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No. 84

MONOPOLY AND ALLOCATIVE EFFICIENCY WITH STOCHASTIC DEMAND*

by

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* I am grateful to Fredrik Bergholm for helpful comments.

CONTENTS

INTRODUCTION

The objections usually raised to monopoly are the negative distributional and allocational effects, of which we will focus on the latter. The allocational distortions brought about by monopoly are well-known. Since price is higher than marginal cost a monopolist will produce less than a competitive industry or a public utility. This is often referred to as a loss of the consumer surplus triangle or welfare triangle. The quantitative importance of the allocational inefficiency has been treated in a number of studies, beginning with Harberger (1954). However, the estimates of the welfare losses are still controversial, as exemplified by the debate between Cowling & Mueller (1978, 1981) and Littlechild (1981).

There is undoubtedly a gap between empirical studies of the welfare effects of monopoly and theoretical models of the conduct of firms. Empirical studies typically analyse firms with constant costs and linear demand in a deterministic environment. On the theoretical side the firm under uncertainty has received increased attention during the last ten fifteen years. This applies to the competitive firm as well as monopoly and public utility. In this paper we try to bridge the gap between application and theory in one important aspect, namely regarding uncertainty in demand. We take the often used model with constant costs and linear demand as a starting point, but add to it a stochastic component in demand. The price and capacity are assumed to be determined before the actual demand is known.

To evaluate the welfare loss from monopoly we use a welfare-maximizing public utility as benchmark instead of a competitive industry. This approach permits us to analyse the effects of different nonprice rationing schemes. It also has two further advantages. Firstly, we do not have to deal with questions of cost-efficiency, since there are no good reasons to believe that this should be different between monopoly and public utility. Secondly, the comparison is not affected by differences in size between firms which may cause differences in attitudes towards risk and in informational activities.

We concentrate on static efficiency and refrain from an analysis of the complicated question of dynamic efficiency. Furthermore, we limit the scope of the analysis to the loss of the welfare triangle. The only difference between monopoly and public utility in the model is that the former maximizes expected profits while the latter maximizes expected welfare. In this pure comparison we can isolate the effects of uncertainty in a model frequently used in empirical applications.

Several questions are treated. Is monopoly capacity relative to welfare-maximizing capacity in the stochastic case larger or smaller than in the deterministic case? Is it possible for monopoly capacity to exceed welfare-maximizing capacity in the stochastic model? Is the welfare loss from monopoly relative to monopoly profits in the stochastic case larger or smaller than in the deterministic case? These questions are clearly relevant if we believe that a model with stochastic demand can give better predictions about important economic variables than a model with deterministic demand. It may then be crucial which model policy implications are based on, e.g. in antitrust cases.

In Section I and II the model and the first order conditions for optimum are presented. The comparison between monopoly and public utility is made in Section III. Section IV contains a summary and conclusions.

I THE MODEL

We assume that the demand for a non-storable good Q takes on two different levels. A single price p and the capacity $\bar{Q}$ has to be determined before actual demand is known. We assume that the demand functions are linear and have the same slope.

$$Q_i = A_i - Bp \qquad \text{for } i = 1,2 \tag{1}$$

where subscript i=1,2 denote high and low demand, respectively and A_i, $B > 0$.

The known probabilities of high and low demand are s and 1-s, respectively. For s=1 or s=0 the model is reduced to the deterministic case with high or low demand.

The marginal operating cost is b per unit of output and the marginal capacity cost is β per unit of capacity. We assume that there are no problems of indivisibilities. This kind of model, which is in the tradition of Brown & Johnson (1969), was used by Andersen (1974) and Sherman & Visscher (1977, 1979). Although the model is simple, it captures the essential stochastic element in demand.

II OPTIMAL PRICE AND CAPACITY

Following Andersen and Sherman & Visscher we derive the optimal price and capacity which are used in the comparison in Section III.

II.1 Public Utility

We take the objective function of the public utility to be the expected value of total benefits minus total costs, E(W)[1]. When welfare is maximized we need some assumptions about the nonprice rationing scheme since demand may exceed capacity at the preset price. Two alternative rationing cases are considered. Consumers are served either in order of highest or lowest willingness to pay.[2] Thus we get the extremes of the rationing schemes.

Firstly, we consider the case when those who receive the largest consumer surplus are served first. The optimal price is either equal to the marginal operating cost b or equal to the market-clearing price with low demand $(A_2-\bar{Q})/B$.[3] The objective function is:

$$E(W) = s[(A_1/B-(A_1-\bar{Q})/B)\bar{Q}/2+((A_1-\bar{Q})/B-b)\bar{Q}] \qquad (2)$$

$$+(1-s)[(A_2/B-p)(A_2-Bp)/2+(p-b)(A_2-Bp)]-\beta\bar{Q}$$

We insert the price solutions in (2) and maximize with respect to $\bar{Q}$.

$$p_w^* = b \qquad \bar{Q}_w^* = A_1- B(b+\beta/s) \qquad (3)$$

$$p_w^* = (A_2-\bar{Q}_w^*)/B \qquad \bar{Q}_w^* = sA_1+(1-s)A_2-B(b+\beta) \qquad (4)$$

where asterisks denote optimal values and subscript w denote welfare-maximizing values. Solution (3) is optimal for $\min(s_k,1) \leq s \leq 1$, while solution (4) is optimal for $0 \leq s \leq \min(s_k,1)$, where s_k is a value of s for which the expected value of the two solutions are equal.

Secondly, we assume that consumers who gain the smallest consumer surplus are served first. In this case the optimal price is either equal to the market-clearing price with high demand $(A_1-\bar{Q})/B$ or equal to the market clearing price with low demand $(A_2-\bar{Q})/B$.[4] The objective function is:

$$E(W) = s[(A_1/B-(A_1-\bar{Q})/B)\bar{Q}/2+(p-b)\bar{Q}] + (1-s)[(A_2/B-p)(A_2-Bp)/2+(p-b)(A_2-Bp)]-\beta\bar{Q} \qquad (5)$$

To get the optimal capacity we insert the price solutions in (5) and maximize with respect to $\bar{Q}$.

$$p^*_w = (A_1-\bar{Q}^*_w)/B \qquad \bar{Q}^*_w = A_1-B(b+\beta) \qquad (6)$$

$$p^*_w = (A_2-\bar{Q}^*_w)/B \qquad \bar{Q}^*_w = A_2-B(b+\beta) \qquad (7)$$

In this case the optimal price always equals $b+\beta$. Solution (6) is optimal for $s_\ell \leq s \leq 1$, while solution (7) is optimal for $0 \leq s \leq s_\ell$, where s_ℓ is analogous to s_k.

II.2 Monopoly

The function to be maximized is taken to be expected profits, $E(\Pi)$. Therefore we do not need any assumption about the nonprice rationing scheme. The optimal price is either equal to the market-clearing price with high demand or equal to the market-clearing price with low demand.[5] The objective function is:

$$E(\Pi) = s[(p-b)\bar{Q}] + (1-s)[(p-b)(A_2-Bp)] - \beta\bar{Q} \tag{8}$$

The optimal price and capacity is:

$$p_m^* = (A_1-\bar{Q}_m^*)/B \qquad \bar{Q}_m^* = [A_1+(1-s)(A_1-A_2)-B(b+\beta)]/2 \tag{9}$$

$$p_m^* = (A_2-\bar{Q}_m^*)/B \qquad \bar{Q}_m^* = [A_2-B(b+\beta)]/2 \tag{10}$$

where subscript m denote monopoly values. Solution (9) is optimal for $s_m \leq s \leq 1$, while solution (10) is optimal for $0 \leq s \leq s_m$, where s_m is analogous to s_k.

III COMPARISON

From the deterministic model with linear demand and constant costs we know that monopoly price is higher than the welfare-maximizing price and that monopoly capacity is 50 per cent of welfare-maximizing capacity. In the stochastic model it is obvious that the monopoly price is higher than the welfare-maximizing price. However, this does not prove that monopoly capacity is smaller than welfare-maximizing capacity, since the monopoly price-capacity combination and the welfare-maximizing price-capacity combination do not necessarily correspond to the same demand function. In solution (3) the price-capacity combination do not even correspond to any demand function.

It is evident that if solution (10) is optimal, then monopoly capacity is smaller than welfare maximizing capacity, since $\bar{Q}_W^* \geq A_2 - B(b+\beta)$. In the Appendix we show that if solution (9) is optimal it is also true that monopoly capacity is below welfare-maximizing capacity. To prove this we compare solution (9) with each of the welfare-maximizing solutions and show that if monopoly capacity is larger than or equal to welfare-maximizing capacity, then some other assumptions in the comparison are contradicted. Thus, the standard result from the case with deterministic demand is still valid, i.e. the monopolist chooses a higher price and a smaller capacity than the welfare-maximizer.[6]

In the stochastic model we note that monopoly capacity is larger than or equal to 50 percent of welfare-maximizing capacity if solution (9) is optimal, while it is smaller than or equal to 50 percent if solution (10) is optimal. This observation is valid irrespective of the nonprice rationing scheme considered. Thus, monopoly capacity relative to welfare-maximizing capacity in the stochastic model may be larger as well as smaller than in the deterministic model. This is exemplified in Table 1 in which monopoly capacity relative to welfare-maximizing capacity ranges from 28 to 69 percent.

Although monopoly capacity is smaller than welfare-maximizing capacity, as shown in the Appendix, the percentage difference between the capacities can be small for suitable values on the parameters in the model. The 69 percent recorded in Table 1 is far from extreme. However, even if monopoly capacity relative to welfare-maximizing capacity in the stochastic model is larger than in the deterministic model, this does not imply that the welfare loss from monopoly relative to the optimal welfare or maximum profits is smaller (see Table 1).

Another conclusion from the numerical example is that monopoly does not necessarily perform worse when the nonprice rationing scheme is imperfect than when it is perfect. In Table 1 it is only for $s = 0.4$ that the welfare loss from monopoly relative to optimal welfare or maximum profits is larger for the rationing case when those who receive the smallest consumer surplus are served first than for the case when those who receive the largest consumer surplus are served first.

In the deterministic model the welfare loss from monopoly can be calculated equivalently as 25 percent of the level of welfare in optimum or as 50 percent of monopoly maximum profits. In the stochastic model these percentages are no longer valid. In the numerical example in Table 1 we see that the welfare loss in percentage of the optimal level of welfare ranges from 25 to 50 percent. The welfare loss in percentage of maximum profits varies from 50 to 152 percent. The often used model with constant costs and linear demand is apparently sensitive to the introduction of a stochastic component in demand. Thus, if there is uncertainty in demand the common practice of evaluating monopoly welfare losses as 25 percent of optimal welfare or 50 percent of maximum profits may lead to considerable miscalculations.

Table 1. Monopoly welfare losses relative to optimal welfare and maximum profits for different nonprice rationing schemes

Probability of high demand	Monopoly capacity/ welfare-maximizing capacity (%)		Welfare loss/ optimal welfare (%)		Welfare loss/ maximum profit (%)	
	(a)	(b)	(a)	(b)	(a)	(b)
0.2	43	50	32	25	87	50
0.4	38	28	39	50	134	152
0.6	69	59	30	26	78	65
0.8	58	54	26	25	59	55

(a): Consumers who receive the largest consumer surplus are served first

(b): Consumers who receive the smallest consumer surplus are served first

Note: $A_1=12$; $A_2=8$; $B=1$; $b=1$; $\beta=2$; $s_k=0.5$; $s_\ell\approx0.22$; $s_m\approx0.41$.

IV SUMMARY AND CONCLUSIONS

Welfare losses from monopoly are often evaluated using a model with constant costs and linear demand. We studied the robustness of this model when a basic assumption was changed, namely the deterministic property of demand. In the frequently used model we inserted an additive stochastic component. The price and capacity was assumed to be determined before actual demand was known. We compared a monopolist and a welfare-maximizer between which the only difference was that the former maximized expected profit while the latter maximized expected welfare.

In the deterministic model monopoly capacity is 50 percent of welfare-maximizing capacity. In the stochastic model we demonstrated that monopoly capacity relative to welfare-maximizing capacity may be larger as well as smaller than in the deterministic case. However, the standard result from the case with deterministic demand is still valid, i.e. the monopolist chooses a higher price and a smaller capacity than the welfare-maximizer.

In the deterministic model the conventional welfare loss due to monopoly is 50 percent of monopoly profits or equivalently 25 percent of the level of optimal welfare. We showed that this is no longer true when there is uncertainty in demand. In a numerical example the welfare losses varied from 50 to 150 percent of monopoly profits. The welfare losses relative to the optimal level of welfare ranged from 25 to 50 percent.

Thus, if observed profits are generated according to a model with stochastic demand while a deterministic model is applied, this may lead to considerable miscalculations of monopoly welfare losses. Therefore the empirical results from the frequently applied deterministic model must be interpreted with great cautiousness.

APPENDIX

1. Proof of $\bar{Q}^*_m < \bar{Q}^*_w$ when solution (3) and (9) are compared.

We study if $\bar{Q}^*_m \geq \bar{Q}^*_w$ at the same time as $E(\Pi)$ in solution (9) $\geq E(\Pi)$ in solution (10). From the first-order conditions for maximum and the objective functions these two inequalities are solved for s. We then study if there is an s in $[0;1]$ which satisfies these inequalities.[7]

$$0 \leq s \leq s_{\bar{Q}} = x/2 + (x^2/4+2y)^{1/2} \quad (1)$$

$$1 \geq s \geq s_m = x + (x^2 + 4y)^{1/2} \quad (2)$$

$$s_{\bar{Q}} \geq s_m \quad (3)$$

where $x = (-A_2+Bb-B\beta)/(A_1-A_2)$ and $y = B\beta/(A_1-A_2)$ are two variables introduced for ease of notation and $s_{\bar{Q}}$ is the value of s for which $\bar{Q}^*_m = \bar{Q}^*_w$ in solutions (3) and (9). We require that $x \leq 0$ and $y \leq 0$. The restriction on x is reasonable since we are interested in cases in which $A_2-Bb \geq 0$.

If $x = 0$ then $y = 0$, which is an economically uninteresting case. If $x < 0$ we rewrite (1) and (2) as:

$$s_{\bar{Q}} = x/2 + |x/2|(1+8y/x^2)^{1/2} \quad (1.b)$$

$$s_m = x + |x|(1+4y/x^2)^{1/2} \quad (2.b)$$

Let $h = 2y/x^2$ and make a first order Taylor series expansion of the square roots at $h = 0$.

$$s_{\bar{Q}} = x/2 + |x/2|(1+2h+R_{\bar{Q}}) \quad (1.c)$$

$$s_m = x + |x|(1+h+R_m) \quad (2.c)$$

where $R_{\bar{Q}}$ and R_m are the remainders which are nonpositive and increase more than proportionally to h. We have $s_m - s_{\bar{Q}} = |x|(R_m - R_{\bar{Q}}/2) \geq 0$. This means that $s_{\bar{Q}} \geq s_m$ only when $y=0$. Thus, we have proved that $\bar{Q}_m^* < \bar{Q}_w^*$ for $y > 0$.

2. <u>Proof of $\bar{Q}_m^* < \bar{Q}_w^*$ when solution (4) and (9) are compared.</u>

In the preceding paragraph we proved that $s_m > s_{\bar{Q}}$ for $y > 0$. Solution (3) is preferred to or indifferent to solution (4) if $\min(s_k,1) \leq s \leq 1$. If $0 \leq s \leq \min(s_k,1)$ then $\bar{Q}_w^*$ in solution (4) $\geq \bar{Q}_w^*$ in solution (3). This means that $s_{\bar{Q}}' \leq s_{\bar{Q}}$ where $s_{\bar{Q}}'$ is the value of s for which $\bar{Q}_m^* = \bar{Q}_w^*$ in solution (4) and (9). Thus, $s_m > s_{\bar{Q}}'$ for $y > 0$ also in solution (4) and $\bar{Q}_m^* < \bar{Q}_w^*$.

3. <u>Proof of $\bar{Q}_m^* < \bar{Q}_w^*$ when solution (6) and (9) are compared.</u>

This is obvious since $p_m^* > b+\beta$ and both solutions correspond to the same demand function.

4. Proof of $\bar{Q}_m^* < \bar{Q}_w^*$ when solution (7) and (9) are compared.

We use three inequalities:

$$0 \leqslant s \leqslant s_\ell = 2y/(1-2x-2y) \tag{4}$$

$$1 \geqslant s \geqslant s_m = x + (x^2+4y)^{1/2} \tag{5}$$

$$s_m \leqslant s_\ell \tag{6}$$

For $s_\ell = 1$ we have $A_1-B(b+\beta)+A_2-B(b+\beta)=0$, which is economically uninteresting. For $s_\ell<1$, we have $y<(1-2x)/4$. Solving (6) for y we get $(1-2x)/4 \leqslant y \leqslant 1-x$. Thus, $s_m > s_\ell$ if $s_\ell < 1$ and therefore $\bar{Q}_m^* < \bar{Q}_w^*$.

NOTES

[1] The usual assumptions for the applicability of partial welfare analysis are made.

[2] Alternative rationing methods are discussed by Holt & Sherman (1982).

[3] See Andersen (1974) and Sherman & Visscher (1977). Optimal prices below b are disregarded.

[4] See Sherman & Vischer (1977).

[5] See Sherman & Vischer (1979).

[6] This result seems to contrast with Appelbaum & Lim (1982) who finds that monopoly capacity may exceed the capacity of a competitive industry when demand is stochastic. In comparing monopoly and competition they assume that price is set ex post. However, we observe that their result is due to differences in size between the monopolist and the competitive firms which are not present in the comparison of monopoly and public utility.

[7] It is only the positive roots that are relevant for $s_{\bar{Q}}$ and s_m.

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