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FACTOR DEMAND IN SWEDISH MANUFACTURING:

Econometric Analyses

by
Joyce Dargay

THE INDUSTRIAL INSTITUTE FOR
ECONOMIC AND SOCIAL RESEARCH

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Abstract
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The empirical studies presented in this thesis investigate the factors influencing input demand in Swedish manufacturing industries: the price-sensitivity of factor demand, the substitution possibilities amongst inputs and the impact of technical change. As opposed to most earlier studies of factor demand in Swedish industries, which have taken the "value-added" approach considering only capital and labour inputs, all factors of production are considered simultaneously. The analysis is based on four aggregate inputs: labour, capital, energy and materials. Energy is treated as a input separate from other intermediate goods because of its unique price development and because of the effects this has been shown to have on the economy, particularly since the first oil crisis.

Three different models describing factor demand and production relationships are estimated. The point of departure is a full static equilibrium model, in which all inputs—including physical capital—are assumed to adjust instantaneously and costlessly to changes in relative factor prices and output demand. Subsequently, more realism is introduced by allowing for the imperfect flexibility of capital, in the context of both a partial static equilibrium and a dynamic cost of adjustment model. By recognising the possibility of disequilibrium, the latter two models provide a basis for estimating both short- and long-run adjustment possibilities and price effects. The various models and the underlying assumptions are assessed in terms of their theoretical plausibility and their empirical performance.

The empirical results of this investigation indicate that factor demand is sensitive to changes in relative factor prices, although the price elasticities are generally rather low, even in the long run. In general, only minimal differences are noted between short- and long-run elasticities. Although the results vary for the individual production sectors as well as for the various models, capital-labour substitutability and capital-energy complementarity prevail. Technological progress is shown to have been strongly labour-saving and capital-using, thus leading to a considerable substitution amongst inputs. Although the results suggest that adjustment of the capital stock is not instantaneous, none of the models appear to provide an adequate description of long-run production possibilities.

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CONTENTS

Foreword

A major part of IUI research has always been directed towards the empirical analysis of economic development and the performance of Swedish manufacturing. The evaluation of technical progress and factor demand, within the framework of production theory has been one recurrent theme in this research. In ''Energy in Swedish manufacturing'', published by IUI in 1983, different ways of measuring and analyzing energy demand were analyzed. Joyce Dargay subsequently expanded her study for that book to include a simultaneous analysis of the demand for labor, capital, energy and materials during 1952—83. Various competing models were tested.

The results are presented in this volume. They explicitly account for the economic flexibility and technical progress in Swedish manufacturing. They also include an evaluation of recently developed methods of production analysis. The study raises intriguing questions concerning, inter alia, the difference between short-run and long-run flexibility, and the marginal gains from using more complex and dynamic models of analysis.

We hope that this book will serve not only as a reference for the analysis of industrial development and policy but also as a starting-point for further econometric research on the mechanisms of industrial resource allocation.

Stockholm in August 1988

Gunnar Eliasson

PREFACE

In the course of writing this thesis, I have benefited from a number of institutions and a great many people have contributed with their suggestions and advice. I would like to take this opportunity to extend my thanks.

Work on this thesis was initiated, much of the actual empirical work carried out and earlier versions of many of the chapters written whilst I was at the Energy Systems Research Group at the University of Stockholm. I have benefitted greatly from seminars and informal discussions with other researchers there over the years, and I would like to thank all of them not only for their suggestions on matters relating to my research, but also for helping to create an exceedingly pleasant environment in which to work. Particularly, I would like to express my gratitude to Alf Carling and Åsa Sohlman, former research leaders, both of whom have provided me with a great deal of inspiration and encouragement.

Much of the background material for this thesis was obtained, and preliminary studies carried out, while I was visiting at the Industrial Institute for Economic and Social Research in Stockholm and at the National Bureau of Economic Research in New York. The opportunity of utilising the resources of these two institutions has been of great help to me. I would also like to thank Professor Erik Ruist at the Stockholm School of Economics for his useful comments and advice during earlier phases of my thesis work and for all the time and effort he has given to help me along the way.

The final version of this thesis has benefitted considerably from the seminars held at the Department of Economics at Uppsala University during the past year. I would like to thank all participants for their contributions to the discussions and for pointing out the many errors I seem to have overlooked. A special note of thanks is due Villy Bergström for his diligent and meticulous reading of various versions of all the chapters. His knowledgeable comments and suggestions have been invaluable in preparing the final version of this thesis.

My greatest debt is to my thesis advisor, Professor Bengt-Christer Ysander. The energy he has devoted to reading and discussing all of my papers, painstakingly correcting my errors and providing me with new ideas and inspiration is over and above what any student could demand. Without his constant encouragement and help in solving my seemingly insurmountable problems this thesis would have remained a collection of disparate working papers.

Hopefully my work reflects his influence, although it would have undoubtedly been better had I the perserverence to follow all his advice.

I would like to acknowledge with appreciation the kind and efficient assistance of Eva Holst with typing and preparing the manuscript, despite its oftentimes rather confused state. My thanks are also extended to Göran Östblom for introducing me to economics in Sweden and for all his help over the years, and to Erik Mellander for many useful discussions and advice on econometric and programming problems. Last, but not least, a special thanks is due Jenny Bancroft and Susan John for helping to create a pleasant atmosphere in which to work during the last phases of writing this thesis. Without their help and moral support it may have remained unfinished.

Finally, I would like to greatfully acknowledge the financial support of the Swedish Energy Research and Development Commission during the years I have worked on this thesis and on the research leading up to it.

Uppsala in April, 1987

Joyce Dargay

I. INTRODUCTION

The initial motivation for the studies presented in this volume stems from questions raised concerning the effects of the dramatic energy price rises in the 70's and how these could be studied on the basis of econometric models. In particular two questions pertaining to the industrial sector were posed for investigation. The first had to do with the effects of the price rises on energy demand. The manufacturing sector accounts for nearly 40% of Swedish energy utilisation. It is clear that the extent to which industries could reduce energy use in response to the price increases is of crucial importance in determining Sweden's dependence on imported energy sources as well in forecasting future energy requirements. The second question had to do with the effects of the price rises on production itself and consequently on economic growth. If industry could adjust smoothly and rapidly, higher energy prices could be absorbed with minimal effects on production. Energy costs would not increase in proportion to energy prices since a less energy-intensive production technique would be used. On the other hand, if adjustment possibilities are limited or if the adjustment process is slow, the full extent of the price rise would be felt on production costs—at least in the short run. How this affects the demand for Swedish goods or the profitability of Swedish firms depends on whether the price rises are international or domestic and on the ability of competing firms to adjust to the price increases.

In order to examine these adjustment possibilities it is necessary to have some conception of the role of energy in production. It is obvious that it is the services that energy performs—heat, light and motive power—that define its use in the production process. These services can, of course, only be realised in conjunction with some sort of capital equipment, be it a light bulb or a blast furnace. By the same token, the services of capital equipment can only be actualised in connexion with labour or energy, or more generally both. In an historical perspective, the role of energy in production is apparent: man's ability to harness energy sources led to an increased mechanisation of production which in turn has resulted in an increased energy demand and an increase in labour productivity. The mechanisation of production which began with the industrial revolution and still continues today can be interpreted as the long term substitution of both capital and energy for labour. The extent to which this apparent factor substitution has been induced by changes in relative factor prices as opposed to being the effect of an autonomous, biased technical progress is of prime importance for determining the effects of further factor price changes on factor demand.

Because of the complexity of relationships defining the production process and the interrelatedness of the various inputs, it can be argued that it is fruitless to attempt to analyse the demand for a single production factor in isolation from the production process. This has been the stance taken in the majority of empirical studies of industrial energy demand from the mid-seventies to the present day.[1] With the dramatic oil price hikes providing a particularly good potential for studying factor substitution and adjustment mechanisms and with the crucial importance of these questions creating an impetus for research efforts, the development of multifactor production models suitable for econometric estimation has progressed rapidly during the last decade. This development was aided by the increased use of cost functions rather than production functions for empirical work and the specification of these by 'flexible functional forms' which came into favour in the early seventies. A multitude of empirical studies have followed in the wake, predominately based on Berndt and Wood's KLEM[2] categorisation of production factors but on various disaggregates of these as well.[3]

The majority of these models were static by nature, derived from the assumptions of, and hence defining production relationships in, full equilibrium. Static models continue to form the basis for most empirical studies of production today, despite the exceedingly stringent assumptions required and the recognised inconsistencies of applying equilibrium models to historical data. The most blatant problem concerns the implicit assumption of the instantaneous adjustment of physical capital. This is not only a questionable assumption on the grounds of it being an over-simplification of reality, it is also contrary to the economic theory of investment and the empirical evidence obtained on the basis of investment models. The next stage of model development took the obvious route: by recognising the imperfect flexibility of physical capital and incorporating dynamic adjustment mechanisms, the investment process was implicitly or even explicitly allowed for in the derivation of production and factor demand models.[4]

By recognising the possibility of disequilibrium, these models provide not only a far more realistic, but also a much more theoretically justifiable and

[1] See Berndt and Wood (1975) for one of the earliest examples of such studies.

[2] Instead of resting on the value added concept of production, these studies take all inputs into consideration. KLEM denotes the production factors included: capital (K), labour (L), energy (E) and materials or intermediate goods (M) which designates all else.

[3] For example, L can be disaggregated into skilled and unskilled labour; K into equipment and structures.

[4] Allowing for disequilibrium in empirical factor demand models is not a new idea. The literature on labour demand provides numerous examples. The innovative work of Nadiri and Rosen (1969), however, is one of the first examples of incorporating the notion of disequilibrium in an interrelated factor demand model.

richer basis for studying factor demand and adjustment mechanisms. Differences in short- and long-run relationships are given a concrete meaning in that they are motivated in terms of the inability to rapidly adjust certain inputs, particularly physical capital. The ability to analyse adjustment possibilities and price effects in both the short and the long run brings us a step closer towards understanding the dynamic reality of the economy.

The object of the studies collected in this volume is twofold. The first is to investigate the factors influencing input demand. As opposed to most earlier studies of factor demand in Swedish industries, which have taken the "value-added" approach considering only capital and labour inputs, all factors of production are considered simultaneously. The analysis is based on four aggregate inputs: labour, capital, energy and materials. Energy is treated as an input separate from other intermediate goods because of its unique price development and because of the effects this has been shown to have on the economy.

One of the major questions addressed concerns the price-sensitivity of demand for various inputs. To what extent do firms react to relative factor price changes? How can this response be explained in terms of the substitution possibilities amongst inputs? How do these vary from input to input and from industry to industry? Is adjustment to changes in relative factor prices achieved rapidly or are long time delays involved?

Another question concerns the effect of technical progess. What influence has technical change had on the use and productivity of different production factors? Has it affected factor-mix or has it been neutral in this respect? Have the consequences of technological development been similar in all industries or are there notable differences? Is it possible to distinguish between factor substitution arising from autonomous technical progress from that induced by changes in relative factor prices?

The other objective is to assess the performance of different types of models in analysing production relationships. All empirically tractable models are of necessity based on numerous simplifying assumptions, many of which are exceedingly difficult, if not impossible, to test. Most models can, however, generally be extended in one direction or another in order to relax specific assumptions that seem particularly questionable in a given application or that appear not to hold in the empirical data sample. By introducing more realism and detail, the models will become more difficult to analyse on the basis of the available statistical data. The complexity of the relationships we hope to investigate may be blurred by aggregation or the approximations used in constructing the data. This may limit the fruition of attempts to incorporate a richer economic structure in econometric models.

The direction of investigation chosen here is the same as that described earlier. The point of departure is a full static equilibrium model, in which all inputs—including physical capital—are assumed to adjust instantaneously and costlessly to changes in relative factor prices and output demand. Subsequently, more realism is introduced by allowing for the imperfect flexibility of capital.

The outline of this study is as follows. Chapter II examines the development of factor prices, factor usage and output in Swedish manufacturing sectors during the post-war period. The object of this historical overview is to aid in the interpretation of the empirical results in the following chapters and to provide a description of the underlying statistical data. A detailed discussion of data sources and variable construction is contained in the appendix. Readers familiar with this material may wish to move directly on to the next chapter, returning as the need arises.

A survey of factor demand models is found in Chapter III. Both static and dynamic formulations are reviewed and examples of empirical applications are discussed. The choice of models for empirical implementation is taken up in the concluding pages.

The following three chapters contain the econometric applications of the various models to total Swedish manufacturing and to nine manufacturing subsectors. With a full static equilibrium model as the point of departure in Chapter VI, the models estimated in both subsequent chapters relax the assumption of full static equilibrium by allowing for the imperfect flexibility of the capital stock. The results of a partial static equilibrium model are discussed in Chapter V. The question of the short-run inflexibility of labour is also addressed, and an attempt is made to estimate a model in which both capital and labour are treated as fixed in the short run. Finally, a dynamic cost of adjustment model is specified and estimated in Chapter VI. The empirical chapters are organised as follows: first, the econometric model is formulated and elasticity measures derived, the estimation method and empirical results follow and finally the implied elasticities are discussed. An exhaustive comparison of the results with those obtained in other studies is not attempted. Instead, reference is made to some of the relevant studies in the discussion of the empirical results in each chapter.

In the final chapter, the various models are compared and evaluated. A discussion of the shortcomings of and the questions raised by the empirical analyses concludes the chapter.

II. FACTOR USAGE AND FACTOR PRICES IN SWEDISH MANUFACTURING INDUSTRIES

In order to estimate the production models refered to in the previous chapter, one must rely on available data on factor usage, factor prices and other relevant variables. Before embarking on an indepth discussion of the theoretical models which can be used in factor demand analysis it is advantageous to determine exactly what sort of data is at our disposal. There is a clear relationship between model development and the availability of empirical data. Simple models generally place little requirements on data, as these can generally be estimated on the basis of a few variables. The more economically realistic and the more complex the theoretical model, the more stringent the requirements on the statistical material which is to serve as a basis for the estimation. Far more variables may be needed and the quality of the data may need to be better as more information must be squeezed out of them.

Since aggregate time-series data are most easily accessible, we limit ourselves in the following chapters to models which can be estimated on the basis of such data. Although some of the models, and particularly the static ones, may be better suited for firm data, the compilation of such data is in itself a monumental task and beyond the scope of the present study. It would, of course, be a most valuable excercise to compare the results of a model estimated on both time series and cross section data for a particular production sector, but this must be left to the future.

In the following, we will examine the statistical data that are used in the empirical studies presented in the following chapters. Total manufacturing as well as nine manufacturing subsectors are considered. The subsector classification, a detailed description of the data sources and the construction of the price and quantity series are contained in the appendix of this chapter. The remainder of the chapter is devoted to a graphical presentation and discussion of this data. Since we shall attempt to explore trends in factor usage with reference to trends in factor prices, we will begin with a description of the development of factor prices.

2.1 *The Development of Factor Prices*

The nominal prices, constructed as detailed in the appendix, of energy (E), labour (L), capital (K) and materials (M) in total manufacturing for the years 1952—83 are shown in figure 2.1. All prices are normalised to equal 1.0 for 1967, the approximate mid-point of the observation period. As expected, the nominal prices of all production factors increase over time. During the 50's and 60's the increase is rather gradual, reflecting the low over-all rate of inflation characteristic of this period. During the 70's and 80's, the average rate of inflation increased to nearly 10% per annum from an average of under 3% in the previous two decades. It is obvious from the diagramme that the prices of production factors displayed the same tendency.

Up until about 1973, we find a rather smooth development in factor prices. The small fluctuations in the user cost of capital during this period are primarily explained by changes in the real interest rate as a result of differences in the annual rate of inflation. From 1952—62, the nominal prices of energy, labour and capital increased more rapidly than the price of manufacturing output, which rose by only 1% annually during this period. Unit labour costs rose by about 7% per year, the user cost of capital by 5% per year, while aggregate energy prices and the price of materials increased in nominal terms by 1.7% and 0.4% per annum respectively. The rise in energy prices was mainly due to an increase in the price of electricity, as even the nominal price of fuel oils decreased during this period.

A similar factor price development is noted for the following decade, 1962—72, when the price of manufacturing output rose by an average of 2.7% annually. Labour costs increased by 10% per annum, while the rate of increase in capital costs was much the same as in the previous ten year period. Aggregate energy prices rose at an annual average of only about 1.2%, which marks a fall in real energy prices. This is explained mainly by a sharp decrease in nominal electricity prices during the early 60's.

To summarise the development up until 1973, we can say that the price of labour rose relative to all other inputs, while the price of capital increased with respect to the prices of energy and materials. The price of energy rose in comparison to the price of materials during the 50's, while the reverse was true for the 60's.

After 1973, and the first oil crisis, the picture changes considerably. Not only does the price of energy rise, but the prices of all inputs increase at an unprecedented rate. Over the period 1972—83, the aggregate energy price increased by an average of around 18% per year, the most substantial part of this being attributed to the dubbling of oil prices in 1974 and the annual oil price rises of around 30% in 1978, 1979 and 1980. During the same period, electricity prices went up by on an average of 13% per year.

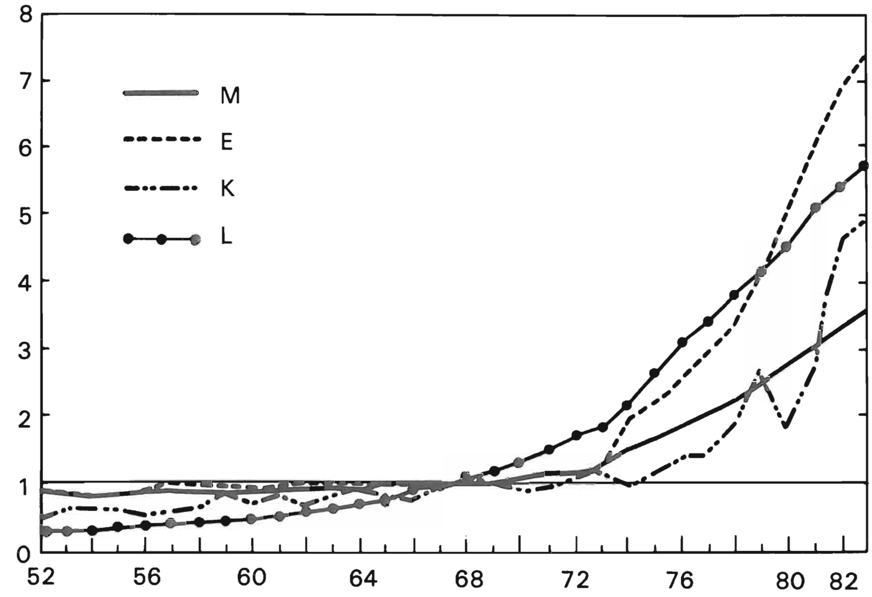

Figure 2.1. *Total Manufacturing*. Factor Prices 1967 = 1.

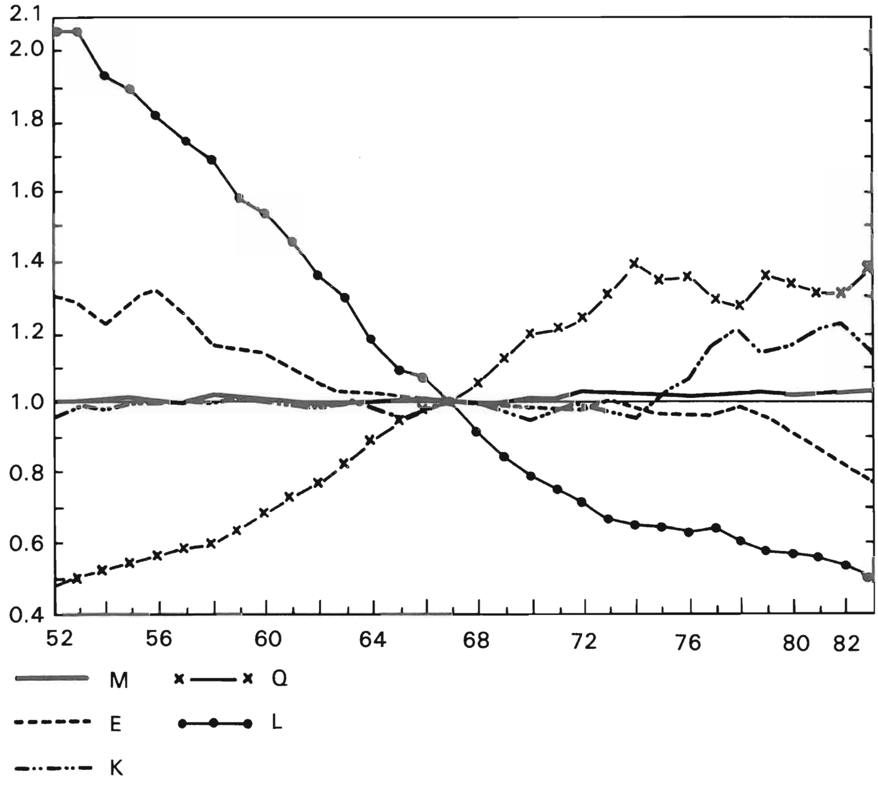

Figure 2.2. *Manufacturing*. Output and Input-Output Ratios. 1967 = 1.

Even the user cost of capital increased considerably—by about 14% annually—during this period taken as a whole, primarily as a result of the comparatively high real interest rates in 1982 and 1983. A combination of higher nominal interest rates and a lower rate of inflation than was the case for the previous two years caused real interest rates to reach unprecedented levels.

Variations in the user cost of capital over time are determined by three factors: changes in the price of investment goods, changes in the nominal interest rate and changes in the rate of inflation (see equation 2.2 in the appendix). From 1972—83, the price of investment goods increased by about 11% annually, as compared to 3% for the previous two decades. This more-or-less followed the general price development during these periods. Nominal interest rates rose to levels of over 10% during the 70's and 80's, compared to rates of less than 6% in the 50's and 60's. Inflation increased substantially, however, as well, so that negative real interest rates are noted for the latter 70's.

Labour costs continued to increase during the 70's and 80's, nominally by about 12% annually, but in real terms far less than in the previous two decades. The real price increase was reduced from an average of 7% per year in the 50's and 60's to less than 2% during the 70's.

The price development noted above has led to a change in relative factor prices from the previous decades. The price of energy rose relative to all remaining inputs, while the price of capital rose relative to labour and materials. Again, however, we find that the price of material inputs increased less rapidly than the other aggregate inputs considered.

The price development illustrated in the diagramme and discussed above pertains to inputs in total manufacturing. Although the rates of change of factor prices vary somewhat among the individual industries, the same general trends in relative factor prices are evident in all of the subsectors, so that it would be redundant to present the branch specific prices diagrammatically or to discuss them in any detail. One observation should, however, be made about the development of aggregate energy prices since 1973. Although this varies a bit for the individual industries, there is no clear cut relationship between price increase and energy-intensity. Up until 1975 we find that aggregate energy prices rose more in the energy intensive industries than in others.[1] For the period up until 1983 the differences diminish substantially, and in most instances disappear totally. Although electricity prices rose more rapidly for energy intensive sectors over the entire period, the primary differences in the development of aggregate energy prices among sectors is explained by differences in the composition of energy usage.

[1] The differences in the development in energy prices among industries for the period 1968—75 are examined in Dargay (1983a).

2.2 *Production and Factor Usage*

To explore the trends in the use of different inputs in production, we will look at the development of input-output ratios over the last thirty years in total manufacturing as well as in the various subsectors. While doing so, it would be helpful to have a clear conception of how output has evolved over this period. Our discussion of factor usage in the various industries will therefore include a description of the development of real output.

2.2.1 Total Manufacturing

In figure 2.2, the inputs of energy (E), labour (L), capital (K) and materials (M) per unit output (Q) in the manufacturing sector are shown along with real output for the years 1952—83. All data series are normalised to equal 1.0 in 1967. We find that output increases more or less continually until 1974. In the first two decades output increased at an annual average rate of 4.6% and 4.8% respectively. During this period, the effects of the business cycle are noticable, but minimal. After 1974, and the first oil crisis, the picture is totally different. The resulting recession is far deeper and of longer duration than those experienced earlier. We find that it took until 1983 before output finally reached the same level as it was in 1974. The troughs of the post -73 recession are obvious, marked by a decline in real output in 1977—78 and 1981—82. After the attempt at recovery in 1979, the economy was thrown into recession once again by a new wave of oil price shocks, only to commence recovery again in 1983.

Regarding the development of input-output ratios, the most striking feature is the dramatic decline in labour-intensity over the entire period. Labour productivity increased by an average of 4% per year from 1952—62 and by 6.3% per year during the next 10 year period. After 1974 we find virtually no change in productivity growth until 1978—79, when a renewed increase is noted. Productivity growth, however, soon begins to approach zero again in the 80's, only to increase in 1983, the last year of the data sample. From 1972—83 the average annual decrease in labour intensity was approximately 3%. The decline in labour-intensity over the entire period corresponds rather well to the increase in real labour costs noted previously.

Regarding the diagramme, we find a strong inverse relationship between labour-intensity and output level, particularly for the pre-1974 period. It appears also that the post-74 recession has had a negative effect on labour productivity growth. Particularly, we find that labour productivity only begins to increase when output rises.

This is an indication that the labour-output ratios for this period do not reflect long-term productivity, but rather short-term under-utilisation of labour. This may be due to labour-hoarding on the part of firms or simply their inability to cut employment in response to production decreases. Strong trade unions and job security legislation in Sweden make the latter a clear possibility.

The development of the energy-output ratio is also shown in the figure. A decrease in specific energy use is evident for the period taken as a whole, with the greatest decrease occurring in the latter 50's and in the post-1978 period. It is apparent, however, that the decline in energy-intensity during the period 1952—78 is rather minimal in comparision to the dramatic fall in labour-intensity. During the period 1952—72 energy intensity decreased by about 1% per year, which can hardly be explained in terms of relative factor price changes as energy prices decreased relative to the prices of all other inputs. From 1978—83, on the other hand, specific energy use declined more rapidly than the input-output ratios for the other production factors as a result of the enormous relative energy price rises. Interpreting changes in energy intensity in terms of price changes, it would appear that the 1979 oil price increase has had a greater impact on energy demand that did the events of 1974. However, we may also be witnessing a lagged response to the earlier price increases.

The development of specific energy use can be better explained in terms of the use of individual energy forms. During the 1950's, the specific use of electricity and oil products rose substantially. This increase was, however, more than compensated for by a 50% decrease in the use of solid fuels. Oil consumption per unit output increased at a slower rate during the 60's and by 1970 had begun to fall. Between 1973 and 1974 specific oil use fell sharply in response to the exceptionally large price increases and shortages associated with the first oil crisis. This downward trend continued through 1975, after which the oil-output ratio remained more or less constant until a new and far more dramatic drop in response to the oil price hikes of 1979—80. Electricity usage follows a somewhat different development. After a rapid increase in the 50's the specific use of electricity remained at a constant level during most of the 60's. From the mid-seventies there is once again a trend towards increasing electricity intensity. This may, however, partially be a reflection of the low capacity utilisation associated with the post-74 recession.

Finally, we find that energy intensity bears little relation to production level. The troughs of the post-1974 recession are not as obviously marked by an increase in energy intensity as was the case with labour, because of the comparative ease with which energy can respond to changes in production level.

Regarding capital, we see from the diagramme that the capital output ratio has remained more-or-less constant until the early 70's, while an apparent increase is found for the remainder of the period. For the period 1972—83 capital intensity increased by an average of 1.3% per annum. This cannot easily be explained in terms of relative factor prices, as the user cost of capital rose in relation to labour costs. It did, on the other hand, decrease relative to the price of energy, so we may be witnessing a substitution of capital for energy. Although industry may actually have become more capital intensive, it is likely that much of this increase in the capital-output ratio can be explained in terms of the low production levels associated with the post-74 period. The pattern after 1974 follows exactly, but of course inversely, the peaks and troughs in real output, which is a rather strong indication that what we are actually observing is an under-utilisation of the capital stock.

Concerning the use of intermediate inputs per unit output, we find little variation over the observation period, suggesting the existence of a Leontief-type technology for the aggregate level of these inputs. We see, too, that there is little, if any, correspondence between material-output ratios and production level. As would be expected intermediate inputs respond quickly to output changes, so that the post-74 recession has no noticable impact.

The general trends of the input-output ratios depicted in the figure can partially be explained in terms of the effects of factor substitution and technical change. Although a complete analysis of the factors influencing factor demand requires a model which considers all explanatory factors simultaneously, a few tentative observations could be made solely on the basis of the data at hand. Firstly, a continual rise in the energy-, capital- and material-labour ratios is apparent for the period 1952—73, which is clearly a reflection of the substitution of energy, capital and materials for labour in manufacturing production. During this period the price of labour rose relative to all other inputs. It would appear too that the availability of cheap energy has led to the introduction of less labour-intensive capital equipment. After 1973 the capital-labour ratio continues to increase, while the energy-capital and energy-labour ratios decline. Since the price of energy rose more rapidly than the prices of labour and capital during this period, this can reflect a substitution of capital—and perhaps even labour—for the now relatively expensive energy.

Finally, we note that the inputs of capital, labour, energy and materials increased at a slower rate than output during the 50's and 60's. Technological development seems to have led to an increased efficiency in the use of all production factors. After 1973, the development is somewhat different. Specific energy use fell more rapidly than the labour-output ratio while production appears to have become more capital intensive. These observations, and particularly the noted rise in the capital-output ratio, must be interpreted with caution, however. The period after 1973 is one of economic recession. Manufacturing output has risen much more slowly than previously and has even decreased in some years, so that plants have not been operating at full capacity. Long-run changes in factor usage—at full capacity utilisation—are most certainly quite different than those observed here.

2.2.2 Manufacturing Subsectors

The observed changes in input-output ratios in manufacturing reflect not only an increased efficiency in factor usage and substitution amongst inputs but also the changing composition of manufacturing output. The last three decades have witnessed considerable variation in output growth in the individual industries. A number of industries—Food, Textiles, Printing and Non-Metallic Minerals—grew less rapidly than total manufacturing so that the shares of these sectors in total manufacturing output decreased. The most substantial changes have been in the Textile and Food industries which decreased from about 9.4% and 21.6% of manufacturing output respectively in 1952 to 2.4% and 12.7% in 1983. The greatest increases in output shares over the period taken as a whole are attributed to the Chemical, Engineering and Primary Metal industries. Taken together, these sectors accounted for 31% of manufacturing output in 1952. By 1983, the combined share had risen to nearly 50%. As the individual sectors have specific production processes, and hence specific input requirements, at least some of the changes in factor demand noted for total manufacturing can surely be attributed to structural change.[2]

In order to distinguish between the influence of changes in product composition and changes in factor use, it is essential to study the development of input-output ratios on a more disaggregated level. In the following we shall examine the 9 subsectors of the manufacturing industry defined in table 2.A.1 in the appendix. The development of real output and the input-output ratios are shown in figures 2.3—2.11, which are constructed in a similar way as figure 2.2 above. These figures shall only be commented upon briefly.

Regarding production, we find that nearly all sectors have experienced a continual growth in real output up until somewhere between 1973 and 1976. The only exception is the textile industry which began contracting by the end of the 60's. Further, we find that the cyclic pattern of the post-73 recession noted for total manufacturing is apparent in all sectors. After the first oil crisis, output reaches a minimum sometime between 1974 and 1978, begins an upswing a year or so later and peaks between 1979 and 1981. A new recessiom follows the oil price hikes of 1979—80, and output reaches a minimum once again between 1981 and 1982. Finally, 1983 marks a renewed recovery in all industries and output once again reaches its pre-recession level. The only exceptions here being the Textile, Rubber and Mineral industries in which production has yet to reach pre-1973 levels.

Regarding the trends in input-output ratios in the individual industries, we

[2] See Östblom (1986) for an analysis of the effects of structural change on input demand.

find a certain degree of similarity to the development noted earlier for total manufacturing. In particular, all industries show a substantial decline in the labour-output ratio over the entire observation period. As was the case with total manufacturing, labour productivity growth was somewhat greater during the 60's than in the 50's. In the early 70's, however, growth begins to slow down substantially, so that during the last 10 year period labour productivity growth had declined to about half of its previous rate.

The increase in labour productivity varies somewhat across industries. For the pre-1973 period we find that growth was exceptionally rapid in the Paper and Pulp and Primary Metal industries, while below average growth was experienced in the Food and Printing industries. In most sectors, slowdowns in productivity growth—or in some cases even absolute reductions in productivity—are noted during the seventies, which generally correspond rather well to the troughs in production during the post-1974 recession. Again we find a tendency towards labour hoarding or a sluggishness in the adjustment of labour.

Energy-intensity has also decreased in the majority of industries over the entire period. The most obvious exception is the printing industry in which energy-intensity has risen substantially. As in total manufacturing, energy-intensity decreases most rapidly during the 50's and the period subsequent to 1974. Again, the 1979—80 oil price hikes seem to have had the most appreciable effects on energy demand.

The development of the capital-output ratio shows the greatest dissimilarites amongst industries. For the period 1952—72, capital-intensity increased in the Food, Textile, Pulp & Paper, Printing and Rubber industries, while the opposite is noted for Chemicals, Primary Metals and Engineering. The most rapid growth in capital-output ratios during this period has been experienced by the Food, Printing and Rubber industries, all of which have decreased their shares in total manufacturing output.

During the post-74 recession, a rise in the capital-output ratio is apparent in all industries. Once again the time pattern corresponds inversely to the cyclical output pattern, suggesting that much of the rise in the capital-output ratio can be explained by the low capacity utilisation associated with this period.

Finally, the development in materials-output ratios varies among industries, but in all cases the changes are rather minimal in comparison to those noted for the other factors of production. The only sector which displays a continual increase is the Food industry, while both the Printing and Mineral industries show a significant increase during the 50's. For most other sectors, there are no significant trends one way or the other. Considering the high level of aggregation of this input and the diversity of goods and services of which it is composed, this is perhaps not surprising.

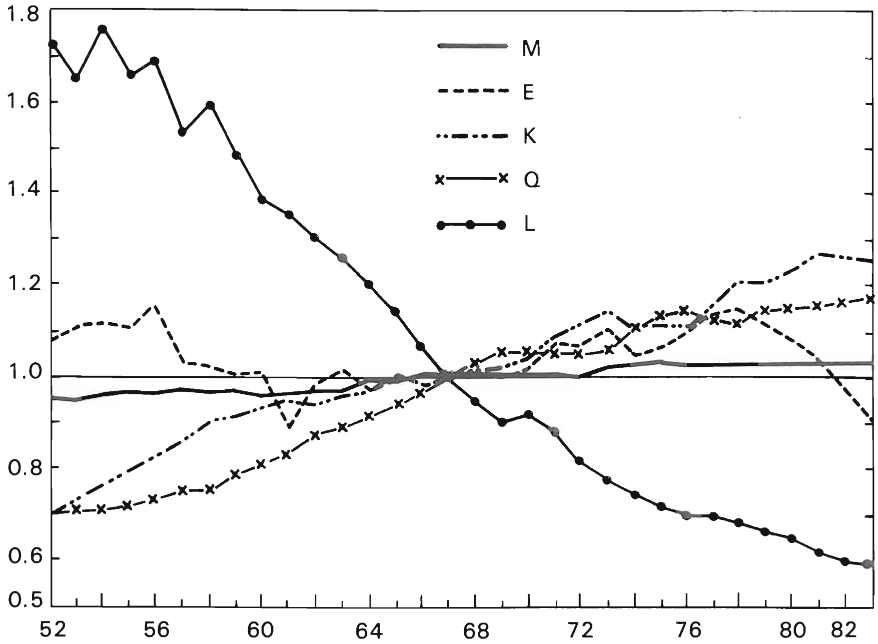

Figure 2.3. *Food.* Output and Input-Output Ratios. 1967 = 1.

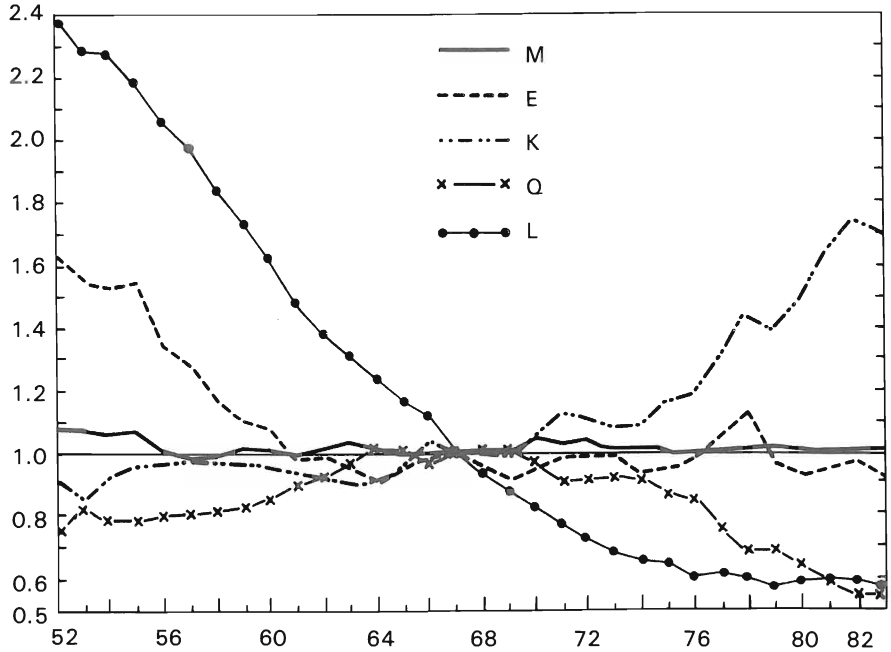

Figure 2.4. *Textiles.* Output and Input-Output Ratios. 1967 = 1.

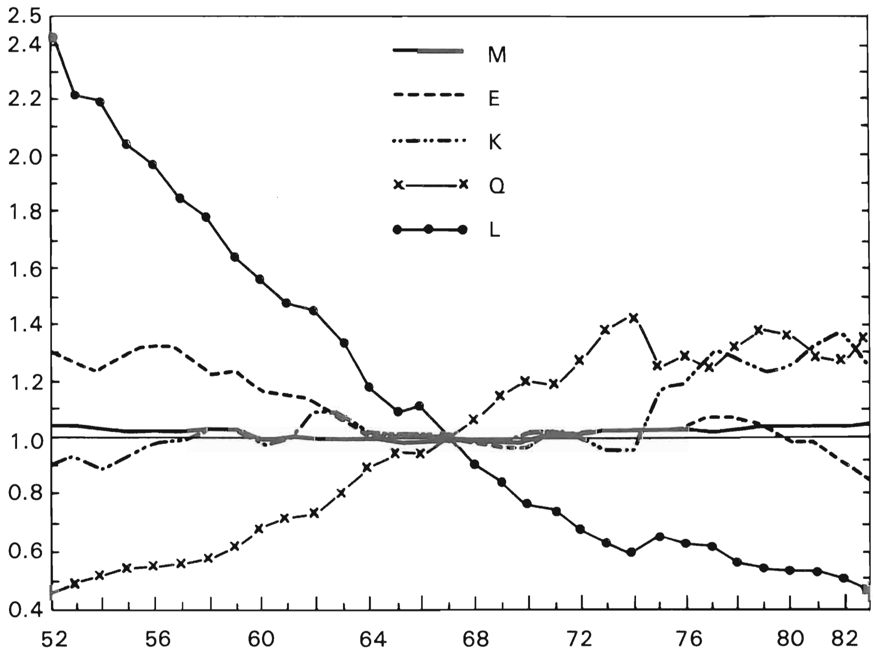

Figure 2.5. *Pulp and Paper*. Output and Input-Output Ratios. 1967 = 1.

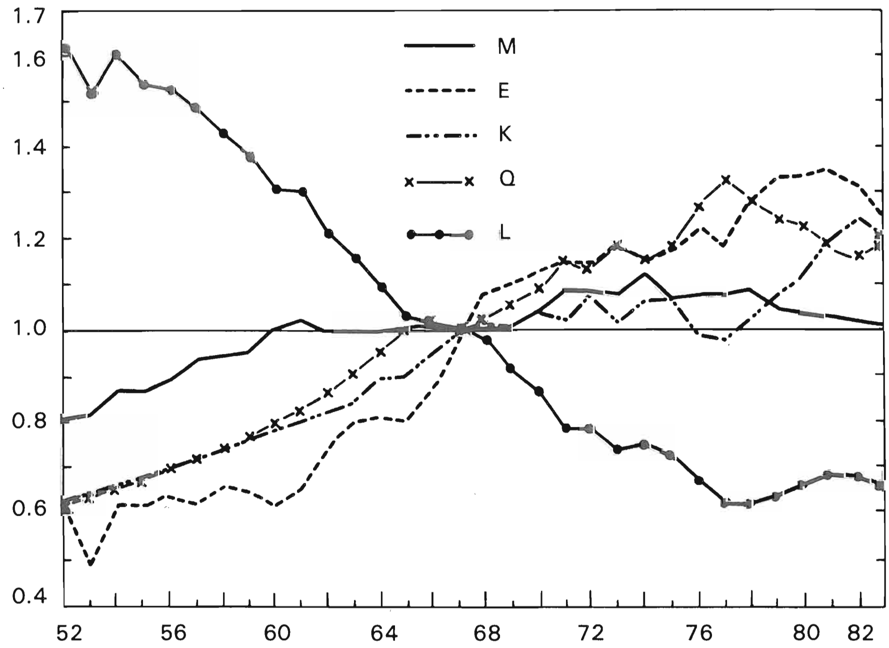

Figure 2.6. *Printing*. Output and Input-Output Ratios. 1967 = 1.

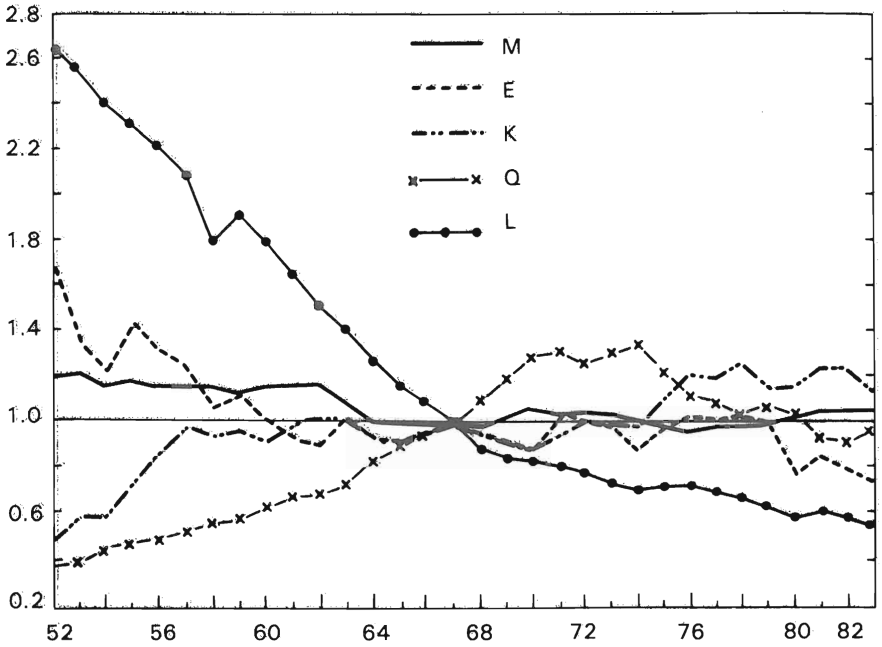

Figure 2.7. *Rubber*. Output and Input-Output Ratios. 1967 = 1.

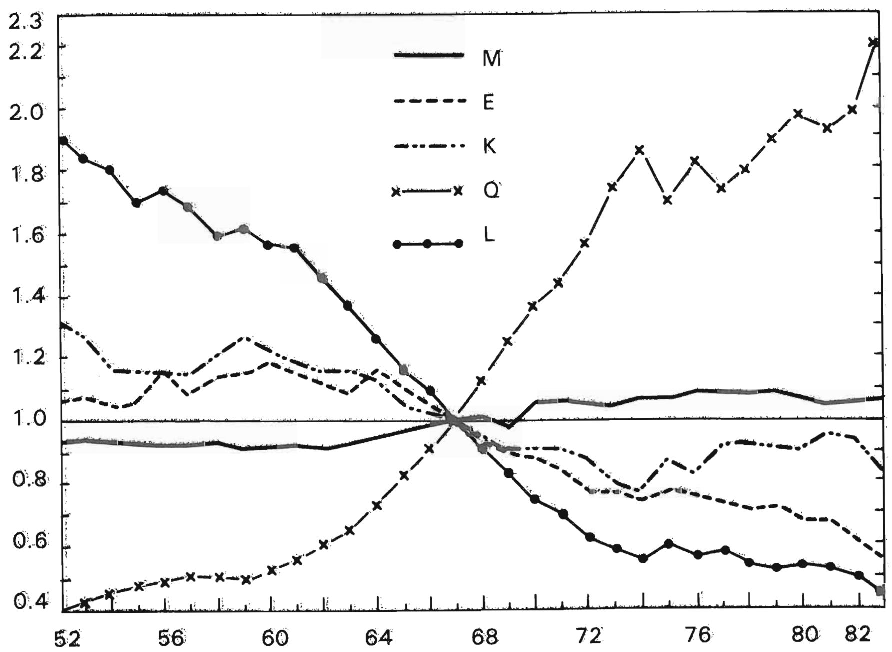

Figure 2.8. *Chemicals*. Output and Input-Output Ratios. 1967 = 1.

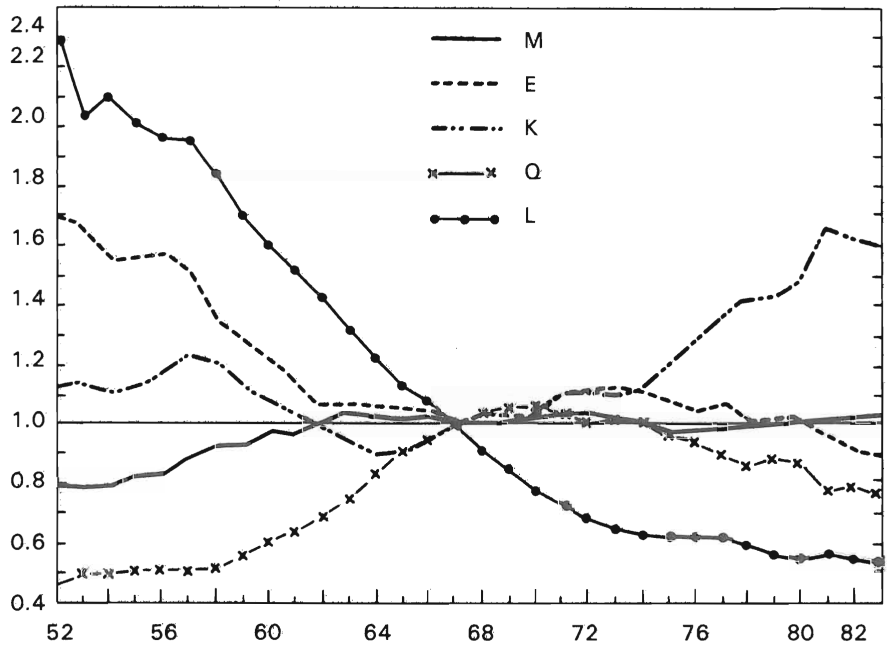

Figure 2.9. *Minerals*. Output and Input-Output Ratios. 1967 = 1.

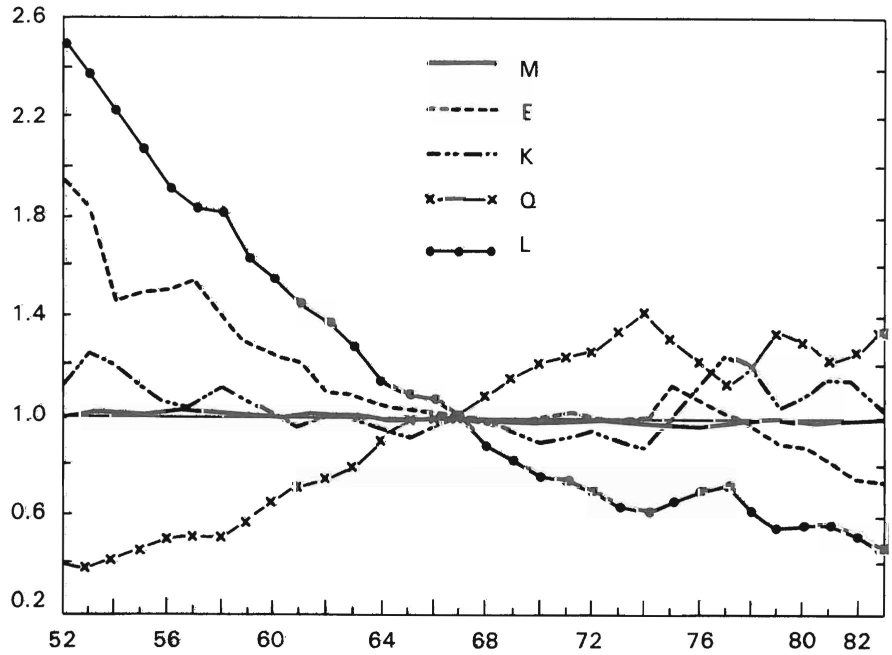

Figure 2.10. *Primary Metals*. Output and Input-Output Ratios. 1967 = 1.

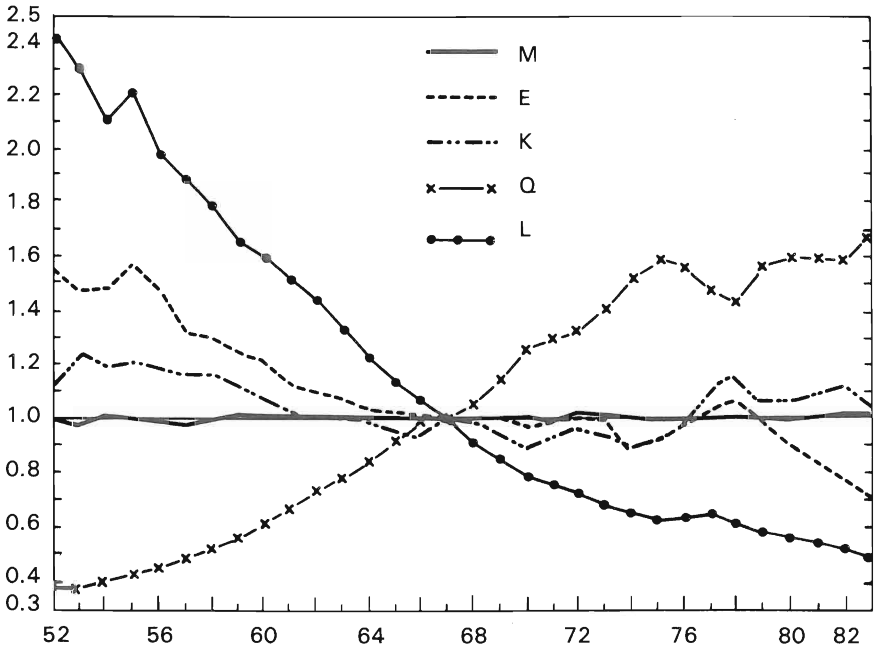

Figure 2.11. *Engineering*. Output and Input-Output Ratios. 1967 = 1.

2.3 *Factor Costs*

Although the diagrammes and discussion of the previous section present a clear picture of the development of the use of each production factor over time, they give us no indication of the relative importance of the individual inputs in the production processes of the various industries. In this final section, we will look at the shares of each input in total production costs. These are shown in table 2.1 for the years 1952 and 1983, the first and last years of our data sample.

Although the shares vary amongst industries, the predominant pattern that emerges is that $S_M > S_L > S_K > S_E$, where S_i denotes the share in total production costs for input i. For the most energy intensive industries—Paper, Minerals and Metals—we find that energy costs were greater than capital costs in 1952. This was, however, reversed by the end of the observation period because of the exceptionally high interest rates. Intermediate goods, which are primarily comprised of agricultural products, account for the greatest proportion of costs in the Food industry. The Food, Chemical and Metal industries appear to be the least labour intensive, while the Mineral and Metal industries tend to use comparatively large amounts of capital. Further, we notice that

Table 2.1. Input Shares.

	Labour		Capital		Energy		Materials	
	52	83	52	83	52	83	52	83
Food	.12	.14	.02	.07	.02	.03	.85	.77
Textiles	.31	.31	.02	.12	.02	.03	.65	.54
Paper	.20	.19	.06	.12	.05	.07	.69	.62
Printing	.39	.31	.03	.08	.01	.01	.58	.60
Rubber	.29	.30	.07	.13	.03	.04	.61	.53
Chemicals	.24	.21	.06	.12	.05	.06	.66	.61
Minerals	.40	.28	.06	.19	.12	.10	.42	.43
Metals	.22	.17	.04	.22	.11	.07	.63	.54
Engineering	.35	.31	.03	.09	.02	.02	.60	.58
Total	.27	.24	.03	.11	.04	.04	.66	.60

the cost shares for capital have risen significantly by 1983, resulting in a decline in the shares for the remaining inputs. The high interest rates, and thus capital costs, of the early eighties are largely responsible. A return to lower real interest rates would certainly change the picture considerably.

APPENDIX: *The Statistical Data*

The data used in this chapter and in the empirical studies in the following chapters have been obtained primarily from Swedish Manufacturing Statistics and National Accounts published by the Swedish Central Bureau of Statistics (SCB). The analysis has been limited to total manufacturing (excluding energy producing sectors) and 9 manufacturing subsectors. The subsector classification used is shown in table 2.A.1.

Table 2.A.1. Sector Classification.

LU	Sector	Swedish industry nomenclature (1952–1967)	ISIC SNI (1968-)
4&5	Food	7	31
7	Textiles	9a-d,f-r,10a-d,i	32
8	Wood, Paper and Pulp	4, 5	33, 341
9	Printing	6	342
10	Rubber Products	10g,h	355
11	Chemicals	9e,11a-d,g-m	351,352,356
13	Non-Metallic Mineral Products	3d-k	36
14	Primary Metals	2a, b	37
15	Engineering	2c-e,g-i,l,m	38 excl. 3841

In addition to these industries, Total Manufacturing also includes Beverages and Tobacco (SNI 313—314), Shipbuilding (SNI 3841) and Miscellaneous Manufacturing (SNI 39).

The data include annual observations on production volume and on the quantities and prices of four inputs: labour, capital, energy and intermediate goods. The time period under consideration covers the years 1952—83. With the exception of the capital variables, the data sources and variable construction are primarily the same as those used in my earlier studies.[3] The data have, however, been updated and revised in accordance with revisions done by SCB as of 1985.

Production Volume. Data on gross production in producers' prices, both current and constant, for each sector are obtained from the National Accounts of Sweden (SCB). Output is defined as gross production in constant prices. The output price index is taken as the ratio of the current to constant price series.

Labour. Total labour costs and the number of hours worked in each industry are also taken from the National Accounts. Total labour costs include wages plus social security charges and other wage fees and taxes paid by employers. The price of labour is defined as the total labour cost divided by the number of hours worked.

Capital. The definition of capital itself and the construction of variables pertaining to capital present considerable problems. Basically the data needed are measures of the capital stock and the user cost of capital. Neither of these can be taken directly from published material. The data on the capital stock available in the National Accounts are of little use to us. These data pertain to gross capital, which is calculated by allowing each individual capital object to be included in the total stock at full value over its entire life-time. As this definition ignores physical and economic depreciation, it provides a poor measure of the actual capital available for productive purposes. The net capital stock, which takes this depreciation into consideration is more appropriate for our purposes. The Swedish Bureau of Statistics has provided unpublished material on this measure of capital for the period 1950—1983.

A widely used manner of calculating the capital stock is the perpetual inventory method. Here, gross investments are accumulated over time taking into account the depreciation of older capital. The capital stock is calculated as

$$K_{t+1} = (1-\delta) K_t + I_t \qquad (2.1)$$

[3] See Dargay (1983a).

where K_t is the capital stock at the beginning of period t, I_t is gross investment during the period and δ is the depreciation rate. Given information on the rate of depreciation, a series on gross investment and the value of the net capital stock for a particular year (benchmark), the net capital stock can be constructed for any time period. This is in principle the method used by the Statistical Bureau in constructing the capital stock. Of course, this is done on a very dissaggregated level using investment data and survival curves for individual types of capital. For this reason, the depreciation rate of the aggregate capital stock is not a constant, but instead varies reflecting the changes in the composition of capital.

Besides a measure of physical capital, we also need some measure of its price or user cost. The simplest way of defining the user cost of capital, u_k, is

$$u_k = p_i (r+\delta) \qquad (2.2)$$

where p_i is the price of investment goods, r is the rate of return and δ is the rate of depreciation. Capital costs are obtained by multiplying the capital stock by the user cost.

We see that the depreciation rate δ occurs in the definition of both the capital stock and the user cost. In constructing these series it is necessary that the same depreciation rate be used, otherwise the data will be inconsistent. Using the net capital stock data provided by SCB along with data on gross investments, an implicit depreciation rate can be calculated for each year using the perpetual inventory equation (2.1). The user cost of capital can then be calculated from equation (2.2), given data on the price of investment goods, p_i, and the rate of return, r. Of course, the depreciation rate will not be a constant. An alternative method is to calculate an average depreciation rate based on the SCB data, reconstruct the capital stock by the perpetual inventory method using this average depreciation rate, a benchmark taken from the SCB data and gross investment. The user cost is then calculated based on this constant rate of depreciation. Since there are certain benefits in assuming a constant rate of depreciation, the latter method has been chosen.

Data on gross investment in structures and machinery in constant and current prices are taken from the National Accounts. Using the constant price data in combination with SCB's data on net capital stocks, implicit depreciation rates are calculated for each type of capital and for each sector. The average depreciation rates are shown in table 2.A.2. We see that although the depreciation rate for structural capital varies only slightly among industries, the depreciation rate for machinery varies considerably—from 4.9% per year in the Primary Metal Industry to over 14% in the Rubber Industry. Capital intensive process industries—Paper, Minerals and Metals—generally have lower rates of depreciation.

Table 2.A.2. Average Implicit Depreciation Rates.

	Structures	Equipment
Food	2.9	12.0
Textiles	2.8	11.5
Paper	2.6	7.4
Printing	2.6	6.5
Rubber	2.4	14.2
Chemicals	2.5	10.5
Minerals	2.9	6.9
Metals	2.4	4.9
Engineering	2.8	8.5
Total	2.6	8.1

Using these depreciation rates, the data on gross investment and SCB's net capital stocks for the year 1975 as benchmarks, the stocks of structures and equipment are constructed according to equation (2.1). The user cost of each type of capital is then calculated using equation (2.2) with these same depreciation rates and the investment price indicies referred to above. The interest rate on long-term government bonds minus the rate of inflation plus an arbitrary risk premium of 3% to insure positivity is taken as the firm's required real rate of return. As mentioned earlier, equation (2.2) is a rather simple definition of capital costs. Corporate taxation, for example, is not taken into consideration; nor are the effects of investment subsidies. The former appears to have little effect, however, as the resulting price series does not differ vastly from that of Bergström (1982), which does include taxation. As for the latter, the difficulties involved in quantifying such subsidies make a consideration of these hardly possible in the present analysis.

Finally, the aggregate capital stock is the sum of equipment and structures and the user cost of the aggregate is calculated as cost-share weighted averages of the user costs of the two types of capital.

Energy. Quantities and costs of energy consumed in Swedish manufacturing subsectors are taken from the Official Statistics of Sweden: Manufacturing, annual reports 1952—58 (Board of Trade) and 1959—83 (SCB). The data include quantities (1952—83) and costs (1962—83) for individual fuels: motor gasoline, fuel oils, gas oil, coal, coke and wood fuels, costs (1952—83) for aggregate fuels and quantities and costs (1952—83) for electricity. Fuels and electricity produced and consumed at the same plant are not included. Most data pertain to establishments with five or more persons employed.

Expenditures for electricity and total fuels in current and constant prices were also supplied by the National Accounts Department of the Swedish Central Bureau of Statistics. These are based on the Manufacturing statistics above, but in addition include information on establishments with less than five persons employed.

Prices for electricity and each fuel are calculated for each subsector on the basis of the costs and quantities obtained from the manufacturing statistics. Since costs for the individual fuels are not available for the pre-1962 time period, the subsector fuel prices for these years were constructed using average fuel prices for industrial consumers. This was done by assuming that the relationship between sector price and average price for each fuel noted for the 1962—70 time period was the same for 1952—61. Average prices of oils and motor fuels are provided by the Swedish Petroleum Institute while average prices of coal, coke and wood fuels are taken as implicit import prices calculated from the data reported in the Official Statistics of Sweden: Foreign Trade, annual reports 1952—83 (SCB).

The prices for the energy aggregates in each sector are calculated as a cost-share weighted average of the price indicies of the individual energy forms. Aggregate energy quantities are taken as total costs in constant prices deflated by the aforementioned aggregate price index.

Intermediate Goods. Data on costs for goods and services purchased in each sector in current and constant prices are obtained from the National Accounts of Sweden (SCB). Costs for intermediate goods are obtained by subtracting energy costs. Implicit price indicies for intermediate goods are formed by using the current and constant price data adjusted for energy inputs.

III. A SURVEY OF ECONOMETRIC APPROACHES TO PRODUCTION ECONOMICS

Over the years an enormous wealth of econometric studies has been undertaken in order to investigate factor demand and substitution relationships. A variety of empirical models have been developed for this purpose and implemented for different countries and industries. It is the object of this chapter to survey the various approaches to the analysis of factor demand that have been suggested in the economic literature.

The discussion is limited to the group of models which can be characterised as neoclassical production models based on the notion of homogeneous inputs. Vintage models are therefore omitted as are other models allowing for qualitative differences in inputs over time. The models presented are of varying degrees of complexity, from single equation models describing the demand for a single input in isolation from others to full systems of demand equations in which the demands for all inputs are treated simultaneously. Static model specifications describing either short- or long-run demand relationships are discussed as well as dynamic models characterising factor demand in different time perspectives.

The choice of empirical model depends, naturally, on the particular questions one wants to answer and the availability of suitable statistical data. In many cases a trade-off must be made. Models rich in economic content generally require larger and more detailed data bases and are often more difficult to implement statistically. The models presented in the following are all possible to estimate on the basis of the available aggregate time-series data.

In Section 3.1, static model formulations are discussed, while Section 3.2 deals with various types of dynamic models. The choice of models for the empirical analysis of Swedish data is discussed in the concluding pages.

3.1 *Static Factor Demand Models*

3.1.1 Long Run Equilibrium Models

Until quite recently, the majority of studies of the demand for factors of production which have included more than two inputs have been based on static long-run equilibrium production models. The theoretical basis of the static equilibrium model stems from neoclassical production theory—the existence of a production function relating the firm's output to the use of various inputs, the assumption of cost-minimising or profit-maximising behaviour on the part of the firm and the theory of production duality which establishes the relationship between economic variables—such as production costs and factor prices—and production technology. The most common approach has been to specify a cost function relating production costs to the prices of aggregated production factors: energy, labour, capital etc.

By assuming that for a given level of output and given factor prices firms choose that input-mix which corresponds to minimum production costs, demand equations for each input are derived from the cost function. Thus, the demand for each factor of production is considered in the context of the production process and is estimated simultaneously with the demand for other inputs. Because of the duality between production technology and production costs, the technology underlying the economic model—the substitution possibilities amongst inputs, economies of scale, production elasticities etc.—can be reconstructed and derived in terms of the estimated coefficients.

One begins by assuming that technology can be represented by a production function which relates the input of aggregated production factors, X_i, $i = 1$ to n, to a unique maximum quantity of output Q

$$Q = F(x) \tag{3.1}$$

where x is a vector of the n inputs X_i which are assumed to be continuously variable and substitutable in the production process. As mentioned above each production factor is in fact an aggregate of a variety of inputs. For example, capital can be composed of various sorts of machines and buildings and labour can represent both skilled and unskilled labour. In specifying the aggregate inputs, it is implicitly assumed that the production function is weakly separable in the X_i aggregates, that is to say, that the marginal rates of substitution between individual types of X_i are independent of the composition of the remaining aggregated inputs X_j. The existence of aggregated inputs thus rests on these being linearly homogeneous functions of their individal components.

Assuming the production function is a continuous, single-valued and twice-differentiable function of the input quantities, the following must hold for the production function to be a description of a reasonable technology

$$\partial F/\partial X_i > 0 \qquad \partial^2 F/\partial X_i^2 < 0. \tag{3.2}$$

The signs of these partial derivatives ensure that the marginal products of each factor are positive and decreasing.

The production function merely summarises the efficient production possibilities open to the firm. In order to apply the production function to economic data and to study the relationship between economic variables, such as factor prices, and factor utilisation, additional assumptions must be made concerning the firm's economic behaviour. This is done by assuming that firms are either profit maximising or cost minimising. In the first instance, we assume that the firm is a price-taker in both output and factor markets. The firm's profit optimisation problem is to choose the levels of output and inputs which maximise revenues minus costs given the technological constraints contained in the production function.

In the second instance we need only assume that the firm is a price-taker on input markets. The optimisation problem facing the firm is to choose its input levels so as to minimise the total costs of producing a given, exogenously determined, level of output. Again, this is done subject to the technological constraints represented in the production function. Although one can use either the profit-maximisation or cost-minimisation approach, the assumption of cost-minimisation is most commonly chosen for empirical studies. The reason for this is basically that data on costs are more reliable than those on profits and that one need not make assumptions concerning the determination of output level or output price. In the following we will thus limit ourselves to the cost-minimisation approach.

Under the assumptions stated above, the optimisation problem facing the firm can be written

$$\begin{aligned} &\text{Min } p'x \\ &\quad \text{s.t. } Q = F(x) \end{aligned} \tag{3.3}$$

where p is a vector of factor prices, x is a vector of factor quantity levels and F is the production function in (3.1). The first order conditions characterising the solution to this problem are

$$p = \lambda \, D \, F(x) \tag{3.4}$$

where λ is the Lagrange multiplier of the constraint and D F(x) is the gradient vector of F at the optimal factor levels x^*. If one substitutes for λ this condition reduces to the requirement that the technical rate of substitution between each factor pair is equal to the ratio of their respective prices. Finally, the second order condition for cost minimisation requires the production function to be locally quasi-concave.

Given a particular functional form for the production function, one can solve the minimisation problem and derive the cost function which contains all the economically relevant characteristics of the underlying technology. The cost function relates minimal production cost, C, to input prices and output level.

$$C = G(p,Q) \tag{3.5}$$

The cost function will be positive, continuous, nondecreasing, linearly homogeneous and concave in factor prices and nondecreasing in the level of output.

Although the cost function can be derived from the production function in the above manner, one generally begins by specifying the cost function (3.5) directly and using the duality between production costs and technology to derive the characteristics of the underlying technology. It can be shown that if the cost function satisfies the conditions stated above it must necessarily arise from some technology.

By applying Shephard's Lemma,[1] the cost-minimising factor demand equations can be derived by partial differentiation of the cost function with respect to input prices

$$\partial C/\partial P_i = \partial G(p,Q)/\partial P_i = X_i\,(p,Q). \tag{3.6}$$

Factor substitution, price-responsiveness, economies of scale and the effects of technical progress can be studied through the cost function. The most commonly used measure of factor substitution is the Allen partial elasticity of substitution.[2] In this definition factor substitution is measured at a constant output level when all other inputs are adjusted optimally to the price change. Uzawa[3] has shown that this elasticity can be computed from the partial derivatives of the cost function according to

$$\sigma_{ij} = \frac{C(\partial^2 C/\partial P_i \partial P_j)}{(\partial C/\partial P_i)(\partial C/\partial P_j)} \tag{3.7}$$

while the own- and cross-price elasticities are calculated as

[1] Shephard (1953).
[2] Allen (1959).
[3] Uzawa (1962)

$$\varepsilon_{ij} = \frac{\partial X_i}{\partial P_j} \frac{P_j}{X_i} = \frac{\partial^2 C}{\partial P_i \partial P_j} \frac{P_j}{X_j} = S_j \sigma_{ij} \tag{3.8}$$

where $S_j = X_j P_j / C$ is the share of factor j in total costs.

The effects of production scale on input demand and the economies of scale of the production function can also be analysed through the cost function. The effect on input demand of output changes are given by

$$\upsilon_{iq} = \frac{\partial X_i}{\partial Q} \frac{Q}{X_i} = \frac{\partial^2 C}{\partial P_i \partial Q} \frac{Q}{X_i}. \tag{3.9}$$

If υ_{iq} is greater than/ equal to/ less than 1, we say we have diminishing/ constant/ increasing returns to factor i. Economies of scale can be studied using the elasticity of cost with respect to output

$$\eta_c = \frac{\partial C}{\partial Q} \frac{Q}{C}. \tag{3.10}$$

Following Ohta,[4] the dual rate of returns to scale, η_Q, can be defined as the inverse of this elasticity

$$\eta_Q = 1 / \eta_c. \tag{3.11}$$

If η_Q is less than/ equal to/ greater than 1, the underlying production function exhibits decreasing/ constant/ increasing returns to scale.

If the production function is homothetic, the cost function can be written as a separable function of output and factor prices.

$$C = G(p,Q) = f(Q)h(p). \tag{3.12}$$

If the cost function is homothetic, the cost shares of the individual inputs are unaffected by changes in output level. Further, a homothetic cost function is homogeneous if the elasticity of cost with respect to output is a constant. If this constant is equal to 1, the cost function is linearly homogeneous and the underlying production function is characterised by constant returns to scale.

It is also possible to investigate effects of disembodied technical progress by including a time trend, t, in the cost function. Doing so would correspond to allowing for a shift factor in the production function. The effects of disem-

[4] Ohta (1974).

bodied technical change are then measured by the partial derivatives of the cost function with respect to the trend variable. Given constant factor prices, the input demand functions are allowed to shift over time as a result of an exogenous development in technology. The biases in technical change can be measured by the derivatives of the demand equations with respect to t

$$\tau_i = \frac{\partial X_i}{\partial t} \frac{1}{X_i} = \frac{\partial^2 C}{\partial P_i \partial t} \frac{1}{X_i} \qquad (3.13)$$

which gives the percentage change in factor i resulting from technical progress. If technical progress affects all factors equally so that the input-mix remains unaffected, it is said to be Hicks neutral. In this case, technical change has no effect on the cost shares of the various inputs, $\partial S_i/\partial t = 0$. On the other hand, technical progress is said to be factor i-using if $\partial S_i/\partial t > 0$ and factor i-saving if $\partial S_i/\partial t < 0$.

Finally, the influence of technical change on total production costs is obtained by partial differentiation of the cost function with respect to t. The rate of technical progress or total-factor productivity growth can be computed as[5]

$$\Delta\ TFP = -(\partial \ln C/\partial t)/(\partial \ln C/\partial \ln Q). \qquad (3.14)$$

This approach to modelling factor demand has its attractive features. Firstly, the econometric model is based on an economic theory that results in a clear interpretation of the estimated parameters and allows for the testing of various assumptions regarding the characteristics of the production process. The existence of economies of scale, the effects of technological change on the use of different production factors as well as productivity development can be investigated. Further, the model allows not only the analysis of the price-sensitivity of input demand but also explains this response in terms of the substitution relationships between production factors. Thereby the effects of higher input prices on the economy—on investment, employment and productivity—can be studied. Finally, by the specification of a flexible functional form[6] for the cost function few a priori restrictions need be placed on the characteristics of the underlying production structure. This allows the elasticities of substitution to take on a wide range of values from strong complementarity to strong substitutability and at the same time permits them to

[5] Ohta (1974).

[6] For a survey of alternative functional forms, see Diewert (1973) and Fuss, McFadden and Mundlak (1978).

vary over the data sample. The use of these generalised functional forms is thus a marked improvement over the traditional forms used in estimating production functions—the Cobb-Douglas or the CES—which restrict the elasticities of substitution to a specific value or to a constant.

Many examples of applications of this type of static long-run equilibrium model can be found in the literature.[7] Since the pioneering study of U.S. industrial energy demand by Berndt and Wood[8] based on the translog cost function in 1975, variations of this basic model have been estimated for a number of countries, including Sweden,[9] for disaggregated industrial sectors as well as on the basis of both time-series and cross-section data. The results of these studies, although providing some information as to the demand relationships, are however not wholly satisfactory. We find, for example, that in some instances the explanatory power of the model is poor or that the estimates are highly unstable. This is especially true when the years after 1974 are included in the estimation. Often, some of the estimated values are implausible from an economic point of view, e.g. positive own-price elasticities, negative scale elasticities, nonconcavity of the cost function. This leads one to question the suitability of the underlying model.

An important prerequisite in econometric studies is that the theoretical model conforms to the data which it is meant to explain, i.e. that the assumptions of the model are consistent with the observations we have on economic phenomena from which inferences about economic behaviour are to be drawn. As mentioned earlier, static cost-minimisation models are derived from producer equilibrium under the assumption that production technology —i.e. the inputs of energy, labour, capital etc.—is fully optimised with respect to production level and the prevailing factor price relationships. Only if this condition holds in the empirical sample can the estimated parameters be interpreted as shifts from one equilibrium to another and the estimated production relationships be considered long run. Estimation of the model requires, in principle, that the empirical data include combinations of production techniques and factor prices that represent points on a long-run cost function. In practice, however, the data are usually limited to historic observations of production techniques which are not necessarily optimised with respect to prevailing factor prices.

The majority of studies of factor demand based on cost-minimisation models have been estimated with time-series—generally annual—data for an individual country. These data can be assumed to approximate observa-

[7] A survey of some of these studies can be found in Dargay (1983b).
[8] Berndt and Wood (1975).
[9] Dargay (1983b, 1983c) and Sjöholm (1981), Bergström and Panas (1985).

tions of different long-run equilibria only under the condition that all inputs fully adjust to output and price changes within one time period, i.e. generally within one year. This instantaneous adjustment implies that all factors of production, including physical capital, are perfectly variable and that firms can switch from a given production technique to the optimal unhindered by economic, institutional or physical considerations. In the context of energy demand, for example, this would require that energy-using capital equipment could immediately be replaced or retrofitted in response to changes in energy prices. This instantaneous adjustment to price or output changes is hardly realistic. On the contrary, long time-lags are generally associated with the investment in physical capital which is necessary for changes in the production process. It is therefore rather dubious that time-series observations of factor prices and production techniques can be assumed to represent points on a long-run cost function.

Because of this inconsistency between the assumptions of the theoretical model and the underlying data, the resulting estimates of price, substitution and output elasticities are exceedingly difficult to interpret. On the one hand, they could hardly be considered as long-run responses. On the other hand, interpreting them as short-run adjustments poses problems, as well, as the theoretical model explicitly assumes optimising behaviour and disequilibrium is not taken into account.

Other studies have attempted to circumvent assuming instantaneous adjustment and capture long-run relationships by basing the analysis of factor demand on a combination of time-series and cross-section data for different regions or countries. The argument here is that observed cross-sectional variation in production techniques, being the result of long-standing differences in factor price relationships, will tend to reflect long-run adjustment possibilities. Although cross-section data may be preferable to time-series data as a basis for estimating equilibrium models, there are still a number of drawbacks to this approach. Firstly, the assumption that cross-section data capture long-run relationships is justified only under the condition that the observed differences in factor prices have been of a long-standing character. In practice, this is not necessarily the case. Further, as the use of international cross-section observations strongly limits the data sample, it is common to include time-series observations for the individual countries as well. This again poses the disequilibrium problems discussed earlier. Finally, as international studies are based on countries with considerable differences in production structure, the estimated elasticities for e.g. aggregate manufacturing will tend to reflect substitution among different types of manufacturing output. As such, they must be considered as representing adjustment in the very long run, when not only production technology, but even product-mix is optimised. It can therefore be difficult to relate these elasticities to the conditions in individual countries, where production structure is partially determined by factors not included in the demand model. Regional data within a country—states in the U.S. for example—provide a much better basis for cross-section studies.

Finally, even if it were the case that purely long-run relationships could be estimated on the basis of static models, there is no indication of the time-span required for the adjustment to equilibrium. This limits the usefulness of such results in forecasting factor demand or as a tool in economic policy-making in which not only the level of the response, but also the time factor involved, is of importance. Neither do static-equilibrium models provide a basis for determining short-run responses. This is a considerable shortcoming as the immediate impact of economic disturbances poses the most severe problems for the economy.

3.1.2 Partial Static Equilibrium Models

A rather simple method of improving upon the full static equilibrium model by dropping the assumption of the perfect flexibility of all inputs is provided by the notion of partial static equilibrium. Here it is assumed that certain factors of production are fixed in the short run, while all other inputs are optimised conditional on the levels of the fixed inputs. By using the envelop theorem, the long-run relationships can be retrieved. The following contains a theoretical presentation of partial static equilibrium. Much of this is based on Brown and Christensen (1981) and Berndt and Wood (1983).

The basis of the partial static equilibrium model is the restricted variable cost or restricted variable profit function developed by Lau.[10] As opposed to the pure static equilibrium model, in which all factors of production are assumed to be at their optimal levels, it recognises the fact that some inputs may be fixed in the short run so that they cannot readily adjust to changes in prices or output demand. Factor inputs are thus designated as either variable or quasi-fixed depending on their short-run flexibility. Partial static equilibrium is just the conditional optimisation of the variable inputs given the level of the quasi-fixed factors. The complete, or long-run, adjustment of the variable inputs is, however, also dependent on the adjustment of the quasi-fixed factors to optimum levels.

The derivation of the model is similar to that of the full-static equilibrium model presented in the previous section. In this case, however, one assumes that for a given level of the quasi-fixed inputs $X_i = \bar{X}_i$, the short-run production possibilities are given by

[10] Lau (1976).

$$Q = F(x) \tag{3.15}$$

where x is a vector of both quasi-fixed and variable inputs and $X_i = \bar{X}_i$ for all quasi-fixed inputs.

One begins by specifying a short-run variable cost (or profit) function which represents the technological constraints facing the firm when certain inputs are fixed in the short run. The firm's objective is to minimise variable costs given the levels of the quasi-fixed factors.

For the sake of simplicity in the presentation of the model, we will assume that only one input is fixed in the short run while all other inputs are perfectly variable. The model could, however, easily be extended to the case of more than one fixed factor. Since it is apparent that capital cannot as quickly adjust to changes in output demand or factor price relationships as can other production factors, capital is specified as quasi-fixed. The remaining inputs —energy, labour etc.—are assumed to be perfectly variable. Given these assumptions, the variable cost function takes the following form:

$$C_V = G(p,Q,X_K) \tag{3.16}$$

where p is a vector of prices of the variable inputs, Q is the level of output and X_K is the quantity of the quasi-fixed input, capital. C_V is the sum of variable costs i.e. $\Sigma_i P_iX_i$, where, of course, the summation is taken only over the variable factors. If costs are minimised with respect to the variable inputs conditional on the level of output and the capital stock, this variable cost function is dual to the short-run production function given in (3.15). The variable cost function must be monotonically increasing in factor prices and output, decreasing and convex in the level of the quasi-fixed input and linearly homogeneous and concave in factor prices. The total short-run cost function is the sum of variable costs and the costs for the quasi-fixed factor, capital:

$$C_T = G(p,Q,X_K) + u_KX_K \tag{3.17}$$

where u_K is the rental price, or user cost, of capital.

The demand equations for the variable inputs can be derived using a variant of Shephard's Lemma. In this case, short-run variable costs are minimised and the cost-minimising demand levels are equal to the first partial derivative of the variable cost function with respect to the prices of the variable inputs

$$X_i = \partial C_V/\partial P_i = \partial G(p,Q,X_K)/\partial P_i = H_i\ (p,Q,X_K). \tag{3.18}$$

Thus, the short-run demand for the variable inputs is not only a function of the prices of all variable inputs and the quantity of output, but is also dependent on the levels of the inputs that are fixed in the short run, in this case, the capital stock.

The reduction in variable costs achieved by a marginal increase in the capital stock—or the negative of the shadow value of capital, R_K—is obtained by differentiating the variable cost function with respect to capital,

$$-R_K = \frac{\partial C_V}{\partial X_K} < 0. \tag{3.19}$$

The short-run variable cost function permits investigation of the flexibility of the variable inputs at a given level of the capital stock. The short-run Allen elasticities of substitution between the variable inputs can be obtained from the following formulae:[11]

$$\sigma_{ij}^{s} = \frac{C_V(\partial^2 C_V/\partial P_i \partial P_j)}{(\partial C_V/\partial P_i)(\partial C_V/\partial P_j)} \tag{3.20}$$

while the short-run own- and cross-price elasticities can be computed as

$$\varepsilon_{ij}^{S} = \frac{\partial X_i}{\partial P_j}\,\frac{P_j}{X_i} = \frac{\partial^2 C_V}{\partial P_i \partial P_j}\,\frac{P_j}{X_i} = S_j \sigma_{ij}^{s} \tag{3.21}$$

where S_j is the share of factor j in variable costs.

The short-run effects on the demand for the variable factors of changes in output or the capital stock can also be studied. The short-run output quantity elasticities can be obtained by taking the partial derivative of the demand equations for the variable inputs with respect to output, holding the capital stock constant. The short-run capital quantity elasticities are obtained by differentiation with respect to the capital stock, holding output constant. Finally, the variable cost elasticity

$$\eta_{cv} = (\partial C_V/\partial Q)(Q/C_V) \tag{3.22}$$

can be derived. The inverse of the cost elasticity can be taken as a measure of short-run scale economies.[12] The above elasticities are, of course, valid only for the levels of the quasi-fixed factor, i.e. the capital stock, at which they are calculated. Price, substitution and output elasticities with respect to capital are by definition zero in the short run.

[11] Uzawa (1962).
[12] Hanoch (1975).

One of the most interesting features of the restricted variable cost function, however, is that it also provides information about long-run production characteristics. In long-run equilibrium, changes in the capital stock will serve neither to increase nor diminish total costs so that $\partial C_T/\partial X_K = 0$. The optimal level of the quasi-fixed factor can therefore be defined by the first order condition, which for the total cost function given in (3.17) implies that

$$\partial C_T/\partial X_K^* = \partial C_V/\partial X_K^* + u_K = 0 \tag{3.23}$$

where X_K^* indicates the equilibrium level of the capital stock. It should be noted that in long-run equilibrium, the user-cost of capital, u_K, should be equal to the shadow value R_K which appears in (3.19). If $R_K > u_K$, the reduction in variable costs obtained by employing an additional unit of capital is greater than its rental cost, which would be an incentive for firms to invest in capital in order to further decrease total costs. On the other hand, if $R_K < u_K$, the marginal cost reductions of additional capital are less than the cost of capital. In this case the firm would be motivated to reduce the level of capital.

By the envelop condition, for a given level of output the short- and long-run cost curves will be tangent at the point where $X_K = X_K^*$ or in other words where $R_K = u_K$. The long-run cost function can thus be constructed from these points of tangency by substituting the optimal level of the fixed factor, X_K^* (p,Q,u_K), implied by (3.23) into the short-run cost function (3.17).

$$C_T^* = G[p,Q,X_K^*(p,u_K,Q)] + u_K X_K^*(p,u_K,Q). \tag{3.24}$$

The characterisation of production technology in long-run equilibrium—the elasticities of substitution, price elasticities, output elasticities and the long-run returns to scale—can be obtained by replacing C_V and X_K in the equations for the short-run elasticities with C_T^* and X_K^* and evaluating all endogenous variables at X_K^*. In practice, the long-run relationships are retrieved in the following manner. First, the optimal levels of the quasi-fixed factor are calculated by solving (3.23) for X_K^*. These values are then substituted into the demand equations for the variable inputs (3.18) to obtain the equilibrium levels X_i^* for these inputs. Long-run price and output responses for the variable factors are calculated by differentiating these optimal demand equations, while relationships involving the quasi-fixed factor are obtained from the appropriate differentiation of (3.23).

Empirical implementation of the restricted variable cost function is straightforward. A functional form is specified for the cost function (3.16) and the short-run demand functions for the variable inputs are derived using (3.18). The coefficients obtained by estimating the resulting system of equations are then substituted into equations (3.20—3.22) to calculate the short-run substitution, price and output elasticities.

The link between the short- and long-run cost functions rests on being able to solve the envelop condition (3.23) for the optimal level of the quasi-fixed factor. Whether or not an analytic solution exists depends, of course, on the particular functional form specified for the variable cost function. As mentioned in the context of the long-run equilibrium model discussed earlier, it is preferable to choose a "flexible" functional form which places minimal restrictions on the characteristics of production. In deriving the long-run elasticities from the variable cost function it is also advantageous that the related envelop condition can be solved analytically for X_K^* and differentiated with respect to the exogenous variables. The quadratic form fills these requirements as does the variant of the generalised Leontief used by Mork.[13] In contrast, the translog function most commonly used in static long-run studies cannot be solved analytically for X_K^*. However, as shown by Brown and Christensen[14] in a study of U.S. agriculture based on the translog form, numerical procedures can be used to obtain a solution for X_K^* and to calculate approximate derivatives.

The restricted variable cost function can also be used to test whether selected inputs are at long-run optimum levels. One test procedure, outlined in Nadiri and Schankerman (1980), is based on the null hypothesis that all factors are in long-run equilibrium, i.e. that the envelop condition holds for the observed levels of the quasi-fixed factors. Under this assumption, equation (3.23) is estimated simultaneously with the variable cost function and the demand equations for the variable factors. This provides a basis for statistically testing whether the computed optimal quanties of the quasi-fixed factor are significantly different from the observed values.

Until quite recently very few studies based on partial static equilibrium models could be found in the literature. One of the earliest is an application to U.S. agriculture by Brown and Christensen.[15] Here, family labour is treated as a quasi-fixed factor while capital, hired labour and materials are treated as variable factors. Comparing their results to those obtained with the same data using a full-static equilibrium model, they find that the assumption of partial static equilibrium produces somewhat more plausible results.

Berndt and Hesse (1986) utilise a partial static equilibrium model to measure capacity utilisation in the manufacturing sectors of nine OECD countries. Capital is treated as a quasi-fixed factor and a translog variable cost

[13] Mork (1973).
[14] Brown and Christensen (1981).
[15] Brown and Christensen (1981).

function is estimated. Capacity output is defined as the point of tangency of the short- and long-run average cost curves. Instead of solving for the optimal capital stock given the level of output, the optimal level of output at the given level of the capital stock is calculated. Capacity utilisation is then measured as the ratio of actual to capacity (or optimal) output.[16]

The main advantage of partial static equilibrium models is that both short- and long-run demand relationships can be estimated fairly easily, and without explicitly specifying the adjustment process. The disadvantage, however, is that no information is given either as to the speed of adjustment to long-run equilibrium or to the factors influencing the adjustment process. The model is basically one of comparative statics and as such is not very useful when one is explicitly concerned with the dynamics of behaviour.

3.2 *Dynamic Factor Demand Models*

3.2.1 Single Equation Models

Until the early 1970's, the majority of empirical studies of dynamic factor demand were based on single equation models describing the demand for a particular production factor. These studies generally follow the approach originally developed for and commonly employed in the analysis of investment demand, in which the dynamic element of the adjustment process is accounted for by assuming that the effect of the exogenous variables is distributed over several time periods. There are a number of different ways in which time lags can be introduced into econometric models and a variety of alternative lag-distributions are available.[17] The most widely used are the lag schemes introduced by Almon and Koyck.

Almon's approach permits a wide range of possible response profiles by representing the lag-distribution by a low-degree polynomial. The degree of the polynomial and the maximum lag must be specified in advance, but various alternatives can be estimated. The advantages of the Almon scheme are that no a priori assumptions about the form of the lag-distribution need be made and that the method permits the individual explanatory variables to assume different lag-forms. However, in small data samples, the time length of the lag must often be limited.

The lag-scheme proposed by Koyck, although not assuming a finite maxi-

[16] See Berndt (1984) for an interpretation of the short-run cost function.

[17] See, for example, Almon (1965), Koyck (1954), Jorgenson (1967). A survey can also be found in Nerlove (1972).

mal lag, is based on a far stronger assumption about the form of the lag-distribution. The lag-profile is given the explicit form of a geometric structure which approaches zero asymptotically. Thus, the most recent events have the greatest effects on current behaviour, and the effects of less recent events dwindle in importance as the time lapse increases. Although this is a rather restrictive assumption, it greatly facilitates the estimation of the lag response.

The incorporation of lags in the demand model allows one to calculate both long- and short-run elasticities. The short-run elasticities are defined as the first period response whereas the long-run elasticities are given by the response over the entire lag-length. The effects of the explanatory variables in various time intervals can also be calculated so that information is also obtained concerning the speed of adjustment.

Numerous examples of applications of distributed lag models can be found in the literature. The empirical models used, however, are in most cases extremely simplified, with the explanatory variables generally being limited to a lag-function of prices and output or income. Little information is therefore gained concerning the mechanism behind the demand response, particularly about the substitution relationships between various factors of production.

Another, more theoretically grounded, approach is based on the partial adjustment models employed in investment studies. These models are generally based on the notion that the actual demand at time t may differ from the desired or long-run equilibrium level because of constraints imposed by the technical characteristics of the present capital stock and the rate at which it could be replaced.

One typically begins with a static formulation wherein the desired or long-run demand function is specified. In the simplest case, each factor of production is viewed independently of other inputs, and the desired demand is specified as a function of its own price and of output. Other studies have attempted to take into account the effects of the prices of substitute or complementary inputs in a more or less intuitive fashion by including for example the prices of these goods as explanatory variables. Still others have used a more formal approach by specifying a production function and deriving the cost-minimising demand for an individual input as a function of factor prices and output.

A major drawback of the partial adjustment model is that the economic meaning of the adjustment mechanism is not explicitly stated in the model formulation. Instead, an intuitive discussion of time delays, installation costs, etc. is used to append an ad-hoc adjustment hypothesis onto an essentially static model in order to account for disequilibrium. It is thus not possible to analyse how various factors influence the adjustment process. On the contrary, the rate of adjustment is assumed to be constant and independent of the levels of, or changes in, factor prices or output. Further, there is no theoretical justification for the lag-structure employed. In many cases a geometrically declining lag-distribution may be too restrictive. Finally, the assumption that the lag-response be identical for all explanatory variables can often lead to implausible results.[18]

Although single equation demand models may be useful in some applications—the demand for individual consumer goods, total aggregate demand—they appear to be less suitable for the study of industrial factor demand. A significant shortcoming is that each input is considered more or less in isolation from its role in the production process. As each production factor is only one of a series of inputs in production, the demand for each factor should be considered in conjunction with the demand for all other inputs. The adjustment of demand to its optimal level depends, too, on the ability of other inputs to adjust. Ideally, this interdependence should be taken into consideration in the model formulation.

3.2.2 Interrelated Factor Demand Models

One of the first attempts to combine the notions of interrelated factor demand and dynamic adjustment is the general disequilibrium study of Nadiri and Rosen (1969 and 1973). Their approach is basically a generalisation of the Koyck single equation adjustment mechanism discussed earlier to the case of multiple inputs. Since the different production factors are treated simultaneously, input adjustments are interdependent. In this way, disequilibrium in one factor market is related to the extent of disequilibrium in the demand for all other factors.

Illustration of the Nadiri-Rosen model is simplified by the use of vector and matrix notation. Let x_t denote a vector of the observed quantities of n inputs at time t, and x_t^* denote the desired or long-run equilibrium levels. The actual change in factor demand between time t-1 and time t is related to the divergence from the desired demand by the n x n adjustment matrix B as follows:

$$x_t - x_{t-1} = B(x_t^* - x_{t-1}). \tag{3.25}$$

This can also be written as

$$x_t = Bx_t^* + (I-B)x_{t-1} \tag{3.26}$$

[18] For a discussion of these problems, see Berndt, Morrison and Watkins (1981).

where I is an identity matrix of order n. It is easily seen that if B = I, the actual demand is equal to the desired demand, i.e. $x = x^*$. In this case the model reduces to the case of instantaneous adjustment.

The interrelatedness of this demand system can more easily be seen by a simple example. Suppose we have three inputs: energy (X_E), labour (X_L) and capital (X_K). Then (3.26) becomes the following three-equation system:

$$X_{E,t} = b_{EE}X^*_{E,t} + (1\text{-}b_{EE})X_{E,t-1} + b_{EL}(X^*_{L,t}\text{-}X_{L,t-1}) + b_{EK}(X^*_{K,t}\text{-}X_{K,t-1})$$

$$X_{L,t} = b_{LL}X^*_{L,t} + (1\text{-}b_{LL})X_{L,t-1} + b_{LE}(X^*_{E,t}\text{-}X_{E,t-1}) + b_{LK}(X^*_{K,t}\text{-}X_{K,t-1}) \qquad (3.27)$$

$$X_{K,t} = b_{KK}X^*_{K,t} + (1\text{-}b_{KK})X_{K,t-1} + b_{KE}(X^*_{E,t}\text{-}X_{E,t-1}) + b_{KL}(X^*_{L,t}\text{-}X_{L,t-1}).$$

The demand for energy, for example, in period t is not only dependent on the level of demand in the previous period $X_{E,t-1}$ and on the equilibrium level $X^*_{E,t}$ but also on the extent of disequilibrium in the labour ($X^*_{L,t} - X_{L,t-1}$) and capital ($X^*_{K,t} - X_{K,t-1}$) markets. Thus, even if only one factor cannot adjust instantaneously, all factors could diverge from their long-run desired values. It is this interdependence that distinguishes the Nadiri-Rosen model from the single factor partial adjustment models discussed earlier. As is seen from the above equations, if B is a diagonal matrix, i.e. if $b_{ij} = 0$ for all $i = j$, the interaction terms cancel out and the demand functions reduce to a system of simple partial adjustment equations.

The next step is to specify the desired demand in terms of exogenous variables. These can be derived from a long-run production or cost function under the assumption of cost-minimizing behaviour. Let us suppose for now, however, that equilibrium factor demand is some function of factor prices and output level:

$$X^*_i = f(P_E, P_L, P_K, Q) \text{ for } i = E,K,L \qquad (3.28)$$

where the P_i are the prices of energy, labour and capital and Q is output level. Substitution of these desired factor demand functions into the adjustment equations given in (3.27) results in a system of equations relating the actual demand for each input to factor prices, output and the lagged quantities of all inputs. Thus, the number of parameters to be estimated becomes quite large in the case of multiple inputs. This can pose problems in small data samples. Further, it is not unlikely that multicollinearity among the exogenous and lagged variables will decrease the precision of the estimation, resulting in large sampling variances for the estimated coefficients.

The long-run elasticities are determined by appropriately differentiating the equilibrium demand functions (3.28). Short-run elasticities, being defined as the first period response, are obtained from differentiation of the functions given in (3.27). As opposed to the partial adjustment model discussed earlier, the difference between short- and long-run responses does not depend solely on the adjustment parameter b_{ii} but also on all cross-adjustment coefficients b_{ij}. The short-run cost function which characterises the disequilibrium responses is, however, not explicitly stated.

The degree of complexity and flexibility of the model depends, of course, on the specification of the equilibrium factor demand functions (3.28). Typically, these have been derived from Cobb-Douglas production functions[19] and have thus been highly restrictive in terms of the permissible substitution relationships. Recently, however, in the study of energy demand, attempts have been made to combine the Nadiri-Rosen model with more flexible functional forms. A detailed discussion of the theoretical considerations and empirical problems involved can be found in Berndt, Fuss and Waverman (1977).

The authors suggest that the disequilibrium model given in (3.25) above can be specified in terms of the adjustment of input levels, input ratios, input-output ratios or cost shares. This choice depends on whether the cost or production function is used to determine optimal factor demand and on the particular functional form chosen for estimating purposes. The authors point out, however, that adjustment to cost shares, input ratios or input-output ratios does not necessarily imply that input levels are adjusting to equilibrium values. In order to insure level adjustment, output quantity restraints must be imposed.

The properties of both the translog and the generalised Leontief forms are investigated. For the translog cost share specification, the authors find that the major problems are that identification of all parameters of the B matrix is not possible unless prior restrictions are placed on B and that short-run elasticities can be greater than long-run elasticities in absolute value. Using the translog function and assuming instead adjustment to input levels results in a system of nonlinear equations without intercepts which would be very cumbersome to estimate. The generalised Leontief functional form is found to be preferable on many of these points. Assuming adjustment to input-

[19] An application of such a model to Swedish manufacturing data can be found in Kanis (1979).

output ratios or input levels, the authors find that the B matrix is identified so that the adjustment mechanism is more flexible than in the translog case. The resulting equation systems are also found to be easier to estimate. The major problem with both these functional forms is that output feasibility constraints are not satisfied throughout the disequilibrium process. With the absence of these constraints, the disequilibrium factor levels derived from the model may not necessarily be sufficient to produce the observed output.

The dynamic adjustment model described above has been estimated for U.S. manufacturing by Berndt, Fuss and Waverman (1977). Four inputs are considered: energy, labour, capital and materials. Both translog cost share and generalised Leontief input-output formulations were used with four alternative adjustment specifications ranging from instantaneous to general disequilibrium. For both functional forms, the instantaneous and the diagonal adjustment specifications are rejected in favour of the general disequilibrium models. The estimated adjustment matrices, however, fail to satisfy conditions for stability and convergence of the adjustment process. The translog function produces "well-behaved" estimates of substitution possibilities, while the curvature conditions are not satisfied for any of the generalised Leontief specifications.

Another variant of the disequilibrium model has been suggested by Norsworthy and Harper (1981). As noted earlier, only the long-run cost function is specified in the models discussed above. Disequilibrium is accounted for by appending an ad-hoc adjustment mechanism directly onto the equilibrium demand functions, so that the short-run cost function is not explicitly stated. Norsworthy and Harper attempt to present a more theoretically justifiable model by specifying a dynamic cost function that incorporates both short- and long-run situations. This is achieved by including a disequilibrium term in the long-run (translog) cost function in order to account for short-run departures from cost-minimisation. The disequilibrium portion of the cost function vanishes at long-run equilibrium. The dynamic factor demand equations are then derived directly from the disequilibrium cost function.

Norsworthy and Harper estimate this model for U.S. manufacturing and compare the results with those obtained from the translog version of the Nadiri-Rosen model as well as with a static long-run translog specification. The authors find that estimation results favour the dynamic over the equilibrium models. They report, however, difficulties in estimating the disequilibrium models. The estimated variances are extremely large, particularly for the disequilibrium cost function model, resulting in statistically insignificant estimates for 9 out of 18 parameters. This can have to do with the large number of parameters estimated and the multicollinearity problem mentioned earlier.

Although the Nadiri-Rosen specification of the adjustment process in combination with flexible functional forms represents a major innovation in the analysis of factor demand, a number of problems still remain. Because of the large number of parameters to be estimated, the models can be difficult to implement empirically. More important, perhaps, is that the theoretical derivation of the model is somewhat tenuous. Although the long-run demand functions are derived from cost-minimising principles, the short-run or disequilibrium demand levels are not well defined in terms of the cost function. The specification developed by Norsworthy and Harper partially solves this problem, but the link between the short- and long-run cost functions is still basically ad-hoc. The motivation for non-instantaneous adjustment is not explicitly stated in the derivation of the model. Instead, an ad-hoc adjustment mechanism is used to approximate the effects of any of a number of factors. Another limitation concerns the assumption of a constant adjustment matrix which is independent of the levels of the exogenous variables. As pointed out by Berndt, Fuss and Waverman, a more realistic and theoretically justifiable specification would permit the adjustment path itself to be determined endogenously.

3.2.3 Cost of Adjustment Models

The majority of the models discussed in previous sections have been based on some sort of static optimisation theory. This is of course true for the long-run model and partial static equilibrium model discussed in Section 3.1, but it is also the case for the dynamic models presented in the previous pages. The partial adjustment and general disequilibrium models, for example, resort to static optimisation to derive the long-run desired demand functions. The dynamic element is then appended in a more or less ad-hoc fashion. In contrast, the model presented in the following is explicitly derived from dynamic optimisation theory.

As discussed in Section 3.1, the major drawback with full static equilibrium models is the inherent assumption that the capital stock is perfectly flexible. Thus, given changes in factor prices or output demand it is assumed that the firm can buy, sell, install, set into or take out of production any capital equipment instantaneously[20] and without added costs so that at all times the optimal capital stock and factor mix is employed. The firm is not bound by previous investment decisions nor does it have to be concerned with present investments affecting future production possibilities. Since capital equipment

[20] Or at least within the time increments represented by the data, which is generally one year in studies based on time series data.

can be changed quickly and costlessly, expectations concerning future output or factor prices are rendered irrelevant. The firm can adjust to these events as they arise, without economic loss. This is, of course, hardly in keeping with our knowledge of the firm's investment process and particularly of the nature of investment goods.

Although the other models presented—the partial static equilibrium model in Section 3.1.2 and the dynamic models of Sections 3.2.1 and 3.2.2—do allow for the inflexibility of capital, the mechanism behind capital accumulation is not explicitly formulated.

The point of departure for the derivation of the model presented in this section is in some ways similar to that behind the partial static equilibrium model described in Section 3.1.2. Both rest on the distinction between variable and quasi-fixed inputs. Here, too, the concept of restricted cost and profit functions plays an important role in the theoretical derivation. The model presented in the following, however, goes quite a bit further. Instead of just assuming certain inputs to be fixed in the short run, the inflexibility of these inputs is motivated in terms of the economic costs involved in quickly adjusting to long-run optimal levels. These costs of adjustment are an integral part of the underlying economic theory and are explicitly incorporated into the model formulation.

Adjustment costs can be defined as those costs incrued by the firm in the buying, selling, installation or productive implementation of production factors over and above their normal price.[21] These can be the costs associated with investment planning and financing, with the early retirement of capital equipment or in retraining, reorganisation or other troubles involved in the installation of new equipment. The existence of adjustment costs, however, does not necessarily prevent the firm from adjusting rapidly to changes in factor prices or output demand. In fact, if adjustment costs are linear or decreasing at the margin, the total response to changes in market conditions for a profit maximising firm would take place in a single period. It would pay the firm to make the required changes in production technology quickly. By assuming, however, that marginal adjustment costs are increasing (or convex) the optimal response of the firm would be to distribute investment over several time periods. This strict convexity of adjustment costs implies that rapid or large changes in the use of a particular production factor are more costly than if the change took place more slowly.

Some examples of adjustment costs pertaining to capital can be given:

1. The marginal cost of financing investment rises as more and more capital

[21] A discussion of the motivations behind internal adjustment costs can be found in Rothschild (1971) and Nickel (1978).

is raised. As the firm needs to raise larger sums of capital during a given time period the costs of both the debt and equity components begin to rise and the average cost of capital also rises. The firm cannot generally raise unlimited amounts of capital at a constant cost. If, for example, the growth rate were such that the firm is required to sell new common stock, the marginal cost of capital would rise. Also the interest rate on debt commonly increases after a certain amount. This could limit the rate of expansion to a slower pace than would be dictated by purely rational profit-maximising behaviour.

2. Analysing capital expenditure proposals is itself not a costless operation. Although small investments for capital replacement (depreciation) are usually based on a simple decision process, cost-reduction replacements, expansion of existing product lines and investments into new products are more complex. The larger the investment, the more detailed the analysis needed to make the decision. Speeding up this process could entail added costs.

3. Generally, a rapid increase in the demand for a particular good may not be met by a corresponding increase in its supply. This could lead to an increase in the price of the good in question. Although the firm is generally considered to be a price taker on all factor markets, there are some indications that a large increase in factor demand can affect supply prices. This may particularly be true for capital equipment which, outside of vehicles and some simpler machinery, is highly industry specific. In this case 'rush orders' can entail additional costs, that could be expected to rise with the size of these orders. On the other hand, the practice of quantity discounts tends to have an opposite effect. The relative importance of these two effects is difficult to ascertain.

4. The costs involved in the installation of new capital equipment—reorganisation of production lines, retraining of staff, work stopages—may in some cases also be increasing at the margin, but on the other hand, needn't be. There are examples where such costs can be diminishing, e.g. retraining 10 workers to operate a new machine may cost less per worker than retraining 1 worker at a time.

5. Selling of used capital equipment can also be costly for the firm since it is unlikely that such equipmement could be resold at a value equalling purchase price minus depreciation. Since most capital equipment is highly specific to a particular firm, no working second hand market exists for many of these goods. The investment decision becomes irreversible in the sense that economic loss may be unavoidable if the equipment is to be replaced before its physical life is exhausted. This prevents the firm from adjusting rapidly to changes in market conditions. Accelerating capital retirement would increase the losses so that such costs could be considered to be increasing at the margin. Even in the case of the existence of second hand markets—such as with vehicles and some more common machinery—it is probable that the early retirement of such equipment would entail certain economic costs for the firm.

In theoretical as well as empirical models adjustment costs are assumed to be either internal of external to production activity. Internal adjustment costs affect current production activity in the sense that resources must be taken from production in order to carry out the investment process. Examples here could be (2) and (4) above. External adjustment costs do not affect current production, but are only an added cost. For example, (1), (3) and (5).

The theoretical basis of models incorporating adjustment costs and dynamic optimisation is found in the works of Lucas and Treadway.[22] Although there are some essential differences in the treatment of adjustment costs, the general approach followed by these authors is similar. In both cases, the firm's objective function is formulated to incorporate the costs of factor adjustment as well as the relationships defined by the production function. The objective function is stated in terms of the firm's net cash flow (profits) or production costs. The firm is then assumed to maximise (minimise) the present value of the future stream of profits (costs) subject to its expectations concerning future factor prices and given its initial stocks of factor inputs. Static price expectations are generally assumed so that adjustment is considered to be towards a fixed target or equilibrium position. The optimal time paths of the endogenous variables are obtained by solving the optimisation problem.

The major difference between the two approaches has to do with the rationale behind the adjustment costs and therefore the manner in which these are incorporated into the economic model. Lucas assumes that adjustment costs are separate from the production relationship. The costs of adjustment are external to production activity because although the adjustment process results in an increased cost for the firm, current production possibilities are not affected. In Treadway's model, on the other hand, the costs of adjustment enter directly into the firm's production function. A change in the stock of the quasi-fixed factors is thereby assumed to affect current production possibilities. The motivation for these internal adjustment costs is, as discussed earlier, that a part of the firm's resources must be spent on changing the stock so that current production is reduced. The more rapid the investment the greater is the reduction in current production possibilities.

The theoretical derivation of dynamic cost of adjustment models as well as their adaptation to econometric estimation has been developed by Berndt, Fuss and Waverman.[23] On the basis of Lucas' and Treadway's work, models are derived in terms of both profit maximisation and cost minimisation. These models are adapted for econometric implementation and restricted profit and

[22] Lucas (1967a, b) and Treadway (1971, 1974).
[23] Berndt, Fuss and Waverman (1977).

cost functions are used to derive dynamic demand functions suitable for econometric estimation.

The following presentation of the cost of adjustment model is based partially on the derivation given in Berndt, Fuss and Waverman. The model derived here, however, is more general in that it is based on both internal and external adjustment costs, with the simpler models as special cases. As previously, the derivation is based on the assumption of cost-minimising behaviour[24] on the part of firms. Adjustment costs are specified as a function of net investment, that is, it is assumed that investment for depreciation does not give rise to adjustment costs.[25] We will suppose that the firm's production process involves the inputs of variable factors the levels of which are designated by the vector v, and one quasi-fixed factor[26] denoted as x. The relative prices of the variable inputs given by the vector p, the relative purchase price q of the quasi-fixed factor and its rate of return r, as well as output Q are exogenously determined. As a further simplification static expectations are assumed, so that future output and prices are known with certainty and expected to remain constant over time.[27] Given these conditions, the firm is assumed to behave so as to minimise the present value of the future stream of costs. The dynamic optimisation problem facing the firm can be written as:

$$L(0) = \int_{t=0}^{\infty} e^{-rt}\{p'v + qI + c(\dot{x})\}\, dt \qquad (3.29)$$

where $I = \dot{x} + \delta x$ is gross investment in physical capital; $\dot{x}$ denotes the change in the capital stock or net investment and δ is the rate of depreciation. The function $c(\dot{x})$ represents the external adjustment costs dependent on net investment. These costs are assumed to be positive and marginally increasing, so that $\partial c/\partial|\dot{x}| > 0$ and $\partial^2 c/\partial|\dot{x}|^2 > 0$.

Given the initial conditions on the capital stock x(0) and the expectational values of factor prices and output, the firm's objective is to choose the time paths of the variables v(t), x(t) and $\dot{x}(t)$ that minimise L(0), subject to the technological constraints imposed by the production function.

Assume that the firm's production possibilities are given by:

$$Q_v = F(v,x,\dot{x}). \qquad (3.30)$$

[24] One can also assume profit maximisation. The main difference is that with profit maximisation the level of output is endogenous. This somewhat complicates the estimation.

[25] Adjustment costs could also be specified as a function of gross investment. The derivation is similar to that presented here.

[26] More than one quasi-fixed factor can be specified, but the derivation becomes rather more complicated. We will return to this later on.

[27] Static expectations need not necessarily be assumed. For examples of models based on non-static expectations, see Pindyck and Rotemberg (1982), Prucha and Nadiri (1982) and Morrison (1986).

The net change in the capital stock $\dot{x}$ is included in the production function to account for the internal costs involved in changing the stock. The accumulation or decumulation of capital is assumed to decrease current production possibilities so that $\partial Q/\partial|\dot{x}| < 0$. Further, since the amount of output foregone is directly related to the rate of investment, the marginal costs of adjustment are increasing, i.e. $\partial^2 Q/\partial|\dot{x}|^2 > 0$.

If costs are minimised with respect to the variable inputs conditional on the level of output, the level of the quasi-fixed factor x and its rate of change $\dot{x}$, then there exists a restricted variable cost function that is dual to (3.30):

$$C_V = p'v = G(p,x,\dot{x},Q) \tag{3.31}$$

where G(·) is the minimum variable cost obtainable under the given restrictions.

Under the regularity conditions on the production function F, it can be shown that G is increasing and concave in p, decreasing and convex in x, increasing and convex in $\dot{x}$ and increasing with Q. Further, according to Shephard's Lemma, the partial derivative of the variable cost function with respect to the price of each variable input is equal to the short-run cost-minimising demand for these inputs:

$$v_i = \partial C_V/\partial P_i = \partial G(p,x,\dot{x},Q)/\partial P_i = H_i\,(p,x,\dot{x},Q). \tag{3.32}$$

Substituting the optimal restricted variable cost function (3.31) into the objective function (3.29), we have:

$$L(0) = \int_{t=0}^{\infty} e^{-rt}\{G(p,x,\dot{x},Q) + qI + c(\dot{x})\}dt. \tag{3.33}$$

Since the restricted variable cost funtion incorporates the optimal demand for the variable factors conditional on x and $\dot{x}$, the firm's optimisation problem is reduced to choosing the time paths x(t) and $\dot{x}$(t) to minimise (3.33). Using the relation $I = \dot{x} + \delta x$, the Euler necessary condition for a minimum can be written as:

$$\partial G/\partial x + r\partial G/\partial\dot{x} - \partial^2 G/\partial\dot{x}^2\ddot{x} - \partial^2 G/\partial x\partial\dot{x}\quad \dot{x} + u - \partial^2 c(\dot{x})/\partial\dot{x}^2\ddot{x} + r\partial c(\dot{x})/\partial\dot{x} = 0 \tag{3.34}$$

where $\ddot{x}$ denotes the second partial derivative with respect to time and $u = q(r+\delta)$ is the user cost of capital. At steady state $\dot{x} = \ddot{x} = 0$ so that the solution x* satisfies:

$$\partial G^*/\partial x + r\partial G^*/\partial\dot{x} + u + r\partial c(\dot{x})/\partial\dot{x} = 0 \tag{3.35}$$

where G^* is evaluated at x^* and $\dot{x} = 0$, and x^* is unique as long as $|\partial G^{*2}/\partial x^2 + r\partial^2 G^*/\partial x \partial \dot{x}| = 0$. The steady state solution for the variable factors v^* can be obtained by substituting x^* for x in equations (3.32).

Taking a linear approximation to (3.34) around $(x = x^*, \dot{x} = 0)$ and using the steady state result in (3.35), we have after a few manipulations[28]

$$f(x,\dot{x}) = \ddot{x} - r\dot{x} - \frac{\partial^2 G/\partial x^2 + r\,\partial^2 G/\partial \dot{x}\,\partial x}{\partial^2 G/\partial \dot{x}^2 + \partial^2 c(\dot{x})/\partial \dot{x}^2}\{x - x^*\} = 0. \quad (3.36)$$

This second order differential equation can be easily solved to yield

$$\dot{x} = \lambda\,(x^* - x) \quad (3.37)$$

where λ, the adjustment parameter, is the stable root defined by

$$\lambda = -1/2[r - \{r^2 + 4\,(\frac{\partial^2 G/\partial x^2 + r\partial^2 G/\partial \dot{x}\partial x}{\partial^2 G/\dot{x}^2 + \partial^2 c(\dot{x})/\partial \dot{x}^2})\}^{1/2}]. \quad (3.38)$$

To be meaningful in an economic sense, the adjustment coefficient λ must lie between 0, which would indicate a perfectly fixed factor, and 1, which would be the case were adjustment instantaneous.

The equilibrium level of the quasi-fixed factor x^* is implied by the steady state solution to the Euler condition (3.35). By definition, at steady state $x = x^*$ and $\dot{x} = 0$, which implies that $\partial G^*/\partial \dot{x} = \partial c(\dot{x})/\partial \dot{x} = 0$. Inserting these values in (3.35) yields

$$\partial C_V^*/\partial x + u = \partial G^*/\partial x + u = 0 \quad (3.39)$$

which is just the requirement that the savings in variable costs resulting from an additional unit of the quasi-fixed factor be equal to its price.

Treadway links this model to the flexible accelerator literature by showing that the short-run demand for the quasi-fixed factor x can be generated from equations (3.34) and (3.35) as an approximate solution in the neighbourhood of x^* to the linear differential equation given in (3.37). The difference between the flexible accelerator model given in (3.37) and Nadiri-Rosen specification (3.26) has to do with the characteristics of the adjustment parameters. In (3.26) the elements of B are constrained to be constant. In (3.37) λ is endogenous and dependent on the rate of return r and technological parameters, i.e. the derivatives of the cost function. Also, the adjustment parameters in

[28] And also assuming that all second partial derivatives with respect to x and $\dot{x}$ are constants.

the Nadiri-Rosen model are specified for all inputs, whereas in the present model they are only given for the quasi-fixed factors.

The cost of adjustment model is summarised by the restricted cost function (3.31), the short-run demand equations for the variable inputs (3.32) and the investment function (3.37) and (3.38). As mentioned previously, this model includes both internal and external adjustment costs. In the case of purely internal adjustment costs, the only difference is that the adjustment coefficient is somewhat simplified in that $\partial^2 c(\dot{x})/\partial \dot{x}^2 = 0$. On the other hand, were all adjustment costs assumed to be external to production activity, the model would simplify considerably: the variable cost function and the equations for the variable factors would no longer contain the net investment terms[29] and the last term in the adjustment parameter would contain two derivatives instead of four.

The system of equations derived above completely characterises the dynamic time paths of factor demand and from these equations all the elasticities discussed earlier can be derived. The short-run elasticities with respect to the variable inputs are defined as the response of the variable factors when x is fixed ($x = \bar{x}$), whereas the long run is given by the total impact when x has fully adjusted to x^* and $\dot{x} = 0$. Intermediate-run elasticities can be defined as the impact when x has partially adjusted, for example after the first period. Using these definitions, the short- ε^S, intermediate- ε^I and long-run ε^L price elasticities can be derived as:

$$
\begin{aligned}
\varepsilon_{ij}^S &= [\partial v_i/\partial P_j|_{x=\bar{x}}]\ [P_j/v_i] \\
\varepsilon_{ij}^I &= [\partial v_i/\partial P_j|_{x=\bar{x}} + (\lambda\ \partial v_i/\partial x^*)(\partial x^*/\partial P_j)]\ [P_j/v_i] \qquad (3.40) \\
\varepsilon_{ij}^L &= [\partial v_i/\partial P_j|_{x=\bar{x}} + (\partial v_i/\partial x^*)(\partial x^*/\partial P_j)]\ [P_j/v_i].
\end{aligned}
$$

Substitution elasticities and elasticities involving other exogenous variables can be derived in a similar manner using the definitions given earlier. The effects of scale, for example, are obtained by taking the appropriate derivatives with respect to Q. In order to implement the model empirically, two further steps must be taken. Firstly, the continuous time adjustment process given in (3.37) must be converted into discrete time intervals.[30] This is done by assuming that changes in the capital stock K can be represented by the discrete

[29] These would, in fact, be identical with those for the partial static equilibrium model. Compare with equations (3.16) and (3.18) in Section 3.1.3.

[30] For an example of a model derived directly from a discrete time specification of the intertemporal optimisation problem, see Prucha and Nadiri (1982).

approximation $K_t - K_{t-1} = \Delta K$. Secondly, it is necessary to specify a functional form for the restricted cost function. Again, it is advantageous to use a flexible functional form. Choice of functional form has, as in the case of the partial static equilibrium model, distinct implications for the estimation of the model and the derivation of the production elasticities. For example, the Euler condition cannot be solved analytically for the translog function. However, as Pindyck and Rotemberg (1982) point out, one could estimate the first order conditions directly for any functional form. The adjustment coefficient cannot, however, be calculated and the model becomes rather complicated to estimate. On the other hand, if a quadratic approximation is used for the cost function the investment function and adjustment coefficient can be obtained from (3.37) and (3.38), at least in the case of one quasi-fixed factor. As Berndt, Fuss and Waverman (1980) point out, the quadratic form has two major advantages. Firstly, the linkage between the short- and long-run responses is simplified because the second partial derivatives are constants. Secondly, the characterisation of the optimal path for the quasi-fixed factors is globally optimal since the quadratic approximation underlying the differential equations is linear.

Finally, it is also worth noting the similarity between the partial static equilibrium model discussed in Section 3.1.2 and the short-run components of the cost of adjustment model. Both are based on the assumption of cost-minimisation with respect to the variable inputs given that certain factors of production are fixed in the short run. The short-run production relationships are in both cases specified by a restricted variable cost function. The major difference is that in the formulation of the partial static equilibrium model presented earlier, no adjustment costs are specified. If one assumes external adjustment costs only, the short-run factor demand equations are identical. Of course, no investment function is estimated in the partial static equilibrium formulation. Instead the optimal capital stock is calculated by applying the envelop condition to the cost function. Solving this equation for the optimal capital stock K^* yields precisely the relationship obtained for the cost of adjustment model (3.39). The equation for the short- and long-run elasticities derived from these two models are also the same. However, the estimated coefficients, and thus the values of the elasticities, need not be identical. The cost of adjustment model contains the additional information of the investment function, and with internal adjustment costs, the investment term in the cost function, so that the estimated parameters can be quite different.

Since the cost of adjustment model discussed above is a relatively new development, only a few empirical implementations can be cited. Berndt, Fuss and Waverman (1980) estimate a non-constant returns to scale version of the internal adjustment cost model for total U.S. manufacturing and for 2-digit manufacturing sectors. Three variable inputs are considered: energy, labour and materials, and one quasi-fixed factor, capital. For total manufacturing, energy is further disaggregated into electricity and fuels. The authors find that adjustment is not instantaneous and that the dynamic model outperforms a static specification in forecasting post-1974 energy demand. The same model is applied to a combination of time-series and cross-section data for Canadian manufacturing by Denny, Fuss and Waverman (1980). Here, a two-stage procedure is employed in order to analyse the demand for various forms of energy as well as for capital, labour and materials. More recently, in a study by Morrison (1986), cost of adjustment models with non-static expectations are estimated. Here, too, capital is treated as quasi-fixed, while energy, labour and materials are specified as variable factors. Models based on various expectational assumptions are formulated and estimated. In both of these studies, however, the differences between the short- and long-run own price elasticities for the variable factors are surprisingly small.

An application of a constant returns to scale variant to U.S. manufacturing can be found in Morrison and Berndt (1981). Both the four factor KLEM model and a five factor model are estimated. In the latter case, aggregate labour is decomposed into skilled and unskilled. Skilled labour is treated alternatively as a variable factor and as a quasi-fixed factor along with capital. However, in order to derive and estimate the model with two quasi-fixed factors, the authors are forced to assume that skilled labour and capital are independent in production. The estimated elasticities are found to be rather sensitive to the specification of inputs as variable or quasi-fixed. Only in the two quasi-fixed factor case are there substantial differences between short- and long-run elasticity estimates.

Another example of an empirical study based on a cost of adjustment model with two quasi-fixed factors can be found in Pindyck and Rotemberg (1982). The authors use a translog functional form and directly estimate the Euler conditions for the two quasi-fixed factors, capital and labour. They find, however, that adjustment costs for labour are insignificant. A different approach is taken by Mohnen, Nadiri and Prucha (1986) in a study of U.S., German and Japanese manufacturing. Labour and materials are specified as variable factors, while capital and R & D are considered quasi-fixed. The cost function is specified as a quadratic approximation and the model is solved by assuming a constant discount rate.

Although the empirical results thus far obtained on the basis of cost of adjustment models may not be wholly satisfactory, these models are, in theory, a marked improvement over the models generally used in empirical studies of industrial factor demand. The advantages of the cost of adjustment model in analysing dynamic factor demand are manifold. The model is very rich in economic structure which allows the testing of a wide variety of hypotheses concerning production relationships. Since the model is based on intertemporal optimisation, the path of adjustment to long-run equilibrium is clearly defined in terms of short-, intermediate- and long-run elasticities. Further, estimation of the model provides information as to the speed of adjustment. As opposed to other dynamic specifications the speed of adjustment is time-varying and dependent on the exogenous variables as well as on the technological characteristics of the production process.

It is evident that the cost of adjustment model provides a more economically realistic description of the factors influencing input demand than the majority of the models discussed previously. The very complexity of the model, however, makes it somewhat more difficult to implement empirically. The investment function is nonlinear and the number of parameters to be estimated is large. Furthermore, the existence of lagged variables in the demand equations may give rise to estimation problems, particularly in the presence of autocorrelated error terms. There is some indication of this in the applications cited above as well as in preliminary estimations based on Swedish data.[31]

Another problem is of an economic nature and concerns the assumption of static expectations with regard to input prices and output level. Since the theoretical model is based on intertemporal cost-minimisation, the firm's expectations concerning the future values of these variables play a decisive role in the derivation of the investment function. A more realistic specification of these expectations would therefore be preferable.[32] Finally, although the explicit intoduction of internal and external adjustment costs provides some explanation to the delays involved in the adjustment of quasi-fixed factors to optimal levels, these costs do not seem totally to account for the inflexibility of the capital stock.

3.3 *Conclusions*

In the preceding sections alternative formulations of factor demand models have been surveyed. Particular consideration was given to those models found in the economic literature that are suitable for the study of the effects of input price changes on factor demand and production in different time perspectives.

[31] See Dargay (1984).

[32] There have been attempts to relax the assumptions of static expectations in empirical studies. Pindyck and Rotemberg (1982) estimate a model consistent with rational expectations and Shankerman and Nadiri (1982) use data on investment plans as expectational variables. Morrison (1986), experiments with various expectational assumptions.

An attempt has been made to assess the various models in terms of their economic content as well as their performance empirically. This assessment serves as a basis for further empirical study of factor demand in Swedish industries.

Analysis of factor demand in Swedish manufacturing has thus far been based primarily on static equilibrium models of the type discussed in Section 3.1.1. The majority of the existing studies to date rely on long-run models and time-series data for their empirical analysis. Although the results of these studies are in many ways quite plausible, there is little concensus concerning the magnitude of the price response or the character of the substitution relationships. On the contrary, there is a strong indication that the estimated elasticities are highly sensitive to the underlying assumptions about the representation of technology, the particular functional form chosen for estimating purposes and the observation period of the statistical sample. Although further investigation on the basis of static equilibrium models may resolve some of these difficulties, the basic problem of the inconsistency between the assumption of long-run equilibrium or instantaneous adjustment and the empirical data still remains. Because of this the interpretation of the resulting elasticities in terms of long- or short-run responses becomes extremely tenuous. Further analysis of factor demand should therefore be based on theoretical models in which there is a clear distinction between short- and long-run production relationships and which conform more realistically to the available empirical data.

The partial static equilibrium model discussed in Section 3.1.2 provides a reasonable alternative, at least for studying short-term price-responsiveness and substitution possibilities. It also provides a comparatively simple method of retrieving the long-run relationships with the added advantage that no assumptions need be made concerning the nature of the adjustment process. Although this model contains less information about the path and speed of adjustment than the truly dynamic specifications, its simplicity makes it far easier to implement empirically.

Of the dynamic factor demand models, the single equation formulations presented in Section 3.2.1 can be dismissed on the grounds that they do not necessarily provide estimates of input demand relationships which are consistent with the observed levels of other production factors and because they provide no information about substitution relationships. This is because the demand for each production factor is considered in isolation from, instead of in conjunction with, the demand for other inputs in the production process. Both the variants of the Nadiri-Rosen model discussed in Section 3.2.2 and the cost of adjustment model presented in Section 3.2.3 represent definite improvements since the demands for all factors of production are treated simultaneously. The cost of adjustment model has the advantages of being derived directly from intertemporal optimisation as well as explicitly incorporating the costs of adjustment to equilibrium. The results obtained from the empirical implementation of the two models also tend to favour the cost of adjustment model.

For the reasons given above, the analysis of the determinants of factor demand in Swedish industries is based on three model formulations: the full static equilibrium model, the partial static equilibrium model and the cost of adjustment model. Of the latter two models, the partial static equilibrium model has the advantage of being much simpler to estimate but the latter contains far more information about the dynamics of the adjustment process. Furthermore, the results obtained from the partial static equilibrium model should provide a good starting point for estimation of the cost of adjustment model as both are based on the restricted variable cost function. The estimates of the full static equilibrium model are included as a basis of comparison with the results of these two models as well as with those of other studies.

The available statistical data described in chapter II—time series observations of input levels and factor prices—should provide an adequate empirical basis for the estimation of these models. Questions concerning the stochastic specification of the models, estimation techniques and the statistical problems involved will be taken up in conjunction with the empirical studies in the following chapters.

IV. ESTIMATION OF A STATIC EQUILIBRIUM MODEL

The majority of empirical studies of production relationships have been based on some type of static equilibrium model. This is true for the early studies based on Cobb-Douglas production functions as well as for the multitude of cost function studies based on flexible functional forms that have been published since the early seventies. A number of applications of these models to Swedish manufacturing can be found,[1] including a few of my own studies.[2] The model presented in the following sections has previously been estimated for Swedish manufacturing sectors for data covering the years 1952—1976, and the results have been reported in Dargay (1983b). In the present analysis, the observation period is extended to 1983. Since the economic situation has changed considerably since 1974, one might expect estimates based on the longer time period to be somewhat different than those based primarily on pre-1974 data.

4.1 *The Econometric Model*

The theoretical derivation of the long-run equilibrium model has been presented in Section 3.1.1. For purposes of empirical implementation it is necessary to specify an explicit functional form for the cost function. It is desirable to choose a functional form which places minimal a priori restrictions on the characteristics of the production function, and in particular on the elasticities of substitution. Several functional forms fulfilling these requirements have been proposed in the literature; among these are the translog, generalised Leontief, generalised Cobb-Douglas, quadratic and the square-root quadratic.[3] All of these forms provide a local approximation to an ar-

[1] For a recent study, see Bergström and Panas (1985).

[2] See Dargay (1983b) and (1983c).

[3] The generalised Leontief, Cobb-Douglas and square-root quadratic forms have been introduced by Diewert (1971, 1973, 1974) and the translog by Christensen, Jorgenson and Lau (1973).

bitrary cost function, but their global properties are not generally well known and there are no theoretical grounds for choosing among them.[4] In the following the translog form is chosen because its characteristics are comparatively well known[5] and in order to facilitate comparison with other studies which are most commonly based on this functional form.

The translog cost function can be interpreted as a second order approximation in logarithms to an arbitrary cost function. Using our previous nomenclature for costs, prices and output and letting t be a time trend representing technical change, the translog cost function takes the following form

$$\text{Ln } C = \alpha_o + \alpha_Q \text{Ln}Q + \Sigma_i \alpha_i \text{Ln}P_i + \alpha_t t + 1/2[\gamma_{QQ}(\text{Ln}Q)^2 + \gamma_{tt}t^2 + \Sigma_i \Sigma_j \gamma_{ij} \text{Ln}P_i \text{Ln}P_j] + \Sigma_i \gamma_{iQ} \text{Ln}Q\text{Ln}P_i + \Sigma_i \gamma_{it} t \text{Ln}P_i \qquad i,j = E,L,M,K \qquad (4.1)$$

where $\gamma_{ij} = \gamma_{ji}$. In order to assure that the underlying production function is well-behaved, the cost function must be homogeneous of the first degree in input prices. This requirement assures that, for a given level of output, an equi-proportionate change in all factor prices results in a proportionate change in total production costs. This implies the following relationships among the parameters:

$$\Sigma_i \alpha_i = 1$$

$$\Sigma_i \gamma_{ij} = \Sigma_j \gamma_{ij} = \Sigma_i \gamma_{iQ} = \Sigma_i \gamma_{it} = 0 \qquad i,j = E,L,M,K. \qquad (4.2)$$

Without any further restrictions on the parameters, the cost function as specified in (4.1) above allows for non-constant returns to scale, non-homotheticity and non-neutral technical change. The translog approximation is homothetic if it could be written as a separable function of output and factor prices, that is if $\gamma_{iQ} = 0$ for all i. In terms of the cost function, homotheticity implies that the cost-minimising input-mix is determined solely by input prices and technical change and is independent of the level of produc-

[4] The choice of functional form has been the subject of a number of articles. See, for example, Berndt and Khaled (1979) and Appelbaum (1979).

[5] A Monte Carlo study by Guilkey and Lovell (1980) indicates that the translog function provides adequate estimates of quite complex technologies. The accuracy of the estimates decreases, however, when the elasticities of substitution differ greatly from unity.

tion. Further, a homothetic cost function is homogeneous if the elasticity of cost with respect to output is constant, i.e. if $\gamma_{QQ} = 0$. Given the above restrictions, the degree of homogeneity of the cost function is determined by the coefficient α_q. Thus, if $\alpha_Q = 1$, the cost function is linearly homogeneous and the underlying technology is characterised by constant returns to scale. Finally, the inclusion of the γ_{it} terms in the cost function allows for biases in technical change, so that even with constant factor prices, the cost-minimising factor-mix can be altered by technical change. Technical change is said to be Hicks neutral if $\gamma_{it} = 0$ for all i.

The assumptions of Hicks neutrality, homotheticity or constant returns can be tested using a simple likelihood ratio test, $-2Ln(L_r/L_u)$, where L_r and L_u are the maximum likelihood values for the restricted and unrestricted specifications respectively. This statistic is asymptotically distributed as Chi-square under the null-hypothesis of the more restricted model with degrees of freedom equal to the number of parameters being tested.

Although it is, in principle, possible to analyse the structure of production by estimating the cost function alone, the number of parameters to be estimated is quite large and multicollinearity among exogenous variables is surely to be a problem, resulting in imprecise parameter estimates. It is common practice, therefore, to base empirical studies not on the cost function alone, but in conjunction with the derived demand equations. These are derived in terms of cost shares S_i according to Shepard's Lemma by partial logarithmic differentiation of the cost function with respect to prices. We have

$$S_i = \alpha_i + \Sigma_j \gamma_{ij} LnP_j + \gamma_{iQ} LnQ + \gamma_{it} t \qquad i,j = E,L,M,K \qquad (4.3)$$

where $\Sigma_i S_i = \Sigma_i P_i X_i / C = 1$.

The characteristics of the underlying production technology, in terms of price, substitution, scale and technology elasticities, can be derived for the translog function according to the formulae presented in Section 3.1.1. The Allen partial elasticities of substitution are calculated as

$$\sigma_{ij} = (\gamma_{ij} + S_i S_j) / (S_i S_j) \qquad i \neq j$$

and (4.4)

$$\sigma_{ii} = (\gamma_{ii} + S_i^2 - S_i) / S_i^2 \qquad i,j = E,L,M,K$$

while the price elasticities of demand for the factor inputs according to (3.8) become

$$\varepsilon_{ij} = S_j \, \sigma_{ij} \qquad i,j = E,L,M,K. \tag{4.5}$$

Since $\gamma_{ij} = \gamma_{ji}$, the elasticities of substitution are symmetric. The cross-price elasticities ε_{ij} are, however, not. We see from equation (4.5) that the elasticity of demand for factor i with respect to the price of factor j depends on the share of factor j in total costs and vice-versa for the elasticity of demand for factor j. The cross-price elasticity ε_{ij} will thus necessarily be greater than ε_{ji} if $S_j > S_i$.

Furthermore, it should be noted that the translog function does not constrain these elasticities to be constant. As functions of the cost shares, they are dependent on the level of factor prices, output and technology. Thus, the estimated elasticities vary over the observation period. A disadvantage of the translog function is that one cannot test for global zero substitution between factor pairs directly from the estimated equations. It is clear from expression (4.4) that the elasticity of substitution between factors i and j is equal to unity if $\gamma_{ij} = 0$. Thus if all $\gamma_{ij} = 0$, the translog cost function corresponds to a Cobb-Douglas production structure.

The economies of scale of the production technology can be calculated as the inverse of the cost elasticity given in (3.10). For the translog function, we have:

$$\eta_Q = 1/\eta_c = 1/(\alpha_Q + \gamma_{QQ}LnQ + \Sigma_i\gamma_{iQ}LnP_i) \qquad i,j = E,L,M,K. \tag{4.6}$$

Using this relation, the output elasticities for the various inputs are:

$$\upsilon_{iQ} = \eta_c + \gamma_{iQ}/S_i. \tag{4.7}$$

The effects of technical change on the individual inputs can be measured by the technology elasticities (3.13), which give the annual percentage change in the use of each production factor resulting from disembodied technical progress:

$$\tau_{it} = \gamma_{it}/S_i + (\alpha_t + \gamma_{tt}t + \Sigma_j\gamma_{jt}LnP_j) \qquad i,j = E,L,M,K. \tag{4.8}$$

The term in parenthesis represents the neutral component of technical change which affects all factors of production equally, while the biases are determined by the first term on the r.h.s. Technical change is said to be factor i-using if $\gamma_{it} > 0$, i-neutral if $\gamma_{it} = 0$ and i-saving if $\gamma_{it} < 0$. Finally, the growth rate of total factor productivity is obtained from (3.14) as

$$\Delta TFP = -(\alpha_t + \gamma_{tt}t + \Sigma_i\gamma_{it}LnP_i)/\eta_Q. \tag{4.9}$$

It can be shown that ΔTFP can be written as the negative of a cost-share weighted average of the technology elasticities given above divided by the returns to scale, η_Q.

4.2 *The Empirical Results*

Empirical implementation of the translog cost function entails estimation of the input demand equations (4.3) together with the cost function (4.1) subject to the restrictions imposed by linear homogeneity in prices (4.2). The stochastic model includes the specification of additive disturbances for each of the equations. These disturbances may be interpreted alternatively as random errors in cost-minimising behaviour or as the random influence of excluded explanatory variables. In either case, it is likely that these disturbances are related for the different equations and allowance should be made for non-zero contemporaneous correlation among them.

Letting $\tilde{\omega}_t$ denote the vector of error terms for the five equations, we assume that $\tilde{\omega}_t$ is joint normally distributed with zero mean and variance-covariance matrix Ω, that is

$$\tilde{\omega}_t \sim N(0,\Omega) \text{ s.t. } E(\tilde{\omega}_t\tilde{\omega}_t') = \delta_{ts}\Omega\,\delta_{ts} = \begin{matrix} 1 \text{ if } t = s \\ 0 \text{ if } t \neq s \end{matrix} \qquad (4.10)$$

This specification implies that the error terms have a constant variance-covariance matrix and allows for non-zero correlation between contemporaneous error terms of the different equations. Zero intertemporal correlation is assumed between all error terms.

Since the share equations must sum to unity, the estimated disturbance covariance matrix is singular. The most common method of dealing with this problem is to delete one of these equations from the system and choose an estimation procedure which is invariant to which equation is deleted. In this study a full-information maximum likelihood estimation procedure is employed.[6] The equation for intermediate goods is dropped from the estimation and the coefficients for that equation are calculated from the identities given in (4.2).

Various versions of the translog cost function presented in Section 4.1 have been estimated for total Swedish manufacturing and for the 9 manufacturing

[6] A micro-computer version of CONRAD was used which entails maximisation of the concentrated likelihood function. See Jansson and Mellander (1984).

subsectors as defined earlier. The data include the annual observations on quantities, costs and prices for energy, labour, intermediate goods and capital and gross output in constant prices for the time period 1952—83 described in Chapter II. The results based on these data, however, had to be rejected. The concavity requirements were not met for any of the observations for any of the industries. In all cases the own-price elasticities for capital were positive. As these results are totally unacceptable from an economic point of view, they are not presented here.

The inability of the long-run equilibrium model to provide a reasonable description of technology indicates that the assumptions on which the model are derived are not met in the empirical data sample. Recall that the cost function and the demand equations describe fully optimised equilibria, so that estimation of the model must be based on observations of equilibrium input-ouput-price combinations. Hence, the data sample must be such that all factors of production are at their fully adjusted cost-minimising levels at the current price relationships and level of output demand. In terms of our annual data this implies that full adjustment to year-to-year price and output changes is achieved within one year. As discussed earlier, our knowledge of the investment process gives us good cause to question this assumption, as did our examination of input-output and factor price data in Chapter II. The poor performance of the static equilibrium model also gives us empirical evidence of the unsuitibility of the assumption of instantaneous adjustment, at least in the case of capital.

It is interesting to note that these results are not at all in conformity with those obtained from the estimates based on the 1952—76 time period.[7] There it was found that the concavity requirements were generally met and that the own-price elasticities for capital lie in an interval between −.1 and −.4. A probable explanation for these results can be found in the considerable changes that have occurred in the economic environment since 1974. Recall from Chapter II the large fluctuations in the price of capital since the latter seventies (figure 2.1) and the increase in capital intensity (figures 2.2—2.11). The steady increase in output demand that characterised the 50's and 60's halted in 1974. The subsequent years witnessed wide fluctuations in real output and even absolute decreases, which were virtually unheard of in the previous two decades. It can be noted, too, that the price of capital increased relative to the price of labour during this period, while the reverse was true for the years prior to 1974.

In some sense, it would appear that the assumption of static equilibrium

[7] See Dargay (1983b).

may not be entirely unreasonable for the period before 1974, even if we reject the instantaneous adjustment of capital. The gradual development in factor prices and the more-or-less continual growth in output demand would require only gradual increases in the capital stock, so that adjustment costs would be minimal. The irreversibility of investment decisions (or lack of second hand markets for capital goods) would also be irrelevant, since output was continually expanding and there was no need to disinvest. Finally, because the firm's expectations of future prices and output demand were more likely to be correct, the importance of delivery lags is also limited. Firms would be able to order new capital goods in advance, so that at every point in time the actual capital in place would correspond with the desired.

If we accept these arguments, it would be justifiable to apply the static equilibrium model to pre-1974 data. However, as the economic environment has changed so since then, it is difficult to apply the results obtained to the present situation. Most econometric models estimated on the basis of pre-1974 data perform very poorly in predicting post-74 relationships, and the same is surely the case for this model.

Since our main reason for rejecting the static equilibrium model is that it results in own-price elasticities for capital which are contrary to economic theory, it would be interesting to see if we could improve these results by slightly changing the assumptions, and particularly those concerning the price of capital. As shown in equation (2.2) in the appendix to chapter II, the user cost of capital is based on the price of investment goods, the expected rate of return and the rate of depreciation. In the construction of capital costs, the expected rate of return was defined as the return on safe investments, or the real interest rate. Alternatively, we can assume that firms base their investment decisions not on the actual short-run possible rate of return defined as the real interest rate—which as we have seen has fluctuated enormously due to changes in the inflation rate during the last ten years, but instead on a long-run rate of return, which we can assume to be constant. Since the depreciation rate is constant, variations in the user cost of capital based on this r will be totally determined by variations in the price of investment goods. Changing the definition of the expected rate of return has minimal consequences for the price of capital for the pre-1974 period, while it serves to smooth out and damper the development after 1974.

By using the long-run expected rate of return, our data sample may be more consistent with the equilibrium model. However the implications of doing so are not totally obvious. What we are saying is basically that we have the wrong data for the model, and instead of changing the model to suit the data we are taking the alternative course of trying to define the data to be more consistent with the assumptions of the model. The observed input quantities are assumed to be equilibrium values, not at the actual rate of interest, but at some long-run expected rate of return—a sort of Walrasian equilibrium capital price. If capital could be adjusted completely and without added cost to all price changes, then cost-minimisation would imply that the firm would react even to temporary fluctuations in the interest rate. Assuming, instead, as we do that it does not, would imply in some sense that it cannot. In other words, adjustment of the capital stock cannot be totally costless. Although the short-run inflexibility of the capital stock is not explicitly built into the model, it is implicit in our definition of the cost of capital. In the remainder of this chapter, we will present the results of the static equilibrium model estimated on the basis of this definition of the cost of capital. As we shall see, they are a clear improvement over those obtained on the basis of the original user cost measure.

The translog model was estimated on the basis of these data under three alternative assumptions concerning the effects of production scale and technical change: 1. non-homothetic and non-neutral, 2. homothetic and non-neutral and 3. non-homothetic and neutral technical change. We can thus test these hypotheses using a likelihood ratio test. The log-likelihood values and the test statistics are shown in table 5.A.1 in the appendix to this chapter. We see that likelihood ratio test statistics are all well above 12.84, the .005 χ^2 critical value with 3 degrees of freedom. We can therefore reject the assumptions of homotheticity and Hicks neutral technical change for all industries, and will concentrate on the results obtained from the most general model.

The estimated parameters along with their asymptotic standard errors are shown in tables 5.A.2 and 5.A.3. We see that the majority of the estimated parameters are significantly different from zero in all of the industries. Judging from the significance of the coefficients of the time trend, technical change has had a substantial effect on over-all factor usage.

Goodness of fit measures for the individual equations are also given in the tables. These are calculated as the squared cosine of the angle between the actual and predicted values of the endogenous variables (if the equation contains no intercept) or between the same vectors measured as deviations from their respective means (if the equation contains an intercept).[8] The latter variant applied here is equal to the squared correlation coefficient between the actual and predicted values of the endogenous variables. The advantage of this measure over R^2 is that it lies within the interval (0—1), whereas the R^2 for an individual equation of a simultaneous system can take on negative values. Interpretation of $\cos^2$ is similar to R^2, a larger value indicates better fit.

From the table, we see that the $\cos^2$ are greater than .8 and in many cases

[8] See Haessel (1978) for a discussion of this measure.

above .9 for all of the individual equations. Judging from these values, we can conclude that the explanatory power of the model is rather good. It should also be mentioned that the Durbin-Watson statistics are somewhat low for a few of the estimated equations, but overall it does not appear that serial correlation is particularly severe. Attempts at allowing for a more general stochastic specification which allows for intertemporal as well as contemporaneous correlation between the error terms have not proved very fruitful. In most cases, serial correlation could be rejected or the estimated autoregressive process proved not to be stationary.

In order to analyse price-responsiveness and factor substitution possibilities we compute the Allen partial elasticities of substitution (σ_{ij}) and the own-price elasticities (ε_{ii}) for all observations according to equations (4.4) and (4.5). Since the resulting elasticities vary somewhat over the observation period, the values for 1960, 1970 and 1980 are presented.

The own-price elasticities of demand for energy, labour, materials and capital are shown in table 4.1. The values in parentheses are not significantly different from zero at the 5% level.[9] In accordance with cost-minimising principles we would expect these elasticities to be negative, which they are in the overwhelming majority of cases. The only instances of positive elasticities which are significantly different from zero are those for energy for the mid-years of the data sample in the Total Manufacturing and in the Food, Paper and Mineral industries. Regarding the own-price elasticities for capital, we find that although these are all negative, they are not significantly different from zero in some instances, and particularly for the earlier years of the data sample. As opposed to the estimates referred to earlier, the concavity requirements are generally met for all industries.

Furthermore, the estimated own-price elasticities are less than unity for all inputs and all sectors, indicating that input demand is inelastic. Although the elasticities do vary for the individual industries, a few general trends are apparent. Firstly, we find that the own-price elasticities for energy first decrease and then rise again over the observation period, while those for capital generally appear to increase. The decreasing energy elasticities correspond with a period of declining real energy prices which suggests that energy price rises have a greater effect on energy demand than do price cuts. For labour and materials, on the other hand, the elasticities show no appreciable time-trends. The variation of the elasticities over time actually follows from the definition of the elasticities and the characteristics of the data. With a few manipulations

[9] Approximate standard errors are calculated under the assumption that the input shares are non-stochastic.

one can see from (4.4) and (4.5) that the effect on the own-price elasticities of an identical absolute change in cost shares will be greater for those inputs with small cost shares. As the cost shares for energy and capital are very small in relation to those for labour and materials, the elasticities will necessarily be less stable as cost shares change over time.

The second point concerns the relative price-sensitivity of the various inputs. For the earlier part of the observation period, we find energy to be the most price-sensitive production factor in about half of the industries, whereas in 1980 it is so in 7 out of 10. The own-price elasticity for labour is under —.2 for all but 3 sectors—Textiles, Printing and Minerals. In virtually all other industries capital is more price-sensitive than labour. Regarding material inputs, we find that the price elasticities are very near zero in the majority of industries. The only exceptions are again Textiles, Printing and Minerals. The predominant ranking of inputs in terms of price sensitivity is $E>K>L>M$. It can be noted that these results agree rather well with those obtained for the 1952—76 time period.[10]

Finally, one further point can be noted regarding the estimated elasticities. The results indicate that energy is less price-elastic in aggregate manufacturing than it is in virtually all of the component industries. For the other inputs, however, the elasticities for the aggregate lie within the range given by the individual subsectors. Although the evidence is somewhat conflicting, it does suggest that one should be extremely careful of applying results obtained for total manufacturing to individual industries.

Next, we turn to an examination of the substitution possibilities among inputs. For this purpose the Allen-Uzawa elasticity of substitution is calculated for each input pair. These elasticities are shown in table 4.2. We see that although the substitution relationships between the inputs vary from sector to sector, we do find a good deal of agreement regarding the nature of these relationships for at least some of the input pairs.

The results indicate that capital and labour are substitutes in virtually all of the industries, which is well in keeping with the results of other studies.[11]

[10] Although homotheticity and neutral technical change have been rejected, it would be interesting to investigate the effects of the imposition of these restrictions on the estimated elasticities. Although the effects vary from sector to sector, a few predominant trends deserve to be commented upon. Firstly, in both cases, many of the own-price elasticities increase somewhat when the restrictions are imposed. This is particularly apparent for capital and labour for which the price-sentitivity increases in 9 out of 10 sectors. In many of the industries, capital is now the most price-elastic input. Secondly, the different assumptions have opposite effects on the elasticities for energy: homotheticity generally decreases the elasticities, while neutral technical change tends to increase them.

[11] See for example Bergström and Panas (1985).

The elasticity is quite near unity in about half of sectors, but it is rather smaller than one would expect in a few cases. Further, in 6 of the 9 industries we may conclude that energy and capital are complements, while only the Rubber industry exhibits a high degree of substitutibility. The results obtained for Total Manufacturing suggest that energy-capital complementarity switches to substitutibily for the more recent years of the data sample, a result not noted for any of the individual industries. We see also that energy and labour are substitutes in total manufacturing as well as in 8 of the 9 subsectors. For energy and materials, however, the results are not at all clear cut: Total Manufacturing shows complementarity, while substitutibility predominates in the individual subsectors. Generally, we find the substitution elasticities between materials and the remaining inputs to be rather low and to show no predominant sign pattern. Again, these results correspond rather well to those based on the 1952—76 time period.

The estimated output elasticities and the implied returns to scale of the underlying production function are presented in table 4.3. The results indicate that a marginal change in output demand is met by a less than proportionate change in the use of most inputs ($\upsilon_{iq} < 1$), suggesting increasing returns for all factors. Only intermediate goods exhibit decreasing returns in a few cases ($\upsilon_{MQ} > 1$), but the output elasticities for this input are generally quite near unity. The negative output elasticities found for energy in many of the industries can obviously not be interpreted in any reasonable manner, and thus must be rejected. For Total Manufacturing and four of the subsectors —Paper, Chemicals, Minerals and Metals—all of the elasticities take on quite acceptable values. The general pattern found in these industries is $\upsilon_{MQ} > \upsilon_{EQ} > \upsilon_{LQ} > \upsilon_{KQ}$. Finally, the results indicate increasing returns to scale ($\eta_Q > 1$) for all industries.

The final results presented here concern the effects of technical change on factor use and the estimated rates of total factor productivity growth. These are shown in table 4.4. The effects of technical change on the individual production factors are measured as the annual percentage change in the use of the particular input resulting from technical progress. We see from the table that technical change has led to a decrease in the use of labour and an increase in the use of capital in the majority of industries, implying an increase in the capital-labour ratio. The results for Total Manufacturing indicate that technical change has led to a decrease in labour intensity of about 3% per year and an increase in capital intensity of about 1.5% per year. The effects of technical change on energy and materials vary from sector to sector. The decrease in energy use noted for Total Manufacturing does not seem to agree with the results obtained for the individual industries, and can reflect changes in the composition of manufacturing output towards the less energy-intensive sectors. From the estimated γ_{it} parameters, the most prevelant pattern is that technical change has been labour and energy saving and capital and materials using.

The rate of growth in total factor productivity is also shown in the table. We see that this appears to have been rather constant over the observation period. A slow-down in total factor productivity growth during the seventies is thus not supported by our estimates. Taken over the entire period, total factor productivity has increased on an average of about 1.3% per year. The most rapid growth is noted for the Printing industry and lowest rates are found in the Food and Textile industries.

4.3 *Conclusions*

The application of the static equilibrium model to Swedish manufacturing has given us a quite plausible description of production technology. In order to achieve this, however, it has been necessary to make some additional assumptions concerning the user cost of capital, specifically the assumption that the firm bases investment decisions on a long-run constant rate of return instead of on costs as reflected by the real interest rate. We have seen that use of the capital price based on the real interest rate as defined in Chapter II has led to economically unrealistic estimates. Although this gives reason to believe that our 'new' data more closely reflect long-run relationships, we cannot be perfectly satisfied as there do seem to be some inconsistancies between the assumptions of the model and the evidence provided by casual observation of the empirical data. Specifically, we know that plants have not been operating at full-capacity during the post-74 recession, which is evidenced by the rise in capital intensity during this period. Our model, however, cannot explain changes in capacity utilisation. On the contrary, we assume that the observations represent long-run input-output relationships which by by definition are characterised by full-capacity utilisation.

Regarding the estimated elasticities, energy and capital appear to be the most price-sensitive production factors, while materials is the least responsive to price changes. Although the substitution relationships between inputs tend to vary among the individual industries, we find a predominance of capital-labour and energy-labour substitutability and energy-capital complementarity. Further, the results indicate increasing returns to nearly all inputs and over-all increasing returns to scale. Technical change is shown to have been labour-saving and capital-using in the majority of industries. The estimates of total factor productivity growth generally seem quite low and give no evidence of a slow-down during the post-1974 period. As the indicated effects of production scale and technical change are implausible in many instances, these results should be interpreted with caution. The high degree of collinearity over much of the observation period between output and the time trend used to represent technical change makes it difficult to distinguish between the effects of scale and technological progress.

Table 4.1. Own-Price Elasticities. Static Equilibrium Model.

	Year	ε_{EE}	ε_{LL}	ε_{MM}	ε_{KK}
Food	60	−.13	−.08	−.01	(.05)
	70	.35	−.15	−.02	(−.08)
	80	−.40	−.24	−.06	−.31
Textiles	60	−.47	−.35	−.15	(−.01)
	70	−.30	−.35	−.18	−.22
	80	−.59	−.35	−.19	−.39
Paper	60	(.09)	−.19	(.00)	(.07)
	70	.35	−.23	(−.01)	−.20
	80	−.21	−.18	(−.01)	−.24
Printing	60	−.36	−.28	−.19	(−.02)
	70	−.48	−.28	−.19	(−.06)
	80	−.81	−.28	−.17	(−.01)
Rubber	60	−.81	(−.05)	(.01)	−.44
	70	−.78	−.12	(.00)	−.51
	80	−.85	−.11	(.00)	−.52
Chemicals	60	−.44	(.01)	(.07)	−.17
	70	−.35	(−.04)	(.07)	−.19
	80	−.51	(.03)	(.07)	−.17
Minerals	60	−.22	−.26	−.49	−.68
	70	.31	−.26	−.46	−.69
	80	−.28	−.25	−.47	−.70
Metals	60	−.29	(−.01)	(.02)	(.10)
	70	(−.10)	(−.02)	(.02)	(.10)
	80	−.34	(.02)	(.02)	(−.01)
Engineering	60	−.67	−.07	−.02	−.10
	70	−.55	−.09	−.03	−.10
	80	−.69	−.07	−.03	−.21
Total	60	(.02)	−.09	.05	(−.08)
	70	.40	−.12	.05	−.14
	80	−.18	−.11	.04	−.25

Note: Values in parenthesis are not statistically different from 0.0 at the 0.05 significance level.

Table 4.2. Elasticities of Substitution. Static Equilibrium Model.

	Year	σ_{EL}	σ_{EM}	σ_{LM}	σ_{EK}	σ_{LK}	σ_{MK}
Food	60	2.20	−.02	.08	−3.57	−.48	.07
	70	2.74	−.65	.16	−5.35	−.15	.19
	80	1.62	.25	.25	−.95	−.31	.37
Textiles	60	1.10	.17	.50	.41	.45	−.22
	70	1.12	−.22	.52	.39	.63	−.02
	80	1.07	.30	.48	.75	.72	.24
Paper	60	.89	−.18	.09	−2.08	1.13	−.15
	70	.88	−.59	.14	−2.18	1.10	.10
	80	.92	.13	−.01	−.64	1.10	.10
Printing	60	1.18	.89	.47	−9.93	.21	.02
	70	1.12	.86	.47	−12.30	.24	.06
	80	1.16	.92	.45	−7.93	.10	.07
Rubber	60	.49	.48	−.09	4.63	1.10	.01
	70	.54	.23	.00	4.60	1.06	.00
	80	.72	.57	−.06	2.95	1.06	.03
Chemicals	60	.78	.29	−.24	.75	1.05	−.28
	70	.77	.09	−.17	.79	1.04	−.18
	80	.81	.39	−.29	.80	1.05	−.28
Minerals	60	.01	.91	.55	−1.45	.25	1.75
	70	−.71	.85	.59	−3.27	.26	1.75
	80	−.06	.92	.49	−.77	.30	1.57
Metals	60	.72	.57	−.22	−1.99	.82	−.17
	70	.65	.41	−.15	−3.34	.88	−.14
	80	.74	.58	−.31	−1.61	.88	−.05
Engineering	60	.87	1.42	.02	−7.99	.83	−.09
	70	.83	1.61	.05	−11.58	.84	−.08
	80	.88	1.40	−.01	−6.21	.85	.05
Total	60	1.09	−.46	−.05	−.16	1.32	−.42
	70	1.11	−1.12	.01	−.50	1.27	−.37
	80	1.06	−.24	−.06	.28	1.24	−.18

Table 4.3. Output Elasticities and Returns to Scale. Static Equilibrium Model.

	Year	υ_{EQ}	υ_{LQ}	υ_{MQ}	υ_{KQ}	η_Q
Food	60	−.42	.43	.89	.35	1.26
	70	−1.02	.61	1.02	.55	1.07
	80	.00	.67	1.04	.66	1.06
Textiles	60	−.09	.90	.96	.20	1.13
	70	−.37	.97	1.03	.43	1.06
	80	−.37	.97	1.03	.43	1.06
Paper	60	.64	.56	.86	.10	1.37
	70	.71	.67	.96	.30	1.23
	80	.78	.67	.99	.36	1.20
Printing	60	.33	.42	.83	.75	1.51
	70	−.08	.10	.50	.42	2.95
	80	−.03	−.03	.39	.33	4.23
Rubber	60	−.17	.44	.65	.55	1.79
	70	−.19	.60	.80	.69	1.45
	80	.11	.52	.72	.61	1.62
Chemicals	60	.44	.29	1.37	.43	1.04
	70	.12	.15	1.18	.28	1.34
	80	.27	.02	1.12	.20	1.41
Minerals	60	.61	.63	.96	.27	1.38
	70	.66	.78	1.09	.43	1.15
	80	.70	.69	1.02	.44	1.25
Metals	60	.37	.45	.92	.16	1.42
	70	.33	.53	.99	.24	1.29
	80	.45	.49	.97	.27	1.33
Engineering	60	.14	.54	1.03	.09	1.27
	70	−.16	.51	.99	.04	1.35
	80	.11	.46	.96	.11	1.40
Total	60	.68	.54	.96	.16	1.28
	70	.66	.58	.99	.23	1.24
	80	.71	.55	.98	.31	1.26

Table 4.4. Effects of Technical Change and Total Factor Productivity Growth. Static Equilibrium Model.

	Year	τ_{Et}	τ_{Lt}	τ_{Mt}	τ_{Kt}	ΔTFP
Food	60	1.80	−2.21	.50	3.69	−.38
	70	2.35	−2.31	.15	2.88	.06
	80	.67	−2.25	−.14	1.76	.38
Textiles	60	−3.53	−3.66	−1.49	.59	2.40
	70	−3.92	−3.92	−1.32	.07	2.12
	80	−2.72	−3.09	−.95	−.15	2.32
Paper	60	−.75	−2.98	.42	3.19	.25
	70	−1.09	−2.85	.27	2.38	.46
	80	−.68	−3.22	.35	2.30	.34
Printing	60	3.27	.05	1.59	3.06	−1.65
	70	2.83	−.97	.58	1.97	.07
	80	1.00	−1.91	−.33	1.17	3.14
Rubber	60	4.44	−2.40	1.53	.58	−.66
	70	3.62	−3.35	.15	−1.11	1.85
	80	.56	−4.44	−.81	−2.07	3.62
Chemicals	60	2.04	−.75	−.76	1.36	.45
	70	2.75	−.46	−.53	1.37	.25
	80	2.14	−.26	−.26	1.77	−.08
Minerals	60	.01	−2.87	−.22	3.21	1.11
	70	.22	−3.32	−.82	2.61	1.54
	80	−.68	−3.89	−.88	1.61	1.81
Metals	60	.69	−2.90	.74	2.89	−.16
	70	.22	−3.44	.08	2.24	.69
	80	−.42	−4.12	−.30	1.51	1.25
Engineering	60	2.84	−2.51	−.04	3.36	.84
	70	4.13	−2.51	−.14	3.24	1.05
	80	2.71	−2.44	.08	2.87	.80
Total	60	−1.69	−2.90	.42	1.89	.56
	70	−2.39	−2.86	.24	1.51	.81
	80	−1.62	−2.95	.29	1.20	.78

APPENDIX

Table 4.A.1. Log-Likelihood Values and Likelihood Ratio Test Statistics. Static Equilibrium Model.

	Log-likelihood			Test statistic	
	NH-NNTC	H-NNTC	NH-NTC	H-NNTC	NH-NTC
Food	626.1	613.7	593.2	24.8	65.8
Textiles	567.8	533.0	547.5	69.6	40.6
Paper	546.6	527.7	498.3	37.8	96.6
Printing	599.4	592.2	546.5	14.4	105.8
Rubber	480.4	496.6	448.9	21.6	63.0
Chemicals	536.2	500.2	517.2	72.0	38.0
Minerals	470.1	460.2	427.2	19.8	85.8
Metals	493.8	452.6	441.9	82.4	103.8
Engineering	606.3	576.7	546.8	59.2	119.0
Total	598.8	568.4	544.8	60.8	108.0

Note:
NH-NNTC: Non-homothetic, Non-neutral Technical Change
H-NNTC: Homothetic, Non-neutral Technical Change
NH-NTC: Non-homothetic, Neutral Technical Change

Table 4.A.2. Parameter Estimates. Static Equilibrium Model.

	Food	Textiles	Paper	Printing	Rubber
α_E	0159	0185	0468	0068	0274
	(.0004)	(.0003)	(.0007)	(.0001)	(.0006)
γ_{EE}	.0130	.0100	.0492	.0045	.0041
	(.0009)	(.0017)	(.0038)	(.0006)	(.0048)
γ_{EL}	.0022	.0006	−.0011	.0005	−.0037
	(.0017)	(.0024)	(.0048)	(.0018)	(.0051)
γ_{EK}	−.0023	−.0005	−.0124	−.0046	.0073
	(.0018)	(.0022)	(.0034)	(.0014)	(.0055)
γ_{Et}	.0002	−.0003	−.0003	.0002	.0010
	(.0001)	(.0002)	(.0003)	(.0000)	(.0003)
γ_{EQ}	−.0185	−.0193	−.0038	−.0023	−.0181
	(.0053)	(.0019)	(.0084)	(.0026)	(.0046)
α_L	.1678	.3637	.2323	.3619	.4115
	(.0019)	(.0028)	(.0017)	(.0021)	(.0028)
γ_{LL}	.0959	.1049	.1321	.1298	.1929
	(.0082)	(.0226)	(.0111)	(.0198)	(.0159)
γ_{LK}	−.0060	−.0080	.0026	−.0179	.0024
	(.0039)	(.0045)	(.0057)	(.0059)	(.0117)
γ_{Lt}	−.0030	−.0049	−.0064	−.0040	−.0081
	(.0005)	(.0016)	(.0005)	(.0006)	(.0007)
γ_{LQ}	−.0436	.0067	−.0353	−.0936	−.0350
	(.0200)	(.0216)	(.0206)	(.0400)	(.0144)
α_K	.0505	.0717	.1038	.0587	.1060
	(.0006)	(.0005)	(.0008)	(.0006)	(.0011)
γ_{KK}	.0340	.0435	.0721	.0539	.0386
	(.0069)	(.0069)	(.0070)	(.0048)	(.0135)
γ_{Kt}	.0011	.0013	.0029	.0012	.0002
	(.0002)	(.0003)	(.0003)	(.0002)	(.0005)
γ_{KQ}	−.0150	−.0316	−.0526	.0052	−.0006
	(.0074)	(.0034)	(.0078)	(.0119)	(.0080)
α_0	10.0200	8.9388	10.4160	9.2284	7.5228
	(.0025)	(.0055)	(.0028)	(.0050)	(.0080)
α_t	−.0026	−.0188	−.0032	−.0033	−.0189
	(.0008)	(.0032)	(.0007)	(.0012)	(.0013)
α_Q	.9401	.8793	.8238	.2811	.6595
	(.0732)	(.0805)	(.0289)	(.0762)	(.0665)
γ_{QQ}	.6173	.5291	.2428	−.8346	.2386
	(.2207)	(.1806)	(.0692)	(.2727)	(.1448)
γ_{tt}	−.0002	.0005	.0003	−.0008	−.0008
	(.0001)	(.0004)	(.0001)	(.0003)	(.0006)

Note: Approximate asymptotic standard errors are in parenthesis.

Table 4.A.3. Parameter Estimates. Static Equilibrium Model.

	Chemicals	Minerals	Metals	Engineering	Total
α_E	.0450	.0810	.0791	.0128	.0309
	(.0006)	(.0022)	(.0017)	(.0002)	(.0005)
γ_{EE}	.0248	.0667	.0528	.0054	.0312
	(.0043)	(.0124)	(.0123)	(.0010)	(.0033)
γ_{EL}	−.0027	−.0366	−.0052	−.0008	.0007
	(.0049)	(.0120)	(.0080)	(.0020)	(.0041)
γ_{EK}	−.0011	−.0262	−.0247	−.0088	−.0024
	(.0050)	(.0078)	(.0040)	(.0018)	(.0026)
γ_{Et}	.0012	.0008	.0005	.0006	−.0004
	(.0003)	(.0005)	(.0006)	(.0001)	(.0003)
γ_{EQ}	−.0256	−.0114	−.0280	−.0110	−.0033
	(.0038)	(.0176)	(.0121)	(.0018)	(.0064)
α_L	.2649	.3550	.2604	.3676	.3000
	(.0021)	(.0025)	(.0022)	(.0025)	(.0020)
γ_{LL}	.1961	.1377	.1725	.2021	.1722
	(.0140)	(.0236)	(.0113)	(.0217)	(.0138)
γ_{LK}	.0011	−.0307	−.0025	−.0034	.0055
	(.0082)	(.0112)	(.0035)	(.0039)	(.0032)
γ_{Lt}	−.0008	−.0076	−.0068	−.0065	−.0065
	(.0008)	(.0008)	(.0006)	(.0006)	(.0006)
γ_{LQ}	−.1745	−.0358	−.0563	−.0867	−.0678
	(.0168)	(.0244)	(.0120)	(.0220)	(.0216)
α_K	.0917	.1221	.0980	.0607	.0761
	(.0007)	(.0011)	(.0007)	(.0003)	(.0003)
γ_{KK}	.0671	.0226	.0904	.0480	.0547
	(.0098)	(.0170)	(.0028)	(.0051)	(.0061)
γ_{Kt}	.0015	.0044	.0025	.0023	.0015
	(.0005)	(.0005)	(.0002)	(.0002)	(.0002)
γ_{KQ}	−.0463	−.0495	−.0486	−.0400	−.0398
	(.0040	(.0125)	(.0044)	(.0032)	(.0042)
α_0	9.3936	8.6081	9.8208	11.0990	12.1600
	(.0028)	(.0062)	(.0045)	(.0054)	(.0037)
α_t	−.0008	−.0141	−.0087	−.0072	−.0069
	(.0011)	(.0011)	(.0008)	(.0012)	(.0008)
α_Q	.7341	.8318	.7491	.7210	.8016
	(.0274)	(.0661)	(.0403)	(.0346)	(.0357)
γ_{QQ}	.0054	.3005	.1741	.0297	.1451
	(.0515)	(.2623)	(.0513)	(.0601)	(.0830)
γ_{tt}	.0002	.0000	−.0002	.0003	.0003
	(.0001)	(.0002)	(.0002)	(.0002)	(.0001)

Note: Approximate asymptotic standard errors are in parenthesis.

Table 4.A.4. Goodness of Fit. Static Equilibrium Model.

	Energy	Labour	Capital	Costs
Food	.97	.96	.99	.99
Textiles	.97	.85	.99	.99
Paper	.96	.90	.96	.99
Printing	.91	.85	.80	.99
Rubber	.91	.96	.85	.99
Chemicals	.98	.90	.94	.99
Minerals	.85	.90	.95	.99
Metals	.83	.89	.98	.99
Engineering	.96	.80	.97	.99
Total	.93	.84	.99	.99

Note: Calculated as the squared cosine of the angle between the actual and predicted values of the exogenous variables. Lies within the interval (0,1).

V. ESTIMATION OF A PARTIAL STATIC EQUILIBRIUM MODEL

Although the application of the full static equilibrium model presented in the previous chapter resulted in a fully acceptable description of technology, it is questionable whether the resulting elasticities represent truly long-run relationships. Since the model is static and does not explicitly allow for the inflexibility of capital, there is no way of distinguishing between the short and the long run. For this reason, the resulting elasticities are difficult to interpret.

Because of the durability of capital equipment, the long planning horizons involved in the investment process, delivery lags—just to name a few—changes in the capital stock are limited in the short term so that the size of plant and the amount of equipment are more-or-less given. The only opportunities for factor substitution in this time frame have to do with changes in the degree of capacity utilisation and the ex-post substitution among the variable inputs within a given production technique. Both of these possibilities would appear to be limited. For this reason, adjustments made in response to factor price changes can be considered rather small in the short term. In the long run, the possibility of changing the capital stock and the production technique provides greater opportunities, not only for substitution between capital and the variable inputs but also among the variable inputs themselves. We would thus expect the response to factor price changes to be significantly greater in the long run than in the short run.

A comparatively simple method of recognising the inflexibility of capital is provided by the notion of partial static equilibrium. In the application presented in this chapter, it is assumed that the capital stock is fixed in the short run, while all other inputs are optimised conditional on the given level of capital. The model does not specify any particular adjustment mechanism, nor rely on any specific justification for the inflexibility of the capital stock. Instead, by using the envelop theorem, the optimal capital stock is calculated for each combination of factor prices and output level. Using the estimates of the short-run cost function and the optimal capital stocks, the long-run relationships can be retrieved.

5.1 *The Econometric Model*

The theoretical derivation of the partial equilibrium model has been presented in Section 3.1.2. To implement this model empirically, a functional form must be specified for the variable cost function (3.16) and the short-run demand functions for the variable inputs must be derived using equation (3.18). The translog function is chosen because it has been used in the long-run static equilibrium studies presented in Chapter IV and because its characteristics are comparatively well known. The coefficients obtained by estimating the resulting system of equations can then be used to calculate the short-run substitution, price and output elasticities.

As mentioned in Chapter III, the envelop condition (3.23) for the optimal level of the quasi-fixed factor based on the translog function cannot be solved analytically. It is therefore necessary to use numerical procedures to obtain a solution for the optimal level of the capital stock and to calculate approximate derivatives.

Using our previous nomenclature for prices and output and letting K denote the level of the capital stock, the variable cost function takes the following form

$$\begin{aligned} \text{Ln } C_V = {} & \alpha_o + \alpha_Q \text{LnQ} + \Sigma_i \alpha_i \text{LnP}_i + \alpha_K \text{LnK} + \alpha_t \text{Lnt} + 1/2\, [\gamma_{QQ}(\text{LnQ})^2 \\ & + \Sigma_i \Sigma_j \gamma_{ij} \text{LnP}_i \text{LnP}_j + \gamma_{KK}(\text{LnK})^2] + \Sigma_i \gamma_{iQ} \text{LnQLnP}_i + \gamma_{KQ} \text{LnQLnK} \\ & + \Sigma_i \gamma_{iK} \text{LnKLnP}_i + \gamma_{Kt} \text{LntLnK} + \Sigma_i \gamma_{it} \text{LntLnP}_i + \gamma_{Qt} \text{LntLnQ} \\ & \qquad i,j = E,L,M \qquad (5.1) \end{aligned}$$

where t is a time trend representing technical change. The variable cost function as specified above allows for non-constant returns to scale, non-homotheticity and non-neutral technical change. Technical change has been specified as a logarithmic time trend since a linear trend produced a high degree of multicollinearity among the independent variables.

For the cost function to be well-behaved, the matrix of second order partial derivatives must be symmetric so that $\gamma_{ij} = \gamma_{ji}$ and the cost function must be homogeneous of the first degree in prices given K, Q and t. This implies the following restrictions:

$$\Sigma_i \alpha_i = 1$$

$$\Sigma_i \gamma_{ij} = \Sigma_j \gamma_{ij} = \Sigma_i \gamma_{ik} = \Sigma_i \gamma_{iQ} = \Sigma_i \gamma_{it} = 0 \qquad i,j = E,L,M. \qquad (5.2)$$

From (3.18) the demand equations for the variable factors can be obtained by differentiating the cost function with respect to prices. For the translog cost function the following share equations are obtained:

$$S_i = \alpha_i + \Sigma_j\gamma_{ij}LnP_j + \gamma_{iK}LnK + \gamma_{iQ}LnQ + \gamma_{it}Lnt \qquad i=E,L,M \quad (5.3)$$

where $S_i = P_iX_i/C_V$ is the share of input i in variable costs. Similarly, the relationship for the shadow value is derived from (3.19) as

$$S_K = \frac{-R_KK}{C_V} = \alpha_K + \Sigma_i\gamma_{iK}LnP_i + \gamma_{KK}LnK + \gamma_{KQ}LnQ + \gamma_{Kt}Lnt. \quad (5.4)$$

The short- and long-run production relationships implied by the partial static equilibrium model can be obtained by estimating the variable cost function (5.1) and the share equations for the variable factors (5.3). Of course, we would have more information on which to base our estimates if the shadow value relationship (5.4) could also be estimated. Unfortunately, we have no appropriate measure of the shadow value. Although value added minus labour costs can be taken as a measure of the average ex-post return to capital, the shadow value is a marginal relationship and we have no justification to assume that these are equal. The model is thus estimated solely on the basis of (5.1) and (5.3).

The short-run elasticities of substitution and price elasticities for the variable inputs are calculated according to (3.20) and (3.21). For the translog variable cost function, these are identical to equations (4.4) and (4.5) in the previous chapter, with S_i now denoting input shares of variable costs. The short-run elasticities of the variable factors with respect to the level of the capital stock are obtained by partial differentiation of the demand equations with respect to capital holding output constant:

$$\upsilon_{iK} = (\partial X_i/\partial K)(K/X_i) = (S_KS_i + \gamma_{iK})/S_i \qquad i = E,L,M \quad (5.5)$$

where S_K is the shadow value calculated from (5.4).

Finally, given the level of the capital stock, short-run economies of scale can be calculated as the inverse of the variable cost elasticity given in (3.22). For the translog function, we have:

$$\eta^s_Q = 1/\eta^s_{cv} = 1/(\alpha_Q + \gamma_{QQ}LnQ + \Sigma_i\gamma_{iQ}LnP_i + \gamma_{KQ}LnK + \gamma_{Qt}Lnt).$$

$$i = E,L,M. \quad (5.6)$$

Using this relation, the short-run output elasticities for the variable factors become:

$$\upsilon^{s}_{iQ} = \eta^{s}_{cv} + \gamma_{iQ}/S_i. \qquad i = E,L,M. \qquad (5.7)$$

As stated in Section 3.1.2, when K is at its long-run equilibrium value, $\partial C_T/\partial K = 0$ or $R_K = u_K$, the user cost of capital. This gives us the following relationship

$$\frac{-u_k K}{C^*_V} = \alpha_K + \Sigma_i \gamma_{iK} \mathrm{Ln} P_i + \gamma_{KK} \mathrm{Ln} K^* + \gamma_{KQ} \mathrm{Ln} Q + \gamma_{Kt} \mathrm{Ln} t \qquad (5.8)$$

where K* is the optimal level of the capital stock and C^*_V is the variable cost function evaluated at K*. By solving this equation for K* using the estimated parameters and the values of the exogenous variables, the characteristics of the long-run cost function can be derived. However, since K* enters equation (5.8) in both logarithmic and natural units, it is not possible to obtain a closed-form analytic solution for K*. Instead, one must use iterative techniques[1] to solve numerically for K*. Since the second order condition for a mimimum is $\partial^2 C_T/\partial K^2 > 0$, and since $\partial^2 C_T/\partial K^2 = \partial^2 C_V/\partial K^2$ and C_V is convex in K, we can be assured that this is a global minimum.

Using the values of K* thus obtained, the long-run elasticities can be derived as explained previously. It should be noted that all long-run elasticities are calculated at the optimal values of C_T, C_V and S_i.

The long-run substitution elasticities for the variable factors are

$$\sigma^L_{ii} = \frac{C_T}{C_V}\left[\sigma^s_{ii} - \frac{(S_i S_K + \gamma_{iK})^2}{(\gamma_{KK} + S^2_K - S_K)S^2_i}\right] \qquad i,j = E,L,M \qquad (5.9)$$

$$\sigma^L_{ij} = \frac{C_T}{C_V}\left[\sigma^s_{ij} - \frac{(S_i S_K + \gamma_{iK})(S_j S_K + \gamma_{jK})}{(\gamma_{KK} + S^2_K - S_K)S_i S_j}\right]$$

whereas those for capital and between capital and the variable factors are

$$\sigma^L_{KK} = -\frac{C_T}{C_V}\left[\frac{1}{\gamma_{KK} + S^2_K - S_K}\right]$$

$$\sigma^L_{iK} = -\frac{C_T}{C_V}\left[\frac{\gamma_{iK} + S_i S_K}{(\gamma_{KK} + S^2_K - S_K)S_i}\right] \qquad i = E,L,M. \qquad (5.10)$$

The long-run price elasticities become

$$\varepsilon^L_{ij} = S^T_j \sigma^L_{ij} \qquad i,j = E,L,M,K \qquad (5.11)$$

[1] This is shown in Brown and Christensen (1981).

where the S_j^T now designate the shares of the inputs in long-run total costs.

The long-run economies of scale and the effects of technical change and can also be calculated from the parameter estimates and values of K*. Long-run returns to scale can be measured as the inverse of the derivative of the total cost function with respect to output

$$\eta_Q^L = C_T/C_V(\alpha_Q + \gamma_{QQ}LnQ + \Sigma_i\gamma_{iQ}LnP_i + \gamma_{KQ}LnK^* + \gamma_{QT}Lnt).$$

$$i = E,L,M. \qquad (5.12)$$

The long-run impact of technical change on the capital stock is calculated as

$$\tau_{Kt} = \frac{\partial LnK}{\partial t} = \frac{(\gamma_{Kt} + S_K\partial LnC_V/\partial t)}{(S_K - \gamma_{KK} - S_K^2)} \qquad (5.13)$$

and the effects on the variable factors as

$$\tau_{it} = \frac{\partial LnX_i}{\partial t} = \frac{\gamma_{it}}{S_i} + \frac{\partial LnC_V}{\partial t} + S_K\frac{\partial LnK}{\partial t} + \frac{\gamma_{iK}\partial LnK}{S_i\partial t}$$

$$i = E,L,M. \qquad (5.14)$$

The long-run impact of technical change on the variable factors is composed of a direct effect and an indirect effect. The first two terms on the r.h.s. of equation (5.14) represent the direct effect which is comprised of a neutral component and a bias component determined by γ_{it}. The indirect effect is seen in the last two terms of the equation. In the long run, technical change affects the use of capital, which in turn changes the demand for the variable factors. The differences of this effect on the variable factors is determined by the degree of substitutability or complementarity between the variable factors and the capital stock, γ_{ik}. Finally, the growth in total factor productivity can be measured by

$$\Delta TFP = (-C_V(\alpha_t + \gamma_{tQ}LnQ + \Sigma_i\gamma_{it}LnP_i + \gamma_{Kt}LnK^*\ \gamma_{Qt}LnQ)/C_T)\eta_Q^L.$$

$$i = E,L,M. \qquad (5.15)$$

Without any further restrictions on the parameters, the translog function as stated above in (5.1) is in its most general form. Constraints could, however, be placed on the parameters to simplify the model. For example, long-run constant returns to scale implies the following restrictions:

$$\begin{array}{lll} \alpha_Q + \alpha_K = 1 & & \\ \gamma_{QQ} + \gamma_{QK} = 0 & \gamma_{QK} + \gamma_{KK} = 0 \rightarrow \gamma_{QQ} = \gamma_{KK} & \\ \gamma_{Qt} + \gamma_{Kt} = 0 & \gamma_{iQ} + \gamma_{iK} = 0 & i=E,L,M. \end{array} \qquad (5.16)$$

5.2 *The Empirical Results*

The translog version of the restricted variable cost function presented in Section 5.1 has been estimated for total Swedish manufacturing and for the 9 manufacturing subsectors as defined earlier. The data include annual observations on quantities, costs and prices for energy, labour, intermediate goods and capital and gross output in constant prices for the time period 1952—83. A description of the data sources and construction is found in the appendix to Chapter II.

The variable factor shares (5.3) were estimated along with the variable cost function (5.1). Since the variable factor shares must sum to unity, one of these is redundant. The share equation for intermediate goods was deleted and the model was estimated using a full-information maximum likelihood estimation procedure[2].

In the stochastic specification additive error terms were appended to all equations, under the assumption of non-zero contemporaneous correlation between error terms of the different equations. Zero intertemporal correlation was, however, assumed for all equations. In all cases the model was estimated under the restrictions implied by homogeneity in prices (5.2).

The model as specified above allows for a full representation of non-neutral technical change. An alternative model is also estimated which assumes a variant of neutral technical change. In both cases, the model was estimated under the assumption of long-run constant returns to scale, that is, by imposing the restrictions given in equations (5.16). Long-run constant returns was imposed in order to reduce the number of parameters to be estimated and to minimise multicollinearity. A particular reason for assuming constant returns has to do with the difficulties in statistically distinguishing between the effects of scale and technical change noted in Chapter IV as well as in numerous other studies based on similar models.[3] This specification does, however, allow for non-constant returns in the short run.

The estimated coefficients along with their asymptotic standard errors are shown in tables 5.A.1 and 5.A.2 in the appendix to this chapter. We see that the majority of the estimated parameters are quite well determined. All γ_{EE} and all but one γ_{LL} are significantly different from zero. The interaction terms between energy and labour are also significant in the majority of industries. The success of the model in determining the long-run relationships, however, is much poorer. Although nearly all the parameters defining interac-

[2] A micro-cumputer version of CONRAD was used. See Jansson and Mellander (1984).

[3] Both Bergström and Panas (1985) and Dargay (1982) find that the most general models often lead to implausible estimates for scale and/or technology elasticities.

tions between capital and the variable factors are significantly different from zero, only about half of the γ_{kk} are. This is rather unfortunate as it is this parameter which determines the long-run price elasticity for capital and the difference between the short- and long-run price elasticities for the variable inputs. Judging from the significance of the coefficients of the time trend, technical change has had a substantial effect on over-all factor usage.

Goodness of fit measures are shown for the individual equations in table 5.A.3. These are calculated as the squared cosine of the angle between the deviations of the actual and predicted values of the endogenous variables from their respective means and can be interpreted similarly to common R^2 values.[4] Judging from the values for the different equations, we see that the explanatory power of the model is rather good for the variable cost function and energy share equation. The fit of labour share equation is, however, rather poor for some of the industries, and particularly so for Primary Metals.

As mentioned above, the model was also estimated under the assumption of Hicks neutral technical change, that is by imposing the restrictions $\gamma_{Et} = \gamma_{Lt} = \gamma_{Kt} = \gamma_{Mt} = 0$. The log-likelihood values and the likelihood ratio test statistics for the different sectors are shown in table 5.A.4. Testing for neutral technical change, the calculated likelihood-ratio test statistics fall in the interval 14—80, while the 0.005 chi-square critical value with 3 degrees of freedom is 12.84. Thus the assumption of Hicks-neutral technical change can be rejected for all industries.

Before going on to a discussion of the resulting elasticities, one further point about the quality of the estimates should be mentioned. The Durbin-Watson statistics are very low for the majority of the estimated equations, suggesting that serial correlation may be a problem. This is not particularly surprising, particularly considering the long time period included in the estimation. A correction for autocorrelation is obviously in order, which from previous experience may improve our results. This, however, would not only complicate the estimation procedure, but would also make it far more difficult to solve for the optimal capital stocks. For this reason, no attempt to allow for non-zero intertemporal correlation has been made.

Keeping this in mind, we will proceed to look at the implications of the estimates in terms of the elasticities discussed in the previous section. Since neutral technical change has been rejected, we will concentrate on the results for the more general model.

In order to represent a reasonable technology, the variable cost function must be increasing and concave in factor prices and decreasing and convex in the level of the capital stock. The concavity requirements are generally not

[4] See Section 4.2 and Haessel (1978).

met in the Paper and Metal industries and for a few years in the Food and Mineral industries. For the Paper and Metal industries the convexity requirements are also not met, which seriously questions the applicability of the model to these industries. The elasticities based on these estimates are therefore uninterpretable and must be rejected. Otherwise it can be noted that in the remaining sectors the concavity and convexity requirements are met for nearly all observations.

The short-run elasticities of the variable inputs with respect to output and the capital stock along with the calculated short-run returns to scale are shown in table 5.1 below. It should be pointed out that the imposition of long-run constant returns to scale places certain restrictions on these elasticities, i.e., that $\upsilon_{iK} + \upsilon_{iQ} = 1$, so that the effects on the variable inputs of changes in capital and output are not independent. Also from eqs. (5.5) and (5.10) we see that $\upsilon_{iK} < 0$ implies $\sigma^{L}_{iK} > 0$, or that factor i and capital are substitutes. Because of the relationship between υ_{iK} and υ_{iQ}, this will necessarily imply that $\upsilon_{iQ} > 1$, or that the production function displays short-run decreasing returns to factor i.

For all industries, we find that increases in the capital stock, at constant output, lead to an increase in energy usage. The opposite is true for labour, with the only exceptions being in the Rubber and Chamical industries. For materials, however, the elasticities with respect to the capital stock are generally quite small and the effects vary among the sectors with no discernable sign pattern. We will return to the significance of these results later on in our discussion of the long-run price elasticities.

The short-run output elasticities are also shown in the table. We generally find increasing returns to energy ($\upsilon_{EQ} < 1$) and decreasing returns to labour ($\upsilon_{LQ} > 1$). For intermediate goods, the results indicate both increasing and decreasing returns, but in most cases the output elasticity is quite near 1. In the short run when capital is fixed, an increase in output requires a proportionately greater increase in labour but a less than proportionate increase in energy. These results are quite interesting and warrant further comment.

Regarding energy, the short-run increasing returns noted for the majority of industries is rather what one would expect. A significant proportion of energy utilisation is fixed in the short run—space heating and lighting—and independent of marginal changes in output. As the utilisation of capital increases, energy usage need not increase to the same degree. The negative values found in a few of the industries are, of course, totally unrealistic.

Short-run decreasing returns to labour seems also to be quite reasonable. When capital is fixed and output demand rises, more labour must be employed—generally as overtime—to produce the additional output. Especially if capital is fully utilised, the efficiency of labour is bound to decrease with increasing output. Since long-run constant returns are assumed,

Table 5.1 Short-Run Capital and Output Elasticities. Partial Static Equilibrium.

	Year	υ^s_{EK}	υ^s_{LK}	υ^s_{MK}	υ^s_{EQ}	υ^s_{LQ}	υ^s_{MQ}	η^s_Q
Food	60	.72	−.71	.07	.28	1.71	.93	.97
	70	.94	−.72	.06	.06	1.72	.94	.96
	80	.35	−.68	.05	.65	1.68	.95	.95
Textiles	60	1.01	−.23	.00	−.01	1.23	1.00	.95
	70	1.14	−.23	−.01	−.14	1.23	1.01	.93
	80	.78	−.22	−.01	.22	1.22	1.01	.94
Paper	60	.17	−1.30	.09	(.85)	2.30	.91	.94
	70	.19	−.76	.08	(.81)	1.76	.92	.92
	80	.03	−1.03	.03	(.97)	2.03	.97	.88
Printing	60	1.87	(−.19)	.03	−.87	(1.19)	.97	.97
	70	2.49	(−.19)	.03	−1.49	(1.19)	.97	.95
	80	1.76	(−.18)	.04	−.76	(1.18)	.96	.97
Rubber	60	.62	.12	−.19	.38	(.88)	1.19	.93
	70	.56	.05	−.24	.44	(.95)	1.24	.91
	80	.36	.05	−.24	.64	(.95)	1.24	.91
Chemicals	60	.42	.73	−.38	.58	.27	1.38	.93
	70	.49	.62	−.40	.51	.38	1.40	.92
	80	.33	.69	−.37	.67	.31	1.37	.94
Minerals	60	.48	−.24	−.12	.52	1.24	1.12	.90
	70	.77	−.26	−.15	.35	1.26	1.15	.88
	80	.38	−.26	−.12	.62	1.26	1.12	.90
Metals	60	(.21)	(−.17)	.16	(.79)	(1.17)	.84	1.07
	70	(.27)	(−.13)	.17	(.73)	(1.13)	.83	1.09
	80	(.20)	(−.14)	.18	(.80)	(1.14)	.82	1.09
Engineering	60	.68	(−.23)	−.12	.32	(1.23)	1.02	.93
	70	1.88	(−.16)	.01	−.88	(1.16)	.99	.96
	80	2.53	(−.14)	.04	−1.53	(1.14)	.96	.98
Total	60	.54	−.55	.05	.46	1.55	.95	.93
	70	.76	−.44	.06	.24	1.44	.94	.93
	80	.52	−.40	.08	.48	1.40	.92	.95

Note: Values of υ^s_{EK}, υ^s_{LK} and υ^s_{EQ}, υ^s_{LQ} in parenthesis are not significantly different from 0 and 1 respectively. Approximate standard errors are calculated under the assumption that the factor shares are non-stochastic.
Standard errors are not available for υ^s_{MK}, υ^s_{MQ} and η^s_Q.

our results imply that long-run labour productivity is greater than short-run labour productivity, which does not seem totally unreasonable. It is interesting to note, that according to the specification of our model, this must be the case if $\upsilon_{LK} < 0$, which in turn implies that labour and capital are long-term substitutes. This is, of course, exactly what one would expect and is well in keeping with other empirical evidence.

As for the short-run materials-output elasticity, it is difficult to have any a priori notion of what this should be, so that a value near 1.0 is perhaps easiest to interpret. The decreasing returns noted for a few of the industries can suggest that these firms respond to a short-run increase in output demand by increasing the purchase of semi-finished goods or services which are normally produced or carried out within the firm.

Finally, the over-all returns to scale η_Q are found to be diminishing in the short run. This is exactly as one would expect in the presence of a fixed production factor. It is, however, somewhat surprising that the estimated short-run returns to scale are so close to unity. Unfortunately, we cannot test whether η_Q are statistically different from 1. The formula for the standard errors of these elasticities are rather complicated and calculation of these requires information about the covariances of the estimated parameters. As there was no simple way to retrieve the standard errors, we are unable to present them here.

As mentioned in the previous section, it is necessary to solve the envelop condition for the optimal capital stocks in order to calculate the long-run elasticities. Using the values of the estimated parameters and the observations on the exogenous variables, equation (5.8) was solved iteratively for the optimal capital stock for each year of the data sample. Since equation (5.8) is derived from the cost function, which was assumed to have a stochastic error term, one would also expect equation (5.8) to contain a stochastic component. In the iteration procedure it was necessary, however, to assume that the equation holds exactly. Even so, k^* is stochastic as it is a function of the estimated parameters. Estimates of the variance of k^* are however not obtainable since the equation cannot be solved analytically for k^*. We can therefore not test whether the estimated departure from equilibrium ($k \neq k^*$) is actually statistically significant, nor can we calculate the standard errors for the long-run elasticities.

The short- and long-run elasticities of substitution are presented in table 5.2. Again, the significance levels of the various elasticities are not presented. Although approximate standard errors for the short-run elasticities can be calculated under the assumption that the variable cost shares are non-stochastic, the long-run elasticities are non-linear functions of the estimated parameters so that calculation of approximate standard errors for these would be a rather complicated procedure.

Table 5.2. Elasticities of Substitution. Partial Static Equilibrium.

	Year	σ^S_{EL}	σ^L_{EL}	σ^S_{EM}	σ^L_{EM}	σ^S_{LM}	σ^L_{LM}	σ^L_{EK}	σ^L_{LK}	σ^L_{MK}
Food	60	1.29	2.60	−.14	−.25	.55	.71	−1.57	2.02	−.17
	70	1.39	3.16	−.86	−1.04	.62	.74	−2.49	1.70	−.15
	80	1.14	1.78	.21	.17	.65	.76	−.91	1.52	−.12
Textiles	60	.35	.44	.37	.39	.78	.82	−.31	.07	.00
	70	.27	.38	.14	.15	.79	.85	−.40	.07	.00
	80	.59	.68	.52	.56	.79	−.84	−.21	.07	.00
Paper	60	.34	−1.03	−.10	09	−.05	−.79	1.99	−6.62	1.01
	70	.42	−1.17	−.50	−.28	.16	−.50	2.97	−6.59	1.22
	80	.63	−.28	.17	.28	.04	−.98	1.50	−8.55	1.51
Printing	60	.66	.78	.19	.19	.91	.96	−.45	.06	−.01
	70	.61	.74	.10	.09	.91	.95	−.52	.06	−.01
	80	.67	.77	.25	.24	.91	.94	−.43	.06	−.01
Rubber	60	1.66	1.75	−.28	−.25	.40	.45	−.28	−.06	.10
	70	1.57	1.71	−.57	−.56	.46	.51	−.29	−.04	.13
	80	1.34	1.46	−.11	−.08	.45	.50	−.17	−.02	.13
Chemicals	60	−.13	−.26	.43	.55	.66	.84	−.21	−.34	.23
	70	−.26	−.42	.31	.44	.67	.86	−.25	−.31	.24
	80	.15	.06	.52	.61	.66	.86	−.17	−.31	.23
Minerals	60	−.44	−.39	.85	.99	.91	.98	−.39	.22	.11
	70	−1.73	−1.72	.70	.92	.92	1.01	−.90	.23	.13
	80	−.45	−.40	.87	1.00	.91	98	−.36	.23	.11
Metals	60	.60	.09	−.01	.20	−.08	−.48	2.14	−2.74	1.66
	70	.57	.16	−.35	.03	.08	−.21	2.39	−1.76	1.64
	80	.70	.40	.03	.35	.02	−.26	1.86	−1.65	1.66
Engineering	60	−.10	.62	.86	1.00	.04	.02	−3.23	.95	.09
	70	−.45	.56	.75	.69	.14	.15	−6.34	78	−.07
	80	.08	.71	.84	.72	.12	.15	−4.31	.75	−.17
Total	60	−.65	−.30	.68	.69	.05	.08	−.75	.71	−.07
	70	−.83	−.40	.47	.43	.21	.26	−1.17	.56	−.08
	80	−.10	.17	.68	.67	.18	.23	−.68	.55	−.11

We see that the substitution relationships between the variable inputs vary from sector to sector, although we do find a predominance of labour-materials and energy-materials substitutibility. These inputs tend also to be substitutes in the long run, and in the overwhelming majority of cases the long-run substitutability is greater than it is in the short run. One surprising result is that many of the industries show energy-labour complementarity, even in the long run. Concerning the (long-run) substitution relationships between the variable inputs and capital, we find that these vary accross industries as well. Energy and capital are strong complements in virtually all of the industries. The only exceptions are in the two most energy-intensive sectors—the Paper and Pulp and the Primary Metal industries. However, since the estimates for these two sectors did not satisfy the convexity requirements, we cannot place any weight on these results. As for the relationship between capital and materials, we find that the elasticities are generally rather low.

Finally, the results indicate that capital and labour are substitutes in the majority of the industries, which is well in keeping with the results of studies based on static models. The degree of substitutibility is, however, rather less than one would expect, particulary in the Textile and Printing industries where the elasticities are close to zero. If we ignore the results for the Paper and the Metal industries since these are based on implausible estimates, the only sectors that display capital-labour complementarity are the Rubber and Chemical industries. Here, too, the elasticities are quite low. It is certainly not easy to explain these differences in terms of the production processes in the individual industries, but variation in the magnitude and pattern of substitution has been noted in other studies, both for Sweden and for other countries.

The short- and long-run own-price elasticities are shown in table 5.3. These are negative in conformity with economic theory for all inputs and all industries with but two exceptions—the Pulp and Paper and the Primary Metal industries. We see from the table that the estimated elasticities are less than unity for all inputs and all sectors, indicating that input demand is inelastic both in the short and long run. Comparing the magnitudes of the short-run elasicities for the various sectors, the results indicate that, in the majority of industries, labour is the most price-elastic of the variable inputs, materials the least and energy somewhere in between. The only exceptions are Total Manufacturing and the Rubber and Engineering sectors, in which energy is found to be the most price-sensitive of the variable inputs. In the long run, we find the same ranking of the variable inputs regarding price-elasticities, while capital is the least price-sensitive production factor in virtually all industries. In fact, the own-price elasticities for capital are very near zero in most cases, which may be considered a rather questionable result. As we shall see, this has definite implications for all of the long-run elasticities.

Table 5.3. Own-Price Elasticities. Partial Static Equilibium.

	Year	ε^S_{EE}	ε^L_{EE}	ε^S_{LL}	ε^L_{LL}	ε^S_{MM}	ε^L_{MM}	ε^L_{KK}
Food	60	−.04	−.05	−.49	−.69	−.07	−.11	−.08
	70	.52	.49	−.53	−.70	−.08	−.13	−.10
	80	−.36	−.36	−.55	−.70	−.11	−.17	−.13
Textiles	60	−.35	−.36	−.51	−.51	−.27	−.27	−.02
	70	−.19	−.19	−.48	−.49	−.30	−.31	−.02
	80	−.53	−.54	−.49	−.49	−.31	−.31	−.02
Paper	60	.00	.02	.02	.88	.02	.38	.48
	70	.21	.23	−.12	−.81	−.03	.57	.87
	80	−.28	−.28	−.08	2.72	−.02	1.85	3.16
Printing	60	−.38	−.38	−.54	−.55	−.37	−.37	−.02
	70	−.30	−.30	−.55	−.56	−.36	−.36	−.01
	80	−.41	−.41	−.56	−.56	−.35	−.36	−.01
Rubber	60	−.36	−.36	−.31	−.31	−.12	−.12	−.04
	70	−.32	−.32	−.30	−.30	−.17	−.17	−.05
	80	−.53	−.54	−.29	−.29	−.20	−.20	−.05
Chemicals	60	−.25	−.25	−.43	−.48	−.21	−.22	−.04
	70	−.12	−.13	−.43	−.47	−.22	−.23	−.05
	80	−.37	−.37	−.43	−.48	−.24	−.26	−.03
Minerals	60	−.24	−.26	−.39	−.41	−.46	−.48	−.09
	70	.42	.38	−.37	−.39	−.45	−.47	−.11
	80	−.27	−.29	−.40	−.42	−.45	−.46	−.09
Metals	60	−.12	−.09	.00	.14	.02	.02	−.72
	70	.09	.13	−.09	−.01	.00	.01	−.77
	80	−.20	−.17	−.08	−.01	−.01	−.01	−.79
Engineering	60	−.54	−.58	−.02	−.09	−.03	−.08	−.28
	70	−.26	−.36	−.08	−.12	−.06	−.10	−.19
	80	−.52	−.59	−.07	−.12	−.07	−.10	−.11
Total	60	−.33	−.34	−.01	−.10	−.04	−.08	−.09
	70	−.03	−.05	−.11	−.19	−.08	−.13	−.09
	80	−.40	−.42	−.11	−.18	−.09	−.13	−.07

Although slight differences between short- and long-term elasticities can be noted in a few cases, in general, we find that the difference between the short and long run is negligible for the majority of inputs and industries. This is explained by the weak interactions estimated between capital and the variable factors that were noted above as well as the insensitivity of capital to changes in its own price. To see why this is the case, we can write the long-run price elasticity for the variable inputs as

$$\varepsilon_{ii}^{L} = \varepsilon_{ii}^{s} + \sigma_{KK}^{L} \cdot \upsilon_{iK}^{2} \qquad i=E,L,M. \qquad (5.17)$$

Since ε_{KK}^{L} must be negative, the second term on the r.h.s. is also negative, assuring that the long-run own-price elasticity is greater in absolute value than the short run. From this formula it is apparent that if a variable input is not highly substitutable with or complementary to capital (υ_{iK} is small), the adjustment of capital to its optimal level will have little influence on the long-term demand for that input. In the extreme case where $\gamma_{iK} = 0$, the elasticity of substitution between input i and capital is zero, and the long-run own-price elasticity for that input will be equal to the short-run price elasticity. Furthermore, if capital is not very sensitive to changes in its own price ($\varepsilon_{KK}^{L} \cong 0$), the elasticities of substitution between capital and the variable inputs, $\sigma_{iK}^{L} = \sigma_{KK}^{L} \cdot \upsilon_{iK}$, will be small, as will the difference between short- and long-run own-price elasticities for the variable factors. Again, in the extreme case where $\varepsilon_{KK}^{L} = 0$, short- and long-run elasticities are identical.

There are a number of reasons which might explain the difficulties in capturing the effects of price-induced factor substitution in the long run. These have primarily to do with the specification of the model and the nature of capital. There is, first of all, the problem of indivisibilities of capital which restricts the scope of adjustment to changes in relative factor prices. Another, perhaps even more serious problem has to do with the assumption of homogeneous capital. A change in relative factor prices could only increase or decrease this homogeneous mass. In actuality, factor substitution is generally achieved by the introduction of new production techniques so that new capital equipment is qualitatively different from the existing stock. Such effects cannot be captured in our measurement of capital. This problem is, however, shared by all investigations that assume homogeneous capital, and we will return to it in Chapter VII.

Finally, the results regarding the effects of technical change on factor use and the estimated rates of total factor productivity growth are presented in table 5.4. The effects of technical change on the individual production factors are measured as the annual percentage change in the use of the particular input resulting from technical progress. It follows from the table that technical change has been highly labour-saving in all of the industries, with an average

Table 5.4. Effects of Technical Change and Total Factor Productivity Growth. Partial Static Equilibrium Model.

	Year	τ_E	τ_L	τ_M	τ_K	ΔTFP
Food	60	−.1	−1.9	−.0	3.2	.2
	70	−.3	−.9	−.0	3.2	.3
	80	−.3	−.6	−.3	9	.3
Textiles	60	−1.0	−2.6	−2.2	2.5	2.1
	70	−1.2	−2.0	−1.8	1.2	1.7
	80	−1.3	−1.6	−1.5	.8	1.4
Paper	60	.7	−11.8	1.6	9.1	.7
	70	.4	−5.7	.5	6.0	.7
	80	−.0	−8.2	.5	9.7	.4
Printing	60	3.0	1.5	−.4	3.1	.5
	70	.5	−1.8	−1.3	1.5	1.3
	80	−.3	−1.6	−1.2	1.0	1.3
Rubber	60	−5.2	−4.2	−2.1	2.7	2.5
	70	−3.3	−2.6	−1.6	1.2	1.8
	80	−2.1	−1.9	−1.3	.8	1.4
Chemicals	60	−.8	−2.7	−.7	−1.7	1.3
	70	−.7	−1.6	−.4	−1.6	1.0
	80	−.6	−1.1	−.5	−.5	.7
Minerals	60	2.2	−3.2	−.5	4.2	.8
	70	1.6	−2.5	−1.4	1.8	1.3
	80	−.4	−2.0	−1.2	1.2	1.1
Metals	60	−3.1	−9.6	.1	7.7	3.0
	70	−2.1	−4.1	−.3	2.8	1.8
	80	−1.2	−2.7	−.2	1.7	1.2
Engineering	60	−8.1	−3.9	.3	−9.9	1.8
	70	−8.2	−2.7	−1.0	−5.5	1.9
	80	−4.5	−2.2	−1.1	−3.9	1.7
Total	60	−.9	−4.6	−.2	−1.2	1.3
	70	−.7	−2.1	−.4	−.6	.9
	80	−.6	−1.5	−.3	−.4	.7

rate of decrease in labour of about 2% per year. This coincides rather well with the results of most Swedish studies. Technical change has also been energy-saving, but at a somewhat lower rate than for labour. The influence of technical progress, however, seems to be diminishing over time. For materials, we find that technical change plays a far smaller role although it has lead to an decrease in the use of materials. Finally, technical change has led to an increasing capital intensity in all sectors except Chemicals, Engineering and Total Manufacturing, again with a diminishing influence over time.

The estimates suggest that biased technical change has played an important role in inducing changes in relative factor use. Judging from the low values of the own-price elasticities in table 5.3, it might appear that autonomous biased technical progress has been more significant than price-induced factor substitution. However, considering our rather crude specification of technical change and the correlation between the capital stock and time, one should be cautious about interpreting these results. Particularly, we may be confusing the effects of labour-saving technical change with those of K-L substitutability. Our results for the model assuming neutral technical change indicate a higher degree of K-L substitutability in many of the industries and in general, the results are rather sensitive to the specification of technical change.

The rate of growth in total factor productivity is also shown in the table. We see that this too appears to be diminishing over time, which is generally in keeping with other evidence. Taken over the entire period, total factor productivity growth has been greatest in the Textile, Rubber and Engineering industries and lowest in the Food industry.

5.3 *The Imperfect Flexibility of Labour*

One of the assumptions that was made in the application of the partial static equilibrium model was that all sluggishness in factor adjustment was due to the imperfect flexibility of capital. All other inputs are assumed to be perfectly variable, so that the rationale behind differences in short- and long-run adjustment possibilities, and thus substitution and price elasticities, lies soley in the difficulties in rapidly adjusting the capital stock. Although there seems no obvious reason for questioning perfect flexibility in the case of energy and other intermediate goods, this assumption seems somewhat more dubious when applied to labour.

One argument against the assumption of the perfect flexibility of labour, which immediately springs to mind in the case of Sweden, has to do with the existence of labour security legislation and strong trade unions. This creates a situation in which a rise in the wage rate or a decrease in output demand cannot immediately be met by the reduction in the workforce that would be dictated by pure optimising behaviour. Adjustment may take time if the firm is to wait for 'natural' retirements or job changes, or may be costly if this process is to be speeded up by monetary incentives for early retirement or voluntary redundancy. Labour security legislation has also been used as an argument for careful hiring policies. An increase in production is more safely met by an increase in overtime than by hiring new workers whom the firm will be forced to sustain even if output demand decreases. The actual behaviour of the firm with respect to hiring and firing policies is, as in the case of investment in physical capital, obviously dependent on the firm's expectations of future output demand and relative factor prices.

Another motivation for the imperfect flexibility of labour is the cost involved in staff-training. These costs are over and above the wage and arise with each increase in the work force. As far as reductions in the work force are concerned, however, these costs are only relevant in cases where the reductions are not expected to be long lived. For example, if output demand is expected to increase in the near future, so that new hiring will be necessary, the future costs of training the new employees may be higher than the costs of retaining present employees regardless of the current decrease in production.

Although the arguments for the partial inflexibility of labour are not as strong as those pertaining to the fixity of the capital stock, they may nonetheless be of some relevance in explaining the results obtained from the partial static equilibrium model. If labour is not, in fact, perfectly variable as the model assumes, the specification errors involved could lead to misleading results all around.

As mentioned in Chapter III, it is quite simple to extend the partial static equilibrium model to include any number of quasi-fixed factors. To specify labour as quasi-fixed, the wage rate is replaced by the quantity of labour in the variable cost function and the demand equations for the variable factors are derived. The demand for the variable factors now become a function of the relative prices of the variable factors, output and the quantities of capital and labour. The number of equations to be estimated is, however, reduced by one, so that equally many parameters must be estimated on the basis of less information than in the case where the input is treated as variable. Given the parameters estimated from the variable cost function, the optimal levels of capital and labour can be calculated from the envelop condition and the long-run elasticities derived.

An attempt was made to estimate the partial static equilibrium model with both labour and capital designated as quasi-fixed. Both translog and quadratic functional forms were used to specify the cost function. The quadratic specification has the advantage that the envelop condition can be solved explicitly for the optimal levels of labour and capital, which is not the case for the translog form.

The data are as defined earlier, with L being measured as total hours worked. As this encompasses both hours worked per employee as well as changes in the number employed, it is not really the correct measure for our purposes. In accordance with the arguments presented for the inflexibility of labour, one would expect the number of hours worked per employee to be variable (up to a limit) and only the number employed to be fixed in the short run. Only if average hours worked per employee is a constant are the number employed and total labour hours equivalent. This, however, is not the case. Data on total hours and number employed indicate a continual reduction of average hours per employee over the entire observation period. Since this reduction is the result of a shortened work week, average hours worked measured in this manner cannot be considered as perfectly variable. Given this situation and the lack of data on actual overtime, specifying labour as total hours worked seems the best alternative, and is thus that chosen for the empirical analysis.

The translog model is obtained by replacing the wage rate, P_L, in the variable cost function (5.1) and the share equations (5.3) with the quantity of labour, L. Of course, the cost shares are only derived for the variable factors, energy and materials. The model is estimated under the restrictions implied by linear homogeneity in prices (5.2), where now i,j = E, M and the additional constraint $\Sigma_i \gamma_{iL} = 0$ is included. Long-run constant returns to scale are also imposed.

Given the same assumptions, the quadratic functional form of the variable cost function can be written as

$$\begin{aligned} C_V/Q &= \alpha_o + \alpha_E P + \alpha_K K/Q + \alpha_L L/Q + \alpha_t t + 1/2[\gamma_{EE} P^2 + \gamma_{KK}(K/Q)^2 \\ &+ \gamma_{LL}(L/Q)^2 + \gamma_{tt} t^2] + \gamma_{EK} P(K/Q) + \gamma_{EL} P(L/Q) + \gamma_{Et} Pt \qquad (5.18) \\ &+ \gamma_{LK}(K/Q)(L/Q) + \gamma_{Kt}(K/Q)t + \gamma_{Lt}(L/Q)t \end{aligned}$$

where $P = P_E/P_M$ is the relative price of the variable factors and the other variables are defined as previously. The short-run demand equations for the variable factors are derived using Shephard's Lemma (3.32). For energy, we have

$$\partial C_V/\partial P = E/Q = \alpha_E + \gamma_{EE} P + \gamma_{EK} K/Q + \gamma_{EL} L/Q + \gamma_{Et} t. \qquad (5.19)$$

Since the variable cost function is normalised by the price of intermediate goods, the demand for this input is expressed in terms of the cost function and the demand equation for energy according to the relation:

$$M/Q = C_V/Q - P\ E/Q. \tag{5.20}$$

Both functional specifications of the model were estimated for Total Manufacturing and the 9 manufacturing subsectors. As for the model with one quasi-fixed factor, a full-information maximum likelihood estimation procedure was employed for the estimation.

For both functional forms and for all industries the resulting estimates were shown not to be in conformity to the economic theory underlying the model. Although the concavity requirements were generally met assuring the negativity of the short-run own-price elasticities for the variable factors, the convexity requirements were not. It should also be pointed out that in many cases the estimates displaying the wrong sign were significantly different from zero. In general, however, the estimated parameters were rather poorly determined and particularly those relating to labour and capital.

The fact that the convexity requirements are not fullfilled seriously impairs us from drawing any economially meaningful conclusions on the basis of the estimated model. Since the resulting own-price elasticities for capital and labour are positive, all other elasticity measures based on the estimates must be disregarded. To avoid wasting space on a lot of meaningless numbers, neither the estimated parameters nor resulting elasticities are presented here.

Because the estimated model results in a totally unreasonable description of technology, it does not give us a valid basis for hypothesis testing. The derivation of the model is based on the existence of a production function which displays certain characteristics. If the variable cost function is not convex in the levels of the quasi-fixed factors, the long-run total cost function will not be concave in the prices of all inputs. Such a cost function cannot arise from a reasonable technology—or production function—which was one of the premises of the model. Given that the estimates contradict the premises of the model, the model taken as a whole must be rejected.

Perhaps these results are not particularly surprising. Even in the case of one quasi-fixed factor reported earlier, we noted that the convexity requirements were not met in all industries. We found there, as well, that the estimated parameters—and particularly those defining the interaction between capital and the variable inputs—were poorly determined in many instances. There it was argued that a possible explanation for the poor performance of the model was that the data might not provide adequate information for constructing the long-run cost function on the basis of the variable cost function alone. With an additional quasi-fixed factor, the difficulties are surely to increase.

Another factor which may have contributed to the poor performance of the model can be the specification of the labour variable. As mentioned above, it would be preferable to distinguish between normal hours worked which would be treated as fixed in the short run, and overtime which would be considered as variable. This would, however, require additional data which are not readily available.

5.4 *Conclusions*

The application to Swedish manufacturing data of the partial static equilibrium model with capital specified as a quasi-fixed factor has resulted in quite reasonable estimates of the short-run production relationships. The majority of the own-price elasticities are negative, but less than 1 in absolute value, indicating an inelastic factor demand. In the short-run, labour and energy appear to be the most price-sensitive production factors, while materials is the least price-sensitive. Regarding the short-run substitution relationships, we find that these tend to vary among the individual industries, although E-M and L-M substitutability prevail.

Technical change is shown to have been labour- and energy-saving and capital-using in the majority of industries. The estimates of total factor productivity growth indicate that productivity has been increasing over the entire 1952—83 time period, but at a decreasing rate.

Estimation of the model provides evidence of short-run increasing returns to energy, and decreasing returns to labour. Diminishing returns to scale in the short run is also supported, although the elasticities do not differ vastly from one. The comparison of the short- and long-term scale effects is, however, not empirically motivated, as long-run constant returns to scale is assumed rather than tested statistically.

The model, however, appears to be far less successful in determining long-run production relationships. The results indicate that capital is rather insensitive to price changes, so that the long-run price elasticities for the variable inputs are nearly identical to their short-run counterparts. Changes in factor prices have limited effects on the level of the capital stock, so that price-induced substitution between capital and the variable inputs is minimal.

Attempts to extend the model to include inflexibilities in labour have not led to any interpretable results. Although this may primarily be explained by an inappropriate definition of the labour variable, there is also reason to believe that the data do not contain sufficient information to allow retrieval of long-run production relationships solely on the basis of the short-run cost function.

APPENDIX

Table 5.A.1. Parameter Estimates. Partial Static Equilibrium Model.

	Food	Textiles	Paper	Printing	Rubber
α_E	.0358	.0404	.0617	.0160	.0659
	(.0075)	(.0017)	(.0090)	(.0024)	(.0118)
γ_{EE}	.0147	.0126	.0489	.0048	.0161
	(.0006)	(.0012)	(.0042)	(.0008)	(.0041)
γ_{EL}	.0006	−.0044	−.0072	−.0011	.0057
	(.0008)	(.0009)	(.0042)	(.0014)	(.0031)
γ_{Et}	−.0027	−.0027	−.0023	−.0007	−.0094
	(.0008)	(.0006)	(.0027)	(.0005)	(.0032)
γ_{EK}	.0102	.0221	.0143	.0118	.0159
	(.0055)	(.0020)	(.0080)	(.0020)	(.0074)
α_L	.0450	.3444	.5179	.3822	.6353
	(.0478)	(.0282)	(.0353)	(.0649)	(.0919)
γ_{LL}	.0472	.0521	.1723	.0222	.1201
	(.0059)	(.0138)	(.0134)	(.0262)	(.0219)
γ_{Lt}	.0098	.0037	−.0859	−.0085	−.0548
	(.0052)	(.0085)	(.0099)	(.0127)	(.0249)
γ_{LK}	−.0959	−.0581	−.1406	−.0573	.0618
	(.0352)	(.0151)	(.0523)	(.0545)	(.0556)
α_0	.4479	2.7312	.4454	4.0175	1.9561
	(.3250)	(.6432)	(.0806)	(.6579)	(.1620)
α_t	−.1653	−.7749	−.1904	−1.0232	−.6105
	(.0423)	(.1769)	(.0233)	(.1588)	(.0446)
α_K	.6368	4.1796	−.3270	5.3981	2.2627
	(.5158)	(1.0574)	(.2747)	(.8629)	(.5784)
γ_{KK}	.3805	3.4921	−.1951	3.3149	1.9474
	(.4125)	(.6465)	(.3586)	(.5891)	(.8192)
γ_{Kt}	−.1202	−.8127	.1017	−.9423	−.5082
	(.0365)	(.2354)	(.0633)	(.1371)	(.1190)

Note: Approximate asymptotic standard errors in parenthesis.

Table 5.A.2. Parameter Estimates. Partial Static Equilibrium Model.

	Chemicals	Minerals	Metals	Engineer-ing	Total
α_E	.0401	.0391	.1314	.0161	.0288
	(.0040)	(.0185)	(.0256)	(.0035)	(.0066)
γ_{EE}	.0368	.0753	.0694	.0086	.0233
	(.0022)	(.0075)	(.0094)	(.0009)	(.0027)
γ_{EL}	−.0169	−.0669	−.0077	−.0069	−.0148
	(.0023)	(.0096)	(.0095)	(.0017)	(.0036)
γ_{Et}	.0055	.0126	−.0140	.0025	.0039
	(.0010)	(.0055)	(.0067)	(.0011)	(.0022)
γ_{EK}	.0232	.0625	.0109	.0163	.0235
	(.0057)	(.0159)	(.0206)	(.0025)	(.0096)
α_L	.3892	.5713	.6128	.7670	.5652
	(.0546)	(.0450)	(.0294)	(.0669)	(.0509)
γ_{LL}	.0817	.0837	.1704	.2097	.1830
	(.0274)	(.0237)	(.0128)	(.0225)	(.0119)
γ_{Lt}	−.0066	−.0486	−.0959	−.1207	−.0850
	(.0135)	(.0133)	(.0081)	(.0136)	(.0098)
γ_{LK}	.1910	−.0546	−.0676	−.0488	−.1128
	(.0701)	(.0251)	(.0545)	(.0802)	(.0793)
α_0	.2286	.4840	.9119	.6126	.4838
	(.0670)	(.0710)	(.0690)	(.1502)	(.1842)
α_t	−.0539	−.1653	−.2842	−.2027	−.1777
	(.0287)	(.0260)	(.0207)	(.0379)	(.0447)
α_K	.0451	1.0108	−.2058	−.5172	−.0774
	(.3606)	(.3373)	(.2494)	(.3751)	(.4814)
γ_{KK}	1.7876	1.1017	−.0301	.1760	.7272
	(.5542)	(.3594)	(.4146)	(.6278)	(.6361)
γ_{Kt}	.2738	−.4731	.0806	.2141	.0797
	(.1385)	(.1169)	(.0547)	(.0772)	(.0909)

Note: Approximate asymptotic standard errors in parenthesis.

Table 5.A.3 Goodness of Fit. Partial Static Equilibrium Model.

	Cost function	Energy	Labour
Food	.99	.97	.91
Textiles	.97	.98	.85
Paper	.99	.95	.16
Printing	.99	.66	.24
Rubber	.99	.85	.74
Chemicals	.99	.93	.34
Minerals	.97	.81	.53
Metals	.99	.85	.07
Engineering	.99	.85	.45
Total	.99	.84	.43

Note: Calculated as the squared cosine of the angle between the actual and predicted values of the exogenous variables. Lies within the interval (0,1).

Table 5.A.4 Log-Likelihood Values and Likelihood Ratio Test Statistics. Partial Static Equilibrium Model.

	Log-likelihood		Test statistic
	Unrestricted model	Neutral technical change	Neutral technical change
Food	423.3	413.4	19.8
Textiles	355.0	348.0	14.0
Paper	330.5	292.6	75.8
Printing	364.2	354.6	19.2
Rubber	299.8	285.1	29.4
Chemicals	333.9	315.7	36.0
Minerals	284.8	267.6	34.4
Metals	292.2	252.4	79.6
Engineering	259.7	322.2	75.0
Total	352.8	322.4	60.8

VI. ESTIMATION OF A DYNAMIC COST-OF-ADJUSTMENT MODEL

The empirical studies presented in the previous chapters have been based on models derived from static optimisation theory. Although the results of these studies provide some insight into the price-sensitivity of demand and the substitution relationships, virtually nothing can be said about the dynamics of the adjustment process, i.e. about the price-response and substitution possibilities in different time perspectives. On the contrary, the application of static equilibrium demand model in Chapter IV implicitly assumes that adjustment to price or output changes is more or less instantaneous so that no distinction is made between short- and long-run responses. In effect, all inputs are assumed to be perfectly variable so that firms can switch from one production technique to another without any time delays or added costs. This is clearly unrealistic particularly in the case of physical capital, i.e. machinery and structures. The durability of the capital stock may make immediate replacement economically unfeasible. Further, the investments necessary are often associated with a delay of several years from the time an investment decision is made to the completion of installation and the productive use of capital equipment.

The stringent assumptions inherent in full static equilibrium were relaxed in the application of the partial static equilibrium model presented in Chapter V by recognising that physical capital cannot as quickly be adjusted to changes in factor prices or output as can other, more variable, production factors. The capital stock was assumed to be given in the short run, and three variable inputs—energy, labour and intermediate goods—were assumed to adjust quickly to their optimum (short-run) levels given the level of the capital stock. The complete or long-run adjustment of the variable inputs was dependent on the adjustment of the capital stock to its optimum level. By using the envelop condition, the long-run optimal capital stock and demand for the variable factors were derived. Although both short- and long-run elasticities could be calculated, no information is given concerning the speed of adjustment or the factors influencing the adjustment process. We have seen, however, in the application to Swedish industry data, that the ability of the model to capture differences in short- and long-run production relationships was quite poor. In fact, short- and long-run elasticities were very nearly identical. In the world of our model this suggests that capital is always optimised, or at least very nearly so, however dubious this may seem. An alternative explanation can be that the statistical data do not contain the necessary information for retrieval of the long-run cost function on the basis of estimates of the variable cost function alone. Further assumptions concerning the nature of the adjustment mechanism are needed.

The model applied in this chapter provides just that. The notion of adjustment costs is used to motivate the fixity of the capital stock[1] and the adjustment mechanism is derived by intertemporal cost-minimisation.

6.1 *The Econometric Model*

The economic model underlying this study is the internal cost of adjustment model developed by Berndt, Fuss and Waverman (1977). The theoretical derivation of dynamic cost of adjustment models has been discussed thoroughly in Section 3.2.3 so only a brief presentation will be included here.

Cost of adjustment models motivate the inflexibility of certain inputs in the short run in terms of the economic costs involved in quickly adjusting to long-run optimum levels. These costs can be assumed to be either internal or external to production depending on whether or not they affect current production possibilities. The internal cost of adjustment model[2] applied in this study assumes that a change in the capital stock affects current production because part of the firm's resources must be taken from production activities in order to implement the required changes in capital.

Adjustment costs are specified as a function of net investment only, so that investment related to capital depreciation is assumed not to give rise to adjustment costs. This seems reasonable if the depreciation rate is assumed constant.[3] It is further assumed that the marginal costs of investment are increasing: the more rapid the change in capital, the greater the reduction in current output. If this were not the case, the firm would have no economic incentive to distribute investment over time.

The derivation of the model is based on dynamic optimisation theory. In

[1] The short-run inflexibility of labour could also be analysed in the context of a cost of adjustment model. As discussed in Chapter 5.3, however, additional data on overtime would be needed to estimate a reasonable formulation of such a model.

[2] One could also have chosen an external cost of adjustment model or a model including both types of adjustment costs (see Section 3.2.3).

[3] Of course, in some instances, such as in the case of sudden and rapid technological innovation where capital quickly becomes obsolete, it may be better to allow adjustment costs to depend on gross investment. The depreciation rate must then be allowed to vary.

the variant applied here, the firm's objective function is stated in terms of production costs and is formulated to include the costs of adjusting the capital stock as well as the relationships defined by the production function. The firm is assumed to minimise the present value of the future stream of costs given its initial stocks of capital and its expectations concerning future factor prices and output demand. For the sake of simplicity, static expectations are assumed, so that relative factor prices, the purchase price of capital and its rate of return as well as output demand are known with certainty and expected to remain constant over time.

The technological constraints of the production process are incorporated into the objective function in the form of a restricted variable cost function. Under the assumption of cost-minimising behaviour, the theory of duality between production and costs assures that the restricted cost function is a complete representation of the available technology at a given level of the quasi-fixed factor, in this case, the capital stock. By solving the intertemporal optimisation problem, the time path of the capital stock which minimises the present value of future costs is obtained in terms of an investment function.

Suppose that the firm's output is produced by the inputs of three variable factors: energy E, labour L and intermediate goods M, and one quasi-fixed factor, capital K. Changes in the capital stock K are represented by the discrete approximation $K_t - K_{t-1} = \Delta K_t$. It is assumed that output Q in period t is determined solely by the capital stock in place at the beginning of the period K_{t-1}. Changes in the capital stock that take place during the period ΔK_t are thus assumed to contribute to productive capacity only in the following period.

As a further simplification, we will assume long-run constant returns to scale so that the variable cost function can be written in terms of average variable costs:

$$C_V/Q = G(\cdot)/Q = H(P_E, P_M, K_{-1}/Q, \Delta K/Q, t). \tag{6.1}$$

In order to insure homogeneity in prices, variable costs C_V and the prices of all inputs are normalised by the wage rate $\bar{P}_L$, i.e. $P_i = \bar{P}_i/\bar{P}_L$ for i = E,M,K where the $\bar{P}_i$ are nominal prices and $\bar{P}_K$ is the asset or purchase price of capital goods. A time trend t is included in the cost function to account for the effects of disembodied technical change.

For empirical implementation, it is necessary to specify a particular functional form for the cost function. As discussed in Section 3.2.3, the majority of flexible functional forms will not allow an analytic solution to the Euler equations (3.34). Since the quadratic form does allow an alalytic solution, it is the one most commonly used in empirical studies based on cost of adjustment models. For this reason, the quadratic form is chosen for this study.

Taking a quadratic approximation to the normalised average variable cost function (6.1) around 0, we have:

$$
\begin{aligned}
G/Q &= (L+P_E E+P_M M)/Q = \alpha_0 + \alpha_E P_E + a_M P_M + \alpha_K K_{-1}/Q \\
&+ \alpha_{\dot{K}} \Delta K/Q + \alpha_t t + 1/2\, [\gamma_{EE} P_E^2 + \gamma_{MM} P_M^2 + \gamma_{KK}(K_{-1}/Q)^2 + \gamma_{\dot{K}\dot{K}}(\Delta K/Q)^2 \\
&+ \gamma_{tt} t^2] + \gamma_{EM} P_E P_M + \gamma_{EK} P_E K_{-1}/Q + \gamma_{E\dot{K}} P_E \Delta K/Q + \gamma_{Et} P_E t \\
&+ \gamma_{MK} P_M K_{-1}/Q + \gamma_{M\dot{K}} P_M \Delta K/Q + \gamma_{Mt} P_M t + \gamma_{K\dot{K}} K_{-1} \Delta K/Q \\
&+ \gamma_{tK} t K_{-1}/Q + \gamma_{t\dot{K}} t \Delta K/Q \qquad (6.2)
\end{aligned}
$$

where the internal costs of adjustment $C(\Delta K)$ are represented by:

$$
\begin{aligned}
C(\Delta K) &= \alpha_{\dot{K}} \Delta K/Q + 1/2\, \gamma_{\dot{K}\dot{K}}(\Delta K/Q)^2 + \gamma_{E\dot{K}} P_E \Delta K/Q + \gamma_{M\dot{K}} P_M \Delta K/Q \\
&\quad + \gamma_{K\dot{K}} K_{-1} \Delta K/Q + \gamma_{t\dot{K}} t K/Q. \qquad (6.3)
\end{aligned}
$$

Assuming that marginal adjustment costs $\partial C(\Delta K)/\partial \Delta K$ are equal to zero at $\Delta K = 0$, we have:

$$
\begin{aligned}
\partial C(\Delta K)/\partial \Delta K &= \alpha_{\dot{K}} + \gamma_{\dot{K}\dot{K}} \Delta K + \gamma_{E\dot{K}} P_E + \gamma_{M\dot{K}} P_M + \gamma_{K\dot{K}} K_{-1}/Q + \gamma_{t\dot{K}} t \\
&= 0. \qquad (6.4)
\end{aligned}
$$

This will hold for all values of the exogenous variables if and only if:

$$\alpha_{\dot{K}} = \gamma_{E\dot{K}} = \gamma_{M\dot{K}} = \gamma_{K\dot{K}} = \gamma_{t\dot{K}} = 0. \qquad (6.5)$$

Incorporating these restrictions the variable cost function becomes:

$$
\begin{aligned}
G/Q &= \alpha_0 + \alpha_E P_E + \alpha_M P_M + \alpha_K K_{-1}/Q + \alpha_t t + 1/2[\gamma_{EE} P_E^2 + \gamma_{MM} P_M^2 \\
&+ \gamma_{KK}(K_{-1}/Q)^2 + \gamma_{\dot{K}\dot{K}}(\Delta K/Q)^2 + \gamma_{tt} t^2] + \gamma_{EM} P_E P_M + \gamma_{EK} P_E K_{-1}/Q \\
&+ \gamma_{Et} P_E t + \gamma_{MK} P_M K_{-1}/Q + \gamma_{Mt} P_M t + \gamma_{tK} t K_{-1}/Q. \qquad (6.6)
\end{aligned}
$$

Economic theory imposes certain restrictions on some of the parameters of the cost function. Variable costs must increase with the prices of the variable inputs and own-price elasticities must be negative (G concave in variable input prices) so that $\alpha_E > 0$, $\alpha_M > 0$ and γ_{EE} and $\gamma_{MM} < 0$. An increase in the level of capital should result in a decrease in variable costs so that $\alpha_K < 0$; but costs should decrease at a decreasing rate which implies that $\gamma_{KK} > 0$. Further, the theoretical derivation assumes that the marginal costs of investment are increasing so that $\gamma_{\dot{K}\dot{K}} > 0$. Finally, technical change must be cost saving which requires that $\alpha_t < 0$.

The short-run demand functions for the variable factors energy and intermediate goods can be derived from (6.6) using the relationship given in Shephard's Lemma (3.32) as:

$$\partial G/\partial P_E = E/Q = \alpha_E + \gamma_{EE}P_E + \gamma_{EM}P_M + \gamma_{EK}K_{-1}/Q + \gamma_{Et}t \qquad (6.7a)$$

$$\partial G/\partial P_M = M/Q = \alpha_M + \gamma_{EM}P_M + \gamma_{MM}P_M + \gamma_{MK}K_{-1}/Q + \gamma_{Mt}t \qquad (6.7b)$$

The variable cost function is normalised by the price of labour so that the short-run demand function for this factor is obtained in a different manner. Since the cost function incorporates short-run cost-minimisation, the demand function for labour can be expressed in terms of the cost function and the demand equations for the remaining variable inputs according to:

$$L/Q = G/Q - P_E E/Q - P_M M/Q. \qquad (6.8)$$

Substituting the cost function in (6.6) for G/Q and the expressions for E/Q and M/Q (6.7) into the above, the short-run demand for labour can be written as

$$L/Q = \alpha_0 + \alpha_t t - 1/2\,[\gamma_{EE}P_E^2 + \gamma_{MM}P_M^2 - \gamma_{KK}(K_{-1}/Q)^2 - \gamma_{\dot{K}\dot{K}}(\Delta K/Q)^2 - \gamma_{tt}t^2] + \gamma_{tK}tK_{-1}/Q + \alpha_K K_{-1}/Q. \qquad (6.9)$$

The demand equation for capital is obtained in terms of an investment function by utilising the results for the optimal path of the quasi-fixed factor given by the flexible accelerator in equations (3.37) and (3.38) in Section 3.2.3. Expressing (3.37) in discrete terms, taking the appropriate derivatives of G in (3.38) and noting that $\partial^2 G/\partial K \partial \Delta K = \gamma_{K\dot{K}} = 0$ from (6.5), we have:

$$K - K = \Delta K = \lambda(K^* - K_{-1}) \qquad (6.10)$$

where

$$\lambda = -1/2[r - (r^2 + 4\gamma_{KK}/\gamma_{\dot{K}\dot{K}})^{1/2}]. \qquad (6.11)$$

To be meaningful in an economic sense, the adjustment coefficient λ should lie between 0 (no adjustment) and 1 (instantaneous adjustment). Since r, γ_{KK} and $\gamma_{\dot{K}\dot{K}}$ must be positive, λ must be positive. Further, it can be shown that if $0 < \gamma_{KK} < \gamma_{\dot{K}\dot{K}}$, then $0 < \lambda < 1$ for any positive r.

The equilibrium capital stock K^* is determined as the steady state solution to the Euler condition for cost-minimisation (3.35). Noting that at steady state $K_{-1} = K^*$ and $\Delta K = 0$, it follows that $\partial G^*/\partial \Delta K = 0$. Taking the derivative of G^* with respect to K^* from (6.6), substituting this into Euler condition and solving for K^* gives the optimal demand for capital:

$$K^* = 1/\gamma_{KK}(-\alpha_K - \gamma_{EK}P_E - \gamma_{MK}P_M - \gamma_{tK}t - u)Q \qquad (6.12)$$

where u is the normalised user cost or service price of capital. By combining (6.10), (6.11) and (6.12) and dividing by Q, the expression for capital investment becomes:

$$\Delta K/Q = -1/2[r-(r^2 + 4\gamma_{KK}/\gamma_{\dot{K}\dot{K}})^{1/2}][(1/\gamma_{KK}(-\alpha_K - \gamma_{EK}P_E - \gamma_{MK}P_M - \gamma_{tk}t - u) - K_{-1}/Q]. \qquad (6.13)$$

The model thus consists of four equations: the short-run demand functions for the variable factors (6.7a), (6.7b) and (6.9) and the capital accumulation function (6.13). Empirical estimation of this simultaneous equation system provides information concerning the characteristics of production in the short and long run as well as the speed of adjustment to equilibrium. The own- and cross-price elasticities can be calculated according to the formulae in equations (3.40) and elasticities with respect to output can be derived similarly. The effects of technical change on factor demand and production costs and the short-run returns to scale are obtained by taking the appropriate derivatives. As a presentation of all the elasticities would be far too space consuming, only a few of these will be given below for illustrative purposes.

The short-run Allen elasticities of substitution (AES) between factor pairs can be calculated from equation (3.20) in the same manner as for the partial static equilibrium model. In the long run, these elasticities are obtained by replacing variable costs, C_V, with total costs, $C_T = C_V + uK^*$, and evaluating the derivatives at the optimal values of the quasi-fixed factor K^* given by (6.12). For the quadratic cost function given in (6.6) the short- and long-run AES between the variable factors can be written as:

$$\sigma^S_{EM} = C_V\gamma_{EM}Q/EM$$

$$\sigma^L_{EM} = C_T(\gamma_{EM} - \gamma_{EM}\gamma_{MK}/\gamma_{KK})Q/EM \qquad (6.14)$$

$$\sigma^S_{iL} = -C_V(P_E\gamma_{Ei} + P_M\gamma_{iM})Q/LX_i \qquad \text{for } i = E,M$$

$$\sigma^L_{iL} = C_T[P_E\gamma_{Ei} + P_M\gamma_{iM} + (\alpha_K + \gamma_{KK}K^*/Q + \gamma_{Kt}t)\gamma_{iK}/\gamma_{KK}]Q/LX_i.$$

The long-run AES between energy and capital can be calculated as

$$\sigma^{L}_{EK} = -(\gamma_{EK}/\gamma_{KK})\,(C_V + uK^*)\,Q/K^*E. \tag{6.15}$$

Energy and capital are long-run substitutes if $\sigma^{L}_{EK} > 0$ and complements if $\sigma^{L}_{EK} < 0$. Since C_V, K^*, E, Q, u and $\gamma_{KK} > 0$, the relationship between energy and capital is determined by the sign of γ_{EK}. If $\gamma_{EK} > 0$, these two inputs are complements; if $\gamma_{EK} < 0$, they are substitutes. Finally, if $\gamma_{EK} = 0$ energy and capital are independent in production.

The own-price elasticities for energy in the short, intermediate and long run are calculated as:

$$\begin{aligned} \varepsilon^{S}_{EE} &= \gamma_{EE}QP_E/E \\ \varepsilon^{I}_{EE} &= (\gamma_{EE} - \lambda\,\gamma^{2}_{EK}/\gamma_{KK})QP_E/E \\ \varepsilon^{L}_{EE} &= (\gamma_{EE} - \gamma^{2}_{EK}/\gamma_{KK})QP_E/E. \end{aligned} \tag{6.16}$$

From the above we see that the difference between the short- and long-run price elasticities is $\varepsilon^{L}_{EE} - \varepsilon^{S}_{EE} = (-\gamma^{2}_{EK}/\gamma_{KK})$. Since $\gamma_{KK} > 0$, this term is negative regardless of the sign of γ_{EK}. If, however, energy and capital are independent in production, i.e. $\gamma_{EK} = 0$, then $\varepsilon^{L}_{EE} = \varepsilon^{S}_{EE}$. Thus, the long-run own-price response is always greater than or equal to the short-run impact. This is of course as it should be according to economic theory. It can also be noted that the difference between the short and long run does not depend on the adjustment coefficient λ. Thus even if the capital stock does not adjust instantaneously, i.e. $\lambda \neq 0$, individual production factors can adjust instantaneously if they are independent of capital i.e. if $\gamma_{iK} = 0$. The dependence of the variable factors on the capital stock can be investigated by statistically testing whether the γ_{iK} are significantly different from zero.

The cross-price elasticities between the variable factors are also obtained from the relationships given in Section 3.2.3. For example, the effects of changes in energy prices on the demand for intermediate goods in the short and long run are:

$$\begin{aligned} \varepsilon^{S}_{ME} &= \gamma_{EM}P_EQ/M \\ \varepsilon^{L}_{ME} &= (\gamma_{EM} - \gamma_{EK}\gamma_{MK}/\gamma_{KK})P_EQ/M. \end{aligned} \tag{6.17}$$

It is easily seen that the difference between the short- and long-run responses is determined by the signs of γ_{EK} and γ_{MK}, the same parameters that define the substitution relationships between energy and capital and materials and capital. If energy and materials are short-run complements ($\gamma_{EM} < 0$), they can be either complements or substitutes in the long run depending on the signs and magnitudes of γ_{EK} and γ_{MK}. If both energy and materials are substitutable (complementary) with capital, γ_{EK}, $\gamma_{MK} < 0$ (γ_{EK}, $\gamma_{MK} > 0$), then energy and materials will be complements in the long run as well. However, if only one of these inputs is substitutable with capital, energy and materials can be long-run substitutes. Thus, the nature of the substitution relationships need not be the same in different time perspectives.

The effects of other exogenous variables—the rate of return, output, technical change, etc.—can be investigated in a similar fashion. The few examples given above clearly illustrate the wealth of information contained in the model.

6.2 *Empirical Results*

The dynamic cost of adjustment model is applied to Swedish manufacturing data by estimating the demand equations for the variable factors together with the capital accumulation equation. The stochastic specification includes the specification of additive disturbances for each of the equations. The disturbances are assumed to be joint normally distributed with zero mean and constant variance-covariance matrix. Allowance for non-zero contemporaneous correlation between the error terms of the different equations is made, but the specification assumes that the error terms are serially independent.

It can be seen that the equation system is non-linear in its parameters, thus requiring a non-linear estimation procedure. Also, the system is seen to be simultaneous since K appears as a regressor in the labour equation and also as an independent variable in the investment function. However, the system is recursive so that a seemingly unrelated equations estimation procedure is valid and results in full information maximum likelihood estimates. All estimations were carried out using nonlinear iterative Zellner techniques.[4] It should be pointed out that the estimation procedure for non-linear equations does not provide analytic expressions for the estimated parameters. Instead an iterative process must be used to attain coefficient estimates that maximise the likelihood function. The convergence of such iterative processes (within a

[4] All estimations were carried out using the non-linear equations procedure in the microcomputer version of SHAZAM. See also Zellner (1962) and (1963).

reasonable number of iterations) often depends on the starting values used for the parameters. Although convergence generally can be attained, there is no assurance that a global optimum is reached. In the event of multiple local maxima, the process would converge to one of them depending on the starting values. Several sets of starting values were tried in order to confirm the estimates presented here, but a full investigation of the characteristics of the likelihood function would be impracticable.

The cost of adjustment model described in the previous sections was estimated for total Swedish Manufacturing (excl. energy producing sectors) and for 9 manufacturing subsectors: Food, Textiles, Pulp and Paper, Printing, Rubber Products, Chemicals, Non-Metallic Mineral Products, Primary Metals and Engineering Products. The data include annual observations on quantities and prices of energy, labour and intermediate goods, the net capital stock, prices and values of investments in machinery and structures, and gross output in current and constant prices for the years 1952—83. A complete description of the data sources and the construction of the variables can be found in the appendix to Chapter II.

The resulting parameter estimates and standard errors are shown in tables 6.A.1 and 6.A.2 in the appendix to this Chapter. In order to conform to economic theory, some of the parameters must fulfill certain constraints. Particularly, in order for the own-price elasticities for energy and intermediate goods to be negative it is required that γ_{EE} and $\gamma_{MM} < 0$. We see that this condition is filled, and generally significantly, for all industries except for Pulp and Paper, where these coefficients are not significantly different from 0. The t-values of the coefficients of the capital stock terms, γ_{EK} and γ_{MK}, indicate, however, a significant relationship between the capital stock and the variable factors in only about half of the industries. As these parameters determine the difference between short- and long-run elasticities, these results are not very encouraging.

Finally, as mentioned earlier, the adjustment coefficient must lie between 0 (no adjustment) and 1 (instantaneous adjustment). This requires that $0 < \gamma_{KK} < \gamma_{\dot{K}\dot{K}}$. We see that this condition is fulfilled for all industries. At first glance, it would appear that the estimates are not totally unrealistic from an economic point of view. We will now turn to an examination of their statistical quality, by examining the goodness of fit of the separate equations using the $\cos^2$ measure defined in Chapter IV. These are given in table 6.A.3. for each industry. Comparing the different equations we see that the fit of the energy, labour and investment equations is quite good, whereas the demand equation for intermediate goods is rather poorly explained in at least half of the industries. The extremely low values for this equation show, in fact, that in many cases a constant materials output ratio would do at least as well as the specified model. This lack of fit is not quite as serious as it may seem since all parameters of the model, excluding γ_{Mt}, appear in the energy and labour equations and can be estimated from these equations alone.

It should also be mentioned that the Durbin-Watson statistics for the individual equations are generally rather low. Although this test is not really applicable for equations containing lagged values of dependent variables among the explanatory variables, it will generally detect the presence of autocorrelation when the coefficient of autocorrelation (ρ) is high. As the Durbin-Watson statistics are less than 1 for many of the equations, this is certainly indicative of a high degree of serial correlation. The presence of autocorrelation is quite serious, as in combination with lagged dependent variables, simple estimation techniques will produce biased estimators. Attempts to correct for autocorrelation have, however, had little effect on the estimated parameters.

We shall proceed, then, to investigate the implications of estimates in terms of elasticities. Since all but the short-run elasticities are non-linear functions of estimated parameters, the variances of these elasticities are not easily computed. The significance levels of the elasticities are therefore not reported.

To begin with, we will look at the implied adjustment coefficients for the different industries which are shown in table 6.1. These are not constant over time, but are dependent on variations in the rate of return to capital. Since the calculated values are rather stable over the observation period, only those for 1970 are presented.

We see that the adjustment coefficients, λ, are quite different for the various branches. The most rapid adjustment is found for the Non-Metallic Mineral Products, Printing and Engineering industries, where more than 25% of the adjustment of the capital stock is shown to occur within one year. The slowest adjustment is indicated for Food, Textiles and Pulp & Paper, where less than 10% occurs within the first year. The adjustment coefficient for Total Manufacturing of 11% lies somewhere in between these two groups. Although it is rather difficult to judge the validity of these results, in some cases they may appear rather questionable. One might, for example, expect heavy process industries such as Primary Metals and Non-Metallic Minerals to adjust more slowly than smaller less capital-intensive industries such as those typical for the Food and Textile sectors, whilst the contrary is indicated by the estimates. On the other hand, the Food and Textile industries are contracting sectors, so perhaps we could expect adjustment to be slower for those than for expanding industries such as Chemicals, Engineering and Primary Metals as the estimates suggest.

The estimated short-, intermediate- and long-term own-price elasticities for the years 1960, 1970 and 1980 are shown in tables 6.2a and 6.2b. Since these depend on the variation in prices and input-output ratios, they tend to vary considerably over the data sample. We find that all price elasticities decrease over time, except those pertaining to energy. This is primarily a result of

Table 6.1 Estimated Adjustment Coefficients, 1970.

Food	.08
Textiles	.08
Pulp & Paper	.09
Printing	.26
Rubber	.16
Chemicals	.17
Non-Metallic Minerals	.28
Primary Metals	.22
Engineering	.26
Total Manufacturing	.11

decreasing input-output ratios, particularly in the case of labour. The own-price elasticities for energy decrease up until the mid-seventies, thereafter to increase again. The only exception is in the Printing industry, where a high elasticity during the 60's is reduced considerably in the 70's and 80's. The changes in price sensitivity follow from the definition of the elasticities. As is seen in equation (6.16), the elasticity is directly related to factor price and inversely related to input-output ratio so that if an increase in factor price is matched by a corresponding greater decrease in factor intensity, the elasticity will rise. This suggests that the substantial energy price rises of the latter seventies have served to make energy more price sensitive than previously.

Further inspection of the own-price elasticities for the four inputs shows that these are negative for all but the variable factors in the Paper Industry. Also, the price-response generally tends to be below unity, even in the long run. In fact, for many of the inputs the elasticities are identical, or very nearly so, in both the short and long run. This is rather disconcerting, as one would not expect the full adjustment of the variable factors to be instantaneous. As pointed out in the discussion of the estimated coefficients, the nonsignificance of the interaction terms between the variable factors and capital, γ_{EK} and γ_{MK}, in many of the industries necessarily leads to this result.

Regarding the individual industries, we find that the long-run own-price elasticity for energy varies considerably, falling in an interval from 0 to -1.1. Surprisingly, one of the most energy-intensive sectors—Pulp & Paper—appears to be totally insensitive to energy-price changes. Energy appears to be particularly price-sensitive over the entire observation period in the Textile, Rubber, Primary Metal and Engineering industries, with elasticity estimates greater than $-.5$. The high energy price elasticity in the Textile industry is perhaps somewhat surprising, as energy accounts for such a small share of production costs. Again, there is little difference between short- and long-run values for the majority of the sectors. It would appear that adjustment to

Table 6.2a. Own-Price Elasticities for Energy and Materials.

	Year	ε^S_{EE}	ε^I_{EE}	ε^L_{EE}	ε^S_{MM}	ε^I_{MM}	ε^L_{MM}
Food	60	−.13	−.13	−.13	−.05	−.05	−.06
	70	−.05	−.05	−.05	−.03	−.03	−.03
	80	−.06	−.06	−.06	−.01	−.01	−.02
Textiles	60	−.78	−.79	−.89	−.15	−.15	−.16
	70	−.35	−.35	−.36	−.06	−.06	−.06
	80	−.50	−.50	−.51	−.05	−.05	−.05
Paper & Pulp	.60	.02	.02	.01	.00	−.01	−.08
	70	.01	.01	.00	.00	.00	−.03
	80	.01	.01	.01	.00	.00	−.03
Printing	60	−.64	−.64	−.65	−.28	−.28	−.28
	70	−.11	−.11	−.11	−.16	−.16	−.16
	80	−.10	−.10	−.10	−.15	−.15	−.15
Rubber	60	−1.07	−1.07	−1.07	−.09	−.11	−.24
	70	−.41	−.41	−.41	−.03	−.04	−.09
	80	−.74	−.74	−.74	−.03	−.03	−.07
Chemicals	60	−.20	−.20	−.20	−.13	−.13	−.14
	70	−.13	−.13	−.13	−.04	−.04	−.04
	80	−.25	−.25	−.25	−.03	−.03	−.03
Non-Metallic minerals	60	−.04	−.14	−.39	−.33	−.40	−.61
	70	−.02	−.06	−.17	−.15	−.19	−.28
	80	−.03	−.11	−.30	−.12	−.16	−.23
Primary Metals	60	−.53	−.55	−.63	−.08	−.10	−.18
	70	−.33	−.34	−.39	−.04	−.05	−.09
	80	−.47	−.49	−.56	−.03	−.04	−.97
Engineering	60	−.70	−.70	−.71	−.21	−.21	−.21
	70	−.34	−.34	−.35	−.10	−.10	−.10
	80	−.50	−.50	−.51	−.08	−.08	−.80
Total Manufacturing	60	−.12	−.12	−.13	−.05	−.05	−.12
	70	−.06	−.06	−.06	−.02	−.03	−.06
	80	−.09	−.09	−.09	−.02	−.02	−.04

Table 6.2b. Own-Price Elasticities for Labour and Capital.

	Year	ε^{S}_{LL}	ε^{I}_{LL}	ε^{L}_{LL}	ε^{L}_{KK}
Food	60	−.31	−.31	−.34	−.38
	70	−.15	−.15	−.15	−.19
	80	−.06	−.06	−.07	−.11
Textiles	60	−.22	−.23	−.34	−.23
	70	−.07	−.08	−.13	−.12
	80	−.06	−.06	−.09	−.08
Paper & Pulp	60	.02	.02	.01	−.45
	70	.01	.01	.00	−.23
	80	.01	.01	.01	−.13
Printing	60	−.40	−.41	−.44	−.25
	70	−.21	−.21	−.22	−.10
	80	−.24	−.25	−.25	−.05
Rubber	60	−.03	−.04	−.06	−.36
	70	−.01	−.01	−.01	−.21
	80	−.01	−.01	−.02	−.15
Chemicals	60	−.39	−.40	−.46	−.39
	70	−.11	−.12	−.18	−.26
	80	−.16	−.17	−.19	−.18
Non-Metallic minerals	60	−.25	−.25	−.26	−.40
	70	−.14	−.14	−.14	−.16
	80	−.08	−.08	−.09	−.09
Primary Metals	60	−.25	−.28	−.36	−.32
	70	−.14	−.15	−.20	−.17
	80	−.19	−.23	−.33	−.08
Engineering	60	−.31	−.32	−.34	−.11
	70	−.14	−.14	−.15	−.06
	80	−.11	−.12	−.12	−.04
Total Manufacturing	60	−.11	−.11	−.12	−.24
	70	−.05	−.05	−.05	−.12
	80	−.04	−.04	−.05	−.07

energy price changes is very rapid, the only exception being in the Non-Metallic Mineral industry.

The short-run own-price elasticities for intermediate goods are very close to zero for many of the industries and less than −.3 for all but the Non-Metallic Minerals industry, indicating a constant materials-output ratio in the short run. Even in the long run, however, we find that intermediate goods are not very sensitive to price-changes in the majority of sectors. The low price-sensitivity of intermediate goods is in agreement with the findings of most other empirical studies.

The results indicate that the demand for labour is not very sensitive to price changes in the short or the long run. Again, the price response varies considerably for the different industries. In all sectors, however, the price elasticity for labour is considerably below unity even in the long run. Labour appears to be the most price-sensitive in the Printing, Chemical and Primary Metal industries—all of which have become particularly capital intensive during the period studied.

The price elasticity for capital is, by definition, zero in the short run but is considerably less than unity even in the long run. Comparing the long-run elasticities for the various inputs, the results suggest—perhaps surprisingly—that energy is the most price-sensitive production factor in many of the industries. For Total Manufacturing, however, capital is the most price-sensitive production factor for all but the most recent years. In general, we find that the price elasticities for Total Manufacturing are much lower than they are for the individual industries, suggesting that elasticities based on the aggregate tend to underestimate the flexibility of factor demand.[5] Finally, materials are least price-sensitive in about half of the industries.

In order to investigate the substitution relationships between inputs, the Allen elasticities of substitution between factor pairs are presented in tables 6.3a and 6.3b. Again, we find a good deal of variation over the observation period. The results suggest that materials and energy and materials and labour are substitutes in the overwhelming majority of industries, while energy and labour are found to be substitutes for all years in only 3 of the 10 industries. In most cases, however, the complementarity becomes weaker over time and in a few cases switches to substitutability for the most recent years.

Regarding the relationships between capital and the variable factors, we find that materials and capital are predominantly substitutes while energy and capital are generally complementary, the only exceptions being in the Rubber,

[5] The same result was noted in the models estimated in the previous two chapters.

Chemical and Primary Metal industries where substitutability prevails. Finally, we see that in about half of the sectors, capital and labour are substitutes over the entire period. The complementarity between capital and labour in Total Manufacturing, Primary Metals and the Rubber industry is rather difficult to explain as one would expect these inputs to be substitutes. Capital-labour complementarity has, however, also been reported in other studies based on cost of adjustment models,[6] and even been noted in the application of the partial static equilibrium model in Chapter 5. As was pointed out there, K-L substitutability in a two factor model need not necessarily imply K-L substitutability when more factors of production are considered jointly. Another explanation can have to do with the aggregation of capital. Although we would expect labour and equipment to be substitutes, this may not be true for labour and structures. In any case, the aggregation of equipment and structures implicitly assumes that these are perfect substitutes. A study by Bergström and Panas (1985) of Swedish manufacturing industries based on a static model indicates that structures and equipment are complementary or only weak substitutes in the majority of cases. This, of course, challenges the validity of our aggregation of capital.

Although the estimated elasticities are not totally unreasonable, a rather surprising result is the small differences between short- and long-run elasticities. This is rather disconcerting since it is the dynamics of the adjustment process and the difference between short- and long-run production relationships that motivate the application of this model. Although the adjustment coefficients indicate that adjustment of the capital stock is not instantaneous, the price response is rather small and the relationship between capital and the variable inputs is often very weak so that the adjustment of the capital stock plays an insignificant role in determining the long-term response of the other production factors. Although this may seem rather unreasonable, similar results have been found in applications of cost of adjustment models by other authors.[7]

Next, let us consider the effects of production scale on factor demand. The estimated short-run output elasticities for energy, materials and labour and scale economies for the years 1960, 1970 and 1980 are shown in table 6.4. These elasticities give the percentage change in the variable factors that results from a 1% change in output, the capital stock remaining constant. Since the model formulation assumes long-run constant returns to scale, the output elasticities for all production factors are equal to unity in the long run. In the

[6] For example, in Berndt, Fuss and Waverman (1980).

[7] Berndt, Fuss and Waverman (1980) and Denny, Fuss and Waverman (1980).

Table 6.3a. Elasticities of Substitution for the Variable Factors.

	Year	σ^S_{EM}	σ^L_{EM}	σ^S_{EL}	σ^L_{EL}	σ^S_{ML}	σ^L_{ML}
Food	60	.28	.32	−.92	−.62	.38	.30
	70	.16	.18	−.60	−.46	.19	.15
	80	.08	.09	−.04	.02	.07	.05
Textiles	60	1.62	1.10	−.70	−.15	.36	.30
	70	.80	.52	−.34	−.06	.13	.11
	80	.62	.40	.34	.37	.08	.06
Paper & Pulp	60	.01	.11	−.09	−.05	−.02	−.05
	70	.00	.05	−.04	−.00	−.01	−.04
	80	.00	.04	−.06	−.06	−.01	.00
Printing	60	.88	.61	.30	.42	.68	.44
	70	.29	.20	−.14	−.04	.37	.23
	80	.22	.15	−.08	−.03	.40	.26
Rubber	60	2.19	1.75	−1.03	−.82	.09	.19
	70	.98	.69	−.32	−.23	.03	.03
	80	.86	.60	.63	.44	−.02	.01
Chemicals	60	−.19	−.16	1.14	.91	.50	.38
	70	−.09	−.08	.59	.44	.13	.09
	80	−.10	−.08	1.12	.88	.14	.10
Non-Metallic Minerals	60	.59	1.42	−.58	−.31	.64	.41
	70	.32	.78	−.35	−.28	.31	.23
	80	.27	.68	.28	−.08	.24	.13
Primary Metals	60	.29	−.11	1.13	1.71	.22	.39
	70	.19	−.07	.80	1.09	.12	.22
	80	.16	−.06	1.50	1.83	.06	.20
Engineering	60	1.21	.83	−.09	.08	.51	.36
	70	.72	.48	−.22	−.07	.23	.16
	80	.57	.38	.41	.34	.17	.12
Total Manufacturing	60	.04	.14	.32	.20	.15	.17
	70	.02	.07	.14	.08	.06	.07
	80	.02	.06	.26	.17	.05	.05

Table 6.3b. Elasticities of Substitution for Capital.

	Year	σ^L_{EK}	σ^L_{ME}	σ^L_{LE}
Food	60	−1.39	.28	1.76
	70	−.74	.14	.62
	80	.39	.07	.43
Textiles	60	−1.24	−.14	1.79
	70	−.60	−.06	.69
	80	−.52	−.05	.58
Paper & Pulp	60	−48	.55	.07
	70	.27	.25	.07
	80	.22	.20	−.02
Printing	60	−1.39	.04	.64
	70	−.39	.02	.22
	80	−.27	.02	.11
Rubber	60	.08	.65	−.38
	70	.04	.30	−.03
	80	.03	.24	−.08
Chemicals	60	.16	.14	.42
	70	.11	.06	.28
	80	.12	.05	.20
Non-Metallic Minerals	60	−2.43	.99	.05
	70	−1.19	.39	−.01
	80	−.97	.31	.04
Primary Metals	60	1.58	.56	−.40
	70	1.06	.30	−.24
	80	.81	.21	−.28
Engineering	60	−.58	−.02	.79
	70	−.35	−.01	.37
	80	−.31	−.01	.29
Total Manufacturing	60	−.52	.41	−.25
	70	−.29	.20	−.10
	80	−.23	.14	−.11

short run, however, these elasticities may either be less than or greater than unity depending on whether the particular input increases less than or more than proportionately to the short-run increase in output. If the use of a particular production factor increases proportionately less than output, the input-output ratio decreases and the short-run returns to that factor will be increasing. Conversely, if the use of the production factor increases proportionately more than the increase in output, the input-output ratio will rise and the returns to that factor will be decreasing.

Inspection of the table shows that the elasticities have the correct signs, being positive in all but one case. We see that 5 of the industries are characterised by diminishing returns to labour ($\upsilon_{LQ} > 1$) for the entire observation period, while the remaining sectors show diminishing returns for a large part of the period but increasing returns for the most recent years.

Regarding the effects of production scale on short-run energy use, the estimates indicate that the short-run effects of an increase in output are a less than proportionate increase in energy usage for the majority of industries. For most industries the effect is smaller towards the end of the period. This increasing returns to energy can probably be explained by the fact that a significant proportion of energy utilisation is fixed (e.g. space heating) and independent of marginal changes in output. The only industry which shows an energy-output elasticity appreciably above unity is Primary Metals, a heavy process industry in which the proportion of fixed energy utilisation is relatively small.

For the majority of the industries, the materials-output elasticity is close to unity. Only a few industries, and particularly Non-Metallic Mineral Products, show a short-run output elasticity appreciably greater than the long- run value of one. This short-run overshooting may be explained by the fact that these firms respond to an output increase in the short run by increasing the purchase of semi-finished goods or services that are generally produced within the firm.

In the last column of the table, the over-all short-run returns to scale are presented. This is calculated as the inverse of the variable cost elasticity, which measures the effects of an increase in output on variable costs given level of the capital stock. We would, of course, expect to find diminishing returns to scale ($\eta_Q^S < 1$) in the presence of a fixed production factor, which is generally supported by our results. We do find, however, that four industries display short-run increasing or nearly constant returns in the most recent years. In fact, even for those industries showing consistently diminishing returns, η_Q^S is nearer to unity for the mid-seventies onwards. The reason for this is obviously to be found in the low level of capacity utilisation evident during this period.[8] If the capital stock is fixed and utilised to near full capacity, it

[8] See Chapter II.

Table 6.4. Short-Run Output Elasticities and Returns to Scale.

	Year	υ^S_{EQ}	υ^S_{MQ}	υ^S_{LQ}	η^S_Q
Food	60	.89	1.05	1.28	.93
	70	.87	1.04	1.49	.91
	80	.77	1.04	1.49	.90
Textiles	60	.65	.97	1.36	.91
	70	.59	.94	1.52	.87
	80	.31	.91	.02	1.78
Paper & Pulp	60	.90	1.15	1.28	.85
	70	.91	1.13	1.86	.75
	80	.81	1.13	1.11	.91
Printing	60	.63	.97	1.17	.94
	70	.63	1.04	1.19	.91
	80	.65	1.03	1.08	.96
Rubber	60	1.15	1.15	.98	.91
	70	1.18	1.23	1.46	.75
	80	1.13	1.24	.57	1.03
Chemicals	60	.93	1.11	1.21	.88
	70	1.17	1.02	1.75	.79
	80	.98	1.02	1.25	.92
Non-Metallic Minerals	60	.12	1.34	1.01	.92
	70	−.22	1.39	1.57	.73
	80	−.51	1.48	.76	1.05
Primary Metals	60	1.63	1.19	.93	.86
	70	1.87	1.20	1.13	.81
	80	1.54	1.18	.71	.90
Engineering	60	.53	.97	1.46	.89
	70	.62	1.01	2.03	.72
	80	.33	.98	1.01	1.02
Total Manufacturing	60	.83	1.15	1.13	.89
	70	.85	1.15	1.62	.77
	80	.69	1.15	.89	.95

is clear that an increase in output will lead to a greater than proportional increase in at least some of the variable inputs, and hence in variable costs. The constraint of the capital stock is binding so that $\eta_Q^S < 1$. On the other hand, if the capital stock is operating far under full-capacity, it would be reasonable to assume that an increase in output could be achieved by increasing the variable inputs more or less proportionately so that variable costs increase less than they would have done at full-capacity. This would imply that η_Q^S at full-capacity is less than η_S^Q at low-capacity utilisation, and that the latter would approach unity at very low utilisation levels.

This same argument can be used to explain the decrease in scale elasticities for energy and labour noted earlier. At low-capacity utilisation an increase in output would require less of an increase in the inputs of labour and energy than if the capital stock were operating at capacity.

Finally estimation of the model allows us to analyse the impact of disembodied technological change on the use of factor inputs. For this purpose, the long-run technology elasticities for each production factor are calculated and presented in table 6.5. This elasticity measures the percentage change in the use of each input that can be attributed to technical progress.

We see that in all cases, the estimates indicate that technological development has led to a significant reduction in the use of labour. This is as one would expect from casual observation of the technological advances of the time period studied. Further, we find that technological change has also led to a decrease in energy use, though to a smaller degree, in all but the Food, Printing and Rubber industries, in which technical change has been energy-using. The impact of technical progress on the use of intermediate goods has, on the other hand, been minimal in all industries. Finally, the results indicate that the effects of technical change on capital usage has varied across industries, in some cases leading to an increase, in others to a decrease.

The estimated average rate of growth in total factor productivity is given in the last column of the table. The results indicate an average total factor productivity growth of about 2% per annum overall which agrees quite well with other evidence. The lowest productivity growth is noted for the Food industry, which is nearly zero, while productivity growth rates of more than 3% per year are indicated for the Textile, Engineering, Non-Metallic Mineral and Rubber industries. Comparing the average rates with those for the individual production factors, we can say that technical progress has on the whole been labour-saving, and capital- and materials-using, while the effects on energy use vary from sector to sector. These results are well in conformity with those reported in other empirical studies for Swedish industries.

Table 6.5. Long-Run Impacts of Technological Progress, 1970.

	Energy	Materials	Labour	Capital	ΔTFP
Food	.3	.21	.9	0.0	.1
Textiles	−1.2	−.9	−4.9	−2.0	3.7
Pulp & Paper	−1.3	.4	−5.6	−.7	1.9
Printing	1.8	−.2	−1.9	.5	1.3
Rubber	1.5	−.2	−6.0	−1.1	3.8
Chemicals	−3.4	.2	−4.0	−2.2	2.0
Non-Metallic Minerals	−.9	−.4	−5.7	1.0	3.5
Primary Metals	−4.3	−.5	−6.2	.5	2.5
Engineering	−1.7	−.7	−5.0	−1.4	3.6
Total Manufacturing	−1.8	.0	−5.6	−.0	2.3

6.3 *Conclusions*

The results obtained from the estimation of the dynamic cost of adjustment model for Swedish industrial sectors generally conform to economic theory. On the whole, the results indicate that factor demand is rather insensitive to relative price changes. For nearly all inputs and all industries the own-price elasticity of demand is under −0.5 even in the long run. Energy is found to be the most price-sensitive production factor, but even here the price-elasticity is generally quite low. The insensitivity of factor demand to price changes is a reflection of limited possibilities for factor substitution: in most cases the results indicate only weak substitutability—and in many cases complementarity—between factor pairs.

The sharp decrease in the labour-output ratio and increase in the capital-labour ratio during the time period studied is thus hardly explained by changes in relative factor prices. The results indicate, however, that much of this capital-labour substitution can be explained by an autonomous labour-saving, capital-using technical change.

The empirical application of the model supports the hypothesis that the cost of adjusting the capital stock is a significant factor in the investment process and hence in explaining the inflexibility of capital. We find that between 8 and 28% of full adjustment of the capital stock occurs within one year. However, since capital is rather insensitive to price changes and substitution between capital and the variable inputs limited, we find little—if any—difference between short- and long-run substitution possibilities and hence price elasticities.

Judging from these results, it appears that the cost of adjustment model provides little information in addition to that obtained on the basis of the static models. We will return to a more detailed comparison of the results of the various models, along with a critical evaluation of their performance in Chapter VII.

APPENDIX

Table 6.A.1. Cost of Adjustment Model. Parameter Estimates.

	Food	Textiles	Paper	Printing	Rubber
α_E	.0107	.0081	.0363	.0038	.0294
	(.0069)	(.0007)	(.0113)	(.0015)	(.0065)
γ_{EE}	−.0006	−.0066	.0004	−.0006	−.0113
	(.0005)	(.0007)	(.0024)	(.0002)	(.0020)
γ_{EM}	.0010	.0068	.0001	.0010	.0107
	(.0008)	(.0009)	(.0019)	(.0005)	(.0015)
γ_{ET}	.0000	.0000	−.0005	.0001	.0004
	(.0001)	(.0001)	(.0002)	(.0000)	(.0001)
γ_{EK}	.0063	.0167	.0071	.0038	−.0009
	(.0180)	(.0048)	(.0114)	(.0027)	(.0067)
α_M	.7271	.5272	.6673	.6711	.5955
	(.0187)	(.0502)	(.0211)	(.0414)	(.0235)
γ_{MM}	−.0110	−.0257	.0010	−.0894	0.0119
	(.0026)	(.0070)	(.0034)	(.0098)	(.0043)
γ_{MT}	.0011	−.0044	.0014	−.0007	−.0021
	(.0003)	(.0016)	(.0005)	(.0005)	(.0009)
γ_{MK}	−.0624	.0491	−.0912	−.0200	−.1478
	(.0417)	(.0821)	(.0219)	(.0599)	(.0259)
α_O	.3596	.7673	.4972	.6861	.7544
	(.0747)	(.1395)	(.0751)	(.1310)	(.0873)
α_T	−.0036	−.0401	−.0202	−.0048	−.0380
	(.0022)	(.0085)	(.0031)	(.0030)	(.0042)
α_K	−.8593	−1.3538	−.5050	−1.1427	−.7978
	(.3344)	(.4922)	(.1548)	(.4157)	(.2110)
γ_{KK}	1.6132	2.1012	.4212	1.7421	.9999
	(.7610)	(.8665)	(.1634)	(.6685)	(.2759)
γ_{KT}	.0000	.0251	.0033	−.0050	.0096
	(.0053)	(.0178)	(.0040)	(.0048)	(.0054)
$\gamma_{\dot{K}\dot{K}}$	168.37	219.15	39.74	23.18	31.95
	(67.02)	(70.73)	(15.34)	(8.12)	(8.39)

Note: Standard errors given in parenthesis. The coefficient γ_{tt} was non-significant and thus set equal to 0.

Table 6.A.2. Cost of Adjustment Model. Parameter Estimates.

	Chemicals	Minerals	Metals	Engineer-ing	Total
ε_E	.0551	−.0400	.1729	.0064	.0246
	(.0098)	(.0236)	(.0459)	(.0032)	(.0123)
γ_{EE}	−.0065	−.0019	−.0295	−.0043	−.0020
	(.0029)	(.0100)	(.0089)	(.0004)	(.0021)
γ_{EM}	−.0025	.0116	.0079	.0044	.0004
	(.0030)	(.0099)	(.0060)	(.0007)	(.0026)
γ_{Et}	−.0016	−.0019	−.0036	−.0001	−.0006
	(.0003)	(.0008)	(.0007)	(.0001)	(.0002)
γ_{EK}	−.0032	.0897	−.0503	.0121	.0098
	(.0128)	(.0203)	(.0340)	(.0051)	(.0162)
α_M	.6004	.6722	.7566	.5574	.6715
	(.0273)	(.0354)	(.0332)	(.0512)	(.0408)
γ_{MM}	−.0205	−.0564	−.0181	−.0413	−.0093
	(.0062)	(.0104)	(.0049)	(.0097)	(.0053)
γ_{Mt}	.0003	.0003	−.0024	−.0035	.0001
	(.0008)	(.0010)	(.0004)	(.0010)	(.0006)
γ_{MK}	−.0221	−.1549	−.0973	.0120	−.1251
	(.0365)	(.0294)	(.0262)	(.0979)	(.0556)
α_O	.5318	.8018	.5800	.9963	.6630
	(.0747)	(.1047)	(.1349)	(.1465)	(.1346)
α_t	−.0264	−.0154	−.0176	−.0410	−.0198
	(.0051)	(.0043)	(.0048)	(.0052)	(.0043)
α_K	−.5823	−.6713	−.5123	−.0173	−.9169
	(.1729)	(.1545)	(.2028)	(.5016)	(.3484)
γ_{KK}	.5350	.4926	.4360	3.0004	1.0863
	(.2106)	(.1229)	(.1553)	(.8811)	(.4614)
γ_{Kt}	.0118	−.0064	−.0029	.0271	.0003
	(.0053)	(.0033)	(.0032)	(.0076)	(.0057)
$\gamma_{\dot{K}\dot{K}}$	15.12	5.54	7.73	38.59	71.56
	(5.33)	(1.72)	(2.02)	(16.83)	(34.19)

Note: Standard errors given in parenthesis. The coefficient γ_{tt} was non-significant and thus set equal to 0.

Table 6.A.3. Cost of Adjustment Model. Goodness of Fit.

	Energy	Materials	Labour	Capital
Food	.15	.93	.98	.85
Textiles	.95	.03	.98	.65
Pulp & Paper	.76	.01	.88	.77
Printing	.97	.89	.97	.62
Rubber	.78	.50	.94	.82
Chemicals	.81	.77	.95	.61
Non-Metallic Minerals	.74	.75	.95	.49
Primary Metals	.40	.05	.96	.57
Engineering	.95	.08	.94	.84
Total Manufacturing	.84	.12	.94	.89

Note: Calculated as the squared cosine of the angle between the actual and the predicted values of the endogenous variables. Lies within the interval (0,1).

VII. CONCLUSIONS

In the preceding chapters a number of different models describing factor demand and production relationships have been presented and estimated for Swedish manufacturing sectors. With the assumption of full static equilibrium (FSE) as the point of departure, each subsequent model attempted to introduce slightly more realism by allowing for the inflexibility of particular inputs. For the obvious reasons discussed throughout, we concentrated mainly on the capital stock, first in the context of a simple partial static equilibrium model (PSE) in Chapter V and then by incorporating the notion of increasing marginal costs of adjustment (COA) in Chapter VI. Also, in Chapter V an attempt was made to extend the partial static equilibrium model to allow for inflexibilities in labour as well as capital.

In this concluding chapter, we will summarise and evaluate the results obtained from the various models, examine the conclusions that can be drawn on the basis the empirical investigation and present suggestions for the possible improvement of the empirical results. We will begin with a statement of the assumptions that underlie the analysis. The implications of these assumptions will be discussed and the results of the various models assessed.

The models presented and estimated in the preceding chapters all fall under the paradigme of orthodox, neoclassical theory. As is well known, this theory rests on a number of assumptions or theoretical constructs, which, although in themselves empirically unverifiable, serve as a basis for arriving at testable propositions concerning economic phenomena. Two of these assumptions which are relevant to our analysis are:

— Efficient production techniques can be summarised in the form of a production function which relates the quantities of each good produced to all minimal combinations of inputs which can be used to produce that good
— The firm's decision making process is characterised by rational, optimising behaviour.

Along with these, other assumptions are needed in order to formulate and derive the theoretical models. Those assumptions common to most production theory and maintained in all of the models employed here can be summarised as follows:

— The production function is continuous and twice-differentiable
— Perfect competition reigns on both product and factor markets. Firms are price-takers
— The firm tries to minimise the costs of producing a given output
— Inputs, output and production functions can be aggregated. Capital has the same productive characteristics whatever its vintage or age. As it ages it evaporates at a constant rate, so that it can be aggregated over time.

Of course, not all of these assumptions are strictly necessary. For example, the assumption of perfect competition can be dropped as in monopoly and oligopoly models, and vintage models can be developed which consider changes in the productive characteristics of capital over time.

Additional, more specific assumptions are required to arrive at a model which is possible to implement empirically. These generally concern the specific functional form chosen for the production or cost function, assumptions regarding the nature of technical progress and returns to scale and the specific aggregation of inputs. In all these cases there are many alternatives which could be chosen or tested. Although there is little, if any, theoretical justification for choosing amongst alternative specifications, it is generally only practicable to use some subset of these in a particular application. Those assumptions used in the studies presented here are:

— The cost function can be represented by a Translog approximation (FSE and PSE models) and a quadratic approximation (COA model)
— Technical change can be represented by an exponential (FSE), linear (PSE) or quadratic (COA) time trend
— The production function is characterised by long-run constant returns to scale (PSE and COA models)
— Inputs can be aggregated into four categories : capital, labour, energy and materials.

The differences in the functional specification of the models estimated in the previous chapters were based on practical considerations. For instance, the quadratic functional form was chosen for the cost of adjustment model because the Euler equations could not be solved analytically for the translog form. The assumption of long-run constant returns to scale in the partial static equilibrium and cost of adjustment models was used to limit the number of parameters to be estimated and to reduce multicollinearity amongst the independent variables.

The major difference between the estimated models concerns the assumptions made regarding the flexibility of the various inputs. In the full static equilibrium model, all inputs are assumed to be perfectly variable within the time intervals given by the observational data. In both the partial static equilibrium and cost of adjustment models, capital is assumed to be fixed in the short run, while the varible inputs—labour, energy and materials—are assumed to adjust to their short-run optimal levels under the constraints imposed by the capital stock. In another variant of the partial static equilibrium model, labour as well as capital is assumed to be fixed in the short run.

The difference in the PSE and COA models has to do with the connection between short- and long-run production relationships. In the partial static equilibrium model long-run relationships are inferred solely on the basis of the variable cost function and the envelop condition relating short- and long-run production costs. The rationale behind the imperfect flexibility of capital is not explicitly stated in the model formulation so that the model is rather general and allows for any of a number of justifications for this inflexibility. The cost of adjustment model is far more specific in that the inflexibility of capital is explained and formulated in terms of the costs entailed in rapidly adjusting the capital stock. The adjustment mechanism is then derived from intertemporal cost minimisation given the specification of these adjustment costs and the assumption of static expectations with regard to future output demand and factor prices.[1] Since the adjustment path from short to long run is explicitly stated, the model is far richer in economic detail. Of course, there is a price to be paid in loss of generality.

Finally, in order to estimate the models, observations on the relevant variables are obtained and the stochastic model specified. Regarding data, there are a number of choices to be made in the definitions of the appropriate variables, the measurement of the raw data, the interpretation of the various measures and the aggregation and indexing methods used. All along the line, many assumptions are made which may have some bearing on the results obtained. The construction of the time series data used to estimate the various models is essentially the same and was detailed in the appendix to Chapter II. Concerning the stochastic specification, additive disturbances were appended to all of the equations under the assumption of nonzero contemporaneous correlation and zero intertemporal correlation between error terms.

Although the economic assumptions outlined on the previous pages can neither be validated nor falsified on the basis of the empirical evidence resulting from our studies, they do have a number of implications which can be investigated. As pointed out in earlier chapters, given the assumptions on which the models are derived, the estimated cost fuction must display certain characteristics if it is to represent a plausible technology. Particularly, the cost funtion must be nondecreasing and concave in factor prices, decreasing and

[1] Expectations are irrelevant in both the FSE and PSE models, so neither of these models includes assumptions regarding these.

convex in the levels of the quasi-fixed factors (if any are specified) and nondecreasing in the level of output. The assumptions of the cost of adjustment model require additionally that adjustment costs are convex and that the adjustment coefficient lie between 0 and 1. Finally, by definition, the effects of technical progress on production costs must be nonincreasing. If the estimated models do not fulfill these requirements, we are unable to draw any economically meaningful conclusions on the basis of the results obtained. Of course, either the model or the data could be erroneous. In any case, we must conclude that the model is unable to provide a plausible interpretation of the statistical data employed.

Although some authors attempt to impose concavity and convexity conditions in the estimation of such models, this approach does not seem justifiable. It is too much like making the data say what we want it to say, instead of finding out what it actually does say, about our models.

We have seen in Chapter IV that we were forced to reject the 'pure' FSE model on the grounds that the concavity requirements were not met for any of the observations in any of the industries. The resulting positive own-price elasticities for capital are not only contrary to our a priori conceptions, but more importantly they do not conform to the theory on which the model is based. We have seen that by changing the assumptions regarding the rate of return—from the actual interest rate to a long term, constant, expected rate of return—the concavity requirements are generally met and the results more plausible. On this basis, we selected the estimates of the FSE model based on the redefined rate of return as the preferred. Even with that specification, we find, however, that the concavity requirements are not met for all observations in all industries.

Neither do the estimates of the PSE or COA models always fulfill the necessary conditions. The validity of the PSE model was questioned for the Paper and Metal industries since neither concavity nor convexity requirements are met for a large part of the data sample. In this respect, the COA model performs somewhat better particularly for the above-mentioned industries. The convexity of adjustment costs and the requirements on the parameters determining the adjustment coefficient in the COA model are fulfilled in all cases. Furthermore, all estimated models provide plausible interpretations of the effects of production scale and technical change.

The estimates of the PSE model with both labour and capital specified as quasi-fixed failed to satisfy the convexity requirements in all instances. As discussed in Chapter V, we could not on the basis of these results alone reject the notion of the inflexibility of labour altogether, since the specification of the labour variable was somewhat questionable. Without additional data which can distinguish between normal and overtime hours worked, no conclusions can be drawn concerning the flexibility of labour.

The empirical results obtained on the basis of the different models and the implications in terms of price and substitution elasticities as well as the effects of technical change and production scale were discussed thoroughly in each chapter. Each model was discussed separately, however, and little, if any, attempt was made to compare the results obtained on the basis of the different formulations. Although a detailed comparison of the all the elasticities for all the sectors would serve little purpose, it would be worth summarising some of the general trends indicated.

The resulting own price elasticities for the year 1980 for the full-static equilibrium, partial static equilibrium and cost of adjustment models are shown in table 7.1. As far as the actual values of the elasticities are concerned, we find that there is very little agreement amongst the models, although the inelasticity of input demand is supported by all of them. Unfortunately, we cannot test if the noted differences are statistically significant as we were unable to calculate confidence intervals for the PSE and COA estimates.

Although the PSE and COA models are based on more realistic assumptions concerning the adjustment of capital, the results do not show that this has much of an effect. The inflexibility of capital was assumed to explain differences in short- and long-run adjustment possibilities (price elasticities). The results of both models suggest that these differences are minimal. Also, the long-run elasticities for capital are generally smaller than those based on the FSE model, which again is not as one would expect.

In all of these models the price sensitivity of factor demand is explained solely in terms of the substitution possibilities amongst the various inputs. It was shown in the empirical chapters that the nature and magnitude of the elasticities of substitution varied from factor to factor and from industry to industry. The variation is all the more obvious when one attempts to compare the different models. A few predominant trends can, however, be noted and we will attempt to see if there is any consensus regarding these.

Firstly, all models show a predominance of capital-labour substitutability and capital-energy complementarity. The elasticity of substitution between labour and capital is generally less than unity, and often greater in the FSE than in the other models. Secondly, all models suggest that most inputs are only weakly substitutable with materials. This is rather as one would expect, given the diversity of this aggregate input. The most notable difference between the models concerns the relationship between labour and energy. These two inputs are predominantly substitutes in the FSE and PSE models, but complements in the COA model. Otherwise, the models agree quite well if we are looking for general characteristics. We have, of course, no a priori basis for preferring one set of results to another for the individual industries.

A hypothesis that is statistically tested and rejected in all models is that of Hicks neutral technical change. The models agree very well on two further

Table 7.1. Own-Price Elasticities, 1980.

	Energy					Labour					Capital		
	FSE	PSE		COA		FSE	PSE		COA		FSE	PSE	COA
		SV	LR	SV	LR		SV	LR	SV	LR		LR	LR
Food	−.40	−.36	−.36	−.06	−.06	−.24	−.55	−.70	−.06	−07	−.31	−.13	−.11
Textiles	−.59	−.53	−.54	−.50	−.51	−.35	−.49	−.49	−.06	−.09	−.39	−.02	−.08
Paper	−.21	−.28	−.27	.01	.01	−.18	−.08	2.72	.01	.01	−.24	3.16	−.13
Printing	−.48	−.41	−.41	−.10	−.10	−.28	−.56	−.56	−.24	−.25	−.01	−.01	−.05
Rubber	−.85	−.53	−.54	−.74	−.75	−.11	−.29	−.29	−.01	−.02	−.52	−.05	−.15
Chemicals	−.51	−.37	−.37	−.25	−.25	−.03	−.43	−.48	−.16	−.19	−.17	−.03	−.18
Minerals	−.29	−.27	−.29	−.03	−.30	−.25	−.40	−.42	−.08	−.09	−.70	−.09	−.09
Metals	−.34	−.20	−.17	−.47	−.56	.02	−.08	−.01	−.19	−.33	−.01	−.79	−.08
Engineering	−.69	−.52	−.59	−.50	−.51	−.07	−.07	−.12	−.11	−.12	−.21	−.11	−.04
Total	−.18	−.40	−.42	−.09	−.09	−.11	−.11	−.18	−.04	−.05	−.25	−.07	−.07

points relating to this: technical change has been labour-saving and capital-using and has led to considerable substitution amongst inputs.

The final elasticities implied by the model are those relating to production scale. Constant returns was statistically tested and rejected in the FSE model. Returns to scale as well as returns to each input were shown to be increasing. However, in some cases rather implausible (negative) output elasticities resulted. In spite of this rejection of constant returns in the FSE model, long-run constant returns was imposed without testing in the PSE and COA models. This may seem a bit inconsistent and perhaps it is. One reason for doing so is that the FSE model was not thought to capture 'true' long-run relationships so that increasing returns in this model need not be inconsistent with long-run constant returns in the other models. The actual motivation for imposing long-run constant returns was, however, based on more practical considerations: the need to reduce the number of parameters to be estimated, the implausible output elasticities noted for the FSE model and the difficulties in distinguishing between the effects of scale and technical change noted in that and other studies.

Short-run scale effects are, of course, not constrained to be constant in the PSE and COA models. In both applications short-run increasing returns to energy and diminishing returns to labour are supported. Short-run returns to scale are, as expected, found to be diminishing, but are rather too near unity to be totally acceptable.

Although some concensus can be found in the results obtained from the various models, many discrepancies can be noted concerning the nature of the substitution relationships in the individual industries as well as in the magnitudes of the particular elasticities. The question naturally arises of whether we have any basis of preferring one set of results (or model) to the others.

First of all, since the models are not nested, we cannot choose amongst them on the basis of statistical tests. Casual observation of the goodness of fit measures of the different models suggests that the fit of the partial static equilibrium and cost of adjustment models is somewhat poorer than that of the full-static equilibrium model, but of course we have no grounds to compare these.

Unfortunately, we cannot even assess the predictive power of the estimated models. In order to have as many observations as possible, the entire data sample was used in the estimation. With the availability of more recent data, an attempt to compare the performance of the different models outside of the estimation sample should be made.

The only criterion available to us for evaluating the alternative model specifications is the conformity of the results to our existing knowledge of economic relationships. As was stressed throughout the preceding chapters, there is good reason to challenge the instantaneous adjustment of capital. Since the use of other factors of production is determined by the characteristics of the capital stock, the reponse to changes in factor prices is also dependent on the adjustment of capital. In the short run, with a given capital stock, the possibilities for factor substitution are minimal. Over time, old capital equipment can be replaced with technologies that are more efficient given the new factor price relationships. It follows that subsitution possibilities and price elasticities should vary considerably in different time perspectives.

The motivation behind the application of the PSE and the COA models was to distinguish between these short- and long-term substitution possibilities and price elasticities, by recognising the inflexibility of the capital stock. Neither of these models succeeded satisfactorily in this undertaking. Although the results suggest that capital does not adjust instantaneously—which is also supported by the poor performance of the "pure" FSE model—virtually no differences were noted between short- and long-run elasticities. In this respect, the FSE model based on our specification of a long-run rate of return performs equally well as the more complicated models, and there is no reason to reject it on the basis of the results of the more realistic models. If we are looking for simplicity the FSE model is to be preferred.

What conclusions, if any, can be drawn from the fact the the PSE and COA models do not provide an adequate description of short- and long-run production relationships? Does it give us any justification to reject the models or the assumptions on which they are based? Or does the explanation lie with the data employed in the empirical analysis? In the remainder of this chapter we will attempt to answer these questions.

As was outlined in the introductory pages of this chapter, a number of assumptions were made in both the theoretical derivation and the empirical implementation of the models employed. Taken together, this leaves us with a rather simplistic view of the production process. This is obviously unavoidable once one departs from the realm of pure theory and enters the practical world of econometrics.

The results obtained from the various models give us no empirical justification for refuting any of the underlying assumptions. Taken individually, they are not empirically testable in the context of the estimated models. We have seen that only the implications deduced from all of these assumptions can be tested empirically. Other than this, the conformity of the results with other empirical evidence or observational facts is the only criterion by which the validity of the model can be assessed. This type of empirical verification does not, however, prove the theory or the assumptions to be correct, it merely provides support for them. If, on the other hand, the necessary implications are not supported by the empirical data and the results are contrary to other evidence, the validity of the assumptions on which the model is based or the adequacy of the statistical data are brought into question. Since we have no reason to believe that the inability of the estimated models to distinguish between short- and long-run production relationships can be explained solely by the quality of our data, a closer examination of some of the assumptions made in the derivation of the models may provide some possible explanations.

Many of the assumptions that underlie our models have come under attack at one time or another. The most severe and common criticism has been aimed at the aggregation assumption, and particularly as it relates to capital.[2] It is argued that the heterogeneity of capital makes it particularly difficult—or even logically impossible—to aggregate, so that no satisfactory measure can be constructed. If this is the case, the whole concept of aggregate (or industry level) production functions becomes meaningless.

Although the aggregation of capital has most often come under attack as it poses particular problems, there is no theoretical reason why the aggregation of capital need be more difficult than the aggregation of other inputs or of output. According to the Leontief condition, aggregation is only possible if the marginal rates of substitution between the inputs to be aggregated are independent of the level of output and the quantities of the other inputs. Bliss (1975) has shown that there are some special cases where this condition will hold: if the inputs are perfect substitutes, if they are always used in fixed proportions, and if they are used to produce an intermediate service. Although intermediate goods may be thought to be used in fixed proportions,[3] this seems less reasonable in the case of capital, labour and even energy. Nor does it seem possible that the components of any of the aggregates are perfect substitutes.

It is obvious that these are very stringent conditions which are very unlikely to be met in most applications. The condition for aggregation of 'less-aggregate' inputs can, in fact, be tested empirically. For example, a cost function can be estimated with capital separated into equipment and structures or labour broken down into white/blue collar workers, men/women etc. Most such studies indicate that the conventional aggregation of these inputs is highly questionable.[4] In the case of capital, a study by Bergström and Panas shows the elasticity of substitution between machine and building capital to be close to zero or negative in half of the industries investigated, suggesting that the criteria for the aggregation of these inputs are not met. They also

[2] For a survey of the so called Cambridge controversies, see Harcourt (1972).
[3] This is, of course, the assumption used in input-output analysis.
[4] For some Swedish studies, see Bergström and Panas (1985) and Panas (1985).

report that the elasticity of substitution between machine capital and labour is greater than that between building capital and labour in all cases. If these results are correct, substitution relationships estimated on the basis of an aggregate measure of capital—as that used in our studies—can be misleading. Obviously more work needs to be done on this question.

The aggregation of output can also present problems in the multiproduct industry groupings studied here. If output is made up of commodities which require different amounts of the various inputs, a change in the composition of output (at a constant aggregate level), will be reflected on the aggregate level as factor substitution. Since changes in the composion of aggregate output can be thought to occur smoothly over time, it may be difficult to distinguish between this effect and that of autonomous, biased technical change, which is often specified as some type of time trend. Measurement of total factor productivity growth will likewise be affected.

The assumption that allows the aggregation of capital over time is also very important to our analysis. If capital has the same productive characteristics regardless of its age, it is just 'more' or 'less' of the same thing. It is also perfectly malleable, so that factor price changes affect the average technique of the entire stock, regardless of its age distribution. A far more plausible description of the available technology would distinguish amongst various vintages of capital, where the efficiency of each vintage in terms of the various inputs reflects the relative price expectations held when it was purchased. Because of changes in relative factor prices over time, the new vintages will be the most efficient and will be characterised by production techniques different from those of the previously installed capacity. Since the different vintages are qualitatively dissimilar, they cannot be aggregated into a single measure of capital by normal aggregation methods. The errors introduced by this aggregation will be particularly serious when there are large changes in relative factor prices—as was experienced during the 70's—since new vintages may be characterised by vastly different production techniques than those of the existing capacity. The profitability of older, less-efficient, vintages will decrease, accelerating the economic obsolesence of previously installed capital. Results from studies based on capital aggregated in the conventional way must therefore be interpreted with caution.

The long-run substitution possibilities amongst inputs are obviously not determined solely by the amount of a homogeneous capital, but primarily by the new techniques provided by investment in different types of capital. It would appear that a good deal of this substitution may be lost by the approximation and aggregation errors involved in our measurement of capital. These errors become more serious when one attempts to account for the inflexibilty of capital and estimate long-run relationships and may be a possible explanation for the similarity of the short- and long-run elasticities estimated on the basis of the PSE and COA models. Increasing the 'realism' of the model by allowing for the inflexibility of capital may require more 'realism' in our measure of capital. The vintage concept may provide the way forward.

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