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IMPERFECT INFORMATION EQUILIBRIUM
existence, configuration and stability

by
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Abstract

In this paper we examine the stability properties of price dispersion equilibrium in a market where individuals and firms have to make efforts to collect information about the prices charged by firms and demand resulting from consumer behavior. The housholds are searching according to sequential stopping rules and the firms are making experiments in order to find out their demand curve We show that the price dispersion equilibrium in such a market is stable

1. Introduction

From the very beginning of research in the field of economics until today, nothing has been of more fundamental interest than the allocation of resources and the determination of prices. In the theory of prices from Smith via Walras and Arrow-

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*The Industrial Institute for Economic and Social Research, Stockholm, Sweden. I am indebted to Harald Lang, who helped me with some of the mathematical intricacies.

Debreu, the main question has been whether or not the decentralized market economy is capable of attaining an equilibrium where a variety of goods is produced and consumed. A further question has been whether or not this equilibrium is stable.

The general approach to the analysis of these questions has been to construct a reaction pattern for households and firms to events in the world outside them.

In the competitive analysis, the agents of both sides of the market are assumed to take the prices as given. For a single market this gives rise to the functions:

$$D = D(p) \qquad (1)$$

$$S = S(p) \qquad (2)$$

where D is demand, S is supply and p is price. To obtain a solution to this system of two equations and three unknowns, the equilibrium condition

$$D = S \qquad (3)$$

is added. This seems, perhaps, self-evident. However, a closer inspection shows that it is not. For an equilibrium to be of any interest the system must have a tendency to approach it, at least if the variables are near their equilibrium values. This is the same as to say that the equilibrium must be locally stable.

One way to guarantee this is to introduce the condition:

$$\dot{p} = f(D-S) \qquad (4)$$

Where $f' > 0$ and $f(0) = 0$, which says that if there is excess demand the price must rise. If the excess demand is negative, the price must fall.

Now, while this seems fairly logical and reasonable, it happens to be inconsistent with the assumption which gave rise to equations (1) and (2). If the price at any moment is below the price which equates demand and supply, there is excess demand in the market and the market price must increase. But (1) and (2) are derived under the assumption that all agents take price as given, households and firms deciding only how much to consume and produce, respectively. Firms in this model do not concern themselves with the setting of prices.

In the aggregate, however, there will be greater demand than supply if the price is below the equilibrium price. Some demand will be unsatisfied. There is thus a possibility for anyone firm to raise its price without risking loss of demand. But then producers are no longer price-takers in the market - they no longer face an infinitely elastic demand curve, and the conditions, necessary to derive a supply curve [equation (2)], are no longer valid. When demand does not equal supply the firms in the market behave monopolistically, although all conditions for perfect compoetition are fulfilled. It is well known that supply curves do not exist in any case other than in perfect competition. But the truth is still more depressing than that. Supply curves do not exist even in perfect competition other than in equilibrium. As early as 1959 Kenneth Arrow pointed this out in an article. [Arrow(1959)]. Already Havelmoo stressed this fact.

We may ask, what can save equations (1)-(4)? The answer is not encouraging. The invisible hand will have to become quite visible and more than that. Only the introduction of an auctioneer with more power than any imaginable price-controlling authority can save the theory of demand and supply. This auctioneer is assumed to announce a price in the market. He then collects bids from individuals and households, comparing total desired supply with total desired demand. If the supply exceeds demand he decreases the price, and vice versa, and asks for a new round of demand and supply bids. This process continues until total demand equals total supply. He then permits agents to trade as desired at these prices. This is what we call a recontracting or a tâtonnement process.

There are no trades out of equilibrium. If the prices are disequilibrium prices, the auctioneer merely collects the wishes from the two sides of the market but does not permit trade to take place. Otherwise conditions will change and nothing guarantees that the tâtonnement process will converge.

Thus, the search for a story which tells how the competitive market ends up at an equilibrium leaves us with the auctioneer. This is quite upsetting, especially when the competitive analysis is thought to be the cornerstone in the theory of the market mechanism.

The desired process of price adjustment must result from an analysis of how prices are set and changed by those who actually set them, namely the agents of the market. Let us look a little closer at a disequilibrium situation. Let us say that all firms in a market charge the same price, but one which is below the market-clearing price, implying unsatisfied excess demand. Any one firm could, in this situation, raise price without loosing all its demand. Firms face less than infinitely elastic demand curves and are thus monopolists in at least the sense that they have some choice in the setting of prices. Normally, a profit maximizing firm confronting a demand curve with finite elasticity aspires to set a price such that marginal revenue equals marginal cost. There are, however, several problems with the description of the price-setting behavior of this kind for a competitive firm out of equilibrium. A firm in the traditional pure competitive situation has to have knowledge only of a single price, the market price. But a competitive firm in a disequilibrium situation requires knowledge of a whole demand curve.

Furthermore, this demand curve is not independent of the behavior of the other firms in the market.

There is a unique shape for the demand curve corresponding to each distribution of other firms' prices.

If there is excess demand in the market one firm could increase profit by means of increasing price. If this increase is sufficient to equalize marginal revenue and marginal cost, the market is cleared in some sense. But the market is self-evidently not in equilibrium. Any other firm could also profit from a similar price increase. The price adjustment process can then be described as a large number of monopolists trying to adjust their prices in order to increase profits. However, there is a great deal of information that each firm must have in order to make a correct decision. It must know not only a whole demand curve instead of just the market price, but also the price strategies of all other firms and their impact on its own demand curve. It is obviously quite unrealistic to believe that any firm could possess all this information. Assuming this would hardly be an improvement over the story of the auctioneer. The main reason why it is unrealistic to think that a firm could have all this information is that it is costly to collect information. It could not be optimal for a firm to try to obtain perfect information - even if this were possible.

In short, when firms charge prices below the competitive equilibrium price, they all try to make profits from their monopoly positions by means of price increases. The size of these increases will differ among firms, according to their beliefs on the shape of the demand curve and on their forecasting of what other firms will do.

Price changes will differ between firms, because different firms have different sets of information, as the information is incomplete and comes from stochastically governed market experiences, and furthermore different firms will have different expectations about other firms behavior, and the effects from this. As a result, one can expect considerable price dispersion among firms during the adjustment process mainly due to the situation of limited information. In addition, one must take into account the consumers' situation. In the traditional view of the consumer in perfect competition, he or she buys at a given and constant market price. Any shop would charge exactly the same price as any other. However, in the disequilibrium situation, when different firms charge different prices, there is a benefit from finding a firm charging a low price relative to the other firms in the market. A searching for such a low price is, however, not costless. It demands resources from the consumer, especially in the form of time needed for making contact with different firms.

In order to examine the stability of the competitive equilibrium, which is the same as constructing a theory of price adjustment in an atomistic market without introducing an auctioneer, one must analyze how small monopolistic firms adjust their prices when they have incomplete information about the demand function and when the consumers at the same time do not have full information about which firms are charging which prices.

Although on the surface there would seem to be forces that make the market's firms increase their prices if the price is less than the competitive equilibrium price and then reduce it if the price is above, information costs for firms - as well as for consumers - will make the price adjustment process considerably different. Instead, if the process ever converges the prices will approach either the monopoly price[1] or a situation with price dispersion equilibrium.

In this article we present a theory of pricing when prices are set by the agents of the market who incur costs in connection with the collecting of information. We study the pricing in one particular market exclusively. The reason for this is not that interdependence among markets is unimportant, but that in order to construct a theory of general equilibrium (or morel correctly of general interdependence), we must first understand better the interaction among agents within a single market. The equilibrium analysis is briefly reviewed because it has been presented in more detail elsewhere. Here we present a proof for the stability of the equilibrium of such a market.

Equilibrium

The equilibrium properties of such a market has been analysed elsewhere (Axell 1977)[2]. Here we give a brief summary of the result.

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[1] Which is the price that a profit maximizing monopolist would charge if h controlled the wholed market.

[2] See also Axell, B (1976), and Burdett, K and Judd, K (1979). The model is a generalization of the model in Diamond (1971).

Notations

c = search cost

$f(p)$ = pdf for firms over prices

$F(p) = \int_0^p f(s)ds$

$\tilde{F}(p) = \int_0^p F(s)ds$

$\gamma(c)$ = pdf for search costs

$\Gamma(c) = \int_0^c \gamma(s)ds$

$g(p)$ = pdf for reservation prices

$G(p) = \int_0^p g(s)ds$

$\omega(p)$ = pdf for consumers' stopping prices

$\Omega(p) = \int_0^p \omega(s)ds$

Consumers are searching from a known distribution of firms [pdf f(p)] They do not know, however, which firm is charging which price. They are searching according to a sequential stopping rule, which means that they decide after each search step whether to stop and accept the offer or to continue the search. The marginal condition, then, is

$$c = \int_0^{p_m} (p_m - p) f(p)dp = \int_0^{p_m} F(p)dp \qquad (5)$$

where p_m is the price quotation just received.

The right hand side of (5) greater than c implies continued search and vice versa.

We can also solve for what is called the reservation price by the equation

$$c = \tilde{F}(R), \tag{6}$$

where R is the reservation price. R is clearly a monotonic function of c.

The consumers search in accordance to the above mentioned search rule and, after having found an acceptable price buy one unit and leave the market. They don't come back until their particular information has become worthless.

Consumers have different search costs according to the pdf $\gamma(c)$. Hence, they have different reservation prices. The reservation price distribution [pdf g(p)] is then

$$g(p) = G'(p) = \gamma[\tilde{F}(p)]F(p). \tag{7}$$

Each consumer will, however, buy at a price below his reservation price. On average this is the expected price given that it is below the reservation price. Hence, the distribution of purchasing consumers over prices is

$$\omega(p) = \Omega'(p) = f(p) \int_{p}^{\infty} \gamma[\tilde{F}(s)]ds. \tag{8}$$

Hence the demand curve q(p) facing a firm is

$$q(p) = \frac{K}{m} \int_{p}^{\infty} \gamma[\tilde{F}(s)]ds, \tag{9}$$

where m is the number of firms and K is the number of consumers per period.

We can then construct the profit function $\Pi(p)$ which, for a cost function $c(q) = c_1 + mc \cdot q$ where c_1 and mc are positive constants, is

$$\Pi(p) = \frac{K}{m}(p-mc) \int_{p}^{\infty} \gamma[\tilde{F}(s)]ds - c_1. \tag{10}$$

We can find equilibrium in either of two ways. One is to apply an equilibrium condition to a static model, i e, the noncooperative Nash condition. Another is to construct a dynamic model and solve it for the case when the variables' time derivatives equal zero.

The question of stability of an equilibrium is always related to a dynamic counterpart of a model. In this paper we are mainly interested in whether or not the static equilibrium is stable for an imagined situation where firms are experimenting with different prices.We want to determine whether the static equilibrium is stable if we construct a corresponding dynamic model?

First, let us look at the configuration of a static equilibrium. There are two equilibria in this market. One is a degenerated distribution at the monopoly price. The other is a price dispersion equilibrium. In this paper we are looking at the price dispersion case.[1]

The condition for equilibrium is then that profits are the same at all prices in the support of f(p).

Hence,

$$\frac{d\pi}{dp} = \int_{p}^{\infty} \gamma[\tilde{F}(s)]ds + \gamma[\tilde{F}(p)](mc-p) = 0 \qquad p>mc. (11)$$

[1] We already know that the monopoly equilibrium is stable.

The solution to equation (11) is

$$\int_p^\infty \gamma[\tilde{F}(s)]ds = \frac{B}{p-mc} \qquad p>mc, \tag{12}$$

where B is a positive constant equal to

$$B = \frac{m(\pi+c_1)}{k}. \tag{13}$$

Then there exists an pdf $f^*(\cdot)$ which satisfies (12) if and only if pdf $\gamma(\cdot)$ satisfies

i) γ is defined on $(0,\infty)$

γ is twice differentiable

$\gamma' < 0$

$\gamma'' > 0$

$\gamma(c) \to 0$ when $c \to \infty$

$\gamma(c) \to \infty$ when $c \to 0+$

ii) $\dfrac{\gamma(c)^{3/2}}{\gamma'(c)}$ is decreasing

iii) $\lim\limits_{c\to\infty} \dfrac{\gamma(c)^{3/2}}{\gamma'(c)} = -\dfrac{\sqrt{B}}{2}$

vi) $\lim\limits_{c\to 0+} \dfrac{\gamma(c)^{3/2}}{\gamma'(c)} = 0$

If $\gamma(c)$ is such that the above necessary and sufficient conditions are fulfilled, the density function of firms over prices, $f(p)$, have the following properties in equilibrium:

1. $f' < 0$
2. $f'' > 0$

$f(p) \to 0$ when $p \to \infty$

$f(p) \to \infty$ when $p \to mc+$.

Figures 1a) and 1b) illustrate $\gamma(\cdot)$ and $f(\cdot)$ respectively in equilibrium.

Figure 1a)

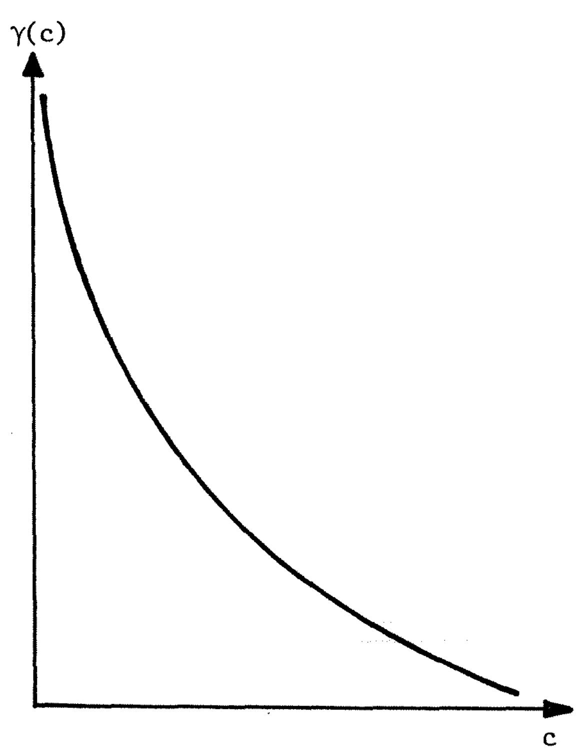
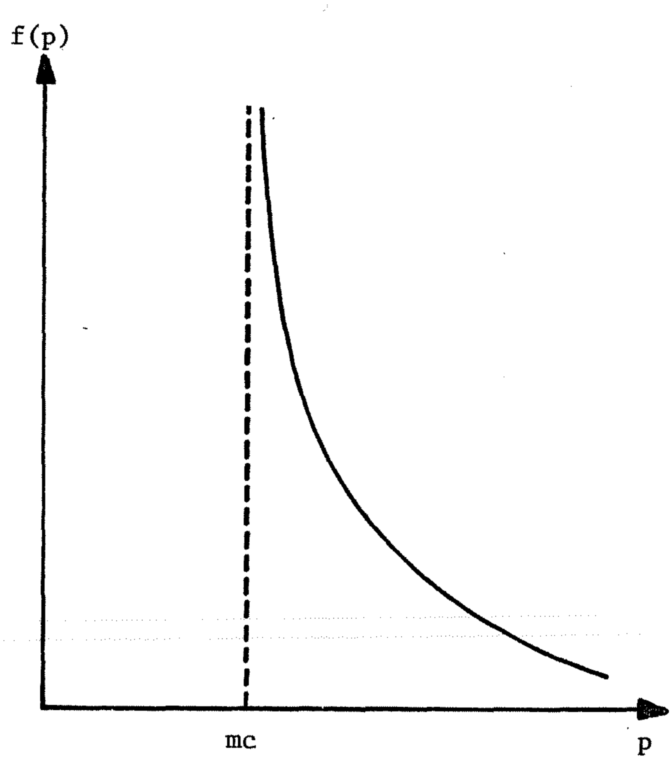


Figure 1b)



Stability of equilibrium

Consider now a situation which fulfills the above conditions for a price dispersion equilibrium. The question is: Is this a stable equilibrium? Hence we ask: If the market just by chance happens to be in an equilibrium, and then is pushed a little bit out of it, will the market then turn back to this equilibrium?

This question can, of course only have an answer if we specify the dynamic behavior of the agents. Here we will specify an experimental behavior of the firms. Firms are assumed to experiment with price changes, trying to figure out if they can increase their profit by means of a price increase or decrease.

Firm behavior

Given the assumed behavior of consumers and the associated demand curve, each firm has to decide what price to charge in order to maximize profits. Parallel with the situation of consumer, firms lack perfect information both about exact consumer behavior and the resulting demand curve. However, each firm may have good information about the demand at the price it charges itself during a period. Also, it may obtain information about the shape of the demand curve by experimenting with price changes. The parallel with the consumer's search activity is obvious; firms risk loosing profits by "searching out" the demand at other prices.

How would one design an optimal "search" or "experimenting" procedure for firms? It is obviously an extremely difficult task. Which prices would one experiment with? For how long?

Firms will get more information out of the prices they set the more dispersed these prices are. However, cost in terms of foregone profits will be greater the greater the deviation is between the price charged and the optimal price.

However, following the tradition of rational expectations, we assume that the information obtained by the firms from their experiments or investigations, gives on average correct information about the direction in which price should be changed to get higher profits. Furthermore, we assume that the probability of getting correct information is positively correlated with the slope of the profit function as a function of price.

A particular story of firms' experimenting which will givet this result and which will make firms move in a profit increasing direction is the following.

In this section we will derive an expression for the firms' price change based on assumptions of experimental behavior of the firms.

Each firm faces a stochastic demand curve in each period. There are two reasons for this. In the first place, the stopping distribution $\omega(p)$ is the expected distribution. Normally ω at p will differ from its expected value, causing the demand at p to be stochastic. In the second place, even if the number of buying consumers in the interval $(p, p + \Delta p)$ is equal to the expected number, the consumers need not be uniformly distributed among firms in this interval, because the number of consumers per firm need not be large.

We can introduce this stochastic element into the firm's environment by adding a stochastic term to the demand function. We are then in position to derive the stochastic profit function. The stochastic term could in principle be derived from consumer search behavior. This, however, would be a very difficult task. For simplicity we assume instead that the stochastic environment of the firms can fairly well be described by adding a stochastic term u to the profit function. Then profit as a function of price is

$$\pi(p) = pq(p) - C\big(q(p)\big) + u, \tag{14}$$

where the demand q(p) is mathematically expected demand. $C\big(q(p)\big)$ is the cost function, which is taken to be the same for all firms in the market. u is a stochastic term which is added to expected profit.

Let us now describe the firm's experimental behavior. We assume that:

1. All firms are risk neutral.
2. A firm knows the expected demand (and thereby the expected profit) at the price it has charged itself during period t.
3. A firm does not know the demand at prices other than that which it has charged itself.

Let us regard a particular firm i charging price p in period t. The firm will, during this period, register the demand q. The firm realizes that it is facing a finitely elastic demand curve, but it does not know whether p is the very best price or whether it could raise its profit by increasing or decreasing the price. However, the fact that it has chosen p reveals that it has no reason to believe that a lower price is likely to be better than a higher price.

We now assume that if the firm undertakes an experiment with a price change, then it will be equally probable for it to try a price raise as to try a price cut. Further, we make the assumption that all firms are experimenting.

Consider a firm charging p. It receives a profit of $\pi(p) + u$, where u is a stochastic term. Let us assume that u shows the variability in profit during a which is the optimal experimental period. We also assume that the profit function is homoscedastic, i e u has the same distribution at all prices. If the firm remains at p for a longer period, say a month or two, it will get a fairly good picture of the expected profit $\pi(p)$. If during a short subperiod the firm tries another price, for instance $p + \Delta p$, then it will get the profit $\pi(p+\Delta p) + u$ at that price.

Given the experimental price increase Δp, what is then the probability of an increase in profit? Thus we ask what is the probability of the following relationship:

$$pr\{\pi(p+\Delta p) + u \geq E[\pi(p)]\} \qquad (15)$$

where $E[\pi(p)] = \pi(p)$.

Let us call this probability υ^+. Making a Taylor expansion of $\pi(p)$ around p_i we get

$$\pi(p+\Delta p) = \pi(p) + \pi'(p)\ \Delta p + 1/2\ \pi''(p)\Delta p^2 +$$

$$+.....+.....$$

Linearizing in the interval, i e, dropping terms of second degree and higher, the probability (15) is

$$\upsilon_1^+ = pr[u \geqslant -\pi'(p)\Delta p] \tag{16}$$

We see that υ^+ is the probability that the stochastic term does not reduce the profit at p+Δp from its expected value more than the actual difference in expected profit, expressed by means of the slope of π at p times Δp. In terms of figure 1 it is the probability of falling within α during the experiment with p+Δp.

If the profit function is homoscedastic, i e, the stochastic term u has the same probability density function at all prices, where this density function is ξ(u) with the cumulative distribution Z(u), we get

$$\upsilon^+ = 1 - \int_{-\infty}^{-\pi'(\pi)\Delta p} \xi(u)du = 1 - Z[\pi'(p)\Delta p].$$

Since ξ(u) is not derived from consumer search, we have to assume a reasonable shape for it. The normal density function is perhaps a good choice, but in a complicated interdependent analysis it will cause great analytical problems. Simple expressions will appear if we assume instead that the stochastic profit term is uniformly distributed. Let us thus assume that ξ(u) is a rectangular distribution with limits -a and +a, i e,

$\xi(u) = \frac{1}{2a}$ Then

$$Z(u) = \int_{-a}^{u} \frac{1}{2a}\, ds = \frac{u + a}{2a} \qquad -a \leq u \leq a.$$

The probability υ_i^+ is then

$$\upsilon^+ = \begin{cases} \frac{1}{2} + \frac{1}{2}\frac{\Delta p}{a}\pi'(p) & \text{if } -a \leq \pi'(p)\,\Delta p \leq a, \\ 1 & \text{if } \pi'(p)\,\Delta p > a, \\ 0 & \text{if } \pi'(p)\,\Delta p < -a. \end{cases} \qquad (17)$$

We see that this probability depends positively on the slope of the profit function and on the size of the price jump, but negatively on the variance of the stochastic term, as can be observed in figure 1.

If, instead, a firm tries the price $p-\Delta p$, then the probability of a profit increase is

$$\upsilon^- = \begin{cases} \frac{1}{2} - \frac{1}{2}\frac{\Delta p}{a}\pi'(p) & \text{if } -a \leq \pi'(p)\,\Delta p \leq a, \\ 0 & \text{if } \pi'(p)\,\Delta p > a, \\ 2 & \text{if } \pi'(p)\,\Delta p < -a. \end{cases} \qquad (18)$$

We see that $\upsilon^- = 1-\upsilon^+$. Note that this follows from the approximation to a linear profit function in the interval $(p-\Delta p,\ p+\Delta p)$, evaluated at p.

Change in the distribution of prices

In the previous section we derived the probability that a price change experiment will lead to increased profits at the experimental price. We now wish to study how firms actually change prices over time. In particular, we want to describe the aggregate effect of the behavior of individual firms i.e., how the distribution of firms over prices will change over time.

We assume the following behavior of firms (in addition to the earlier assumptions): If a firm, charging the price p during a given period, experiments with the price $p+\Delta p$ during a sub-period and registers a higher profit at $p+\Delta p$, then it will charge the price $p+\Delta p$ during the next period; otherwise it will return to p. From this follows that the probability that a firm charging p will raise its price to $p+\Delta p$ is $1/2\upsilon^{+}$.[1]

We have assumed the market to be an atomistic market, i.e., one in which the number of firms is very great. Then the probability of changing the price from, for instance, p to $p+\Delta p$ will show

[1] Note that we have assumed that half of the firms at p_i tries prices increases.

the proportion of firms at p changing price in that direction.

The frequency of firms charging p at time t is $f_t(p)$. The frequency of firms charging p at time t+1 is the share of those at $p-\Delta p$ at t which experimented with a price increase (i.e.,one half) and obtained positive information (i.e.,profit increase) plus the share of those at $p+\Delta p$ which experimented with a price decrease and obtained positive information, plus those at p that experimented with a price decrease or a price increase and obtained negative information. We thus get[1]

$$f_{t+1}(p) = \frac{1}{2} f_t(p-\Delta p)\, \upsilon + \frac{1}{2} f_t(p_t+\Delta p)(1-\upsilon) +$$

$$+ \frac{1}{2} f_t(p)\, \upsilon + \frac{1}{2} f_t(p)(1-\upsilon),$$

which is

$$f_{t+1}(p) = \frac{1}{2} f_t(p-\Delta p)\, \upsilon + \frac{1}{2} f_t(p+\Delta p)(1-\upsilon) +$$

$$\frac{1}{2} + f_t(p) \tag{19}$$

Writing $f_t(p-\Delta p)$ with the help of Taylor expansion we get

[1] Note that we have changed notation of υ slightly. Here we think, for simplicity, that the linearization around the experimental price and the ordinary price does not differ too much.

$$f_t(p-\Delta p) = f_t(p) - f_t'(p)\Delta p + \frac{1}{2} f_t''(p)\Delta p^2 - \ldots +$$

In a corresponding way we have for $f_t(p_i+\Delta p)$:

$$f_t(p+\Delta p) = f_t(p) + f_t'(p)\Delta p + \frac{1}{2} f_t''(p)\Delta p^2 + \ldots -$$

Disregarding terms of second degree and higher, we can write expression (19) as

$$f_{t+1}(p) - f_t(p) = \frac{1}{2}[f_t(p) - f_t'(p)\Delta p]\ \upsilon +$$

$$+ \frac{1}{2}\left[f_t(p) + f_t'(p)\Delta p\right] (1-\upsilon) - \frac{1}{2} f_t(p),$$

which can be simplified to

$$f_{t+1}(p) - f_t(p) = f_t'(p)\ \Delta p(\frac{1}{2} - \upsilon)$$

If we now substitute for the expression for υ derived earlier in (17) and (18) we have

$$f_{t+1}(p) - f_t(p) = f_,(p)\ \Delta p[\frac{1}{2} - \frac{1}{2} - \frac{1}{2}\frac{\Delta p}{a}\pi'(p)]$$

which is

$$f_{t+1}(p) - f_t(p) = \frac{1}{2} f'(p)\ \pi'(p)\ \frac{\Delta \bar{p}^2}{a} \qquad (20)$$

The expression for the profit function is

$$\pi(p) = pq(p) - C[q(p)],$$

where

$$q(p) = \frac{k}{m} \int_{p}^{\infty} \gamma \ [\tilde{F}(s)]ds.$$

We get,

$$\frac{d\pi}{dp} = \frac{k}{m}\left(\int_{p}^{\infty} \gamma[\tilde{F}_t(s)]ds - p\cdot\gamma[\tilde{F}_t(p)]\right) + \frac{dC}{dq}\ \frac{k}{m}\gamma[\tilde{F}_t(p)].$$

Rearranging terms we get

$$\frac{d\pi}{dp} = \frac{k}{m}\left\{\int_{p}^{\infty} \gamma[\tilde{F}_t(s)]ds + \gamma[\tilde{F}_t(p)]\left(\frac{dC}{dq} - p\right)\right\}. \tag{21}$$

The complete expression for the change of the price distribution will then be

$$f_{t+1}(p) - f_t(p) = -\frac{k\cdot\Delta p^2}{m\cdot 2a}\ \frac{df}{dp}\left\{\int_{p}^{\infty} \gamma[\tilde{F}(s)]ds + \gamma[\tilde{F}(p)](mc-p)\right\}, \tag{22}$$

where mc is a constant marginal cost.

The question is now: If f(p) and γ(c), both pdf, are consistent with the necessary and sufficient conditions for an equilibrium, is this equilibrium stable? In equilibrium f(p,t) , i e (22) , is zero. Hence, f(•) and γ(•) are such that the { } in (22) is zero. The question of stability is: If f(p) at p increases a bit (and decreases at some other price - f(p) is a pdf), what will be the sign of f(•), i.e., $\{\quad\}$ in (22)? I want to show that if f(p) increases then the sign of $\{\quad\}$ is negative and if f(p) decreases it is positive (note that $\frac{df}{dp} < 0$ in equilibrium).

First let us derive the general expression for stability. We want to make a small change in the equilibrium configuration. Let us introduce the function

function h(p) with the property $\int_0^\infty h(s)ds = 0.$

Substitute the function f(p) with f(p) + δ h(p) in (22).

The $\{\quad\}$ in (22) then becomes:

$$\int_p^\infty \gamma[\widetilde{F}(s) + \delta\widetilde{H}(s)]ds + \gamma[\widetilde{F}(p) + \delta\widetilde{H}(p)](mc-p) \qquad (23)$$

where $\widetilde{H}$ is defined in the same way as F, i.e.:

$$H(p) = \int_0^p h(s)ds$$

$$\widetilde{H}(p) = \int_0^p H(s)ds$$

Taking the derivative of (23) with respect to δ and then setting δ=0 gives:

$$\int_p^\infty \gamma'[\tilde{F}(s)]\tilde{H}(s)ds + \gamma'[\tilde{F}(p)]\tilde{H}(p)(mc-p). \tag{24}$$

<u>The general condition for stability is then that this expression have the opposite sign to h(p)</u>.

Now let h(p) have the particular form

$$h(p) = \varepsilon\delta_{p_1}(p) - \varepsilon\delta_{p_2}(p),$$

which is the sum of two Dirac delta functions, where we assume that $p_1 < p_2$ and ε is positive <u>or negative</u>.

Since (24) is homogenous in h we can have $\varepsilon=1$. Then we have

$$\tilde{H}(p) = \begin{cases} 0 & \text{if } p \leq p_1 \\ (p_2-p_1)>0 & \text{if } p \geq p_2 \end{cases} \tag{25}$$

We want to show that (24) is < 0 at p_1 and > 0 at p_2. The first condition obviously holds, since the integral (the first term) is < 0, because $\gamma' < 0$ and the second term is $= 0$, because $\tilde{H}(p_1) = 0$.

Let us go to the case when $p = p_2$.

At first, observe that lim.inf. of (24) when $p \to \infty$ is ≥ 0, because the integral $\to 0$ and the second term is always positive ($\gamma' < 0$, $(mc-p) < 0$, and $\tilde{H}(p) > 0$).

We want to show that (24) is > 0 for $p = p_2$.

ence, it is sufficient to show that (24) is decreasing. Differentiating (24) gives:

$$- \gamma'[\tilde{F}(p)]\tilde{H}(p) + \gamma''[\tilde{F}(p)]F(p)\tilde{H}(p)(mc-p) -$$
$$- \gamma'[\tilde{F}(p)]\tilde{H}(p)$$

which is

$$-2\gamma'[\tilde{F}(p)]\tilde{H}(p) + \gamma''[\tilde{F}(p)]F(p)\tilde{H}(p)(mc-p) \qquad (26)$$

For stability we want (26) to be < 0.

Because $\tilde{H}(p) = p_2 - p_1 > 0$ we want to show that:

$$2\gamma'[\tilde{F}(p)] - \gamma''[\tilde{F}(p)]F(p)(mc-p) > 0 \qquad (27)$$

We can solve for $\gamma[\tilde{F}(p)]$ from equation (12).

We get

$$\gamma[\tilde{F}(p)] = \frac{B}{(p-mc)^2}$$

Differentiating we get:

$$\gamma'[\tilde{F}(p)]F(p) = \frac{-2B}{(p-mc)^3} \qquad (28)$$

Differentiating again we get:

$$\gamma''[\tilde{F}(p)][F(p)]^2 + \gamma'[\tilde{F}(p)]f(p) = \frac{6B}{(p-mc)^4}$$

or, in other words:

$$\gamma''[\tilde{F}(p)][F(p)]^2 = \frac{6B}{(p-mc)^4} - \gamma'[\tilde{F}(p)]f(p) \qquad (29)$$

Now, multiplying (27) with the positive F(p) and substituting for (28) and (29), we get:

$$\frac{-4B}{(p-mc)^3} - \{\frac{6B}{(p-mc)^4} - \gamma'[\tilde{F}(p)]f(p)\}(mc-p) =$$

$$= \frac{2B}{(p-mc)^3} + \gamma'[\tilde{F}(p)]f(p)(mc-p) \qquad (30)$$

Stability requires that this expression is > 0. It obviosly is. The first term is > 0 (p>mc). The second term is > 0, too, because $\gamma' < 0$, $f(p) > 0$ and $(mc-p) < 0$ $(p > mc)$

Q.E.D.

Conclusions

In this paper we have shown that if the price dispersion equilibrium in a search market (Axell 1977) is forced out of equilibrium, it will return to the equilibrium configuration. In other words: the equilibrium is stable!

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