employment increase their wages while those with net gains keep their wages constant, according to the conjecture on the optimal policy in the recruitment-cum-quit model (cf. Section 5 of the Technical Supplement). As we discuss in the next section, even stronger effects may result from the interaction between the distributions of reservation wages and wage offers in connection with the turnover of employees.

In addition it should be noted that a change in the quit intensity affects the optimal wage, regardless of any quits actually occurring. The conjectured result of such a sensitivity analysis carried out on the recruitment-cum-quit model is that the optimal wage is non-decreasing in the quit intensity, if the quit intensity changes uniformly because of a change in the wage offer intensity (we have the quit intensity $\mu(w) = \Theta[1 - G(w)]$, where Θ is the wage offer intensity, and we see that the conjecture is consistent with the effect of a change in Θ on the expression for $W_T'(n,w)$ in Section 6). Consequently, the occurrence of a slump, decreasing the wage offer and hence the quit intensity, introduces an additional disequilibrium effect at the unchanged wage level, an effect that is reversed when the wage offer intensity increases at the recovery.

The above sketch of the cyclical pattern of wage and employment dynamics under downward wage rigidity must suffice for the time being. It must be seen as a conjecture of what more rigorous and detailed analysis can yield in the future. The most complicated parts of the aggregate, interdependent process are the simulataneous changes that occur in the wage distributions as recruitment proceeds, a problem hardly touched upon in the preceding discussion. The matching of vacant jobs and job applicants implies that job searchers and vacancies are successively eliminated. This process does not occur uniformly, however. In expectation vacancies with higher wage offers and job applicants with lower reservation wages should be eliminated more quickly. As the matching process goes on, both the wage offer and the reservation wage distribution change systematically (the existence of on-the-job search and quits adds to this tendency). It is consequently not consistent with the model to think of the wage distributions as unchanged to the firm or to the job searchers when recruitment takes place in the labour market.

It is on the other hand virtually impossible to establish analytically the result of such simultaneous changes in the relevant wage distributions (quite apart from the fact that the result is a stochastic variable), without assuming what is to be shown. For example, recruitment means, ceteris paribus, that the unemployed become fewer and that, through self-selection, those remaining should be distributed according to a reservation wage distribution that has shifted to the right. At the same time, the number of vacancies has been reduced and those with high wage offers most rapidly at that. Consequently, the wage offer intensity goes down and the wage offer distribution shifts to the left to the effect that the unemployed job applicants adjust their reservation wages downwards. The total effect on the reservation wage distribution is unclear. Hence the effect on the wage decision of a single firm is unclear, too, especially if we take into account that the reduced contact intensity of the unemployed is in expectation balanced by an increase in the employment volume of the firm, corresponding to its 'share' of total realized recruitment.

These observations point to the need for a formidable future research program, in which an extended recruitment-cum-quit stochastic decision model is coupled to a job search model of the type presented in Albrecht, Holmlund and Lang (1986) to form a model of stochastic interdependence in a search labour market. Such a model will hardly be analytically tractable within a foreseeable future, but conjectures may be formed as to its structural properties and subjected to tests in the form of numerical simulations.

Before concluding this section on the cyclical pattern we should add a remark. As the reader has observed we have treated the changes in parameter values as completely unexpected, applying the traditional method of sensitivity analysis in a dynamic context. Clearly this procedure may appear somewhat unsatisfactory, especially as we have assumed that the firm is fully informed about the current value of all relevant parameters. Again we want to stress that the discussion in this section is conjectural and intended to indicate plausible results of a more rigorous treatment. That the existence of wage disequilibria is plausible, even if changes in parameter values were foreseen, is indicated by its existence at employment expansion in our basic model of firm behaviour; there the future change in the employment state - or rather its probability law - is perfectly known by the firm [cf. also Leban (1982b) in which the firm is supposed to know the pattern of parameter changes during the first business cycle in the future].

To specify a known pattern of parameter changes is of course another and quite natural extension of the present model of the firm's optimal decision problem. In technical terms such an extension would lead to a Markov decision process with uncontrolled deterministic drift, if the parameters are specified as deterministic functions of time. Such general processes have been especially considered in Vermes (1980). Restrictions must clearly be put on the simultaneous changes in the parameters, so that the dimension of the state space, which now contains the parameters, does not become intractably large.

An extension in the opposite direction is to relax the assumption that the firm possesses full knowledge of the current values of the parameters. Here the problem is not essentially that a certain value of the parameter has to be replaced with a probability distribution over different values; it is rather the specification of the distribution and its changes, when new information is gathered by the firm, which causes trouble. This latter issue does not seem to have attracted much attention in the Markov decision literature. In Yeh and Thomas (1983), a simple queue control model is modified in an 'adaptive' way; the (uncontrolled) arrival intensity, corresponding to our contact intensity, is a stochastic variable, the distribution of which changes as the process goes on, but in a very mechanical way, it seems. There is certainly no lack of challinging research opportunities within this area.

How the current values of the parameters and their changes in the future are perceived is of importance when empirical applications of the model are considered. In principle it is possible to overcome the difficulties of analyzing theoretically the interdependent market process by observing the evolvement over time of the relevant parameters, including the different wage distributions. In order to make inferences from these observations, however, we must also make an assessment of the way in which the firms and the job applicants have themselves observed the state of the system and formed expectations on its future state.

8 The Phillips curve issue reconsidered

The Phillips curve issue has been a recurrent theme in this chapter and concludes it fittingly. Up to now we have argued that a causal relation of the Phillips type exists on the firm level in the sense that the firm increases its wage in the face of a decrease in unemployment. A necessary condition for this to happen is that the firm is in a 'high wage' position, where the wage sensitivity of recruitment of unemployed job applicants is low. Such a position is a plausible result of an optimal response to the recruitment (and turnover) conditions in the labour market during a business cycle peak and downward wage rigidity preserves such a position during the downturn of the cycle.

We have also argued that the seemingly strong impact of unemployment (and other labour market variables) on the rate of wage increases may be due to the timing of wage-increasing events during the business cycle. Decreases in unemployment occur at the end of a series of such events and when downward wage rigidity prevails, the early events in the series just eliminate existing wage disequilibria, while the decrease in unemployment can exercise its increasing impact on the wage level unhamperedly. It also seems plausible - though this is a conjecture on the basis of a conjecture - that labour turnover between firms replaces changes in unemployment as the strongest wage increasing factor in the mature phase of the boom.

Besides the further complexities that may arise when all interdependencies in the composite processes are taken into account, it is striking how strong the conditions are that must hold in order to yield a stable relation between wage increases and unemployment decreases. It is clear already from the basic decision model that unemployment exerts its influence through its impact on $\lambda_U(w) = \gamma_U F_U(w) = U \cdot \theta_U \cdot F_U(w)$. If there is fall in γ_U, reflecting either a decrease in unemployment or a decrease in the search intensity of the unemployed, the firm reacts in the same way. Similarly, if $F_U(w)$ decreases, reflecting an increase in the reservation wages of the unemployed, the same wage response follows. All these changes show up as a decrease in $\lambda_U(w)$ or, equivalently, as an increase in the expected duration of a vacancy.

Making use of Swedish data Schager (1981) has shown that a positive relation between the rate of wage increases and the mean duration of vacancies recovers much of the regularity that is lost when unemployment is substituted for the vacancy duration measure. Together with the results reported in Chapter I, these empirical findings must clearly be seen as an indication that our argument in favour of a labour market influence on the wage formation process is relevant.

As we demonstrated earlier, our results are crucially depending on a clear division between the reservation wage distributions of the unemployed and the employed job applicants. If these distributions begin to shift in relation to each other, another source of instability is introduced. Consider, e.g., the effect of a right hand shift in the reservation wage distribution of the unemployed. The firm in a 'high wage' position will now experience a decrease in $\lambda(w)$ and an increase in $\lambda'(w)$, both changes calling for an increase in its wage. If such a shift in the reservation wage distribution takes place over time, e.g. because of more generous unemployment compensation rules, wage increases may be observed without any decreases in unemployment. In fact, we should expect the effect in the labour market of such a more fastidious attitude on the part of the unemployed to be an increase in the stock of unemployment. Higher unemployment will cause counteracting responses, that is true, but it should be noted that they are weakened as $f_U(w)$ is increased at the optimal wage. In the extreme case, where the two reservation wage distributions coincide, the qualitative properties of the model are changed drastically. Higher unemployment now gives rise to higher wages. With reference to our earlier discussion we can say that the Mortensen effect has replaced the Pissarides effect of unemployment. It is not unlikely that something of this kind has happened in many Western economies since the late sixties. To validate this suggestion, however, would require empirical evidence that is not at hand (and cannot be recovered for past periods).

The observant reader has noticed that on the basis of the model of firm behaviour there is a relation between the level of unemployment [through its impact on the values of $\lambda_U(w)$ and $\lambda_U'(w)$] and the wage level of the firm. The Phillips curve as originally conceived is a relation between the level of unemployment and the rate of wage increases in the labour market. At an early stage of the debate on the Phillips curve it was argued, though, that the rate of change of unemployment should be included as another explanatory factor [cf. Phelps (1970), Section 3, with references]. Clearly our model supports this view. The level of unemployment may turn up as a seemingly 'explanatory' factor in the market process, however, along the following lines.

Consider any wage-increasing parameter change affecting the firms, when most of them are in wage equilibrium. The firms increase their wages, so the wage offer distribution shifts to the right. As a response to this the unemployed increase their reservation wages, so this distribution will also shift to the right. The change in the reservation wage distribution will in turn cause a new shift to the right in the wage offer distribution and so on, until possibly the two distributions become consistent with each other.

The stock of unemployment will presumably decrease as a response to the first change in the wage offer distribution, as the unemployed will partly take advantage of the better offer conditions by reducing their expected sojourn time in unemployment (in the nowadays received version of the Phillips curve mechanism they do so fully, but only because and as long as they have not realized that the wage offer distribution has changed). This is not essential, however. Our point is that a wage increasing change at the critical state of the market process, where firms have generally achieved wage equilibrium, will trigger off a selfsustaining sequence of right-hand shifts in the wage distributions. The time-path of unemployment is at this stage of importance only in so far as it does not dampen the process of wage increases, which it should not do if our partitioning of the reservation wage distribution holds.

Recall from our discussion on the business cycle pattern that the stock of unemployment goes down as the firms are on average gradually diminishing their wage disequilibria. This means in practice that more and more firms will increase their wages at any further reduction of the unemployment stock. Eventually increases will be as prevalent as to influence the wage offer distribution significantly. The reservation wage distribution will be affected as we just described. Although there might be additional reductions in unemployment, the increase in the reservation wage distribution per se indicates that the unemployed are not too keen to reduce further their search time in unemployment. Consequently, the shifts in the wage distributions begin to gain momentum at the same time as the stock of unemployment approaches its minimum; at least we may think of the duration of unemployment approaching its minimum. What happens is then that low stocks of unemployment accompany (but essentially do not cause) selfsustaining changes in the wage distributions. Clearly this would also explain the asymptotic behaviour of the Phillips curve at high rates of wage increases and low levels of unemployment.

The behaviour of the reservation wages of the employed and of labour turnover between firms plays presumably an important role for the speed of the change in the wage distributions. When the stock of unemployment is very low, the composite reservation wage distribution facing the firms is in effect dominated by the employed job searchers. In principle the reservation wages of the employed should follow closely the changes in wage offers and in paid wages. Consequently, rapid successive adjustments in the wage distributions are then to be expected. Otherwise, the process by which firms recruit employees from each in a situation where the stock of the unemployed is low is complicated to penetrate. It is clearly a most important subject to investigate.

One remark should be made, though. In our formal analysis, we have worked with an upper bound on the reservation wage distribution and no firm has an incentive to offer a wage beyond it. In a dynamic setting this wage cannot be regarded as an upper limit to the sequence of shifting distributions of wage offers and of reservation wages, however. If sufficiently many firms begin to offer a wage in the immediate neighbourhood of the originally highest reservation wage and recruit (as is likely) to that wage, this must produce employed job applicants with reservation wages equal to that wage plus the transition premium. Consequently, the upper bound is moved upwards through time and cannot per se constitute an ultimate limit to a sequence of righthand shifting wage distributions.

In our model with downward rigid wages the value added per employee (in the most productive firms) sets such an ultimate limit, however. No firm will ever offer as high a wage as this value. To get an inflationary process moving, new demand must be injected in the product market. In this respect our model is no different from other search theory applications, that concentrate on the labour market. Siven (1979) in his analysis of both a search labour and a search product market argues that a 'quasi-equilibrium' - the concept is borrowed from the celebrated study on inflation theory by Hansen (1951) - with positive (unexpected) rates of wage and price increases can exist, when the two markets interact. His conclusion appears attractive, but is tentative, because, as Siven himself points out, it is based on an aggregation of the individual firms, not on an analysis of their interaction.

As far as more rigorous interdependence models of search in a labour and product market are concerned, it has recently been established as a significant result that a two-point wage and price distribution can sustain itself (is locally stable) under partly very weak conditions [Albrecht, Axell and Lang (1986)]. Clearly a search theoretical model of inflation with all complicated dynamic distributional changes carefully analyzed is not near at hand. In the meantime there is of course no reason to let search theory be firmly associated with any particular macroeconomic inflation theory, whether monetaristic or not.

EXCURSUS TO CHAPTER II

ON THE STRUCTURAL SIMILARITIES BETWEEN A STOCHASTIC RECRUITMENT CONTROL AND A DETERMINISTIC CONTROL OF RECRUITMENT-CUM-QUITS

In this excursus we substantiate our claim in Section 6 of this chapter that the formal structure of our stochastic decision process of recruitment is very similar to that of a deterministic control of both recruitment and quits.

In order to do so we shall apply the case of downward rigid wages, but use a slightly modified version of the decision process presented in Section 3 of the Technical Supplement. Instead of having a constant intensity of contacts γ, we assume that the contact intensity is falling linearly in the employment state i so that $\gamma_i = (S-i)\gamma$. Such a structure of contacts will arise, e.g., if the intensity is linearly increasing in the number of vacancies announced by the firm. If S is the optimal stopping state of the firm, the optimal number of vacancies at state i is (S-i), so the probability of a hire at the firm depends on its number of vacancies as well as on its wage level.

Expressed in other words, in the process analyzed in the Supplement, the vacancies work in series, so that it takes (in expectation) twice as long a time to fill two vacancies instead of one; in the new process the vacancies work in parallel, so that any number of vacancies are (in expectation) filled within the same time period. With reference to the terminology of Chapter I the first process is an I-E-process of vacancies, the second one is an I-D-process.

When the number of vacancies influences the contact intensity, the determination of S is not only the determination of an optimal stopping state; S affects the contact intensity as well. Ruling out the possibility of 'false vacancies', which are never meant to be filled, there is still the possibility that the optimal S will not coincide with any of the endpoints N_k of the productivity intervals. Given that an optimal S exists, as it must clearly do when capacity is bounded, it has been shown by the author in an unpublished extension of the analysis in the Technical Supplement, that the optimal structure of the wage policy is the same as when the contact intensity is constant, i.e. the optimal wage trajectory is constant at employment expansion.

We will not present the analysis of this decision process here. Our intention is to use it in a comparison with a deterministic control process, and to that end we postulate that $S = N_k$, $pa_{k+1} \leq$ $\leq w_{N_k} < pa_k$, $k=\ell \ldots \ell'$, just as in the process with constant contact intensity (while an optimal $S > N_k$ is plausible for n sufficiently smaller than N_k).

Without utilizing the established result on the structure of the optimal wage policy, we can state the formal decision problem, when $\gamma_i = (N_k-i)\gamma$, as

$$\left|\begin{array}{l} H_k(i,w_{i-1}) = \max\limits_{x_i \geq 0} \{H_k(i,w_{i-1},x_i)\} \\[2ex] H_k(i,w_{i-1},x_i) = \dfrac{r(i,w_{i-1}+x_i)}{\lambda_i(w_{i-1}+x_i) + \alpha} + \dfrac{\lambda_i(w_{i-1}+x_i)}{\lambda_i(w_{i-1}+x_i) + \alpha} \cdot H_k(i+1,w_i), \\[2ex] \qquad\qquad \lambda_i(\cdot) = \gamma_i \cdot F(\cdot) \; ; \; w_{i-1}+x_i = w_i; \; i=0,\ldots,N_k-1 \\[2ex] H_k(N_k,w_{N_k-1}) = \dfrac{r(N_k,w_{N_k})}{\alpha}, \quad pa_{k+1} \leq w_{N_k} < pa_k, \quad k = \ell \ldots \ell' \end{array}\right.$$

The reader can compare this functional equation to (3.2) in the Technical Supplement and verify that they are almost identical; $\lambda_i(w_i) = \gamma \cdot (N_k-i) \cdot F(w_i)$ is just substituted for $\lambda(w_i) = \gamma \cdot F(w_i)$.

Let us now turn to the deterministic control of both hires and quits.

Typically, the deterministic models of labour flow control assume that the flows are continuous in the state space. We can adhere to that assumption but in order to maintain the discrete nature of realized employment states, we assume that there exists a critical level B such that $b_i \cdot t = B$ implies a transition from employment state i to i+1 and $b_i \cdot t < B$ implies that the firm remains in state i; b_i is the continuous net flow of labour to the firm when it is in state i and t is the time it has spent in that state.

The standard way of specifying b_i is

$$b_i(v) = h(v) - i \cdot q(v)$$

where h is the hiring rate, $h(v) \geq 0$; $h'(v) \geq 0$, and q is the quit rate, $q(v) \geq 0$: $q'(v) \leq 0$.

Let the firm be in state (n,w_{n-1}) and suppose that the firm only considers such wage policies that are consistent with net expansion, so that $b_n(w_n) > 0$, $w_n = w_{n-1} + x_n$. The firm will (with certainty) remain in employment state n for a period of time $[0,T_n]$, $T_n(w_n) = B/b_n$ and move to state n+1 at the end of that period.

If the firm earns as before a profit rate $r(n,w_n)$ when it has n employees and pays the wage w_n, the total discounted profits over an infinite horizon $P(n,w_{n-1},x_n)$ must obey the functional equation of dynamic programming

$$\left| \begin{array}{l} P(n,w_{n-1}) = \max\limits_{x_n \geq 0} \{P(n,w_{n-1},x_n)\} \\ \\ P(n,w_{n-1},x_n) = r(n,w_n) \cdot \int_0^{T_n(w_n)} e^{-\alpha t} dt + e^{-\alpha T_n(w_n)} \cdot P(n+1,w_n) \\ \\ w_n = w_{n-1} + x_n \end{array} \right.$$

The terminal conditions remain to be specified. There is little difference in this respect, however, between the stochastic control problem as presented above and the deterministic one, provided that the production function is the same in both cases. The expansion should stop at any of the $\ell-\ell'+1$ endpoints N_k of the productivity intervals, w_{N_k} such that $pa_{k+1} \leq w_{N_k} < pa_k$.

The only qualification to add is that it is conceivable that expansion cannot occur up to all N_k's, because of the requirement that $b_{N_k-1}(w_{N_k}) > 0$, i.e., $h(w_{N_k}) - (N_k-1) \cdot q(w_{N_k}) > 0$. The left hand boundary of this feasibility region defines N_k as an increasing function of w_{N_k}, while the condition for optimal expansion defines N_k as a (stepwise) decreasing function of w_{N_k} ; it serves as a right hand boundary of the region of possible optimal states. Consequently, the effect is to eliminate all N_k, $k=\ell''\ldots\ell'$, ℓ'' being the lowest k such that $b_{N_k-1}(w_{N_k}) \leqq 0$, $w_{N_k} < pa_k$, as feasible terminal employment states and replace them with terminal states (S,w_S) defined by $b_{S-1}(w_S) > 0$; $b_S(w_S) \leqq 0$ for all $w_S < pa_{\ell''}$.

So in the terminal state we have

$$P(N_k,w_{N_k}) = \frac{r(N_k,w_{N_k})}{\alpha}; \; k = \ell\ldots\ell''-1; \; pa_{k+1} \leqq w_{N_k} < pa_k$$

$$P(S,w_S) = \frac{r(S,w_S)}{\alpha}; \; \frac{h(w_S)}{q(w_S)} \leqq S < \frac{h(w_S)}{q(w_S)} + 1; \; w_{n-1} \leqq w_S < pa_{\ell''}$$

Consider again the expression for $P(n,w_{n-1},x_n)$; evaluating the integral, we have

$$P(n,w_{n-1},x_n) = \frac{r(n,w_n)}{\alpha} \; [1 - e^{-\alpha T_n(w_n)}] + e^{-\alpha T_n(w_n)} P(n+1,w_n)$$

A first order approximation of $e^{-\alpha T_n}$ yields

$$P(n,w_{n-1},x_n) \approx \frac{r(n,w_n)T_n}{1 + \alpha T_n} + \frac{1}{1 + \alpha T_n} P(n+1,w_n) =$$

$$= \frac{r(n,w_n)}{T_n^{-1} + \alpha} + \frac{T_n^{-1}}{T_n^{-1} + \alpha} P(n+1,w_n) =$$

$$= \frac{r(n,w_n)}{B^{-1}[h(w_n) - nq(w_n)] + \alpha} + \frac{B^{-1}[h(w_n) - nq(w_n)]}{B^{-1}[h(w_n) - nq(w_n)] + \alpha} P(n+1,w_n)$$

Compare this expression with the functional equation for the stochastic decision process above, which, substituting n for i and writing down $\lambda_n(w_n)$ explicitly, reads

$$H_k(n, w_{n-1}, x_n) = \frac{r(n, w_n)}{\gamma[N_k F(w_n) - nF(w_n)] + \alpha} +$$

$$+ \frac{\gamma[N_k F(w_n) - nF(w_n)]}{\gamma[N_k F(w_n) - nF(w_n)] + \alpha} H_k(n+1, w_n)$$

With the exception of a few minor differences the structure of the two functional equations is the same. The terminal conditions are partly identical; the difference which appears for low wages introduces nothing new in principle.

We can consequently establish the strong formal resemblance between a stochastic model of pure recruitment and a deterministic model of recruitment-cum-quits, given net expansion. The critical line as to tractability is not to be found between these two approaches but is encountered when hires and quits are both treated as realizations of independent, simultaneous stochastic processes.

The deterministic process contains an implicit "$\lambda_i(v)$" $= [T_i(v)]^{-1} = B^{-1}[h(v) - iq(v)]$, which is decreasing in i and increasing in v, so we should consequently not be surprised to find that downward wage rigidity implies the same structure of the optimal wage policy in that process as in our stochastic processes; in Virén (1979), Ch. 4, a model is developed which has essentially the same properties as the deterministic model presented here and it is shown that downward wage rigidity implies an optimal policy such that the firm makes one immediate wage increase and establishes a constant wage level to hold along the employment expansion path.

TECHNICAL SUPPLEMENT TO CHAPTER II

STRUCTURES OF MARKOV DECISION PROCESSES OF AN EMPLOYMENT-EXPANDING FIRM

1 Introduction

In this Supplement we analyze the structure of a firm's optimal wage and hiring policy under the assumption that the firm faces a flow of job applicants, that forms a Markov process. The process of contacts with job applicants can be transformed by the firm into a process of hires, the intensity of which is increasing in the wage offered by the firm. The recruitment of new employees increases the reward (profit) rate earned by the firm, while the wage reduces it. The optimization problem of the firm is a well defined Markov decision process and the concepts and tools of such processes can be applied to establish the structure of the optimal policy.

Those acquainted with the area of optimal control of queues will easily recognize that the present analysis is an application of methods used in that theory. The queue control theory has traditionally been occupied with specific problems that appear quite naturally in a queuing context. For our subject of study some of these problems are of little interest, while we want to focus on some features that have not received much attention in the queue control literature. For example, optimal queue control through pricing is usually not considered: Low (1974) was one of the first to address this issue. A series of articles by Deshmukh and Chitke

(1976), Deshmukh and Winston (1979) and Lippman (1980) treats the problem of an industry regulating the flow of new entrants by its pricing policy. We have drawn upon their method in analyzing the case with fully flexible wages. As to the investigation of the case, when wages are downward rigid, we have applied a more original approach [Schager (1986a)].

As shown below strong results are obtainable as to the structure of the optimal policy in a pure recruitment model, where no quits (departures) of employees are allowed. That is because the process of hires is stopped at a well-defined absorbing employment state, from which induction can easily be applied. Unfortunately, the state of the art is not so developed as to the structure of optimal policies of birth-cum-death-processes, the application of which is required in order to deal with quits also [cf. Deshmukh and Winston (1977)]. I do not doubt that a concentrated effort should overcome the obstacles connected with such an extended model but this ambitious aim will not be pursued here. Some conjectures as to the structure of the optimal policy in a recruitment-cum-quit model will nevertheless be presented in Section 5.

Otherwise, the basic assumptions of the model are given in Section 2. Section 3 investigates the decision process, when wages are downward rigid, and Section 4 the decision process, when they are flexible. The main results of the analysis are found in Theorem 3.2 and in Theorem 4.2, respectively.

An appendix presents the device of virtual decision epochs, applied to the decision process of Section 4.

A list of notation concludes the Supplement as well as the chapter as a whole.

2 Basic assumptions

In this section the assumptions of the model are succinctly stated in a formal fashion. A more roundabout presentation of the assumptions with references to economic theory is given in Section 2 of the main text.

The model describes a profit maximizing firm, which expands employment through recruitment from a stationary stochastic arrival process of job applicants in continuous time.

Production technology and profit (reward) rate

Number of employees $i \in I = \{0,1,2...\}$

For every $i>0$, $i \in [N_{\ell-1}+1,\ N_\ell]$, $\ell=1, \ldots L+1$, such that if $i \in [N_{\ell-1}+1,\ N_\ell]$, the i:th employee produces a_ℓ units of the product per unit of time, $a_1 \geq a_2 \geq a_L \geq a_{L+1} = 0$, L bounded. $[N_{\ell-1}+1,\ N_\ell]$ is called the ℓ:th productivity interval.

Production per unit of time

$$q(i) = \sum_{j=1}^{\ell-1} \Delta a_j \cdot N_j + a_\ell \cdot i, \qquad \Delta a_j = a_j - a_{j+1} \geq 0$$

Wage level $v \in R_+$

Profit rate at i and v per unit of time $r(i,v) = p \cdot q(i) - i \cdot v$, p exogeneously given.

Objective function

The firm maximizes expected total discounted profits over an infinite horizon

$$E\left[\int_0^\infty r[i(t),v(t)]e^{-\alpha t}dt\right],$$

$i(0)$, $v(0)$ and $\alpha>0$ exogeneously given.

The recruitment process

Job applicants arrive at the firm according to a stationary Poisson process with intensity γ, $\gamma>0$. $F(\cdot)$ is the distribution function of the reservation wages of job applicants with the range $[\underline{v}, \bar{v}]$, $\underline{v}\geq 0$; $\bar{v}\leq\infty$. $F(\cdot)$ has a differentiable density function $f(\cdot)$.

The firm can choose to hire job applicants who arrive (stopping control $\delta=1$). The hiring or recruitment process is then Poisson with intensity $\gamma\cdot F(v)$ at the wage offer v. The firm can also choose to reject job applicants ($\delta=0$), by which the recruitment process is stopped.

As the arrival process of job applicants as well as the time preferences of the firm are stationary, there is no loss in generality in assuming that decisions are taken by the firm only at changes in the employment state. Policy trajectories of type $v(t)$, $\delta(t)$ are replaced by sequences of decisions $\{w_i\}$, $\{\delta_i\}$. We can analyze the decision problem of the firm as a Markov decision process. The method of dynamic programming applied to such a process gives rise to functional equations, the structure of which we will study in the following sections in order to establish the properties of the optimal policy of the firm.

3 The structure of the optimal wage and vacancy policy: downward wage rigidity

In this section we analyze the structure of the optimal wage and recruitment policy of the firm under the assumption that the firm cannot lower its nominal wage level over time. Only non-decreasing wage policies are feasible.

The existence of downward wage rigidity affects the formal structure of the decision problem. We must now consider a decision process in two state variables, the number of employees and the wage level, and two control variables, the stopping or vacancy control and the non-negative wage change, which we denote x_i; x_i is the wage increase to be chosen at the point of time state $(i,\ w_{i-1})$ is entered.

The functional equation of our decision problem reads:

$$(3.1)\quad \left|\begin{array}{l} H(i,w_{i-1}) = \max\limits_{\substack{x_i\geq 0\\ \delta_i=0,1}} \{H(i,w_{i-1},x_i,\delta_i)\} \\[2ex] H(i,w_{i-1},x_i,\delta_i)=\dfrac{r(i,w_{i-1}+x_i)}{\delta_i\cdot\lambda_i(w_{i-1}+x_i)+\alpha}+\dfrac{\delta_i\cdot\lambda(w_{i-1}+x_i)}{\lambda(w_{i-1}+x_i)+\alpha}\cdot H(i+1,w_i\ ), \\[2ex] \qquad\qquad\qquad\qquad w_i\ = w_{i-1}+x_i;\quad i = 0,1\ldots \\[2ex] \delta_i\ = \{0,1\};\ x_i \geq 0 \end{array}\right.$$

The existence of a solution to (3.1) is not immediately guaranteed. In the case of flexible wages to be considered in Section 4, the existence of a solution is easily verified, as the state space is countable and the wage control influences continuously the profit rate and the transition intensities in such a way that a maximum must obtain at a bounded wage level. In the present case the wage state variable is not countable and its value is directly changed by the decision variable x (some authors prefer to call such a decision variable not a control but an intervention).

Nevertheless, a solution to (3.1) exists. Suppose that H(i+1,v) is realvalued, continuous and non-increasing in v. Then $H(i,w_{i-1}, x_i, \delta_i)$ is continuous in x_i and strictly decreasing in x_i for $x_i \geq \bar{v}-w_{i-1}$ and $\delta_i \in \{0,1\}$. Consequently, a realvalued function H (i,v) exists and it is continuous in v according to the Maximum Theorem. Furthermore, an increase in v must have a non-increasing impact on H(i,v). According to Theorem 3.1 below, there exists a stopping state (S,v) such that H(S,v) has the properties that were supposed to hold for H(i+1,v). Induction with respect to i establishes the existence of a solution to (3.1) for every $i \in \{0,1...S\}$.

We now demonstrate the existence and character of an optimal stopping state (S, w_S).

<u>Theorem 3.1</u>. There exists an optimal stopping state (S, w_S) as a part of the solution to the decision problem (3.1) such that S belongs to the set of righthand limits $[N_\ell ... N_{\ell'}]$ of the and productivity intervals, where ℓ is given by $i \in [N_{\ell-1}+1, N_\ell]$ and ℓ' by $w_{i-1} \in [pa_{\ell'+1}, pa_{\ell'}]$; $\ell, \ell' = 1...L$. Furthermore, $S=N_k$, $k=\ell ... \ell'$, implies $w_S=w_{N_k} \in [pa_{k+1}, pa_k)$.

Proof. An optimal stopping state must always exist, because expansion into the $(\ell'+1)$th productivity interval can only yield lower profit rates in the future with $w_{i-1} \geq pa_{\ell'+1}$ acting as a lower bound on all feasible wage levels. Regardless of the non-negative value of w_{i-1} such a situation must at least occur when N_L is reached as $pa_{L+1} \equiv 0$.

To show that any optimal stopping employment state must be N_k, $k = \ell \ldots \ell'$, let us suppose that the firm has reached states $(N_{k-1}, w_{N_{k-1}-1})$ and is about to expand into the kth productivity interval. For such an expansion to be optimal it must hold that $w_{N_{k-1}-1} < pa_k$. It is clear that a policy which keeps that wage level constant and allows expansion up to the employment state N_k yields higher expected discounted profits that any feasible policy which stops the process before N_k is reached. This follows from the inequality

$$\frac{\alpha}{\lambda(w_{N_{k-1}-1})+\alpha} \cdot \frac{r(i,w_{N_{k-1}-1})}{\alpha} + \frac{\lambda(w_{N_{k-1}-1}) \cdot r(i+1,w_{N_{k-1}-1})}{\alpha(\lambda(w_{N_{k-1}-1})+\alpha)} > \frac{r(i,w_i)}{\alpha}$$

where the L.H.S. is the value of applying the constant-wage policy, expand to the next employment state and then stop and the R.H.S. is the value of stopping immediately at any feasible wage level w_i. The inequality holds for any $i \in [N_{k-1}, N_k-1]$ by virtue of $r(i,v)$ being strictly decreasing in v, strictly increasing in i on the kth productivity interval (with $pa_k > w_{N_{k-1}-1}$) and the feasibility condition $w_i \geq w_{N_{k-1}-1}$. Consequently, any stopping of the process before N_k is reached is not an optimal policy.

By the same argument it is even more obvious that any increase of the wage level up to pa_k during the expansion in the k:th interval is inoptimal (implying an implicit stop and useless decrease of immediate profits). Hence the firm will reach the state (N_k, w_{N_k-1}) with $w_{N_k-1} < pa_k$ and stop if and only if $w_{N_k-1} \geq pa_{k+1}$ as we noted above. When the process is stopped, no wage increase is warranted, so $w_{N_k} = w_{N_k-1}$. Hence the optimal stopping state (S, w_S) must be one of the states (N_k, w_{N_k}), $pa_{k+1} \leq w_{N_k} < pa_k$, $k = \ell \ldots \ell'$.

$\square$

We proceed by analysing the optimal wage increase policy conditional on the assumption that the recruitment process stops at (N_k, w_{N_k}). This conditional decision problem can be formulated

$$
\text{(3.2)} \quad \left|
\begin{array}{l}
H_k(i, w_{i-1}) = \max\limits_{x_i \geq 0} \{H_k(i, w_{i-1}, x_i)\} \\[2ex]
H_k(i, w_{i-1}, x_i) = \dfrac{r(i, w_{i-1}+x_i)}{\lambda(w_{i-1}+x_i)+\alpha} + \dfrac{\lambda(w_{i-1}+x_i)}{\lambda(w_{i-1}+x_i)+\alpha} \cdot H_k(i+1, w_i) \\[2ex]
\qquad\qquad w_i = w_{i-1}+x_i; \quad i=0\ldots N_k-1 \\[2ex]
H_k(N_k, w_{N_k-1}) = \dfrac{r(N_k, w_{N_k})}{\alpha}, \qquad pa_{k+1} \leq w_{N_k} = w_{N_k-1} < pa_k
\end{array}
\right.
$$

In order to find the structure of this decision process, we will first consider the case where the firm applies a very special wage (increase) policy: to make an increase at the initial state and to keep the established wage level constant during expansion up to employment state N_k. Of course, the class of such policies does not necessarily contain the solution to (3.2). We do require, however, that the constant wage level policy is compatible with optimal vacancy creation in so far as the chosen wage level, denoted by w, shall lie in the interval $[w_{i-1}, pa_k)$.

The problem of selecting the best future constant wage level in the initial state (n, w_{n-1}) corresponds to the following functional equation system:

$$
(3.3) \quad \left|
\begin{array}{l}
V_k(n, w_{n-1}) = \max\limits_{w \geq w_{n-1}} \{V_k(n,w)\} \\
V_k(i,w) = \dfrac{r(i,w)}{\lambda(w)+\alpha} + \dfrac{\lambda(w)}{\lambda(w)+\alpha} \cdot V_k(i+1,w), \\
\qquad\qquad w_{n-1} \leq w < pa_k, \quad i=n\ldots N_k-1 \\
V_k(N_k,w) = \dfrac{r(N_k,w)}{\alpha}
\end{array}
\right.
$$

Note that the maximization in (3.3) is only carried out in state (n, w_{n-1}); the following states (i,w), $i=n+1\ldots N_k$ are not allowed to have any feedback on the wage level.

We now intend to show that the systems (3.2) and (3.3) are equivalent for $pa_{k+1} \leq w < pa_k$ and that the easily found closed form solution to (3.3) is the solution to our decision problem (3.2). To do this we need the following

Lemma 3.1

Let the firm be in state (n, w_{n-1}) and decide on a constant wage level $w_{n-1} \leq w < pa_k$ to hold for all possible future employment states i as given by (3.3).

Then $\alpha \cdot V_k(n,w) = p \cdot \sum_{j=1}^{\ell-1} \Delta a_j \cdot N_j +$

$$+ n(pa_\ell - w) + p \cdot \sum_{j=\ell}^{k-1} \Delta a_j \cdot \sum_{h=1}^{N_j-n} \left(\frac{\lambda}{\lambda+\alpha}\right)^h + (pa_k - w) \sum_{h=1}^{N_k-n} \left(\frac{\lambda}{\lambda+\alpha}\right)^h$$

$$\lambda = \lambda(w), \quad n \in [N_{\ell-1}+1, N_\ell], \ \ell = 1...k$$

Furthermore, $V_k'(n,w)$, the derivative of $V_k(n,w)$ with respect to w,is strictly decreasing in n.

Proof. We recall that the profit rate $r(n,w) =$ $= p \sum_{j=1}^{\ell-1} \Delta aj \cdot N_j + n(pa_\ell - w)$. The two first terms in the expression for $\alpha \cdot V_k(n,w)$ state that the n employees already in place will contribute this profit rate immediately and for all future time, when the constant wage level w is chosen. The last serial term in $\alpha \cdot V_k(n,w)$ is the basic profit rate (pa_k-w) that will be contributed by all (N_k-n) future employees, evaluated with respect to the expected discounted waiting time until they are hired. The third serial term is the additional profit rate from those (N_j-n) employees which will be hired at earlier stages of the recruitment process and hence will be employed in intervals $j=\ell...k-1$ of higher productivity, evaluated in the same way.

A more formal demonstration applies the standard methods of solving first order linear difference equations to (3.3) as in Schager (1986a). The technique used there is directly applicable if the productivity intervals are treated separately. For a comparison note also that

$$\frac{\lambda}{\alpha}[1 - (\frac{\lambda}{\lambda+\alpha})^{N_j-n}] = \sum_{h=1}^{N_j-n} (\frac{\lambda}{\lambda+\alpha})^h .$$

To show that $V_k'(n,w)$ is decreasing in n we first rewrite $V_k(n,w)$, utilizing the relation $a_\ell = \sum_{j=\ell}^{k-1} \Delta a_j + a_k$, as

$$\alpha \cdot V_k(n,w) = p \cdot \sum_{j=1}^{\ell-1} \Delta a_j \cdot N_j + p \cdot \sum_{j=\ell}^{k-1} \Delta a_j [n + \sum_{h=1}^{N_j-n} (\frac{\lambda}{\lambda+\alpha})^h] +$$

$$+ (pa_k - w)[n + \sum_{h=1}^{N_k-n} (\frac{\lambda}{\lambda+\alpha})^h].$$

From this expression it is immediately clear that

$$\alpha[V_k(n+1,w) - V_k(n,w)] = p \cdot \sum_{j=\ell}^{k-1} \Delta a_j [1 - (\frac{\lambda}{\lambda+\alpha})^{N_j-n}] +$$

$$+ (pa_k - w)[1 - (\frac{\lambda}{\lambda+\alpha})^{N_k-n}] > 0$$

as $\Delta a_j \geqq 0$, $pa_k - w > 0$.

This relation holds even if $n = N_\ell$ so that $n+1$ does not belong to $[N_{\ell-1}+1, N_\ell]$. We just have to replace ℓ with $\ell+1$, which is a matter of formalism.

Consequently, by differentiation with respect to w, which is clearly admissible for $w < pa_k$ as $\lambda'(w) = \gamma \cdot f(w)$, we get

$$\alpha[V'_k(n+1,w) - V'_k(n,w)] = -p \cdot \sum_{j=\ell}^{k-1} \Delta a_j (N_j - n) \cdot (\frac{\lambda}{\lambda+\alpha})^{N_j-n-1} \cdot$$

$$\cdot \frac{\alpha \cdot \lambda'}{(\lambda+\alpha)^2} - (pa_k - w) \cdot (N_k - n) \cdot (\frac{\lambda}{\lambda+\alpha})^{N_k-n-1} \cdot$$

$$\cdot \frac{\alpha \cdot \lambda'}{(\lambda+\alpha)^2} - [1 - (\frac{\lambda}{\lambda+\alpha})^{N_k-n}] < 0$$

as $\Delta a_j \geqq 0$; $pa_k - w > 0$; $\lambda'(w) = \gamma \cdot f(w) > 0$.

$\square$

We are now in the position to state the following

Theorem 3.2

Let the firm be in the initial state (n, w_{n-1}) and face the decision problem (3.2). Then the optimal wage (increase) policy is to make a wage increase $x_n^* \geqq 0$ and to keep the wage level $w_k^* = w_{n-1} + x_n^*$ fixed for all future, regardless of the realization of higher employment states i, $i = n+1 \ldots N_k$, i.e. the optimal wage increases $x_i^* = 0$ for $i = n+1 \ldots N_k$. Furthermore, w_k^* is the solution to $\max_{pa_{k+1} \leqq w < pa_k} \{V_k(n,w)\} =$ $H_k(n, w_{n-1})$ and $H_k(i, w_{i-1}) = V_k(i, w_k^*)$, $i = n+1 \ldots N_k$.

Proof: As was established in the formulation of (3.2) $x^*_{N_k} = 0$. When the firm has entered the state $(N_k - 1, w_{N_k-2})$ it will make a wage increase x_{N_k-1} and establish the wage level $pa_{k-1} \leqq$ $\leqq w_{N_k-1} = w_{N_k-2} + x_{N_k-1} < pa_k$ to hold for all future. Consequently, recalling the definition of V_k (i,w), we have

$$H_k(N_k-1,\ w_{N_k-2},\ x_{N_k-1}) = V_k(N_k-1,\ w_{N_k-2} + x_{N_k-1})$$

and x_{N_k-1} is to be chosen such that

$$H_k(N_k-1,\ w_{N_k-2}) = \max_{x \geqq \max\{0,\ pa_{k+1} - w_{N_k-2}\}} \{V_k(N_k-1,\ w_{N_k-2} + x)\} =$$

$$= V_k(N_k-1,\ w_{N_k-2} + x^*_{N_k-1})$$

Thus the theorem is true for $n = N_k - 1$.

Let us assume that the firm has entered state (i,w_{i-1}), $n \leq i \leq N_k-2$ and that it has been established that the optimal policy when state $(i+1,w_i)$ is reached is to make one wage increase x^*_{i+1} and to keep the wage level $w_i+x^*_{i+1} = w_{i+1}$ constant for all future. This corresponds to the following formulation of (3.2)

$$\left|\begin{array}{l} H_k(i,w_{i-1}) = \max\limits_{x_i\geq 0}\{H_k(i,w_{i-1},x_i)\} \\[2ex] H_k(i,w_{i-1},x_i) = \dfrac{r(i,w_{i-1}+x_i)}{\lambda(w_{i-1}+x_i)+\alpha} + \dfrac{\lambda(w_{i-1}+x_i)}{\lambda(w_{i-1}+x_i)+\alpha}\cdot V_k(i+1,w_i+x^*_{i+1}) \\[2ex] \quad w_{i-1}+x_i = w_i\;; \qquad pa_{k+1} \leq w_i + x^*_{i+1} < pa_k \end{array}\right.$$

As it holds that

$$V_k(i+1,w_i+x^*_{i+1}) = \int_0^{x^*_{i+1}} V'_k(i+1,w_i+x)dx + V_k(i+1,w_i)$$

and from (3.3) that

$$V_k(i,w_{i-1}+x_i) = \frac{r(i,w_{i-1}+x_i)}{\lambda(w_{i-1}+x_i)+\alpha} + \frac{\lambda(w_{i-1}+x_i)}{\lambda(w_{i-1}+x_i)+\alpha}\cdot V_k(i+1,w_{i-1}+x_i)$$

we obtain

$$H_k(i,w_{i-1},x_i) = V_k(i,w_{i-1}+x_i) + \frac{\lambda(w_{i-1}+x_i)}{\lambda(w_{i-1}+x_i)+\alpha}\cdot\int_0^{x^*_{i+1}} V_k'(i+1,w_i+x)dx$$

An optimal choice of $x_i=x^*_i$ implies that $H_k(i,w_{i-1},x_i)$ is maximized.

Let $\hat{x}_i \geq 0$ be any choice of x_i that establishes a $\hat{w}_i=w_{i-1}+\hat{x}_i$ such that the corresponding $x^*_{i+1}=\hat{x}^*_{i+1}>0$. The value of applying such a policy is

$$H_k(i,w_{i-1},\hat{x}_i) = V_k(i,\hat{w}_i) + \frac{\lambda(\hat{w}_i)}{\lambda(\hat{w}_i)+\alpha}\cdot\int_0^{\hat{x}^*_{i+1}} V_k'(i+1,\hat{w}_i+x)dx$$

Consider now another, strictly different policy $x_i = \hat{x}_i + \hat{x}^*_{i+1}$; $x_{i+1} = 0$. It must yield the value

$$V_k(i,w_{i-1}+\hat{x}_i+\hat{x}^*_{i+1}) = V_k(i,\hat{w}_i) + \int_0^{\hat{x}^*_{i+1}} V_k'(i,\hat{w}_i+x)dx$$

The difference in value between the first and the second policy is consequently

$$H_k(i,w_{i-1},\hat{x}_i) - V_k(i,w_{i-1}+\hat{x}_i+\hat{x}^*_{i+1}) =$$

$$= \frac{\lambda(\hat{w}_i)}{\lambda(\hat{w}_i)+\alpha} \int_0^{\hat{x}^*_{i+1}} V_k'(i+1,\hat{w}_i+x)dx - \int_0^{\hat{x}^*_{i+1}} V_k'(i,\hat{w}_i+x)dx < 0$$

because

$\int_0^{\hat{x}^*_{i+1}} V_k'(i+1,\hat{w}_i+x)dx \geq 0$ by the assumption that $\hat{x}^*_{i+1}$ is a strictly positive, optimal wage increase in state $(i+1,\hat{w}_i)$,

$$0 < \frac{\lambda(\hat{w}_i)}{\lambda(\hat{w}_i)+\alpha} < 1$$

and

$$\int_0^{\hat{x}^*_{i+1}} V_k'(i+1,\hat{w}_i+x)dx < \int_0^{\hat{x}^*_{i+1}} V_k'(i,\hat{w}_i+x)dx$$

as $V_k'(i,w)$ is strictly decreasing in i for all w, $w_{n-1} \leq w < pa_k$ according to Lemma 3.1.

Thus we conclude that $\hat{x}_i$ cannot be an optimal wage increase policy. But $\hat{x}_i$ is any policy in state (i, w_{i-1}) which establishes a wage level $\hat{w}_i$ such that in state $(i+1,\hat{w}_i)$ the optimal increase is $\hat{x}^*_{i+1} > 0$.

Consequently, an optimal increase x_i^* in state (i,w_{i-1}) must imply $x_{i+1}{}^* = 0$, provided that it is an optimal policy to keep the established wage level $w_i+x_{i+1}^* = w_{i+1}$ constant for all future.

Furthermore we have already shown that it is optimal to keep the wage level $w_{N_k-1}=w_{N_k-2}+x_{N_k-1}$ constant for all future. We conclude that $x_{N_k-1}^* = 0$ if x_{N_k-2} is optimally chosen as an element of the solution to the decision problem. Consequently it is optimal to keep the wage level w_{N_k-2} constant for all future. Induction then establishes the sequence of optimal wage increases $(x_{N_k}^*=)x_{N_k-1}^*= =\ldots = x_{n+1}^* = 0$, when (n,w_{n-1}) is the initial state, in which the optimal policy is determined. As w_{n-1} is by definition given and not determined as a part of the optimal policy, $x_n^* \geqq 0$.

Consequently, we can restrict the search for the optimal wage policy to the class of constant wage level policies $w=w_{n-1}+x_n$.

For N_k to be the optimal stopping state, it must hold that $pa_{k+1} \leqq w_{n-1} + x_n = w < pa_k$. Consequently,

$$H_k\,(n,w_{n-1}) = \max_{pa_{k+1}\leqq w<pa_k} V_k(n,w) = V_k(n,w_k^*) \qquad k = \ell\ldots\ell'-1,$$

$$H_{\ell'}(n,w_{n-1}) = \max_{w_{n-1}\leqq w<pa_{\ell'}} V_{\ell'}(n,w) = V_{\ell'}(n,w_{\ell'}^*)$$

and

$$H_k(i,w_{i-1}) = H_k(i,w_k^*) = V_k(i,w_k^*), \qquad i = n+1\ldots N_k$$

$\square$

We have shown that the solution to the decision problem (3.2) is a simple one, implying the choice of a constant wage level to hold for all future. The policy is conditional on the optimal stopping employment state being N_k , $k = \ell \ldots \ell'$, with the accompanying restriction that $\max \{w_{n-1}, pa_{k+1}\} \leqq$ $\leqq w_{N_k} < pa_k$.

The condition $\max\{w_{n-1}, pa_{k+1}\} \leqq w_k^* < pa_k$ is necessary and clearly also sufficient for $V_k(n,w_k^*)$ to be a part of solution of the unconditional decision problem (3.1), where optimal stopping is fully applied. We can directly state the following

Theorem 3.3

Let the firm be in the initial state (n,w_{n-1}) and face the decision problem (3.1). Then the optimal wage policy is to choose a wage level $w^* \geqq w_{n-1}$ and keep it constant for all future, w^* being one of the $(\ell-\ell'+1)$ solutions to

$$H_k(n,w_{n-1}) = \max_{pa_{k+1} \leqq w < pa_k} \{V_k(n,w)\}, \qquad k = \ell \ldots \ell'-1$$

$$H_{\ell'}(n,w_{n-1}) = \max_{w_{n-1} \leqq w < pa_{\ell'}} \{V_{\ell'}(n,w)\}$$

such that $\max\limits_{k=\ell \ldots \ell'} \{H_k(n,w_{n-1})\}$ is obtained.

Proof: The theorem follows directly from Theorems 3.1 and 3.2.

$\square$

We end this section by pointing to some interesting features of the optimal solution to the vacancy and wage decision problem of an employment-expanding firm under downward wage rigidity.

The wage and vacancy decisions are closely interlinked in the case of downward wage rigidity. The optimal wage level depends on the set of possible optimal vacancy levels N_k-n at the same time as it determines which one of them is optimal. A change in the initial employment state n might easily change the optimal vacancy level not only directly but also by changing the optimal stopping employment state N_k^* through its influence on the optimal wage level.

It is a nice feature of the solution that the vacancy decision is reflected explicitly in the initial wage decision by virtue of the future constancy of the wage level. If the optimal wage level belongs to $[pa_{k+1}, pa_k)$ the optimal vacancy level is N_k-n (and vice versa).

An important consequence of the assumption of downward wage rigidity is that a successfully recruiting firm will find itself increasingly in 'disequilibrium' with respect to its wage level, as higher employment states are realized. This follows from the crucial property that $V_k'(n,w)$ is strictly decreasing in n. Let the optimal wage level $w^*(n)$ constitute an interior solution to (3.1) so that $w^*(n) > w_{n-1}$; let us further denote w_{n-1} by w_o to indicate that it is not determined as a part of the optimal solution. Then it must hold that $w^*(n+1) <$ $< w^*(n)$, if w_o were the same at the two employment states. However, in the optimal two-state decision process (3.1) the lower bound in employment state

$i=n+1$ is not w_o but $w^*(n)$ and the best the firm can do is to choose that wage level. $V_k'(i,w)$ will be negative at $w^*(n)$ for $i = n+1...N_k^*$ and increasingly so as i increases.

We can formulate the effects of downward wage rigidity by saying that the firm, in order to be able at an initial stage to recruit as rapidly as it wants to, must make itself able recruit more rapidly than it wants to at a later stage. Due to the vacancy control it must never recruit more employees than it wants to at the chosen wage level, however. On the contrary, one probable effect of the higher wage level is clearly that less vacancies will be announced and fewer job applicants hired as the higher wage will make expansion into employment intervals with lower productivity unprofitable. Again the firm will enter the state i, i>n, but the historically given wage level $w^*(n) \in [pa_{k+1}, pa_k)$ forbids it to choose $w^*(i) \in [pa_{k'+1}, pa_{k'})$ $k'>k$ and to announce $N_{k'}-i$ vacancies instead of the lower number N_k-i.

We demonstrated in Theorem 3.1 that the firm will never choose $w=pa_k$ after that it has entered productivity interval k. We will now show that $w=pa_k$ will not be chosen in productivity interval k-1 either, so that the optimal wage level is never equal to the static marginal product of labour (unless w_{n-1} is chosen and by chance happens to be equal to $pa_{\ell'+1}$). At the same time we present some analytical properties of the value function which are implicit in Theorem 3.3.

As is clear from Theorem 3.3, there exists a single-valued function of w, defined for all $w \in [w_{n-1}, pa_\ell)$

$$V(n,w) = V_k(n,w), \qquad \begin{array}{ll} pa_{k+1} \leq w < pa_k, & k=\ell \ldots \ell'-1 \\ w_{n-1} \leq w < pa_{\ell'}, & k=\ell' \end{array}$$

as for every value of $w \in [w_{n-1}, pa_\ell)$ there exists one and only one single-valued function V_k. Theorem 3.3 is equivalent to

$$H(n,w_{n-1}) = \max_{w \geq w_{n-1}} \{V(n,w)\} = V(n,w^*)$$

It is easily verified that $V(n,pa_k) = V_{k-1}(n,pa_k) = \lim_{w \to pa_k^-} V(n,w) = \lim_{w \to pa_k^-} V_k(n,w)$, so $V(n,w)$ is continuous everywhere on $[w_{n-1}, pa_\ell)$.

$V'(n,w)$ evaluated at $w=pa_k$ becomes

$$\alpha \cdot V'(n,pa_k) = \alpha \cdot V'_{k-1}(n,pa_k) =$$

$$= p \cdot \sum_{j=\ell}^{k-2} \Delta a_j \cdot \frac{\alpha \cdot \lambda'}{(\lambda+\alpha)^2} \sum_{i=1}^{N_j-n} i \cdot \left(\frac{\lambda}{\lambda+\alpha}\right)^{i-1} + (pa_{k-1}-pa_k) \cdot$$

$$\cdot \frac{\alpha \cdot \lambda'}{(\lambda+\alpha)^2} \sum_{i=1}^{N_{k-1}-n} i \cdot \left(\frac{\lambda}{\lambda+\alpha}\right)^{i-1} - \left[n + \sum_{i=1}^{N_{k-1}-n} \left(\frac{\lambda}{\lambda+\alpha}\right)^{i}\right] =$$

$$= p \cdot \sum_{j=\ell}^{k-1} \Delta a_j \cdot \frac{\alpha \cdot \lambda'}{(\lambda+\alpha)^2} \sum_{i=1}^{N_j-n} i \cdot \left(\frac{\lambda}{\lambda+\alpha}\right)^{i-1} -$$

$$- \left[n + \sum_{i=1}^{N_{k-1}-n} \left(\frac{\lambda}{\lambda+\alpha}\right)^{i}\right], \qquad \lambda = \lambda(pa_k), \quad \lambda' = \lambda'(pa_k).$$

$V'(n,w)$ evaluated in a left hand neighbourhood of pa_k is

$$\alpha \cdot V'(n,w) = \alpha \cdot V'_k(n,w) =$$

$$= p \cdot \sum_{j=\ell}^{k-1} \Delta a_j \cdot \frac{\alpha \cdot \lambda'}{(\lambda+\alpha)^2} \sum_{i=1}^{N_j-n} i \cdot \left(\frac{\lambda}{\lambda+\alpha}\right)^{i-1} + (pa_k - w) \cdot$$

$$\cdot \frac{\alpha \cdot \lambda'}{(\lambda+\alpha)^2} \sum_{i=1}^{N_k-n} i \cdot \left(\frac{\lambda}{\lambda+\alpha}\right)^{i-1} - \left[n + \sum_{i=1}^{N_k-n} \left(\frac{\lambda}{\lambda+\alpha}\right)^{i}\right]$$

$\lambda = \lambda(w)$; $\lambda' = \lambda'(w)$

So

$$\lim_{w \to pa_k^-} \alpha \cdot V'(n,w) = p \cdot \sum_{j=\ell}^{k-1} \Delta a_j \cdot \frac{\alpha \cdot \lambda'}{(\lambda+\alpha)^2} \sum_{i=1}^{N_j-n} i \cdot$$

$$\cdot \left(\frac{\lambda}{\lambda+\alpha}\right)^{i-1} - [n + \sum_{i=1}^{N_k-n} \left(\frac{\lambda}{\lambda+\alpha}\right)^{i}],$$

$\lambda = \lambda(pa_k)$; $\lambda' = \lambda'(pa_k)$

Consequently, $\alpha V'(n,pa_k) - \lim_{w \to pa_k^-} \alpha \cdot V'(n,w) =$

$= \sum_{i=N_{k-1}+1}^{N_k} \left(\frac{\lambda}{\lambda+\alpha}\right)^{i} > 0$, so $V'(n,w)$ is discontinous at pa_k, $k=\ell+1 \ldots \ell'$ and exhibits an upward jump as w increases to pa_k.

The reason for this behaviour of $V'(n,w)$ is simply that by raising the wage level to almost pa_k the firm has eliminated the profits from employment interval k but has still to pay the wages for the employees in that interval. By raising w to exactly pa_k (and by correspondingly stopping expansion into employment interval k) the firm suddenly saves the wage costs of those employees without any further loss in profits.

For $w = pa_k$ to be an interior solution on $[w_{n-1}, pa_\ell)$ as described by Theorem 3.3 it must hold that $V'(n,pa_k+\epsilon) - V'(n,pa_k-\epsilon) < 0$ for a sufficiently small $\epsilon > 0$. But as we showed above, just the opposite is true. Hence $w = pa_k$, $k = \ell \ldots \ell'-1$, cannot be an optimal solution to the decision problem (3.1).

The further properties of $V(n,w)$ and of $H(n,w_{n-1})$ are crucially dependent on the characteristics of the reservation wage distribution

F($\cdot$). This issue is discussed in Section 4 of the main text of this chapter. The discussion there is based partly on results from Schager (1986b), where a slightly simplified version of the present model is further analysed.

4 The structure of the optimal wage policy: flexible wages

In this section we analyze the structure of the optimal wage policy of a recruiting firm under the assumption that wages are fully flexible, i.e. both increasing and decreasing wage paths over time are feasible.

As before we need to take only stationary wage policies into account and as wages are fully flexible, the state is unambiguously defined by the employment state i. When the firm is in state i, it chooses a wage level w_i and makes a transition to state i+1 after a sojourn time that is exponentially distributed with expectation $[\lambda(w_i)]^{-1}$ = = $[\gamma \cdot F(w_i)]^{-1}$. [Note the difference in notions between this section and section 3: here w_i is a control variable, the value of which is chosen when state i is entered; in section 3, w_i is a state variable and x_i is the control variable, the value of which is chosen when state (i,w_{i-1}) is entered, so $w_i=w_{i-1}+x_i$ is the wage state during the sojourn time in employment state i.]

The optimal wage policy is to satisfy the following functional equation of dynamic programming:

$$
(4.1)\quad
\begin{cases}
L(i) = \max\limits_{w_i \geq \underline{w}} \{L(i,w_i)\} \\[2ex]
L(i,w_i) = \dfrac{r(i,w_i)}{\lambda(w_i)+\alpha} + \dfrac{\lambda(w_i)}{\lambda(w_i)+\alpha}\, L(i+1), \quad i=0\ldots S-1 \\[2ex]
L(S) = \max\limits_{w_S \geq \underline{w}} \{\dfrac{r(S,w_S)}{\alpha}\} = \dfrac{r(S,\underline{w})}{\alpha}
\end{cases}
$$

We have introduced $\underline{w}$ as the lower bound on the interval of feasible wage levels, because wages cannot be treated as unbounded downwards, even if the wage level is 'flexible'; for example, the wage cannot attain negative values. As to the properties of the optimal stopping state S, they differ according to whether $\underline{w}>\underline{v}$ or $\underline{w}\leq\underline{v}$ ($\underline{v}$ is, we recall, the lower bound of the range of the reservation wage distribution).

If $\underline{w}>\underline{v}$, the determination of S is simple. If wage costs cannot be further reduced, clearly the firm should aim at expanding up to a terminal state where the profit earned on the latest hired employee is positive and the profit earned on the next employee is non-positive. The concavity of the production function, the flexibility of the wage level and the feasibility of expansion for all $w_i\geq\underline{w}$ ensures that the optimal terminal state $S=N_{\ell'}$, where ℓ' is defined by $pa_{\ell'+1} \leq \underline{w} < pa_{\ell'}$. When $N_{\ell'}$ is reached, the firm should use the vacancy control to stop the otherwise automatic expansion into unprofitable productivity intervals. In this case a vacancy control is needed, but its optimal value is immediately given by the parameters of the decision problem, regardless of the determination of the optimal sequence of wage controls.

If $\underline{w} \leq \underline{v}$, any vacancy control is obsolete, because if the firm wants to stop the recruitment process at employment state S, it is always to its advantage to put $w_S = \underline{w}$, by which $L(S, w_S)$ is maximized at the same time as the process stops automatically. This situation can be called the pure case of wage flexibility in the sense that no quantity control of the process is needed in order to obtain its maximum value.

An optimal stopping state S - or rather an optimal wage $w_S^* = \underline{w}$ - must exist, because the firm is not willing to expand employment beyond $N_{\ell''}$, $pa_{\ell'+1} \leq \underline{w} <$ $< pa_{\ell'}$; further expansion is bound to decrease total profits. S is not necessary equal to $N_{\ell'}$, however; S is the lowest employment state i, in which $w_i^* = \underline{w}$, so S is determined implicitly by the sequence of optimal wages. Although there is no vacancy control, there are certainly vacancies, the number of which is (S-i) in employment state i.

As S is not determined by any independent control, its derivation is completely contained in the system (4.1) and no further information is required. We should note that the decision process (4.1) is defined for all $i \leq N_{\ell'}$, i.e. there exist an optimal wage w_i^* and a maximum value L(i) for all $i \leq N_{\ell'}$, regardless of whether or not a $w_j^* = \underline{w}$, $j < i$, exists. Consequently, there will be no anomalies in (4.1), which disturb a straightforward analysis. One question, that will be answered in the following investigation, is whether or not $w_i^* = \underline{w}$ implies $w_{i+1}^* = \underline{w}$, i.e. whether or not there may exist several non-adjacent stopping states on $[0, N_{\ell'}]$, so that S may depend on the initial employment state n.

We can consequently reformulate the terminal condition as

$$L(N_{\ell'}) = \frac{r(N_{\ell'}, \underline{w})}{\alpha}$$

regardless of whether $\underline{w} > \underline{v}$ or $\underline{w} \leq \underline{v}$. In the latter case $S \leq N_{\ell'}$, S being the lowest $i \geq n$ such that $w_i^* = \underline{w}$.

In establishing the properties of the solution to (4.1) we follow the approach of Lippman (1980), who in turn develops a model of competition control presented in Deshmukh and Chitke (1976) and in Deshmukh and Winston (1979). The latter authors applied the device - originally proposed by Lippman in an earlier methodological contribution Lippman (1975) - of analyzing the functional equation for the finite horizon case, establishing its properties by induction and showing that they hold when the horizon goes to infinity. As Lippman (1980) reveals, stronger results are obtainable with the direct approach where the value function is explicitly specified.

Before stating our principal results, we introduce the following simplifying notation: $N_{\ell'} = N$; $\lambda(w_i^*) = \lambda_i^*$.

<u>Theorem 4.1</u>: $L(n) = \sum_{i=n}^{N-1} \prod_{j=n}^{i-1} \frac{\lambda_j^*}{\lambda_j^* + \alpha} \cdot \frac{r(i, w_i^*)}{\lambda_i^* + \alpha} + \prod_{j=n}^{N-1} \frac{\lambda_j^*}{\lambda_j^* + \alpha} \cdot \frac{r(N, \underline{w})}{\alpha}$

is positive and increasing in n.

<u>Proof</u>: L(n) is the expected total discounted profits when the optimal sequence of wages $\{w_i^*\}_n^N$ is applied. Given the exponentially distributed sojourn time in state i, in which the profit rate $r(i, w_i^*)$ is earned and which is followed by a

transition to state i+1, and given that the process stops at no higher state than N, the above formula follows from immediate calculation. Note that if $w_S^* = \underline{w} \leq \underline{v}$ for some $S<N$, $\lambda_S^* = 0$ and all terms with indices $i>S$ become zero and the last non-zero term becomes

$$\prod_{j=n}^{S-1} \frac{\lambda_j^*}{\lambda_j^*+\alpha} \cdot \frac{r(S,\underline{w})}{\alpha} .$$

L(n) must be positive, because - disregarding the trivial case where $\underline{w} > pa_\ell$ - the firm can always choose a sequence of wages that ensures a positive profit rate in every state.

L(n) is increasing in n, because

$$L(n) = \sum_{i=n}^{N-1} \prod_{j=n}^{i-1} \frac{\lambda_j^*}{\lambda_j^*+\alpha} \cdot \frac{r(i,w_i^*)}{\lambda_i^*+\alpha} + \prod_{j=n}^{N-1} \frac{\lambda_j^*}{\lambda_j^*+\alpha} \cdot \frac{r(N,\underline{w})}{\alpha} =$$

$$= \sum_{i=n-1}^{N-2} \prod_{j=n-1}^{i-1} \frac{\lambda_{j+1}^*}{\lambda_{j+1}^*+\alpha} \cdot \frac{r(i+1,w_{i+1}^*)}{\lambda_{i+1}^*+\alpha} + \prod_{j=n-1}^{N-2} \frac{\lambda_{j+1}^*}{\lambda_{j+1}^*+\alpha} \cdot \frac{r(N,\underline{w})}{\alpha} \geq$$

$$\geq \sum_{i=n-1}^{N-2} \prod_{j=n-1}^{i-1} \frac{\lambda_j^*}{\lambda_j^*+\alpha} \cdot \frac{r(i+1,w_i^*)}{\lambda_i^*+\alpha} + \prod_{j=n-1}^{N-2} \frac{\lambda_j^*}{\lambda_j^*+\alpha} \cdot \frac{r(N,\underline{w})}{\alpha} >$$

$$> \sum_{i=n-1}^{N-2} \prod_{j=n-1}^{i-1} \frac{\lambda_j^*}{\lambda_j^*+\alpha} \cdot \frac{r(i,w_i^*)}{\lambda_i^*+\alpha} + \prod_{j=n-1}^{N-2} \frac{\lambda_j^*}{\lambda_j^*+\alpha} \cdot \frac{r(N,\underline{w})}{\alpha} >$$

$$> \sum_{i=n-1}^{N-1} \prod_{j=n-1}^{i-1} \frac{\lambda_j^*}{\lambda_j^*+\alpha} \cdot \frac{r(i,w_i^*)}{\lambda_i^*+\alpha} + \prod_{j=n-1}^{N-1} \frac{\lambda_j^*}{\lambda_j^*+\alpha} \cdot \frac{r(N,\underline{w})}{\alpha} = L(n-1)$$

The equality signifies just a change of index, the weak inequality follows from the application of the wage policy $\{w_i^*\}_{n-1}^{N-2}$ instead of the optimal one

$\{w_i^*\}_n^{N-1}$, the first strict inequality follows from the replacement of $r(i+1,\ w_i^*)$ with $r(i,w_i^*)$, $r(i+1,\ w_i^*) > r(i,w_i^*)$, and the second strict inequality follows from the identity

$$\frac{r(N,\underline{w})}{\alpha} \equiv \frac{r(N,\underline{w})}{\lambda^*_{N-1}+\alpha} + \frac{r(N,\underline{w})}{\alpha} \cdot \frac{\lambda^*_{N-1}}{\lambda^*_{N-1}+\alpha}$$

and the inequality

$$\frac{r(N,\underline{w})}{\lambda^*_{N-1}+\alpha} > \frac{r(N-1,w^*_{N-1})}{\lambda^*_{N-1}+\alpha} .$$

□

In order to establish the structure of the optimal wage policy, we must show that $L(n)$ increases proportionally less than $(n-1)$. This follows from the following

<u>Lemma 4.1</u>: $L(n)/n$ is decreasing in n.

<u>Proof</u>: We first note that

$$r(i,w_i) \cdot \frac{n-1}{n} < r(i-1,w_i) \quad i \geq n,$$

because

$$\frac{n-1}{n} \cdot r(i,w_i) = \frac{n-1}{n} \cdot p \cdot \sum_{j=1}^{\ell-1} \Delta a_j \cdot N_j + \frac{n-1}{n} \cdot i\ (pa_\ell - w_i) <$$

$$< p \cdot \sum_{j=1}^{\ell-1} \Delta a_j \cdot N_j + (i-\tfrac{i}{n})\ (pa_\ell - w_i) < p \sum_{j=1}^{\ell-1} \Delta a_j N_j + (i-1)(pa_\ell - w_i)$$

$$= r(i-1,w_i)$$

Hence

$$\frac{n-1}{n} L(n) = \sum_{i=n}^{N-1} \prod_{j=n}^{i-1} \frac{\lambda_j^*}{\lambda_j^*+\alpha} \cdot \frac{r(i,w_i^*)}{\lambda_i^*+\alpha} \cdot \frac{n-1}{n} + \prod_{j=n}^{N-1} \frac{\lambda_j^*}{\lambda_j^*+\alpha} \cdot \frac{r(N,\underline{w})}{\alpha} \cdot \frac{n-1}{n} <$$

$$< \sum_{i=n}^{N-1} \prod_{j=n}^{i-1} \frac{\lambda_j^*}{\lambda_j^*+\alpha} \cdot \frac{r(i-1,w_i^*)}{\lambda_i^*+\alpha} + \prod_{j=n}^{N-1} \frac{\lambda_j^*}{\lambda_j^*+\alpha} \cdot \frac{r(N-1,\underline{w})}{\alpha}$$

Further it holds that

$$\frac{r(N-1,\underline{w})}{\alpha} \leq L(N-1) = \frac{r(N-1,w_{N-1}^*)}{\lambda_{N-1}^*+\alpha} + \frac{\lambda_{N-1}^*}{\lambda_{N-1}^*+\alpha} \cdot \frac{r(N,\underline{w})}{\alpha}$$

so we have

$$\frac{n-1}{n} L(n) < \sum_{i=n}^{N-1} \prod_{j=n}^{i-1} \frac{\lambda_j^*}{\lambda_j^*+\alpha} \cdot \frac{r(i-1,w_i^*)}{\lambda_i^*+\alpha} + \prod_{j=n}^{N-1} \frac{\lambda_j^*}{\lambda_j^*+\alpha} \cdot \frac{r(N-1,w_{N-1}^*)}{\lambda_{N-1}^*+\alpha} +$$

$$+ \prod_{j=n}^{N-1} \frac{\lambda_j^*}{\lambda_j^*+\alpha} \cdot \frac{\lambda_{N-1}^*}{\lambda_{N-1}^*+\alpha} \cdot \frac{r(N,\underline{w})}{\alpha} \leq L(n-1)$$

because the expression above is the expected value of going from state n-1 to N by applying the inoptimal wage w_i^* in state i-1, i=n...N-1.

Hence $\frac{L(n)}{n} < \frac{L(n-1)}{n-1}$

$\square$

<u>Corollary 4.1</u>: L(n)/(n-1) is decreasing in n.

Proof: $\frac{L(n)}{n} < \frac{L(n-1)}{n-1}$; $n \cdot L(n) - L(n) < n \cdot L(n-1)$;

$n \cdot L(n) - 2L(n) < n \cdot L(n-1) - L(n-1)$ as $L(n) > L(n-1)$;

$\therefore \quad \frac{L(n)}{n-1} < \frac{L(n-1)}{n-2}$

$\square$

We return to the functional equation (4.1)

$$L(n,w_n) = \frac{r(n,w_n)}{\lambda(w_n)+\alpha} + \frac{\lambda(w_n)}{\lambda(w_n)+\alpha} L(n+1)$$

$$L(n) = \max_{w_n \geq \underline{w}} \{L(n,w_n)\}$$

$L(n)$ is obtained either at an interior solution $w_n^* > \underline{w}$, where the conditions $L'(n,w_n^*) = 0$; $\int_{\underline{w}}^{w_n^*} L'(n,w)dw \geq 0$; $L''(n,w_n^*) < 0$ hold, or else at $w_n^* = \underline{w}$.

We have

$$L'(n,w_n) = \frac{\alpha \cdot \lambda'(w_n)}{(\lambda(w_n)+\alpha)^2} \left(L(n+1) - \frac{r(n,w_n)}{\alpha}\right) + \frac{r'(n,w_n)}{\alpha(\lambda(w_n+\alpha)}$$

which is negative outside the range $(\underline{v},\bar{v})$, where $\lambda'(w_n)=0$. Any optimal value w_n^* must lie in the interval $[\underline{w},\bar{v})$. We can now establish the structure of the optimal wage policy by the following

Theorem 4.2: The solution to the decision problem (4.1), when the firm is in the initial employment state n, is to choose a non-increasing sequence of wages $\{w^*_i\}_n^N$ to hold in employment states i. Let $\underline{i}$, $n \leq \underline{i} \leq N$ be the smallest i for which $w_{\underline{i}}^* = \underline{w}$; then $\{w_i^*\}_n^{\underline{i}}$ is a strictly decreasing sequence of wages.

Proof: The theorem is proved if we can show that $L'(n,w_n)/n$ is strictly decreasing in n.

Denoting $p \sum_{j=1}^{\ell-1} \Delta a_j N_j = A_{\ell-1}$ for simplicity, we have

$$L(n,w_n) = \frac{A_{\ell-1} + (pa_\ell - w_n)n}{\lambda(w_n)+\alpha} + \frac{\lambda(w_n)}{\lambda(w_n)+\alpha} L(n+1)$$

or

$$L(n,w_n) - \frac{A_{\ell-1}}{\alpha} = \frac{(pa_\ell - w_n)n}{\lambda(w_n)+\alpha} + \frac{\lambda(w_n)}{\lambda(w_n)+\alpha} \left(L(n+1) - \frac{A_{\ell-1}}{\alpha} \right)$$

Now it is clear that the firm will always earn a total discounted profit from its n initial employees equal to $A_{\ell-1}/\alpha$, irrespective of the action taken; it constitutes a lump-sum contribution to $L(i)$ and $L(i,w_i)$ in every state i, $n \leqq i \leqq N$. Consequently, the optimal policy will be unaffected if we consider the process, the state values of which are $\bar{L}(i) = L(i) - A_{\ell-1}/\alpha$ and $\bar{L}(i,w_i) = L(i,w_i) - A_{\ell-1}/\alpha$, respectively. $\bar{L}(i)$ will inherit the properties of $L(i)$ according to Theorem 4.1, Lemma 4.1 and its Corollary, including the property of being greater than zero, which does not necessarily follow from a lump-sum reduction in general. (Note that the transformed process has a profit rate $\bar{r}(i,w_i) = r(i,w_i) - A_{\ell-1}$, $n \leqq i \leqq N$, with the same structure as $r(i,w_i)$.)

Consequently we can consider the functional equation

$$\bar{L}(n,w_n) = \frac{(pa_\ell - w_n)n}{\lambda(w_n)+\alpha} + \frac{\lambda(w_n)}{\lambda(w_n)+\alpha} \bar{L}(n+1)$$

and differentiating with respect to w_n, we get,

$$\bar{L}'(n,w_n) = - \frac{n}{\lambda(w_n)+\alpha} - \frac{(pa_\ell - w_n)n\cdot\lambda'(w_n)}{[\lambda(w_n)+\alpha]^2} + \frac{\alpha\cdot\lambda'(w_n)}{[\lambda(w_n)+\alpha]^2}\cdot \bar{L}(n+1) =$$

$$= n\,[- \frac{1}{\lambda(w_n)+\alpha} - \frac{(pa_\ell - w_n)\cdot\lambda'(w_n)}{[\lambda(w_n)+\alpha]^2} + \frac{\alpha\cdot\lambda'(w_n)}{[\lambda(w_n)+\alpha]^2}\cdot \frac{\bar{L}(n+1)}{n}]$$

The only term within brackets that depends on n is positive and according to the Corollary to Lemma 4.1 it is strictly decreasing in n. Hence the expression in brackets is strictly decreasing in n for any value of w_n and we can denote $\bar{L}'(n,w_n) = n\cdot K(n,w_n)$, where $K(n,w_n)$ strictly decreasing in n.

Let $w_n^* > \underline{w}$ be the optimal wage in employment state n so that (i) $K(n,w_n^*) = 0$; (ii) $K'(n,w_n^*) < 0$;

(iii) $n\cdot\int_{\underline{w}}^{w_n^*} K(n,v)\,dv \geq 0$;

(iv) $n\int_{w_{n,max}}^{w_n^*} K(n,v)\,dv \geq 0$; $w_{n,max} < w_n^*$;

(v) $n\cdot\int_{w_n^*}^{w_{n,max}} K(n,v)\,dv \leq 0$; $w_{n,max} > w_n^*$,

where $w_{n,max}$ is any feasible $w_n \neq w_n^*$ which satisfies (i)-(iii). Because of the properties of $K(n,w_n)$, including the continuity of $K(n,w_n)$ and $K'(n,w_n)$ in w_n, it is clear that $K(n-1,v)$, $K'(n-1,v)$ and $(n-1)\int K(n-1,v)dv$ must attain such values that conditions (i)-(v) are satisfied for a $v > w_n^*$. Consequently, in employment state n-1 there exists an optimal wage $w_{n-1}^* > w_n^* > \underline{w}$.

Let us now assume that $w_{n-1}^* = \underline{w}$ so that

$(n-1)\int_{\underline{w}}^{w_{n-1}} K(n-1,v)dv \leq 0$ for any $w_{n-1} > \underline{w}$. This implies that $n\int_{\underline{w}}^{w_n} K(n,v)dv < 0$ for any $w_n > \underline{w}$ so in state n the optimal wage $w_n^* = \underline{w}$.

We conclude that the sequence of optimal wages $\{W_i^*\}_n^N$ will be nonincreasing in such a way that the sequence $\{w_i^*\}_n^{\underline{i}}$, $n \leq \underline{i} \leq N$ is strictly decreasing and the sequence $\{w_i^*\}_{\underline{i}}^N = \{\underline{w}\}_{\underline{i}}^N$, $\underline{i}$ being the lowest i such that $w_{\underline{i}}^* = \underline{w}$.

$\square$

By establishing Theorem 4.2 we have also shown, that there can exist only one optimal stopping state $S=\underline{i}$, on $[n,N]$ in the case where $\underline{w} \leq \underline{v}$. Unless the process is stopped at the initial employment level n, so that no expansion is optimal, it follows that the optimal stopping state is independent of n.

In establishing some general properties of the optimal path of employment expansion we may keep to the transformed value function $\bar{L}(n,w_n)$; we have

$$\bar{L}(n,w_n) = \frac{(pa_\ell - w_n)n}{\lambda(w_n)+\alpha} + \frac{\lambda(w_n)}{\lambda(w_n)+\alpha} \cdot \bar{L}(n+1) = \frac{(pa_\ell - w_n)n}{\alpha} + \frac{\lambda(w_n)}{\lambda(w_n)+\alpha}(\bar{L}(n+1) - \frac{(pa_\ell - w_n)n}{\alpha}$$

The second term in the expression to the right is the expected additional contribution to $\bar{L}(n,w_n)$ of not stopping at n and earning the profit rate $(pa_\ell - w_n)n$ forever after. Taking the derivative with respect to w_n, we get

$$\bar{L}'(n,w_n) = \frac{\alpha}{(\lambda(w_n)+\alpha)^2}\{\lambda'(w_n)\cdot(\bar{L}(n+1) - \frac{(pa_\ell - w_n)n}{\alpha}) - \frac{n}{\alpha}\cdot\lambda(w_n) - n\}$$

(As it should be, the right hand side of the equation remains the same if L(n+1) is substituted for $\bar{L}(n+1)$ and $(pa_\ell - w_n)n + A_{\ell-1}$ for $(pa_\ell - w_n)n$.)

$$\bar{L}(n+1) - \frac{(pa_\ell - w_n)n}{\alpha} \geq \frac{(pa_\ell - \underline{w})(n+1)}{\alpha} - \frac{(pa_\ell - w_n)n}{\alpha} > 0$$

so the first term in $\bar{L}'(n,w_n)$ is always non-negative and strictly positive for $\underline{v}<w_n<\bar{v}$. For n=0 the optimal policy is very clear-cut: an interior maximum exists and it is situated at $w_0^*=\bar{v}$, the upper bound of the range of the reservation wage distribution. It is always optimal to get business started and to do so as soon as possible.

This result illustrates conspicuously the variability of the wage level which is possible when wages are assumed to be downward flexible; wages decrease from $\bar{v}$ to $\underline{w}$ when employment expands from zero to the capacity limit. It is in sharp contrast to the situation where wages are downward rigid and the optimal wage level remains fixed. Furthermore, there is no guarantee that the profit rates are positive at the early employment states in the recruitment process.

Without more information about the parameters of the decision problem, we cannot say whether there exists an interior solution $w_n^*>\underline{w}$ for n>0. The conditions that must hold for such solutions enable us to establish some of its properties, however.

Let $w_n^*>\underline{w}$; then

$$\bar{L}'(n,w_n^*) = 0 \Rightarrow \bar{L}(n+1) - \frac{(pa_\ell - w_n^*)\cdot n}{\alpha} = \frac{n(\lambda(w_n^*)+\alpha)}{\alpha\cdot\lambda'(w_n^*)}$$

Substitution into $\bar{L}(n,w_n)$ at $w_n=w_n^*$ gives

$$\bar{L}(n) = \frac{(pa_\ell - w_n^*)n}{\alpha} + \frac{n\cdot\lambda(w_n^*)}{\alpha\cdot\lambda'(w_n^*)},$$

and, consequently,

$$\bar{L}(n+1) - \bar{L}(n) = \frac{n}{\lambda'(w_n^*)}$$

This condition has a straightforward economic interpretation: in optimum the marginal effect of the wage on the speed of recruitment multiplied with the increase in value of recruiting one more person should equal the marginal cost of the wage, i.e. the number of employees.

It is immediately obvious that $\lambda'(w_n)$ must be decreasing at w_n^* and that is confirmed by the second-order condition

$$\bar{L}''(n,w_n^*) = \frac{\alpha\cdot\lambda''(w_n^*)}{\lambda(w_n^*)+\alpha}\left(\bar{L}(n+1) - \frac{(pa_\ell - w_n^*)n}{\alpha}\right) < 0 \Rightarrow \lambda''(w_n^*) < 0$$

Unless $w_n^*=\underline{w}$, the optimal wage must be situated at a point where the probability density function of the reservation wage distribution is falling. (Note that this is not a necessary condition for the optimal wage level in the case of downward wage rigidity.)

The condition $\lambda''(w_n^*)<0$ does not necessarily imply $\lambda'(w_n^*) > \lambda'(w_{n-1}^*)$, although it holds that $w_{n-1}^*>w_n^*$, because w_n^* may not be the result of (the discretization of) a continuous variation from w_{n-1}^*. Utilizing the established properties of $\bar{L}(n)/n$, we are nevertheless able to show a slightly stronger result.

From the relations $\frac{\bar{L}(n+1)}{n+1} < \frac{\bar{L}(n)}{n} < \frac{\bar{L}(n-1)}{n-1}$,

the concavity of L(n) follows, as they easily yield

$$\bar{L}(n+1) - \bar{L}(n) < \frac{\bar{L}(n)}{n} < \bar{L}(n) - \bar{L}(n-1).$$

We just showed that

$$\bar{L}(n+1) - \bar{L}(n) = \frac{n}{\lambda'(w_n^*)} \ ; \ \bar{L}(n) - \bar{L}(n-1) = \frac{n-1}{\lambda'(w_{n-1}^*)}$$

Hence $\frac{n}{\lambda'(w_n^*)} < \frac{n-1}{\lambda'(w_{n-1}^*)}$; $\lambda'(w_n^*) > \frac{n}{n-1}\lambda'(w_{n-1}^*)$

Together with the result $w_0^*=\bar{v}$, this optimality condition indicates that it is very likely that recruitment from low levels of employment is connected with fairly large decreases in the wage level (unless the pdf of the reservation distribution falls very steeply at the right end of its range).

Further results on the structure of the optimal wage policy are obviously dependent on the properties of the reservation wage distribution. We consider some cases of relevance in the main text of this chapter. Nevertheless we have been able to establish that under weak assumptions the optimal wage decreases from $\bar{v}$ to $\underline{w}$ as recruitment goes from zero to the optimal stopping state. If the pdf of the reservation wage distribution is not falling everywhere on $(\underline{w},\bar{v})$, the optimal wage will exhibit a downward jump at one or several employment states on (0,S).

5 Conjectural remarks on the structure of optimal wage policies in a recruitment-cum-quit model

The Markov model of pure recruitment analyzed in this chapter is a second-best alternative to a complete model, in which both new hires and quits can occur. Provided that both types of labour flows are Markov processes, which can be controlled through the firm's wage level, we face the problem of solving the following functional equations

$$\left|\begin{array}{l} H_T(n,w_{n-1}) = \max\limits_{\substack{x_n \geq 0 \\ \delta_n = 0,1}} \{H_T(n,w_{n-1},x_n,\delta_n)\} \\ \\ H_T(n,w_{n-1},x_n,\delta_n) = \dfrac{r(n,w_{n-1}+x_n)+\delta_n\lambda_n(w_{n-1}+x_n)\cdot H_T(n+1,w_{n-1}+x_n)+\mu_n(w_{n-1}+x_n)\cdot H_T(n-1,w_{n-1}+x_n)}{\delta_n\lambda_n(w_{n-1}+x_n)+\mu_n(w_{n-1}+x_n)+\alpha} \end{array}\right.$$

and

$$\left|\begin{array}{l} L_T(n) = \max\limits_{\substack{w_n \geq \underline{w} \\ \delta_n = 0,1}} \{L_T(n,w_n,\delta_n)\} \\ \\ L_T(n,w_n,\delta_n) = \dfrac{r(n,w_n)+\delta_n\lambda_n(w_n)\cdot L_T(n+1)+\mu_n(w_n)\cdot L_T(n-1)}{\delta_n\lambda_n(w_n)+\mu_n(w_n)+\alpha} \end{array}\right.$$

for the cases of downward wage rigidity and of wage flexibility, respectively.

$\lambda_n(w_n)$ is as before the recruitment intensity and the standard assumptions are $\lambda_n(w_n)=\lambda(w_n)$; $\lambda'(w_n)\geq 0$. $\mu_n(w_n)$ is the quit intensity and in accordance with usual deterministic models we can assume $\mu_n(w_n)=n\cdot\mu(w_n)$; $\mu'(w_n)\leq 0$. These properties follow, for example, if each employee at the firm searches over the wage offers of other firms at an intensity θ, if these offers are distributed according to the distribution function $G(\cdot)$, and if each employee's reservation wage is an increasing function $h(w)$ of his present wage. Then clearly $\mu_n(w_n)=n\cdot\theta\cdot[1-G\{h(w_n)\}]$ and $\mu'(w_n)=-\theta\cdot g\cdot h'(w_n)\leq 0$ as $g=G'\geq 0$.

The functional equations, especially in the case of flexible wages, do not look too deterrent compared to what can be found in decision process literature. Nevertheless, a strict analysis does not easily yield structural results as to the optimal wage policy. In Deshmukh and Chitke (1977) a model of optimal product pricing to control the flow of 'loyal customers' to and from the firm is presented, which shows strong similarities to the recruitment-cum-quit model (with flexible wages). Their findings on the optimal price structure are not very comprehensive.

An additional difficulty in the recruitment-cum-quit model lies in the determination of the optimal stopping state. It is by no means as straightforward as in the case of pure recruitment, where changes in employment can occur in one direction only. It is plausible that it is optimal to let employment expand into productivity intervals where the marginal profit rate is negative, because of the positive probability that an employee will quit in the near future. Such a result offers an explanation of the existence of labour hoarding in a firm, when the environment is characterized by stochastic labour flows. It has a direct counterpart in queueing models, in which a service station may prefer to have direct access to the customers in the 'waiting-room', although it means the incurrence of some immediate costs. The optimal volume of labour hoarding, i.e. the optimal stopping state, is likely to be dependent on, among other things, the basic contact and search intensities.

Nevertheless, on the basis of the findings on the recruitment model of this paper, we can conjecture the structure of the optimal wage policy in the recruitment-cum-quit model, thus indicating results, which we hope future research will be able to establish. In the case of complete wage flexibility we have reason to believe, that the property of the optimal wage policy of being non-increasing in employment states is preserved, given otherwise the same structure of the decision problem as in our recruitment model.

When wages are downward rigid, the optimal wage structure seems to be more complex. Let $w^*(n)$ denote the optimal wage level in an initial employment state n, when no historically given wage level w_0 exists (or $w_0 \leqq \underline{v}$). The conjecture is that $w^*(n)$ is non-increasing in n and strictly decreasing for $n=0...\underline{i}$, $\underline{i} \geqq 0$. Consequently, the firm will pay the wage $w^*(n)$, when it is in a subsequent state i, $i \geqq n$, where n is the lowest employment state it has ever experienced. As soon as state n-1 is entered, $w^*(n-1) \geqq w^*(n)$ is the new optimal wage level to be paid in every subsequent state $i \geqq n-1$.

As a result of the conjectured structure of the optimal wage policy, employment expansion always takes place at an unchanged wage level, while employment contraction may or may not be connected with wage increases. In terms of the disequilibrium notion discussed in Section 3, we can say that expansion always increases the disequilibrium at the optimal wage level, while contraction always decreases it or preserves an existing equilibrium. There is consequently an interesting asymmetry in the decision process, the effect of which can be described as follows: let initially a given number of employees be shared between two firms, that are in 'equilibrium' with respect to their optimal wages; let recruitment and quits redistribute the employees between the firms, so that one firm has expanded and the other one contracted their employment volume with the same amount at a later point of time; then the wage of the expanding firm must have remained unchanged, while the wage of the contracting firm has increased; moreover, any way of return to the initial employment distribution cannot restore the initial wage relation. So labour turnover, even as a purely stochastic phenomenon, may on the average initiate wage increases in a recruitment-cum-quit model with downward wage rigidity, if our conjecture is correct. It is a feature of the model, the empirical relevance of which might well deserve attention.

We conclude this tentative section by pointing out that it is not always necessary to know the structure of the optimal policy in order to see how the optimal value of the control variable is affected by changes in the parameters of the problem. The functional equation may fairly easily enable one to reach conclusive results of such a sensitivity analysis, as the presentation in Deshmukh and Winston (1977) gives examples of.

Appendix to Section 4 of the Technical Supplement

SENSITIVITY ANALYSIS OF MARKOV DECISION PROCESSES USING THE DEVICE OF VIRTUAL DECISION EPOCHS

In his contribution Lippman (1975) Lippman demonstrated that the structural properties of the functional equation of many Markov decision processes are more easily revealed, if one introduces the device of virtual transition epochs. Such an epoch is defined as an exponentially distributed sojourn time of the process, the expected value of which is independent of the state of the process and the decision taken. It should also hold that the expected value of the virtual transition epoch is no greater than the expected value of any real transition epoch (at any state and decision). In our recruitment model with Poisson arrivals this requirement is fulfilled if the virtual epoch has an expected duration Λ^{-1}, $\Lambda \geq \gamma \geq \lambda(v)$.

The next step in the approach is to consider the decision process, in which decisions are taken only at those points of time at which a virtual transition takes place. It can be shown that this new decision process is equivalent to the original one, so that the essential properties of the optimal policy that are established for the new process hold for the original process as well.

In general the device of virtual decision epochs is applied in the context of a finite expected horizon. The time until the horizon is given as the number of remaining virtual epochs and the functional equation expresses the value of the process at any state when m epochs remain in terms of the values of the process when m-1 epochs remain. When m=0 the values are by definition zero and when m=1 they are easily identified as the reward (profit) rate divided by $\Lambda + \alpha$. Induction can then be used to establish structural properties of the value function for any m. Under conditions that can in most cases be safely assumed to hold, there exists a solution to the corresponding infinite horizon decision problem, the functional equation of which is the result of a uniform convergence of that of the finite horizon one, when $m \to \infty$, so its structural properties are preserved. The interested reader is referred to Lippman (1975) and to Deshmukh and Winston (1979) for a comprehensive presentation as well as several applications of the method.

We will make use of the device of virtual decision epochs in order to establish some sensitivity analysis results for our recruitment model in the case where wages are fully flexible. Although the value function has been explicitly derived, it is less cumbersome to work with the functional equation by induction, once it has been transformed to the virtual decision epoch version. An additional advantage of applying the method to our recruitment model is the existence of a terminal employment state, which can replace the terminal states at the end of the horizon in the induction arguments. We can consequently work with the infinite horizon problem directly.

Consider the functional equation of the decision process (4.1)

$$\begin{cases} L(n) = \max\limits_{w_n \geq \underline{w}} \{L(n,w_n)\} \\ \\ L(n,w_n) = \dfrac{r(n,w_n)}{\lambda(w_n)+\alpha} + \dfrac{\lambda(w_n)}{\lambda(w_n)+\alpha} L(n+1) \qquad 0 \leq n \leq N-1 \\ \\ L(N) = \dfrac{r(N,\underline{w})}{\alpha} \end{cases}$$

Recalling that $\lambda(w_n) = \gamma \cdot F(w_n) \leq \gamma$ we may choose $\Lambda = \gamma$, so that the virtual decision epochs are in fact the interarrival times of job applicants.

It is easily established that (4.1) implies

$$L(n,w_n) = \frac{r(n,w_n)}{\gamma+\alpha} + \frac{\gamma \cdot F(w_n)}{\gamma+\alpha} \cdot L(n+1) + \frac{\gamma\{1-F(w_n)\}}{\gamma+\alpha} L(n,w_n)$$

This is just a reformulation of the middle equation in (4.1), but it is also the structure of the decision process, in which a "transition" takes place after an exponentially distributed sojourn time with expectation γ^{-1}. At the end of that epoch the firm has made a transition to employment state n+1 with probability $F(w_n)$ and remains in state n with probability $1-F(w_n)$. Applying an optimal policy yields the value L(n+1) in state n+1; but it is equally true that a wage policy such that L(n) is obtained should be applied in state n. The firm faces no other decision problem at the end of the virtual transition epoch than at its beginning, given that the employment state has not changed (and given an infinite horizon).

Consequently the following decision process is equivalent to (4.1).

$$\begin{cases} L(n) = \max\limits_{w_n \geq \underline{w}} \{\hat{L}(n,w_n)\} \\ \hat{L}(n,w_n) = \dfrac{r(n,w_n)}{\gamma+\alpha} + \dfrac{\gamma \cdot F(w_n)}{\gamma+\alpha} L(n+1) + \dfrac{\gamma\{1-F(w_n)\}}{\gamma+\alpha} \cdot L(n) \\ L(N) = \dfrac{r(N,\underline{w})}{\alpha} \end{cases}$$

We have

$$\hat{L}'(n,w_n) = \frac{1}{\gamma+\alpha} \{-n + \gamma \cdot f(w_n)[L(n+1) - L(n)]\},$$

and the properties of the optimal solution indicated by the properties of $\hat{L}'(n,w_n)$ are those of the solution to 4.1.

As is clear from the expression for $\hat{L}'(n,w_n)$, the effect on the optimal policy of a parameter change is crucially dependent on its effect on $L(n+1)-L(n)$. In some cases this effect is easily evaluated and so we can establish that an increase in p or in a_k, $k = \ell \ldots \ell'$ has a non-decreasing impact on $\hat{L}'(n,w_n)$ and hence on the optimal wage sequence. The case of a change in α or in γ requires more elaboration.

<u>Theorem</u>: The optimal wage sequence $\{w_1^*\}_n^N$ is non-increasing in α and non-decreasing in γ.

<u>Proof</u>: From the functional equation of (4.1) we immediately have

$$\alpha L(n,w_n) = r(n,w_n) + \frac{\lambda(w_n)}{\lambda(w_n)+\alpha} [\alpha \cdot L(n+1) - r(n,w_n)] =$$

$$= r(n,w_n) + \lambda(w_n)[L(n+1) - L(n,w_n)]$$

We first consider a change in α. From the first equality it is clear that $\alpha \cdot L(n+1)$ non-increasing in α implies $\alpha\, L(n,w_n)$ and hence

$$\alpha\, L(n) = \max_{w_n \geq \underline{w}} \{\alpha\, L(n,w_n)\} \text{ decreasing in } \alpha. \quad \alpha \cdot L(N) = r(N,\underline{w})$$

is non-increasing in α so by induction it holds that $\alpha\, L(n,w_n)$ and $\alpha\, L(n)$ are decreasing in α for $n < N$.

From the second equality and the fact that $\alpha \cdot L(n,w_n)$ is decreasing in α it follows that $L(n+1) - L(n,w_n)$ is decreasing in α. Hence $L(n+1) - L(n) = \min_{w_n \geq \underline{w}} \{L(n+1) - L(n,w_n)\}$ is also decreasing in α.

Returning to $\hat{L}'(n,w_n) = \frac{1}{\gamma+\alpha} \{-n + \gamma \cdot f(w_n)\,[L(n+1) - L(n)]\}$ we observe that $L(n+1) - L(n)$ decreasing in α implies that the expression in curly brackets decreases in α. Hence $\hat{L}'(n,w_n)$ must change in such a way that w_n^* is non-increasing in α for $n < N$ (cf. the argument in the proof of Theorem 4.2). If $w_n^*(\alpha_1) > \underline{w}$, $w_n^*(\alpha_2) < w_n^*(\alpha_1)$, $\alpha_2 > \alpha_1$.

Considering the case of a change in γ, we rewrite the equalities as

$$L(n,w_n) = \frac{r(n,w_n)}{\alpha} + \frac{\gamma \cdot F(w_n)}{\gamma \cdot F(w_n)+\alpha}\left[L(n+1) - \frac{r(n,w_n)}{\alpha}\right] =$$

$$= \frac{r(n,w_n)}{\alpha} + \gamma \cdot \frac{F(w_n)}{\alpha}\,[L(n+1) - L(n,w_n)]$$

From the first equality it is clear that $L(n+1)$ non-decreasing in γ implies that $L(n,w_n)$ and $L(n)$ are increasing in γ. $L(N) = \frac{r(N,\underline{w})}{\alpha}$ is non-decreasing in γ, so by induction it holds that $L(n,w_n)$ and $L(n)$ are increasing in γ for $n < N$.

From the second equality and the fact that $L(n,w_n)$ is increasing in γ it follows that $\gamma\,[L(n+1) - L(n,w_n)]$ is increasing in γ. Thus $\gamma\,[L(n+1) - L(n)]$ increases in γ.

This result establishes that the expression within curly brackets in $\hat{L}'(n,w_n) = \frac{1}{\gamma+\alpha}\{-n + f(w_n)\cdot\gamma[L(n+1)-$ $-L(n)]\}$ increases in γ and hence w_n^* must be non-decreasing in γ for $n < N$. If $w_n^*(\gamma_1) > \underline{w}$, $w_n^*(\gamma_2) >$ $> w_n^*(\gamma_1)$, $\gamma_2 > \gamma_1$.

$\square$

LIST OF NOTATION

Variables and parameters

i	employment state
n	initial employment state
S	stopping employment state
N_k	employment state at end of productivity interval k
N_L	physical capacity limit (in terms of employment)
N_{ℓ},	economic capacity limit at lowest feasible wage
w_i	wage level in employment state i
x_i	wage increase at the entrance of employment state i
w_0	$= w_{n-1}$ wage level at the entrance of employment state n
w	constant future wage level (trajectory)
$w_{max}(n)$	local maximum of $V(n,w)$
$w_{i,max}$	local maximum of $L(i,w_i)$
$\underline{w}$	minimum wage
δ_i	stopping (vacancy) control at employment state i
$F(\cdot)$	reservation wage distribution
$\bar{v}$	upper bound on $F(\cdot)$
$\underline{v}$	lower bound on $F(\cdot)$
$F_U(\cdot)$	reservation wage distribution of the unemployed
$F_E(\cdot)$	reservation wage distribution of the employed
$f(\cdot)$	$= F'(\cdot)$
$f_U(\cdot)$	$= F'_U(\cdot)$
$f_E(\cdot)$	$= F'_E(\cdot)$
γ, γ_i	contact intensity of job applicants (at employment state i)

γ_U contact intensity of unemployed job applicants

γ_E contact intensity of employed job applicants

$\lambda(\cdot)$ $= \gamma \cdot F(\cdot)$ recruitment intensity at the wage argument; $\lambda(\cdot)$ may be indexed as γ and $F(\cdot)$

$G(\cdot)$ wage offer distribution

$g(\cdot)$ $= G'(\cdot)$

θ search intensity of an employee at the firm

θ_U search intensity to the firm of an unemployed job searcher

θ_E search intensity to the firm of an employed job searcher

U stock of unemployed job searchers

E stock of employed job searchers

$\mu(\cdot)$ $= \theta[1-G(\cdot)]$ quit intensity of an employee at the firm

$\mu_i(\cdot)$ $= i \cdot \mu(\cdot)$ quit intensity at the firm in employment state i

p product price

a_k marginal labour productivity in productivity interval k

q(i) production in employment state i

$r(i,w_i)$ $= p \cdot q(i) - i \cdot w_i$ profit rate in employment state i and at wage level w_i

α discount intensity (instantaneous discount rate)

* indicates optimal value of a decision variable

Note especially

w_k^* optimal value of w when $S = N_k$

$w^*(n)$ optimal value of w at initial employment state n, when S is optimally chosen

λ_i^* $= \lambda(w_i^*)$

Functions

$H(i,w_{i-1},x_i,\delta_i)$	value at unspecified wage increase and stopping policy, downward rigid wages
$H(i,w_{i-1})$	$= H(i,w_{i-1},x_i^*,\delta_i^*)$
$H_k(i,w_{i-1},x_i)$	as $H(i, w_{i-1}, x_i, \delta_i)$ but stopping at N_k
$H_k(i,w_{i-1})$	$= H_k(i,w_{i-1},x_i^*)$
$V(i,w)$	value at constant future wage level, stopping optimal
$V_k(i,w)$	value at constant future wage level, stopping at N_k
$W(i,w)$	as $V(i,w)$, when never stopping is optimal
$L(i,w_i)$	value at unspecified wage level, flexible wages
$L(i)$	$= L(i,w_i^*)$
$\bar{L}(i,w_i)$; $\bar{L}(i)$	as $L(i,w_i)$; $L(i)$, but subject to a lump-sum reduction
$\hat{L}(i,w_i)$	$L(i,w_i)$, transformed to virtual decision epoch form
$L_T(i,w_i)$; $L_T(i)$	as $L(i,w_i)$; $L(i)$, extended to recruitment-cum-quit
$H_T(i,w_{i-1},x_i,\delta_i)$; $H_T(i,w_{i-1})$	as $H(i,w_{i-1},x_i,\delta_i)$; $H(i,w_{i-1})$, extended to recruitment-cum-quit
$W_T(i,w)$	as $W(i,w)$,extended to recruitment-cum-quit
$P(i,w_{i-1},x_i)$	value in deterministic recruitment-cum-quit model, downward rigid wages, wage increase unspecified
$P(i,w_{i-1})$	$= P(i,w_{i-1},x_i^*)$

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